

# GIRR Model Solutions

## Spring 2023

### 1. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

### Learning Outcomes:

- (2b) Describe the different types of exposures used for conducting actuarial work.
- (2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.
- (2d) Adjust historical earned premiums to current rate levels.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 12 and 13.

### Commentary on Question:

*This question tests the candidate's understanding of the earnings of policies with different policy terms, as well as adjusting earned premiums to current rate level for ratemaking purposes.*

### Solution:

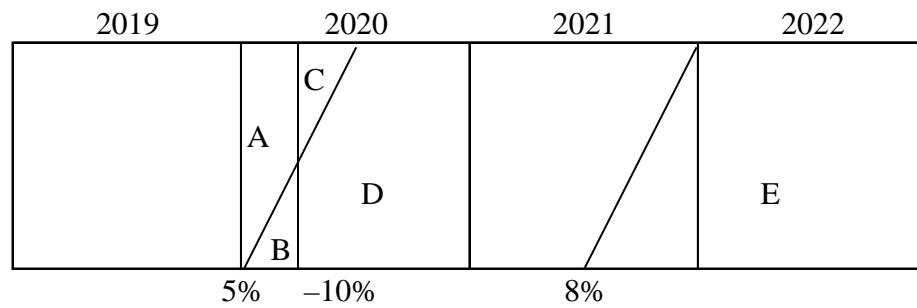
- (a) State the two key assumptions of the parallelogram method.
  - Policies are written evenly over the experience period
  - Exposures are earned evenly over the policy term

# 1. Continued

- (b) Calculate the calendar year 2020 on-level premium to be used for a ratemaking analysis.

6-month policies:

- in force premium on Jan 1, 2020: 3,000,000
- annualized premium: 6,000,000



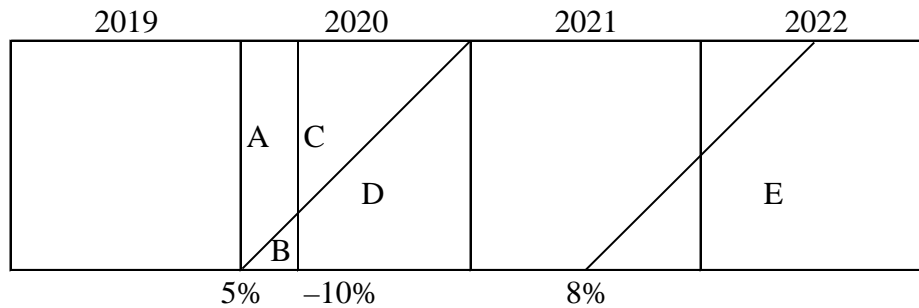
Section	Rate Level Index	Percent Premium Earned in CY2020 at Rate Level
A	1.00000	18.75%
B	1.05000	6.25%
C	0.90000	6.25%
D	0.94500	68.75%
E	1.02060	
Average rate level:		0.9591
On-level factor:		1.0642
On-level earned premium:		6,384,985

e.g.,  $1.0642 = 1.0206 / 0.9591$

# 1. Continued

12-month policies:

- in force premium on Jan 1, 2020: 9,000,000



Section	Rate Level Index	Percent Premium Earned in CY2020 at Rate Level
A	1.00000	21.88%
B	1.05000	3.13%
C	0.90000	28.13%
D	0.94500	46.88%
E	1.02060	

Average rate level: 0.9477  
 On-level factor: 1.0770  
 On-level earned premium: 9,692,755

Total CY2020 earned premium at current rate level for ratemaking:  
 $= 6,384,985 + 9,692,755 = 16,077,740$

- (c) Provide two examples of general insurance policies where exposures are not usually earned evenly throughout the policy term.

Any two of the following are acceptable:

- Policies covering seasonal risks
- Warranty
- Financial guarantee
- Property catastrophe and aggregate stop-loss reinsurance
- Retrospectively-rated policies
- Policies with reinstatement premium

## 2. Learning Objectives:

1. The candidate will understand the key considerations for and key concepts underlying general insurance actuarial work.
2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

### Learning Outcomes:

- (1g) Identify different types of data used for actuarial work.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 4 and 11.

### Commentary on Question:

*This question tests the candidate's understanding of the different types of data used for actuarial work, as well as adjusting development triangles of claims and counts from changes in transactions.*

### Solution:

- (a) Provide one advantage and one disadvantage to aggregating claims data by policy year.

Advantage: There is a precise matching of the premiums and the claims arising from those premiums.

Disadvantage: There is a time lag associated with this type of aggregation.

- (b) Provide one disadvantage to aggregating claims data by report year.

Disadvantage: It does not capture claims that have been incurred but not yet reported (pure IBNR).

## 2. Continued

(c) Construct new data triangles with corrections for this claim file.

