Weighted Fund Style Analysis of Variable Annuity

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Abstract

In this paper, we propose weighted fund style analysis as a better method to hedge market risks of the variable annuities. Two typical weights, geometric and power weights, are considered. The optimal weight parameter, rolling window size, and benchmark indices are selected based on the rolling window cross validation method. We provide models that can be easily implemented by insurance and reinsurance companies. A real-world example demonstrates that the algorithm significantly reduces the fund basis and improves the hedge effectiveness.

JEL Classification: C52, C58

Keywords: Market Risks, Variable Annuity, Weighted Fund Style Analysis, Model Selection, Cross Validation

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1 Introduction

Variable annuities (VAs) are long-term and popular investment vehicles offered by life insurance companies. There are two phases (see [**Haefeli (2013)**]) in VAs, the accumulation phase and the withdrawal phase. The policyholders may invest in mutual funds in a separate account. The policyholders can purchase riders, such as Guaranteed Minimum Accumulation Benefit (GMAB), Guaranteed Minimum Death Benefit (GMDB), Guaranteed Minimum Income Benefit (GMIB) and Guaranteed Minimum Withdrawal Benefit (GMWB) by paying rider fees. These GMxBs are linked to benefit bases. The benefit bases are typically set at the higher of the current account value and previous year's benefit base on the fund anniversary. Therefore, the benefit bases typically never drop, but the account value could drop in a bear market. When the stock market crashes, the insurance/reinsurance company has to make up the difference between the account value and benefit base.

Key risks associated with VAs are insurance risks, such as longevity risk for GMIB, mortality risks for GMDB, market risks, and policy holder behavior risks, such as persistency risk and benefit utilization risk. To manage the insurance risks, insurance/reinsurance companies need to perform experience studies and set assumptions on mortality. To manage the policy holder risks, they need to monitor the lapse and utilization experience and reset the lapse and utilization assumptions.

To mitigate the market risks, the insurance/reinsurance companies need to hedge the market risks. Unlike Equity Index Annuities(EIA) (see [**Bernar and Boyle (2011)**]), there are exchange traded contracts (future/options) for the index (typically S&P 500 index) underlying the EIA. However, exchange traded contracts do not exist for mutual funds underlying VAs. These VAs cannot be hedged from the market risks directly. Therefore, the mutual fund returns must be mapped onto indices' returns. Then the VAs could be hedged using the corresponding indices. The fund style analysis (mapping model) is often used to determine the fund's exposure to these indices. The difference between fund returns and benchmark returns used for the fund style analysis is called fund basis risk. Fourteen of the largest North American Variable Annuity writers have average account value in-force of $ 50 Billion according to the last survey by Towers Watson (2013). Because of a large account value, the fund basis risk could be very large in dollar amount. Therefore, a better fund mapping method is needed for insurance and reinsurance companies to manage the market risk. This helps them to reduce the downturn risks and smooth out the earnings.

Fund style analysis was first introduced by William Sharpe at Stanford University. It quickly became a powerful tool for analyzing mutual fund returns. The style analysis is used to map the fund returns onto indices' returns; thus the fund manager's style (e.g., conservative, moderate or aggressive) could be determined. Moreover, the asset allocation in the corresponding indices could also be obtained. In practice, exclusive indices, such as large cap index, small cap index, developed market index, emerging market index and bond index are often used. Since the fund manager typically cannot take a short position, all the factors' weights are non-negative. Due to budget constraints, the sum of these weights is equal to one. Equal time weights are applied when the style analysis is performed by Sharpe's method. Therefore, it is a long-term average asset allocation over the given period. It is well known that fund managers may change their style or asset allocation (see [**Henriksson and Kiernan**]). Using these average weights to hedge the market risks of

VAs may produce large basis risks due to large account values.

The basic fund style analysis provides a reasonably good model for fund mapping. However, there are some aspects of the model that can be improved so that it can better capture the fund managers' style. We summarize the recent work of Sharpe's model extensions in this paragraph. Bodson et al. (2010) considered varying the time weights in the benchmark indices. A Kalman filter algorithm is used to select the appropriate benchmark indices. Numerical examples show that on average the Kalman filter model has a smaller Mean Squared Error (MSE) than the typical Sharpe's model. However, one needs a special algorithm to calibrate the constrained Kalman filter model proposed by Gupta and Raphael (2007). Sen and Chaudhuri (2016) proposed analysis of the mapping of mutual fund based on time series decomposition of the price movements. Gallagher et al. (2016) proposed a new fund mapping method where the benchmark indices are six equity style factors, three currency style factors and an extra alpha factor that captures the fund manager's performance. Fukui et. al. (2016) proposed fund mapping using a state space model and Monte Carlo filter. They estimate the parameters using a generalized simulated annealing. Raza and Mohsin (2015) proposed a new method measuring the fund manager's performance using modern style tilts. Bubley and Burch (2016) investigated the benchmark drift by performing regression analysis. Corbett (2016) investigated the style rotating funds using a dynamic state space factor model and a holding based approach. Faff et al. (2012) compared the asset allocation strategies across different style groups of the Australian managed and superannuation funds using a rolling window method. Gallagher et al. (2015) proposed a style rotation model to investigate the style factor timing. Buncic et al. (2015) proposed a new style profiling approach using linear combinations of the shelf stock market indices.

