PRINCIPLES UNDERLYING ACTUARIAL SCIENCE

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ABSTRACT

This paper was developed over an extended period of time, during which the authors were members of the Casualty Actuarial Society (CAS) and Society of Actuaries (SOA) Joint Committee on Actuarial Principles. Drafts were exposed for comment on several occasions. Although the Boards of the two societies decided that at this time they were no longer interested in developing a statement of general principles of actuarial science under their auspices, they granted the committee members permission to publish the document under the authors' names. The authors are pursuing publication of this document because they believe it will provide significant educational benefit to actuaries and the publics they serve.

The authors believe that the principles presented in this paper are based on fundamental concepts that are relevant to most actuaries' work. These principles are grounded in observation and experience and are intended to strengthen the reader's understanding of the intellectual foundations of actuarial science, such as the role of models, the effect of preferences regarding risk and timing, the nature of actuarial risk and the basis for risk classification and financial security systems. This paper is the result of a long-term, dedicated effort to articulate those principles for educational purposes.

The presentation of principles is divided into four sections that range from the theoretical to the applied:

- statistical framework,
- economic and behavioral framework,
- risk management and modeling, and
- financial security systems.

INVITATION TO COMMENT

This version of the paper is being circulated by the authors for commentary from actuaries and other professionals who might have an interest in actuarial science. Comments received will be considered by the authors in producing a final paper on the principles underlying actuarial science. It is the intention of the authors to submit a final version of the paper, possibly revised to reflect comments and suggestions received, to an appropriate refereed publication. Comments may be submitted to Stuart Klugman at stuart.klugman@drake.edu.

BACKGROUND, SCOPE AND LIMITATIONS

Background

In October 1991, the Society of Actuaries (SOA) Board of Governors accepted the statement of the SOA Committee on Actuarial Principles titled "Principles of Actuarial Science," which had been revised in response to the comments received during an exposure process that included both a discussion draft and an exposure draft. The SOA Board authorized the statement as an expression of opinion by the Committee. It was not an expression of opinion by the SOA. The statement was published in *Transactions of the Society of Actuaries* XLIV, 1992, along with a note indicating that the statement's application was limited to the areas of actuarial practice that fall within the purview of the SOA, and that the paper was not intended to address the areas of property and casualty insurance.

Subsequently, the Casualty Actuarial Society (CAS) and SOA Principles Committees worked together to produce an April 30, 1997 discussion draft of "General Principles of Actuarial Science" and related companion documents. The SOA Board and the CAS Board authorized the distribution of these documents to members of the actuarial profession and other interested parties. Comments received in response to that discussion draft and its companion documents were considered in the development of a second discussion draft authorized for release in August 1998. Comments received in response to the second discussion draft were considered in the development of an exposure draft that was released on Oct. 15, 1999. A large number of comments were received, and from that time through 2007, the Joint CAS, SOA Committee on Actuarial Principles worked on revisions. Changes to the environment in which actuaries practice also were reflected in these revisions.

Scope and Limitations

To clarify the intended scope of this paper, it is important to emphasize the distinction between principles underlying the scientific framework of actuarial work and the standards that apply when performing such work.

The intent of the articulation of the principles included in this paper is to describe the key elements of actuarial science. These principles do not represent guidelines to be followed in carrying out actuarial work, but, rather, they indicate the authors' current understanding of key concepts and phenomena often relevant to the application of actuarial science. The paper contains statements grounded in observation, experience and current understanding. As our observations, experience and understanding continue to develop, the authors recognize that the articulation of the principles as presented in this paper might need to change to reflect those developments.

In addition to its scientific framework, the actuarial profession makes use of actuarial standards of practice. These standards represent normative guidance, reflecting currently expected actuarial practice, regulatory constraints and other external conditions. Such standards might guide the actuary in the selection of appropriate models and assumptions. New or revised standards might be introduced as actuarial practice evolves. This paper does not represent standards of practice or another form of guidance for actuarial practice. Further discussion of standards or actuarial practice guidance is outside the scope of this paper.

It is important to note that some of the principles included in this paper have been taken from other disciplines and are not intended to represent a complete formulation of the science or practice in those disciplines.

In addition, this paper is not intended to be a scientific treatise that presents a formal, axiomatic structure, but, rather, an identification and discussion of only those principles that the authors consider significant and are within the scope of the current and expected future practice of actuaries.

In the paper "The Methodology of Actuarial Science," presented to the Institute of Actuaries in October 1998[1], J.M. Pemberton asserts that, "Actuarial science is concerned with the development of models which approximate the behaviour of reality and have a degree of predictive power, not the truth."

Pemberton notes that "simple laws do not adequately describe complex realities," and, thus, "actuarial science deals directly with low-level generalisations, recognising the limited nature of available regularities." Further, there is "need for knowledge of the variations in the specific locality of interest."

The above excerpts summarize the nature of actuarial science and, thus, the limitations of any paper, including this one, that attempts to articulate its general principles. These excerpts also suggest that any such articulation would include some principle statements with words like "tend" or "generally," because they do not hold or apply in all cases.

It also is unlikely that principles of the same order of certainty as those used in physics and chemistry can ever be articulated for actuarial science. This means that these principles generally will not satisfy criteria that demand that the principles be falsifiable. The authors believe it is sufficient that, for now, they have yet to be invalidated.

[1] *British Actuarial Journal*, 1999, Vol. 5, pp. 115-196

During the past 20 years or so, the CAS, SOA and the American Academy of Actuaries (AAA) have published several statements of actuarial principles. They have primarily related to more specific aspects of actuarial science or practice than this paper covers. The principles in this paper are intended to serve a different purpose and the authors do not believe there are any conflicts with those statements of actuarial principles.

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INTRODUCTION

Actuarial science is an applied science based on concepts and observations distilled from the experience of practitioners and from other sciences. This paper presents concepts extracted from this experience and from related fields, including mathematics, statistics, economics and finance.

The objective of *Principles Underlying Actuarial Science* is to articulate the current understanding of the significant principles that form the scientific framework underlying all areas of actuarial practice. The intended audience includes practicing actuaries, researchers and others, such as representatives of standard-setting organizations. This articulation is not immutable and should be revised if changes arise in our understanding of the key concepts and phenomena relevant to actuarial science, or if a better expression of the ideas is developed.

Nature of Actuarial Science and Actuarial Practice

Actuarial science is primarily concerned with the study of consequences of events that involve risk and uncertainty. Actuarial practice identifies, analyzes and assists in the management of the outcomes — including costs and benefits — associated with events that involve risk and uncertainty. Understanding the principles underlying actuarial science enables actuaries to develop models of such events and other techniques to solve practical problems.

To gain insights about future possibilities, the actuary depends on observation and wisdom gained through prior experience. Actuaries use these observations and experience to construct, validate and apply models. Actuaries continually incorporate additional observations and insights into their models. This feedback cycle systematically addresses discrepancies between these models and observed reality.

Actuarial practice, in turn, is concerned with the assessment of the economic consequences associated with phenomena that are subject to uncertainty. This practice requires an understanding of the principles underlying several fields, including statistics, economics and risk management, as well as the principles of modeling, valuation and risk classification.

Actuaries solve business problems in which the mitigation of negative consequences and the exploitation of positive consequences of risk play major roles. Thus, actuaries must be familiar with the principles of the management of fields in which they work, including insurance, health care and retirement systems, investment portfolios and the risks individuals face. Actuarial models can be developed to solve most of the typical problems that require analysis of the consequences of risk and uncertainty.

Reasons for Articulation of Principles

The reasons for articulating these principles include the following:

- describing and strengthening the intellectual foundation of the actuarial profession;
- providing a foundation for the extension of actuarial models to new applications;
- providing a basis for formulating sound and consistent standards;
- providing a foundation for the articulation of practice-specific principles;
- furthering actuarial education;
- focusing research efforts; and
- aiding in strategic planning for the profession by providing a foundation upon which actuaries can offer their services in current and potential areas of actuarial practice.

Organization of This Paper

This paper identifies those significant principles that form a foundation for actuarial science. They include principles from mathematics, statistics, economics and finance. When these principles are considered in conjunction with the actuarial risk variables of frequency, timing and severity, models of particular interest to actuaries can be constructed.

Combining these principles with observations about human behavior, actuaries formulate models to assess the implications of future events. In most cases, these models are applied to the risk-management process and to financial security systems.

Thus, the organization of the remainder of this paper is:

Section 1. Statistical Framework

Section 2. Economic and Behavioral Framework

Section 3. Principles Underlying Risk Management and Actuarial Modeling

Section 4. Principles Underlying Financial Security Systems.

For reference, glossaries of the Principles and Definitions are included at the end of the paper.

1. STATISTICAL FRAMEWORK

When actuaries use historical observations to make inferences, certain insights derived from probability and statistics are used. This section provides definitions and principles related to those insights.

DEFINITIONS

Phenomena are occurrences that can be observed. An *experiment* is an observation of a given phenomenon under specified conditions. The result of an experiment is called an *outcome*; an *event* is a set of one or more possible outcomes. A *stochastic phenomenon* is a phenomenon for which an associated experiment has more than one possible outcome. An event associated with a stochastic phenomenon is said to be *contingent*.

For example, the rolling of two fair dice is a stochastic phenomenon. The observation of the sum of the numbers on the faces of two such dice is an experiment. The possible outcomes are the numbers two through 12. One contingent event is that the total is an odd number. The definition of an event can include a specification of severity and timing. For example, the event could be a $100,000 loss incurred within a particular calendar year.

DEFINITIONS

Probability is a measure of the likelihood of the occurrence of an event, measured on a scale of increasing likelihood from zero to one. A person's *knowledge* about a phenomenon is the person's assessment of the probabilities associated with the phenomenon. *Data* are observed occurrences or other facts related to a phenomenon. *Information* is data that changes or has the potential to change a person's knowledge.

For example, a person might assign a probability of 1/6 to each face of a die appearing on a single roll. Upon observing 100 rolls in which a six appears 30 times, this information is likely to change that person's probability assignments.

DEFINITIONS

A *random variable* is a function that assigns a numerical value to every possible outcome. The probability-weighted average of the numerical values taken on by a random variable, if the average exists, is called the *expected value* of the random variable. An *estimator* is a function of one or more random variables associated with an experiment. An *estimate* is a value produced by an estimator.

The random variables used to define an estimator associated with an experiment might be the outcomes themselves. The estimator is defined to provide insights concerning some property (for example, the expected value) of the random variable. An estimate is the particular numerical value that arises when the outcomes are obtained. An estimate is not a prediction concerning future outcomes; rather, it is a statement about the random variables that produced the outcomes that have already occurred. To the extent that the random variables to be observed in the future are similar to those that produced the current outcomes, the estimate can lead to inferences about future outcomes.

DEFINITIONS

The *degree of uncertainty* of a random variable is a measure of the variation of the values taken on by the random variable from its expected value. Two random variables are said to be positively (negatively) *correlated* if, for a given outcome, a positive deviation of one random variable from its expected value tends to be accompanied by a positive (negative) deviation of the other random variable from its expected value. A *correlation coefficient* is a measure of the extent and direction of correlation of two random variables.

Degree of uncertainty and correlation can be defined more generally in terms of deviations from a central value. For example, the median could be used. If observing the value taken on by one random variable does not change a person's knowledge relative to a second random variable, the two random variables are said to be "independent." Independence implies no correlation, but the converse is not true.

For certain collections of phenomena, it is of interest to compare the respective expected values of a given random variable. For example, the number of times a six occurs when a die is thrown 1,000, 2,000 and 3,000 times are three separate phenomena. Nevertheless, the expected number of sixes for the 1,000-throw phenomenon, the 2,000-throw phenomenon and the 3,000-throw phenomenon would be expected to be in proportions of 1:2:3. For these phenomena, the number of throws acts as a scaling factor that relates the expected values. When a scaling factor exists, the proportion of occurrences of a specific event from experiments of various phenomena can be compared by using the scaling factor. In this way, a wider array of data can be used to develop a specific estimate.

DEFINITION

An *exposure measure* is a scaling factor that relates the expected value of one or more random variables over a collection of phenomena.

Principle 1.1 is motivated by the law of large numbers. It might appear at first that this law does not apply to a field like insurance, because the experiment whose outcomes are the occurrence or nonoccurrence of the insured event can rarely be precisely repeated. For example, the survival to a specified age, the death of a specific individual or the destruction of a particular automobile can only be observed once. But if the experiment is defined as observing whether in the next year a randomly selected, 40-year-old nonsmoking male dies or a randomly selected car of a given make is destroyed, this experiment can be repeated. In these cases, it is likely that the proportion of deaths or destructions will converge to a limit as the number of observations increases. The principle is not stated as strongly as the law of large numbers. Actuaries understand that even when generalized as above, experiments still cannot be repeated under exactly identical circumstances. Nevertheless, numerous phenomena exist that are sufficiently well behaved to allow various quantities to be studied.