What's in the current data:

AY		12	24	36	48
2019	Cumulative paid claims	0	1,500	2,500	60,000
2019	Case estimate	0	900,000	900,000	400,000
2019	Reported Claims	0	901,500	902,500	460,000
2019	Reported Counts	0	1	1	1

What should have been in the data:

AY		12	24	36	48
2019	Cumulative paid claims	0	1,500	2,500	60,000
2019	Case estimate	90,000	90,000	90,000	40,000
2019	Reported Claims	90,000	91,500	92,500	100,000
2019	Reported Counts	1	1	1	1

Accident	Revised Reported Claims					
	12	24	36	48	60	72
2017	2,147,785	3,025,674	3,620,901	4,136,684	4,362,359	4,382,594
2018	2,219,814	3,071,925	3,876,926	4,331,668	4,596,920	
2019	2,432,602	3,344,013	4,112,135	4,714,225		
2020	2,591,328	3,398,123	4,339,405			
2021	2,582,962	3,768,518				
2022	2,735,738					

Accident	Revised Reported Counts					
	12	24	36	48	60	72
2017	729	895	998	1,082	1,119	1,122
2018	727	900	1,019	1,089	1,130	
2019	744	911	1,022	1,102		
2020	765	902	1,042			
2021	763	939				
2022	767					

There is no change to paid claims and no change to closed counts.

## 2. Continued

- (d) Calculate calendar year 2022 reported claims, based on corrected data.

Change in reported for accident years 2016 and prior: 7,200

Sum of latest diagonal of adjusted reported claims triangle: 24,537,400

Sum of previous diagonal of adjusted reported claims triangle: 18,787,247

Calendar year 2022 reported claims:

$$= 24,537,400 - 18,787,247 + 7,200 = 5,757,353$$

### 3. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

#### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

#### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 15.

#### Commentary on Question:

*This question tests the candidate's understanding of development method for estimating ultimate claims.*

#### Solution:

- (a) State the two key assumptions of the development method.
- Historical experience is predictive of future experience.
  - Activity observed to date is relevant for projecting future activity.
- (b) Describe an advantage of using paid claims instead of reported claims when applying the development method.
- Paid development patterns are not influenced by changes in philosophy or processes regarding case estimates.
- (c) Describe an advantage of using reported claims instead of paid claims when applying the development method.
- Reported claims are often used instead of paid claims as there tends to be less volatility and more credibility associated with the selection of development factors for reported claims.
- (d) Describe one way you might account for the presence of large claims in the data when applying the development method.
- Remove the large claims from the data triangle so that the development pattern is not distorted by the presence of large claims.

### 3. Continued

- (e) Describe two ways you might account for limited credibility of the data when applying the development method.
- When the credibility of the data is more limited, use longer-term averages, which frequently demonstrate greater stability than shorter-term averages.
  - Could look to industry data for development patterns.
- (f) Calculate projected ultimate claims for all accident years using the paid development method.

AY	Paid Claims Age-to-age factors						
	12-24	24-36	36-48	48-60	60-72	72-84	84-ult
2016	2.339	1.481	1.306	1.218	1.133	1.047	
2017	1.931	1.613	1.332	1.186	1.131		
2018	2.767	1.510	1.317	1.213			
2019	2.774	1.517	1.353				
2020	2.269	1.598					
2021	2.210						
2022							
Simple 3	2.418	1.542	1.334	1.206	1.132	1.047	
Simple All	2.382	1.544	1.327	1.206	1.132	1.047	
Vol Wtd 3	2.385	1.542	1.334	1.205	1.132	1.047	
Vol Wtd 5	2.352	1.544	1.328	1.205	1.132	1.047	
Vol Wtd All	2.350	1.544	1.328	1.205	1.132	1.047	
Selected:	2.350	1.544	1.328	1.205	1.132	1.047	1.015
Age-to-Ult.	6.983	2.972	1.925	1.450	1.203	1.062	1.015

Recommend volume-weighted average of all years to address the variability.

#### *Algebraic Method for Paid Claims*

##### *Tail Factor:*

AY	Ultimate Reported Claims	Paid Claims Developed to 84 months	Implied Tail Factor
2016	2,513,084	2,487,315	1.010
2017	2,665,698	2,625,300	1.015
2018	2,809,772	2,760,204	1.018
Average			1.015



### 3. Continued

Paid Claims	CDF	Ultimate Paid Claims
2,487,315	1.015	2,523,552
2,507,208	1.062	2,663,547
2,328,436	1.203	2,800,417
2,091,115	1.450	3,031,578
1,650,625	1.925	3,177,499
1,140,537	2.972	3,389,654
408,139	6.983	2,850,185
12,613,375		20,436,433

#### 4. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

#### Learning Outcomes:

- (6d) Quantify different types of expenses required for ratemaking including expense trending procedures.

#### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 30.

#### Commentary on Question:

*This question tests the candidate's understanding of different types of expenses required for ratemaking.*

#### Solution:

- (a) Recommend the total variable expense ratio to use in ratemaking. Justify your recommendation.

