LTAM Exam

Spring 2020

Solutions to Written Answer Questions

Question 1 Model Solution

Learning Outcomes: 2(b), 3(a), 4(a), 5(b), 5(c)

Chapter References: 2nd Ed. Chapters 4, 6, 9 and 12. AMLCR (3rd Ed.) Chapters 4, 6, 10 and 13.

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a) Due to independence:

$$_{10}p_{70:70} = ({}_{10}p_{70})^2 = \left(\frac{l_{80}}{l_{70}}\right)^2 = \left(\frac{75,657.2}{91,082.4}\right)^2 = 0.689972$$

Comment: This part was done correctly by almost all candidates.

b)

$E[L_0] = 100{,}000A_{70:70:\overline{10|}} + 400 - G(0.98\,\ddot{a}_{70:70:\overline{10|}} - 0.48)$ where

$\ddot{a}_{70:70:\overline{10|}} = 7.2329$ (from SULT)

$$A_{70:70:\overline{10|}} = 1 - d\ddot{a}_{70:70:\overline{10|}} = 1 - (0.04762)(7.2329) = 0.65557$$

Alternatively,

$$\begin{aligned} A_{70:70:\overline{10|}} &= A_{70:70} + {}_{10}p_{70:70}\ v^{10}\,(1 - A_{80:80}) \\ &= 0.52488 + 0.689972\,(1.05)^{-10}\,(1 - 0.69146) \\ &= 0.65557 \end{aligned}$$

Then

$$\begin{aligned} E[L_0] &= 65{,}557 + 400 - 10{,}658\,(0.98(7.2329) - 0.48) \\ &= -4{,}474 \end{aligned}$$

Comment: Once again, this part was done correctly by almost all candidates.

c)

i) $$^2A_{70:70:\overline{10|}} = {}^2A_{70:70} + {}_{10}p_{70:70}\ v^{20}\,(1 - {}^2A_{80:80})$$
$$= 0.30743 + 0.689972\,(1.05)^{-20}\,(1 - 0.50165)$$
$$= 0.437$$

ii) $$L_0 = 100{,}000\,v^{min(K_{70:70}+1,10)} + 400 - G\left(0.98\,\ddot{a}_{\overline{min(K_{70:70}+1,10)|}} - 0.48\right)$$
$$L_0 = 100{,}000 v^{\min(K_{70:70}+1,10)} + 400 - G\left((0.98)\left(\frac{1-v^{\min(K_{70:70}+1,10)}}{d}\right) - 0.48\right)$$
$$L_0 = \left(100{,}000 + \frac{0.98\,G}{d}\right) v^{min(K_{70:70}+1,10)} + 400 - G\left(\frac{0.98}{d} - 0.48\right)$$

$$Var[L_0] = \left(100{,}000 + \frac{0.98\,G}{d}\right)^2 Var\left[v^{min(K_{70:70}+1,10)}\right]$$
$$= \left(100{,}000 + \frac{0.98\,G}{d}\right)^2 \left({}^2A_{70:70:\overline{10|}} - (A_{70:70:\overline{10|}})^2\right)$$
$$= \left(100{,}000 + \frac{0.98(10{,}658)}{0.04762}\right)^2 \left(0.437 - (0.65557)^2\right)$$
$$= 737{,}080{,}000$$

$$SD[L_0] = 27{,}150$$

Comment: Your answer will be slightly different if you carry more decimal places. The majority of students did well on this part with many receiving full credit.

d)

Let $L_{0,i}$ be the loss at issue for policy i, i=1,2,…,100.
The aggregate loss at issue is $= \sum_{i=1}^{100} L_{0,i}$.
Then,
$$E[L] = 100\,E[L_0] = -447{,}400$$
$$Var[L] = 100 Var[L_0]$$
$$SD[L] = 10 SD[L_0] = 271{,}500$$

$$P[L > 0] \approx 1 - \Phi\left(\frac{0 - (-447{,}400)}{271{,}500}\right)$$
$$\approx 1 - \Phi(1.648)$$
$$\approx 0.05$$

Comments: Candidates also did well on this part of the question.

e)

The total profit realized in this sub-portfolio will be **<u>greater</u>** than its expected value.

<u>Justification:</u>

There were two death benefits paid during the term of the contract and 8 endowment benefits paid at maturity (time 10).

The expected number of death benefits in the sub-portfolio was 10 $(1 - {}_{10}p_{70:70})$ which represents 3.1 death benefits.