1.1 PRINCIPLE (Statistical Regularity). Phenomena exist such that, if a sequence of independent experiments is conducted under conditions that are substantially similar to a set of specified conditions, the proportion of occurrences of a given event converges as the number of experiments becomes large.

DEFINITION

A phenomenon to which Principle 1.1 applies is said to display *statistical regularity*.

Actuaries use this principle in two ways, which reflects the frequentist and subjectivist approaches to assigning probabilities. The frequentist approach uses the principle to note that if sufficient data are observed under conditions of statistical regularity, the proportion of occurrences provides an estimate of the probability of the occurrence of the event. If probabilities are assigned in this way, then probability theory can be used to determine quantities of actuarial interest.

The subjectivist approach assigns the probabilities of occurrence using one or more sources, including judgment. If the probabilities are assigned in a manner consistent with the axioms of probability theory, future observations are expected to exhibit statistical regularity. Thus, models and conclusions determined from these probabilities could be evaluated against data as they emerge.

The numerical average of the values taken on by a random variable during a sequence of experiments of the kind described in Principle 1.1 will in most, but not all, cases converge to the expected value of the random variable. An example of non-convergence is when the expected value of a random variable does not exist. In this case, there is no value to which the sample mean can converge.

DEFINITIONS

A *scientific model* is an abstract and simplified representation of a given phenomenon. A *mathematical model* is a scientific model in which the representation is expressed in mathematical terms. A *stochastic component* of a mathematical model is a representation of an aspect of a given phenomenon expressed in terms of random variables. A *deterministic component* of a mathematical model is a representation of an aspect of a given phenomenon expressed without the use of random variables. A *deterministic model* is a mathematical model that consists exclusively of deterministic components. A *stochastic model* is a mathematical model that has at least one stochastic component.

A deterministic component might be derived from a stochastic component by replacing each random variable in that component with a fixed value, such as its expected value. For example, a model might have a component that represents the survival of a group by use of a life table, in which the entries are the expected frequencies of deaths at each age. If the lives are represented as expiring at exactly the rate given by the table, the component is deterministic. If the table entries are assumed to be the expected value of Bernoulli-distributed random variables, the component is stochastic.

More information is generally required to construct a stochastic component than is needed to construct a deterministic component of the same aspect of a phenomenon. Actuaries often construct deterministic components first, and proceed to stochastic components only when the necessary information becomes available. Moreover, representation of the probabilistic aspects of a given phenomenon might not be essential for the purposes for which a model is to be used. For example, actuaries once used only deterministic components to represent interest rates. Subsequently, a number of stochastic components have been developed. Nevertheless, deterministic components are still used in cases where additional complexity adds little value.

Principle 1.1 states that there exist phenomena that can be modeled. Statistical regularity is generally easier to demonstrate for physical phenomena than for biological, social and behavioral phenomena, where experiments often cannot be replicated and where data numerous enough to be useful have to be gathered from diverse sources over extended periods of time. Thus, models based on the assumption of statistical regularity must be applied with care to actuarial problems. Nevertheless, such models are in widespread use by actuaries and are found to produce useful results in many situations.

Principle 1.2 states that it is possible to construct models of stochastic phenomena. Prior observations can be used to determine the nature of the model and also the particulars (often parameters) of the model. Data from related experiments can also be incorporated. For example, experience of drivers in a given city is relevant to the construction of a model of automobile losses in that city. It might be possible to improve the model by incorporating data from similar cities. The construction of the model also might reflect prior knowledge or understanding of the phenomenon. For example, the apparent symmetry of a six-sided die could be taken into account in constructing a model of die-throwing. Also, a model constructed from data could be modified or adjusted to account for known changes in the phenomenon that might affect future results.

1.2 PRINCIPLE (Basis for Model Construction). A model of a specific stochastic phenomenon can be based on the outcomes of experiments performed on that phenomenon, on observations of related phenomena, on knowledge related to the phenomenon itself, or on a combination of all three.

Model-building often is viewed as an iterative process. The starting point can be data collected from observations of the phenomenon being studied (for example, the company's recent experience), observations on a related phenomenon (for example, an industry study), or an understanding of the phenomenon (for example, adjustments made by an underwriter or educated guesses concerning phenomena for which little or no data exist).

These approaches support both the frequentist and subjectivist views of probability. The frequentist would normally collect data, use them to formulate a model and then revise the model as more information is obtained. The subjectivist approach allows for the formulation of a model with no data or an ad-hoc summary of prior experience. The model can be revised as additional information is obtained.

When several estimates of a particular quantity are available, these estimates can be combined as a weighted average, where each weight represents an assessment of the relative importance of that estimate. This assessment might involve statistical analysis or be based entirely on judgment, and can vary according to the purpose for which the model is to be used.

1.3 PRINCIPLE (Credibility). It is possible to determine estimators of the relative weights to be assigned to each of several estimates of a particular quantity to optimize a given measure of the accuracy of the weighted average.

DEFINITION

The *credibility* of a particular estimator is the relative weight assigned to it when constructing a weighted average.

The uncertainty associated with modeling stochastic phenomena has three distinct sources:

- the inherent variability of the phenomenon;
- knowledge based on incomplete information or inaccurate representations of the probabilities of alternative sets of outcomes; and
- the risk that the model adopted is not a perfect, or even a good representation of the underlying situation.

Sometimes these sources of uncertainty are referred to as "process risk," "parameter risk" and "model risk," respectively.

The process of checking the results produced by a model for consistency with available observed information is commonly referred to as "validation."

DEFINITIONS

A mathematical model is said to be *valid within a specified degree of accuracy* relative to certain observed results if it can reproduce these results within that degree of accuracy. A mathematical model is *potentially valid* if it produces results that are consistent with available observations of the modeled phenomena or of similar phenomena and is capable of being validated relative to the specified observed results when sufficient data are available.

Observed results involving the phenomena represented by a model might not be available or sufficiently voluminous to allow the model to be validated within a specified degree of accuracy. In this case, the appropriateness of the model can be established initially by comparing its results with those from the observation of similar phenomena. The validity of such "judgmentally validated" models can be tested when sufficient information related to the modeled phenomena becomes available.

The phrase "within a specified degree of accuracy" is a crucial part of the definition. For example, a lognormal model might be used to represent automobile physical damage claims even if it is known that such claims are not lognormal. If sufficient data are collected, it will be confirmed that the claims are not lognormal. What is important is that data confirm that the lognormal model is good enough for its intended purposes. It might be sufficient that it closely (but not exactly) matches the mean, variance and 90th percentile of the phenomenon's random variable.

Not all observable aspects of the modeled phenomena must be reproduced for a model to be valid. For example, a model used in the appraisal of an insurance company might be validated with respect to only a few quantities, such as aggregate liabilities and total number of inforce insurance contracts.

2. ECONOMIC AND BEHAVIORAL FRAMEWORK

Economics can be used to help explain behavior in terms of definable, and often quantifiable, incentives and disincentives. Because an organization's actions are the result of the actions of the individuals it comprises, its behavior is subject to similar incentives and disincentives.

DEFINITIONS

An *economic system* consists of (i) a group of persons, called *participants*; (ii) a set of items, called *economic goods (goods)*, each of which belongs, at a given time, to one of the participants; and (iii) a mechanism, called a *market*, through which participants can exchange the economic goods they currently hold for other economic goods.

"Goods" include services, and "person" might refer to an individual human being, an organization, or a collection of individuals and organizations. A collection of economic goods is itself an economic good.

DEFINITIONS

An *economic transaction (transaction)* is an exchange of economic goods among two or more participants, called *parties to the transaction*. A *contract* is a mutual commitment by two or more participants in an economic system, called *parties to the contract,* to carry out certain actions.

Transactions are often effected through contracts.

Not every economic good can be exchanged. In some cases, this is due to legal or other restrictions, as in the case of social insurance rights. In other cases, there might not be a second party to effect a transaction.

The actions taken by a person who has been offered several potential transactions reflect that person's relative preferences for the economic goods to be exchanged. The potential transactions available to a participant in a given market are affected by the ability and willingness of other market participants to offer goods desired by the first party in return for the goods offered by the first party. Thus, the outcomes of potential transactions between two participants reflect the relative preferences of the two participants.

DEFINITIONS

A participant *prefers* one economic good to a second economic good if he or she is unwilling to exchange the first good for the second, but is willing to exchange the second for the first. A participant is *indifferent* between two economic goods if the participant neither prefers the first to the second, nor the second to the first.

A participant who is indifferent between two goods is willing to exchange either one for the other one, but has no incentive to do so. A participant's relative preferences depend on many aspects of the participant's current situation, including economic goods currently held. Thus, the occurrence of a transaction can change the participant's relative preferences.

It is possible that two non-identical goods can be found such that the participant is unwilling to exchange the first good for the second and is also unwilling to exchange the second good for the first, but Principle 2.1 indicates that this situation is rare and can usually be ignored.

2.1 PRINCIPLE (Preference or Indifference). At a given time, for almost all pairs of economic goods, a participant either prefers one economic good to the other or is indifferent between them.

DEFINITION

A *desirable (undesirable) economic good* is an economic good that a participant prefers to receive (dispose of) rather than dispose of (receive).

An example of an undesirable economic good is (for most participants) garbage.

A given participant is able to set preferences or declare indifference, but Principle 2.2 indicates that participants need not agree with each other.

2.2 PRINCIPLE (Diversity of Preferences). In most economic systems, there are pairs of economic goods for which some participants prefer the first good to the second, and others prefer the second good to the first.

In addition to exchanges of currently held economic goods, transactions might also involve exchanges of the future possession of such goods.

DEFINITIONS

An *economic asset (asset)* is the right to possess a specified desirable economic good upon the occurrence of a specified condition. An *economic liability (liability)* is an obligation to hold an undesirable economic good or to transfer a specified asset to another participant upon the occurrence of a specified condition. The *deferral period* of an economic asset or an economic liability is the period of time from the present until possession of the specified economic good is to commence, or transfer of the specified economic asset is to occur. An *economic option (option)* is a right, but not an obligation, to exchange one set of economic assets or economic liabilities for another set of economic assets or liabilities at a specified future time or during a specified time interval. An *economic swap* (*swap)* is an agreement to exchange specified economic assets or economic liabilities for other economic assets or economic liabilities at one or more specified future times. An *economic instrument* is a set of rights and obligations to hold, receive or transfer one or more economic goods currently, or at specified future times.

Economic assets, liabilities, options and swaps are all economic instruments and are themselves economic goods. The specified condition for an economic asset or economic liability could be a known point in time, or the emergence of the right or obligation could depend on a particular event taking place.

The terms "asset," "liability," "option" and "swap" might be defined somewhat differently when used in the context of a specific accounting system, body of law or regulation. Unless such context is specifically noted, these terms, when used below, should be taken to mean *economic* assets, liabilities, options and swaps, as defined above.

Principle 2.3 indicates that a participant's preferences might be affected by the period of time until possession.

2.3 PRINCIPLE (Time Preference). Participants tend to prefer shorter deferral periods for otherwise identical economic assets and longer deferral periods for otherwise identical economic liabilities.

There are classes of economic goods for which almost all participants agree on relative preferences. For example, for a class made up of members representing 1 ounce of gold, 2 ounces of gold, etc., almost all participants would agree that a member of the class representing more ounces of gold is preferred to a member of the class representing fewer ounces of gold.

DEFINITIONS

A *commodity* is a class of desirable economic goods such that (i) all participants agree to the assignment of the same nonnegative number to each member of the class; (ii) given two members of the class that have been assigned different numbers, almost all participants prefer the member with the higher number; and (iii) all participants are indifferent between one set consisting of members of the class and any other such set, as long as the sum of the numbers assigned to the members of the first set equals the sum of the numbers assigned to the members of the second set. The *amount* of the commodity represented by a given member of the class is the number assigned to that member.

For example, ground beef, measured in pounds, is a commodity. A person is likely to be indifferent between possessing five 1-pound packages and one 5-pound package. However, the latter might cost less. Most services are not commodities. A person might prefer two hours of time from one actuarial consultant to one hour of time from each of two consultants, even if the total costs are the same.

Participants' preferences also reflect their attitudes toward uncertainty. Because commodities are available in various amounts, situations with several possible outcomes, each resulting in the possession of a specified amount of the commodity, can be used to assess these attitudes.

DEFINITIONS

A *wager* is an agreement to disburse or receive, at a specified time, an amount of a given commodity determined by the outcome of a stochastic phenomenon. The *payoff* associated with a wager is a random variable that assigns to each outcome a number representing the amount of the commodity to be disbursed (if negative) or received (if positive). The *expected payoff* of a wager is the expected value of the payoff.

A common type of wager has two possible outcomes with complementary probabilities, one outcome resulting in the possession of an amount of the commodity at a specified time and the other outcome resulting in no possession.

Principle 2.4 indicates that, in comparing situations where the outcomes and their probabilities are known and measurable, most participants prefer less uncertainty.

2.4 PRINCIPLE (Risk Aversion). For a given measure of the degree of uncertainty, when choosing between two wagers with the same expected payoff, participants tend to prefer the wager with the lower degree of uncertainty.