Calendar Year	(1) Earned Exposures	(2) Direct Written Premium	(3) Direct Earned Premiums	(4) Total Commission Expenses and Premium Taxes	(5) General Expenses
2019	8,700	7,447,430	7,377,050	670,269	243,420
2020	9,150	7,895,360	7,846,640	710,582	253,065
2021	9,340	8,112,390	8,090,270	730,115	260,640
2022	9,240	8,097,340	8,083,570	728,761	268,436
2023 Budget	9,120	8,050,000	8,048,900	724,500	285,000

Calendar Year	(6) = (5) × 75% General Expenses	(7) = (6) / (3) As a % of Earned Premiums	(8) = (4) / (2) Commission and Premium Tax Expense Ratio
2019	182,565	2.47%	9.00%
2020	189,799	2.42%	9.00%
2021	195,480	2.42%	9.00%
2022	201,327	2.49%	9.00%
2023 Budget	213,750	2.66%	9.00%
Recommended		2.66%	9.00%

#### 4. Continued

Total variable expense ratio = 2.66% + 9.00% = 11.66%

Justification: There is a significant increase expected from budget, so give more consideration to the budget.

- (b) Recommend the fixed expense per exposure to use in ratemaking. Justify your recommendation.

Calendar Year	Fixed General Expense (000)	Fixed General Expense Per Exposure
2019	60,855	6.99
2020	63,266	6.91
2021	65,160	6.98
2022	67,109	7.26
2023 Budget	71,250	7.81

$$(10) = (5) \times 25\% \quad (11) = (10) / (1)$$

Selection: 7.81

Justification: There is a significant increase expected from budget, so give more consideration to the budget.

Provision for new system = 2,500,000 / 9,120 / 4 = 68.53 (amortized over 4 years)

Total: 7.81 + 68.53 = 76.34

## 5. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.
6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.
- (6j) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 27 and 32.

### Commentary on Question:

*This question tests the candidate's understanding of trending premiums and indicated rate changes using claim ratios.*

### Solution:

- (a) Recommend the annual premium trend due to the shift in policy limits to use for ratemaking. Justify your recommendation.

### Commentary on Question:

*The year-to-year change in **average** increased limit factor (ILF) needs to be analyzed for the trend due to shift in policy limits.*

## 5. Continued

Experience Period	Weighted Average ILF	Annual Trend Due to Shift in ILF
2015	1.00018	
2016	1.00270	0.25%
2017	1.00603	0.33%
2018	1.00877	0.27%
2019	1.01202	0.32%
2020	1.01500	0.29%
2021	1.01769	0.27%
2022	1.01924	0.15%
Average:		0.27%
Average excluding high & low:		0.28%

Recommended trend: 0.28%

Justification: average excluding high and low removes the outliers, especially 2022.

- (b) Calculate the indicated rate level change for this line of business using a claims ratio approach. Justify any selection(s).

Average earned premium dates in 2022:	Jul. 1, 2022	
Effective date of new rates:	Sep. 1, 2023	# of months
Average earned premium dates in future rating period:		<u>trending period</u>
for 12-month policies	Sep. 1, 2024	26
for 6-month policies	Jun. 1, 2024	23
Average:		25.25

Annual premium trend =  $(1 + 0.28\%)(1 + -0.1\%) - 1 = 0.180\%$

Annual pure premium trend =  $(1 + 6\%)(1 + -1.2\%) - 1 = 4.728\%$

Accident Year	(1) Earned Premiums	(2) Trending Period (months)	(3) Premium On-Level Factors	(4) = (1.0018) <sup>(2)/12</sup> Premium Trend Factors	(5) = (1)(3)(4) Earned Premiums Trended at Current Rates
2018	15,804,847	73.25	1.064	1.01102	17,001,688
2019	15,333,428	61.25	1.106	1.00921	17,114,913
2020	15,526,085	49.25	1.104	1.00740	17,267,582
2021	16,625,910	37.25	1.049	1.00559	17,538,061
2022	17,102,494	25.25	1.026	1.00379	17,613,581

Accident Year	(6) Ultimate Claims	(7) = (1.04728) <sup>[(2)/12]</sup> Claim Trend Factors	(8) = (6)(7) Trended Ultimate Claims	(9) = (8) / (6) Claim Ratio
2018	8,703,669	1.32577	11,539,025	67.87%
2019	9,184,011	1.26591	11,626,161	67.93%
2020	9,602,493	1.20876	11,607,137	67.22%
2021	10,401,614	1.15419	12,005,466	68.45%
2022	11,309,041	1.10209	12,463,536	70.76%
Average:				68.45%
Average latest 3 years:				68.81%

Selected claim ratio = 68.81%

Justification for selected claim ratio: Increasing in most recent years, so give more weight to more recent 3 years.

Indicated rate change:  $[0.6881(1 + 0.07) + 0.05]/(1 - 0.23 - 0.04) = 7.71\%$

- (c) Describe one reason why an indicated rate change using a pure premium approach may not result in the same result as part (b).

The premium on-level factors are an approximation used to restate historical earned premiums as if they were at the current rate level for the forecast period.

- (d) Calculate the profit and contingencies to premium ratio implied by a 3% rate increase using your colleague's indicated rate change.

Claim ratio implied by the 6% rate indication:

$(\text{Claim ratio} + 0.05)/(1 - 0.23 - 0.04) - 1 = 0.06 \rightarrow \text{claim ratio} = 72.38\%$

Profit margin implied by a 3% rate change:

$(0.7238 + 0.05)/(1 - 0.23 - Q) - 1 = 0.03$

$\rightarrow Q = 1.87\%$

- (e) State two actions the company can take that could help achieve the target profit, given the 3% rate increase.

