

# GIRR Model Solutions

## Fall 2020

**1. Learning Objectives:**

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

**Learning Outcomes:**

- (2a) Create development triangles of claims and counts from detailed claim transaction data.

**Sources:**

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 10.

**Commentary on Question:**

*This question tests the constructions of claims data triangles as well as the candidate's ability to recognize inconsistencies with claims data triangles.*

**Solution:**

- (a) Identify the inconsistencies in the data triangles.

Reported Claims = Cumulative Paid + Case Estimates

Accident Year	Reconciled Reported Claims (000)			
	12	24	36	48
2016	12,800	16,380	18,350	<b>18,680</b>
2017	13,700	17,810	<b>19,550</b>	
2018	15,200	<b>19,070</b>		
2019	<b>14,720</b>			

The most recent diagonal does not reconcile, as Reported claims > Paid claims + Case estimates.

- (b) Provide one potential cause for the data issue identified in part (a).

There are either paid claims or case estimates missing from the latest calendar year.

# 1. Continued

- (c) Construct revised paid claims and case estimates triangles incorporating this additional information.

Changes to cumulative paid:

Increase in AY2017 @ 12 months by 11,000 {both indemnity + ALAE}  
 Increase in AY2017 @ 24 months by 25,000 {both indemnity + ALAE}  
 Increase in AY2017 @ 36 months by 25,000 {both indemnity + ALAE}  
*{Note: no payments and no case adjustment in CY2019, so still the same values by the end of 2019}*

Changes in case estimates:

Increase in AY2017 @ 12 months by 30,000 {latest case estimate}  
 Increase in AY2017 @ 24 months by 20,000 {latest case estimate}  
 Increase in AY2017 @ 36 months by 20,000 {latest case estimate}  
*{Note: no payments and no case adjustments in CY2019, so still the same values by the end of 2019}*

Accident Year	Restated Paid Claims (000)			
	12	24	36	48
2016	9,730	14,580	17,430	18,300
2017	9,461	15,345	18,435	
2018	10,940	16,090		
2019	11,100			

Accident Year	Restated Case Estimates (000)			
	12	24	36	48
2016	3,070	1,800	920	380
2017	4,280	2,510	1,160	
2018	4,260	2,980		
2019	3,620			

e.g., Restated paid claims for AY2017 @ 12 months = 9,450 + 11,000 / 1,000

## 1. Continued

- (d) Calculate the calendar year 2018 reported claims using the revised triangles from part (c).

CY reported = (Case estimate at end of year) – (Case estimate at beginning of year) + (Paid claims during the year)

$$\text{Case estimate at the end of 2018} = 4,260 + 2,510 + 920 = 7,690$$

$$\text{Case estimate at the end of 2017} = 4,280 + 1,800 = 6,080$$

$$\text{Paid during 2018} = 10,940 + 15,345 - 9,461 + 17,430 - 14,580 = 19,674$$

$$\text{CY 2018 reported claims} = 21,284$$

## 2. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 17.

### Commentary on Question:

*This question tests the candidate's understanding of the Bornhuetter Ferguson and Benktander methods for estimating ultimate claims.*

### Solution:

- (a) Calculate the total ultimate claims using the Bornhuetter Ferguson method applied to the following two claim amounts:
- (i) Paid claims
- (ii) Reported claims

Accident Year	(1) Actual Claims as of Dec. 31, 2019		(3) Ultimate Claims from Development Method on		(5) Expected Claims
	Paid	Reported	Paid	Reported	
2016	889,190	898,170	916,755	916,133	889,488
2017	916,340	964,570	1,014,895	1,003,537	998,479
2018	824,940	959,230	1,065,872	1,077,820	1,113,814
2019	586,850	838,362	1,140,237	1,139,829	1,142,919

## 2. Continued

Accident Year	(6) = (3)/(1) (7) = (4)/(2) Age-to-Ultimate Dev. <u>Factors Based on</u>		(8) (9) BF Estimate <u>Ultimate Claims</u>	
	Paid	Reported	Paid	Reported
	2016	1.0310	1.0200	915,935
2017	1.1076	1.0404	1,013,301	1,003,341
2018	1.2921	1.1236	1,076,709	1,081,780
2019	1.9430	1.3596	1,141,539	1,140,646
Total			4,147,484	4,141,378

Notes: (8) = (1) + (5)[1 - 1/(6)]  
(9) = (2) + (5)[1 - 1/(7)]

- (b) Evaluate the reasonableness of the inputs for the Bornhuetter Ferguson method in part (a) by comparing the following two amounts:
- (i) Actual paid claims to expected paid claims
  - (ii) Actual reported claims to expected reported claims

Accident Year	(10) = 1/(6) (11) = 1/(7) Expected % Dev. At <u>Dec. 31, 2019</u>		(12) = (5)(10) (13) = (5)(11) Expected Claims <u>Developed</u>		(14) = (1) - (15) = (2) - (13) Difference <u>Actual and Expected</u>	
	Paid	Reported	Paid	Reported	Paid	Reported
	2016	0.9699	0.9804	862,743	872,047	26,447
2017	0.9029	0.9612	901,518	959,708	14,822	4,862
2018	0.7740	0.8900	862,045	991,264	-37,105	-32,034
2019	0.5147	0.7355	588,230	840,634	-1,380	-2,273
Total			3,214,536	3,663,654	2,784	-3,322

Overall, it appears reasonable, but there are some AY's that are not reasonable (e.g., 2016 & 2018).

## 2. Continued

- (c) Identify two reasons that might cause the differences shown in part (b).

Any two of the following are acceptable:

- development pattern
- trend rate
- selected values for expected claims
- existence of unusually large claims

- (d) Describe a reason why the Benktander method might be preferred to estimate ultimate claims.

A situation where you would want to put more weight (confidence) on the development method but still give consideration to the Bornhuetter Ferguson method.

- (e) Calculate the total ultimate claims applied to paid claims using one iteration of the Benktander method.

$$(16) = (1) + (8)[1 - 1/(6)]$$

Accident Year	Benktander Estimate
2016	916,730
2017	1,014,740
2018	1,068,322
2019	1,140,869
Total	4,140,661

### 3. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

#### Learning Outcomes:

- (6q) Distinguish occurrence-based and claims-made based coverage.  
 (6r) Calculate rates for claims-made coverage as well as claims-made maturity and tail factors.

#### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34.

#### Commentary on Question:

*This question tests the candidate's understanding of claims-made ratemaking.*

#### Solution:

- (a) Describe why the risk of reserve inadequacy is greatly reduced for claims-made policies compared to occurrence policies.

Claims-made policies incur no liability for pure IBNR claims.

- (b) Calculate the total reported claims for each of the following:
- (i) A first-year claims-made policy effective January 1, 2013
  - (ii) A third-year claims-made policy effective January 1, 2015
  - (iii) A tail policy purchased after the third-year claims-made policy from part (b)(ii)

Accident Year Lag by Report Year Matrix of Ultimate Claims								
Accident Year Lag	Report Year							
	2011	2012	2013	2014	2015	2016	2017	2018
0	160	168	176	185	194	204	214	225
1	240	252	265	278	292	306	322	338
2	240	252	265	278	292	306	322	338
3	160	168	176	185	194	204	214	225

(i)  $C_{0,3}$  176

(ii)  $C_{0,5} + C_{1,5} + C_{2,5}$  778

(iii)  $C_{1,6} + C_{2,6} + C_{3,6} + C_{2,7} + C_{3,7} + C_{3,8}$  1,577

### 3. Continued

(c) Calculate each of the following factors for this coverage:

(i) A second-year claims-made step factor

(ii) A mature claims-made tail factor

Accident Year Lag	Reported Years							
	1	2	3	4	5	6	7	8
0	200.00	220.00	242.00	266.20	292.82			
1	181.82	200.00	220.00	242.00	266.20	292.82		
2	165.29	181.82	200.00	220.00	242.00	266.20	292.82	
3		165.29	181.82	200.00	220.00	242.00	266.20	292.82

$$(i) \text{ second-year claims-made step factor} = \frac{266.20 + 242.00}{266.20 + 242.00 + 220.00 + 200.00} = 0.54751$$

$$(ii) \text{ mature claims-made tail factor} = \frac{266.20 \times 3 + 242.00 \times 2 + 220.00}{266.20 + 242.00 + 220.00 + 200.00} = 1.61883$$



#### 4. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

#### Learning Outcomes:

- (3k) Estimate ultimate claims by layer using common methods.  
(3l) Understand the differences in development patterns and trends for various claim layers.

