Closing-out the Algerian life tables: for more accuracy and adequacy at old-ages

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Abstract: Due to data unavailability or irregularity at old ages in developing countries, the model life tables are used to estimate old ages mortality. Such procedure is not supposed to provide country-specific estimates, especially when the model life tables are not correctly used. Thereby, extrapolating the old ages mortality based on the trend observed at younger ages is assumed to provide more consistent results. In this paper, we compare some models to extend mortality rates beyond the age of 80 for the Algerian population. The resulted life expectancy at birth display an average gap of one year compared to official statistics.

Key-words: mortality rates, goodness of fit, predictive capacity, old ages, life expectancy, Algeria

1 Introduction

The improvement of life expectancy in Algeria during the recent decades resulted from a reduction of mortality rates at all ages. Consequently to this improvement, the probability of surviving until the age of 60 years has improved from 0.82 in 1977 to 0.96 in 2014. Parallel to this, the size of the population aged 60 and over rose from 790,000 in 1966 to more than 2.5 million in 2008 according to the population censuses data. The size of the elderly will increase continuously during the coming decades. Thus, the old age mortality should be attentively considered as well as the mortality at adult ages.

A life table describes the probability of dying of a cohort of individuals, at different ages, from birth till total extinction. The lack of mortality data and the weakness of the exposure to death risk beyond a certain age lead to some irregularities in mortality curves. For a long time, some classical mortality models initially performed to graduate mortality rates at adult ages, have been used to extend mortality to old ages. The most practical example in this sense is the fact that a Makeham-type function was used by the United Nations Population Division to extend the model life tables (MLT) till the age of 85 years (UN, 1982). For the Coale-Demeny MLT revised in 1983, a Gompertzian function was used to extend mortality rates until 100 years old (Coale *et al.,* 1983). Such a practice assumes that mortality rates will keep growing at old ages following the trend observed at adult ages. As mentioned in Heligman & Pollard (1980), after childhood and young ages, mortality rates can be represented by an exponential function as that proposed by Gompertz (1825). Parallel to this, a deceleration of mortality rates at old ages was noticed in many works starting from Gompertz (1825) and then Perks (1932) (Gavrilova & Gavrilov, 2014). By the late of the 20^{th} century, the improvement of population life conditions in developed countries resulted in the improvement of life expectancy and the growth of the population of elderly. Furthermore, the improvement of data quality provided consistent databases for old ages mortality modeling. Coale & Guo (1989) confirmed mortality deceleration on the observation of mortality rates until the age of 100 for some countries (i.e., Netherlands, Japan, France, West Germany, Austria, Sweden, and Norway). This deceleration becomes evident starting from the age of 80 or 85 years (Coale & Kisker, 1990).

For Algeria, the national life tables being published by the Office of National Statistics (ONS) starting from 2010 were closed-out at the age interval of 85 and older. For the period before 2010, this closure age was set at 80 years and older or lower. Based on the selected MLT, the corresponding life expectancy at the closure age is estimated. Combined with the younger ages mortality rates, the life expectancy at birth is concluded.

The first step in all this process is to select an adequate MLT among the Coale-Demeny (CD) or the United Nations (UN) MLTs. The selection process is based on the comparison of the national mortality data, at adult ages, to the mortality pattern given by the different types of MLT. Between the two, African countries prefer to use the first ones to complete their mortality data ES84.

Mortality of the northern African countries is usually represented by the south type of CD life tables while the North type is used to describe the mortality pattern of sub-Saharan countries. However, this procedure implies much inadequacy compared to the use of the ordinary least squared deviations method (Ekanem & Som, 1984). This inadequacy emerges when joining the two mortality curves, at adult and older ages, coming from national statistics and MLT respectively.

Our main objective is to reduce the variation of life expectancy due to changes in the closing out method and to give more homogeneous old age mortality surface for the Algerian population. The second objective is to provide readers with a general methodology to extend mortality to old ages for the Algerian population starting from adult mortality data.

To this end, some old age mortality models will be evaluated and compared to extend mortality beyond the age of 80 years. Usually, starting from a certain age, e.g. 35, 40, or 45, mortality rates grow following an exponential function. This regularity of the observed trend allows for extending mortality rates to older ages. The quality of the extrapolation is related to the quality of the existing data and also to the age range used for model calibration. In the absence of observed data to be used for comparison, other criteria can be used to orient models calibration. Usually, an assumption about the surviving age limit can be defined. The age mortality pattern of the Algerian population has changed many times over the observed period. Thus, it is tough to find a unique model providing good fitting quality over the whole period and ensuring some adequacy either between males and females or regarding the year-to-year variation.

2 Old ages mortality extrapolation methods

Historically, different models have been proposed to extrapolate mortality rates to old ages. In what follows, we give a general overview of these methods.

2.1 Gompertz-Makeham model (GPZ, MKM)

For a long time, the classical mortality models have been used to extend mortality to old ages. Gompertz (1825) discovered that the force of mortality μ_x evolves exponentially with age (x):

$$\hat{\mu}_x = \alpha * \beta^x \tag{1}$$

Makeham (1867) enhanced this last model by adding a constant term (c) representing the risk of death by accident not age dependent:

$$\hat{\mu}_x = c + \alpha * \beta^x \tag{2}$$

A Makeham-type function was used to extend the United Nation's MLT beyond the age of 80 (UN, 1982). The used function was expressed as:

$$\ln \frac{{}_n\hat{q}_x}{1-{}_n\hat{q}_x} = A + B * x \quad (3)$$

With ${}_n\hat{q}_x$ representing the probability of dying between age x and $x+n$.

The observed trend of ${}_nq_x$ between ages 50 and 75 was used to extend mortality rates to old ages. At higher ages, the risk to die by accident becomes negligible because old people are usually apart from any risky activity Gavrilov & Gavrilova (2011). In such a case, Makeham model does not provide any added value compared to the Gompertz model.

When we extend mortality only till the age of 100 years, the choice of the extrapolation method is of a little importance. Since the 1980s, life expectancy has improved as well as the number of centenarians (Buettner, 2002 ; Robine & Vaupel, 2001). Hereafter, the need became apparent for more adapted tools for old age mortality extrapolation.

