Spring 2020, Multiple Choice Solutions

MC1: Answer C

MC2: **Answer D**

$$ {}_{0.8}q_{[80]+0.6} = 1 - \frac{l_{[80]+1.4}}{l_{[80]+0.6}} = 1 - \frac{0.6 l_{[80]+1} + 0.4 l_{82}}{0.4 l_{[80]} + 0.6 l_{[80]+1}} $$

$$ = 1 - \frac{0.6(37,700) + 0.4(33,862)}{0.4(40,695) + 0.6(37,700)} = 1 - 0.9297 = 0.070 $$

MC3: **Answer D**

Present Value of Premiums = Present Value of Benefits

$$ P\left(1 + {}_1p_x \cdot v + {}_2p_x \cdot v^2\right) = 100,000\left(q_x \cdot v + 2\,{}_1p_x \cdot q_{x+1} \cdot v^2 + 3\,{}_2p_x \cdot q_{x+2} \cdot v^3\right) $$

$$ P = 100,000\left(\frac{0.03v + (2)(0.97)(0.05)v^2 + (3)(0.97)(0.95)(0.07)v^3}{1 + 0.97v + (0.97)(0.95)v^2}\right) = \frac{27,711.05}{2.73523} = 10,131 $$

The easiest way to get the reserve is to use the recursive formula

$$ {}_1V = \frac{P(1+i) - q_x\,(\textit{Death Benfit})}{p_x} = \frac{(10,131)(1.06) - (0.03)(100,000)}{0.97} = 7978 $$

MC4: **Answer E**

The probability of withdrawal or death between age 60 and 61 is $e^{-\mu_x^{02} - \mu_x^{03}} = e^{-0.07} = 0.93239$.

The probability of retirement at 61 is 0.80 .

The probability of withdrawal between age 61 and 62 is $\int_0^1 \mu_{x+t}^{02}\, e^{-0.07t}\, dt = 0.05\left(\frac{1 - e^{-0.07}}{0.07}\right) = 0.048290$

$\Rightarrow$ Prob of withdrawal between ages 61 to 62 is $(0.93239)(0.8)(0.048290) = 0.036020$

$\Rightarrow$ Expected number of withdrawals $= 3000 \times 0.036020 = 108.06$

MC5: **Answer C**

$$H(y_5) = H(y_4) + \frac{d_5}{r_5} \Rightarrow 0.57000 = 0.29727 + \frac{3}{r_5} \Rightarrow r_5 = 11$$

$$r_4 = r_5 + 9 + d_4 = 21$$

MC6: **Answer C**

$$C_a(40,t) = at^3 + bt^2 + ct + d; \quad C_a'(40,t) = 3at^2 + 2bt + c$$

$$C_a(40,0) = d = \varphi(40,0) = 0.035 \quad \text{and} \quad C_a'(40,0) = \varphi'(40,0) = c = 0$$

$$C_a(40,3) = 27a + 9b + 3c + d = \varphi(40,3) = 0.005 \Rightarrow 27a + 9b = -0.03$$

$$C_a'(40,3) = 27a + 6b + c = \varphi'(40,3) = 0 \Rightarrow 27a + 6b = 0$$

$$\Rightarrow 3b = -0.03 \Rightarrow b = -0.01 \Rightarrow a = 0.06/27 = 0.00222$$

$$\Rightarrow C(40,2) = 0.00222(8) - 0.01(4) + 0.035 = 0.0128$$

MC7: **Answer A**

$$p_{x:y} = 0.77; \quad \Pr[\text{(y) survives, (x) dies}] = 0.9 - 0.77 = 0.13$$

$$\Rightarrow \text{required probability } = 0.77 \times 0.13 = 0.1001$$

MC8: **Answer E**

The premium for the second year and later is $P_{[x]+1}^{FPT} = \dfrac{S \cdot A_{[68]+1}}{\ddot{a}_{[68]+1}} = \dfrac{S(1 - d\ddot{a}_{[68]+1})}{\ddot{a}_{[68]+1}}$

$$= \frac{100,0000[1 - (0.05/1.05)(12.3355)]}{12.3355} = 3344.78$$

$${}_2V = 100,000A_{70} - 3344.78a_{70} = (100,000)(0.42818) - (3344.78)(12.0083) = 2653$$

MC9: **Answer A**

$$EPV = 50,000\,p_{80}^{01}v + 50,000\,p_{80}^{00}\,p_{81}^{01}v^2 + 50,000\,p_{80}^{01}\,p_{81}^{12}v^2 + 100,000\,p_{80}^{02}v + 100,000\,p_{80}^{00}\,p_{81}^{02}v^2$$

$$= 15,964$$

MC10: Answer C

Present Value of Premiums = Present Value of Benefits

$$12P\ddot{a}^{(12)}_{65:\overline{10|}} = {}_{20}E_{65} \times 50000 \times \ddot{a}^{(2)}_{85}$$

$$\ddot{a}^{(12)}_{65:\overline{10|}} = 1.0002(7.8435) - 0.46651(1 - 0.55305) = 7.6366$$