**Commentary on Question:**

*Other actions are possible.*

- decrease expenses
- decrease claims (e.g., changing mix of business, better risk selection)

## 6. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 19.

### Commentary on Question:

*This question tests the candidate's understanding of the Cape Cod method for estimating ultimate claims.*

### Solution:

- (a) Describe why the Cape Cod method may not be appropriate for coverages such as property or automobile collision.

The development factor may be less than 1, which will result in used-up exposures that are greater than the original exposures.

- (b) Calculate projected ultimate claims using the Cape Cod method applied to paid claims.

Accident Year	(1) On-Level Earned Premiums	(2) Paid Cumulative Development Factors	(3) = 1/(2) Expected % Paid	(4) = (1)(3) Used-Up On-Level Earned Premiums	(5) Actual Paid Claims
2017	14,304,922	1.048	95.42%	13,649,735	8,573,426
2018	14,662,414	1.097	91.16%	13,365,920	8,699,818
2019	14,826,526	1.326	75.41%	11,181,392	7,732,920
2020	15,064,165	1.847	54.14%	8,156,018	5,857,706
2021	15,448,284	3.146	31.79%	4,910,453	3,561,183
2022	15,630,481	9.473	10.56%	1,650,003	1,395,852
Total				52,913,520	35,820,905

## 6. Continued

Accident Year	(6) Claim Trend Factors	(7) Tort Reform Factors	(8) = (5)(6)(7) Adjusted Claims	(9) = (A)(1)/[(6)(7)] Expected Claims	(10) = (5) + (9)[1 - (3)] Ultimate Claims
2017	1.2763	0.800	8,753,684	9,438,074	9,005,704
2018	1.2155	0.800	8,459,747	10,157,636	9,597,986
2019	1.1576	0.800	7,161,457	10,784,894	10,384,410
2020	1.1025	0.950	6,135,215	9,688,961	10,300,884
2021	1.0500	1.000	3,739,242	9,911,179	10,321,955
2022	1.0000	1.000	1,395,852	10,529,475	10,813,802
Total			35,645,197	60,510,219	60,424,742

Adjusted Expected Claim Ratio =  $35,645,197 / 52,913,520 = 67.37\%$  (A)

e.g., 2020 tort reform factor:  $25\%(0.8) + 75\%(1.0) = 0.95$

- (c) Describe two situations that could result in such a difference in Cape Cod projections.

**Commentary on Question:**

*Other situations are possible.*

Any two of the following are acceptable:

- Decrease in the adequacy of case reserves in the latest diagonal
- Change in the settlement rates resulting in higher paid claims than in past
- Unusual payment of large claims where the case is low
- Change in environment (internal or external) that is reflected in case estimates but not yet seen in paid claims that lag the reporting of claims



## 7. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

### Learning Outcomes:

- (3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.
- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 21.

### Commentary on Question:

*This question tests the candidate's understanding of the appropriateness of various methods of estimating ultimate claims under changing conditions.*

### Solution:

Recommend a different estimation method to use with each of the following four independent books of business. Justify your recommendations.

#### Commentary on Question:

*The Berquist-Sherman adjustments are adjustments to data and not a method. Candidates needed to also recommend the estimation method if they recommended Berquist-Sherman adjustments.*

- (i) A long-tailed book where the case estimates were strengthened in 2018.

The paid development is responsive to the case change, but it will be too leveraged for 2022 so avoid this method. Recommend adjusting for the case change using Berquist-Sherman approach and then use the reported development method to estimate ultimate claims.

- (ii) A book that has unstable development patterns and experience that has been improving.

Recommend using the frequency-severity method to separately analyze claim counts and average severity. This might give better insights as to what patterns are changing and where the deterioration is coming from (i.e., frequency or severity or both).

## 7. Continued

- (iii) A quickly growing book of business that has only been writing business for three years.

***Commentary on Question:***

*The frequency-severity method is also acceptable, but not if already selected in part (ii), as the question asks for a different method for each part.*

Any of expected, Bornhuetter Ferguson or Cape Cod methods for a new line of business and also the significant growth.

- (iv) A medium-tailed book of business where the policy limit was increased from 2 million to 3 million, effective January 1, 2019.

The expected method (if the expected method was not chosen in part (iii)) because both the pattern and experience will change at mature (in the tail) periods which will take many years to figure out.

## 8. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.
6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

### Learning Outcomes:

- (5a) Identify and describe the influences of portfolio changes on claim frequency and severity.
- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6f) Explain the requirements for loadings for catastrophes and large claims in ratemaking.
- (6g) Calculate loadings for catastrophes and large claims.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 26, 27, and 31.

### Commentary on Question:

*This question tests the candidate's understanding of the loading for catastrophes in ratemaking.*

### Solution:

- (a) Explain why two trend adjustments must be made to the modeled expected earthquake claims to calculate the catastrophe loading for ratemaking.