Following the significant improvement of life expectancy at birth as well as the quality of mortality data at older ages, it became possible to compares models to real data. It turned out that old age mortality does not follow a Gompertzian function, but slows down slightly. This mortality deceleration at older ages was observed on several populations. Many studies have tried to find a convincing explanation for it. Some researchers assumed it to be due to a selection process (Coale & Guo, 1989; Kannisto, 1992; Kannisto *et al.*, 1994). On the other side, Gavrilov & Gavrilova (2011) have shown that there is no mortality deceleration at old ages and it is just the effect of the weakness of data quality at old ages which results in such an effect. In the unavailability of data allowing to verify these two hypotheses for the Algerian population, we assume that the first assumption is nearer to be real.

2.2 Weibull's model (WBL)

Weibull (1951) proposed the following formula:

$$\hat{\mu}_x = \alpha * x^{\beta} \quad (4)$$

2.3 Helligman and Pollard model (HP)

Heligman & Pollard (1980) is the only mortality model which fits mortality rates (q_x) at all ages.

$$\frac{\hat{q}_x}{1-\hat{q}_x} = A^{(x+B)^C} + D * exp(-E(\ln(x) - \ln(F))^2 + G * H^x \quad (5)$$

Each of the three terms of the model fits the mortality at a defined age interval (Heligman & Pollard, 1980). After the age of 40 or 50 years, the two first terms become useless and can be ignored. Then, the model can be written as:

$$logit[\hat{q}_x] = \pi + \theta * x \quad (6)$$

with $\pi = \ln(G)$ and $\theta = ln(H)$.

2.4 Coale-Guo and Coale-Kisker methods (CK)

The main idea of the Coale-Guo model CG89 is that, at old ages, mortality rates keep growing with a decreasing growth rate. This deceleration is supposed to follow a linear trend. The growth rate of the death rates ${}_{n}m_{x}$ from age to another was defined by:

$$k_x = \ln \frac{{}_{5}m_{x}}{{}_{5}m_{x-5}} \tag{7}$$

k_x is supposed to increase between two consecutive ages by a constant R: $k_{x+5} = k_x - R$. For two ages x and $x + 5 * i$, we can write : $k_{x+5*i} = k_x - i * R$.

The Coale-Guo model was first applied to extend death rates beyond the age of 75 years until 110 years. When k_{80} is known, for $x \geq 80$, the death rate at age x can be deduced from that of the previous age interval by the following formula:

$${}_{5}m_{x+5} ={}_{5} m_x * \exp(k_{80} - \frac{(x-80)}{5} * R) \tag{8}$$

As a general formula, we can write:

$${}_{5}m_{80+5*i} ={}_{5} m_{75} * \exp(k_{80} + k_{80} - R + k_{80} - 2R + k_{80} - 3R + \ldots + k_{80} - i * R) \tag{9}$$

This implies:

$${}_{5}m_{x+5*i} ={}_{5} m_{75} * \exp(i * k_{80} - \frac{i(i+1)}{2} * R); i = 1,2,3. .. \tag{10}$$

The last death rate ${}_{5}m_{105}$ can be deduced from ${}_{5}m_{75}$ by using:

$${}_{5}m_{105} ={}_{5} m_{75} * \exp(6. K_{80} - 15 * R). i \tag{11}$$

Authors have arbitrarily imposed the constraint:

$${}_{5}m_{105} - {}_{5}m_{75} = 0.66 \tag{12}$$

Coale & Kisker (1990) adapted the Coale-Guo formula to the case of single age's mortality data :

$$\hat{\mu}_x = \hat{\mu}_{x-1} * \exp(k_{80} + s * (x - 80)); x = 80,81, \ldots ,109. \tag{13}$$

k_{80} represents the growth rate of mortality at 80 years. It is defined to be the average growth rate at the age range $[65, 80[$ and can be calculated as follows:

$$k_{80} = \frac{\ln(\frac{\hat{\mu}_{80}}{\hat{\mu}_{65}})}{15} \tag{14}$$

In some cases, the Coale-Kisker formula can lead to some incoherence regarding the male-female mortality evolution. This incoherence can be seen either as a crossover or an exaggerated divergence between the male and female extrapolated mortality curves. To offset this inconvenient, the authors have arbitrarily fixed the rates for a relatively high age (110 years):

$$\hat{\mu}_{110} = \begin{cases} 1 \quad for \quad males, \\ 0.8 \quad for \quad females, \end{cases} \tag{15}$$

This leads to define S which is equal to:

$$s = -\frac{\ln(\frac{\hat{\mu}_{79}}{\hat{\mu}_{110}}) + 31 * k_{80}}{465} \tag{16}$$

s in the Coale-Kikser model has the same interpretation as R in the Coale-Guo formula.

Finally, both of these methods are known as the Coale-Kisker model (CK) also called the quadratic mortality model (Roli, 2008; Thatcher, 1999). Accordingly, the CK model consists of writing the logarithm of the force of mortality as a quadratic function of age. However, this formulation is not used nor explained in the literature:

$$\ln(\hat{\mu}_x) = a + bx + cx^2 \tag{17}$$

The mortality growth rate can be expressed as:

$$k_{80} = \ln(\frac{\hat{\mu}_x}{\hat{\mu}_{x-1}}) = a + bx + cx^2 - a - b(x-1) - c(x-1)^2 = b + c(2x-1) \tag{18}$$

Considering a starting age of 80 years, we can write:

$$k_x - k_{80} = 2 * c * (x - 80) \tag{19}$$

and that leads to the same formula seen earlier :

$$\hat{\mu}_x = \hat{\mu}_{x-1} * \exp(k_{80} + 2 * c * (x - 80)); \quad x = 80,81,\ldots,109. \tag{20}$$

2.5 Perks, Logistic, Kannisto, Thatcher models (PRK, LOG, KST, THT)

The use of logistic functions to graduate mortality curves was first introduced by Perks (1932). He proposed the following formula to fit the force of mortality with age:

$$\hat{\mu}_x = \frac{\alpha * e^{\beta * x}}{1 + \gamma * e^{\beta * x}} \tag{21}$$