2 Problem Formulation

Insurance and reinsurance companies need to hedge market risks of variable annuities. However, it is difficult to hedge the fund directly; therefore the fund return is mapped onto benchmark indices. These indices represent different market sizes and values. They often have different returns and volatilities. Suppose a fund's monthly return $FR_t, t = 1, 2, \ldots, N$ is given. M indices' monthly returns, $IR_{t,i}, t = 1, 2, \ldots, N \quad i = 1, 2, \ldots, M$ are also chosen. William Sharpe proposed the following fund style analysis model

$$FR_t = \sum_{i=1}^{M} IR_{t,i}\beta_i + \epsilon_t, \quad \text{where } \epsilon_t \text{ is random noise} \tag{1}$$

$$\sum_{i=1}^{M} \beta_i = 1, \quad \beta_i \geq 0 \tag{2}$$

The asset allocation weights of indices, β_i must be non-negative, since the fund manager cannot take the short position. The fund manager also has a budget constraint, so the sum of the weights must be one.

We introduce the following vectors, $IR_t = (IR_{t,1}, IR_{t,2}, \ldots, IR_{t,M})$, $\beta = (\beta_1, \beta_2, \ldots, \beta_M)^T$

and $\mathbf{1} = (1, 1, \dots, 1)$. The model above has the compact form

$$FR_t = IR_t\beta + \epsilon_t \tag{3}$$
$$\mathbf{1}\beta = 1, \quad \beta \geq 0 \tag{4}$$

To find the optimal weights of Sharpe's model, we need to minimize the sum of squared errors

$$\arg\min_{\beta} \sum_{t=1}^{N} (FR_t - IR_t\beta)^2 \tag{5}$$
$$\mathbf{1}\beta = 1, \quad \beta \geq 0 \tag{6}$$

Instead of using equal weights for fund returns from $t = 1, 2, \dots, N$, we propose a weighted linear regression model. In our hedging model, there is a greater emphasis on the latest assets allocation of benchmark indices . We propose two typical weights that are easier to calculate and use. First, we consider the geometric progression weights (see [**Steiner and Mackay (2014)**])

$$w_t = \frac{\lambda^{t-1} * (1-\lambda)}{1-\lambda^N}, \text{where } \lambda \geq 1, t = 1, 2, \dots, N \tag{7}$$

It is easy to see that the sum of the indices weights is equal to 1. If $\lambda = 1$, then $w_t = \frac{1}{N}$. By L'Hospital's Rule, we recover Sharpe's model.

Second, to the best our knowlege, we also first propose the power weights,

$$w_t = \frac{t^k}{\sum_{j=1}^{N} j^k}, \text{where } k \geq 0, t = 1, 2, \dots, N \tag{8}$$

It is easy to see that the sum of the weights is equal to 1. If $k = 0$, then $w_t = \frac{1}{N}$, and we recover Sharpe's model too.

Introducing the following notations: mutual fund returns, $Y = (FR_1, FR_2, \dots, FR_N)^T$, indices returns, $X = (IR_1, IR_2, \dots, IR_N)^T$ and the fund weights, $W = diag(w_1, w_2, \dots, w_N)$, we propose the following weighted factor model:

$$\arg\min_{\beta} (Y - X\beta)^T W (Y - X\beta) \tag{9}$$
$$\mathbf{1}\beta = 1, \quad \beta \geq 0 \tag{10}$$

It is well known that the above optimization problem has no closed form solution due to the inequality of the constraints. This quadratic programming problem may be solved using one of the various algorithms, such as the interior point method, active set method, conjugate gradient method or dual method of Goldfarb and Idnani (see [**Goldfarb and Idnani (1983)**]). We solve this quadratic optimization problem by using the open source R language (see [**R Core Team (2016)**]) package quadprog (see [**Weingessel (2013)**]) with the following setup.

$$\arg\min_{\beta} 1/2\beta^T D\beta - d^T\beta \tag{11}$$
$$A^T\beta \geq b_0 \tag{12}$$

where

$$D = 2X^T W X \tag{13}$$
$$d = 2X^T W Y \tag{14}$$
$$A = [\mathbf{1} \quad I_{M\times M}] \tag{15}$$
$$b_0 = [1 \quad \mathbf{0}_{1\times M}]^T \tag{16}$$

2.1 Data

To illustrate our weighted factor model, we use public data from Yahoo Finance (http://finance.yahoo.com/). The TIAA-CREF Lifecycle 2035 Fund (ticker:TCLRX) is used to decompose into the benchmark indices. The following benchmarks are used: The benchmark indices in table 1 repre-

Index	Ticker	Country/Region
S&P500	GSPC	US large cap index
RUSSELL 2000 Index	RUT	US small cap index
Nikkei 225	N225	Japan equity index
Hang Seng Idex	HSI	Hongkong equity index
SSE Composite Index	SSEC	Shanghai equity index
CAC 40	FCHI	French equity index
DAX	GDAXI	German equity index
iSharesCore US Aggregate Bond	AGG	US bond index

Table 1: Benchmark Indices

sent equity and bond returns and covers different countries and, market capitalization. The monthly index level from December 2010 to November 2016 is used for the analysis. The continuous monthly returns are calculated by using the following formula:

$$FR_t = \ln\left(\frac{\text{Index level}_t}{\text{Index level}_{t-1}}\right) \tag{17}$$

In addition, the following simple return may also be used.

$$FR_t = \frac{\text{Index level}_t}{\text{Index level}_{t-1}} - 1 \tag{18}$$

It is easy to show that the simple return and continuous return are very close to each other by Taylor's formula (see [**Thomas et al. (2013)**]) due to the small monthly returns.

3 Model selection

Model selection plays a central role in the fund mapping analysis. There are several classical methods to select statistical models such as cross validation, bootstrap method and subset selection method (see chapter 5 and 6 of [**James et al. (2016)**]). The weighted fund style model depends on three parameters: the weight matrix (λ and k), rolling windows length (N), and Benchmark indices. We can test the model by choosing the optimal weight matrix, rolling window length and benchmark indices.