#### Sources:

Fundamentals of General Insurance Actuarial Analysis 2019 Supplement, J. Friedland, Appendix I.

#### Commentary on Question:

*This question tests the calculation of claims excess of a limit using various methods.*

#### Solution:

- (a) Calculate the ultimate claims for the layer of claims excess 500,000 using the development method with all-years volume-weighted average development factors.

Accident Year	Reported Claims XS 500,000 Limit (000)					Ultimate Claims
	12	24	36	48	60	
2015	52	1,378	2,011	1,958	1,857	1,857
2016	307	1,098	1,824	2,935		2,784
2017	517	1,690	1,888			2,285
2018	0	975				1,621
2019	667					6,507
All-years volume-weighted average development factors:						15,053
age-to-age	5.869	1.374	1.276	0.948	1.000	
age-ultimate	9.756	1.662	1.210	0.948	1.000	

e.g.,  $1.662 = 1.374 \times 1.276 \times 0.948 \times 1.000$

$1,621 = 975 \times 1.662$

## 4. Continued

- (b) Calculate the ultimate claims for the layer of claims excess 500,000 as the difference between ultimate total limits claims and ultimate 500,000 limit claims.

Age-to-Ultimate Development Factors					
	12-24	24-36	36-48	48-60	60-Ult
Total Limits	1.807	1.100	1.051	1.002	1.000
500,000 Limit	1.609	1.043	1.028	1.010	1.000

Accident Year	Ultimate Claims at		
	Total Limits	500,000 Limit	XS Limit
2015	16,711	14,854	1,857
2016	16,526	13,694	2,832
2017	16,292	13,996	2,296
2018	17,555	15,615	1,941
2019	17,951	14,908	3,043
Total	85,035	73,066	11,970

e.g., for AY 2018:

Total limits:  $17,555 = 15,952 \times 1.100$

500,000 limit:  $15,615 = 14,977 \times 1.043$

XS limit:  $1,941 = 17,555 - 15,615$

- (c) Recommend the ultimate claims for the layer of claims excess of 500,000. Justify your recommendation.

Recommend the ultimate claims from part (b).

The part (a) estimate uses development on the excess claims where there is a lot of uncertainty due to higher volatility from the lower claim volume.

For part (b), the total limits and limited claims have higher volume and therefore lower volatility and therefore less uncertainty.

#### 4. Continued

- (d) Calculate the ultimate claims for the layer of claims excess 500,000 for accident years 2018 and 2019, using the expected method.

(1) Earned Premium	(2) Claim Trend Factor	(3) = 0.12/(2) Trended Claim Ratio	(4) = (1)(3) Expected Claims
30,500	1.050	0.1143	3,486
31,800	1.000	0.1200	3,816

- (e) Describe when the expected method is appropriate to use for determining ultimate claims for excess limits.

For immature accident years.

## 5. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

### Learning Outcomes:

- (6d) Quantify different types of expenses required for ratemaking including expense trending procedures.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 29.

### Commentary on Question:

*This question tests the candidate's understanding of expense ratios used in ratemaking.*

### Solution:

- (a) Calculate the historical trend in fixed expenses.

Calendar Year	(1) Earned Premiums	(2) Earned Premiums at Current Rate Level	(3) Fixed Expenses
2014	4,526,480	5,850,000	172,580
2015	4,830,080	6,166,130	186,220
2016	5,279,580	6,451,780	200,650
2017	5,542,320	6,658,360	214,400
2018	6,139,740	6,901,520	231,200
2019	6,873,650	7,231,270	253,090

Calendar Year	(4) = (3)/(2) Fixed Expense per On-Level Earned Premium	(5) = (4) <sub>i</sub> /(4) <sub>i-1</sub> - 1 Change in Fixed Expense per On-Level Earned Premium
2014	2.95%	
2015	3.02%	2.37%
2016	3.11%	2.98%
2017	3.22%	3.54%
2018	3.35%	4.04%
2019	3.50%	4.48%

Average all years: 3.48%  
Average most recent 3 years: 4.02%

*{Note: averages not needed for part (a) but helpful for part (c)}*

## 5. Continued

- (b) Assess the reasonableness of using the publicly-available cost index for this line of business in comparison to using the historical trend in fixed expenses.

The company trend may have been similar to the publicly available cost index in older years, but the recent increases make that index not reasonable.

- (c) Recommend the annual fixed expense trend. Justify your recommendation.

**Commentary on Question:**

*Other recommendations are acceptable where the required justification matches the outcome of the trend analysis.*

Recommend an annual fixed expense trend of 4.0%.

Justification: There is a clear increasing trend rate so more weight should be given to more recent years.

- (d) Calculate the fixed expense ratio to be used in ratemaking, using a simple average from calendar years 2017, 2018 and 2019.

Rates effective: April 1, 2021

Average incurred date in rating period: April 1, 2022 (i.e., 12 months following the effective date as policies are annual and in effect for 12 months)

Calendar Year	Average Incurred Date		Trend Period (months)	Trend Factors	Trended Fixed Expenses	Fixed Expense per On-Level Earned Premium
	Experience Period	Forecast Period				
2017	2017-07-01	2022-04-01	57	1.2048	258,305	3.88%
2018	2018-07-01	2022-04-01	45	1.1584	267,832	3.88%
2019	2019-07-01	2022-04-01	33	1.1139	281,914	3.90%
					Average:	3.89%

e.g., for CY 2018:

$$\text{Trend factor: } 1.1584 = 1.04^{(45/12)}$$

$$\text{Trended fixed expenses: } 267,832 = 231,200 \times 1.1584$$

$$\text{Fixed expense per on-level earned premium: } 3.88\% = 267,832 / 6,901,520$$

## 6. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.
5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

### Learning Outcomes:

- (3g) Estimate ultimate values using the methods cited in (3e).
- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 14, 15 and 25.

### Commentary on Question:

*This question tests the candidate's understanding of the development method and the frequency-severity claim closure method for estimating unpaid claims. In addition, this question tests the candidate's understanding of calculating claim trend.*

### Solution:

- (a) Calculate the ultimate paid severity for each accident year using the development method.

