QFI Quantitative Finance Exam

# Exam QFIQF

**Date:** Thursday, April 25, 2024

## INSTRUCTIONS TO CANDIDATES

**General Instructions**

1. This examination has 11 questions numbered 1 through 11 with a total of 70 points.

   The points for each question are indicated at the beginning of the question.

2. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

3. Each question part or subpart should be answered either in the Excel document, or in the provided Yellow Answer Booklet. Graders will only look at the work in the Yellow Answer Booklet or Excel document.

4. In the Excel document, answers should be entered in the box marked ANSWER. The box will expand as lines of text are added. There is no need to use special characters or subscripts (though they may be used). For example, $\beta_1$ can be typed as beta_1 (and ^ used to indicate a superscript).

5. Prior to uploading your Excel files, each file should be saved and renamed with your five-digit candidate number in the filename.

6. The Excel file that contain your answers must be uploaded before the five-minute upload period expires.

**Written-Answer Instructions**

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

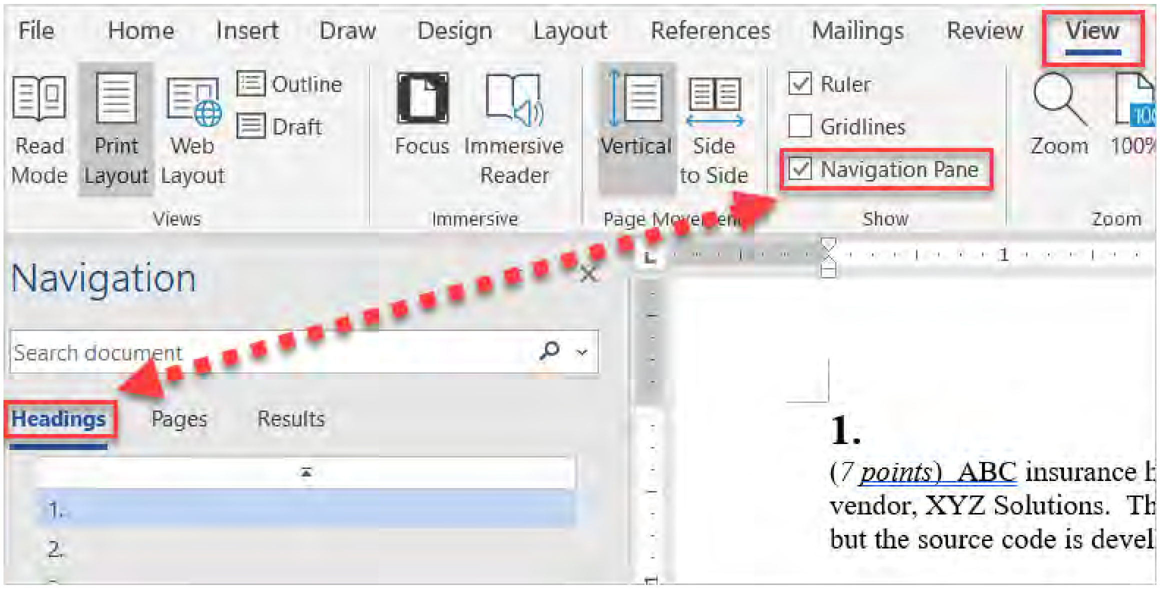
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.

5. When you finish, hand in all your written-answer sheets to the Prometric center staff. Be sure to hand in all your answer sheets because they cannot be accepted later.

**Navigation Instructions**

Open the Navigation Pane to jump to questions.

Press Ctrl+F, or click View > Navigation Pane:



*The responses for all parts of this question are required on the paper provided to you.*

# 1.

(*6 points*) Let $S(0) = 100$ be the price of a security at time 0. After 6 months, the price of the security can be either 120 or 60. Let $r$ be the nominal semi-annual risk-free rate of return.

(a) (*1.5 points*) Determine the range of $r$ such that there are no arbitrage possibilities.

For the remainder of the question, set $r = 0.06$.

(b) (*1 point*) Calculate and interpret the state prices.

(c) (*1.5 points*) Calculate the no-arbitrage price of a European call option with strike price of 100 that expires in 6 months.