Past adjustment: modeled catastrophe claims must be trended from February 1, 2022 to July 1, 2022 for exposure trend to reflect in-force exposures as of July 1, 2022

Future adjustment: modeled catastrophe claims must be trended from July 1, 2022 to mid-point of future rating period for severity trend to reflect the cost level in the future rating period

- (b) Calculate the catastrophe loading to be used for ratemaking, as a claim ratio.

Midpoint of future rating period:	October 1, 2024
Exposure trend period (months): February 1, 2022 to July 1, 2022	5
Exposure trend = $1.01^{(5/12)}$ =	1.00415
Severity trend period (months): July 1, 2022 to October 1, 2024	39
Severity trend = $1.06^{(39/12)}$ =	1.20849
Trended modeled catastrophe claims = $450,000 \times 1.00415 \times 1.20849$ =	546,081.10
Catastrophe loading = $546,081.10 / 15,450,000$ =	3.53%

## 8. Continued

- (c) Describe an additional step or approach that would increase your confidence in the estimate of expected earthquake claims.

Running alternative catastrophe models would increase confidence in the estimate of expected earthquake claims.

- (d) Describe how you would consider the effect of a demand surge in the calculation of the catastrophe loading for ratemaking.

Demand surge can result in a trend rate that is higher post-catastrophe than pre-catastrophe. Therefore, could recognize a demand surge by selecting a higher post event claim severity trend rate.

## 9. Learning Objectives:

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

### Learning Outcomes:

- (4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.
- (4c) Evaluate and justify selections of unpaid unallocated loss adjustment expenses based on ratio and count-based methods.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 23.

### Commentary on Question:

*This question tests the candidate's understanding of estimating unpaid ULAE.*

### Solution:

- (a) Explain why the classical paid-to-paid method may not be appropriate for estimating unpaid ULAE in this case.

The significant inflation in this case will cause a relatively higher increase in the calendar paid ULAE than the calendar paid claims, since inflation can more quickly affect the underlying costs of ULAE, including the salaries of claims adjusters, rent, and utilities. This will overstate the paid ULAE to paid claims ratio, thus overestimating the unpaid ULAE.

## 9. Continued

- (b) Calculate the ULAE ratio for each year using the Mango and Allen smoothing adjustment based on paid and reported claims data.

### Commentary on Question:

*It is recommended to solve this part of the question by displaying the details of the how the expected reported claims for CY 2021 & 2022 are determined to ensure that no report years are excluded.*

First need to calculate the CY2021 & CY2022 expected reported claims:

	Maturity Age in months				
	12	24	36	48	60
Reported CDF	2.306	1.479	1.137	1.023	1.000
% Cumulative Reported	43.4%	67.6%	88.0%	97.8%	100.0%
% Incremental Reported	43.4%	24.2%	20.3%	9.8%	2.2%

e.g., 43.4% = 1/2.306

Report Year	Selected Ultimate Claims	Expected Reported Claims in Calendar Years	
		2021	2022
2017	8,297,960	186,562	
2018	9,230,643	904,692	207,532
2019	10,390,684	2,113,205	1,018,387
2020	11,357,111	2,753,886	2,309,752
2021	12,811,927	5,555,909	3,106,651
2022	14,531,428		6,301,573
Total		11,514,254	12,943,895

e.g., for Report Year 2019:

$$2,133,205 = 10,390,684 \times 20.3\%$$

$$1,018,387 = 10,390,684 \times 9.8\%$$

Calendar Year	Paid ULAE	Expected Claims		Ratio of Paid ULAE to Average of Paid and Reported Claims
		Paid	Reported	
2019	725,000	8,950,624	9,323,021	7.93%
2020	825,176	9,921,833	10,304,355	8.16%
2021	935,423	11,058,159	11,514,254	8.29%
2022	1,062,610	12,393,344	12,943,895	8.39%
Total	3,548,209	42,323,960	44,085,525	8.21%

e.g., 8.29% = 935,423 / ((11,058,159 + 11,514,254) / 2)

## 9. Continued

- (c) Recommend a ULAE ratio to use for this line of business. Justify your recommendation.

Recommended ratio: 8.28%

Justification: using the average of the latest 3 years to reflect the increasing trend and remove the 2019 low outlier value.

- (d) Calculate unpaid ULAE as of December 31, 2022 using the recommended ratio from part (c).

$$\begin{aligned}\text{Calculated unpaid ULAE} &= 8.28\% \times 4,965,557 \times (1 - 0.40) + 8.28\% \times 13,974,912 \\ &= 1,403,552\end{aligned}$$

## 10. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.
4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

### Learning Outcomes:

- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).
- (4f) Calculate claim liabilities.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 22 and 24.

### Commentary on Question:

*This question tests the candidate's understanding of evaluating and justifying selections of ultimate values based on multiple methods, as well as calculating claim liabilities.*

### Solution:

- (a) Evaluate the appropriateness of each of the following methods for estimating ultimate claims:

#### Commentary on Question:

*Evaluation of the appropriateness of a method requires an explanation why it is or is not appropriate.*

- (i) Bornhuetter Ferguson method based on reported claims for AY 2018
  - The Bornhuetter Ferguson method is an acceptable method for this line of business.
  - It is likely not influenced too much by change in case adequacy in AY2018.
  - Therefore, this method is likely appropriate for AY 2018.
- (ii) frequency-severity method based on reported claims for AY 2021
  - This method is good for more recent periods (i.e., 2021).
  - However, since this method is based on reported claims, it could be affected by the change in case adequacy, since there is no indication that the severities have been adjusted for the change in case estimates.
  - Therefore, this method is likely not appropriate for AY 2021.