The EPV of the 2 death benefits was lower than expected.

<u>Or</u>

The mortality experience in this sub-portfolio was better than expected and the death benefits were paid relatively late in the term of the policy (at times 6 and 9).

Consequently, the EPV of the benefits was lower than expected.

Comments:

- *Very few students got this correct.*
- *Even those that determined that the expected value was greater generally were not correct in the justification of the answer.*

Question 2 Model Solution

Learning Outcomes: 2(a), 2(b), 3(a), 4(b), 5(a)

Chapter References: AMLCR (2nd Ed. Chapters 7, 8) (3rd Ed.) Chapters 9 and 7, 8 (reserves)

a)

$$ {}_sp_{40}^{(\tau)} = exp\left\{-\int_0^s \mu_{40+t}^{(\tau)}\, dt\right\} $$

$$ p_{40}^{(\tau)} = exp\left\{-\int_0^1 \mu_{40+t}^{(1)} + \mu_{40+t}^{(2)}\, dt\right\} = exp\left\{-4\int_0^1 \mu_{40+t}^{(1)}\, dt\right\} $$

with

$$ \begin{aligned} exp\left\{-\int_0^1 \mu_{40+t}^{(1)}\, dt\right\} &= exp\left\{-\int_0^1 A + Bc^{40+t}\, dt\right\} \\ &= exp\left\{-A - B\int_0^1 1.12^{40+t}\, dt\right\} \\ &= exp\left\{-0.0001 - 1.075x10^{-5}\left(\frac{1.12^{41} - 1.12^{40}}{ln\ 1.12}\right)\right\} \\ &= exp\{-0.001159182\} \\ &= 0.9988415 = p'^{(1)}_{40} \end{aligned} $$

So,

$$ p_{40}^{(\tau)} = (0.9988415)^4 = 0.995374 $$

Alternatively,

$$ \begin{aligned} p_{40}^{(\tau)} &= exp\left\{-4\int_0^1 \mu_{40+t}^{(1)}\, dt\right\} \\ &= exp\left\{-4\int_0^1 A + Bc^{40+t} dt\right\} = exp\left\{-4\left(0.0001 + 1.075x10^{-5}\left(\frac{1.12^{41}-1.12^{40}}{ln\ 1.12}\right)\right)\right\} \\ &= exp\{-4(0.001159182)\} = 0.995374 \end{aligned} $$

Alternatively,

$$\mu_{40}^{(\tau)} = \mu_{40}^{(1)} + \mu_{40}^{(2)} = 4\mu_{40}^{(1)} = 4\left[0.0001 + 0.00001075(1.12)^{40+t}\right]$$

This is Makeham so from the tables

$$p_{40}^{(\tau)} = \exp\left[-0.0004 - \left(\frac{0.000043}{\ln(1.12)}\right)(1.12)^{40}(1.12^{1} - 1)\right]$$

$$= 0.995374$$

Comments:

- *As a group, students did well on this part.*
- *The most common error was treating decrements as independent instead of dependent. While this resulted in the same answer to four decimal places, points were deducted for an incorrect calculation.*
- *Another common error was assuming that the force of decrement was constant.*

b)

i)

$$EPV = 100,000\sum_{k=0}^{2} v^{k+1} \cdot {}_k p_x^{(\tau)} \cdot q_{x+k}^{(\tau)} = 100,000\sum_{k=0}^{2} v^{k+1}({}_k p_x^{(\tau)} - {}_{k+1} p_x^{(\tau)})$$

$$EPV = 100,000\left[\begin{array}{r}(1-0.995374)v + (0.995374 - 0.99027)v^2 \\ +(0.99027 - 0.98462)v^3\end{array}\right] = 1391.59$$

ii)

$$EPV(premiums) = P(1 + 0.995374v + 0.99027v^2) = P(2.846179)$$

$$P = \frac{1391.59}{2.846179} = 488.93$$

Comments: Candidates did very well on this part of the question with the majority of the students getting full credit.

c)