The general form of the logistic model can be written as $\hat{\mu}_x = \theta + \frac{\lambda * \alpha * e^{\beta * x}}{1 + \alpha * e^{\beta * x}}$. When Thatcher *et al.* (1998) tried to use this model to fit old age mortality pattern of 13 developed countries, they found that λ is very close to 1 (Thatcher, 1999). They concluded that the model can be simply written with 3 parameters:

$$\hat{\mu}_x = \theta + \frac{\alpha * e^{\beta * x}}{1 + \alpha * e^{\beta * x}} \qquad (22)$$

At older ages, θ becomes negligible as the accident risk constant in Makeham's model. Therefore, the previous formula can be simplified to be Kannisto (1992):

$$\hat{\mu}_x \approx \frac{\alpha * e^{\beta * x}}{1 + \alpha * e^{\beta * x}} \qquad (23)$$

in Logit form, we find:

$$logit(\hat{\mu}_x) = \ln(\alpha) + \beta * x \qquad (24)$$

Kannisto proposed this simplified version of the model unintentionally while reporting some findings. This model was reported in Thatcher *et al.* (1998) as Kannisto model.

2.6 Denuit and Goderniaux method (DG)

This method relies on a polynomial formula of order 3 of the log mortality rate:

$$\ln(\hat{q}_x) = a + bx + cx^2 \qquad (25)$$

In the original article, the authors have set the survival age limit at 130 years. To respect this constraint, they imposed : $\hat{q}_x = 1$.

2.7 Comparison

There are different ways to extend mortality to old ages. Two mortality measures can be used for this issue, i.e., μ_x and q_x. Gavrilov & Gavrilova (2011) explained the difference between these parameters and how the choice of the indicator can affect the final extrapolation results. At older ages, mortality rates reach a high level and keep growing slowly until the limit of 1, while μ_x keep growing without any constraints. For this, μ_x is more suitable for extrapolation to older ages. When reported to the semi-log scale, this last can easily be approximated by a straight line more than q_x. Another way to give more regular linear trend to mortality rates over age is to introduce the Logit which allows passing from a variation interval of [0,1] to]-∞, +∞[. The third element is that there are principally two types of models: the transformed linear models and the transformed quadratic ones.

Among the presented models, there are only two which are based on q_x, i.e., HP and DG models. The first proposes an extrapolation based on a linear trend of $\ln(q_x)$, while the second aims to impose a quadratic form to $logit(q_x)$. The other models are all based on μ_x. GPZ, MKM and WBL models express μ_x in a log-linear form. The CK model tries to extrapolate $\ln(\mu_x)$ using a quadratic function, while the logistic models (PRK, LOG, KST, THR models) try to approximate

$logit(\mu_x)$ to a linear function. In addition, two families of models can be specified in function of either any closure constraint is imposed or not. The models GPZ, MKM, WBL, LOG, KST, THT, PRK, and HP are supposed to be behavioral models since the extrapolated rates are just a result of the model calibration at younger ages. The CK and DG models are quadratic transformed models. The set of the age limit that they can lead to is huge and unrealistic in most cases. Thus, a closure age constraint is imposed in order to make the extrapolation results more realistic.

3 Data and Method

In this paper, we use the Algerian life tables published by the ONS during the period from 1977 to 2014. For missing data, we use the estimates of Flici (2014). So, our database is composed of five-age mortality rates from 0 to 75 years during the period [1977, 2014]. Figure 1 shows the mortality surfaces for males and females.

Old ages mortality models are usually based on a single ages mortality description. For this, we interpolate the age-specific mortality rates (ASMRs) from the five-ages ones. Since, there is no perfect model to interpolate ASMRs at all ages, we use a mixture of two methods, namely the Karup-King and the Lagrange's methods. The first one suits mortality at high ages but it gives bad results at young ages while the second is completely the adverse. The idea is to combine the two methods to join the interpolated curve obtained with Karup-King at high ages and that obtained with Lagrange's method at low ages. The junction point is defined at the age providing the smallest distance between the two curves.

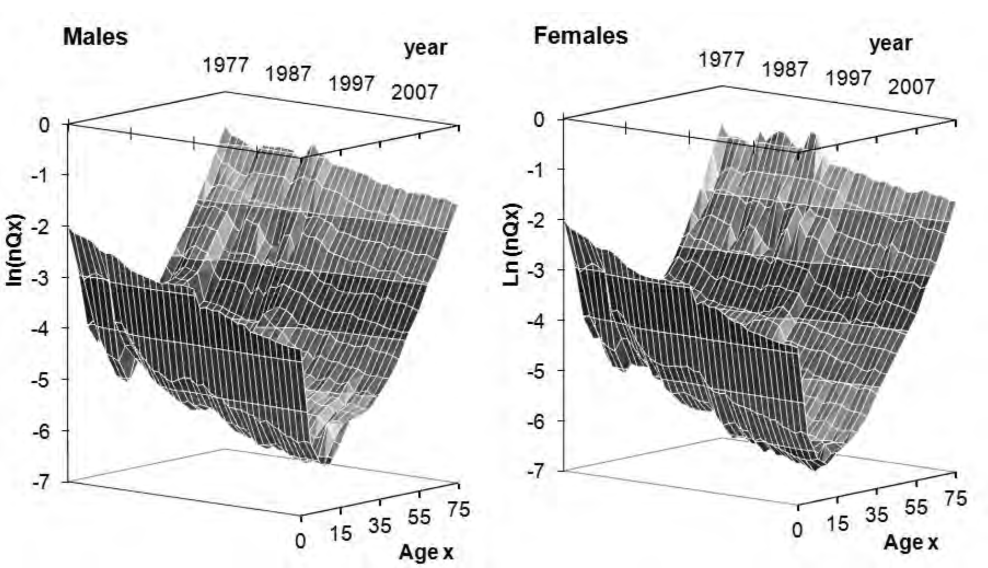

Figure 1: Five-ages mortality surfaces (Algeria: 1977-2014). Source: ONS data completed by Flici (2014)

