ILA LAM Model Solutions
Fall 2023

1. **Learning Objectives:**
1. The candidate will understand, evaluate and use stochastic, generalized linear, multi-state, projection and transition matrix models. The candidate will demonstrate an understanding of their underlying methodologies, strengths, limitations, and applications.

Learning Outcomes:
(1d) With respect to multi-state and transition matrix models:
- Describe and apply the modeling methodology in an LTC product context.

Sources:
LAM-141-19: Case Study: LTC Insurance First Principles Modeling

LAM-142-19: Case Study: LTC Insurance First Principles Modeling: Mortality Assumptions

LAM-143-19: Case Study: LTC Insurance First Principles Modeling: Lapse Assumptions

Stochastic Analysis of Long-Term Multiple-Decrement Contracts, 2008 (excluding Attachments)

Commentary on Question:
Candidates showed a general understanding of the facts and concepts related to First Principles Modeling for LTC. Most candidates struggled a bit in applying the concepts though when asked to critique or assess reasonableness of results.

Solution:
(a) Critique the following statements:

      A.   *A first principles approach to model LTC will always be simpler to understand than a claim cost approach.*

      B.   *A first principles approach tends to have better internal consistency but is difficult to test sensitivity.*

      C.   *Companies should develop their own mortality assumption on healthy lives and disabled lives based on their experience because it is the most consistent with the first principles approach to have assumptions for each cohort.*

1. Continued

D.   *When developing mortality assumptions, the most accurate way to determine the correct mortality is to separate the study into the mortality rate for the total population, the mortality rate for disabled lives, and solve for the mortality rate for healthy lives.*

E.   *It is reasonable to assume the same lapse assumption for the total population and the healthy cohort.*

F.   *If the credibility of data used to generate lapse and mortality assumptions are high, all the information needed to generate the non-economic assumptions for the model are present.*

Commentary on Question:
Most candidates gave reasonable responses and knew that a critique required them to give some assessment of the statements. Often candidates would repeat the same facts and contribute them as answers showing knowledge recall but not full understanding of how to apply the concepts.

A This may not always be the case. A first principles approach brings a lot more detail and for a complicated product this can help simplify the results. However, for a simple product a claims cost approach may be sufficient.

B. The statement is not entirely correct. A first principles approach does tend to have better internal consistency however it is not generally difficult to test sensitivity. The way the model is constructed typically helps with testing sensitivity.

C. False. While it is good to incorporate a company's own experience for mortality assumptions, there is typically benefit of reviewing and including industry data to help improve credibility of results. Generally companies don't have enough experience to just use their own data.

D. This statement is not correct. There are different ways to go about it and the method described is not necessarily the most accurate. Typically, it's hardest to get credible mortality rates for disabled lives so often the total population and healthy population are studied and the disabled lives mortality solved for.

E. This is not a reasonable assumption. When on disabled status receiving claim there is effectively no lapse. So the lapse assumption for the healthy cohort and total cohort (which is healthy + disabled) should not be the same.

F. No all the information is not present to generate the non-economic assumption. Other non-economic assumption like morbidity cannot be inferred just because there is credible mortality and lapse assumption data.

1. Continued

(b)   You are given the following information:

Year	Exposure	Lapses	Disabled Deaths	Total Terminations
1	10,000	60	9	100
2	9,900	40	10	50
3	9,850	10	11	100
4	9,750	10	12	120
5	9,630	15	6	150

- Each year 200 lives become disabled.
- Disablements occur at the beginning of the year.

(i)   Calculate the following for years 1-5:

- Disabled life mortality rate
- Active life mortality rate
- Implied lapse rate

(ii)   Assess the reasonableness of the results.

Commentary on Question:
Many candidates showed a decent understanding of how to calculate the disabled, active and implied lapse rates but often had some calculation mistakes. Candidates struggled with assessing the reasonableness of results and often missed the obvious issues with the results that should be questioned.

(i)
Number of disabled lives a time t = number disabled at t-1 - disabled deaths at t-1 + 200

Number Disabled Lives: Time 1 = 200, Time 2 = 200 – 9 + 200 = 391, Time 3 = 581, Time 4 = 770, Time 5 = 958

Number of health lives at time t = number healthy at t-1 – healthy deaths at t-1 – lapses at t-1 – 200

Number of Health Lives: Time 1 = 10000 – 200 = 9800, Time 2 = 9800 – 31 – 60 – 200 = 9509, Time 3 = 9269, Time 4 = 8980, Time 5 = 8672

1. **Continued**

Disabled Life Mortality = disabled deaths / number disabled
Active Life Mortality = healthy deaths / number healthy
Implied Lapse Rate = lapses / number healthy

Time	Disabled Life Mortality	Active Life Mortality	Implied Lapses
1	9/200 = 4.50%	31/9800 = 0.32%	60/9800 = 0.61%
2	10/391 = 2.56%	0/9509 = 0.00%	40/9509 = 0/42%
3	1.89%	0.85%	0.11%
4	1.56%	1.09%	0.11%
5	0.63%	1.49%	0.17%

(ii) 0% healthy deaths at time 2 is not plausible. This leads us to question the results. It is not reasonable that active life mortality would be 0% at any point so we should question the credibility of the data.

Further at time 5 disabled life mortality is less than active life mortality. This is another result that is just not plausible and not expected which should lead us to question the credibility of the results.

The number of disabled increases by 200 each year, but the number of disabled deaths remains roughly flat and as such the annual disabled life mortality decreases each year. This result is questionable.

With the limited data we're given it's hard to infer whether lapse is reasonable or not.

Given the above points there may be an issue with the data and the data should be re-evaluated.

(c) Critique the following statement:

Our disabled life mortality is very sensitive. If we apply a 30% shock to the mortality rate, we see a 30% increase in the number of disabled lives dying. To ensure we're modeling disabled life mortality appropriately we should be using a stochastic model.

Commentary on Question:
Most candidates suggested stochastic modelling is appropriate but without giving full or good justification as to why. A case for it could be made but it would have required a good justification for the additional complexity.