The goal of our proposed model is to generate better mapping by changing the various parameters. The weight matrix controls the time weights of the observations. We may put larger weights on the most recent observation and determine wheter it generates better mapping. Alternatively, we may equally distribute the weights by setting $\lambda = 1$ or $k = 0$.

The rolling windows also play a central role in the analysis. If it is too short, the model may fit the noise instead of the trend. On the other hand, if it is too long the model captures the long term average, but may not capture the fund manager's latest style and asset allocation. This is important because the fund management and strategy may change over time.

In our proposed model, the more benchmark indices used, the better in sample fit that can be achieved. However, as noted above the model may be subject to over fitting. When it is used to forecast, it may have large basis risks (out of sample error). When the options or futures based on the fund mapping are used to hedge the market risks, these mismatches pose great financial risks to the VAs.

To choose the parameters and predictors, we use a rolling window method that is similar to the cross validation method (see chapter 5 of [**James et al. (2016)**]). The cross validation method randomly splits the data into two data sets, the training set and test set. The model is calibrated using the training set. Then the test error, typically Mean Squared Error (MSE), can be computed based on the test set. The MSE of the test set is typically larger than that of the training set since the model does not have access to the test data. The goal is to strike a balance between in sample fit and out of sample fit. In practice, the leave one out cross validation (LOOCV) and k-fold cross validation are commonly used.

It is well known that fund returns have volatility clustering property (see [**Cont(2005)**]). Large returns typically flow large returns. Small returns typically follow small returns. When we perform fund mapping analysis, we use the time series of fund return data. If we use the LOOCV or K-fold cross validation method, we essentially rearrange the date of the data by randomly dividing the data set into training set and test set. The volatility clustering property may be violated. Therefore it doesn't reflect the nature of the data and hedging business.

The rolling windows method is very similar to the LOOCV method. The training set and test set keep the volatility clustering property. We calibrate the model based on the training set from $t = 1, 2 \ldots, N$ and compute the MSE, mse_1, based on test data at $t = N + 1$. Next, we calibrate the model using the training set from $t = 2, 3, \ldots, N + 1$ and compute the MSE, mse_2 based on test data at $t = N + 2$. Finally we calibrate the model using the training set from $t = k, k + 1, \ldots, N + k - 1$ and compute the MSE, mse_k based on test data at $t = N + k$. The MSE is computed by the following formula

$$MSE = \frac{\sum_{i=1}^{k} mse_i}{k} \tag{19}$$

3.1 Optimal Fund Weight

We choose the optimal fund weight by fixing the rolling window size and the benchmark indices. For example, the rolling window size is set to be 36. We use all the benchmark indices listed in Table 1. We have 72 monthly data points from December 2010 to November 2016. By the rolling window validation method proposed above, we compute the MSE of the test errors based on 36 test errors.

For geometric weights, we let λ increment from 0.1 to 2 with step size=0.1. We plot the MSE at each λ in Figure 1 on page 7.