When acting in accordance with Principle 2.4, people are said to be "risk averse." When people prefer a higher degree of uncertainty, they are said to be "risk seeking." Some people might be risk-seeking in certain situations, such as when making decisions regarding the purchase of lottery tickets in which extremely high return, low-probability outcomes are possible, but are risk averse in most other situations. In most cases of concern to actuaries, risk aversion applies.

Some commodities, such as gold, achieve such wide acceptance within an economic system that, given any one of a wide range of economic goods, most participants are willing to state an amount of the commodity that they would exchange for that good. Commodities with this characteristic play an important role in economic systems, as indicated by Principle 2.5.

2.5 PRINCIPLE (Existence of Money). In many economic systems, there is a commodity such that, at a given time, for most desirable economic goods and for most participants, there is an amount of the commodity such that each participant is indifferent between the economic good and that amount of the commodity.

DEFINITION

Money is a commodity satisfying Principle 2.5.

In a given economic system, several forms of money might be present. Often, one form of money is selected as the preferred means of effecting transactions. At one time, commodities such as gold, pearls or certificates exchangeable for fixed amounts of these commodities were used for this purpose. Eventually, people came to realize that denominated certificates, such as coins and bills issued by governments, as long as they are universally accepted, have all the characteristics of money.

DEFINITIONS

The *currency* of a given economic system is the form of money that is accepted as the preferred means of effecting transactions, if such a preferred means exists. A *monetized* economic system is an economic system with a currency. The *current monetary value* assigned to an economic good at a particular time by a given participant is an amount of currency such that the participant is indifferent between the amount of currency and the economic good. Two economic goods are *equivalent* for a given participant at a given time if they have the same current monetary value.

Because a participant's relative preferences depend on many aspects of the participant's current circumstances, including economic goods held, the current monetary value of each economic good will likely change over time. When a participant prefers one good to another and has assigned current monetary values to both goods, the preferred good will have a greater current monetary value for that participant.

With respect to commodities, the current monetary value is normally an increasing function of the amount of the commodity. However, the second receipt of a certain amount of a commodity might not be valued the same as the first receipt. For example, the first pint of cold drink on a hot day might have more value to a thirsty participant than would a second pint of the same liquid.

DEFINITIONS

A *cash receipt (disbursement)* is an economic good consisting of a single amount of currency received (disbursed) at a specified time. A *cash flow* is a series of cash receipts and disbursements at specified points in time. A *financial instrument*, *financial asset, financial liability, financial option or financial swap* is, respectively, an economic instrument, economic asset, economic liability, economic option or economic swap for which the economic goods are cash flows. A *financial portfolio (portfolio)* is a set of financial instruments. A *financial market* is a market for the exchange of financial instruments. The *sum* of two cash flows is the series of cash receipts and disbursements consisting of the algebraic sums of the amounts of currency received or disbursed at each time under the constituent cash flows, with amounts received taken to be positive and amounts disbursed taken to be negative. The *net cash flow* of a portfolio is the sum of the cash flows associated with the financial instruments in the portfolio.

Financial assets, financial liabilities, financial options and financial swaps are all financial instruments.

Principles 2.3 and 2.4 have implications for a participant's attitude toward currency and financial instruments. When stated in terms of currency and financial instruments, Principle 2.3 expresses the "time-value of money." People tend to prefer receiving a currency payment at an earlier date to receiving the same currency payment at a later date, and disbursing a currency payment at a later date to disbursing the same currency payment at an earlier date. Similarly, Principle 2.4 expresses the observation that, given two wagers with the same positive expected payoff, people tend to assign a greater current monetary value to the wager with a lower degree of uncertainty.

As an illustration of the implications of Principles 2.2, 2.3 and 2.4 on current monetary value, consider the purchase of a 10-year, $5,000 zero-coupon bond issued by a person's local school board. The current monetary value assigned to this bond by the person is influenced by the facts that (i) cash will not be received for 10 years, (ii) there is a possibility that 10 years from now the school board will not be able to repay the $5,000, and (iii) people might place different values on supporting the school board as compared to other investments. A second person might assign a different current monetary value to the same zero-coupon bond due to different preferences and beliefs with respect to timing, risk and other factors.

DEFINITIONS

A *value function* on a set of cash flows is a function that assigns a number to each cash flow in the set such that (i) each cash flow consisting only of cash receipts (disbursements) is assigned a positive (negative) number, and (ii) the number assigned to the immediate receipt (disbursement) of an amount of currency is that amount (the negative of that amount).

A *present value function* on a set of cash flows is a value function on that set such that, if one cash flow consists of receipts identical in amount to, but received earlier than, those of a second cash flow and disbursements identical in amount, but disbursed later than those of the second cash flow, the first cash flow is assigned a higher value. The *present value* of a cash flow relative to a present value function is the number assigned to the cash flow by the present value function.

A familiar example of a present value function is a function that assigns to a receipt (disbursement) the value (the negative of the value) obtained by discounting at a specified interest rate.

A present value function is especially useful when the values it produces for cash flows are consistent with the current monetary values a participant would assign to them. The present value of the sum of two cash flows is not necessarily equal to the sum of the present values of the cash flows. For example, a participant might not perceive currency to be received at a future time as being of equal and opposite value to the requirement to disburse the same amount of currency at that time. However, for many entities of interest to actuaries, especially institutions that invest and borrow on an aggregate basis, the present value of the sum usually equals the sum of the present values.

DEFINITIONS

An *economic scenario (scenario)* for a given economic system is a description representing specified aspects of the economic system at future times. An *economic model* of an economic system is a set of economic scenarios, each assigned a probability of occurrence, such that the probability that some member of the set will occur is one. A *cash flow model* of a portfolio is an economic model that, for each financial instrument in the portfolio, assigns a cash flow to each of the economic scenarios. A *present value model* is a cash flow model that has a present value function associated with each economic scenario. A *present value metric* of a portfolio relative to a present value model is a number-valued function of the scenario present values. The *present value random variable* relative to a present value model is the random variable that assigns to each economic scenario the present value of the cash flow assigned to that scenario for the financial instrument. The *expected present value* of a portfolio relative to a present value model is the expected value of the present value random variable of the portfolio.

The expected present value is a present value metric.

The definition of economic model is very inclusive. Scenarios could be descriptions of future interest rates and equity prices, could represent the possible dates of occurrence and degree of severity of a specified event, or could include some aspects of each. The model might represent several aspects of the economic system, and each aspect might be represented stochastically or deterministically.

Note that the expected present value is a present value metric. So, too, are percentiles such as the median of present values over the scenario set. Such percentiles are often referred to as "value-at-risk." An example of a present value metric of interest to actuaries is the "x% conditional tail expectation" (CTE) metric, which assigns to the portfolio the average of the scenario present values that exceed a threshold set at a corresponding value-at-risk percentile. Conditional tail expectation metrics have been used in determining regulatory liabilities for annuities, and also have been used in company-specific economic capital calculations.

Transactions that take place in the market of an economic system involve the interaction of participants, each with a distinct point of view regarding relative value. Principle 2.1 implies that, in any economic transaction, the parties to the transaction might prefer certain outcomes to others. Participants who carry out transactions in an attempt to achieve outcomes they prefer to their pre-transaction state are said to be "acting in their own self-interest." However, a participant's ability to do this depends on the participant's knowledge.

2.6 PRINCIPLE (Enlightened Self-interest). The parties to an economic transaction act in accordance with their preferences, subject to the knowledge each has about the environment and about the other parties.

Actions each party to a transaction takes reflect, among other factors, that party's time and risk preferences. Each party's actions can, in turn, affect the actions taken by the other parties. For example, the decision of an insurer to incorporate a deductible amount into its insurance contract might influence the subsequent actions of purchasers of the contract, both in the choice of which contract to purchase and in subsequent decisions after its purchase, regarding whether to continue it and whether to submit a claim.

If all parties to the transaction do not have the same information before or after the contract is entered, the decisions made by the parties may be significantly affected.

DEFINITIONS

Information is *relevant information* for a transaction if it can change a party's willingness to complete that transaction on the stated terms. *Information asymmetry* exists if, at the time an economic transaction takes place, there is relevant information that is available to one party to the transaction but not to the other party. A transaction exhibits *adverse selection* if either (1) information asymmetry exists before the transaction is completed or (2) the same relevant information is available to both parties but one party cannot use all that information in deciding whether to complete the transaction on the stated terms, and, in either case, the terms of the transaction are different than what they would have been otherwise.

If, prior to the completion of a transaction, information asymmetry exists or one party is unable to use relevant information that is available to that party, the transaction is said to be "subject to adverse selection," whether or not the terms of the transaction change.

Law, regulation or custom can impose constraints that lead to situations where one party possesses certain information but is barred from using it. Such transactions are subject to adverse selection because, although the information is available to both parties, one party is unable to use it. Differences in knowledge between two parties can arise due to information asymmetry or due to differences in interpretation or application of information available to both parties. The latter situation does not constitute adverse selection.

An example of a transaction subject to adverse selection is the sale of a life insurance contract to someone who knows he or she has a life-threatening disease. If the insurance company does not know that the person has the disease, it will offer a different contract than if it knows. The same result will occur if the insurer is barred by law from testing for the disease or using such information if otherwise obtained. On the other hand, if the information is available to the insurer, but the insurer chooses to ignore it or interpret it differently than does the applicant, there is neither information asymmetry nor adverse selection. Nevertheless, there can be economic consequences to the insurer's choice. This will be discussed further in Section 4.

Another important aspect of transactions involving contracts is whether decisions subsequent to the transaction are affected by the existence of the contract. The existence of a contract might contribute to a change in behavior by one of the parties, which might, in turn, affect the outcomes, or the probability of outcomes, arising from the transaction.

DEFINITION

A transaction involving a contract is subject to *moral hazard* if the existence of the contract creates a potential for actions by one party subsequent to the transaction that have an adverse impact on the other party.

An example of moral hazard is the change in behavior of the purchaser of a homeowner's insurance contract that includes theft coverage. The existence of the insurance contract might result in the purchaser being less vigilant about keeping the doors of the house locked, thereby increasing the probability of a theft claim.

Classical economic theory attempts to understand the interactions of participants in a market. Each participant has different preferences and beliefs, and thus the current monetary values assigned to a given economic good will vary. However, by observing transactions occurring in a market, the amounts of various commodities that are needed, at a particular instant, to complete an exchange for a given economic good can be observed. In this way, the market can provide an objective valuation of economic goods in terms of these commodities. In particular, the market mechanism has been used to assign values expressed in monetary terms to financial instruments such as stocks, bonds and related securities.

DEFINITIONS

A *market trade* is a transaction involving the exchange of an economic good for money in a market. The *market price (price)* of an economic good is the amount of currency exchanged for the good in the most recent market trade. A *market value model* for an economic good is a model that produces an estimate of the price at which a market trade for that good would occur at a specified time. A *market value* is an estimate of price produced by a market value model.

A market value model need not be a mathematical model. When an experienced appraiser estimates the price of a unique antique, the estimate might be a market value, although the appraiser's model is probably not a mathematical model.

A market value for an economic good can be estimated by taking into account the net result of the actions that buyers and sellers are expected to take given their individual preferences and knowledge. As the participants, goods offered and demanded, and available information change from instant to instant, each market participant's optimum position also can change. If there has been a recent transaction, the market value might well be close to the price at which that transaction was made. However, this is not always the case. For example, when new information becomes available, prices might adjust instantaneously and might be very different from the prices realized in market trades in the recent past. Thus, the market value of an economic good can differ significantly from the price paid for the good, even if the transaction occurs immediately before or soon after the market value is determined. The existence of such a difference is not necessarily an indication that the market value model is inappropriate.

The accuracy of the estimates given by a market value model depends on the characteristics of the market. For certain markets, notably those for frequently traded financial instruments, a market value model might prove to be valid within an ascertainable degree of accuracy with respect to prices and other data associated with the economic good being modeled. For other goods in other markets, reasonable estimates of price might be made using potentially valid models — that is, models that are consistent with what is known about the prices of the good and that are capable of being validated if sufficient data become available. An example of a potentially valid market value model is one that estimates the price for one economic good using estimates provided by a market value model for a second good. Such models might be used where trades for the first good are infrequent, or where the market has imperfections. For example, a potentially valid model for a private placement bond might be based on a market value model for a public bond with similar coupon, ratings and other characteristics. To be considered potentially valid, this model would be expected to replicate any known information about the private placement bond.

Market value models that have not been validated against market data might nevertheless be of practical importance. However, the prices of economic goods that are of most interest to actuaries generally can be modeled by market value models that can be validated.

2.7 PRINCIPLE (Market Value Models). Markets exist in which the prices of specified economic goods can be estimated using a market value model that is valid to within a specified degree of accuracy relative to market prices.