Mortality models are conventionally compared based on the goodness-of-fit and the predictive capacity. The first criterion evaluates the fidelity of models estimates to the raw data, while the second one assesses their ability to predict the mortality rates at older ages. In our case, we are not mainly interested in the quality of the fitting itself but more in that of the extrapolation results. Since the goodness-of-fit does not imply necessarily a good predictive capacity, this latter needs to be independently assessed by comparing a part of the extrapolated series to observed data. Unfortunately, data availability does not allow a relevant analysis in this sense. The observed data needs to be arranged in a way to allow evaluating both the goodness-of-fit and the predictive capacity. The more the compared age interval is larger, the more evaluation is relevant. We remind that q_x are available until the age of 79 years until 2009 and until the age of 84 during the last five years. For this, we extend the age interval for model calibration until 74 years while the age interval [75, 84] is used to evaluate the predictive capacity. In regards to this later, we observe that data lengths are different for the periods before and after 2010. For the first period, the age interval to be used for such an evaluation is [75, 79] years while it passes to [75, 84] starting in 2010. Hence, the evaluation criterion needs to be set to consider all the ages with similar weight. We use the Mean Squared Error (MSE) for this issue.

We remind that the six models are based on different mortality measures $\ln(\mu_x)$; $logit(\mu_x)$ and $logit(q_x)$. To ensure the comparability of the models, the MSE is expressed in an unified mortality measure which is $\ln(q_x)$. The goodness-of-fit can be evaluated on the age range $[x, 74]$ by:

$$MSE_{[x,74]} = \frac{1}{\mathrm{x} * \mathrm{t}} \sum_{\mathrm{x}=\mathrm{X}}^{74} \sum_{\mathrm{t}=1977}^{2014} [\ln(\hat{\mathrm{q}}_{\mathrm{xt}}) - \ln(\mathrm{q}_{\mathrm{xt}})]^2$$

To evaluate the predictive capacity of the different models, the MSE is estimated in two steps: First, we calculate the MSE corresponding to each age interval [75, 79] and [80, 84] by:

$$MSE_{[75,79]} = \frac{1}{5 * 38} \sum_{x=75}^{79} \sum_{t=1977}^{2014} [\ln(\hat{q}_{xt}) - \ln(q_{xt})]^2$$

and

$$MSE_{[80,84]} = \frac{1}{5 * 5} \sum_{x=80}^{84} \sum_{t=2010}^{2014} [\ln(\hat{q}_{xt}) - \ln(q_{xt})]^2$$

Then, the weighted MSE (WMSE) corresponding to the age interval [75, 84] is calculated by:

$$WMSE_{[75,84]} = \tfrac{1}{2}(MSE_{[75,79]} + MSE_{[80,84]})$$

Mortality rates will be independently extrapolated for male, female, and global populations. First, the six models are estimated without setting any age limit constraint. Additionally to the goodness-of-fit and the predictive capacity, some qualitative criteria are added to enhance the evaluation aiming to make the extrapolation results consistent with some general rules:

1. Mortality rates keep increasing with age. That allows to write: $\hat{q}_{x+1} > \hat{q}_x; \quad \forall x \geq 75$. We introduce this element because the quadratic transformed models (DG and CK) can lead in some cases to a reversal in the mortality rates trend beyond a certain age;

2. Since extrapolation is made independently for male, female, and global populations, the extrapolated mortality rates for the both sexes population must be closer to the weighted average of male and female rates. Given that the structure of the Algerian population aged 70 and older is equidistributed by sex, we can write, for a fixed year (t): $\hat{q}_x^{both} = \frac{(\hat{q}_x^m + \hat{q}_x^f)}{2}; \quad \forall x \geq 75$. In the case of a change in the sex distribution at very advanced ages in any way, mortality rates of the both-sexes population $\hat{q}_x^{both}$ must be situated in the interval between the male and female rates. That implies : $\hat{q}_x^{both} \in [\hat{q}_x^f - \hat{q}_x^m]$;

3. Male mortality rates are higher than the female ones : $\hat{q}_x^m > \hat{q}_x^f; \quad \forall x \geq 75$.

One last last element which can enhance the evaluation of the extrapolation is the age limit predicted by the model. Theoretically, the age limit (w) is attained when mortality rates rise to the value of 1: $\hat{q}_w = 1$. A model is coherent when the predicted age limit is near to the observed maximum surviving age. For the Algerian population and according to the MICS IV (Multi Indicators Cluster Survey) results, maximal surviving ages of 110.5 and 112.5 years old were observed for males and females respectively (Flici and Hammouda, 2016). That does not represent an estimate of the surviving age limit of the Algerian population, but just a minimal of the interval where this age can be situated.

4 Results and Discussion

4.1 Model selection

4.1.1 Goodness-of-fit and predictive capacity

In most cases, the goodness-of-fit is widely related to the length of age interval used to calibrate a model. Usually, the quality is higher as much the data length is shorter, but at the expenses of robustness. Thus, it is necessary to define a fitting criterion which combines the two. The use of the Bayesian Information Criterion (BIC) is more suitable for such purposes since it considers, in addition to the gap between predictions and observations, the number of parameters and the number of observations. Even if the BIC was first proposed to suit the Likelihood Estimation method (Schwarz, 1978), the formula was adapted later to the Least Squares Errors estimation method (Burnham & Anderson, 1998; Hansen, 2007). The adapted formula can be written as:

$$BIC = n * \ln(\frac{1}{n} SSE) + k * \ln(n) \tag{26}$$

with n representing the number of observations and k the number of parameters in the model.

In our case, the BIC is used to evaluate both the fitting quality and the predictive capacity of the models. For each model, these qualities are related to the age range used for calibration. However, it is not evident to define a common age range which ensures the best quality for all models. Also, there is practically no best model in all situations. Accordingly, each model is calibrated on various age ranges, i.e., [40, 74], [45, 74], [50, 74], [55, 74], and [60, 74], then extrapolated beyond the age 75. The models are ranked according to the two criteria. Results are presented in Table 1.