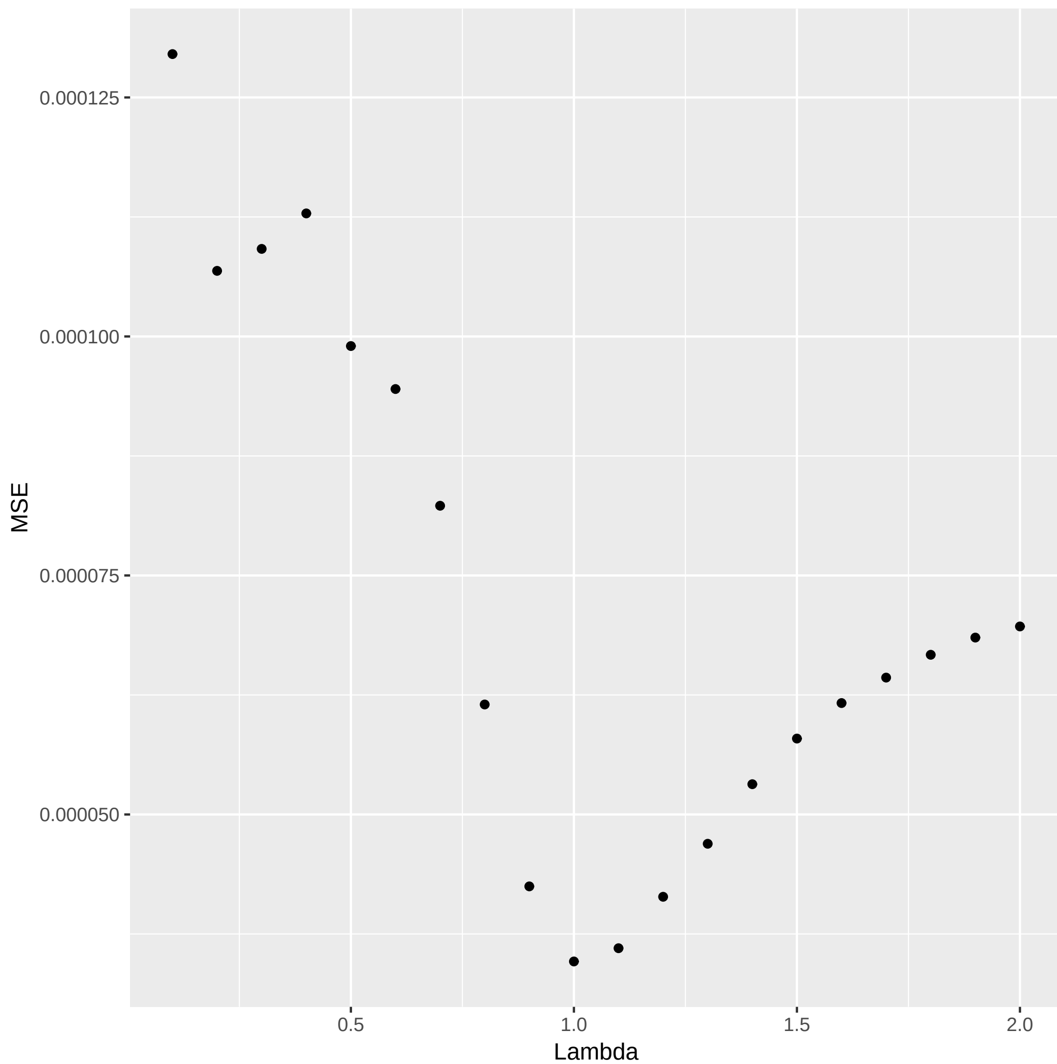

Figure 1: MSE of Geometric Weights

As expected that the MSE is decreasing when the weights of the latest observation are increasing at the beginning . It has the minimum MSE with $\lambda = 1$, i.e. weighted equally. Notably, the MSE is increasing later due to the heavy weight at the last few observations. We summarize the results in Table 2 on page 8. As we expected, when the geometric weights

	λ	MSE
Min	1	3.462537×10^{-5}
Max	0.1	1.295376×10^{-4}
Mean	NA	7.195641×10^{-5}
STD	NA	2.802368×10^{-5}

Table 2: Summary of MSE of Geometric Weights

	k	MSE
Min	0.6	3.370327×10^{-5}
Max	5	4.296243×10^{-5}
Mean	NA	3.758864×10^{-5}
STD	NA	3.129359×10^{-6}

Table 3: Summary of MSE of Power Weights

exponentially grow, the corresponding test errors (MSE) have a wide range with a large standard deviation (STD).

Similarly, for power weights, we iterate k from 0 to 5 with step size 0.1. We plot the test errors at each k in Figure 2 on page 14.

Our model shows that the MSE decrease first, then increase later when the weights of the latest observation is increasing. It has the minimum MSE with $k = 0.6$. We summarize the results in Table 3 on page 8. Note that the MSE of power weights has a narrow range with small STD due to the slow change of the weights of the observations. The MSE of power weights is more stable than that of geometric weights. The optimal weights occurs when the power weight $k = 0.6$ is selected. This value performs best for the mapping.

3.2 Optimal Rolling Window Size

We choose the optimal rolling window size by fixing fund weights and benchmark indices. We use the power weight with $k = 0.6$. We use all the benchmark indices listed above. By the rolling window cross validation method proposed above, we compute the corresponding MSE of the test errors.

We iterate the rolling window size from 12 to 60 with a step size of 1. We plot the test errors at each rolling window size in Figure 3 on page 15.

When the rolling window size is small, the model tends to fit the noises, so the cross validation errors are large. On the other hand, when the rolling window size is large, the model tends to fit the trend instead of noises, therefore it has smaller test errors. But when the windows size is too large, there are few test errors available. In these cases, the MSE tend to have large variation. We summarize the results in the Table 4 on page 9.

	Rolling Window Size	MSE
Min	36	3.370327×10^{-5}
Max	12	5.207435×10^{-5}
Mean	NA	3.839232×10^{-5}
STD	NA	3.78192×10^{-6}

Table 4: Summary of MSE of Rolling Windows

3.3 Optimal Benchmark Indices

We find the optimal benchmark indices by fixing fund weight matrix and rolling window size. We let $k = 0.6$ and rolling window size = 36. By the rolling window validation method proposed above, we compute the corresponding MSE of the cross validation errors. There are 8 indices available. We plot the test errors at each combination of indices in Figure 4 on page 16. The Y-axis shows the MSE. The x-axis shows the MSE of each one-index cases (8 cases), of each two-index case (28 cases), and so on up to the MSE of the eight-index (1 case). There are $2^8 - 1 = 255$ cases. The corresponding x-axis values are from 1 to 255.

When few benchmark indices are used, the MSE is larger since the fund mapping fails to capture the fund managers' style. When more benchmark indices are used, the MSE tends to become smaller as we expected. The optimal benchmark indices are GSPC, RUT, HIS, FCHI and AGG. It covers equities (GSPC, RUT, HIS, FCHI) and bond index (AGG). It has large cap (GSPC) and small cap (RUT). It also has domestic equity (GSPC and RU) and international equities (HIS and FCHI). Care must be taken, when selecting the benchmark indices for mapping. The mutual fund prospectus along with business judgment should also be used to choose the best indices instead of looking only at test errors.

3.4 Optimal Fund Weight, Rolling window Size and Benchmark Indices

The optimal solutions obtained above are local optimal solutions since the other parameters are fixed during the search. The local optimal solution is found in one dimensional space. AS such, there is only one loop to search it in the R codes. To find the global optimal solution, we need to perform a global search in three parameter dimensional spaces, including weight parameter, window size, and benchmark indices. The window sizes are natural numbers only. The benchmark indices are a combination problem. There is no closed form solution available. We must perform a brute-force search in three dimensional spaces by using nested loops in R codes. The parameter spaces are as follows:

- The geometric weight parameters, λ is from 0.1 to 2 with step size 0.1;
- The power weight parameters, k is from 0 to 5 with step size 0.1;
- The benchmark indices: there are 8 indices (GSPC, RUT, GDAXI, N225, HIS, SSEC, FCHI, AGG) to choose.
- The rolling window size, N is from 12 to 60 with step size 1.