(d) (*1 point*) Describe two general situations in which arbitrage opportunities can arise.

Consider the following derivative of the security: the derivative pays 22 in 6 months when the price of the security is 120; the derivative pays 10 in 6 months when the price of the security is 60.

(e) (*1 point*) Construct a replicating portfolio and use it to price the derivative.

*The responses for all parts of this question are required on the paper provided to you.*

# 2.

(*7 points*) Let $(\Omega, \mathcal{F}, \mathbb{Q})$ be a probability space and let $W$ be a standard Brownian motion with respect to the filtration $\{\mathcal{F}_t\}_{t\geq 0}$. Suppose that the risk-free rate is *r* and that $S(t)$ is a risky asset such that:

$$dS(t) = rS(t)dt + \sigma S(t)dW(t), \qquad 0 \leq r, S(0) < \infty$$

where $\sigma > 0$ is a constant.

(a) (*0.5 points*) Derive the closed-form expression for $S(t)$ using Ito's Lemma

You are investigating the discounted payoff process $Y_n(t) = e^{-rt}\big(S(t)\big)^n$ for positive integer $n$ and $0 < t < T$.

(b) (*3 points*)

(i) Derive the stochastic dynamics of $Y_n(t)$ in the form

$$dY_n(t) = \mu\big(t, S(t)\big)dt + \sigma\big(t, S(t)\big)dW(t)$$

(ii) Show that $E_t^{\mathbb{Q}}\big(Y_1(T)\big) = Y_1(t)$.

(iii) Prove that $Y_n(t)$ is a sub-martingale for $n \geq 2$ and $t < \infty$.

(c) (*1 point*)

(i) Determine, using Girsanov theorem, an equivalent martingale measure $\mathbb{Q}^A$ such that $Y_n(t)$ is a martingale with respect to $\mathbb{Q}^A$.

(ii) Verify your result by deriving the dynamics of $Y_n(t)$ under $\mathbb{Q}^A$.

Now assume $\sigma = 1, r = 0$.

(d) (*0.5 points*) Determine, using part (c), the Radon-Nikodym process $Z_t = \left.\frac{d\mathbb{Q}^A}{d\mathbb{Q}}\right|_t$

You are interested in pricing a derivative whose payoff is given by the expression:

$$V(T) = e^{-\frac{49}{8}T - \frac{7}{2}W(T)}\big(S(T)\big)^8$$

(e) (*2 points*) Determine a closed-form expression for today's price of $V(T)$.

*The responses for all parts of this question are required on the paper provided to you.*

# 3.

(*6 points*) You are given the sample space $\Omega = [0,1]$ and the collection of sets $\mathcal{F}_t, 0 \leq t \leq 1$, defined by

$$\mathcal{F}_t = \{[0,s], [0,s), (s,1], [s,1]: 0 \leq s \leq t\} \cup \{\emptyset\}.$$

(a) (*1.5 points*)

(i) Write down the 3 conditions satisfied by a sigma-algebra.

(ii) Explain how a filtration is defined in terms of an indexed collection of sigma-algebras.

(iii) Determine whether $\mathcal{F}_t, 0 \leq t \leq 1$, is a filtration on $\Omega$.

For a standard Wiener process $W_t,\ t > 0$, consider the following process

$$X_t = \int_0^t W_s{}^2 dW_s.$$

(b) (*1 point*) Show that $X_t = \frac{1}{3} W_t{}^3 - \int_0^t W_s \, ds$.

(c) (*1.5 points*) Compute the first two (raw) moments of $\frac{1}{3} W_t{}^3$.

(d) (*2 points*) Show that $Y_t = \int_0^t W_s \, ds$ does not have independent increments.

## 4.

(*6 points*) You are an ALM analyst for a public pension fund, and the fund has just completed an ALM Study. During the study, the main focus was on risk quantification, with a particular emphasis on funding status. In efforts to mitigate interest rate risk, the investment division has begun using derivatives for hedging purposes.

(a) (*1 point*) Recommend whether it would be more appropriate to use a real-world model or a risk-neutral model for the following purposes.