1. **Continued**

In this example it seems that shocks to mortality and the impact to number of disabled lives dying move similarly. Stochastic modelling is used where the variables have a lot of sensitivity. Stochastic modelling often takes effort to setup, test and calibrate properly. In this instance the results don't give enough information to make the leap that a stochastic model is needed and worth the cost. We can still model disabled life properly without a stochastic model.

2. **Learning Objectives:**

2.   The candidate will understand and be able to assess issues and concerns common to actuarial models and their development and management.

Learning Outcomes:

(2b)   Explain and apply the technique for the compression of model data using the "Cluster Analysis Spatial Approach".

(2d)   Describe and evaluate key components of model risk management

Sources:

Reflection of Inflation, Interest Rates, Stock Market Volatility, and Potential Recession on Life Insurance Business, American Academy of Actuaries, 2022

The impact of a rising interest rate environment, Oliver Wyman, 2021

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a)

     (i)   Critique the use of an industry formula in their ESG.

     (ii)   Recommend economic scenarios management should consider when developing their own approach.

     (iii)   Recommend factors management should consider when modeling the scenarios discussed in (ii).

     Commentary on Question:

     Overall, candidates did well on part (i); for part (ii), most of the candidates only got partial credits for mentioning the spike of interest rates but missing the downstream impacts including the inflation spikes and recession. For part (ii), the candidate should recommend a scenario and justify their recommendation. All reasonably justified answers were accepted.

     Candidates interpreted part (iii) several different ways. Credit was awarded to any recommended factors that correctly related to the recommended scenarios in the part a (ii). Recommended factors may be different than the ones listed in the model solution.

     (i)   For general economic considerations, if the industry formula uses studies on inflation versus interest rates that are out of date, the actuary should consider updating the studies in order to justify the inflation assumptions being used.

2. **Continued**

    (ii)   Recommend the actuary performing scenario testing to reflect a temporary spike in inflation and interest rates in the scenario testing done. Inflation can exceed the current interest rates. Interest rates typically do also spike when inflation spikes.

           Or recommend considering economic recession, because high inflation and increasing interest rates can lead to economic recession.
When developing assumptions, the actuary should consider the prevailing market sentiment when assigning a likelihood to a given scenario.
i.e. - If the consensus among most economists as of a valuation date is that a recession is likely, the actuary should consider reflecting a recession in a prudent estimate CPI assumption or other formulae to prudently relate inflation to the assumed interest rates in projections; otherwise, one or more recession scenarios could be modeled as stress scenarios.

           Or, if some economists forecast as of a valuation date that there is a material chance of a near-term recession, then if the actuary does not reflect a recession in a prudent estimate CPI assumption or other formulae to prudently relate inflation to the assumed interest rates in projections, the actuary should consider whether one or more of the stress scenarios to be modeled should be recession scenarios.

    (iii)   Example of factors that management should consider include:
- Variable Products May be More Attractive if sold as an inflation hedge.
- Possible New Products/Shift to Existing Products with Inflation Protection: this is especially true if people are concerned about inflation in an high-inflation scenario
- More Spreads to Offer Additional Options in Products.
- New Money vs. Portfolio Products: If inflation causes interest rates to increase, products that are on a new money basis would have an advantage in that they will likely be able to credit higher rates, which could give them a competitive advantage.
- Need for Non-Guaranteed Element Review: The impact on products is that current inflation and interest environment emphasizes the need for non-guaranteed element (NGE) reviews. Inflation could impact current expense and interest rate assumption for universal life (UL) insurance

  (b)   Analyze how increasing interest rates may affect the profitability of these products.

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2. **Continued**

Commentary on Question:
Most candidates did well on UL; for Trad Whole Life, most of the candidates only got partial credits for realizing the increase of lapse due to the spike of interest rates but missed the impact on mortality due to the anti-selection of lapse; for UL with LTC rider, most of the candidates did not get full credits.

Traditional Whole Life (WL)
Lapse will increase. The richer benefits and reduced cost of new-money products will inevitably drive policyholders to consider surrendering their policies to reinvest the proceeds elsewhere. Policyholders are less likely to value a policy with a fixed payout/face amount if they perceive inflation would erode the value of that future payout.
Mortality becomes more adverse in inforce business, because re-underwriting will only be an option for better risks policies.

Universal Life (UL)
Adjustable products provide more levers to manage profitability, so would be less reactive than Traditional Whole Life.

Credited interest rates are the obvious competitive difference between the in-force block and new money alternatives, and by managing crediting rate strategies insurers can encourage policyholder persistency.

Portfolio yields change slowly as assets turn over and new money is reinvested, causing returns to lag current rates. As a result, increasing credited rates in order to remain competitive with new money products will require compromising on credited rate spreads.

UL with LTC Rider
Policyholders relying on secondary guarantees or other benefits / riders will be far less concerned with credited rates. Since many secondary guaranteed blocks react favorably to increased lapses, care should be taken to not apply accumulation-related dynamic lapse behavior to protection-style products.

If people are more concerned with inflation, there may be a market for products that provide benefits based on inflation. There can be more sales of products such as long-term care (LTC) products that offer cost-of-living adjustments (COLA). Obviously, these will be impacted by inflation. Most of these products have caps; it is likely the caps will be triggered in this environment.

In case of recession, Long-term care insurance/disability claim recoveries may decrease, as there may be fewer jobs and less incentive to go back to work.

2. **Continued**

(c) Recommend two new life insurance products your company can offer. Justify your answer.

Commentary on Question: *One point for each reasonable recommended product up to two points. Candidate must include justification. Recommended products may be different than the ones listed in the model solutions.*

Candidates did well on this question. Most had at least one recommended product with justification.

Traditional Whole Life
Rising rates may enable insurers to redesign products with lower premiums or accelerated cash values, both of which would entice consumers reviewing life illustrations.

Participating products may also become more viable, with higher anticipated asset yields allowing for a more favorable illustrated dividend scale.