Rank	Window Size	Benchmark Indices	Weight Parameter	MSE
1	58	GSPC, GDAXI, HIS, FCHI, AGG	power: $k = 0.4$	1.61185×10^{-5}
2	58	GSPC, GDAXI, HIS, SSEC, FCHI, AGG	power: $k = 0.4$	1.61185×10^{-5}
3	58	GSPC, GDAXI, HIS, SSEC, FCHI, AGG	power: $k = 0.3$	1.61556×10^{-5}
4	58	GSPC, GDAXI, HIS, FCHI, AGG	power: $k = 0.3$	1.61556×10^{-5}
5	58	GSPC, GDAXI, HIS, SSEC,FCHI, AGG	power: $k = 0.5$	1.61680×10^{-5}

Table 5: Summary of MSE of Global Solution

Using the available hardware (ThinkPad laptop with 2.40 GHz CPU and 32GB RAM with Windows 7 operating system), it only takes about three seconds to get the local optimal solution. Whereas, it takes about 22 hours to compute the global optimal solution.

We summarize the top 5 models in Table 5 on page 10. We found that the global optimal solution significantly reduce the test errors. However, it is not easy to plot the graph in four dimensional space. We first fix the benchmark indices (GSPC, GDAXI, HIS, FCHI, AGG) and weight method (power and geometric), then plot the test errors against the window size and weight parameter. The MSE of geometric weights have a larger variation due to the geometric growth of the fund weights (Figure 5). The optimal values of MSE occurs around ($\lambda = 0$), i.e. weighted equally. On the other hand, the MSE of power weights has a smaller variation (Figure 6). The optimizing parameters in the weighted model generate a map that coincides with the trend.

The top 5 models have similar MSE values. To select the best model for hedging VAs, we also need to consider the following factors. The fund prospectus ([**MorningStar**]) shows that the TIAA-CREF Lifecycle 2035 fund covers the US Equity(GSPC), international equities (GDAXI, FCHI), emerging market equities (SSEC, HSI) and bonds (AGG). Another factor to consider is the need to explain the model to management and auditors. Based on these factors, we recommend that using the 5th model since it covers a wide range of equities and bonds. It is also easier to explain the model to management and auditors since the weight is $\sqrt{i}$ that is the square root of the observation index.

4 Conclusion

Insurance and reinsurance companies have large blocks of VA business. The market risks need to be hedged to smooth out the earnings. To hedge these products, Sharpe's fund style analysis is often performed. Instead of using equal time weights for fund mapping, we propose weighted fund style analysis. Two types of weights, power and geometric weights are considered. Besides the weight matrix, we also consider the optimal benchmark indices and the optimal rolling window size. Our goal is to reduce the fund basis risk. The fund basis risk is the difference between fund return and approximate returns using benchmark indices. To select the optimal parameters, we use the rolling window cross validation method. The model is calibrated using the training data set. The mean squared error is calculated using the test data. We choose the optimal parameters that produce the smallest cross validation errors.

These algorithms are easy to implement. The hedging team may run the global optimization monthly to get the exposures to the benchmark indices and model parameters. They may also need to monitor the risk exposures weekly by using the local algorithms. If there are large deviations of the weights of the benchmark indices, they may need to readjust the exposure to mitigate the market risks.

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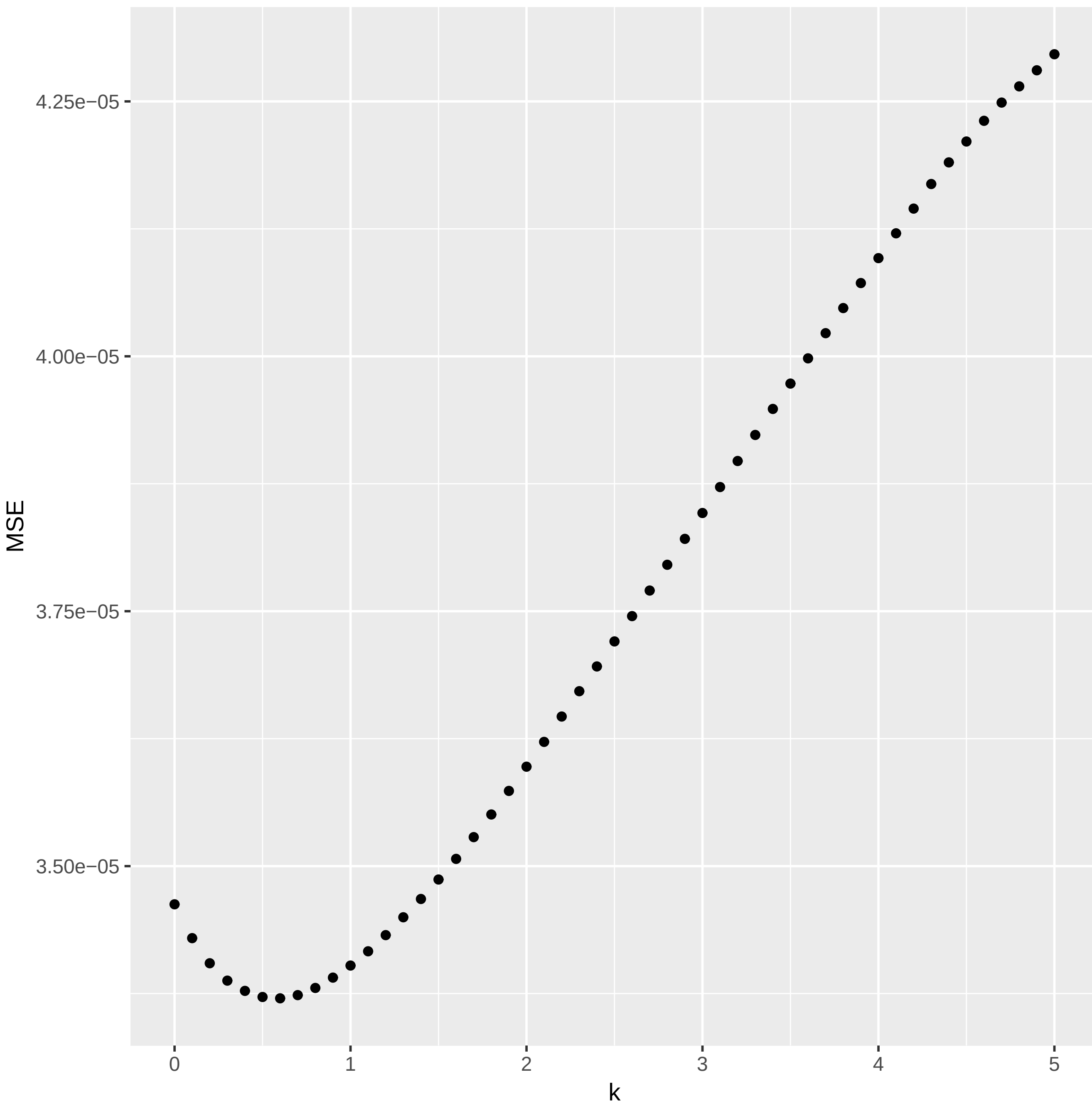