$$\ddot{a}^{(2)}_{85} = 1.00015(6.7993) - 0.25617 = 6.5441$$

$$P = \frac{(0.24381)(50{,}000)(6.5441)}{(12)(7.6366)} = 870.54$$

MC11: **Answer B**

EPV Premiums: $0.95P\ddot{a}_{50:\overline{10|}} = (0.95)(8.0550)P = 7.65225P$

EPV Benefits and expenses: $2{,}000{,}000\,{}_{10}E_{50}A^{1}_{60:\overline{20|}} + 500{,}000\,{}_{30}E_{50} + 260\,{}_{10}E_{50}\,\ddot{a}_{60:\overline{20|}}$

$$= (2{,}000{,}000)(0.60182)(0.41040 - 0.29508) + (500{,}000)(0.34824)(0.50994) + (260)(0.60182)(12.3816) = 229{,}532$$

$$\Rightarrow P = \frac{229{,}532}{7.65225} = 29{,}995$$

MC12: Answer D

$$EPV = 1000\left(1 + {}_{0.5}p_{98}v + p_{98}v^2 + {}_{1.5}p_{98}v^3\right)$$

$${}_{0.5}p_{98} = 1 - \tfrac{1}{3}q_{98} = 0.920955$$

$$p_{99} = 1 - q_{99} = 0.762866$$

$${}_{1.5}p_{99} = 0.762866\left(1 - \tfrac{1}{3}q_{100}\right) = 0.696168$$

$$\Rightarrow EPV = 1000\left[1 + \frac{0.920955}{1.05} + \frac{0.762866}{(1.05)^2} + \frac{0.696168}{(1.05)^3}\right] = 3170$$

MC13: Answer A

Present Value of Premium = Present Value of Benefits $\Rightarrow P\bar{a}^{00}_{55:\overline{10}|} = 50,000\bar{a}^{01}_{55:\overline{10}|}$

$$\bar{a}^{00}_{55:\overline{10}|} = 10.1228 - 0.74091 \times v^{10} \times 6.6338 - 0.11682 \times v^{10} \times 0.0395 = 7.1026$$

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$$\bar{a}^{01}_{55:\overline{10}|} = 2.3057 - 0.74091 \times v^{10} \times 2.8851 - 0.11682 \times v^{10} \times 8.8123 = 0.361405$$

$$\Rightarrow P = \frac{(50,000)(0.361405)}{7.1026} = 2544$$

MC14: Answer D

Due to the interest rate change, we will find the reserve at the end of the 10th year and then use the recursive formula to find the answer.

$${}_{10}V = 1000A_{70} - [(0.9)(36) - 5]\ddot{a}_{70} = (1000)(0.42818) - [(0.9)(36) - 5](12.0083) = 99.15$$

$${}_{10}V = \frac{({}_9V + P - Expenses)(1+i) - (DeathBenefit)(q_{69})}{p_{69}} = \frac{({}_9V + (0.9)(36) - 5)(1.03) - (1000)(0.009294)}{1 - 0.009294}$$

$$\Rightarrow {}_9V = \frac{\left((1000)(0.009294) + (99.15)(1 - 0.009294)\right)}{1.03} + 5 - 0.9 \times 36 = 76.99$$

MC15: Answer A

$$A^{(4)}_{80} = {}_{0.25}q_{80}v^{0.25} + {}_{0.25}p_{80}v^{0.25}A^{(4)}_{80.25}$$

$$= 0.008164v^{0.25} + 0.991836v^{0.25}A^{(4)}_{80.25}$$

$$\Rightarrow A^{(4)}_{80.25} = \frac{0.66227 - 0.008085}{0.991836 \times 0.990243} = 0.666069$$

MC16: Answer B

$$\text{Profit} = 100({}_5V + 0.89P)(1.05) - 100,000 - 99(13,529) = 87,418$$

MC17: Answer B

$$\Pr_2^{(0)} = ({}_1V^{(0)} + 0.98P)(1.08) - p_{x+1}^{01}\left(30,000 + {}_2V^{(1)}\right) - p_{x+1}^{00}({}_2V^{(0)})$$

$$= 6933.6 - 1642.0 - 4921.5 = 370.1$$

MC18: Answer E

$$NC = 0.02 \times 90,000 \times {}_{20}E_{45} \times \ddot{a}_{65}^{(12)} = 8479.0$$

MC19: Answer B

Let S denote the salary from age 45 to age 46.

$$RR = \frac{S\left(\dfrac{(s_{58} + s_{59})}{2s_{45}}\right) \times 0.016 \times 15}{S\left(\dfrac{s_{59}}{s_{45}}\right)} = 0.238$$

MC20: Answer A

$$\frac{d}{dt}\,{}_tV^{(1)} = \delta\,{}_tV^{(1)} - 50,000 - \mu_{x+t}^{10}\left({}_tV^{(0)} - {}_tV^{(1)}\right) - \mu_{x+t}^{12}\left(-{}_tV^{(1)}\right)$$

At $t = 15$ we have:

$$\frac{d}{dt}\,{}_tV^{(1)} = (0.04879)(300,860) - 50,000 - 0.00087(40,942 - 300,860) + 0.04754(300,860)$$

$$= -20,792$$