(i) $\;{}_tq_{40}^{(1)} = \int_0^t {}_sp_{40}^{(\tau)}\, \mu_{40+s}^{(1)}\, ds$

(ii)

$$\begin{aligned} {}_tq_{40}^{(\tau)} &= \int_0^t {}_sp_{40}^{(\tau)}\, \mu_{40+s}^{(\tau)}\, ds \\ &= \int_0^t {}_sp_{40}^{(\tau)} \left(\mu_{40+s}^{(1)} + \mu_{40+s}^{(2)}\right) ds = \int_0^t {}_sp_{40}^{(\tau)} \left(4\, \mu_{40+s}^{(1)}\right) ds \\ &= 4 \int_0^t {}_sp_{40}^{(\tau)} \left(\mu_{40+s}^{(1)}\right) ds \\ &= 4\; {}_tq_{40}^{(1)} \end{aligned}$$

$$\Rightarrow {}_tq_{40}^{(1)} = \frac{1}{4}\, {}_tq_{40}^{(\tau)}$$

(iii) $\;\text{EPV(Disease 1 rider)} = \sum_{k=0}^{2} 50{,}000 v^{k+1}\, {}_{k|}q_{40}^{(1)}$

$$= \sum_{k=0}^{2} 50{,}000\; v^{k+1} \left(\frac{1}{4}\, {}_{k|}q_{40}^{(\tau)}\right)$$

$$= 50{,}000\left[\begin{aligned}(1/4)(1-0.995374)v + (1/4)(0.995374-0.99027)v^2 \\ +(1/4)(0.99027-0.98462)v^3\end{aligned}\right] = 173.95$$

Rider net premium = 173.95/2.846179 since the *EPV(premiums)* is the same as in (b) ii.

P^r = 61.12

Comments

- *Almost all students received credit for Part i.*
- *Many students did some work on Part ii but only received partial credit because they did not prove that this true. For example, many assumed that the force of decrements were constant in their proof but the force of decrement is not constant. These answers only received partial credit.*
- *For Part iii, many candidates skipped this part. Those that attempted to calculate the premium did well. These students recognized that the relationship in Part ii could be used to solve for the premium in a straight forward manner.*

d)

$$ {}_1V = \frac{({}_0V + P + P^r)(1+i) - EPV(db) - EPV(\text{rider benefit})}{p_{40}^{(\tau)}} $$

$$ = \frac{(0 + 488.93 + 61.12)(1.05) - 100{,}000\, q_{40}^{(\tau)} - 50{,}000\, q_{40}^{(1)}}{p_{40}^{(\tau)}} $$

$$ = \frac{(550.05)(1.05) - 462.60 - 57.83}{0.995374} = 57.39 $$

Alternatively

$$ {}_1V = PVFB - PVFP $$

$$ = 100{,}000(vq_{41}^{(\tau)} + v^2 p_{41}^{(\tau)} q_{42}^{(\tau)}) + 50{,}000(vq_{41}^{(1)} + v^2 p_{41}^{(\tau)} q_{42}^{(1)}) - (488.93 + 61.12)(1 + vp_{41}^{(\tau)}) $$

$$ 100{,}000\left[\frac{1 - \frac{0.99027}{0.995374}}{1.05} + \frac{\frac{0.99027}{0.995374}\left(1 - \frac{0.98462}{0.99027}\right)}{1.05^2}\right] $$

$$ + 50{,}000(0.25)\left[\frac{1 - \frac{0.99027}{0.995374}}{1.05} + \frac{\frac{0.99027}{0.995374}\left(1 - \frac{0.98462}{0.99027}\right)}{1.05^2}\right] $$

$$ -550.05\left[1 + \frac{\frac{0.99027}{0.995374}}{1.05}\right] = 57.39 $$

Comments:

- *Many students did not attempt this part. Those that did attempt question still found Part (d) to be very challenging.*
- *The easy way to this part was to use the recursive formula. Only about 10% of the candidates used this method.*
- *Most students calculated the reserve using the alternative method of PVFB – PVFP. Most did not receive full credit as numerical errors or formula errors resulted. A common error was to use the wrong premium.*

Question 3 Model Solution

Learning Outcomes: 1(a), 2(b), 3(a)

Chapter References: AMLCR Chapters 4 and 5

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a)

(i) $Y_1 = \frac{1-Z_1}{\delta}$

where $Y_1 = \bar{a}_{\overline{min(T_x,n)}|}$ and $Z_1 = v^{min(T_x,n)}$

(ii) $Y_2 = \frac{1-Z_2}{\delta}$

where $Y_2 = \bar{a}_{\overline{T_x}|}$ and $Z_2 = v^{T_x}$

Comments:

- *Many candidates skipped this question.*
- *The candidates that did attempt this question did very well.*

b)

(i) $Z_1\, Z_2 = \begin{cases} v^{2\, T_x} & T_x < n \\ v^{T_x+n} & T_x \geq n \end{cases}$

$$E[Z_1\, Z_2] = \int_0^n v^{2t}\ f_x(t)\ dt + \int_n^\infty v^t\ v^n\ f_x(t)\ dt$$