Figure 2: MSE of Power Weights

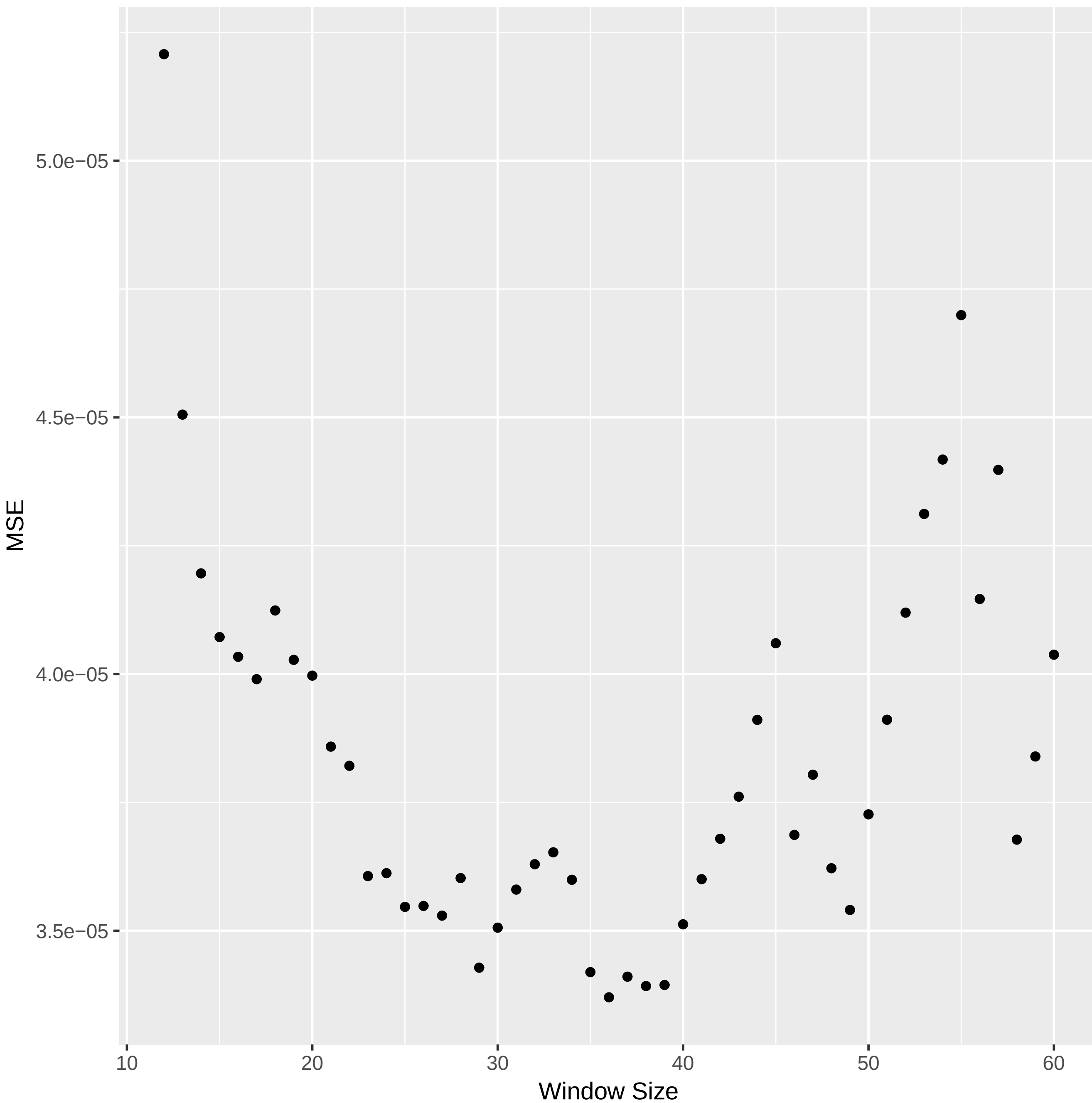

Figure 3: MSE of Rolling Window Size