(i) Measuring a 5% worst-case interest rate level for funding status

(ii) Valuation of derivatives

The current funding ratio of the pension fund stands at 102%. Additionally, 37% of the assets are sensitive to the changes in interest rates and the remaining 63% are in cash. 100% of the liabilities are sensitive to the changes in interest rates. Both interest rate sensitive assets and liabilities have the same positive duration. In order to determine the interest rate sensitivity of the current funding ratio, you use the Vasicek model with the following parameter values:

| Real-World | | Risk-Neutral | |
|---|---|---|---|
| Parameter | Value | Parameter | Value |
| $\gamma$ | 0.3261 | $\gamma^*$ | 0.4653 |
| $r_0$ | 3.02% | $r_0$ | 3.02% |
| $\bar{r}$ | 5.09% | $\bar{r}^*$ | 6.34% |
| $\sigma$ | 2.21% | $\sigma$ | 2.21% |

The interest rate is approximated as:

$$r_{i+1}^{j} = r_i^j + \gamma(\bar{r} - r_i^j)\delta + \sigma\sqrt{\delta}\epsilon_{i+1}^{j},$$
$$where\ time\ step\ \delta = 1, i = time, j = simulation\ path$$
$$and\ \epsilon_i^j\ are\ i.i.d\ and\ standard\ normal$$

(b) (*1 point*) Determine the value of $r_{i+1}$ that would produce the 5th percentile of the distribution of the funding ratio.

**4.** **Continued**

The investment division is currently hedging with bond options. You have been requested to compare the Vasicek model with the Cox-Ingersoll-Ross (CIR) model given by:

$$dr_t = \gamma^*(\bar{r}^* - r_t)dt + \sqrt{\alpha r_t}dX_t \textit{ where X is standard Brownian motion.}$$

The parameters of the CIR model are given below.

| Parameter | Value |
|---|---|
| $\gamma^*$ | 0.25 |
| $\bar{r}^*$ | 6.59% |
| $\alpha$ | 0.0134 |

(c) (*4 points*)

(i) Verify that the calibrated CIR model does not allow negative rates.

(ii) Plot a chart displaying the zero-coupon bond yield for both the Vasicek and CIR models for each maturity from 1-year to 10-years with an increment of 1 year.

*The response for this part is to be provided in the Excel spreadsheet.*

The investment group is considering bond options that have a maturity of 1 year and an underlying bond with a maturity of 10 years.

(iii) Calculate the values of the call option and put option with strike price 64.06 under the Vasicek interest rate model.

*The response for this part is to be provided in the Excel spreadsheet.*

*The responses for all parts of this question are required on the paper provided to you.*

# 5.

(*5 points*) The Vasicek model of interest rates is

$$dr_t = \gamma(\bar{r} - r_t)dt + \sigma dX_t$$

where $\gamma$ and $\sigma$ are positive constants, and $X$ is a standard Brownian motion.

Let $F$ be a filtration generated by $r$.

(a) (*2 points*) Show that

(i) $E(r_t|F_s) = r_s e^{-\gamma(t-s)} + \bar{r}(1 - e^{-\gamma(t-s)})$

(ii) $Var(r_t|F_s) = \frac{\sigma^2}{2\gamma}\left[1 - e^{-2\gamma(t-s)}\right]$

Suppose that under the Vasicek model:

The short rate $r$ is 4% and its real-world process is

$$dr_t = 0.1(0.05 - r_t)dt + 0.01dX_t$$

The risk-neutral process is

$$dr_t = 0.1(0.09 - r_t)dt + 0.01dX_t$$

(b) (*0.5 points*) Determine the market price of interest risk.

The price of a zero-coupon bond with \$1 principal at time t with maturity date $T$ is given by

$$Z(r, t; T) = e^{A(t;T) - B(t;T)r}, \text{where } B(t;T) = \frac{1 - e^{-\gamma(T-t)}}{\gamma}.$$

Let $Z$ be the price process with $T = 10$.

(c) (*1 point*) Compute the drift and the diffusion of $\frac{dZ}{Z}$ for the risk-neutral process.

(d) (*1.5 points*) Compute the drift and the diffusion of $\frac{dZ}{Z}$ for the real-world process.