UL
Insurer can pass some of the increased yield back to the policyholder through credited rates or offer goateed living benefits with richer guarantees.

Indexed products may increase guaranteed minimum account values or use the additional option budget from increased asset yield to raise index caps or participation rates.

(d) In a rising interest rate environment:

(i) Critique the asset portfolio supporting this line of business.

(ii) Critique the lapse assumption.

Commentary on Question:
Candidates did well on part (ii); for part (i), most candidates did not cover all the potential issues and only received partial credit.

(i)
The duration of the asset portfolio is likely longer than the liability portfolio given the assets have WAL of 20 and the deferred annuities are outside the surrender charge period, with rising interest rates these long duration assets will have depressed market values, given that rising interest rates will likely also increase lapse rates the assets will likely have to be sold at a capital loss.

2. **Continued**

(ii)
Policyholder behavior is impacted by the economy and the company should add a dynamic lapse assumption instead of just having a deterministic lapse assumption

As rates rise policyholders will be incentivized to lapse and purchase fixed annuities with higher crediting rates

3.   **Learning Objectives:**

2.   The candidate will understand and be able to assess issues and concerns common to actuarial models and their development and management.

3.   The candidate will understand the principles of Asset-liability Management ("ALM"), and be able to describe and evaluate various techniques for addressing the mitigation of risk.

Learning Outcomes:

(2b)   Explain and apply the technique for the compression of model data using the "Cluster Analysis Spatial Approach".

(3a)   With respect to Asset-Liability Models:
- Describe and apply the fundamental elements of the theory and practice of ALM in an insurance company, including assessing the dangers of mismatched assets and liabilities.
- Describe and demonstrate how ALM can be used to identify and manage product and asset risks, including:
  o Major product risks for which ALM can be a useful tool for their management.
  o Using ALM as a means to manage interest rate risk, equity risk, and risks from optionality.
- Describe how common insurance contracts and variations generate embedded options in an insurer's balance sheet, and assess basic strategies for managing exposures created by such embedded options.
- Describe and apply the basic concepts of cash flow matching, immunization, duration/convexity matching, segmentation.
- Describe and apply Key Rate Durations (KRD) and their use in evaluating interest rate sensitivities of portfolios, including understanding the derivation of KDRs, the profiles of KDRs for selected major asset types, and assessing KRDs in a portfolio context.
- Describe and evaluate the Goldman Sachs' ALM/Strategic Asset Allocation approach for integrating ALM into an enterprise's risk and financial management framework.
- Describe and evaluate ALM modeling considerations in the context of modeling risk aggregation, dependency, correlation of risk drivers and diversification.

3. Continued

(3b) With respect to asset adequacy analysis and cash flow testing, describe and evaluate actuarial practice with respect to:
- Modeling and selecting assets and related assumptions (incl. modeling assets with contingent cash flow risks).
- Handling liability cash flow contingencies and risks.
- Setting up projection model parameters and assumptions.
- Describe how Interest Rate Forwards and Futures and Swaps can be used in ALM, and apply the mathematics in given situations.

Sources:
LAM-146-19: ALM Management of Financial Institutions Ch 16, Tilman, 2003

LAM-140-19: Asset Adequacy Analysis Practice Note, 2004 , questions: 3, 5, 10-16, 18-20, 27, 29-31, 39, 42-60, 65-68, 71-82, 85 & 89

LAM-131-19: Life Insurance Accounting, Asset/Liability Management Ch 22

Commentary on Question:
Commentary listed underneath question component.

Solution:

(a) The CFT results suggest that both a steep increase and a steep decrease of market interest rates could result in additional required reserves.

    (i) Explain how each scenario affects ABC from an asset and liability management perspective.

    (ii) Recommend two derivatives that ABC could use as a hedge for low interest rates.

        Explain how each works.

Commentary on Question:
Part (a) tests candidates' ability to assess the impact of interest rate risk, on both assets and liabilities from a standalone basis, as well as how they interact from an ALM perspective.

For part (i):
Steep increase:
Candidates in general did well in assessing liability impact, e.g., policyholder lapses would increase.

3. Continued

For assets – While most candidates were able to identify that bonds experience a decrease to market value under an interest rate increase scenario, many struggled to reach this same conclusion for mortgages.

Steep decrease:
Candidates in general did well in assessing liability impact.

For assets – Some candidates correctly interpret that an interest rate decrease increases the initial market value of assets, however the key objective is to identify the more material and longer-term risk, which is reinvestment risk, given the lack of ability for ABC to sustain asset returns to pay off future liability claims.

For part (ii):
Some candidates responded with the following combination of derivatives: (1) an interest rate swap and (2) an interest rate swaption, were not awarded full credit, given the lack of differentiation of these two derivatives. As the underlying derivative instrument in both cases is still an interest rate swap.

For the interest rate floor derivative, some candidates did not use the explicit term of an interest rate floor, while still correctly describing its functionality as a put option on an interest rate – this was deemed acceptable.

Part a (i)
<u>Steep increase of market interest rates</u>
Assets:
- Increase in market interest rates reduces current market value of both existing bonds and mortgages.

Liabilities:
- Liabilities remain stable, given ABC is not obligated to adjust current credited rates along with market rate movement, however this results in an increased likelihood of policyholders choosing to surrender their policies, in pursuit of higher credited rates provided by products offered by the market.

Asset & Liability interaction:
- As a result, ABC will be forced to liquidate assets at depressed values to fund surrender values of departing policyholders, resulting in financial loss.

3. Continued

Steep decrease of market interest rates

Assets:
- Asset portfolio experiences an initial increase in value, however emerging asset cashflows are forced to be reinvested at lower returns.

Liabilities:
- Policyholders become less likely to surrender, as they value their minimum guaranteed crediting rates, and extend liability cashflows further into the future.