Accident Year	<u>Paid Severity</u>					
	12	24	36	48	60	72
2014	644.17	1,100.50	1,592.98	1,875.85	2,247.01	2,524.96
2015	676.36	1,177.53	1,704.50	2,007.16	2,359.37	
2016	716.96	1,248.18	1,789.73	2,127.58		
2017	752.79	1,335.56	1,897.10			
2018	797.95	1,415.70				
2019	853.83					

Paid severity = Paid claims / Closed counts

## 6. Continued

Accident Year	Paid Severity Age-to-Age Development Factors					
	12	24	36	48	60	72
2014	1.708	1.448	1.178	1.198	1.124	
2015	1.741	1.448	1.178	1.175		
2016	1.741	1.434	1.189			
2017	1.774	1.420				
2018	1.774					
<u>Age-to-Age Development Factors:</u>						
Simple All	1.748	1.437	1.181	1.187	1.124	1.000
Selected	1.748	1.437	1.181	1.187	1.124	1.000
Age-to-Ultimate	3.957	2.264	1.575	1.333	1.124	1.000

Accident Year	Paid Severity	Age-to-Ultimate Development Factors	Ultimate Severity
2014	2,524.96	1.000	2,524.96
2015	2,359.37	1.124	2,651.21
2016	2,127.58	1.333	2,837.04
2017	1,897.10	1.575	2,988.34
2018	1,415.70	2.264	3,205.31
2019	853.83	3.957	3,378.65

e.g., for AY2018:  $3,205.31 = 1,415.70 \times 2.264$

- (b) State two considerations for deciding how many data points to include when selecting an annual trend.

Any two of the following are acceptable:

- The relevance of the historical experience to the period for which the trend is being determined.
- The reliability of recent data points.
- The purpose for which the trend will be used.
- The goodness of fit statistics from the regression analysis.

## 6. Continued

- (c) Recommend the annual severity trend. Justify your recommendation.

Accident Year	Year-to Year Change
2014	
2015	5.00%
2016	7.01%
2017	5.33%
2018	7.26%
2019	5.41%
Average:	6.00%

Recommended severity trend: 6.00%  
 Justification: The pattern is erratic, so an all-years average is selected.

- (d) Calculate the proportion of closed counts for development months 36 through 72 using a simple average of all years.

Accident Year	Incremental Closed Counts						Ultimate Counts
	12	24	36	48	60	72	
2014	583	130	41	18	20	21	813
2015	539	126	44	19	29		798
2016	450	99	33	14			649
2017	427	95	22				606
2018	332	82					488
2019	405						586

Accident Year	Proportion of Closed Counts					
	12	24	36	48	60	72
2014	0.717	0.565	0.410	0.305	0.488	1.000
2015	0.675	0.486	0.331	0.213	0.414	
2016	0.693	0.497	0.330	0.209		
2017	0.705	0.531	0.262			
2018	0.680	0.526				
2019	0.691					
Average:			0.333	0.243	0.451	1.000

$$\text{e.g., for AY2014 @ 36 months: } 0.410 = \frac{41}{813 - (583 + 130)}$$



## 6. Continued

- (e) Calculate the accident year 2018 unpaid claims, using the results from parts (c) and (d).

**Commentary on Question:**

*It is also acceptable to round the incremental closed counts to the nearest integer value.*

Accident Year	Incremental Closed Counts						Ultimate Counts
	12	24	36	48	60	72	
2018	332	82	24.66	11.97	16.86	20.52	488

Accident Year	Incremental Severity Trended to AY2018						Total
	12	24	36	48	60	72	
2018			10,521.70	12,834.91	11,421.70	8,211.32	
AY2018 Unpaid Claims:			259,418	153,588	192,561	168,487	774,053

e.g.,  $24.66 = 0.333 \times [488 - (332 + 82)]$   
 $10,521.70 = 11,153 \times 1.06$  *{uses the trend recommendation from part (c)}*  
 $259,418 = 24.66 \times 10,521.70$

## 7. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

### Learning Outcomes:

- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 21.

### Commentary on Question:

*This question tests the candidate's understanding of the evaluation and selection of estimated ultimate claims under various circumstances.*

### Solution:

Recommend two methods for projecting ultimate claims that are appropriate for each line of business without repeating any methods. Justify your recommendations for all four methods

#### Line of Business A:

- Frequency-Severity method because it allows separate analysis of claim frequency from claim severity. This method should show increasing claim frequency trend and potential increases in severity from policy limits.
- Cape Cod method because it uses actual claim experience to determine expected claims. The use of expected claims will also moderate some of the volatility and claim trends can be explicitly reflected in expected claims.

#### Line of Business B:

- Development method applied to reported claims because experience is relatively stable and mature enough for development patterns.
- Bornhuetter Ferguson method applied to reported claims because this is a new line of business which means using the a priori expected claim ratio is appropriate. The Bornhuetter Ferguson method will not over-project large losses to date. A priori expectations can reflect industry data if available.

## 8. Learning Objectives:

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

### Learning Outcomes:

- (4e) Describe the components of claim liabilities in the context of financial reporting
- (4g) Describe the components of premium liabilities in the context of financial reporting.
- (4h) Evaluate premium liabilities.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 11, 23, and 24.

### Commentary on Question:

*This question tests the candidate's understanding of premium liabilities.*

### Solution:

- (a) Describe the difference between claim liabilities and premium liabilities.

Claim liabilities relate to claims that occurred on or before the accounting/valuation date, regardless of whether it has been reported or not.

Premium liabilities relate to claims that occurred after the accounting/valuation date and are associated with the unexpired portion of policies effective on or before the accounting/valuation date.

- (b) Describe each of these approaches.

Premium approach: the premium liability equals the net unearned premium liability from financial statements (or net UPR) less anticipated profit margin.

Claim approach: the premium liability is evaluated directly from actuarial analysis of claims experience.

## 8. Continued

(c) Provide one challenge with the premium approach.

Any one of the following is acceptable:

- This approach relies on an up to date pricing basis
- This approach relies on a relatively stable claims and exposure environment (since you are not using claim experience directly in the calculation)
- The unearned premium reserve may not reflect most current view of future claims experience (particularly due to lag between setting rates and effective date)
- It may be difficult to quantify profit margin (particularly for commercial lines)
- Rates and profit margins tend to vary with the underwriting cycle

(d) Calculate the equity in unearned premiums as of June 30, 2020, net of reinsurance.

Line of Business	Gross Written Premium (000)	Gross Expected Claim Ratio incl. ALAE	Gross Unearned Premium (000) as of Jun 30, 2020	Gross Expected Claims (000)
Property	1,305	82%	870.00	713.40
General Liability	1,539	56%	1,026.00	574.60
Automobile	1,244	79%	829.30	655.10
Total	4,088		2,725.00	1,943.00

Remaining time of policies as of Jun. 30, 2020: 0.6667 (8 months)

Expected ULAE for premium liabilities gross of reinsurance = total gross expected claims  $\times$  12.9% = 1,943  $\times$  12.9% = 250.65

expected ULAE for premium liabilities net of reinsurance = expected ULAE for premium liabilities gross of reinsurance = 250.65

## 8. Continued

	Gross of Reinsurance	Net of Reinsurance
Unearned Premiums (000)	2,725.00	2,043.75
Expected Claims (000)	1,943.00	1,457.25
Expected ULAE (000)	250.65	250.65
Total Expected Claims and LAE		1,707.90
Maintenance Expenses = $2,725 \times 16\% \times 30\%$		130.80
Profit-sharing Commissions = $2,725 \times 3.2\%$		87.20
Total Premium Liabilities		1,925.90
Profits (Equity) in the unexpired policy		118.00

## 9. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.
3. The candidate will know how to calculate and evaluate projected ultimate values.

### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.
- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 12, 14 and 18.

### Commentary on Question:

*This question tests the estimating of ultimate claims using the Cape Cod method.*

### Solution:

- (a) Describe why premium on-level factors are typically used in the Cape Cod method but not in the Bornhuetter Ferguson method.

The Bornhuetter Ferguson method uses an external a priori estimate as an expected claim ratio. (This should implicitly be at appropriate rate level).

The Cape Cod method derives one adjusted expected claim ratio from all historical data, so all years need to reflect the same rate level.