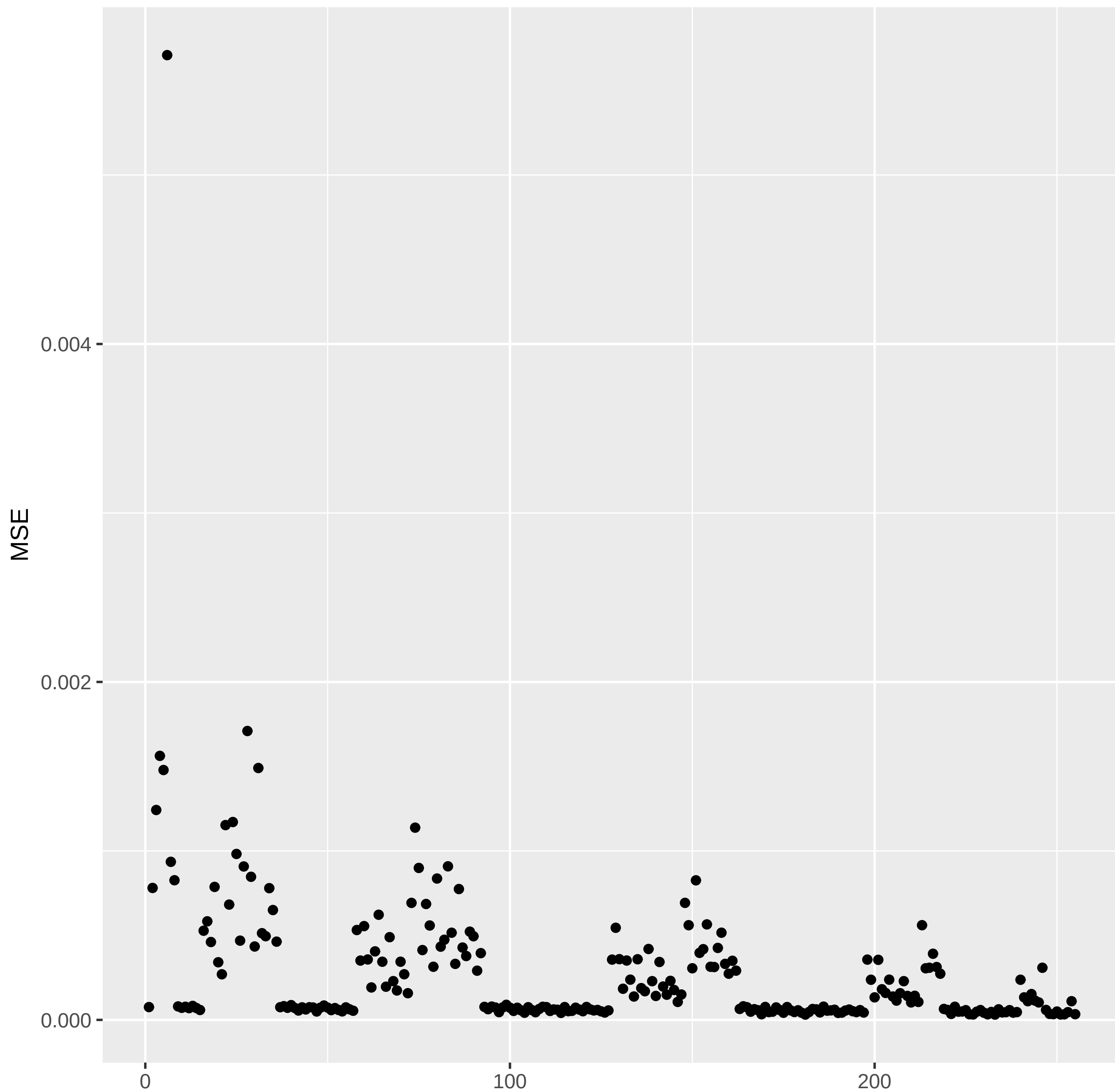

Figure 4: MSE of Different of Indices

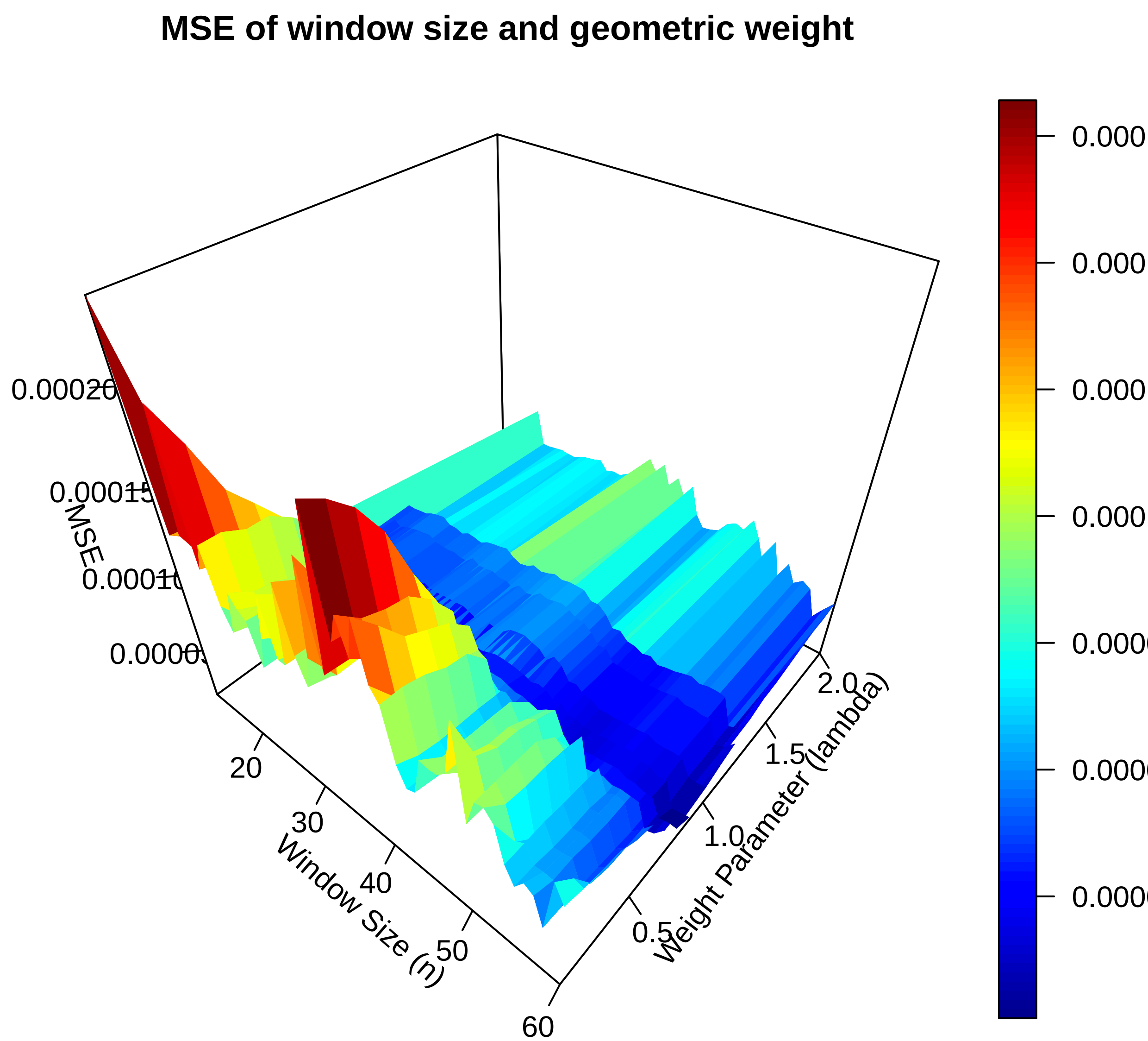

Figure 5: MSE of Window Size