Asset & Liability interaction:
- As a result, ABC is exposed to reinvestment risk.
- ABC faces spread compression and struggles to earn satisfactory returns from its assets to back liability commitments, reducing ABC's profits (or even negative spread leading to losses, when market interest rate drops below the minimum guaranteed crediting rate of 3%).

Part a (ii)

For the low interest rate scenario, ABC can hedge against the associated risk by purchasing (1) an interest rate floor or entering into (2) an interest rate swap.

1. Interest rate floor
   A put option on interest rate – this instrument would pay off when interest rate drops below a predetermined strike rate (e.g., the minimum guaranteed crediting rate of 3%), effectively eliminating reinvestment risk in the scenario where the market interest rate drops below strike rate.

2. Interest rate swap
   Enter into an interest rate swap where ABC would receive the fixed interest rate stream, while paying the floating interest rate stream. In a low interest rate scenario, this instrument will return a higher-than-market interest rate, eliminating reinvestment risk.

(b) You are validating the CFT model.

   (i) List three model validation methods commonly applied to a CFT model.

   (ii) Describe how each method can be applied.

Commentary on Question:
Part (b) tests candidates' knowledge of model validation techniques during asset adequacy testing.

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3. **Continued**

Part b (i)
Three commonly applied model validation methods:
1. Static validation
2. Dynamic validation
3. Attribution analysis

Part b (ii)
1. Static validation
   Applied to ensure the opening balance of certain modeled items matches the
   reported financials (e.g., policy count on the liabilities side; asset market /
   book value).

2. Dynamic validation
   Applied where the actuary compares projections/forecasted results coming
   from current model against recent actual results.

3. Attribution analysis
   Performed as a step-by-step analysis of the change from the prior year's
   modeled results to the current year's to confirm that the model appropriately
   reacts to changes in inforce, actuarial assumptions, and/or macroeconomic
   conditions.

(c) Insurance products are often sold with embedded options for both the
policyholder and the insurance company.

For the following products:

- Deferred annuity with a minimum guaranteed crediting rate
- Participating traditional whole life insurance that provides cash value
  and dividends
- Long-term care product with guaranteed premium

(i) Identify two embedded options offered to the policyholder that are shared
by more than one product.

(ii) Explain which product features are triggered for the embedded option(s) in
part (i).

3. Continued

      (iii)   Identify two embedded options available to the insurance company that are shared by more than one product.

      (iv)   Explain which product features are triggered for the embedded option(s) in part (iii).

Commentary on Question:
This part of the question tests candidates' ability to identify various options embedded in traditional life insurance products. As embedded options bring complex product risks to life companies', thorough understanding of product optionality is essential for managing product risks through ALM practices.

Successful candidates were able to identify correct options shared by correct products, with sufficient justifications provided. Partial marks were awarded to candidates who were able to reasonably explain how each option functions under each circumstance but failed to name the option correctly or identify the correct products. Few points were given to candidates who only listed combinations of options and products with minimal justifications.

Part (i) and (ii) were graded in a consolidated fashion for candidates who answered both parts in the same answer box; same applied to part (iii) and (iv).

For part (i) and (ii), most candidates were able to identify a call and a put option. A number of candidates got partial marks thinking that a call on the value of future payment exists for the participating whole life product. In fact, although whole life product has a fixed premium structure, the insurer has the right to alter dividends, in which case policyholders are exchanging fixed premiums with non-fixed total policy value. Hence, the call does not apply to participating whole life.

For part (iii) and (iv), many candidates were able to identify and explain the two options well. Some candidates failed to justify the call action for the callable bond is only triggered when policyholders fail to pay their premiums due.

Part c (i)
For the three products provided, two embedded options offered to the policyholders are:
1. Call on the value of future payments; shared by deferred annuity and long-term care products.
2. Put on the value of the policy; shared by deferred annuity and participating traditional whole life insurance products.

3.   **Continued**

Part c (ii)
The call on the value of future payments enables policyholders to purchase coverages at a pre-determined price.
- Deferred annuity policyholders have the right to deposit additional premiums into existing fixed-rate deferred annuities.
- Long-term care policyholders have the right to renew their existing policies at guaranteed premiums.

The put on the value of the policy enables policyholders to exit their in-force contracts for a guaranteed level of cash.
- A deferred annuity policyholder has the right to surrender the existing contract for a lump-sum value accumulated at a minimum guaranteed crediting rate.
- A participating whole life policyholder has the right to surrender the existing policy for a guaranteed cash surrender value.

Part c (iii)
For the three products provided, two embedded options offered to the insurance company are:
1. Callable bond; shared by long-term care and participating traditional whole life insurance products.
2. Swaption; shared by deferred annuity and participating traditional whole life insurance products.

Part c (iv)
A callable bond holder has the right to receive coupon cash flows before maturity or the face value of the bond if the bond issuer decides to call the bond.
A similar circumstance happens when an insurer has the right to receive premium cash flows when a policy is in-force or a lump-sum amount when the policyholder fails to pay the premium (or decides to lapse).
- The insurer has the right receive the reserve when a long-term care policyholder lapses.
- The insurer has the right receive the reserve, less any cash surrender values, when a participating whole life policyholder lapses.

A swaption gives the holder the right to exchange variable rate with fixed rate (or vice-versa).
An insurer can achieve similar outcome through strategically managing non-guaranteed elements of its product features.
- For deferred annuity, the insurer has the right to alter the crediting rate with respect to the desired return, given it remains above the minimum guaranteed level.

3. **Continued**

- For participating whole life, the insurer has the right to alter the dividends with respect to the overall return of its participating business.

4. **Learning Objectives:**

3. The candidate will understand the principles of Asset-liability Management ("ALM"), and be able to describe and evaluate various techniques for addressing the mitigation of risk.

5. The candidate will understand the role of the Investment Actuary and the Portfolio Management Process in the Life Insurance company context, as well as the common forms of Fixed income securities and their uses, and the methods and processes used for evaluating portfolio performance and asset allocation.