Economists study the reaction of the prices in market to different kinds of information. Some markets are seen to react more quickly and completely to certain information than other markets. Such markets are said to be more "efficient" with respect to that class of information. For example, the equity markets react almost instantaneously to central bank pronouncements about interest rates, whereas the residential real estate market reacts, but more slowly. The "degree of efficiency" of a market relative to a certain class of information depends upon how quickly and clearly the information is transmitted to participants, the extent to which the information is relevant to the participants, and the ability and willingness of the participants to incorporate the information into their trading activities.

DEFINITIONS

An *equilibrium pricing model* is a market value model that estimates prices based on the assumption that market participants will enter into trades until each participant has optimized his or her position relative to his or her preferences and knowledge.

Market value models might imply a particular degree of efficiency with respect to certain types of information. For example, equilibrium pricing models often imply perfect efficiency with respect to specified information. Such models would not be appropriate representations of a market thought to be inefficient with respect to such information. In fact, empirical tests of market efficiency often consist of the comparison of the prices actually observed to those predicted by an equilibrium pricing model.

Markets vary in their degree of efficiency with respect to specified information. For example, the secondary market for individual life insurance contracts that arises from the existence of contractual cash surrender values exhibits a very low degree of efficiency with respect to information about changes in the insured's health after issue. On the other hand, the reinsurance market for blocks of such insurance contracts might exhibit a high degree of efficiency with respect to information about changes in mortality expectations for the blocks, although not with respect to similar information about individual insurance contracts.

The applicability of a specific class of market value models to a given market depends on the mechanism by which information is assumed to affect prices. For equilibrium pricing models, new information might change the relative preferences and beliefs of each participant and thus affect prices in the market. To use such a model requires extensive knowledge about all market participants. Usually, simplifying assumptions are adopted to make the model more tractable. Such simplifications can reduce the real-world applicability of the models.

Another class of market value models, often requiring much less extensive knowledge, is based on the observation that "arbitrage profits" are not usually available in financial markets.

DEFINITIONS

Arbitrage is the extraction of risk-free profits from a series of market trades that requires no net investment. A *no-arbitrage pricing model* is a market value model in which arbitrage cannot occur.

If a participant can construct a portfolio that requires no net investment and generates a nonnegative return under each of the possible scenarios and a positive return under at least one scenario, this portfolio will produce arbitrage. If a market were segmented, arbitrage profits might be earned by simultaneously purchasing and selling the same, or essentially similar, financial instruments in different segments for different prices. Such segmentation might be transient if it occurs at all.

No-arbitrage models are extensively used for pricing "derivative" financial instruments; that is, financial instruments for which the amounts paid to the instrument owners are based on the values of other financial instruments. Options involving the purchase or sale of financial instruments are examples of derivative financial instruments. The assumption of optimality in the definition of equilibrium pricing models implies that all arbitrage opportunities have been exhausted prior to the establishment of equilibrium; hence, equilibrium pricing models are no-arbitrage models. The reverse is not necessarily true.

DEFINITIONS

A *replicating portfolio* for a financial instrument in a given financial market is a portfolio that does not contain the financial instrument and that provides the same net cash flows as the financial instrument under all possible scenarios. A *basis* for a financial market is a subset of the financial instruments traded in the financial market such that a portfolio of instruments from the proper subset can replicate every such financial instrument not in the subset. A financial market is *complete* with respect to a class of financial instruments if it has a basis for that class.

The market for individual insurance or annuity contracts only would be complete if a basis that includes instruments reflecting all insured contingencies can be found. An individual life insurance contract issued on the life of a specific individual, for example, only could be replicated by a portfolio containing instruments contingent on that individual's survival. Current markets for individual insurance contracts are not complete, although related markets for blocks of such insurance contracts might be. Furthermore, arbitrage opportunities might theoretically exist in the market for individual insurance contracts, but the lack of unconstrained secondary markets for most such instruments limits the possibility of realizing arbitrage profits.

In a no-arbitrage model, the price estimates for two portfolios that have the same net cash flows under all scenarios clearly must be the same. Principle 2.8 states that this result is commonly observed in financial markets.

2.8 PRINCIPLE (Law of One Price). In most financial markets, two portfolios that have the same net cash flows under all possible scenarios will trade at the same price.

3. PRINCIPLES UNDERLYING RISK MANAGEMENT AND ACTUARIAL MODELING

The identification, quantification and management of risk underlie actuarial science. The term "risk" is used to name both a phenomenon (an occurrence that can be observed) and a specific subject (a person, thing or collection of persons and things) in connection with which the occurrence is observed. Thus, the "risk" labeled "the death of John Doe" is a phenomenon that can be observed in connection with the "risk" whose name is John Doe.

DEFINITIONS

An *actuarial phenomenon* is a stochastic phenomenon that comprises a random number of stochastic phenomena, the outcomes of which must occur within specified period(s) of time and are considered desirable or undesirable by one or more participants in an economic system. The *actuarial risk variables* associated with an actuarial phenomenon are: (1) *frequency,* the nonnegative integer-valued random variable representing the number of component stochastic phenomena comprised; and, for each of these component phenomena, (2) *timing,* the nonnegative, possibly multivariate, random variable measuring the time at which outcomes associated with the phenomenon occur and (3) *severity*, a possibly multivariate random variable measuring the undesirability of these outcomes.

To be an actuarial phenomenon, a stochastic phenomenon must occur in the context of an economic system and must have various possible outcomes. Actuaries are typically interested in phenomena with economic consequences for which the outcome, either undesirable or desirable, is measured in monetary units.

The next few paragraphs contain five examples of actuarial phenomena. The first (fire insurance) example exhibits all three actuarial risk variables: frequency, timing and severity. The second (mortality study) example illustrates the existence of frequency without severity. The third (group life insurance) example illustrates that phenomena can have severity without frequency. The fourth (medical malpractice insurance) example illustrates the importance of the time frame in which the outcomes of the phenomena occur. The fifth (residential mortgage-backed security) example illustrates frequency, timing and severity in a different way from the other examples.

Example 1 – Fire insurance

The phenomenon comprises all the fires that strike a particular insured building (the risk subject) in a given period of time. The number of fires is a random variable — the frequency. Each fire is itself a stochastic phenomenon. The outcomes associated with the fire include the ignition and the quenching of the flames, the extent of damage, payments made in connection with the insurance, and payments for the legal defense or the settlement of lawsuits occasioned by the fire. Each of these outcomes must occur within the specified period to be considered part of the fire phenomenon. The timing variable associated with the fire is the multivariate random variable giving the times of certain outcomes, such as the time the fire starts, the time at which it is extinguished, the times any payments are made by the insurer, and the times any lawsuits are settled. Similarly, the severity is a multivariate, random variable giving some measurement of the undesirability of each such outcome; for example the percentage of the building damaged, the amounts of any insurance payments or the amounts paid to settle any lawsuits.

Example 2 – A mortality study

The study involves 100 individuals at age 50 who are each observed for five years. If the phenomenon is specified as being the compound phenomenon consisting of each individual's survival, the frequency is the number of observed deaths and the outcomes of the timing variable are ages at death (or the age at the end of the study for survivors), less 50. There is no severity risk variable, illustrating that not all three of the actuarial risk variables need to be present. This example also illustrates that actuarial phenomena need not involve monetary payments.

Example 3 – Group life insurance

A group life insurance program provides a salary-related death benefit for employees who die in the next calendar year. The actuarial phenomenon, and the corresponding actuarial risk variables, can be specified in more than one way. The phenomenon could be specified as a compound phenomenon, with each component being the survival or death of a specific employee during the calendar year. If the phenomenon is specified in this manner, the example is seen to be analogous to the fire damage example just discussed. Each of the component phenomena has a severity (the employee's benefit amount) and timing (dates of death and benefit payment) associated with it.

Alternatively, the actuarial phenomenon could be specified as the number of deaths within the group during the calendar year. This is a natural way to specify the phenomenon if, for example, the benefit is a flat dollar amount instead of being salary-related. In this case, the phenomenon is not compound and there is no frequency variable. The severity is the number of deaths or, equivalently, the total benefits paid, perhaps accumulated to the end of the year with interest. If the phenomenon is specified in this way, there is no timing variable. The only actuarial risk variable is severity.

Example 4 – Medical malpractice insurance

Consider medical malpractice liability insurance covering a single doctor. One version of such an insurance contract (called an "occurrence" contract) provides coverage for any medical incident occurring within a given calendar year. The actuarial phenomenon can be specified as the set of such incidents occurring within the year that eventually lead to a claim being reported. Each such incident is a stochastic process. The frequency is the number of such incidents. A second version of the insurance contract (called a "claims-made" contract) provides coverage for claims that are reported within a given calendar year, when such claims are made with respect to incidents taking place after a specified date, which might be prior to the beginning of the calendar year. The actuarial phenomenon in this version can be specified as the set of such claims, and the frequency is the number of such claims.

The concept of time is used in two distinct ways: (1) An actuarial phenomenon is associated with a specified period of time, the specification of which can be important, as illustrated in the claims-made insurance contract example. (2) The timing risk variable measures the times of the actual occurrences of events associated with the actuarial phenomenon.

Example 5 – Residential Mortgage-Backed Securities

A portfolio of home mortgages develops cash flows that can be combined to form an asset that can be sold to investors. Each such asset involves a set of actuarial phenomena, one of which is the set of prepayments of principal, and thus has its own sets of random variables tied to frequency, severity and timing. A homeowner chooses to take out a mortgage to pay for a house, which is a contract to pay interest and repay principal at a rate as least as fast as initially specified. Prepayments consist of all payments made on the mortgage in excess of the agreed-to schedule and might occur each month or all at once. From the point of view of the investors, each prepayment is itself a stochastic phenomenon. The number of prepayments is the frequency random variable. The outcomes associated with each prepayment phenomenon are affected by the availability and rates of alternative mortgages and investments, the desire and ability of the homeowner to pay off the loan, and the sale of the home for various reasons. The timing variable associated with the prepayment phenomenon is the random variable representing the borrower's choice of date of prepayment. Similarly, the severity is a multivariate random variable measuring the amount of each prepayment.

An actuarial phenomenon might be related to other phenomena, including other actuarial phenomena. For example, the behavior of policyholders (such as their likelihood of continuing to pay premiums on an insurance contract) might be related to economic conditions (such as the level of stock prices or interest rates).

DEFINITIONS:

An *actuarial risk* of concern to a participant is an actuarial phenomenon together with one or more random variables, called *economic consequences*, that assign a positive (negative) number to outcomes that the participant finds desirable (undesirable) and zero to other outcomes. Two or more actuarial risks are *combinable* if for each economic consequence of one there are corresponding economic consequences of the others such that the sum of each set of corresponding economic consequences is an economic consequence of another actuarial risk, called the *combination* of the actuarial risks. A *net economic consequence* of combinable actuarial risks is the sum of corresponding economic consequences from the actuarial risks. A *risk subject* is a person, thing, or collection of persons or things in connection with which the actuarial phenomenon associated with a given actuarial risk can be observed. Two combinable actuarial risks are *similar* if their associated actuarial risk variables are the same, but they have different risk subjects.

In Example 1 above, the actuarial risk of concern to the insurance company is the actuarial phenomenon of fire in the insured building, together with the economic consequences represented by the payments the insurer is required to make. Examples 3 and 4 also have actuarial risks of concern to an insurance company. In each of these examples, there is another actuarial risk associated with the same phenomenon that is of concern to the participants who are considering purchasing insurance. Note that the severity random variable, if it exists, represents an economic consequence for one or another participant.

An example of combinable actuarial risks is a portfolio of retirement annuity contracts and a second portfolio of life insurance contracts. The payment of premiums and benefits from the two portfolios can be combined into a single portfolio with the same types of cash flows. The individual contracts in each portfolio are similar risks. The actuarial risk variables relating to the insurance coverage are the same, but the outcomes differ by risk subject. Different outcomes can result in this portfolio having positive or negative consequences based on the specific characteristics of the portfolio.

Certain collections of actuarial risks involve the same or related economic consequences. For example, the actuarial risk of claims resulting from $100,000 life insurance contracts issued to selected 45-year-old males and the actuarial risk of claims resulting from $200,000 insurance contracts for similarly selected insureds might be similar except for the severity of the claims that occur. The severity random variable is, at least approximately, scaled by the amount of insurance. Recall that a scaling factor that relates the expected value of a random variable over a collection of phenomena is called an exposure measure.

3.1 PRINCIPLE (Scaling). For many collections of actuarial risks that share a specified economic consequence, there exist one or more exposure measures that scale the economic consequence over the collection of the actuarial risks.

For the example of life insurance covering 45-year-old males, the face amount of the insurance contract is an exposure measure satisfying Principle 3.1. Another example of Principle 3.1 arises in the case of an insurer deciding how many similar automobile drivers it has sufficient resources to cover. The actuarial risks represented by 100, 200, … liability insurance contracts on a class of similar drivers, all offering the same coverage, is a collection of actuarial risks sharing a specified economic consequence (claim payments arising from accidents) that are scaled by the number of covered drivers. The number of drivers is an exposure measure satisfying Principle 3.1. However, the collection of actuarial risks represented by insurance contracts covering losses up to $10,000, $20,000, … on one of these drivers is not scaled by the coverage limit.