*The responses for all parts of this question are required on the paper provided to you.*

# 6.

(*9 points*) You have one-month daily treasury bill yields (annualized) over 500 consecutive trading days in the daily_data table and 20 treasury bond yields with various maturities in the bond_data table. There are 252 trading days per year. You would like to fit the Vasicek model for the data sets.

Partially completed R codes and outputs are attached to the end of this question.

(a) (*1.5 points*) Describe the assumptions made in the chosen real-world parameter estimation method.

You are using the real-world Vasicek model, $dr = \gamma(\bar{r} - r)dt + \sigma\, dX$ where $X$ is standard Brownian motion.

(b) (*2 points*) Estimate the parameters of your model.

(c) (*2 points*) Describe the procedure employed in risk-neutral model calibration.

You are using the risk-neutral Vasicek model, $dr = \gamma(\bar{r} - r)dt + \sigma\, dY$ where $Y$ is standard Brownian motion.

(d) (*2 points*) Estimate the parameters of your new model.

(e) (*1.5 points*) Determine whether the fitted models are adequate.

**6.** **Continued**

*Partially completed R codes and outputs.*

```
#daily_data one-month daily treasury bill yields (annualized)
# over 500 trading days
# display header and first three rows
head(daily_data,3)

##         Yield
## 1 0.03000000
## 2 0.02856146
## 3 0.02899520

# display header and last two rows
tail(daily_data,2)

##           Yield
## 499 0.04868607
## 500 0.04960966

N = nrow(daily_data)
y = daily_data$Yield[2:N]
x = daily_data$Yield[1:(N-1)]
model=lm(y~x)
summary(model)

##
## Call:
## lm(formula = y ~ x)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0042738 -0.0008491 -0.0000248  0.0008307  0.0040188
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.0001815  0.0001620   1.121    0.263
## x           0.9964045  0.0038202 260.822   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0013 on 497 degrees of freedom
## Multiple R-squared:  0.9927,Adjusted R-squared:  0.9927
## F-statistic: 6.803e+04 on 1 and 497 DF,  p-value: < 2.2e-16

sigma(model)

## [1] 0.001300116
```

**6.** **Continued**

*Partially completed R codes and outputs.*

```
# we use sigma value that was estimated in real world situation,
#which is mle.sigma
# display header and first three rows of the data frame
head(bond_data,3)

##   bond.maturities bond.yields
## 1             0.5  0.02452059
## 2             1.0  0.03131300
## 3             1.5  0.03462041

# display header and last two rows of the data frame
tail(bond_data,2)

##    bond.maturities bond.yields
## 19             9.5  0.06028585
## 20            10.0  0.06093331

Vasicek.fit =
  nls(bond.yields~Vasicek.yield(r0=0.02,t=0,T=bond.maturities,
                                gamma,rbar,sigma=mle.sigma),
      start=list(gamma=0.2,rbar=0.02),
      data=bond_data,algorithm = "port",
      lower=list(gamma=0.1,rbar=0.01),
      upper =list(gamma=3,rbar=1),
      nls.control(maxiter = 100, tol = 1e-3, minFactor = 1/1024,
                  printEval = FALSE, warnOnly = FALSE,
                  scaleOffset = 0, nDcentral = FALSE))
sum_mod= summary(Vasicek.fit)
sum_mod$coefficients[,2:4]

##         Std. Error  t value     Pr(>|t|)
## gamma 0.031458254 15.49071 7.513001e-12
## rbar  0.001384681 50.90106 6.590768e-21
```

*The responses for all parts of this question are required on the paper provided to you.*

# 7.

(*5 points*) You are tasked with building a hedging model for QFI's Equity Index Linked Annuity with the following features:

- S&P 500 Price Return Index (assume no dividends)
- Segment duration: 1 year (the period that we measure the index return)
- Strategy option: point-to-point
- Current cap rate: 10%
- Minimum cap rate: 0%
- Risk-free rate: 2%

You are given the following market prices for the index and associated options and times $t = 0$ and $t = 0.5$, where $t$ is in years:

**Table 1 – Index and Options Prices at Time 0 and 0.5**

| | t=0 | t=0.5 |
|---|---|---|
| S&P 500 Index Price | 4,200 | 4,500 |
| 1-year S&P 500 Call Price (Strike 4,200) | 374 | 452 |
| 1-year S&P 500 Call Price (Strike 4,620) | 214 | 229 |
| 1-year S&P 500 Put Price (Strike 4,200) | 291 | 110 |
| 1-year S&P 500 Put Price (Strike 4,620) | 523 | 283 |

**Table 2 – Excerpt from Standard Normal Distribution Table**
**For a standard normal random variable, $Z$, the table displays $Pr(Z \leq z)$**

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|---|
| **0.0** | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 |
| **0.1** | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 |
| **0.2** | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 |
| **0.3** | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 |
| **0.4** | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 |
| **0.5** | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 |

(a) (*1 point*) Determine the cost to replicate the strategy today with a static hedge.

**7.** **Continued**

Your hedging department believes future S&P 500 volatility over the next year will be 25%, higher than volatilities implied by market prices over the segment duration. You decide to hedge the index strategy dynamically using the department's assumption of 25% for future volatility by continuously trading shares of the underlying index.

The P&L of the hedge strategy constructed this way will follow a process given by

$$dP\&L = dV_I + A\,dS + B\,dt$$

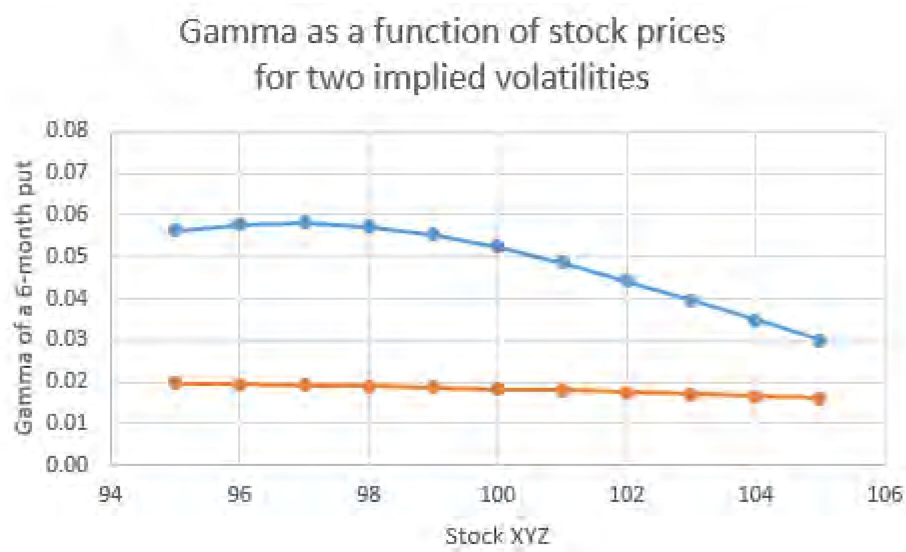
where $dV_I$ is the incremental change in the market value of the strategy under implied volatilities and $dS$ is the incremental change in the underlying index.

(b) (*4 points*) Calculate $A$ and $B$ at time t = 0.5.

**8.**

(*7 points*) Consider a put option on Stock XYZ, a non-dividend-paying stock. Also assume that:

- The spot price of Stock XYZ is 100 and the option strike is 100.
- The continuously compounded risk-free interest rate is 5%.




The chart above shows the option Gamma as a function over a range of stock prices under two different implied volatilities. For part (a), also assume that:

- The time-to-maturity is six months.
- One curve is the option Gamma under 10% implied volatility while the other one is the option Gamma under 30% implied volatility.

(a) (*1.5 points*) Determine which curve corresponds to which implied volatility.

**8.** **Continued**


The chart above shows the option Theta as a function over a range of stock prices under two different implied volatilities. For part (b) and part (c), assume that:

- The spot price of Stock XYZ is 100 and the put option strike is 100.
- The time-to-maturity is one year.
- One curve is the option Theta with a 20% implied volatility while the other curve is the option Theta with a 30% implied volatility.

(b) (*1.5 points*) Determine which curve corresponds to which implied volatility.