Or

$$E[Z_1\, Z_2] = \int_0^n v^{2t}\ {}_tp_x\ \mu_{x+t}\, dt + \int_n^\infty v^t\ v^n\ {}_tp_x\ \mu_{x+t}\, dt$$

(ii) $E[Z_1 \, Z_2] = \int_0^n (v^2)^t \; {}_tp_x \;\; \mu_{x+t} \, dt + \int_n^\infty v^t \, v^n \; {}_tp_x \;\; \mu_{x+t} \, dt$

$$E[Z_1 \, Z_2] = {}^2\bar{A}^{\;1}_{x:\overline{n|}} + \int_n^\infty v^t \, v^n \; {}_tp_x \;\; \mu_{x+t} \, dt$$

Let t=n+s,

$$E[Z_1 \, Z_2] = {}^2\bar{A}^{\;1}_{x:\overline{n|}} + \int_0^\infty v^{n+s} \, v^n \; {}_{n+s}p_x \;\; \mu_{x+n+s} \, ds$$

$$= {}^2\bar{A}^{\;1}_{x:\overline{n|}} + v^{2n} \int_0^\infty v^s \; {}_np_x \; {}_sp_{x+n} \;\; \mu_{x+n+s} \, ds$$

$$= {}^2\bar{A}^{\;1}_{x:\overline{n|}} + v^{2n} \, {}_np_x \int_0^\infty v^s \; {}_sp_{x+n} \;\; \mu_{x+n+s} \, ds$$

$$= {}^2\bar{A}^{\;1}_{x:\overline{n|}} + v^n \, {}_nE_x \;\; \bar{A}_{x+n}$$

$$= \left({}^2\bar{A}^{\;1}_{x:\overline{n|}} + v^n \, {}_nE_x \right) - v^n \, {}_nE_x \; + v^n \, {}_nE_x \;\; \bar{A}_{x+n}$$

$$= {}^2\bar{A}_{x:\overline{n|}} - v^n \, {}_nE_x \; + v^n \, {}_nE_x \;\; \bar{A}_{x+n}$$

$$= {}^2\bar{A}_{x:\overline{n|}} - v^n \, {}_nE_x \left(1 - \bar{A}_{x+n}\right)$$

Comments:

- *This part of the question was not attempted by the majority of the candidates.*
- *For the candidates that attempted this question, very few received full credit but most received significant partial credit.*
- *Candidates did significantly better on Part i when compared to Part ii.*

c)

$$
\begin{aligned}
Cov[Y_1, Y_2] &= Cov\left[\frac{1-Z_1}{\delta}, \frac{1-Z_2}{\delta}\right] \\
&= \frac{1}{\delta^2} cov[Z_1, Z_2] \\
&= \frac{1}{\delta^2}\left[E[Z_1\, Z_2] - E[Z_1]E[Z_2]\right] \\
&= \frac{1}{\delta^2}\left[{}^2\overline{A}_{x:\overline{n}|} - v^n\, {}_nE_x\left(1 - \overline{A}_{x+n}\right) - \overline{A}_{x:\overline{n}|}\,\overline{A}_x\right]
\end{aligned}
$$

Alternative derivation,

$$
\begin{aligned}
Cov[Y_1, Y_2] &= E[Y_1\, Y_2] - E[Y_1]E[Y_2] \\
&= E\left[\frac{1-Z_1}{\delta}\cdot\frac{1-Z_2}{\delta}\right] - E\left[\frac{1-Z_1}{\delta}\right]E\left[\frac{1-Z_2}{\delta}\right] \\
&= E\left[\frac{1-Z_1-Z_2+Z_1\, Z_2}{\delta^2}\right] - \frac{(1-E[Z_1])(1-E[Z_2])}{\delta^2} \\
&= \frac{1}{\delta^2}\{1 - E[Z_1] - E[Z_2] + E[Z_1 Z_2] - (1 - E[Z_1] - E[Z_2] + E[Z_1]E[Z_2])\} \\
&= \frac{1}{\delta^2}\{E[Z_1\, Z_2] - E[Z_1]E[Z_2]\} \\
&= \frac{1}{\delta^2}\left[{}^2\overline{A}_{x:\overline{n}|} - v^n\, {}_nE_x\left(1 - \overline{A}_{x+n}\right) - \overline{A}_{x:\overline{n}|}\,\overline{A}_x\right]
\end{aligned}
$$