Much of actuarial science is devoted to the management of actuarial risks, which involves the identification of risk and its quantification. One particular way to manage an actuarial risk is through the exchange of money or other economic goods in return for the transfer of the actuarial risk. The development and management of systems that facilitate this exchange are of special interest to actuaries.

DEFINITIONS

Risk identification is a process for determining the actuarial risks that are relevant to a given participant in an economic system. *Risk assessment* is a process whereby the economic consequences of an actuarial risk for a given participant are evaluated. *Risk management* is a process intended to mitigate some or all of the undesirable economic consequences or to take advantage of some or all of the desirable economic consequences of an actuarial risk for a given participant. *Risk transfer* is a type of risk management in which another participant assumes certain rights or obligations related to some or all of the economic consequences of an actuarial risk. A *risk-management system* is an arrangement under which risk identification, risk assessment and risk management processes are applied to a set of actuarial risks of concern to a participant or group of participants. A *success criterion* for a risk-management system is a condition required to satisfy the system's purposes. *Failure of a risk-management system relative to a success criterion* occurs when the system first fails to satisfy that success criterion.

Many risk-management systems are primarily concerned with actuarial risks with economic consequences that are uncertain with respect to amount or timing. These risk-management systems often utilize strategies that combine actuarial risks so that the degree of uncertainty of the resulting net economic consequence is less than the degree of uncertainty of any of the components.

3.2 PRINCIPLE (Combinations of Actuarial Risks). The degree of uncertainty of each economic consequence of a combination of actuarial risks reflects both the degree of uncertainty of the economic consequences of the component actuarial risks and their correlation.

Note that Principle 3.2 does not indicate that the uncertainty of an economic consequence is necessarily reduced when actuarial risks are combined.

DEFINITIONS

Pooling is a risk-management process under which similar actuarial risks are combined. *Diversification* is a risk-management process under which combinable actuarial risks with at least one set of corresponding economic consequences having correlation coefficient less than one are combined. *Hedging* is a risk-management process under which combinable actuarial risks with at least one set of corresponding economic consequences having correlation coefficient less than zero are combined. A *hedging instrument* for a specified actuarial risk is an economic instrument with an associated actuarial risk that is used to manage the specified actuarial risk through hedging. A *hedge* is a portfolio of hedging instruments.

Pooling, diversification and hedging are common risk-management strategies. The definition of pooling does not imply that it will always be beneficial. However, the degree of uncertainty is often reduced after pooling. These processes are not mutually exclusive. For example, when pooling is employed to manage risk, as in insurance systems, it might be possible to select risk subjects in such a way as to ensure that the pooled risks are diversified.

In applying the hedging process, a second actuarial risk having economic consequences that are negatively correlated with those of the original actuarial risk is identified and combined with the first risk. For example, financial instruments such as options or swaps having additional risks that partially or fully offset existing risks might be included or added to a portfolio. "Perfect hedging" involves identifying a hedge with economic consequences such that the degree of uncertainty of the combined actuarial risks is zero. "Macro-hedging" utilizes a hedge made up of a portfolio of financial instruments whose economic consequences are negatively correlated with the net economic consequences of a combination of a large number of actuarial risks. An example of macro-hedging is asset-liability matching. Macro-hedging might be contrasted with "micro-hedging" — the use of a single financial instrument to hedge an economic consequence of a single actuarial risk.

Actuaries apply models to assess and understand actuarial risks associated with risk-management systems.

DEFINITIONS

An *actuarial model* is a mathematical model of one or more actuarial risks. An *actuarial value* is a numerical value assigned to a given set of actuarial risks that is determined using an actuarial model of those risks. The *degree of confidence* assigned by a participant to an actuarial value developed by an actuarial model is a measure that assigns higher values to actuarial values that the participant believes are closer to the true values. The *failure probability* for a given risk-management system relative to a specified success criterion is a participant's assessment of the probability that failure relative to that criterion will occur within a specified period of time.

Actuarial phenomena are stochastic by definition, but an actuarial model might represent a given risk either stochastically or deterministically. A stochastic actuarial model will incorporate at least one probability distribution in connection with the representation of its actuarial risk variables. To determine a failure probability, the model must be stochastic. A deterministic actuarial model will incorporate sets of values, often representing the means of the underlying distributions. Principle 1.2 states that models of stochastic phenomena can be based on combinations of the outcomes of experiments performed on the phenomenon, on observations of related phenomena or on knowledge related to the phenomenon itself.

The failure probability need not be an estimate as defined in Section 1. Therefore, the degree of confidence differs from the degree of uncertainty in that it can be subjective and need not involve random variables.

Components of an actuarial model might exhibit a functional dependence upon one another. For example, the underlying actuarial risks might be related (a person in poor health is less likely than a healthy person to surrender a life insurance contract in the next year) or changing one distribution or value might result in a change in another (when the rate of unemployment increases, the rate of disability claims may also increase).

Actuarial risks often have as consequences the creation of rights or obligations that involve cash receipts or disbursements.

DEFINITION

A *financial consequence* of an actuarial risk is an economic consequence that is a financial instrument.

Actuarial models are frequently used by actuaries when estimating the values a participant (for example, an insurance company, a consumer of insurance products, a pension sponsor or a pension plan participant) assigns to financial instruments involving actuarial risks. Estimates developed using actuarial models can be expressed as ranges instead of point values.

Recall that a cash-flow model incorporates economic scenarios and assigns a cash flow to each scenario for each financial instrument that is being modeled. If the cash-flow model is a present value model, it can be used to estimate expected present values for each of the financial instruments.

DEFINITION

An *actuarial present value model* of an actuarial risk is a present value model of financial instruments associated with the actuarial risk.

Note that an actuarial present value model is an actuarial model. Actuarial present value models can be used to calculate values that are functions of the values of financial instruments, such as capital requirements and premiums for insurance systems. Success criteria for risk-management systems can sometimes be stated using values produced by actuarial present value models.

For the actuarial present value model to be useful in estimating values that would be assigned by a participant, its economic scenarios and the cash flows assigned to financial instruments should be consistent with the participant's view of the future. Also, if the participant's assignment of current monetary values to specific financial instruments is known, it should be possible to determine a present value metric that closely reproduces them. Principle 3.3 states that it often is possible to construct such models.

3.3 PRINCIPLE (Actuarial Present Value). If a participant in an economic system has assigned current monetary values to certain financial instruments, it is usually possible to construct an actuarial present value model and to specify a present value metric so that, at a given time, the value obtained by applying the present value metric to each of the specified financial instruments is equal to or nearly equal to the assigned current monetary values.

DEFINITION

The *actuarial present value* of a portfolio for a participant relative to an actuarial present value model and a present value metric satisfying Principle 3.3 is the sum of the values obtained by applying the metric to the financial instruments in the portfolio.

A participant's actuarial present values for a portfolio incorporate the participant's preferences, including time preference and risk preference, as well as the participant's views (perhaps influenced by consultation with an expert, such as an actuary) about the future events described by the scenarios. A non-financial asset or liability that has been assigned a current monetary value could be represented in an actuarial model by a set of cash flows and scenarios with an actuarial present value equal to the current monetary value.

Success criteria for risk-management systems are often stated in terms of an actuarial present value relative to some metric. The expected present value metric, the value-at-risk metric and the conditional tail expectation metric described in Section 2 all have been used for this purpose. For example, a self-insurance plan might state its success criterion as having an amount of assets between the 50^{th} and 75^{th} percentile of the distribution of the corresponding liabilities.

Because actuarial present value models incorporate economic models, by definition they involve probability-weighted sets of economic scenarios as is necessary to calculate expected present values. An actuarial model might also be used in conjunction with a set of scenarios selected to understand the sensitivity of the estimates to the assumptions underlying the scenarios. If a present value function is assigned to each of these scenarios, the sensitivity of present value estimates can be evaluated. The selected scenarios are not necessarily assigned probabilities, as would be the case if an actuarial present value model were used. The set of selected scenarios often includes unlikely scenarios to exhibit the sensitivity of the present value estimates to extreme conditions. Note that the average of the present values of such selected scenarios is not necessarily an expected present value.

Recall that the degree of accuracy of a mathematical model measures how closely values calculated using the model reproduce observed values. As additional data become available or the environment changes, the degree of accuracy might change, perhaps requiring that the model be changed.

3.4 PRINCIPLE (Continued Validity of Actuarial Models). The degree of accuracy of an initially valid actuarial model might change over time depending on changes in factors such as:

a. the environment in which it is applied;

b. knowledge and understanding of this environment; and

c. data available to validate the model.

Changes in the environment might include contractual, regulatory, judicial, financial, economic and social changes. As time passes, additional data might make possible the validation of, and thus the determination of a degree of accuracy for, a model that was initially only potentially valid. Any of the changes listed above might lead to an increase or a decrease in the degree of accuracy of the model. In addition, such changes might lead to an alteration of components of the model or indicate that an entirely new model is needed.

Actuaries use models to estimate failure probabilities and related quantities, and might want to assign a degree of confidence to such estimates. As a first step, it is important to understand the various dependencies of the estimate. Among these are the dependencies of the failure probabilities on the actuarial risks involved, the risk-management techniques applied, and the success criterion specified, as well as the characteristics of the actuarial model.

3.5 PRINCIPLE (Probability of Failure). The degree of confidence assigned by a participant to the probability of failure relative to a specified success criterion that is developed by an actuarial model of a risk-management system depends on the participant's understanding of:

a. the actuarial risks managed by the risk-management system;

b. the characteristics of the risk-management system;

c. the characteristics of the actuarial model, including its representation of the success criterion; and

d. other actuarial risks that affect the success criterion.

To have a high degree of confidence in a failure probability produced by an actuarial model, all the significant and relevant actuarial risks and the relevant success criteria must be represented in a way that is reasonably formulated and quantified. Identification of an actuarial model that adequately represents all significant and relevant actuarial risks, such as those with asymmetrical distributions, can be difficult. Other characteristics of actuarial risks, such as correlations between the risks, might also affect the degree of confidence in model results. Similarly, the set of success criteria of the risk-management system might be difficult to represent faithfully. For the calculated failure probability to be useful, the model must, at a minimum, include distinguishable representations of outcomes that satisfy and fail to satisfy each success criterion. The validity of the model, as measured by its degree of accuracy, might also affect the degree of confidence in the model's failure probability. Principle 3.4 points out that the degree of confidence can vary over time.

4. PRINCIPLES UNDERLYING FINANCIAL SECURITY SYSTEMS

Additional principles apply to a class of risk-management systems that provide benefits related to a specified set of actuarial risks and associated risk subjects in return for one or more payments to the system.

DEFINITIONS

> A *financial security system* is a risk-management system in which participants might transfer one or more actuarial risks, including their associated undesirable economic consequences, to a second party, called a *provider,* in return for a specified payment or set of payments made, in cash or in kind. The *consideration* paid to a financial security system on behalf of a participant in the system is the payment or set of payments made, in cash or in kind, to the provider by or on behalf of that participant.

A participant in a financial security system transfers one or more actuarial risks. For example, the participant might transfer the actuarial risk of loss from damage to a building due to a fire occurring during the next year. The risk subject for this actuarial risk is the building. For certain types of financial security systems, such as life insurance and pension plans, the participant might be the risk subject associated with the actuarial risk.

Examples of financial security systems include insurance, annuity, retirement and health-care systems. Examples of risk-management systems that are not financial security systems are self-insurance and loss-control systems (such as the purchase of automatic sprinkler systems) that mitigate risk but do not involve its transfer to another entity. The definition of a financial security system encompasses both systems having two or more providers (e.g., the insurance industry) and those having a single provider (e.g., a specific insurance company or a social insurance program).

Examples of considerations are the single premium or the set of premium payments made to an insurance company to obtain insurance or annuity coverage, the annual contribution of an employer to a pension plan for its employees and the taxes paid to a government for a government-sponsored health-care financing program such as Medicare in the United States.

Often, at least part of the consideration is available for investment in one or more types of assets for some period of time. Investment income on these assets might represent an additional source of funds for the providers of a financial security system.

DEFINITIONS

A *mandatory financial security system* is a financial security system in which all members of a specified group must participate. A *voluntary financial security system* is a financial security system that is not mandatory.

For a mandatory financial security system, the obligations created by the risk transfer might be specified in a law, regulation or employment agreement. Social insurance systems in which all persons meeting a given set of criteria must participate are mandatory financial security systems. For a voluntary financial security system, the obligations created by the risk transfer are usually described in a contract. Private insurance arrangements are usually voluntary.

The transfer of a specific actuarial risk to a specific provider might be simultaneously part of a mandatory and a voluntary financial security system. For example, an employer can offer a group insurance program in which all employees must participate. The group insurance plan made available to the employees by an insurance company in return for the consideration paid by the employer is a mandatory financial security system, whereas the group insurance market from which the employer selects the provider is a voluntary financial security system.