(c) (*2 points*) Plot the two Theta curves.

*The response for this part is to be provided in the Excel spreadsheet.*

For part (d), you want to evaluate the sensitivity of Rho to the interest rate levels because the interest rate could potentially go much higher or much lower. Assume the following:

- The time-to-maturity is one year.
- Consider the Rho under two different interest rate levels at 3% and 10%, respectively.

(d) (*2 points*) Plot the two Rho curves as a function of stock prices.

*The response for this part is to be provided in the Excel spreadsheet.*

## 9.

(*5 points*) You want to use Monte Carlo simulation to price a 6-month barrier option.

You are given the market implied volatility term structure constructed using the Black-Scholes model from a series of European options traded in the markets, as shown in the following table:

| Tenor (Month) | Market volatility |
|---|---|
| 1 | 27.0% |
| 2 | 28.0% |
| 3 | 26.0% |
| 4 | 24.0% |
| 5 | 22.5% |
| 6 | 21.4% |

For part (a) only, assume no volatility smile.

(a) (*1.5 points*) Calculate the Monte Carlo step volatility based on the market implied volatilities.

*The response for this part is to be provided in the Excel spreadsheet.*

To simulate a volatility smile in the Monte Carlo simulation, you now employ a stochastic volatility model which explicitly includes a stochastic process modelling the fluctuations of volatility.

(b) (*2 points*) Determine whether such a stochastic volatility model creates a volatility smile in the Monte Carlo simulation and how.

(c) (*1.5 points*) List two major disadvantages of using a stochastic volatility model for pricing options.

*The responses for all parts of this question are required on the paper provided to you.*

# 10.

(*5 points*) XYZ is offering a Principal Protected Note (PPN) on a reference equity index, $S$, with the following payout structure at the end of its $T$-year term:

$$Payout = \begin{cases} P, & S_T < K \\ (1+\alpha)P, & S_T \geq K \end{cases}$$

where $P$ is the initial principal paid by the accountholder to the insurer, $\alpha > 0$ is the step-up rate, and $K > 0$ is the strike price that triggers the step-up payment on the PPN.

Assume the Black-Scholes framework: $r$ is risk-free rate, $q$ is dividend yield, $\sigma$ is volatility.

(a) (*1 point*) Show that the value of the PPN is:

$$Pe^{-rT}[1 + \alpha\Phi(d_2)]$$

where $\Phi$ is the normal cumulative distribution function and
$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - q - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}.$$

Define the "fair step-up rate" as the step-up rate that would make the value of the PPN equal to the initial principal paid, $P$.

(b) (*1 point*)

(i) Show that the fair step-up rate is:

$$\alpha = \frac{e^{rT} - 1}{\Phi(d_2)}$$

(ii) Show that in the limit as $K \longrightarrow 0$, the fair step-up rate approaches the theoretical minimum value of $e^{rT} - 1$.

**10. Continued**

Consider the following reference equity index for the PPN:

| | |
|---|---|
| Underlying index value at $t = 0$ ($S_0$): | \$1000 |
| Dividend yield ($q$): | 1% |
| Implied volatility ($\sigma$): | 15% |

The continuously compounded risk-free rate ($r$) is 4%.

Consider a PPN with a 2-year term and a \$1100 strike price.

(c) (*1 point*)

(i) Calculate the fair step-up rate on this PPN.

(ii) Explain whether the fair step-up rate would increase, decrease, or not change if an annual expense charge, $m > 0$, is added to this PPN.

(d) (*1 point*) Critique the following statements made by your analyst.

(i) "Given a fixed step-up rate and strike price, as the risk-free rate increases, the value of the PPN will decrease."

(ii) "Given a fixed step-up rate and strike price, as the implied volatility increases, the value of the PPN will decrease."

Company XYZ wants to offer a guaranteed minimum death benefit (GMDB) rider to this PPN, whereby the initial principal, $P$, is paid in the event of the death of the accountholder at time $t \leq T$, otherwise the accountholder receives the PPN payout.