Alternative expression,

$$
\begin{aligned}
Cov[Y_1, Y_2] &= E[Y_1\, Y_2] - E[Y_1]E[Y_2] \\
&= E\left[\frac{1-Z_1}{\delta}\cdot\frac{1-Z_2}{\delta}\right] - \overline{a}_{x:\overline{n}|}\,\overline{a}_x \\
&= E\left[\frac{1-Z_1-Z_2+Z_1\, Z_2}{\delta^2}\right] - \overline{a}_{x:\overline{n}|}\,\overline{a}_x \\
&= \frac{1}{\delta^2}\left[1 - \overline{A}_{x:\overline{n}|} - \overline{A}_x + {}^2\overline{A}_{x:\overline{n}|} - v^n\, {}_nE_x\left(1 - \overline{A}_{x+n}\right)\right] - \overline{a}_{x:\overline{n}|}\,\overline{a}_x
\end{aligned}
$$

Comments:

- *This part of the question was not attempted by the majority of the candidates.*
- *For the candidates that attempted this question, very few received full credit but most received significant partial credit.*

d)

The covariance is <u>positive</u>.

Reason:

The longer (x) survives:

- the higher the value of the whole life annuity will be;
- the higher the value of the term annuity will be (subject to a maximum after time n).

<u>or</u>

If the whole life annuity payable to (x) gets more expensive, the term annuity either gets more expensive or stays the same.

<u>or</u>

The two random variables never move in opposite directions and move in the same direction in some cases ($T_x < n$), therefore the correlation is positive (so is the covariance).

Comments:

- *As with Parts (b) and (c), this part of the question was not attempted by the majority of the candidates.*
- *For those candidates who correctly identified that the covariance was positive, many were not able to state the reasons why.*

Question 4 Model Solution

Learning Outcomes: 1(a), 1(c), 2(j), 3(a), 4(b), 4(c)

Chapter References: SN LTAM 21-18 **AMLCR (3rd Ed.) Chapter 1 and 8**

(a)

The gross premium rate, P, is such that,

EPV[benefits and expenses] = EPV[premiums]

$$\underbrace{20{,}000\bar{A}^{02}_{x:\overline{10}|}}_{\text{Death ben.}} + \underbrace{1000\bar{a}^{01}_{x:\overline{10}|}}_{\text{Sickness ben.}} + \underbrace{60\left(\bar{a}^{00}_{x:\overline{10}|} + \bar{a}^{01}_{x:\overline{10}|}\right)}_{\text{Maintenance exp.}} + \underbrace{0.05(P)\bar{a}^{00}_{x:\overline{10}|}}_{\text{Commissions}} = P\bar{a}^{00}_{x:\overline{10}|}$$

⇨ $(0.95)P\bar{a}^{00}_{x:\overline{10}|} = 3020 + 684 + 60(5.844 + 0.684)$

$$P = \frac{4095.68}{5.5518} = 737.7211$$

Comments: Candidates did well in this part overall.

(b)

Pr(death within 10 years without becoming sick) is

$$\int_0^{10} {}_tp_x^{\overline{00}}\ \mu_{x+t}^{02}\ dt = \int_0^{10} e^{-(0.03+0.02)t}(0.02)dt$$

$$= \frac{0.02}{0.05}e^{-(0.05)t}\Big|_0^{10} = (0.4)(1 - e^{-0.5}) = 0.15739$$

Comments: Many candidates did well in this part. The most common mistake was confusing ${}_tp_x^{\overline{00}}$ *with* ${}_tp_x^{00}$.

(c)

The *elimination period* or *waiting period* is the time between the beginning of a period of disability/sickness and the start of the disability income payments.

Comments: Candidates did not do well on this part. Common mistakes were:

- *Confusing the definition of elimination period with the definition of off period;*
- *Not clearly stating the trigger of events at the beginning (becoming disabled/sick) and end (benefit payment starting) of the elimination period.*

(d)

$$\bar{a}_{x:\overline{1}|}^{\overline{11}} = \int_0^1 e^{-\delta t}\, {}_tp_x^{\overline{11}}\, dt$$

$$= \int_0^1 e^{-0.07t}\, e^{-(\mu^{10}+\mu^{12})t}\, dt$$

$$= \int_0^1 e^{-0.07t}\, e^{-(0.01+0.05)t}\, dt$$

$$= \int_0^1 e^{-0.13t}\, dt = \frac{1-e^{-0.13}}{0.13} = \frac{1-0.878095}{0.13} = 0.937727$$

Comments: Many candidates did well in this part. Common mistakes include:

- *Using the wrong force of transition; and*
- *Forgetting discounting in the EPV of the annuity.*

(e)

The only change is the duration of the sickness benefit. No change in death benefit, maintenance expenses, commissions.