- (b) Describe a situation in which an actuary may choose to derive an adjusted expected pure premium instead of an adjusted expected claim ratio when using the Cape Cod method.

Any one of the following is acceptable:

- If rate change history is not available/reliable
- If on-level premium adjustment factors are not available/reliable
- If exposure base is not inflation-sensitive, then using exposures simplifies the calculation

## 9. Continued

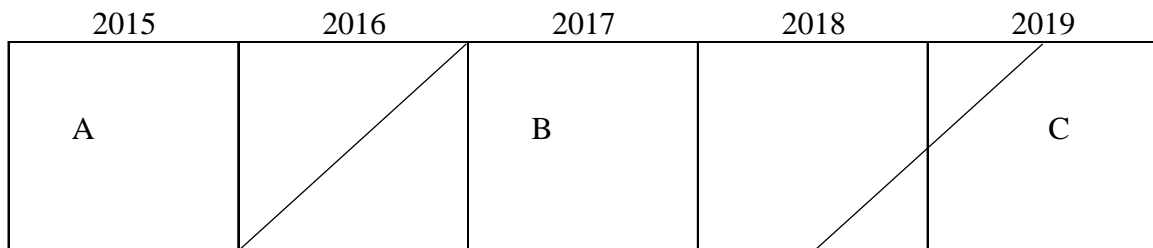
- (c) Explain why confidence in the development method is a consideration in selecting the decay factor.

As the decay factor approaches 0, projected ultimate claims in the Generalized Cape Cod method approach results from the development method. So, an actuary who has significant confidence in the development method can choose a smaller decay factor.

- (d) Calculate premium on-level factors for each accident year, to use in the Cape Cod method as of December 31, 2019.

**Commentary on Question:**

*The diagram is helpful to solve the question but not required for credit.*



Level	Rate Level	<u>Percent Premium Earned in Each CY at Rate Level</u>				
	Index	2015	2016	2017	2018	2019
A	1.00000	100.0%	50.0%			
B	1.06000		50.0%	100.0%	87.5%	12.5%
C	1.11300				12.5%	87.5%
Total		100.0%	100.0%	100.0%	100.0%	100.0%

Average rate level in each CY:      1.00000    1.03000    1.06000    1.06663    1.10638

On-level factors for reserving:      1.1064    1.0742    1.0438    1.0373    1.0000

e.g.,  $1.0373 = 1.10638 / 1.06663$

## 9. Continued

- (e) Calculate the projected ultimate claims for each accident year using the Cape Cod method.

Accident Year	(1) Earned Premiums (EP)	(2) Premium On-Level Factors from part (d)	(3) = (1)(2) On-Level Earned Premiums	(4) Reported CDFs	(5) = 1/(4) Expected % Reported	(6) = (3)(5) Used-Up On-Level EP
2015	16,100	1.1064	17,813	1.030	97.1%	17,294
2016	17,600	1.0742	18,905	1.055	94.8%	17,919
2017	18,300	1.0438	19,101	1.100	90.9%	17,364
2018	19,800	1.0373	20,538	1.300	76.9%	15,798
2019	21,600	1.0000	21,600	1.700	58.8%	12,706
	93,400		97,956			81,082

Accident Year	(7) Actual Reported Claims excluding Large Claim	(8) <u>Claim Adjustment Factors</u> Trend at 5%	(9) Tort Reform	(10) = (7)(8)(9) Adjusted Claims	(11) Expected Claims
2015	11,150	1.2155	0.90	12,198	11,548
2016	11,380	1.1576	0.95	12,515	12,191
2017	11,190	1.1025	1.00	12,337	12,287
2018	10,870	1.0500	1.00	11,414	13,872
2019	9,040	1.0000	1.00	9,040	15,319
	53,630			57,503	65,217

Adjusted Expected Claim Ratio: 70.92% = 57,503 / 81,082

Notes: (7) for AY2018: 10,870 = 11,470 – 800

(11) = [(3) × 0.7092] / [(8)(9)]

Accident Year	(12) Actual Reported Claims	(13) = 1 – (5) Expected % Unreported	(14) = (11)(13) Expected Unreported	(15) = (12) + (14) Projected Ultimate
2015	11,150	2.9%	336	11,486
2016	11,380	5.2%	636	12,016
2017	11,190	9.1%	1,117	12,307
2018	11,470	23.1%	3,201	14,671
2019	9,040	41.2%	6,308	15,348
	54,230		11,598	65,828



## 10. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

### Learning Outcomes:

- (6t) Describe the types of experience rating used with general insurance.
- (6u) Determine experience rating modification factors and experience rating adjustments.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 35.

### Commentary on Question:

*This question tests the candidate's understanding of the NCCI approach to experience rating.*

### Solution:

- (a) Describe how the NCCI split rating experience rating plan differentiates between the frequency and severity of an insured's experience.

The NCCI experience rating plan uses split rating to explicitly reflect the frequency and severity of an insured's experience.

The split rating segregates actual claims into primary claims and excess claims.

- (b) Provide another way that an experience rating formula can differentiate between frequency and severity, other than the approach identified in part (a).

Cap individual claims.

- (c) Calculate the following:

- (i) Total actual excess claims
- (ii) Total expected primary claims
- (iii) Expected excess claims for Classification Code C

## 10. Continued

		(1)	(2)	(3)		
		Actual Claims				
Claims ID		Reported	Primary	Excess		
# 2		15,000	10,000	5,000		
# 4		40,000	10,000	30,000		
# 7		5,000	5,000	0		
Claims less than 1,000		20,000	20,000	0		
Total		80,000	45,000	35,000		
(4)	(5)	(6)	(7)	(8)		
		Expected Claims				
Code	Payroll	ELR (per 100)	D-ratio	Primary	Excess	
A	1,400,000	0.10	0.5	700	700	
B	1,600,000	2.00	0.4	12,800	19,200	
C	1,000,000	1.50	0.3	4,500	10,500	
Total				18,000	30,400	
(i)	45,000					
(ii)	18,000					
(iii)	10,500					

Notes: (2) = (1) capped at 10,000

(3) = (1) – (2)

(7) = (4)(5)(6) / 100

(8) = (4)(5)[1 – (6)] / 100

- (d) Calculate the NCCI experience rating modification factor using  $W = 0.5$  and  $B = 50,000$ .

$$M = \frac{A_P + (1-W) \times E_{XS} + B + W \times A_{XS}}{E_P + (1-W) \times E_{XS} + B + W \times E_{XS}}$$

$$= \frac{45,000 + (1-0.5) \times 30,400 + 50,000 + 0.5 \times 35,000}{18,000 + (1-0.5) \times 30,400 + 50,000 + 0.5 \times 30,400} = \frac{127,700}{98,400} = 1.298$$

- (e) Recommend two ways to increase responsiveness of this experience rating plan.

- Increase the limit (or cap) applied to the claims included in the experience rating formula
- Decrease the number of years in the experience period

## 11. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.
6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6m) Describe key considerations in the analysis of deductible factors and increased limits factors.
- (6n) Calculate deductible factors and increased limits factors.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 25 and 33.

### Commentary on Question:

*This question tests the candidate's understanding of deductible factors used in ratemaking.*

### Solution:

- (a) Describe a potential issue related to the absence of complete data when using reported claim data from recent years.

You may only have access to the claim detail for the portion of the loss that the insurer covers.

- (b) Describe a potential issue related to claim development when using individual reported claim data from recent years.

Claim development factors are selected based on aggregated claim experience by accident year and represent case development as well as pure IBNR. As such, claim development factors are not intended to be used on an individual claim file basis.