Risk subjects associated with a given set of actuarial risks, such as those covered by a financial security system, often share similarities that can be described quantitatively.

DEFINITION

A *risk measure* for a specific quality on a set of risk subjects is a function that assigns a numerical value to each of the risk subjects for that quality. A *risk characteristic* is a quality that can be quantified by a risk measure.

Examples of possible risk characteristics for life insurance include gender, smoking status, blood pressure reading and cholesterol level. For a given risk characteristic, there might be more than one possible risk measure. For example, if systolic blood pressure is a risk characteristic, the risk measure could be the measurement itself. Alternatively, the risk measure could be based on a grouping of systolic blood pressure measurements into low and high ranges, such as 139 or below being assigned a value of 0 and 140 or above assigned a value of 1. A third alternative is a "membership function" that assigns, for example, membership value 0 if the reading is below 100 and membership value (x-100)/x if the reading x is at or above 100.

A quality with two or more discrete states such as gender or smoking status can be assigned values such as 0, 1, … and thus can be a risk characteristic.

The following principle discusses the use of risk characteristics in connection with an actuarial model of a set of combinable actuarial risks.

4.1 PRINCIPLE (Risk Classification). For a given set of combinable actuarial risks, it is possible to identify a set of risk characteristics and risk measures for the associated risk subjects and a valid or potentially valid actuarial model of the actuarial risks such that:

> **a. each actuarial risk is assigned by the model to a set of random variables representing the actuarial risk variables associated with the actuarial risk; and**
>
> **b. all actuarial risks whose associated risk subjects are assigned the same values by the risk measures associated with each of the identified risk characteristics are assigned to the same set of random variables.**

Continuing the earlier blood pressure example, an actuarial model could assign a standard mortality table to those with risk measure 0 (in the example, 139 or below) and a substandard mortality table to those with risk measure 1 (in the example, 140 and above).

DEFINITION

> A *risk-classification system* for a set of combinable actuarial risks is a set of risk characteristics and risk measures for the associated risk subjects, together with an actuarial model, that satisfies Principle 4.1. A *risk class* of a risk classification system is a set of actuarial risks that are represented by the same set of random variables in the actuarial model associated with the risk-classification system.

There might be more than one possible risk-classification system for a given set of actuarial risks and their risk subjects. For blood pressure, the risk classes could be "139 or less" and "140 or more" as in the earlier example. Another possibility could be the ranges "129 or less," "130 through 149," and "150 or more." Additional considerations, such as marketing strategy, the competitive environment, expected participant behavior, social attitudes, and legal or regulatory constraints, have sometimes been taken into account to determine which risk-classification system to use for a given purpose.

DEFINITIONS

> A *refinement of a risk-classification system* is a risk-classification system with the same actuarial model as the original risk-classification system such that each risk class is either

identical to or a subset of a risk class of the original risk-classification system. The refinement is said to be *more homogeneous* than the existing system if there are at least two actuarial risks that were assigned to the same risk class under the original system and are assigned to different risk classes under the refinement.

A refinement of a risk-classification system might be accomplished by changing the risk measure and thus changing the risk classes. The three-class system in the previous blood pressure example is not a refinement of the two-class system because there is no way to form two classes from the three risk classes to create the original two risk classes. A risk-classification system having the risk classes "139 or below," "140 through 169," and "170 or above" is a refinement of the original two-class system. For these classes to be risk classes, different mortality tables must be assigned to each. The refinement will be more homogeneous as long as each of the two new risk classes includes at least one actuarial risk, so that two actuarial risks that formerly were assigned the same mortality table are now assigned different tables.

A risk-classification system having risk classes in one-to-one correspondence with the blood pressure measurement itself is a refinement of a risk-classification system having risk classes in correspondence with the "membership function" discussed above, even though the respective risk measures assign different values to every risk class.

The accuracy of estimates of the economic consequences of an actuarial risk tends to be better for a risk-classification system that is more homogeneous than another. For example, using different random variables for those in one class, such as those with blood pressure from 140 through 169 and those in another class such as those with blood pressure of 170 and over, will usually produce more accurate estimates than using the same random variable for all those with blood pressure higher than 139. However, consistent with Principle 1.1, in most cases less data will be observed for each of the two narrower blood pressure ranges than for the single range. The estimated distribution for each of the two random variables usually will have a higher degree of uncertainty than will the estimated distribution of the random variable for the single range.

4.2 PRINCIPLE (Effect of Refinement). The effect of a refinement of a risk-classification system on the degree of uncertainty of an estimate of an economic consequence associated with an actuarial risk is the combined effect of a reduction in uncertainty due to increased homogeneity and an increase in uncertainty due to reduced data for each of the refined classes.

Principle 4.2 implies that over-refinement of a risk-classification system might lead to less accurate estimates.

Actuaries often are asked to determine a set of considerations for a provider in a financial security system that satisfies certain success criteria. The actuary might begin by developing a risk-classification system for the actuarial risks that are being considered for coverage by the provider. It might be determined that accepting the actuarial risks belonging to one or more of the risk classes for coverage is incompatible with the specified success criteria, such as the long-term financial viability of the provider or the affordability of coverage for the participants. In the absence of external constraints, such actuarial risks might be excluded from coverage. Consideration might then be specified for the remainder of the risk classes in a way that is consistent with the potential obligation created by the transfer of these risks.

DEFINITIONS

The *exposure* of an actuarial risk is the scaling factor assigned to the economic consequences associated with that actuarial risk by an exposure measure. A *rate structure* for a provider in a financial security system is (1) a set of exposure measures, each of which assigns an exposure to each of the actuarial risks transferred to the provider and (2) a rule that assigns a number called a *rate* to each of these actuarial risks so that the consideration for a participant is the sum of the products of the exposure and the rate for each actuarial risk transferred to the provider by the participant.

The exposure can be one of several factors that determine the rate; i.e., a rate structure is not necessarily linear with respect to the exposure. For example, the rate for the first $1 million of exposure might not be the same as the second $1 million of exposure.

DEFINITIONS

A rate structure for a provider in a financial security system is *compatible with* a risk-classification system for a set of actuarial risks transferred by the participants if (1) there is a set of rates for every risk class in the risk-classification system and (2) every actuarial risk in a given risk class is assigned the same rate. A *refinement of a rate structure* for a provider in a financial security system is a rate structure that (1) has the same set of exposure measures as the original rate structure, (2) is compatible with a refinement of a risk-classification system with which the original rate structure is compatible, and (3) assigns the same rates to actuarial risks in any risk class in the refinement that is identical to a risk class in the risk-classification system of the original rate structure, and increased rates to some actuarial risks and decreased rates to other actuarial risks in other risk classes.

As an example, suppose a life insurance rate structure assigns a single rate to all those whose age is younger than 18 and a distinct rate for each age 18 and older. It is subsequently replaced by a rate structure that assigns distinct rates for each age from 0 through 17. For the new rate structure to be a refinement of the original rate structure, the rates for ages 18 and above must be unchanged.

Principle 4.3 applies the concept of enlightened self-interest described in Principle 2.6 to a rate structure for a provider in a voluntary financial security system.

4.3 PRINCIPLE (Antiselection). If the rate structure of a provider in a voluntary financial security system is based on a risk-classification system such that a refinement of the system could result in significant differences in consideration among participants originally assigned to the same risk class, there will be a tendency for relatively greater participation in the system and with the provider by those whose considerations would increase if the refinement were put in place.

DEFINITION

A rate structure of a provider in a voluntary financial security system is *subject to antiselection* when a potential refinement of the current rate structure would produce differences in consideration among participants in an existing risk class.

Entry into a voluntary financial security system occurs by means of a transaction, because it involves the transfer to a provider in the system of risk in return for the payment of a consideration. If relevant information is not available to, or due to law, regulation or custom cannot be used by a provider, entry into the system can be subject to adverse selection. If a provider in a voluntary financial security system is unable to obtain or utilize information about a risk characteristic that otherwise would be used to define its risk-classification system, there will be a tendency for relatively greater participation by those who would have been assigned higher considerations had the information been available, and the provider will be subject to antiselection. However, antiselection also can occur if the provider chooses not to use all the information it has gathered. If, for example, the expected cost differs by gender or by age, and if an insurer does not use age or gender as a risk characteristic, that insurer might encounter an increase in participation by risk subjects whose genders or ages are associated with relatively higher expected cost.

It is not necessary for participants to recognize that a potential refinement of a rate structure exists for antiselection to occur. In a voluntary financial security system, greater participation can be expected from those who perceive that the cost of participation is low relative to its value, compared to those who have the opposite perception. An example is a group insurance program in which young employees perceive that their expected benefits are less than their scheduled considerations. This difference in perception can explain the tendency for greater participation as described in Principle 4.3. The effects of antiselection in a voluntary financial security system usually can be mitigated by the use by providers of rate structures that are sufficiently refined.

Because, in contrast with a voluntary security system, a mandatory financial security system cannot reject potential participants and potential participants must participate; antiselection cannot exist with respect to the risks that the financial security system covers.

A risk-classification system often is designed on the basis of information regarding or related to the actuarial risks of the risk subjects.

DEFINITIONS

The *experience* of a risk-management system is data observed in connection with the operation of the system, including data regarding the associated actuarial risk variables.

Section 2 indicates that a transaction involving a contract is said to be subject to moral hazard if the existence of the contract creates a potential for actions subsequent to the transaction by one party that might have an adverse effect on the other party. The existence of a financial security system might, by itself, change the future experience of that system.

4.4 PRINCIPLE (Moral Hazard). The actuarial risk variables associated with an actuarial risk covered by a financial security system tend to differ from those associated with the same actuarial risk occurring in the absence of any such system.

In the absence of other factors, the availability of certain coverages, such as dental insurance that pays in full for periodic checkups but only in part for other dental procedures, is very likely to cause the frequency both of checkups and of more severe consequential dental incidents to change.

Principle 4.3 observes that antiselection in the context of a voluntary financial security system might change its experience by influencing who will participate, whereas Principle 4.4 observes that the existence of a financial security system might change the behavior of those who participate.

Financial security systems involve the transfer of actuarial risks that have future economic consequences. In some cases these consequences can emerge over many years. In managing these economic consequences, it is necessary to appropriately consider both short-term and long-term perspectives. Actuaries might be asked to review certain practices adopted by the providers in a financial security system and to opine about their effect. Such practices might include underwriting and eligibility rules, rate structures, funding levels, investment guidelines and expense levels.

"Actuarial soundness" is a term that is used in connection with the assessment of the ability of a provider in a financial security system to satisfy its obligations. This term can be used in relation to the financial security system as a whole, a provider in a financial security system, or specific elements of a financial security system that are affected by the actuarial risks assumed by a provider in the system. Actuarial soundness only can be defined in a specific context if one criterion or more related to the ability of the provider to satisfy its obligations is specified.

Principle 4.5 defines actuarial soundness in the context of a provider in a financial security system.

4.5 PRINCIPLE (Actuarial Soundness). For a set of actuarial risks assumed by a provider in a financial security system during a specified period of time, there might be one or more combination of (1) considerations charged or rights to receive considerations and (2) amount and composition of assets held by the provider that will enable the provider to satisfy its obligations associated with the assumption of the actuarial risks.

It is possible that some combinations of considerations and assets are not compatible with satisfying the provider's obligations to pay benefits and expenses for the actuarial risks assumed by the provider. In other words, it might not be possible to find a combination that will ensure that all obligations will be satisfied at all future times. Some practices increase the likelihood of satisfying these obligations, whereas others decrease it. A provider's practices, such as risk selection, rate structures or funding levels, can affect the likelihood that the provider's obligations will be satisfied.

DEFINITION

A *degree of actuarial soundness* of a provider in a financial security system is a measure of the ability of the provider to satisfy the obligations associated with the actuarial risks it has assumed.

Practices that increase the likelihood that the provider will satisfy its obligations increase the degree of actuarial soundness of the provider.

GLOSSARY — THE PRINCIPLES

1.1 PRINCIPLE (Statistical Regularity). Phenomena exist such that, if a sequence of independent experiments is conducted under conditions that are substantially similar to a set of specified conditions, the proportion of occurrences of a given event converges as the number of experiments becomes large.

1.2 PRINCIPLE (Basis for Model Construction). A model of a specific stochastic phenomenon can be based on the outcomes of experiments performed on that phenomenon, on observations of related phenomena, on knowledge related to the phenomenon itself or on a combination of all three.

1.3 PRINCIPLE (Credibility). It is possible to determine estimators of the relative weights to be assigned to each of several estimates of a particular quantity to optimize a given measure of the accuracy of the weighted average.

2.1 PRINCIPLE (Preference or Indifference). At a given time, for almost all pairs of economic goods, a participant either prefers one economic good to the other, or is indifferent between them.

2.2 PRINCIPLE (Diversity of Preferences). In most economic systems, there are pairs of economic goods for which some participants prefer the first good to the second, and others prefer the second good to the first.

2.3 PRINCIPLE (Time Preference). Participants tend to prefer shorter deferral periods for otherwise identical economic assets, and longer deferral periods for otherwise identical economic liabilities.