EPV(sickness benefit) becomes $1000 \int_0^{10} e^{-\delta t} \; {}_tp_x^{00} \;\; \mu_{x+t}^{01} \quad \left(\bar{a}_{x+t}^{\overline{11}} - \bar{a}_{x+t:\overline{1}|}^{\overline{11}}\right) dt$

Where $\bar{a}_{x+t}^{\overline{11}} = \int_0^{\infty} e^{-\delta s} \; {}_sp_{x+t}^{\overline{11}} \, ds = \int_0^{\infty} e^{-0.07s} \, e^{-0.06\,s} \;\, ds = \frac{1}{0.13} = 7.69231$

⇨ EPV(sickness benefit) = $1000\,(7.69231 - 0.937727) \int_0^{10} e^{-\delta t} \; {}_tp_x^{00} \;\; \mu_{x+t}^{01} \, dt$

Since $\int_0^{10} e^{-\delta t} \; {}_tp_x^{00} \;\; \mu_{x+t}^{01} \, dt = \bar{A}_{x:\overline{10}|}^{01}$,

EPV(sickness benefit) = (6754.58) (0.175) = 1182.0515

Premium for the new policy, P^*:

EPV(new premiums) = EPV (benefits and expenses)

⇨ $(0.95)P^*\bar{a}_{x:\overline{10}|}^{00} = 3020 + 1182.0515 + 391.68$

$$P^* = \frac{4593.7315}{5.5518} \; = \; 827.431$$

<u>Another acceptable interpretation was that</u>:

The 60 of maintenance expenses are paid while receiving sickness benefits & elimination period, not just during the first 10 years. In that case,

EPV(maint. Exp.) = $60\,(7.69231)(0.175) = 80.7693$

⇨ $(0.95)P^*\bar{a}_{x:\overline{10}|}^{00} = 3020 + 1182.0515 + 60(5.844) + 80.7693$

$$P^* = \frac{4633.461}{5.5518} \; = \; 834.59$$

<u>*Comments:*</u>

- *This part is very challenging for most candidates.*
- *Candidates had trouble putting together the correct pieces to calculate the EPV of this special new sickness benefit.*
- *One common mistake was confusing ${}_tp_x^{\overline{00}}$ with ${}_tp_x^{00}$. Conceptually, the policyholder can receive the sickness benefit more than once should they become sick multiple times within the 10-year term and each sickness last longer than the elimination period.*

Question 5 Model Solution

Learning Outcomes: 1(j), 3(a), 4(b), 5(a), 5(c)

Chapter References: AMLCR 2nd Ed: Ch. 5, 8 , 9 and 12 3rd Ed: Chapters 5, 8, 10 and 13

(a)

EPV(payments + expenses)=100,000

$2000 + 500 + (B + 25)a_{\overline{65:\overline{10}|}} = 100{,}000$ where $a_{\overline{65:\overline{10}|}} = a_{\overline{10}|} + {}_{10}E_{65}\, a_{75}$

$$a_{\overline{65:\overline{10}|}} = \frac{1-v^{10}}{i} + (0.55305)(9.3178) = 7.72173 + 5.15321 = 12.87494$$

$$\Rightarrow \quad B = \frac{100{,}000 - 2500 - (25)(12.87494)}{12.87494} = 7{,}547.848$$

Comments: Candidates did well on this part.

(b)

Note that at age 73, the annuity is within the 10-year guarantee period. So only states 0 and 1 are possible. Possible transitions are 0→0, 0→1, 1→1.

(i) $p_{73}^{01} = q_{73} = 0.014664$

(ii) $p_{73}^{02} = 0$
Since there is no possible transition from State 0 at 73 to State 2 at age 74since the annuity is within the guarantee period.

(iii) $p_{73}^{12} = 0$ Since payments would continue until age 76 if annuitant died before 73.

Comments: Many candidates answered (ii) and/or (iii) incorrect.