## 11. Continued

- (c) Calculate the indicated deductible factors for deductibles of 500 and 1,000 relative to a base deductible of zero.

Average accident date in future rating period is March 1, 2022.

Claim #	Date of Claim	(1) Ground Up Ultimate Claims	(2) <u>Trending Period</u>		(4) Trend Factor @5%	(5) <u>Trended Ultimate Indemnity</u>		
			Months	Years		No Deductible	Deductible of 500	Deductible of 1000
1	Jan. 1, 2017	7,500	62	5.17	1.2867	9,650.27	9,150.27	8,650.27
2	Jul. 1, 2017	800	56	4.67	1.2557	1,004.55	504.55	4.55
3	Jul. 1, 2017	1,600	56	4.67	1.2557	2,009.11	1,509.11	1,009.11
4	Jan. 1, 2018	2,400	50	4.17	1.2254	2,941.03	2,441.03	1,941.03
5	Jan. 1, 2018	6,700	50	4.17	1.2254	8,210.39	7,710.39	7,210.39
6	Jul. 1, 2018	2,300	44	3.67	1.1959	2,750.57	2,250.57	1,750.57
7	Jan. 1, 2019	700	38	3.17	1.1671	816.95	316.95	0.00
8	Jul. 1, 2019	300	32	2.67	1.1390	341.69	0.00	0.00
9	Jul. 1, 2019	1,100	32	2.67	1.1390	1,252.85	752.85	252.85
10	Jul. 1, 2019	4,500	32	2.67	1.1390	5,125.28	4,625.28	4,125.28
Total						34,102.67	29,260.99	24,944.04
Deductible factor:							0.858	0.731

Notes: (4) =  $1.05^{(3)}$

(5) = (1)(4)

(6) = Greater of 0 and (5) – 500

(7) = Greater of 0 and (5) – 1,000

Deductible factors:

$$0.858 = 29,260.99 / 34,102.67; 0.731 = 24,944.04 / 34,102.67$$

- (d) Evaluate the reasonability of the deductible factors calculated in part (c) by performing a consistency test.

The marginal rate should decrease as the value of the deductible increases.

Deductible	Deductible Factor	Marginal Rate Per Thousand
0	1.000	
500	0.858	0.284
1000	0.731	0.253

Since there is a decrease, the deductible factors are considered to be consistent.

## 12. Learning Objectives:

9. The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

### Learning Outcomes:

- (9b) Apply catastrophe modeling results in ratemaking, loss mitigation, risk selection, and reinsurance.  
(9d) Understand and apply common risk metrics associated with catastrophe modeling results.

### Sources:

Uses of Catastrophe Model Output, American Academy of Actuaries, July 2018.

### Commentary on Question:

*This question tests the candidate's understanding of catastrophe modeling.*

### Solution:

- (a) State four applications of catastrophe modeling for insurance.

Any four of the following are acceptable:

- Ratemaking
- Underwriting and Risk Selection
- Loss Mitigation
- Catastrophe Reinsurance
- State and federal public policymakers use catastrophe models to address public policy issues.
- Capital adequacy (sensitivity) testing
- For reserving purposes

- (b) Recommend which portfolio you would add to the book. Justify your recommendation.

Account Y is recommended because it has a relatively high AAL, but it could be in an area with low concentration in the current book, since it doesn't impact the total book's PML too much.

- (c) Calculate the premium for this other portfolio assuming hurricane shutters are installed on all properties in the portfolio.

$$\begin{aligned}\text{Adjusted AAL} &= \text{AAL} \times (1 - \text{Discount}) = 5,000 \times (1 - 0.137) = 4,315 \\ \text{Premium} &= (\text{Adjusted AAL} + \text{Risk Load}) / (1 - \text{expense load factor}) \\ &= (4,315 + 440) / (1 - 0.27) = 6,514\end{aligned}$$

## 12. Continued

- (d) Provide a consideration in the selection of a risk load in this situation.

The variability (i.e., standard deviation or CV) or uncertainty in the loss estimates.

- (e) Recommend a way this risk could be managed.

The company could manage this exposure by transferred the risks to other parties (e.g., investors or reinsurers with worldwide portfolios).

### 13. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

#### Learning Outcomes:

- (6f) Explain the requirements for loadings for catastrophes and large claims in ratemaking.  
(6g) Calculate loadings for catastrophes and large claims.

#### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 30.

#### Commentary on Question:

*This question tests the candidate's understanding of loadings for large claims for ratemaking.*

#### Solution:

- (a) Explain why actuaries typically conduct separate analyses of property and liability claims for homeowners insurance when determining a loading for large claims.

There are very different forces influencing the claim development, severities, frequencies, and the trends applicable to property and liability coverages.

- (b) Calculate the loadings for 500,000 to total limits for each accident year.

Average accident date in each experience year = July 1

Average accident date in forecast period = April 1, 2022

Severity trend for 1,000,000 limit =  $4.00\% \times 0.60 + 5.00\% \times 0.40 = 4.40\%$

Severity trend for total limit =  $5.00\% \times 0.50 + 6.00\% \times 0.50 = 5.50\%$

Accident Year	(1) Trend Period (months)	(2) <u>Severity Trend at:</u> 4.40%	(3) 5.50%	(4) Trended Claims at 1,000,000 Limit	(5) Total Limit
2016	69	1.281	1.361	9,505	10,816
2017	57	1.227	1.290	9,570	10,510
2018	45	1.175	1.222	9,990	10,622
2019	33	1.126	1.159	10,300	10,798

Notes: (4) = (Selected ultimate claims at 1,000,000 limit)(2)

(5) = (Selected ultimate claims at total limit)(3)

### 13. Continued

Accident Year	(6) = (5) / (4) Loading for 1,000,000 to Total Limit	(7) Loadings for 500,000 to 1,000,000 Limit	(8) = (6)(7) Loadings for 500,000 to Total Limit
2016	1.138	1.182	1.345
2017	1.098	1.185	1.301
2018	1.063	1.270	1.350
2019	1.048	1.285	1.347

- (c) Recommend a loading for 500,000 to total limits for ratemaking purposes. Justify your recommendation.

Average of all years excluding 2017 = 1.348

Justification: 2017 appears to be an outlier so the average of all other years is reasonable.

- (d) Explain why severity trend is used for the part (b) calculation instead of pure premium trend.

Limiting claims to remove the effect of large claims does not affect the frequency of claims on a given portfolio; capping only affects the severities. Therefore, using pure premium trend would have overstated the large claim loading.



## 14. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.
7. The candidate will understand the need for monitoring results.

### Learning Outcomes:

- (3d) Analyze development triangles for investigative testing.
- (7b) Analyze actual claims experience relative to expectations.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 13 and 36.

### Commentary on Question:

*This question tests the candidate's understanding of various diagnostic tests on development triangles. This question also tests the candidate's understanding of monitoring actual versus expected reported claims.*

### Solution:

- (a) Calculate the difference between the actual and expected reported claims from December 31, 2019 through September 30, 2020 for all accident years, using a linear interpolation of the development pattern.