Table 1: Models evaluation and comparison – BIC

Males

	Age Range for model calibration / Age range to evaluate the Predictive Capacity										*Rank*	
Model	[40,74]	[75,84]	[45,74]	[75,84]	[50,74]	[75,84]	[55,74]	[75,84]	[60,74]	[75,84]	GOF	PC
DG	-173.4*	-50.6	-163.8	-50.8	-137.5	-53.8**	-107.7	-45.0	-81.2	-46.1	2	1
HP	-173.0*	-29.2	-165.4	-33.7	-138.1	-35.9	-109.8	-37.3**	-89.8	-36.6	4	4
GPZ	-174.0*	-31.3	-165.9	-35.5	-138.5	-35.8	-110.1	-39.1**	-90.0	-37.8	3	3
WBL	-166.1*	-16.0	-141.7	-16.0	-125.9	-20.3	-105.2	-23.9	-86.8	-27.0**	6	6
CK	-178.3*	-46.9	-157.4	-45.6	-131.9	-45.9	-102.8	-46.9	-75.6	-48.4**	1	2
KST	-171.9*	-28.7	-164.8	-32.1	-137.2	-34.2	-109.6	-35.8**	-89.7	-35.5	5	5

Females

	Age Range for model calibration / Age range to evaluate the Predictive Capacity										*Rank*	
Model	[40,74]	[75,84]	[45,74]	[75,84]	[50-74]	[75,84]	[55,74]	[75,84]	[60,74]	[75,84]	GOF	PC
DG	-152.1*	-24.8	-139.7	-23.9	-114.5	-32.6**	-95.4	-29.5	-82.1	-31.1	2	2
HP	-144.1*	-13.6	-140.4	-16.5	-121.6	-20.1	-98.0	-23.1	-84.4	-26.3**	4	4
GPZ	-144.8*	-14.2	-141.1	-17.1	-122.0	-20.8	-98.2	-23.8	-84.4	-27.0**	3	3
WBL	-115.9	-3.5	-119.3	-7.0	-110.3	-11.0	-93.3	-15.1	-81.9	-19.4**	6	6
CK	-152.3*	-22.8	-139.1	-22.1	-115.3	-33.4**	-95.0	-27.9	-69.8	-27.6	1	1
KST	-141.3*	-13.1	-139.8	-16.0	-121.2	-19.3	-79.8	-22.4	-84.3	-25.6**	5	5

Both sexes

	Age Range for model calibration / Age range to evaluate the Predictive Capacity										*Rank*	
Model	[40,74]	[75,84]	[45,74]	[75,84]	[50,74]	[75,84]	[55,74]	[75,84]	[60,74]	[75,84]	GOF	PC
DG	-169.4*	-36.1	-150.7	-35.2	-124.1	-35.7	-96.7	-36.7	-71.8	-37.5**	1	2
HP	-160.2*	-20.6	-155.9	-23.9	-132.1	-26.9	-105.6	-29.4	-90.7	-31.2**	4	4
GPZ	-161.2*	-21.5	-156.6	-24.9	-132.5	-28.1	-105.9	-30.5	-90.9	-32.2**	3	3
WBL	-126.5	-7.3	-130.9*	-11.2	-118.6	-15.3	-100.0	-19.2	-87.1	-23.0**	6	6
CK	-165.8*	-30.6	-161.3	-38.7	-133.7	-40.4	-103.8	-40.5**	-76.6	-40.0	2	1
KST	-159.2*	-19.7	-155.0	-23.0	-131.5	-25.9	-105.3	-28.3	-90.5	-30.3**	5	5

Each model is estimated and evaluated on 5 different age intervals. The BIC is calculated for the age intervals [x, 74] and [75, 84] as indicators of the goodness-of-fit and the predictive capacity. (*): The age interval providing the best fitting quality. (**): The age interval leading to the best predictive capacity.

According to Table 1, the rank of the six models changes according to the age interval used to calibrate the models. However, some evidence appears clearly: The three best models for fitting and extrapolating mortality rates are the DG, CK, and GPZ models. Since each model leads to its best quality in a specific age interval, which is not common for all models, we propose using l [50, 74] to calibrate all models. That is because it is the only interval which allows keeping the initial models ranking obtained on the three populations. Figure 2 shows the mortality rates extrapolated to old ages with the six models.

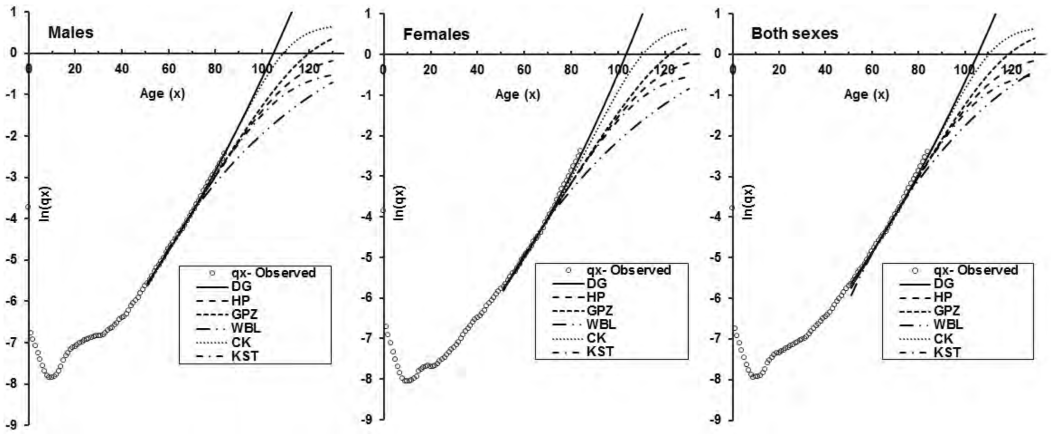

Figure 2: Models comparison

The quadratic models (DG and CK) provide better quality compared to the linear transformed models. However, models comparison needs to be assessed by using complementary criteria as the male-female coherence, the predicted age limit, and the coherence between single sexes and both-sexes extrapolations.

4.1.2 Expected Mortality Sex Ratio

The observation of the Mortality Sex Ratio (MSR) calculated on the extrapolated mortality rates beyond the age of 75 for the period from 1977 to 2014 are shown in Figure 3 separately for each of the six models.