(c)

At time 8, there are 2 years left in the guarantee period.

$${}_8V^{(0)} = EPV(\text{future payments plus maintenance expenses})$$
$${}_8V^{(0)} = (B+25)a_{\overline{65+8:\overline{10-8}|}}$$
$$= (7{,}547.848+25)a_{\overline{73:\overline{2}|}} \quad \text{where}$$
$$a_{\overline{73:\overline{2}|}} = a_{\overline{2}|} + {}_2E_{73}\, a_{75} = \frac{1-v^2}{i} + v^2\, {}_2p_{73}(9.3178)$$

$$= 1.85941 + (0.879036)(9.3178) = 10.050091$$

$$\Rightarrow {}_8V^{(0)} = 76{,}107.81$$

Comments: Many students did not answer this part. Those that did answer, did well.

(d)

In State 1, there are only 2 guaranteed payments to be made.

$${}_8V^{(1)} = (B+25)a_{\overline{2}|}$$
$$= (7547.848+25)(1.85941) = 14{,}081.03$$

Comments: As with Part (c), many students did not answer this part. Those that did answer, did well.

(e)

(i) $Pr_9^{(0)} = {}_8V^{(0)}(i_9 - i)$
$= (76{,}107.81)(0.06-0.05) = 761.08$

(ii) $Pr_9^{(1)} = {}_8V^{(1)}(i_9 - i)$
$= (14{,}081.03)(0.06-0.05) = 140.81$

<u>Alternatively, for (i) and (ii)</u>, using

$${}_9V^{(1)} = (B+25)v$$
$$= (7547.848+25)/1.05 = 7212.236$$

and

$${}_9V^{(0)} = (B+25)v + {}_1E_{74}(B+25)a_{75}$$
$$= 7212.236 + (0.936724)(7547.848+25)(9.3178)$$
$$= 7212.236 + 66{,}097.384 = 73{,}309.62$$

(i) $Pr_9^{(0)} = {}_8V^{(0)}(1 + i_9) - E - p_{73}^{00}\left(B + 25 + {}_9V^{(0)}\right) - p_{73}^{01}\left(B + 25 + {}_9V^{(1)}\right)$

$= 76{,}107.81(1.06) - 0 - (0.985336)(7547.848 + 25 + 73{,}309.62)$

$-(0.014664)(7547.848 + 25 + 7212.236)$

$= 80{,}674.2786 - 79.696.4075 - 216.8085 = 761.06$

(ii) $Pr_9^{(1)} = {}_8V^{(1)}(1 + i_9) - E - \left(B + 25 + {}_9V^{(1)}\right)$

$= (14{,}081.03)(1.06) - 0 - (7547.848 + 25 + 7212.236)$

$= 14{,}925.892 - 14{,}785.084 = 140.81$

(iii) $\pi_9 = {}_8p_{65}Pr_9^{(0)} + (1 - {}_8p_{65})Pr_9^{(1)}$

$= (0.9295525)(761.08) + (0.0704475)(140.81)$

$= 717.38$

Comments: Candidates found this part to be very challenging. Many candidates did not attempt this part of the question. Even those that did attempt part (e), did not have great success.

Question 6 Model Solution

Learning Outcomes: 6(d), 6(e), 6(g)

Chapter References: AMLCR 2nd Ed. Chapters 9, 10 3rd Ed. Chapters 10, 11.

(a)

(i)
Final Average Salary (FAS) between ages 64 and 65 is projected to be
$S_{39}(1.03)^{54-39}(1.025)^{64-54} = 110{,}000(1.557967)(1.2800845) = 219{,}376.23$

$$AL = 219{,}376.23(0.02)(20)\,{}_{25}E_{40}\ddot{a}_{65}^{(12)} = (87{,}750.49)\,{}_{25}E_{40}\ddot{a}_{65}^{(12)}$$

where
$${}_{25}E_{40} = v^{25}\ {}_{25}p_{40} = {}_{20}E_{40}\ {}_{5}E_{60} = (0.36663)(0.76687) = 0.281158$$

$$\ddot{a}_{65}^{(12)} = \ddot{a}_{65} - \frac{11}{24} = 13.5498 - \frac{11}{24} = 13.09147$$

$$\Rightarrow\ AL = (87{,}750.49)(0.281158)(13.09147) = 322{,}989.50$$

$AL_0 = 322{,}989.50$ and $AL_1 = 219{,}376.23(0.02)(21)\,{}_{24}E_{41}\ddot{a}_{65}^{(12)}$

$$NC = v\,p_{40}\,AL_1 - AL_0 = \frac{AL_0}{20}$$

$$NC = \frac{AL}{20} = \frac{322{,}989.50}{20} = 16{,}149.48$$

$$NC\ rate = \frac{NC}{S_{40}} = \frac{16{,}149.48}{110{,}000(1.03)} = 0.142537$$

Comments: Candidates did very well on both parts.