## 10. Continued

- (iii) paid development method for AY 2022
- This method is not affected by change in case adequacy.
  - Leveraged effect for 2022 =  $8,195,915 / 1,312,636 = 6.24$ . This is too high, therefore there is too much uncertainty with this method.
  - Therefore, this method is likely not appropriate for AY 2022.

- (b) Recommend the ultimate claims for AY 2020. Justify your recommendation.

**Commentary on Question:**

*Other recommendations based on paid claims methods are acceptable.*

- Reported methods are affected by the change in case adequacy, so these methods should be avoided.
- The data analysis does not support a change in claim settlement patterns, so paid methods are reasonable.
- Leveraged effect for AY 2020 =  $7,951,950 / 4,015,114 = 1.98$ . This is not too high suggesting that methods based on the paid development method are reasonable.
- Therefore, recommend the average of all paid methods, as all paid methods are determined to be reasonable:

$$= (7,951,950 + 8,011,083 + 7,939,852 + 7,945,960) / 4 = 7,962,211$$

- (c) Calculate the case estimate and IBNR for AY 2020, based on your recommendation in part (b).

- Unpaid claims =  $7,962,211 - 4,015,114 = 3,947,097$
- Case estimate =  $5,866,764 - 4,015,114 = 1,851,650$
- IBNR =  $3,947,097 - 1,851,650 = 2,095,447$

*Also, IBNR =  $7,962,211 - 5,866,764 = 2,095,447$*

## 11. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 20.

### Commentary on Question:

*This question tests the candidate's understanding of a Berquist-Sherman adjustment for a change in claims settlement.*

### Solution:

- (a) Identify two possible reasons for a delay in claims processing.

Any two of the following are acceptable:

- a recent change in the claims processing system
- an increase in volume that creates a backlog of processing claims
- a change in claims personnel

- (b) Calculate the disposal ratio triangle for this line of business.

Accident Year	Disposal Ratios (closed counts / ultimate counts)					
	12	24	36	48	60	72
2017	0.291	0.532	0.735	0.886	0.991	0.996
2018	0.314	0.574	0.799	0.961	0.991	
2019	0.341	0.617	0.824	0.922		
2020	0.334	0.609	0.784			
2021	0.350	0.581				
2022	0.325					

e.g., for AY2020 at 12 months development:  $0.334 = 459 / 1,373$

## 11. Continued

- (c) Interpret the results from part (b).

Accident Year	Change in Disposal Ratios				
	12	24	36	48	60
2017-2018	7.7%	7.7%	8.8%	8.4%	0.0%
2018-2019	8.9%	7.5%	3.1%	-4.0%	
2019-2020	-2.1%	-1.3%	-4.8%		
2020-2021	4.8%	-4.5%			
2021-2022	-7.3%				

The ratios down the column should show noticeable decrease if there is a slowing in settlement patterns. There appears to be a noticeable decrease in the most recent diagonal.

- (d) Calculate the adjusted paid claims triangle.

Selected disposal ratios (most recent diagonal):

12	24	36	48	60	72
0.325	0.581	0.784	0.922	0.991	0.996

Accident Year	Adjusted Closed Counts						Ultimate Counts
	12	24	36	48	60	72	
2017	445	797	1,075	1,265	1,359	1,365	1,371
2018	432	773	1,043	1,227	1,318		1,330
2019	427	764	1,032	1,213			1,315
2020	446	798	1,077				1,373
2021	462	826					1,421
2022	459						1,413

e.g., for AY2020 at 12 months:  $446 = 0.325 / 1,373$

Accident Year	Average Claim Cost					
	12	24	36	48	60	72
2017	3,448	4,443	4,997	5,426	5,714	6,125
2018	3,620	4,665	5,247	5,697	6,000	
2019	3,801	4,898	5,510	5,982		
2020	3,991	5,143	5,785			
2021	4,190	5,400				
2022	4,400					

e.g., AY2020 at 12 months:  $3,991 = 4,400 \times 1.05^{-2}$

## 11. Continued

Accident Year	Adjusted Paid Claims = (Adjusted Closed Counts)(Average Claim Cost)					
	12	24	36	48	60	72
2017	1,535,374	3,540,462	5,374,253	6,861,836	7,763,600	8,360,625
2018	1,563,931	3,606,313	5,474,211	6,989,463	7,908,000	
2019	1,623,608	3,743,923	5,683,096	7,256,166		
2020	1,779,980	4,104,507	6,230,445			
2021	1,934,318	4,460,400				
2022	2,019,600					

e.g., AY2020 at 12 months: 1,779,980 = 446×3,991

## 12. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.
4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

### Learning Outcomes:

- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).
- (4f) Calculate claim liabilities.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 17, 18, and 24.