and Geometric Weight Parameter

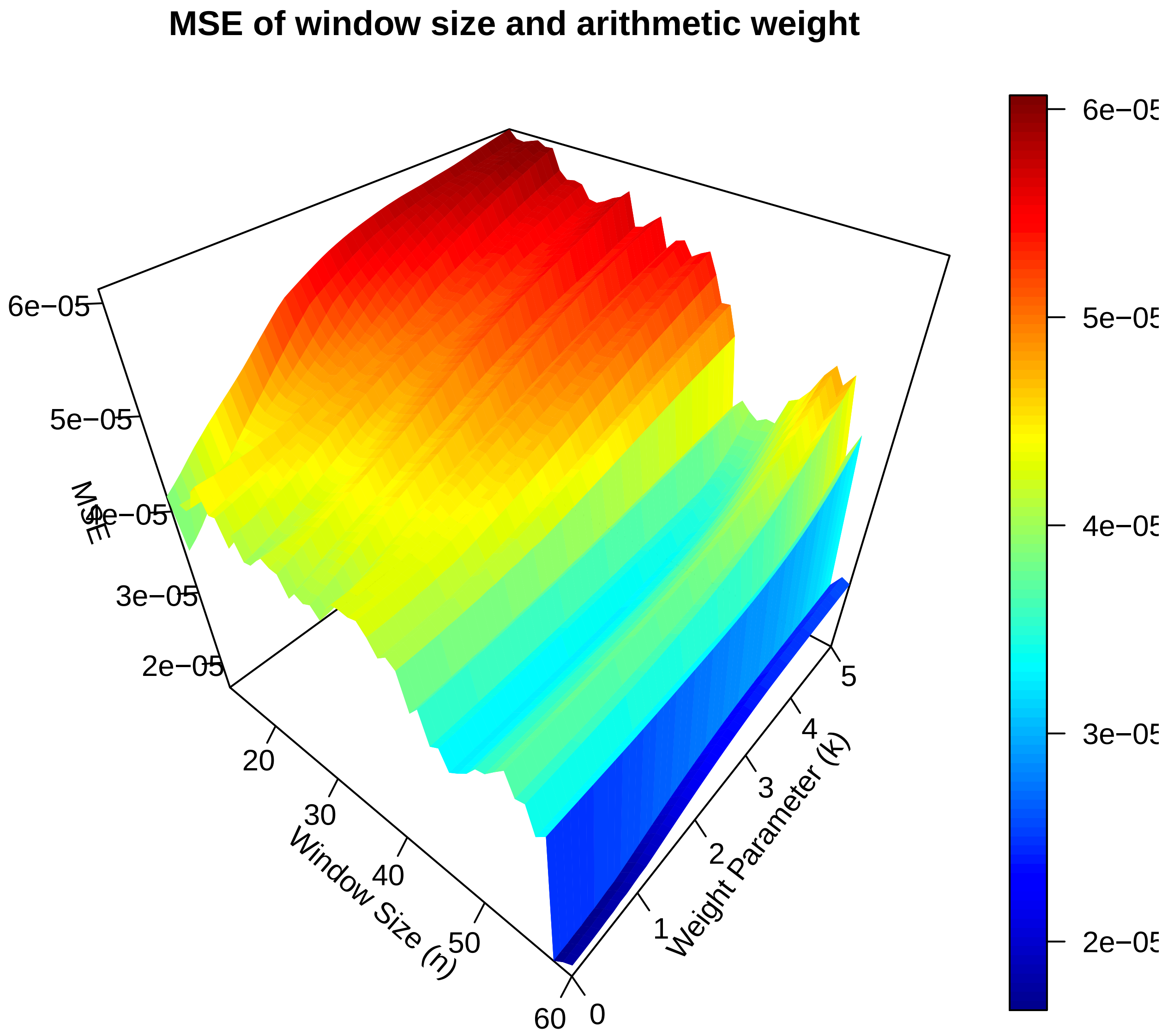

Figure 6: MSE of Window Size and Power Weight Parameter