Learning Outcomes:

(3a) With respect to Asset-Liability Models:
- Describe and apply the fundamental elements of the theory and practice of ALM in an insurance company, including assessing the dangers of mismatched assets and liabilities.
- Describe and demonstrate how ALM can be used to identify and manage product and asset risks, including:
  o Major product risks for which ALM can be a useful tool for their management.
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- Setting up projection model parameters and assumptions.
- Describe how Interest Rate Forwards and Futures and Swaps can be used in ALM, and apply the mathematics in given situations.

4. **Continued**

    (5a)   Describe the portfolio management process in an insurance company, and the role of Investment Policy, the Investment Actuary, and external portfolio managers.

    (5b)   Describe and evaluate how a company's objectives, needs and constraints affect investment strategy and portfolio construction (including capital, funding objectives, risk appetite and risk return tradeoff, tax and accounting, accounting considerations, and constraints such as regulation, rating agency ratings and liquidity.

    (5f)   Describe and apply methods and processes for evaluating portfolio performance, including performance attribution, sources of earnings analysis on investment income, benchmarks, metrics, and risk adjusted performance appraisals (including total return vs reported earnings).

Sources:
Managing Investment Portfolios, Maginn, John L. and Tuttle, Donald L., 3rd Edition, 2007 - Ch. 5: Asset Allocation (sections 2-4)

LAM-140-19: Asset Adequacy Analysis Practice Note, 2004 , questions: 3, 5, 10-16, 18-20, 27, 29-31, 39, 42-60, 65-68, 71-82, 85 & 89

Commentary on Question:
This question is testing candidates' ability to evaluate available asset mixes based on expected utility and safety-first ratio, and understanding with asset modeling considerations and strategic asset allocation considerations.

Solution:
(a)
    (i)   Calculate NDV's expected utility (also referred to as "risk-adjusted return") for each asset mix.

    (ii)   Calculate the safety-first ratio from Roy's safety-first criterion for each asset mix.

    (iii)   Recommend an optimal asset mix. Justify your answer.

Commentary on Question:
For part (i), there is an alternative solution if candidate converts to percentages, Full credit is given for arriving at either solution.

4. **Continued**

For part (ii), full credit is given for correctly state and use the formulas for safety first ratio, identify the shortfall level based on the upcoming cash flow requirements stated in the problem, and arrive at correct calculation.

For part (iii), if candidates provided the alternative solution in (i), then asset mix W would show the highest expected utility and similar SF ratio as mix Y. Full credit is given to candidates who arrives at the alternative solution and recommends mix W with appropriate justification.

Most candidates were able to calculate utility and safety first ratio. Subsequently, most candidates were also able to identify the optimum asset allocation based on their calculations regardless if they calculated the utility/SF Ratio correctly.

(i)
$$U_m = E(R_m) - 0.005 * R_A * \sigma_m^2$$

where
U_m = the investor's expected utility for asset mix m
$E(R_m)$ = expected return for mix m
R_A = the investor's risk aversion
σ_m^2 = variance of return for mix m

Given that $R_A = 4$, then

$$U_m = E(R_m) - 0.005 * R_A * \sigma_m^2 = E(R_m) - 0.005 * 4 * \sigma_m^2 = E(R_m) - 0.02 * \sigma_m^2$$

NDV's expected utility, or risk-adjusted return for each asset mix:
$U_W = E(R_W) - 0.02 * \sigma_W^2 = 9 - 0.02 * 19^2 = 1.8$
$U_X = E(R_X) - 0.02 * \sigma_X^2 = 4.5 - 0.02 * 10^2 = 2.5$
$U_Y = E(R_Y) - 0.02 * \sigma_Y^2 = 7.9 - 0.02 * 14^2 = 4.0$
$U_Z = E(R_Z) - 0.02 * \sigma_Z^2 = 6.9 - 0.02 * 12^2 = 4.0$

(Alternative solution if candidate converts table to percentage)
NDV's expected utility, or risk-adjusted return for each asset mix:
$U_W = E(R_W) - 0.02 * \sigma_W^2 = 9\% - 0.02 * 19\%^2 = 8.9\%$
$U_X = E(R_X) - 0.02 * \sigma_X^2 = 4.5\% - 0.02 * 10\%^2 = 4.5\%$
$U_Y = E(R_Y) - 0.02 * \sigma_Y^2 = 7.9\% - 0.02 * 14\%^2 = 7.9\%$
$U_Z = E(R_Z) - 0.02 * \sigma_Z^2 = 6.9\% - 0.02 * 12\%^2 = 6.9\%$

4. **Continued**

(ii)
Safety-first ratio = $[E(R_P) - R_L] / \sigma_P$

Shortfall level required for safety-first ratio is R_L = \$15M / 320M= 4.69% based on stated requirement to fund upcoming liability cashflow

The safety-first ratio from Roy's safety-first criterion for each asset mix:
W: (9% – 4.69%) / 19% = 22.7%
X: (4.5% – 4.69%) / 10% = -1.9%
Y: (7.9% – 4.69%) / 14% = 22.9%
Z: (6.9% – 4.69%) / 12% = 18.4%

(iii)
I recommend asset mix Y. Asset mix Y would be the most optimal as it has the highest safety-first ratio. While asset mix W provides the highest expected return, asset mix Y's lower volatility increases expected utility and provides same utility as asset mix Z but with a higher safety-first ratio. Unlike asset mix X, still provides enough expected income to meet the excess liability cash flow in 12 months as indicated by the positive safety-first ratio.