### Commentary on Question:

*This question tests the candidate's understanding of the expected method and the Bornhuetter Ferguson method for estimating ultimate claims.*

### Solution:

- (a) Calculate expected claims for each accident year using the expected method.

$$\text{Claim trend} = (1 + 5.5\%)(1 - 0.5\%) - 1 = 4.9725\%$$

Accident Year (AY)	(1) Earned Premiums	(2) Projected Ultimate Claims Based on Paid	(3) Reported	(4) Claim Trend at 4.9725%	(5) Premium On-Level Factors
2017	7,830,576	5,515,481	5,396,582	1.275	1.139
2018	8,092,188	5,886,678	5,758,999	1.214	1.133
2019	8,536,126	6,187,315	6,051,964	1.157	1.089
2020	8,983,907	6,565,775	6,448,346	1.102	1.049
2021	9,288,767	7,041,612	6,913,772	1.050	1.034
2022	9,626,289	7,648,572	7,544,729	1.000	1.000

## 12. Continued

AY	(6) = (2)(4)/[(1)(5)]	(7) = (3)(4)/[(1)(5)]	(8) = (A)(5)/(4)	(9) = (1)(8)
	Trended On-Level Claim Ratio based on		Claim Ratio at Each AY Cost Level	Expected Claims
	Paid	Reported		
2017	78.8%	77.1%	68.1%	5,329,615
2018	78.0%	76.3%	71.1%	5,751,085
2019	77.0%	75.3%	71.7%	6,120,941
2020	76.8%	75.4%	72.5%	6,513,970
2021	77.0%	75.6%	75.0%	6,968,819
2022			76.2%	7,331,874

Average trended on-level claim ratio at AY2022 cost and rate level (excl. 2022)

All Years	77.5%	75.9%
Latest 5 Years	77.5%	75.9%
Latest 5 Years Excl. High and Low	77.3%	75.7%
Latest 3 Years	76.9%	75.4%

Selected expected claim ratio at  
AY2022 cost level

76.17% (A)

Rationale: There is no indication of a significant the difference between reported and paid, so use the average of paid and reported. Use the more recent 3 years average to reflect more recent experience.

- (b) Calculate projected ultimate claims using the Bornhuetter Ferguson method with reported claims and your results from part (a).

Accident Year	(10)	(11) = (3)/(10)	(12) = (10) + (9)[1 - 1/(11)]
	Reported Claims	Cumulative Development Factors	Projected Ultimate Claims
2017	5,313,155	1.0157	5,395,547
2018	5,582,317	1.0317	5,758,756
2019	5,471,143	1.1062	6,058,584
2020	5,175,067	1.2460	6,461,304
2021	4,529,697	1.5263	6,932,754
2022	3,414,718	2.2095	7,428,211

## 12. Continued

- (c) Assess the reasonableness of the inputs to the Bornhuetter Ferguson method in part (b).

$$(13) = 1 / (11) \quad (14) = (9)(13) \quad (15) = (10) - (14)$$

Accident Year	Percent Reported	Expected Reported	Actual and Expected Difference
2017	98.45%	5,247,223	65,932
2018	96.93%	5,574,646	7,671
2019	90.40%	5,533,500	-62,357
2020	80.25%	5,227,733	-52,666
2021	65.52%	4,565,762	-36,065
2022	45.26%	3,318,381	96,337
			18,853

Assessment: overall it appears reasonable, but there are some accident years that might not be reasonable (e.g., 2017, 2019 & 2022) that should be investigated.

- (d) Calculate total IBNR using your results from part (b).

Total ultimate claims	38,035,156
Total reported claims	29,486,097
IBNR = Ultimate Claims – Reported Claims	8,549,059

### 13. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

#### Learning Outcomes:

- (3c) Identify the types of development triangles that can be used for investigative testing.  
(3d) Analyze development triangles for investigative testing.

#### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), Friedland, Chapter 14.

#### Commentary on Question:

*This question tests the candidate's understanding of the types of development triangles that can be used for investigative testing and analyzing development triangles for investigative testing.*

#### Solution:

- (a) Analyze this data for evidence of a change in case reserve adequacy, using two different investigative tests. Justify your conclusion.

Change in average case estimates:

Accident Year	Average Case Estimates = Case Estimates / Open Counts					
	12	24	36	48	60	72
2017	4,401	5,011	5,618	6,147	9,947	0
2018	4,771	5,421	5,923	6,477	0	
2019	5,041	5,844	6,452	0		
2020	5,345	6,083	7,575			
2021	5,636	7,466				
2022	6,801					

Accident Year	Change in average case estimates:				
	12	24	36	48	60
2017-2018	8.4%	8.2%	5.4%	5.4%	n/a
2018-2019	5.7%	7.8%	8.9%	n/a	
2019-2020	6.0%	4.1%	17.4%		
2020-2021	5.4%	22.7%			
2021-2022	20.7%				



### 13. Continued

Change in average reported claims:

Accident Year	Average Reported Claims					
	12	24	36	48	60	72
2017	3,764	4,448	4,801	5,094	5,204	5,231
2018	4,064	4,585	5,074	5,426	5,498	
2019	4,163	4,878	5,333	5,642		
2020	4,601	5,152	5,836			
2021	4,912	5,748				
2022	5,436					

Accident Year	Change in average reported:				
	12	24	36	48	60
2017-2018	8.0%	3.1%	5.7%	6.5%	5.7%
2018-2019	2.4%	6.4%	5.1%	4.0%	
2019-2020	10.5%	5.6%	9.4%		
2020-2021	6.7%	11.6%			
2021-2022	10.7%				

Based on the significant increase in the most recent diagonal of both triangles, there is an indication of a strengthening of case estimates.