Accident Year	(1) Ultimate	(2) Reported Claims @ Dec. 31, 2019	(3) Reported Claims @ Sep. 30, 2020	(4) = (2)/(1) Expected % Reported @ Dec. 31, 2019	(5) Expected % Reported @ Sep. 30, 2020
2014	6,557	6,557	6,557	100.0%	100.0%
2015	7,293	7,242	7,283	99.3%	99.8%
2016	8,087	7,544	7,923	93.3%	97.8%
2017	7,150	5,988	6,572	83.7%	90.9%
2018	7,572	5,018	6,335	66.3%	79.4%
2019	7,875	3,537	5,129	44.9%	60.9%
Total	44,534	35,886	39,799		

e.g., (5) for AY2019:  $60.9\% = (3/12) \times 44.9\% + (9/12) \times 66.3\%$

## 14. Continued

AY	(6) Expected Reported at 9 months	(7) Actual Reported at 9 months	(8) = (7) – (6) Difference
2014	0	0	0
2015	38	41	3
2016	365	379	14
2017	511	584	73
2018	993	1,317	324
2019	1,261	1,592	331
Total	3,168	3,913	745

e.g., (6) for AY2019:

$$\frac{[(1) - (2)] \times [(5) - (4)]}{1 - (4)} = \frac{(7,875 - 3,537) \times (.609 - .449)}{1 - .449} = 1,261$$

- (b) Provide an interpretation of the results for the actual versus expected analysis derived in part (a).

The actual versus expected differences are significant in recent years. This means that the development factor assumptions were not appropriate for this projection.

- (c) Evaluate if the data indicates a possible change in case reserve adequacy using two different diagnostic tests.

### **Commentary on Question:**

*Note: The reported claims for AY 2018 at 12 months was given as 3.292, but it should have been 3,292. Some candidates noticed the error, and some did not. However, credit was given regardless of the value used, provided the work was done correctly.*

## 14. Continued

Accident Year	<u>Ratios of Paid Claims to Reported Claims</u>					
	12	24	36	48	60	72
2014	0.46	0.59	0.71	0.78	0.87	0.92
2015	0.46	0.60	0.72	0.78	0.86	
2016	0.48	0.60	0.72	0.78		
2017	0.45	0.60	0.69			
2018	454.43	0.57				
2019	0.41					

The latest diagonal shows a decrease in ratios which could mean a decrease in settlement (numerator) or increase in case reserve adequacy (denominator).

Accident Year	<u>Average Case Estimates</u>					
	12	24	36	48	60	72
2014	5.508	10.142	12.651	13.500	11.583	9.596
2015	5.775	10.613	13.258	14.154	12.155	
2016	6.042	11.183	13.853	14.928		
2017	6.292	11.815	16.541			
2018	-5.488	13.728				
2019	7.824					

The latest diagonal shows a decrease in ratios which could mean a decrease in settlement (numerator) or increase in case reserve adequacy (denominator).

- (d) Evaluate if the data indicates a possible change in case settlement rates using a diagnostic test different than either of the two tests from part (c).

Accident Year	<u>Reported Counts</u>					
	12	24	36	48	60	72
2014	774	842	853	853	853	853
2015	807	883	890	890	890	
2016	830	927	938	938		
2017	734	797	808			
2018	724	799				
2019	714					

## 14. Continued

Accident Year	Ratios of Closed Counts to Reported Counts					
	12	24	36	48	60	72
2014	0.62	0.79	0.85	0.88	0.92	0.93
2015	0.60	0.79	0.86	0.88	0.91	
2016	0.63	0.79	0.86	0.88		
2017	0.62	0.80	0.86			
2018	0.62	0.80				
2019	0.63					

Closed to reported counts are relatively stable which means a change in settlement rate is not likely.

## 15. Learning Objectives:

1. The candidate will understand the key considerations for and key concepts underlying general insurance actuarial work.
4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (1i) Describe how and why data are segregated and aggregate.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.
- (4c) Evaluate and justify selections of unpaid unallocated loss adjustment expenses based on ratio and count-based methods.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3, 4, and 22.

### Commentary on Question:

*This question tests the candidate's understanding of unpaid ALAE and unpaid ULAE.*

### Solution:

- (a) Describe one way a reinsurer might assess the reasonableness of an estimate of unpaid ULAE.

Either one of the following is acceptable:

- Consider the reinsurer from a run-off perspective.
- Estimate the number of years to run-off the claim liabilities and the estimated cost per year.

- (b) Recommend one of the two approaches from the table above to use in estimating unpaid ULAE. Justify your recommendation.

Either one of the following is acceptable:

- Kittel refinement because it incorporates reported claims which reduces distortion from exposure growth.
- Kittel refinement because the classical paid-to-paid overstates the ULAE ratio (numerator) when exposure is growing.

## 15. Continued

- (c) Estimate unpaid ULAE as of December 31, 2019 using the approach you selected in part (b).

Ratio of ULAE to claims (Kittel refinement): average of 2018 and 2019 = 7.20%  
For the ULAE ratio selection, use the average of the most recent 2 years to reflect the growing exposure base.

$$\begin{aligned}\text{Unpaid ULAE} &= (\text{ULAE ratio} \times \text{pure IBNR}) + [\text{ULAE ratio} \times \text{multiplier} \times (\text{case} \\ &\quad \text{estimates} + \text{development on case estimates})] \\ &= (0.072 \times 1,600,000 \times 0.2) + 0.072 \times 0.75 \times (3,510,000 + 0.8 \times 1,600,000) \\ &= 281,700.\end{aligned}$$

- (d) Determine calendar year 2019 incurred ULAE.

$$\begin{aligned}\text{CY 2019 incurred ULAE} &= \text{2019 paid ULAE} + \text{Change in outstanding in 2019} \\ &= 880,000 + 281,700 - 270,000 = 891,700.\end{aligned}$$

- (e) Critique your colleague's recommendation.

Any two of the following are acceptable:

- ALAE shouldn't be evaluated on a calendar year basis because ALAE reflect development over time.
- ALAE is more directly related to the size of a claim and should be evaluated like claim experience.
- ALAE are directly attributable to claims and should be analyzed similar to claims while ULAE are general and not assigned to claims.
- Accident year detail is recorded for ALAE which allows a deeper analysis.
- ALAE reporting requires accident year detail.

## 16. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.
5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.
6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.
- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6j) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 12, 25, 26, and 31.

### Commentary on Question:

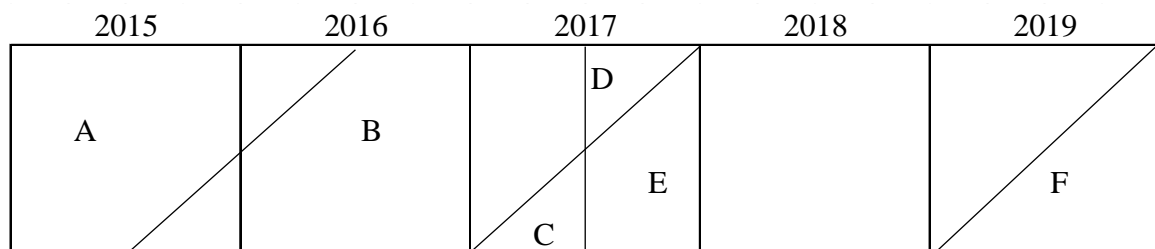
*This question tests basic ratemaking using a claim ratio approach. The candidate also needs to understand earned premiums adjusted to current rate level for ratemaking purposes.*

### Solution:

- (a) Calculate premium on-level factors for accident years 2015-2019 to use for ratemaking purposes.

### Commentary on Question:

*The diagram is helpful to solve the question but not required for credit.*



## 16. Continued

Level	Rate Level Index	<u>Percent Premium Earned in Each CY at Rate Level</u>				
		2015	2016	2017	2018	2019
A	1.00000	87.5%	12.5%			
B	1.08000	12.5%	87.5%	37.5%		
C	1.18800			12.5%		
D	0.86400			12.5%		
E	0.95040			37.5%	100.0%	50.0%
F	0.99792					50.0%
Total		100.0%	100.0%	100.0%	100.0%	100.0%

Average rate level in each CY:            1.01000   1.07000   1.01790   0.95040   0.97416

On-level factors for ratemaking:        0.9880   0.9326   0.9804   1.0500   1.0244

e.g.,  $0.97416 = 0.5 \times 0.95040 + 0.5 \times 0.99792$   
 $1.2044 = 0.99792 / 0.97416$

- (b) Calculate the trended on-level claim ratios for each accident year.