(b)   Critique each of the following statements from the memorandum:

A.   *Only one asset segment exists which supports all interest sensitive liabilities. Each line of business backed by this segment is modeled with a pro-rata allocation of assets since we are not permitted to refine the allocation to optimize asset-liability duration matching.*

B.   *In accordance with ASOP 22 – Statements of Opinion based on Asset Adequacy Analysis by Actuaries for Life or Health Insurers, the moderately adverse scenario chosen is based on conditions (such as rising interest rates) and assumptions (such as higher mortality) which are deemed "moderately adverse", regardless of model results in aggregate.*

C.   *Most callable bonds held at the company include a "make-whole" premium and callable bonds make up a relatively insignificant portion of the portfolio. The call option is still modeled, as required, even though we are unable to capture the "make-whole" premium in our modeling.*

D.   *To ensure our model sufficiently captures prepayment of residential mortgage-backed securities, we validate that prepayment rates increase in falling interest rate scenarios and decrease in rising scenarios. We also validate that prepayment rates maintain at an elevated level as long as the projected interest rates continue to decrease in the falling scenario.*

4. Continued

> E. We calculate the modeled market value of our mortgage-backed securities (MBS) using discounted cash flows assuming interest rates remain level from each valuation point, as opposed to a stochastic method. The stochastic method caused extreme increases in run time, and we determined this would not significantly affect our analysis given the relatively small proportion of MBS in our portfolio.

Commentary on Question:
Full credits are given to candidates opining on the accuracy of each statement and defending their conclusions with information that aligns with source material.

Very few candidates gain full credits for this part of the question. Candidates specifically struggled with the concept of moderately adverse scenarios and modeling of callable bonds/make-whole premium/MBSs.

A. This statement is incorrect. While it is permitted and common to use a pro-rata slice of the entire asset segment to back a given product line within the segment, it is also permitted to allocate assets within a segment based on metrics such as duration, which can be the preferred method to avoid allocating assets that differ in characteristics such as duration from the liabilities they are modeled with.

B. This statement is correct. It is a reasonable interpretation to classify a scenario as "moderately adverse" based on conditions and assumptions chosen. Certain conditions may produce unfavorable results for one line of business and favorable for another, resulting in offsetting impact in aggregate. For example, rising interest rates may improve investment income for one block but adversely increase lapses for another, while higher mortality may be beneficial for annuities while adverse for life.

C. This statement is incorrect. Since make-whole premiums require the issuer to pay the holder an amount to compensate for any loss incurred when called, it is reasonable to instead model these as non-callable. Additionally, it is reasonable to exclude the impact of call features if the callable bond portfolio is insignificant in relation to the total portfolio, though other components should be considered such as the size of the potential gain or loss if the bonds are called.

4. **Continued**

    D. This statement is partially correct. It is true that prepayments would increase as interest rates fall, based on the difference of the coupon rates of the MBS held and the current declining market rate (and the opposite is true for rising interest rates). However, it is appropriate to reflect a "burn-out" factor which reflects how mortgage holders who watch interest rates closely tend to prepay when interest rates are first lowered, while those remaining may not react as much to subsequent interest rate changes.

    E. This statement is correct. It is appropriate to consider alternative methods to a stochastic approach for modeling complex MBS market values, such as using the method described which performs a discounted cash flow calculation assuming rates remain level. This is especially true since the actuary believes the valuation method is not critical to the analysis given the small size of the portfolio.

5. **Learning Objectives:**
4.   The candidate will understand the basic design and function of Economic
        Scenario Generators and Equity Linked Insurance Models.

Learning Outcomes:
(4b)   With respect to Equity-Linked models:
- Describe and apply methods for modeling long-term stock returns and certain
  guarantee liabilities (GMMB, GMDB, GMAB).
- Describe and evaluate the Actuarial and Hedging risk metrics for GMAB and
  GMDB models.
- Describe and apply methods for modeling Guaranteed annuity options and
  Guaranteed Minimum Income Benefits (GMIB), and EIA guarantees.

Sources:
Investment Guarantees Ch 1, 2, 6, 8, Hardy, 2003

LAM -139-19: Simulation of a Guaranteed Minimum Annuity Benefit, Freedman, 2019;
Excel Model - Stochastic Simulation of a GMAB Option (Accompanies Simulation of a
GMAB)

Commentary on Question:
*All candidates made a good attempt at providing a response, but in many instances the
main points were not well captured. Many candidates interpreted the question to be a
true or false question, simply saying whether they agreed or disagreed, with minimal
supporting rationale. There was also a tendency to focus on the literal wording within the
question statements, instead of using the opportunity to demonstrate deeper
understanding on the topic. This was particularly the case for Statement B, where most
candidates interpreted this to be a quick question to see if they were able to spot that
"returns are normal and prices are lognormal", and did not go on to discuss the overall
merits of using a lognormal model. For Statement C, almost all candidates were able to
name and describe tail risk, but very few went on to differentiate the need for looking at
tail risk when evaluating mortality versus investment risk.*

Solution:
(a)   Critique each of the following statements that relate to the modeling of the
        GMMB:

        *A.   When calculating the cost of the GMMB, the model will use implied
                volatility derived from current market statistics instead of historical
                volatility.*

        *B.   When modeling equity returns to project fund values, the model assumes
                that continuous time returns follow a geometric Brownian motion which
                implies returns are lognormally distributed.*

5. **Continued**

> *C.* *It is better to use a deterministic approach instead of a stochastic approach when modeling mortality and investment guarantees. A deterministic approach will sufficiently capture the tail risk of the GMMB.*

A. Implied volatility is appropriate for use when evaluating publicly traded securities. As VA's with guarantees are not traded securities, implied volatility may not be suitable. Furthermore, the effective maturities of VA guarantees are far longer than the maturities of market instruments, as such the volatility observed in current market conditions may not be a good match.

A second consideration is that current market valuations are risk neutral and based on adjusted probability. To analyze future cash flows of an equity-linked contract, it is important to consider true unadjusted probability measures.

A third consideration is that implied volatility reflects current market conditions, and may be extremely volatile depending on the given economic environment at the time. Results would therefore vary greatly depending on the timing of the evaluation.

B. Geometric Brownian motion implies that returns are normally distributed and prices are lognormally distributed. The benefits of using Lognormal models are that it is simple and tractable, and provides a reasonable approximation over the long term, which may be suitable in the case of a long dated guarantee such as a GMMB. Drawbacks of a Lognormal model are that it does not capture extreme price movements, does not capture fat tails, does not allow for autocorrelation in the data, and fails to capture volatility bunching.