At older ages, when mortality rates increase to approximately 1, the male and female mortality rates converge increasingly to each other. Consequently, the MSR must converge to 1. Regarding this criterion, the KST, HP, and GPZ models have led to more coherent results compared to the three other ones.

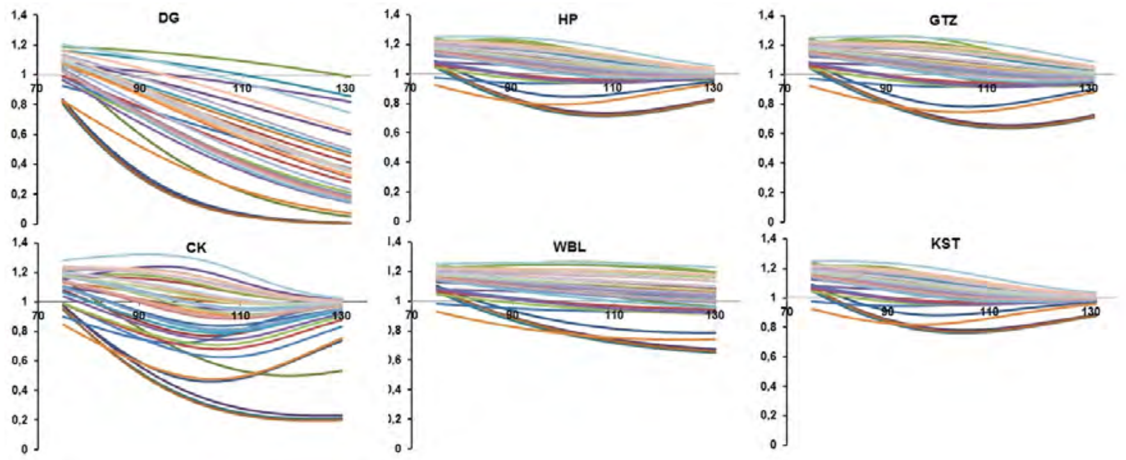

Figure 3: Extrapolated Mortality Sex Ratio with the six models

4.1.3 Coherence between single sexes and both-sexes expected mortality rates

To evaluate the coherence between single sexes and both-sexes estimates, we calculate the part of cases where both-sexes the predicted mortality rates for both-sexes population are situated out of the interval between the male and the female rates. The more this failure ratio is lower, the more the extrapolation is coherent. Results are presented in Figure 4.

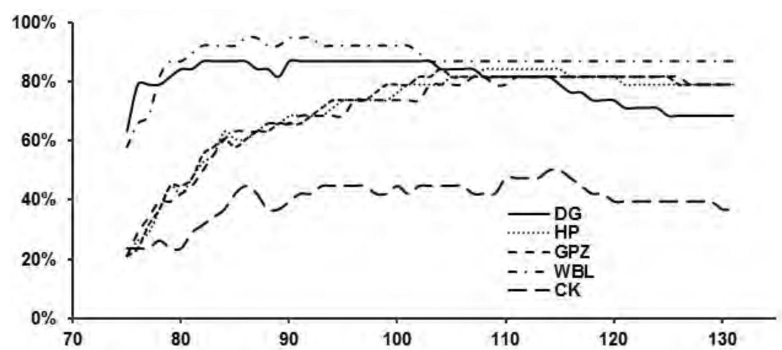

Figure 4: Failure ratio between single-sexes and both-sexes predicted mortality rates

The DG and WBL models led to the most failed results when single sexes are compared to both-sexes predicted mortality rates. The other models give an acceptable failure ratio under 20% increasing beyond the age of 100 to around 70%. The CK model gives the less important failure ratio.

4.1.4 Expected age limit

Another way to evaluate the coherence of the extrapolated mortality rates is to analyze the age limit predicted by the different models. This later represents the age x for which q_x is equal to 1. Results show some differences between the different models. By the age of 130, the HP, WBL, and

KST models give a mortality rate under 1. The age limit predicted by the DG model is 104, 103, and 106 years on average for males, females, and both-sexes populations, respectively. For the CK model, the predicted age limit is situated between 109 and 111 years old. In adverse, the GPZ model leads to a higher age limit equal to 117 on average.

According to the maximum surviving age observed for the Algerian population until now, the GPZ model seems to be the most consistent model according to this criterion. Resulting from the MICS survey, the maximum surviving age was equal to 112.5 for females and 110.5 for males FH16. This survey is far to provide an accurate estimate of the age limit of the Algerian population since it is not exhaustive compared to civil registration data, but it allows to fix the lowest bound of the age limit estimate. Also, we must consider the future evolution of the age limit since longevity keeps increasing. The age limit is defined to be the age which can not be surpassed by any human being in a certain geographic area. Denuit & Goderniaux (2005) supposed this age to be equal to 130 for the developed countries. For developing countries, we can assume a lower age limit. To this end, we consider 120 as a reasonable limit for the Algerian population.

4.1.5 Results discussion

We have seen through this comparison that the quadratic models give a better fitting quality and predictive capacity compared to the linear transformed models. The GPZ model makes an exception of this rule. The study of the age limit predicted by the different models confirmed this finding. The comparison based on other criteria has shown different judgments. Generally, when considering other comparison criteria, i.e., the coherence between the single sexes and both-sexes estimates and the male-female coherence, the rank overturns. The GPZ model displays a good score in all situations.

The quadratic models, of CK and DG, display some incoherence between either males and females or single-sexes and global population estimates. Contrary to the linear transformed models, the quadratic ones can lead to various possible trajectories of mortality rates from age 80 to the age limit. To avoid incoherence, some constraints need to be imposed either as a surviving age limit (Denuit & Goderniaux, 2005) or as a fixed mortality rate at any high age (Coale & Kisker, 1990). Such a constraint might avoid a crossover of the extrapolated mortality curves considered either from a year-to-year evolution or from a gender differential comparison (Buettner, 2002).

In what follows, we keep working with 3 models, i.e., the models of DG and CK with 120 years as an age limit, and the GPZ model.