- *A common error on Part (i) were using the wrong salary increases or not projecting for 25 years (24 or 26 years). A less common error was crediting 45 years of service instead of 20.*
- *Almost all candidates who answered (i) (correctly but even incorrectly) got the Normal Cost correct.*
- *Only very well-prepared candidates calculated the Normal Cost rate correctly. Common errors were not calculating NC rate or using the wrong salary.*

(b)

(i) Per unit of FAS, the EPV of the retirement benefits are

- <u>With partner at retirement</u>, $EPV^p = \ddot{a}_{65}^{(12)} + 0.5\,\ddot{a}_{65|65}^{(12)}$
$= 13.09147 + 0.93335 = 14.02482$

where

$$\ddot{a}_{65|65}^{(12)} = \ddot{a}_{65}^{(12)} - \ddot{a}_{65:65}^{(12)}$$
$$= \left(\ddot{a}_{65} - \frac{11}{24}\right) - \left(\ddot{a}_{65:65} - \frac{11}{24}\right)$$
$$= \left(13.5498 - \frac{11}{24}\right) - \left(11.6831 - \frac{11}{24}\right) = 1.8667$$

- <u>Single</u>, $EPV^s = 1.05\,\ddot{a}_{65}^{(12)} = 13.74604$

$$AL = 219{,}376.23(0.02)(20)\,{}_{25}E_{40}\left((0.9)EPV^p + (0.1)EPV^s\right)$$
$$= 219{,}376.23(0.02)(20)(0.281158)\left((0.9)(14.02482) + (0.1)(13.74604)\right)$$
$$= 219{,}376.23(0.02)(20)(0.281158)(13.99694)$$
$$= 345{,}329.04$$

(ii) The revised NC rate is

$$NC = AL / 20 = 345{,}329.04 / 20 = 17{,}266.45$$

$$NC\ rate = \frac{NC}{S_{40}} = \frac{17{,}266.45}{110{,}000(1.03)} = 0.1524$$

Comment:

- *The reversionary annuity for members with partners was very challenging. Most omitted this part. Many of those that attempted this part appeared not to know what a reversionary annuity was or ran out of time. Only well prepared candidates made a reasonable attempt at valuing the reversionary annuity.*
- *A number of candidates seemed confused by the 90% with partners at retirement and 10% without partners.*
- *A number discounted the benefit using ${}_{25}E_{40:40}$ instead of ${}_{25}E_{40}$ (as if the member and partner had to survive to age 65 to receive any retirement benefit)*
- *NC was done well by those who made attempted (b)(i).*
- *NC rate was not calculated by the vast majority or done incorrectly.*

(c)

The EPV of the reversionary partner benefits would decrease.

Acceptable Reasons:

Since $\ddot{a}_{x|y} = \ddot{a}_y - \ddot{a}_{x:y}$ where $\ddot{a}_y$ does not change, the change in $\ddot{a}_{x|y}$ would be the opposite of the change in $\ddot{a}_{x:y}$.

Fixing the time of death of (x) and considering the two cases:

- (x) dies before (y): positive correlation means (y) will now tend to die younger than under the independence case but since payments stop at the death of (x) => no change
- (y) dies before (x): positive correlation means (y) will now tend to die older (closer to (x)). Here payments stop at the death of (y) => EPV increases on average.

Conclusion: $\ddot{a}_{x|y}$ decreases.

or

Reversionary benefits have value >0 only if (y) survives (x). With positive correlation, (y)'s time of death in that case would tend to be closer to that of (x) => fewer reversionary payments => EPV decreases.

or

If (x) dies young, (y) will tend to die young => fewer payments, possibly large impact.
If (x) dies old, (y) will tend to die old => likely more payments than under independence but small impact.
Overall, EPV of reversionary benefits would decrease.

Comments:

- About 2/3 omitted or gave an incorrect answer (stay the same, or increase) for this part. About 1/3 said it would decrease and gave the broken heart syndrome as a reason to justify their answer, for which they got full credit.
- Most candidates used the broken syndrome as an example of positively correlated lifetimes and argued that the value of the reversionary annuity would decrease.
- A few presented a more general argument (like the first 2 above) to justify the decrease in EPV when lifetimes are positively correlated.