You decide to model the mortality risk assuming a constant force of mortality:

$$ {}_tp_x = \exp(-\mu_x t) $$

Where, ${}_tp_x$ is the probability that an accountholder age $x$ survives to age $x + t$ , and $\mu_x$ is the force of mortality for an accountholder of age $x$.

Assume that the mortality risk and market risk are independent.

(e) (*1 point*) Show that the fair step-up rate for the PNN-GMDB structure is:

$$ \frac{\frac{r}{\mu_x + r}\left[e^{(\mu_x + r)T} - 1\right]}{\Phi(d_2)} $$

## 11.

(*9 points*) Your company currently offers a structured product based variable annuities (spVA) product with a buffered and capped payout. Your company would like to offer a new spVA product that allows a higher participation rate on the upside index returns. You are assigned to value and hedge a new product.

Denote B = buffer %, C = cap rate %, $P^u$ = participation rate on the upside index returns. Suppose the maturity payoff of the new spVA is

$$Payoff_t^{spVA} = \begin{cases} S_0(1+\mathrm{C}) & , \quad S_t \geq S_0\left(1+\frac{C}{P^u}\right) \\ P^u(S_t - S_0) + S_0 & , \quad S_0 \leq S_t < S_0\left(1+\frac{C}{P^u}\right) \\ S_0 & , \quad S_0(1-B) \leq S_t < S_0 \\ S_t + S_0 B & , \quad S_t \leq S_0(1-B) \end{cases}$$

where $0 < P^u < C$, and $0 < B < 1$.

(a) (*3 points*) Show that a portfolio of a risk-free bond (with maturity value of $S_0$) and the following options provides the maturity payoff of this new spVA.

(i) Long $P^u$ units of call option with strike price $S_0$ (i.e., at-the-money ATM)

(ii) Short $P^u$ units of call option with strike price $S_0(1+\frac{C}{P^u})$ (i.e., out-of-money OTM)

(iii) Short a put option with the strike price $S_0(1-B)$ (i.e., OTM)

Given the following market data and parameters for the spVA,

| | |
|---|---|
| Underlying Asset – Current Price ($S_0$) | 100 |
| Dividend Yield (q) | 0% |
| Implied Volatility (σ) | 15% |
| Term (T) | 3 |
| Risk-Free Rate (r) | 4% |
| Buffer Rate% (B) | 10% |
| Cap Rate% (C) | X% |
| Participation rate ($P^u$ ) | Y% |

The risk budget is the amount that is available to invest in the portfolio of options that replicate the payoff of the spVA.

**11. Continued**

(b) (*3.5 points*)

(i) (*0.5 points*) Calculate the risk budget of the spVA for a notional amount of $100.

*The response for this part is to be provided in the Excel spreadsheet.*

(ii) (*2 points*) Calculate the price/cost of the portfolio of options specified in part a) for the following combinations of participation rates and cap rates (table below)

| **Price of Option Portfolio** | **Participation Rate** | | |
|---|---|---|---|
| **Cap rate** | 100% | 110% | 125% |
| 17% | | | |
| 26% | | | |
| 35% | | | |
| 44% | | | |
| 53% | | | |
| 62% | | | |

*The response for this part is to be provided in the Excel spreadsheet.*

(iii) (*0.5 points*) Determine the cap rate % (C) of the spVA with participation rate ($P^u$) = 125% that leads to cost of the portfolio of options in part (b)(ii) that is closest to the risk budget calculated in part (b)(i).

*The response for this part is to be provided in the Excel spreadsheet.*

(iv) (*0.5 points*) Explain whether the cap rate increases, decreases, or remains unchanged while keeping the same cost of portfolio of options and increasing the participation rate.

*The response for this part is to be provided in the Excel spreadsheet.*

## 11. Continued

You are considering adding a floor (F) to the payoff of the spVA in part (a).

As an example, suppose $S_0$ = 100, B = 10% and F = 85% , then the spVA maturity payoff can be illustrated as follows:

| $S_t$ | spVA Maturity Payoff |
|---|---|
| 90 | 100 |
| 80 | 90 |
| 70 | 85 |

(c) (*2.5 points*) Derive the portfolio of options and a bond (with maturity value of $S_0$) that replicate the payoff of this spVA.

****END OF EXAMINATION****