Trend from the average accident date in each AY (i.e., July 1) to the average accident date in future rating period.

Average accident date in future rating period: November 1, 2021

Accident Year	Trending Period in Years	Earned Premiums	<u>Premium Adj. Factors</u>		Trended Earned Prem. at Current Rate Level
			Trend at 1.00%	On-Level	
2015	6.333	11,755,570	1.0650	0.9880	12,370,486
2016	5.333	11,864,520	1.0545	0.9326	11,668,350
2017	4.333	12,406,530	1.0441	0.9804	12,698,923
2018	3.333	12,492,860	1.0337	1.0500	13,559,877
2019	2.333	12,394,530	1.0235	1.0244	12,995,072

e.g., for AY2019:  $1.0235 = 1.01^{2.333}$   
 $12,995,072 = 12,394,530 \times 1.0235 \times 1.0244$



## 16. Continued

Accident Year	Ultimate Claims	Pure Premium Trend Factor at 4.00%	Regulation Adjustment to Claims	Trended Claims	Trended Claim Ratio
2015	8,130,150	1.2820	0.80	8,338,086	67.40%
2016	7,970,110	1.2327	0.80	7,859,570	67.36%
2017	7,781,380	1.1853	0.90	8,300,615	65.36%
2018	8,001,680	1.1397	1.00	9,119,247	67.25%
2019	7,995,960	1.0958	1.00	8,762,239	67.43%

e.g., for AY 2019:  $1.0958 = 1.04^{2.333}$   
 $8,762,239 = 7,995,960 \times 1.0958 \times 1.00$   
 $67.43\% = 8,762,239 / 12,995,072$

- (c) Recommend a trended claim ratio to use for ratemaking. Justify your recommendation.

Accident Year	Trended Claim Ratio	Accident Year Weights
2015	67.40%	10%
2016	67.36%	15%
2017	65.36%	20%
2018	67.25%	25%
2019	67.43%	30%

Weighted average trended claim ratio = 66.96%

Justification: No significant outliers, so average of all years with more weight to more recent experience.

- (d) Calculate the indicated rate change.

Weighted average trended claim ratio	66.96%
Ratio of ULAE to claims	10.00%
Weighted average trended claim ratio including ULAE = $0.6696 \times (1 + 0.10) =$	73.65%
Fixed expenses as ratio to premiums at current rate level	6.00%
Variable expenses (ratio to premiums)	19.00%
Profit and contingencies ratio to premiums	5.00%
Permissible claim ratio = $(1 - 0.19 - 0.05) / (1 + 0.06/0.7365) =$	70.28%
Indicated rate change = $0.7365 / 0.7028 - 1 =$	4.81%

## 16. Continued

- (e) Explain why an indicated rate increase of 5% is not necessarily indicative of deteriorating experience.

We are told that rates were adequate at the time of the rate change. Therefore, if experience does not get better or worse after the change, then experience should change with expected net trend.

$$\text{Net trend} = (\text{claim trend})/(\text{premium trend}) - 1 = (1 + 0.04) / (1 + 0.01) - 1 = 2.97\%$$

Time from the change to the effective date of the new rates = 1.5 years

$$\text{Therefore, experience should change with respect to net trend} = (1 + 0.0297)^{1.5} - 1 = 4.5\%$$

Since this is close to the rate change implemented at that time, this is as expected and does not indicate deteriorating experience.

## 17. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

### Learning Outcomes:

- (3h) Explain the effect of changing conditions on the projection methods cited in (3e).  
(3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 20.

### Commentary on Question:

*This question tests the candidate's understanding of how claims data is affected by various changing conditions and the appropriateness of various methods of estimating ultimate claims under changing conditions.*

### Solution:

- (a) Provide two different examples of changing conditions that are likely to decrease the latest diagonal of a reported claim triangle.

Any two of the following are acceptable:

- Decrease in case reserve adequacy
- Slowdown in settlement pattern
- Tort reform reducing open claims

- (b) Describe how an increase in attachment point for an excess of loss reinsurer could affect a reported claim triangle.

Any two of the following are acceptable:

- Increase in severity
- Increase in severity trend
- Longer pattern/tail
- Could affect row (or multiple rows) depending on implementation period

- (c) Explain what affect the claim ratio deterioration is likely to have on reported claim development factors.

If all other assumptions are steady-state, then deterioration in the claim ratio will not impact the patterns in the claim development triangle.

## 17. Continued

- (d) Explain which of the following two methods is likely to produce a more accurate estimate of ultimate claims in recent accident years in this scenario:
- (i) the development method applied to reported claims, or
  - (ii) the Bornhuetter Ferguson method applied to reported claims.

Since the development factors are not impacted by the claim deterioration, the development method will produce a better estimate than the Bornhuetter Ferguson method which will be understated because the expected claim ratio is not picking up the unexpected deterioration.

## 18. Learning Objectives:

1. The candidate will understand the key considerations for and key concepts underlying general insurance actuarial work.
3. The candidate will know how to calculate and evaluate projected ultimate values.

### Learning Outcomes:

- (1q) Understand the types of reinsurance and key reinsurance terms.
- (1s) Analyze and describe the types of reinsurance.
- (1t) Understand important reinsurance contract provisions that potentially affect actuarial work.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 14 and 16.

Fundamentals of General Insurance Actuarial Analysis 2019 Supplement, J. Friedland, Appendix H.

### Commentary on Question:

*This question tests the candidate's understanding of reinsurance claims experience as well as certain details of reinsurance contracts.*

### Solution:

- (a) Contrast a treaty reinsurance arrangement with a facultative reinsurance arrangement.

Treaty reinsurance – any two of the following are acceptable:

- ceded LOBs are agreed in advance
- all business falling under contract is automatically insured
- involves ongoing relationship between primary insurer and reinsurer
- commonly used for a group of homogeneous risks

Facultative reinsurance – any two of the following are acceptable:

- non-obligatory
- individual underlying insurance contracts (risks) are ceded
- reinsurer underwrites each contract separately
- primary insurer chooses which contracts to submit
- reinsurer can accept or reject submissions
- commonly used for heterogeneous risks with large limits

## 18. Continued

- (b) Calculate Primary Insurance's ultimate claims gross of reinsurance for all accident years, using the development method and 3-year volume-weighted average.

Accident Year	<u>XYZ Re Cumulative Reported Claims</u>					
	12	24	36	48	60	72
2014	6,138	6,740	7,144	7,380	7,505	7,580
2015	6,605	7,279	7,788	8,022	8,182	
2016	7,086	7,816	8,254	8,543		
2017	4,572	5,020	5,311			
2018	4,898	5,808				
2019	5,251					

Accident Year	<u>Gross Cumulative Reported Claims</u>						Ultimate Claims	Quota Share %
	12	24	36	48	60	72		
2014	12,276	13,480	14,288	14,760	15,010	15,160	15,236	50%
2015	13,210	14,558	15,576	16,044	16,364		16,610	50%
2016	14,172	15,632	16,508	17,086			17,664	50%
2017	15,240	16,733	17,703				18,901	30%
2018	16,327	19,360					21,932	30%
2019	17,503						22,424	30%
3-year vol. weighted average		1.131	1.061	1.033	1.019	1.010	1.005	
Age-to-ultimate factors		1.281	1.133	1.068	1.034	1.015	1.005	

e.g., AY2018:

Gross cumulative claims at 24 months:  $19,360 = 5,808 / 0.3$

Ultimate claims:  $21,932 = 19,360 \times 1.133$

## 18. Continued

- (c) Calculate the trended on-level claim ratio at AY 2019 cost and rate level gross of reinsurance, using an all-years simple average.