- (b) Critique your colleague's conclusion.

The case strengthening could cause the most recent diagonal to increase, but a deterioration in claims experience could also cause the increase.

- (c) Describe why an increase in the most recent diagonal of the ratios of paid to reported claims triangle may not give a clear indication of this change.

A decrease in the overall adequacy of case estimates, which would decrease the reported claims, could also be driving the increase in the ratios of paid to reported claims.

### 13. Continued

- (d) Analyze this data for evidence of a change in claim settlement patterns, using an investigative test other than the test described in part (c). Justify your conclusion.

Change in ratios of closed to reported counts:

Accident Year	Closed to Reported Counts					
	12	24	36	48	60	72
2017	0.516	0.719	0.839	0.927	0.994	1.000
2018	0.510	0.720	0.839	0.928	1.000	
2019	0.507	0.717	0.842	1.000		
2020	0.507	0.719	0.893			
2021	0.507	0.768				
2022	0.547					

Accident Year	Change in ratios of closed to reported counts:				
	12	24	36	48	60
2017-2018	-1.2%	0.1%	0.0%	0.1%	n/a
2018-2019	-0.6%	-0.4%	0.3%	n/a	
2019-2020	0.1%	0.3%	6.1%		
2020-2021	0.0%	6.9%			
2021-2022	7.8%				

Based on the significant increase in the most recent diagonal there is an indication of an increase in claim settlement patterns.

## 14. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.
5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

### Learning Outcomes:

- (3g) Estimate ultimate values using the methods cited in (3e).
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16 and 26.

### Commentary on Question:

*This question tests the development-based frequency-severity method for estimating ultimate claims.*

### Solution:

- (a) Recommend an annual claim frequency trend to use for the development-based frequency-severity method. Justify your recommendation.

	(1)	(2)	(3) = (2) / (1)	(4) = (3) <sub>i</sub> / (3) <sub>i-1</sub> - 1
	Ultimate Counts Based on			
Accident Year	Earned Exposures	Development Method	Indicated Frequency	Year-to-Year Change
2017	11,434	1,235	10.80%	
2018	11,635	1,247	10.72%	-0.773%
2019	11,681	1,249	10.69%	-0.234%
2020	11,821	1,260	10.66%	-0.314%
2021	12,044	1,256	10.43%	-2.163%
2022	12,240	1,301	10.63%	1.924%
Average:				-0.312%
Selected frequency trend:				-0.312%

Justification: The year-to-year changes are quite erratic, with an overall decrease over the period. The average of all years provides a reasonable measure of the overall trend.

## 14. Continued

- (b) Estimate ultimate claims for all accident years using the development-based frequency-severity method.

Accident Year	(5) Frequency Trend @ -0.312%	(6) = (3)(5) Trended Frequency	(7) = 10.59%×(1)/(5) Calculated Ultimate Counts
2017	0.98450	10.63%	1,230
2018	0.98758	10.58%	1,248
2019	0.99067	10.59%	1,249
2020	0.99377	10.59%	1,260
2021	0.99688	10.40%	1,279
2022	1.00000	10.63%	1,296

Average excluding 2022

- all years	10.56%
- latest 3 years	10.53%
- excl. hi-lo	10.59%

Selected freq. at 2022 cost level 10.59%

Justification: Excluding high and low values excludes the outlier value in 2021.

Accident Year	(8) Ultimate Severity	(9) Severity Trend @ 7.5%	(10) = (8)(9) Trended Severity	(11) = 5,900.79/(9) Calculated Ultimate Severity	(12) = (7)(11) Projected Ultimate Claims
2017	4,104	1.43563	5,891.82	4,110.25	5,055,292
2018	4,384	1.33547	5,854.70	4,418.52	5,512,721
2019	4,751	1.24230	5,902.15	4,749.90	5,931,044
2020	5,066	1.15563	5,854.40	5,106.15	6,432,161
2021	5,531	1.07500	5,945.83	5,489.11	7,023,037
2022	5,897	1.00000	5,897.00	5,900.79	7,648,692

Average excluding 2022

- all years	5,889.78
- latest 3 years	5,900.79
- excl. hi-lo	5,882.89

Selected severity at 2022 cost level 5,900.79

Justification: Latest 3 years gives more consideration to the increasing more recent experience.

## 14. Continued

- (c) Describe two scenarios when projections from the frequency-severity method are preferred.

Any two of the following are acceptable:

- For immature periods (i.e., most recent accident years)
- Following the introduction of new GI products when limited or no historical experience is available
- Following entry into a new geographical area for which limited or no historical data exists
- If there have been wide-ranging changes, either internally at the insurer or in the external environment, such that historical relationships and development patterns are not a reliable guide to the future