"Stochastic Modeling in Actuarial Science and Financial Mathematics: A Research Experience for Undergraduates"

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Abstract

From June 11 through August 3, 2007, the University of Illinois at Urbana-Champaign (UIUC) hosted a National Science Foundation-sponsored Research Experience for Undergraduates (REU) on the topic "Stochastic Modeling in Actuarial Science and Financial Mathematics." This Experience, provided to 6 undergraduate mathematics students from universities across the United States, was directed by Professor Rick Gorvett, the Director of the UIUC Actuarial Science Program. This report summarizes the program and describes the research projects undertaken.

Overview of Program

Actuarial science has evolved significantly over the last couple of decades, both in terms of its mathematical and financial content, as well as its range of applications. A broad description of actuarial science as being the "analysis of future contingencies" reflects the recognition that we live in a stochastic and ever-riskier world – with risks both natural and man-made – and that the mathematical, financial, and analytical skills of actuarial science are critical for identifying, quantifying, and managing the impact of those risks.

This summer research experience provided a select group of undergraduate students with an opportunity to explore and research the mathematical modeling of stochastic processes in an actuarial and financial context. The program began with the participants obtaining a theoretical background in several key areas: stochastic processes (including Brownian motion, Markov models, and Poisson processes); risk theory (beginning with the classic Cramér-Lundberg

model); and Monte Carlo simulation. Opportunities to explore both mathematical theory and hands-on use of state-of-the-art actuarial simulation models were provided.

Then, building upon their exposure to foundational concepts, participants explored and pursued several possible research directions. Participants, depending upon their specific interests, undertook both mathematical and data-analytical types of research projects. The program culminated with each participant submitting a research paper, and making a presentation to a general admission audience at UIUC.

Program Participants

- Six students, from six different U.S. colleges and universities, were chosen from an extremely competitive applicant pool (which numbered about 40). The participant group was equally represented by gender, and included four students between their junior and senior years, and two between their sophomore and junior years. In general, participants were math majors, with little specific knowledge of, but indeed with interest in, actuarial science and financial mathematics.

Subject Matter and Materials Introduced

The first two or three weeks of the REU involved an intensive introduction to relevant subject matter, and the suggestion and exploration of possible research projects. Specific reading materials, data sources, and software tools that were introduced to all participants during the first third of the REU are documented in Appendix I at the end of this report.

Participants were provided "lecture notes" and other materials as subjects were covered. Topical highlights from this front part of the REU included:

- Review of statistics and probability theory
- Stochastic processes
    - Markov processes
    - Poisson processes
    - Brownian motion
- Risk and ruin theory
- Monte Carlo simulation
- Interest rate modeling
- Option pricing theory

Participant Research Projects

After the introductory materials described above, the participants identified (with the help of the program director) and read the recent relevant research literature, refining and exploring their topics. While joint-participant projects were allowed, in the end each student chose to work on

their own individual project. The program director generally met individually at least once per week with each student. Group gatherings also occurred in several weeks during the last two-thirds of the program; these meetings provided opportunities for the participants to share their research experiences, bounce ideas off of each other, and to communicate items of general interest.

With the exception of two projects with a similar theme (but different application environments), the projects undertaken in this REU were quite varied. Briefly:

- One project examined the historical actuarial discrepancy between life expectancies of white Americans and black Americans. The participant created an Excel-based scenario model to examine the potential impact of reductions in three significant explanatory risk factors: infant mortality, homicide among young adult males, and HIV. This model could serve as a basis for evaluation and cost-benefit analysis of possible future public policy initiatives.

- One project explored the potential actuarial deficit associated with the State of Illinois' prepaid college tuition program. Several stochastic models were developed for simulating, via Monte Carlo techniques, potential future tuition inflation and trust fund investment returns

- Two projects dealt with regime-switching models in financial analysis. One of these involved development of a regime-switching model for the evolution of foreign-exchange (FX) rates. Historical FX rates were classified according to low, medium, and high volatility regimes. One-step-ahead FX predictions were simulated in a discrete Markov chain framework. The other project of these two involved incorporating a regime-switching framework into a stochastic model of interest rate movements.

- One project considered the philosophical issues associated with the Multifractal Model of Asset Returns. This model was applied to a number of financial time series, and the appropriateness of a fractal-geometric approach to risk management was evaluated.

- One project considered the conceptual and motivational foundations for mathematical models of cancer risk. A nine-state Markov chain model for the potential emergence of cancer cells was developed, based on the gene mutation theory of cancer progression. The model reflects the likelihood that over time a cell will mutate and become malignant.

Collectively, these REU participants instigated research and performed analyses that have potential to impact and advance at least the areas of actuarial science, financial scenario modeling, dynamic financial analysis (modeling an insurance organization as being impacted by both stochastic underwriting and investment processes), and enterprise risk management (holistic modeling of risk and risk interactions across an enterprise).

Appendix I

Written Materials, Data, and Software

Book Read and Discussed by Participants

The (Mis)Behavior of Markets: A Fractal View of Risk, Ruin, and Reward, 2004, by Mandelbrot and Hudson

Textbooks Available and Referenced

(1) *Introduction to Probability Models*, Seventh Edition, Sheldon M. Ross

(2) *Introductory Stochastic Analysis for Finance and Insurance*, X. Sheldon Lin

Papers Read and Discussed

(1) "A Comparison of Actuarial Financial Scenario Generators," 2007, by Ahlgrim, D'Arcy, and Gorvett, revised and resubmitted to *Variance: Advancing to Science of Risk*

(2) Report of "Modeling of Economic Series Coordinated with Interest Rate Scenarios," 2004, by Ahlgrim, D'Arcy, and Gorvett, on the Casualty Actuarial Society website at http://www.casact.org/research/econ/ (includes links to the various sections of the report, presentations and papers presented in association with the research, and an Excel-based scenario generator model)

(3) "Foreign Exchange Rate Risk: Institutional Issues and Stochastic Modeling," 2001, by Gorvett, *CAS Discussion Paper Program: Financial and Accounting Systems and Issues Associated with the Globalization of Insurance*, available at http://www.casact.org/pubs/dpp/dpp01/01dpp19.pdf

Software Introduced and Available

(1) TeX and/or LaTeX – mathematical word processing

(2) Microsoft Excel – familiarity with spreadsheet package, particularly functions useful for stochastic simulation {e.g., =RAND(), =NORMINV(prob,mean,stdev)}.

(3) DynaMo – a public-access, Excel-based dynamic financial analysis (DFA) program. Downloadable from http://www.pinnacleactuaries.com/pages/products/dynamo.asp

(4) Financial Scenario Generator – this is the simulation model available as Appendix D in the report listed above as Paper (2), available at http://www.casact.org/research/econ/ .

Financial Data Source Introduced

FRED – Federal Reserve Economic Data –the research database on the St. Louis Fed website, at http://research.stlouisfed.org/fred2/ .