2.4 PRINCIPLE (Risk Aversion). For a given measure of the degree of uncertainty, when choosing between two wagers with the same expected payoff, participants tend to prefer the wager with the lower degree of uncertainty.

2.5 PRINCIPLE (Existence of Money). In many economic systems, there is a commodity such that, at a given time, for most desirable economic goods and for most participants, there is an amount of the commodity such that each participant is indifferent between the economic good and that amount of the commodity.

2.6 **PRINCIPLE (Enlightened Self-interest). The parties to an economic transaction act in accordance with their preferences, subject to the knowledge each has about the environment and about the other parties.**

2.7 **PRINCIPLE (Market Value Models). Markets exist in which the prices of specified economic goods can be estimated using a market value model that is valid to within a specified degree of accuracy relative to market prices.**

2.8 **PRINCIPLE (Law of One Price). In most financial markets, two portfolios that have the same net cash flows under all possible scenarios will trade at the same price.**

3.1 **PRINCIPLE (Scaling). For many collections of actuarial risks that share a specified economic consequence, there exist one or more exposure measures that scale the economic consequence over the collection of the actuarial risks.**

3.2 **PRINCIPLE (Combinations of Actuarial Risks). The degree of uncertainty of each economic consequence of a combination of actuarial risks reflects both the degree of uncertainty of the economic consequences of the component actuarial risks and their correlation.**

3.3 **PRINCIPLE (Actuarial Present Value). If a participant in an economic system has assigned current monetary values to certain financial instruments, it is usually possible to construct an actuarial present value model and to specify a present value metric so that, at a given time, the value obtained by applying the present value metric to each of the specified financial instruments is equal to or nearly equal to the assigned current monetary values.**

3.4 **PRINCIPLE (Continued Validity of Actuarial Models). The degree of accuracy of an initially valid actuarial model might change over time depending on changes in factors such as:**

a. the environment in which it is applied;

b. knowledge and understanding of this environment; and

c. data available to validate the model.

3.5 **PRINCIPLE (Probability of Failure). The degree of confidence assigned by a participant to the probability of failure relative to a specified success criterion**

that is developed by an actuarial model of a risk-management system depends on the participant's understanding of:

a. the actuarial risks managed by the risk-management system;

b. the characteristics of the risk-management system;

c. the characteristics of the actuarial model, including its representation of the success criterion; and

d. other actuarial risks that affect the success criterion.

4.1 PRINCIPLE (Risk Classification). For a given set of combinable actuarial risks, it is possible to identify a set of risk characteristics and risk measures for the associated risk subjects and a valid or potentially valid actuarial model of the actuarial risks such that:

a. each actuarial risk is assigned by the model to a set of random variables representing the actuarial risk variables associated with the actuarial risk; and

b. all actuarial risks whose associated risk subjects are assigned the same values by the risk measures associated with each of the identified risk characteristics are assigned to the same set of random variables.

4.2 PRINCIPLE (Effect of Refinement). The effect of a refinement of a risk-classification system on the degree of uncertainty of an estimate of an economic consequence associated with an actuarial risk is the combined effect of a reduction in uncertainty due to increased homogeneity and an increase in uncertainty due to reduced data for each of the refined classes.

4.3 PRINCIPLE (Antiselection). If the rate structure of a provider in a voluntary financial security system is based on a risk-classification system such that a refinement of the system could result in significant differences in consideration among participants originally assigned to the same risk class, there will be a tendency for relatively greater participation in the system and with the provider by those whose considerations would increase if the refinement were put in place.

4.4 PRINCIPLE (Moral Hazard). The actuarial risk variables associated with an actuarial risk covered by a financial security system tend to differ from those

associated with the same actuarial risk occurring in the absence of any such system.

4.5 PRINCIPLE (Actuarial Soundness). For a set of actuarial risks assumed by a provider in a financial security system during a specified period of time, there might be one or more combination of (1) considerations charged or rights to receive considerations and (2) amount and composition of assets held by the provider that will enable the provider to satisfy its obligations associated with the assumption of the actuarial risks.

GLOSSARY — THE DEFINITIONS

Actuarial Model - An *actuarial model* is a mathematical model of one or more actuarial risks.

Actuarial Phenomenon - An *actuarial phenomenon* is a stochastic phenomenon that comprises a random number of stochastic phenomena, the outcomes of which must occur within specified period(s) of time and are considered desirable or undesirable by one or more participants in an economic system.

Actuarial Present Value - The *actuarial present value* of a portfolio for a participant relative to an actuarial present value model and a present value metric satisfying Principle 3.3 is the sum of the values obtained by applying the metric to the financial instruments in the portfolio.

Actuarial Present Value Model - An *actuarial present value model* of an actuarial risk is a present value model of financial instruments associated with the actuarial risk.

Actuarial Risk - An *actuarial risk* of concern to a participant is an actuarial phenomenon together with one or more random variables, called *economic consequences*, that assign a positive (negative) number to outcomes that the participant finds desirable (undesirable) and zero to other outcomes.

Actuarial Risk Variables - The *actuarial risk variables* associated with an actuarial phenomenon are: (1) *frequency,* the nonnegative integer-valued random variable representing the number of component stochastic phenomena comprised; and, for each of these component phenomena, (2) *timing,* the nonnegative, possibly multivariate, random variable measuring the time at which outcomes associated with the phenomenon occur and (3) *severity*, a possibly multivariate random variable measuring the undesirability of these outcomes.

Actuarial Value - An *actuarial value* is a numerical value assigned to a given set of actuarial risks that is determined using an actuarial model of those risks.

Adverse Selection - A transaction exhibits *adverse selection* if either (1) information asymmetry exists before the transaction is completed or (2) the same relevant information is available to both parties but one party cannot use all that information in deciding whether to complete the transaction on the stated terms, and in either case the terms of the transaction are different than what they would have been otherwise.

Amount - The *amount* of the commodity represented by a given member of the class is the number assigned to that member.

Arbitrage - *Arbitrage* is the extraction of risk-free profits from a series of market trades that requires no net investment.

Asset - See *economic asset (asset)*.

Basis - A *basis* for a financial market is a subset of the financial instruments traded in the financial market such that a portfolio of instruments from the proper subset can replicate every such financial instrument not in the subset.

Cash Flow - A *cash flow* is a series of cash receipts and disbursements at specified points in time.

Cash Flow Model - A *cash-flow model* of a portfolio is an economic model that, for each financial instrument in the portfolio, assigns a cash flow to each of the economic scenarios.

Cash Receipt - A *cash receipt* is an economic good consisting of a single amount of currency received at a specified time.

Combinable - Two or more actuarial risks are *combinable* if for each economic consequence of one there are corresponding economic consequences of the others such that the sum of each set of corresponding economic consequences is an economic consequence of another actuarial risk, called the *combination* of the actuarial risks.

Combination - Two or more actuarial risks are *combinable* if for each economic consequence of one there are corresponding economic consequences of the others, such that the sum of each set of corresponding economic consequences is an economic consequence of another actuarial risk, called the *combination* of the actuarial risks.

Commodity - A *commodity* is a class of desirable economic goods such that (i) all participants agree to the assignment of the same nonnegative number to each member of the class; (ii) given two members of the class that have been assigned different numbers, almost all participants prefer the member with the higher number; and (iii) all participants are indifferent between one set consisting of members of the class and any other such set, as long as the sum of the numbers assigned to the members of the first set equals the sum of the numbers assigned to the members of the second set.

C**ompatible with** - A rate structure for a provider in a financial security system is *compatible with* a risk classification system for a set of actuarial risks transferred by the participants if (1) there is a set of rates for every risk class in the risk-classification system and (2) every actuarial risk in a given risk class is assigned the same rate.

Complete - A financial market is *complete* with respect to a class of financial instruments if it has a basis for that class.

Consideration - The *consideration* paid to a financial security system on behalf of a participant in the system is the payment or set of payments made, in cash or in kind, to the provider by or on behalf of that participant.

Contingent - An event associated with a stochastic phenomenon is said to be *contingent*.

Contract - A *contract* is a mutual commitment by two or more participants in an economic system, called *parties to the contract,* to carry out certain actions.

Correlated - Two random variables are said to be positively (negatively) *correlated* if, for a given outcome, a positive deviation of one random variable from its expected value tends to be accompanied by a positive (negative) deviation of the other random variable from its expected value.

Correlation Coefficient - A *correlation coefficient* is a measure of the extent and direction of correlation of two random variables.

Credibility - The *credibility* of a particular estimator is the relative weight assigned to it when constructing a weighted average.

Currency - The *currency* of a given economic system is the form of money that is accepted as the preferred means of effecting transactions, if such a preferred means exists.

Current Monetary Value - The *current monetary value* assigned to an economic good at a particular time by a given participant, is an amount of currency such that the participant is indifferent between the amount of currency and the economic good.

Data - *Data* are observed occurrences or other facts related to a phenomenon.

Deferral Period - The *deferral period* of an economic asset or an economic liability is the period of time from the present until possession of the specified economic good is to commence, or transfer of the specified economic asset is to occur.

Degree of Actuarial Soundness - A *degree of actuarial soundness* of a provider in a financial security system is a measure of the ability of the provider to satisfy the obligations associated with the actuarial risks it has assumed.

Degree of Confidence - The *degree of confidence* assigned by a participant to an actuarial value developed by an actuarial model is a measure that assigns higher values to actuarial values that the participant believes are closer to the true values.

Degree of Uncertainty - The *degree of uncertainty* of a random variable is a measure of the variation of the values taken on by the random variable from its expected value.

Desirable Economic Good - A *desirable economic good* is an economic good that a participant prefers to receive rather than dispose of.

Deterministic Component - A *deterministic component* of a mathematical model is a representation of an aspect of a given phenomenon expressed without the use of random variables.

Deterministic Model - A *deterministic model* is a mathematical model that consists exclusively of deterministic components.

Disbursement - A *disbursement* is an economic good consisting of a single amount of currency disbursed at a specified time.

Diversification - *Diversification* is a risk-management process under which combinable actuarial risks with at least one set of corresponding economic consequences having correlation coefficient less than one are combined.

Economic Asset (Asset) - An *economic asset (asset)* is the right to possess a specified desirable economic good upon the occurrence of a specified condition.

Economic Consequences - An *actuarial risk* of concern to a participant is an actuarial phenomenon together with one or more random variables, called *economic consequences*, that assign a positive (negative) number to outcomes that the participant finds desirable (undesirable) and zero to other outcomes.

Economic Goods (Goods) - An *economic system* consists of (i) a group of persons, called *participants*, (ii) a set of items, called *economic goods (goods)*, each of which belongs, at a given time, to one of the participants, and (iii) a mechanism, called a *market*, through which participants can exchange the economic goods they currently hold for other economic goods.

Economic Instrument - An *economic instrument* is a set of rights and obligations to hold, receive, or transfer one or more economic goods currently or at specified future times.

Economic Liability (Liability) - An *economic liability (liability)* is an obligation to hold an undesirable economic good or to transfer a specified asset to another participant upon the occurrence of a specified condition.

Economic Model - An *economic model* of an economic system is a set of economic scenarios, each assigned a probability of occurrence, such that the probability that some member of the set will occur is one.

Economic Option (Option) - An *economic option (option)* is a right, but not an obligation, to exchange one set of economic assets or economic liabilities for another set of economic assets or liabilities at a specified future time or during a specified time interval.

Economic Scenario (Scenario) - An *economic scenario (scenario)* for a given economic system is a description representing specified aspects of the economic system at future times.

Economic Swap (Swap) - An *economic swap* (*swap)* is an agreement to exchange specified economic assets or economic liabilities for other economic assets or economic liabilities at one or more specified future times.

Economic System - An *economic system* consists of (i) a group of persons, called *participants*, (ii) a set of items, called *economic goods (goods)*, each of which belongs, at a given time, to one of the participants, and (iii) a mechanism, called a *market*, through which participants may exchange the economic goods they currently hold for other economic goods.

Economic Transaction (Transaction) - An *economic transaction (transaction)* is an exchange of economic goods between two or more participants, called *parties to the transaction*.

Equilibrium Pricing Model - An *equilibrium pricing model* is a market value model that estimates prices based on the assumption that market participants will enter into trades until each participant has optimized his or her position relative to his or her preferences and knowledge.

Equivalent - Two economic goods are *equivalent* for a given participant at a given time if they have the same current monetary value.

Estimate - An *estimate* is a value produced by an estimator.

Estimator - An *estimator* is a function of one or more random variables associated with an experiment.

Expected Payoff - The *expected payoff* of a wager is the expected value of the payoff.

Expected Present Value The *expected present value* of a portfolio relative to a present value model is the expected value of the present value random variable of the portfolio.

Expected Value - The probability-weighted average of the numerical values taken on by a random variable, if the average exists, is called the *expected value* of the random variable.

Experience - The *experience* of a risk-management system is data observed in connection with the operation of the system, including data regarding the associated actuarial risk variables.