Accident Year	On-Level Earned premium XYZ Re share	Quota Share%	On-Level Earned premium 100% gross
2014	14,251	50%	28,502
2015	14,662	50%	29,324
2016	15,105	50%	30,210
2017	9,320	30%	31,067
2018	9,517	30%	31,723
2019	9,750	30%	32,500
	72,605		183,326

Annual claim ratio (pure premium) trend =  $1.04 \times 1.01 - 1 = 5.04\%$

Accident Year	On-Level Earned Premiums	Ultimate Claims	Claim Trend at 5.04%	Gross Reported Claims
2014	28,502	15,236	1.2787	68.35%
2015	29,324	16,610	1.2174	68.96%
2016	30,210	17,664	1.1589	67.76%
2017	31,067	18,901	1.1033	67.13%
2018	31,723	21,932	1.0504	72.62%
2019	32,500	22,424	1.0000	

Average trended on-level claim ratio at AY2019 cost and rate level, excluding 2019 = 68.96%

e.g., AY 2018:  $72.62\% = 21,932 \times 1.0504 / 31,723$

## 18. Continued

- (d) Calculate the total ultimate claims for XYZ Re's share of all accident years as of December 31, 2019.

Accident Year	On-Level	Trend at 5.04%	Claim Ratio	Expected Claims	
	Earned Premiums			Gross	XYZ Share
2014	28,502	1.2787	53.9%	15,372	7,686
2015	29,324	1.2174	56.7%	16,612	8,306
2016	30,210	1.1589	59.5%	17,977	8,988
2017	31,067	1.1033	62.5%	19,418	5,825
2018	31,723	1.0504	65.7%	20,828	6,248
2019	32,500	1.0000	69.0%	22,413	6,724
					43,778

e.g., AY 2018:

$$65.7\% = 68.96\% / 1.054$$

$$20,828 = 0.657 \times 31,723$$

$$6,248 = 20,828 \times 0.30$$

- (e) Estimate XYZ Re's AY 2020 expected claims.

$$\text{AY 2020 claim ratio} = 68.96\% \times 1.0504 = 72.44\%$$

$$\text{Gross expected claims} = 0.7244 \times 33,000,000 = 23,905,186$$

$$\text{XYZ expected claims} = 23,905,186 \times 0.3 = 7,171,556$$

- (f) Estimate the total impact on XYZ Re's AY 2020 expected claims.

$$\text{Revised 2020 earned premium} = 33,000,000 \times (1 - 0.15) = 28,050,000$$

$$\text{Revised claim ratio} = 72.44\% \times (1 - 0.2) \times (1 + 0.1) / (1 - 0.15) = 75.0\%$$

$$\text{Revised XYZ expected claims} = 28,050,000 \times 0.75 \times 0.3 = 6,310,969$$



## 19. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 14 and 19.

### Commentary on Question:

*This question tests the calculation of ultimate claims using the development method applied to claims the Berquist-Sherman adjustment for change in claims settlement.*

### Solution:

- (a) Calculate the triangle of adjusted closed counts.

Accident Year	Adjusted Closed Counts Excluding Large Claim Counts					Ultimate Counts
	12	24	36	48	60	
2015	564	864	1,060	1,187	1,256	1,256
2016	678	1,038	1,274	1,426		1,509
2017	576	882	1,082			1,282
2018	606	929				1,350
2019	699					1,557

e.g., AY2018 @ 12 months:  $606 = 1,350 \times 0.449$

- (b) Calculate total unpaid claims using the development method applied to paid claims, adjusted for changes in settlement rates.

Accident Year	Adjusted Paid Claims Excluding Large Claims				
	12	24	36	48	60
2015	600,585	2,136,841	4,787,346	6,664,813	7,213,000
2016	525,026	2,264,528	5,219,251	8,044,000	
2017	564,056	1,969,044	4,601,000		
2018	698,435	2,145,000			
2019	832,000				

e.g., AY2018 @ 12 months:  $698,435 = 85,287e^{(0.00347 \times 606)}$

## 19. Continued

Development factors (3-year volume weighted average):

	12-24	24-36	36-48	48-60	60-
Age-to-age	3.568	2.293	1.470	1.082	1.000
Age-to-ult	13.017	3.648	1.591	1.082	1.000

Accident Year	Paid Claims	Age-to-ultimate Dev. Factors	Large Claims Reported	Ult. Claims Incl. Large	Large Claims Paid	Unpaid Claims
2015	7,213,000	1.000		7,213,000		0
2016	8,044,000	1.082	801,000	9,506,627	615,000	847,627
2017	4,601,000	1.591		7,319,331		2,718,331
2018	2,145,000	3.648	923,000	8,747,519	297,000	6,305,519
2019	832,000	13.017		10,829,959		9,997,959
	22,835,000			43,616,436		19,869,436

e.g., AY 2018:

$$8,747,519 = 2,145,000 \times 3.648 + 923,000$$

$$6,305,519 = 8,747,519 - (2,145,000 + 297,000)$$

- (c) Assess the appropriateness of relying on the accident year 2019 ultimate claims from part (b) when selecting ultimate claims.

The AY2019 cumulative paid development factor is highly leveraged (13.017). Therefore, we should likely seek other methods for selecting ultimate claims.

## 20. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

### Learning Outcomes:

- (5a) Identify and describe the influences of portfolio changes on claim frequency and severity.  
(5b) Identify the time periods associated with trending procedures.  
(5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).  
(5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.  
(5e) Calculate trend factors for claims and exposures.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 26.

### Commentary on Question:

*This question tests the candidate's understanding of trend on premiums for ratemaking purposes.*

### Solution:

- (a) Calculate the annual premium trend due to the shift in policy limits for each year.

### Commentary on Question:

*The increased limits factors in effect starting on November 1, 2020 need to be used to calculate the weighted average ILFs, as they represent the current rating factors.*

Weighted Average ILF	Annual Trend Due to Shift in ILF
1.0238	
1.0281	0.42%
1.0372	0.89%
1.0465	0.90%
1.0532	0.64%
1.0599	0.64%

## 20. Continued

- (b) Recommend the annual premium trend due to the shift in policy limits to use for ratemaking. Justify your recommendation.

Average all years: 0.70%

Average excluding high/low: 0.72%

Recommend annual trend: 0.72%

Justification: exclude the high and low values because of the volatility.

- (c) Explain why the annual premium trend due to a shift in policy limits tends to be positive while the annual premium trend due to a shift in deductibles tends to be negative.

Over time, policy limits tend to shift to higher limits. The higher limits have higher factors, which results in more premium to the insurer, meaning positive trend.

Over time, deductibles tend to shift to higher deductibles. The higher deductibles have lower factors, which results in less premium to the insurer, meaning negative trend.

- (d) Calculate the calendar year 2017 on-level earned premium trended for ratemaking purposes.

Average earned premium date in future rating period: December 1, 2021

Total premium trend =  $(1 + 0.0072)(1 - 0.004) - 1 = 0.3176\%$

Experience period trend factor (2017 to 2019) =  $1.0599 / 1.0465 = 1.0128$

Forecast period from July 1, 2019 to Dec. 1, 2021: 29 months

CY2017 on-level EP trended to future rating period:

$$= 17,808,000 \times 1.0128 \times (1 + 0.003176)^{(29/12)} = 18,174,778.$$