C. Mortality risk is diversifiable. Given a large enough portfolio of policyholders, the mortality experience will be very close to expected. On a GMMB product, mortality risk is also not the most material risk. Therefore, it is sufficient to evaluate mortality risk in a GMMB using deterministic modelling.

By contrast, investment guarantees are associated with extreme equity movements and tail risk of the stock price distribution, and is not a diversifiable risk. In order to quantify the tail risk exposure and allow scenario analysis, for example via calculations of CTE, stochastic approach is required for evaluating investment guarantees.

(b) Calculate the expected value of the GMMB claims assuming a 1% fund return for all years.

5. **Continued**

> **Commentary on Question**:
> *Most candidates were able to correctly apply the fund returns and fee assumptions, as well as apply the mortality decrements. There were some careless errors, such as applying fees to the beginning of year fund value, or forgetting to include fund return at all. Besides these, there were two other themes. The first is that many candidates were inconsistent in their consideration of the role decrements played in the overall picture, i.e. applied decrements to the AV, but not to the GV, then compared the two numbers. The second issue was that many candidates calculated "cashflows" in each year, when the product is actually a GMMB that matures at the end of ten years, and no interim GV-AV should have been looked at or accumulated into a final total.*

Please see Excel solution for detailed calculations.

The main idea of the solution is that the expected claim at the end of ten years is the comparison of the GV and the AV, for the expected population that remains. The GV would have evolved over time via expected deaths, and the AV would have evolved over time via expected deaths as well as return accumulation and fee deductions.

(c)   Determine whether the rider fee will be sufficient to cover the value of the GMMB using Black-Scholes option pricing.

> **Commentary on Question**:
> *Most candidates were able to recall the formula to calculate d1 and d2, as well as the put option formula. The strike price was missed by most candidates, and very few reflected the concept of decrement. Some candidates were unfamiliar with Excel functionality for calculating normal distribution values, and were unable to complete the calculation without being provided normal distribution tables.*

Please see Excel solution for detailed calculations.

The main idea of the solution is that the strike price takes into account both the M&E fee and the rider fee, with decrements. The put option price is then calculated using this strike price. An annuity factor should also be incorporated to allow for payment over a ten year period. Finally, the put price is compared to the rider fee to conclude that the rider fee is insufficient to cover the exposure.

6.   **Learning Objectives:**

5.   The candidate will understand the role of the Investment Actuary and the Portfolio Management Process in the Life Insurance company context, as well as the common forms of Fixed income securities and their uses, and the methods and processes used for evaluating portfolio performance and asset allocation.

Learning Outcomes:

(5a)   Describe the portfolio management process in an insurance company, and the role of Investment Policy, the Investment Actuary, and external portfolio managers.

(5d)   Describe and assess Fixed Asset Portfolio management methods, and immunization (including derivatives) and cash matching strategies, including:
- Considerations such as managing funds against a bond market index, the classification of possible strategies, the impact of risk factors and tracking risk, and the use of indexing and active strategies.
- Considerations such as managing funds against liabilities, the use of dedication strategies and immunization strategies, the assessment of risk minimization for immunized portfolios, and the use of cash flow matching and combo strategies.
- The use of derivative enabled strategies, and the use of futures, swaps, and options.

(5g)   Describe the principles of Liquidity Risk Management in an insurance company portfolio management context.

(5i)   Describe the attributes of US Treasuries, Agency Debt Securities, Municipal bonds, Corporate bonds, Private Money Market securities, Floating Rate Agreements, Agency Mortgage Backed securities, Agency Collateralized Mortgage securities, Interest Rate Swaps and Swaptions, Credit Derivatives and High Yield Bonds, and the markets they are traded in.

Sources:

Managing Investment Portfolios, Maginn, John L. and Tuttle, Donald L., 3rd Edition, 2007 - Ch. 6: Fixed-Income Portfolio Management (sections 1-5)

LAM-158-23: Managing Liquidity Risk, Industry Practices and Recommendations for CROs, CRO Forum, 2019

Handbook of Fixed Income Securities, Fabozzi, Frank J., 9th Edition, 2021 - Ch. 68: Credit Derivatives (pp. 1657-1671)

LAM-154-23: Ch. 7 (sections 7.2-7.5 & 7A) of Derivatives Markets, McDonald, 3rd Edition

LAM-153-23: Managing your Advisor: A Guide to Getting the Most Out of the Portfolio Management Process

6. Continued

Commentary on Question:
Commentary listed underneath question component.

Solution:
(a)   Calculate the Macaulay Duration of the bond.

Commentary on Question:
Candidates generally did well on this part of the question. Common errors included not halving the coupon rate, discounting at the annual yield instead of the semi-annual yield, and weighting with annual coupons instead of semi-annual coupons (ie. t=1, 2, 3... instead of t=0.5, 1, 1.5,...) and partial credit was given accordingly.

Semi-annual coupons of 250,000 * 4.5% * 0.5 = 5,625
Face at maturity = 250,000
t1 = 0.5, 1, 1.5,…, 10 (weightings)
t2 = 1,2,3,…, 20 (discount periods)
Discount rate = 3% * 0.5 = 1.5% per period
Bond price = sum[(coupons +face) * discount rate] = 282,191
Numerator = sum[t1* (coupons+face) * discount rate] = 2,339,319

Macauley duration = numerator / bond price = 8.29

Detailed calculations below:

t2	Semi-annual Coupons	Discount factor	Bond price	t1	Numerator
1	5,625	0.98522	5,542	0.5	2,771
2	5,625	0.97066	5,460	1	5,460
3	5,625	0.95632	5,379	1.5	8,069
4	5,625	0.94218	5,300	2	10,600
5	5,625	0.92826	5,221	2.5	13,054
6	5,625	0.91454	5,144	3	15,433
7	5,625	0.90103	5,068	3.5	17,739
8	5,625	0.88771	4,993	4	19,974
9	5,625	0.87459	4,920	4.5	22,138
10	5,625	0.86167	4,847	5	24,234
11	5,625	0.84893	4,775	5.5	26,264
12	5,625	0.83639	4,705	6	28,228
13	5,625	0.82403	4,635	6.5	30,128
14	5,625	0.81185	4,567	7	31,967
15	5,625	0.79985	4,499	7.5	33,744
16	5,625	0.78803	4,433	8	35,461
17	5,625	0.77639	4,367	8.5	37,121
18	5,625	0.76491	4,303	9	38,724
19	5,625	0.75361	4,239	9.5	40,271
20	255,625	0.74247	189,794	10	1,897,940

6. Continued

(b)   Calculate the rate of return that can be expected from this strategy.