Experiment - An *experiment* is an observation of a given phenomenon under specified conditions.

Exposure - The *exposure* of an actuarial risk is the scaling factor assigned to the economic consequences associated with that actuarial risk by an exposure measure.

Exposure Measure - An *exposure measure* is a scaling factor that relates the expected value of one or more random variables over a collection of phenomena.

Failure of a Risk-Management System Relative to a Success Criterion - *Failure of a risk-management system relative to a success criterion* occurs when the system first fails to satisfy that success criterion.

Failure Probability - The *failure probability* for a given risk-management system relative to a specified success criterion is a participant's assessment of the probability that failure relative to that criterion will occur within a specified period of time.

Financial Consequence - A *financial consequence* of an actuarial risk is an economic consequence that is a financial instrument.

Financial Instrument, Financial Asset, Financial Liability, Financial Option or Financial Swap - A *financial instrument*, *financial asset, financial liability, financial option or financial swap* is, respectively, an economic instrument, economic asset, economic liability, economic option or economic swap for which the economic goods are cash flows.

Financial Market - A *financial market* is a market for the exchange of financial instruments.

Financial Portfolio (Portfolio) - A *financial portfolio (portfolio)* is a set of financial instruments.

Financial Security System - A *financial security system* is a risk-management system in which participants can transfer one or more actuarial risks, including their associated undesirable economic consequences, to a second party, called a *provider,* in return for a specified payment or set of payments made, in cash or in kind.

Frequency - The *actuarial risk variables* associated with an actuarial phenomenon are: (1) *frequency,* the nonnegative, integer-valued random variable representing the number of component stochastic phenomena comprised; and, for each of these component phenomena, (2) *timing,* the nonnegative, possibly multivariate, random variable measuring the time at which outcomes associated with the phenomenon occur and (3) *severity*, a possibly multivariate random variable measuring the undesirability of these outcomes.

Goods - See *economic goods (goods).*

Hedge - A *hedge* is a portfolio of hedging instruments.

Hedging - *Hedging* is a risk-management process under which combinable actuarial risks with at least one set of corresponding economic consequences having a correlation coefficient of less than zero are combined.

Hedging Instrument - A *hedging instrument* for a specified actuarial risk is an economic instrument with an associated actuarial risk that is used to manage the specified actuarial risk through hedging.

Indifferent - A participant is *indifferent* between two economic goods if the participant neither prefers the first to the second nor the second to the first.

Information - *Information* is data that changes or has the potential to change a person's knowledge.

Information Asymmetry - *Information asymmetry* exists if at the time an economic transaction takes place there is relevant information that is available to one party to the transaction but not to the other party.

Knowledge - A person's *knowledge* about a phenomenon is the person's assessment of the probabilities associated with the phenomenon.

Liability - See *economic liability (liability).*

Mandatory Financial Security System - A *mandatory financial security system* is a financial security system in which all members of a specified group must participate.

Market - An *economic system* consists of (i) a group of persons, called *participants*, (ii) a set of items, called *economic goods (goods)*, each of which belongs, at a given time, to one of the participants, and (iii) a mechanism, called a *market*, through which participants can exchange the economic goods they currently hold for other economic goods.

Market Price (Price) - The *market price (price)* of an economic good is the amount of currency exchanged for the good in the most recent market trade.

Market Trade - A *market trade* is a transaction involving the exchange of an economic good for money in a market.

Market Value - A *market value* is an estimate of price produced by a market value model.

Market Value Model - A *market value model* for an economic good is a model that produces an estimate of the price at which a market trade for that good would occur at a specified time.

Mathematical Model - A *mathematical model* is a scientific model in which the representation is expressed in mathematical terms.

Monetized - A *monetized* economic system is an economic system with a currency.

Money - *Money* is a commodity satisfying Principle 2.5.

Moral Hazard - A transaction involving a contract is subject to *moral hazard* if the existence of the contract creates a potential for actions by one party subsequent to the transaction that have an adverse impact on the other party.

More Homogeneous - The refinement is said to be *more homogeneous* than the existing system if there are at least two actuarial risks that were assigned to the same risk class under the original system and are assigned to different risk classes under the refinement.

Net Cash Flow - The *net cash flow* of a portfolio is the sum of the cash flows associated with the financial instruments in the portfolio.

Net Economic Consequence - A *net economic consequence* of combinable actuarial risks is the sum of corresponding economic consequences from the actuarial risks.

No-arbitrage Pricing Model - A *no-arbitrage pricing model* is a market value model in which arbitrage cannot occur.

Option - See *economic option (option)*.

Outcome - The result of an experiment is called an *outcome*; an *event* is a set of one or more possible outcomes.

Participants - An *economic system* consists of (i) a group of persons, called *participants*, (ii) a set of items, called *economic goods (goods)*, each of which belongs, at a given time, to one of the participants, and (iii) a mechanism, called a *market*, through which participants can exchange the economic goods they currently hold for other economic goods.

Parties to the Contract - A *contract* is a mutual commitment by two or more participants in an economic system, called *parties to the contract,* to carry out certain actions.

Parties to the Transaction - An *economic transaction (transaction)* is an exchange of economic goods between two or more participants, called *parties to the transaction*.

Payoff - The *payoff* associated with a wager is a random variable that assigns to each outcome a number representing the amount of the commodity to be disbursed (if negative) or received (if positive).

Phenomena - *Phenomena* are occurrences that can be observed.

Pooling - *Pooling* is a risk-management process under which similar actuarial risks are combined.

Portfolio - See *financial portfolio (portfolio)*.

Potentially Valid - A mathematical model is *potentially valid* if it produces results that are consistent with available observations of the modeled phenomena or of similar phenomena and is capable of being validated relative to the specified observed results when sufficient data are available.

Prefers - A participant *prefers* one economic good to a second economic good if he or she is unwilling to exchange the first good for the second, but is willing to exchange the second for the first.

Present Value - The *present value* of a cash flow relative to a present value function is the number assigned to the cash flow by the present value function.

Present Value Function - A *present value function* on a set of cash flows is a value function on that set such that, if one cash flow consists of receipts identical in amount to, but received earlier than, those of a second cash flow and disbursements identical in amount, but disbursed later than those of the second cash flow, the first cash flow is assigned a higher value.

Present Value Metric - A *present value metric* of a portfolio relative to a present value model is a number-valued function of the scenario present values.

Present Value Model - A *present value model* is a cash flow model that has a present value function associated with each economic scenario.

Present Value Random Variable - The *present value random variable* relative to a present value model is the random variable that assigns to each economic scenario the present value of the cash flow assigned to that scenario for the financial instrument.

Price - See *market price (price)*.

Probability - *Probability* is a measure of the likelihood of the occurrence of an event, measured on a scale of increasing likelihood from zero to one.

Provider - A *financial security system* is a risk-management system in which participants can transfer one or more actuarial risks, including their associated undesirable economic consequences, to a second party, called a *provider,* in return for a specified payment or set of payments made, in cash or in kind.

Random Variable - A *random variable* is a function that assigns a numerical value to every possible outcome.

Rate - A *rate structure* for a provider in a financial security system is (1) a set of exposure measures, each of which assigns an exposure to each of the actuarial risks transferred to the provider and (2) a rule that assigns a number called a *rate* to each of these actuarial risks so that the consideration for a participant is the sum of the products of the exposure and the rate for each actuarial risk transferred to the provider by the participant.

Rate Structure - A *rate structure* for a provider in a financial security system is (1) a set of exposure measures, each of which assigns an exposure to each of the actuarial risks transferred to the provider and (2) a rule that assigns a number called a *rate* to each of these actuarial risks so that the consideration for a participant is the sum of the products of the exposure and the rate for each actuarial risk transferred to the provider by the participant.

Refinement of a Rate Structure - A *refinement of a rate structure* for a provider in a financial security system is a rate structure that (1) has the same set of exposure measures as the original rate structure, (2) is compatible with a refinement of a risk classification system with which the original rate structure is compatible, and (3) assigns the same rates to actuarial risks in any risk class in the refinement that is identical to a risk class in the risk-classification system of the original rate structure, and increased rates to some actuarial risks and decreased rates to other actuarial risks in other risk classes.

Refinement of a Risk Classification System - A *refinement of a risk-classification system* is a risk-classification system with the same actuarial model as the original risk-classification system such that each risk class is either identical to or a subset of a risk class of the original risk-classification system.

Relevant Information - Information is *relevant information* for a transaction if it can change a party's willingness to complete that transaction on the stated terms.

Replicating Portfolio - A *replicating portfolio* for a financial instrument in a given financial market is a portfolio that does not contain the financial instrument and that provides the same net cash flows as the financial instrument under all possible scenarios.

Risk Assessment - *Risk assessment* is a process whereby the economic consequences of an actuarial risk for a given participant are evaluated.

Risk Characteristic - A *risk characteristic* is a quality that can be quantified by a risk measure.

Risk Class - A *risk class* of a risk-classification system is a set of actuarial risks that are represented by the same set of random variables in the actuarial model associated with the risk-classification system.

Risk-Classification System - A *risk-classification system* for a set of combinable actuarial risks is a set of risk characteristics and risk measures for the associated risk subjects, together with an actuarial model, that satisfies Principle 4.1.

Risk Identification - *Risk identification* is a process for determining the actuarial risks that are relevant to a given participant in an economic system.

Risk Management - *Risk management* is a process intended to mitigate some or all of the undesirable economic consequences, or to take advantage of some or all of the desirable economic consequences of an actuarial risk for a given participant.

Risk-Management System - A *risk-management system* is an arrangement under which risk identification, risk assessment and risk-management processes are applied to a set of actuarial risks of concern to a participant or group of participants.

Risk Measure - A *risk measure* for a specific quality on a set of risk subjects is a function that assigns a numerical value to each of the risk subjects for that quality.

Risk Subject - A *risk subject* is a person, thing, or collection of persons or things in connection with which the actuarial phenomenon associated with a given actuarial risk can be observed.

Risk Transfer - *Risk transfer* is a type of risk management in which another participant assumes certain rights or obligations related to some or all of the economic consequences of an actuarial risk.

Scenario - See *economic scenario (scenario).*

Scientific Model - A *scientific model* is an abstract and simplified representation of a given phenomenon.

Severity - The *actuarial risk variables* associated with an actuarial phenomenon are: (1) *frequency,* the nonnegative, integer-valued random variable representing the number of component stochastic phenomena comprised; and, for each of these component phenomena, (2) *timing,* the nonnegative, possibly multivariate, random variable measuring the time at which outcomes associated with the phenomenon occur and (3) *severity*, a possibly multivariate random variable measuring the undesirability of these outcomes.

Similar - Two combinable actuarial risks are *similar* if their associated actuarial risk variables are the same but they have different risk subjects.

Statistical Regularity - A phenomenon to which Principle 1.1 applies is said to display *statistical regularity*.

Stochastic Component - A *stochastic component* of a mathematical model is a representation of an aspect of a given phenomenon expressed in terms of random variables.

Stochastic Model - A *stochastic model* is a mathematical model that has at least one stochastic component.

Stochastic Phenomenon - A *stochastic phenomenon* is a phenomenon for which an associated experiment has more than one possible outcome.

Subject to Antiselection - A rate structure of a provider in a voluntary financial security system is *subject to antiselection* when a potential refinement of the current rate structure would produce differences in consideration among participants in an existing risk class.

Success Criterion - A *success criterion* for a risk-management system is a condition required to satisfy the system's purposes.

Sum - The *sum* of two cash flows is the series of cash receipts and disbursements consisting of the algebraic sums of the amounts of currency received or disbursed at each time under the constituent cash flows, with amounts received taken to be positive and amounts disbursed taken to be negative.

Swap - See *economic swap* (*swap).*

Timing - The *actuarial risk variables* associated with an actuarial phenomenon are: (1) *frequency,* the nonnegative, integer-valued random variable representing the number of component stochastic phenomena comprised; and, for each of these component phenomena, (2) *timing,* the nonnegative, possibly multivariate, random variable measuring the time at which outcomes associated with the phenomenon occur and (3) *severity*, a possibly multivariate random variable measuring the undesirability of these outcomes.

Transaction - See *economic transaction (transaction).*

Undesirable Economic Good - An *undesirable economic good* is an economic good that a participant prefers to dispose of rather than receive.

Valid within a Specified Degree of Accuracy - A mathematical model is said to be *valid within a specified degree of accuracy* relative to certain observed results if it can reproduce these results within that degree of accuracy.

Value Function - A *value function* on a set of cash flows is a function that assigns a number to each cash flow in the set such that (i) each cash flow consisting only of cash receipts (disbursements) is assigned a positive (negative) number, and (ii) the number assigned to the immediate receipt (disbursement) of an amount of currency is that amount (the negative of that amount).

Voluntary Financial Security System - A *voluntary financial security system* is a financial security system that is not mandatory.

Wager - A *wager* is an agreement to disburse or receive, at a specified time, an amount of a given commodity determined by the outcome of a stochastic phenomenon.