4.2 Adding some constraints

4.2.1 Age limit constraint

Here, DG and CK models are re-estimated by imposing 120 years as an age limit. This additive condition has to impair the fitting quality of these models. To recover such a lack in fitting quality and to keep the two quadratic models within the same performance as the GPZ model, we reduce the length of the age range used to calibrate the two models. We observe that when the DG and CK models are calibrated on the age interval [60, 79], they give approximately the same fitting quality as the GPZ model calibrated on the age interval [50, 79]. The results are given in Figure 5.

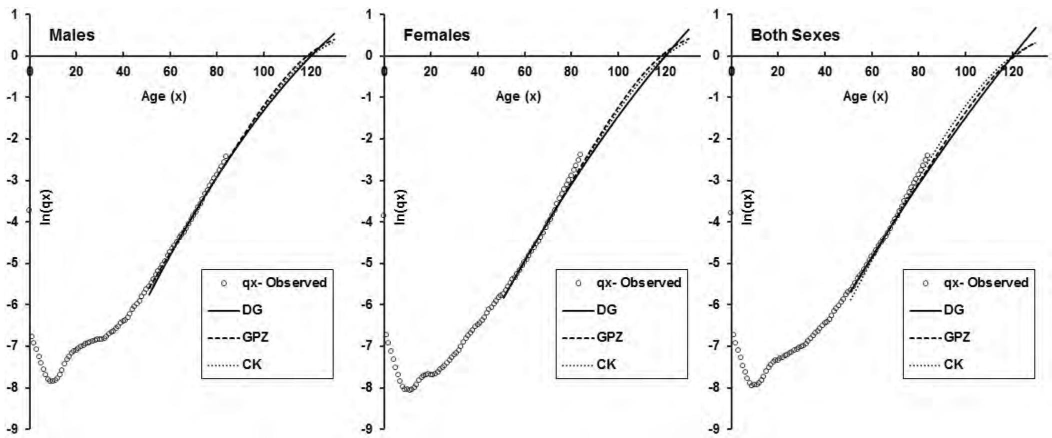

Figure 5: Old age mortality extrapolation with an age limit constraint

The three models lead approximately to similar extrapolation results. The coherence criteria are supposed to perform models comparison. Figures 6 and 7 show respectively the MSR and the Failure ratio obtained after having imposed the age limit constraint.

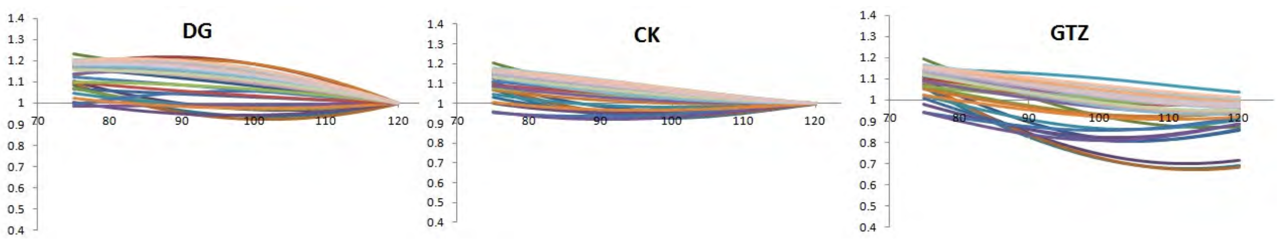

Figure 6: Mortality Sex Ratio under age limit constraint

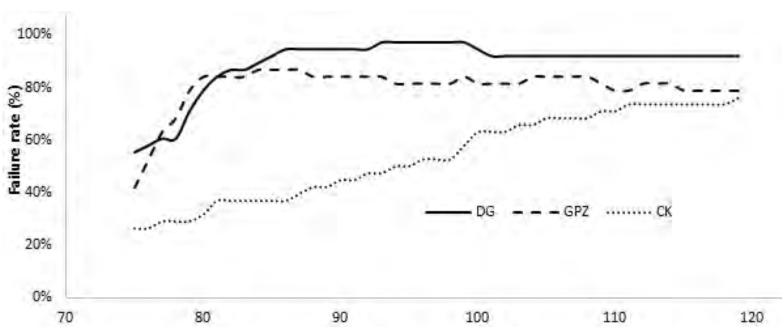

Figure 7: Failure Ratio under age limit constraint

According to the coherence criteria, the model of CK is better than the models of DG and GPZ.

The expected MSR converges to 1 more quickly in the first model compared to the second ones. For the coherence between singles sexes and both sexes extrapolations, the increasing trend of the failure ratio can be explained by the fact that at near the age limit, all mortality rates converge to 1. Consequently, the difference between the male, female, and the combined sexes mortality rates become as smaller as the age limit nears. However, to improve the quality of the extrapolation further, we impose some additional constraints about the comparative evolution of the male, female, and combined sexes extrapolated rates.

4.2.2 Coherence constraints

Here, we impose two additional constraints; the first aims to keep the female mortality under the male mortality while the second tries to keep the combined sexes mortality estimates in between the male and female ones.

We notice that the first constraint could not be fully respected for all years. Some years of the period before 1994 have marked a slight female over mortality, or even not, a significant decrease in the MSR by the end of the age interval used for models calibration. Since extrapolation results are just an extension of what is observed at younger ages, a female over mortality could not be avoided in the extrapolation results.

The second constraint, concerning the coherence between the combined-sexes and the single-sexes estimates, was fully respected without affecting significantly the quality of the fitting.

4.3 Final results

After imposing all the necessary constraints on the extrapolation process, we conclude that the three models: GPZ, CK and CG give similar quality regarding the evaluation criteria. Therefore, it is very difficult to decide whether one model is more appropriate than the others. The quadratic models allow more flexibility in old age mortality extrapolation while ensuring a good fitting. When the age limit constraint is imposed, the extended mortality surface shows a high regularity. The GPZ model have shown a good performance regarding all the selection criteria except that the use of unfitted adult age mortality surface for its calibration have led to unstable expected age limit series. This disadvantage does not appear in the case of quadratic models by imposing a common age limit constraint for all years. Consequently, the GPZ model can be a perfect to extrapolate old age mortality on a fitted mortality surface. As a result and given that the CK model provide better quality compared to the DG model on the basis of the other selection criteria, we decided to adopt the CK model to extrapolate old age mortality for the Algerian mortality surface. The extended mortality surfaces are shown in Figure 8.