Commentary on Question:
Candidates did well on this part of the question. Partial credit was awarded to candidates who identified the correct formula but substituted the incorrect variables.

Return = rate of return + funds borrowed/original investment * (rate of return – interest expense)
= 3% + 10/2 * (3% - 2.8%)
= 4%

(c)   Determine the par value of the new bond needed to keep the duration of the portfolio constant.

Commentary on Question:
Candidates did poorly on this part of the question. The majority of candidates incorrectly calculated dollar duration based on market value instead of price. Partial credit was awarded to those who correctly calculated dollar durations.

Dollar duration(old) = price(old) * duration(old) / 100 = 90 * 5 / 100 = 4.5
Dollar duration(new) = price(new) * duration(new) /100 = 115 * 7 / 100 = 8.05

#units = MV(old) / price(old) = 350,000 / 90 = 3,889

MV(new) = dollar duration (old) / dollar duration (new) * 100 * #units
= 4.5 / 8.05 * 100 * 3,889
=217,391

Par value = MV(new) / price(new) * 100 = 217,391 / 115 * 100 = 189,036

(d)

   (i)   Explain why insurance companies have different liquidity concerns than banks.

   (ii)   Describe two possible liquidity risk metrics your company can use.

Commentary on Question:
Candidates received full credit for identifying at least four of the points below for part (i). Candidates generally did well on part (i) of this question. Most candidates correctly identified the long-term nature of insurance liabilities and upfront premiums leading to insurers being liquidity rich and the contrast with banks facing runs. Fewer candidates mentioned insurance liabilities being less liquid and that liability side risks still exist for insurers.

6. Continued

Candidates generally did very well on part (ii) of this question. Some candidates provided risk measures such as VaR and CTE and did not receive any credit.

(i)
- In the insurance business model, the payments/premiums are collected upfront and services/claims/benefits provided in the future. The cycle makes insurers liquidity rich.
- Insurers, unlike banks, generally have liabilities with a longer maturity than their assets, which makes them less vulnerable to customer runs. In addition, insurers' liabilities are in general less liquid than bank deposits, as the possibilities for savings withdrawals are restricted in most insurance contracts and are also more costly for customers (owing to tax and surrender penalties).
- Liability-side liquidity risks still exist for insurers. For example, life insurers, in particular, face the risk of simultaneous withdrawals or policy surrenders by policyholders.
- Insurance companies are much less interconnected than banks and by pooling a large number of risks and retaining the bulk of the risks underwritten on their balance sheet, potential liquidity issues are likely to be idiosyncratic.
- Liquidity is a key factor in insurers' investment strategies but it is less of a risk for the sector players than for banks which rely primarily on the wholesale funding market and engage in maturity transformation.

(ii)   Metrics include liquidity coverage ratio, a calculation covering an excess or deficit liquidity amount, survival period, asset (only) liquidity, or liability (only) liquidity.

(e)   Critique the following statements:

A.   *For CDS, the buyer pays a regular floating coupon to the seller, the amount of which depends on the market value of the credit risk at that time. These payments continue to the scheduled termination date.*

B.   *A positive upfront payment is required to enter into a CDS.*

C.   *There are two possible ways to hedge credit risk: purchasing corporate bonds or purchasing a CDS. The lower transactions costs of a CDS make it a better strategy.*

D.   *A Standard North American CDS Contract covers all credit events.*

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6. **Continued**

Commentary on Question:
Most candidates correctly identified that the coupon paid should be fixed but less candidates also correctly identified that the payments would cease upon a credit event.

A. The first statement is not true. First, the credit protection buyer pays a regular fixed coupon to protection seller. The price of the credit risk is reflected in the upfront payment, not the coupons. The second sentence is also not true. A key feature of the premium leg in CDS is that the payments terminate following a credit event. This feature of a CDS means that once a credit event has occurred, the premium leg terminates and the contract can be closed out.

The majority of candidates performed well on this part of the question.

B. This is partly true. A payment is required to enter into a contract but it can be positive or negative depending on whether the contract is being bought or sold

Most candidates correctly identified that shorting a corporate bond would not hedge the credit risk. Fewer candidates commented on CDS having higher transaction costs or incorrectly agreed that CDS have lower transaction costs.

C. Both statements are false. Shorting a corporate bond would hedge the credit risk, not purchasing. The reason a CDS is a better strategy to hedge the risk is not due to transaction costs as depending on level of customization, costs can be high. But buying a CDS is much easier than shorting a bond.

Many candidates noted that only hard credit events were covered or soft events were not covered and received partial credit. Full credit was only awarded to those who correctly identified the two credit events covered.

D. This is not true. There are many covered credit events but the Standard North American Contract only covers bankruptcy and failure to pay.

(f) Recommend two strategies utilizing CDS for your company.

Commentary on Question:
Most candidates correctly identified selling CDS as a strategy. Very few candidates identified buying a CDS index as the other.

CDS market allows for speculation on the credit in the market. Given the credit risk is perceived to be lower than rated, a CDS should be sold (selling protection). Your boss believes that a credit event is less likely to happen than the price available in the market and therefore the premiums collected on this should be worth more than the perceived risk.

6. **Continued**

Another strategy is to buy a CDS index. In the CDS index market, the "buyer" is usually an investor who is selling protection on the underlying portfolio of CDS