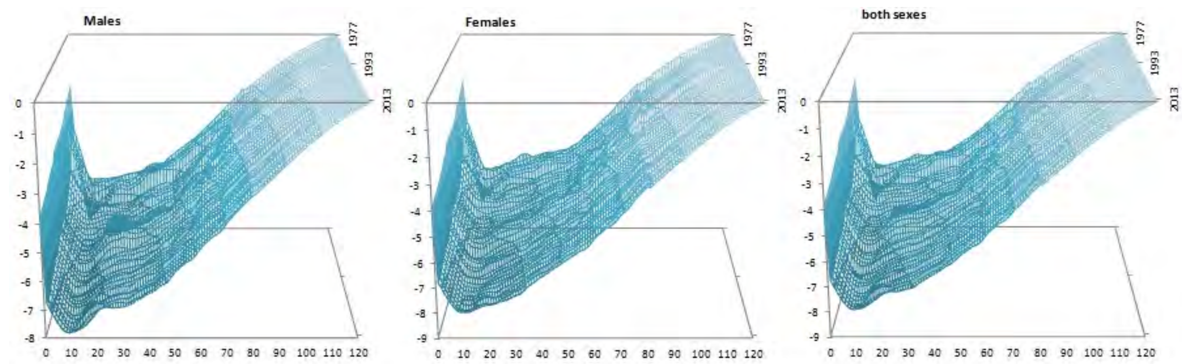

Figure 8: Extended mortality surfaces $ln(q_{xt})$ for males, females and both sexes populations (1977-2014)

As we have already pointed out, the life expectancy at birth is largely related to how the life tables are closed out. Figure 9 shows a comparison of the life expectancy at birth issued from national statistics as well as the re-estimated series.

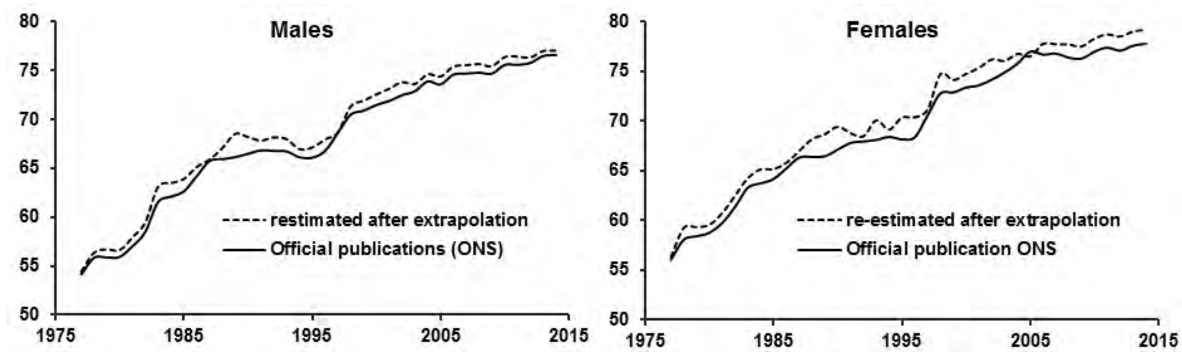

Figure 9: Re-estimated life expectancy compared to official statistics

The re-estimated life expectancy is slightly higher than the values included in the ONS official publications. The average gap in this sense is around 0.9 year for males and 1.2 year for females.

5 Conclusion

We have seen along this paper the advantages of the extrapolation approach to close out the Algerian life tables compared to the use of the MLT as an external reference. The advantage of the first approach is to allow extending the mortality pattern beyond the usual closure age until the surviving age limit. These details are generally needed in actuarial calculations and population forecasts. Because of the information's lack and unreliability, such a detail is usually not included in official life tables which are closed out at early ages. Only the residual life expectancy at the closure age is published to summarize the mortality pattern for the ages beyond. In adverse, the quality of the estimates resulted from the use of the MLT to close-out national life tables is very related to the adequacy of the selected MLT with national data. Since that, a wrong use of the MLT may lead to unrealistic estimated life expectancy at the closure age. We have seen in the introduction of this work some evidences concerning the wrong use of MLT in the case of some African countries (Ekanem & Som, 1984).

In this paper, we proposed another approach to close-out the Algerian official life tables ensuring more accuracy, adequacy and regularity. Our approach was to estimate the old age mortality by extrapolating the observed mortality trend at adult ages. A set of old age mortality models were presented and compared for this issue, i.e., Gompertz (1825),Weibull (1951), Kannisto (1992), Heligman & Pollard (1980), Coale & Kisker (1990), and Denuit & Goderniaux (2005). Models evaluation and selection were based on a set of criteria: the goodness of fit, the predictive capacity, the predicted age limit, the coherence between male and female mortality, and the coherence between single sexes and both sexes' estimates. In the first selection stage, three models were selected, i.e., the CK, DG and GPZ model. To enhance the quality of the estimates, we imposed the age of 120 years old as an age limit constraint for the quadratic models (CK and DG) and other constraints to ensure coherence in male vs female, single vs both sexes extrapolations. In final, we concluded that the three models lead to similar results. On the basis of the age limit expected by the three models, the GPZ model was excluded from the comparison. Among the two quadratic models, the CK model marked better quality than the DG model regarding the male vs female and single vs both sexes coherence. Once the old age mortality rates were extended until the age 120 with the CK model, life expectancy at birth was re-estimated. The comparison to the life expectancy at birth in national statistics showed that our method leads globally to a gain of about 1 year in average on the whole period [1977, 2014]. Also, the obtained series shows more regularity in terms of the time evolution trends. Finally, we would like to highlight the importance of closing out the Algerian life table by extrapolating the observed mortality trend at adult ages rather than the use of MLT as an external reference. This latter approach allows to reduce irregularities in the mortality indicators time evolution series which is supposed to suit perfectly a pertinent analysis of mortality natural evolution. Also, the presented methodology provides readers a way to extrapolate mortality to old ages in the Algerian context.

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