Stochastic Modeling Techniques

Keeping it Real

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- Life can Get Complex
- Uses of "Stochastic-on-Stochastic"
- Policyholder Behavior
- Balancing Run Time & Model Robustness
- Scenario Reduction Techniques

Life can get complex

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Regulatory requirements constantly expand

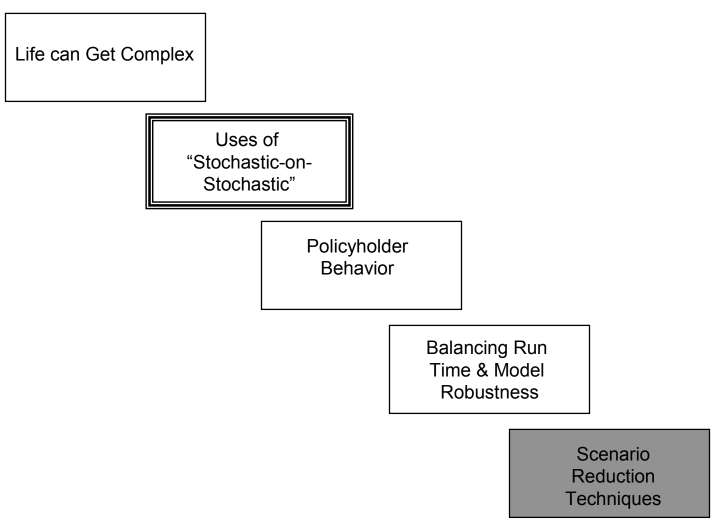
- Many of the newest changes implement new stochastic scenario requirements:
  - Actuarial guideline 39
  - Proposed RBC C3 Phase II (“C3 II”)
  - GAAP SOP 03-1
  - Possible reserve requirement similar to C3 II
- These requirements significantly expand the duties of valuation actuaries

What about product development actuaries?

- Actually, their situation could be even worse
  - Requirements for valuation are often a snapshot
  - Product development actuaries need to be concerned about profitability over products’ lives
- What does that imply?
  - Capital levels must be assessed at future points, not just at issue of a product
  - Ergo, if pricing is done on a stochastic basis, and capital must be done on a stochastic basis, you have the situation known as “stochastic-on-stochastic” projections

Is that all we have to worry about?

- Probably not:
  - Policyholder behavior might also vary by economic scenario
  - That needs to be taken into account over all of your possible paths
- This could easily become a real nightmare
- So how can we get through it?



Clearest Example: Pricing

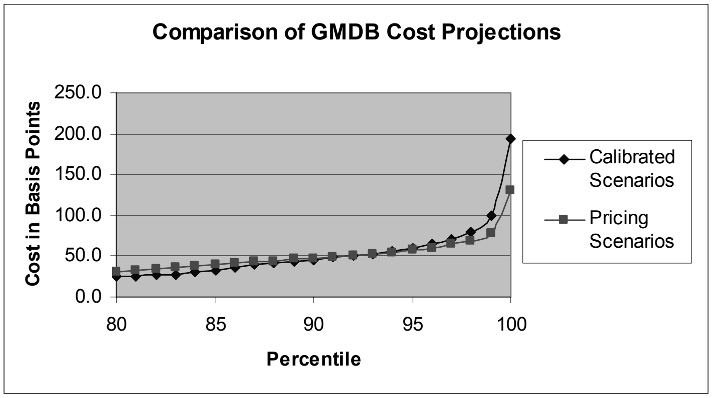
- Pricing product guarantees often involves basic use of stochastic scenarios
  - GMDB on VA or EIA
  - VAGLBs
  - Book value and minimum credited rate guarantees
- If in addition, you feel the need to assess capital strain at issue and through projection, you're there:
  - E.g. 1, basic costing for SPDA, combined with simulated cash flow testing at future points
  - E.g. 2, basic GMDB costing, combined with C3 II
  - E.g. 3, basic VAGLB costing, with AG 39 *and* C3 II
  - The last *could* result in "double SOS "

Looking closer at example 2: what would C3 II tell us to do?

- As a means of setting part of your VA's required capital, you do the following:
  - Calibrate 1,000-10,000 scenarios to C3 II standards
  - Re-project your VA and GMDB over this set
  - Look at your minimum present value of surplus at any duration
  - Set required capital at the average of the worst 10% of your scenario results
- In a stochastic-on-stochastic run, this exercise would be repeated at each of your chosen set of future "valuation dates," to set capital

How do GMDB costs compare?

- Let's look at our scenarios, vs. C3 II scenarios, on a combination (ratchet/rollup) GMDB:



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Any comments on those?

- In this particular case, the costs below the 90th percentile appear to be lower
  - Really no reason they shouldn't be
  - No calibration below the 90th percentile returns
  - Calibration *at* the 90th percentile pretty mild
- The real impact is deep in the tail
- Actual GMDB costs at those points are not the key issue – only surplus shortfall, which definitely could arise, given current typical GMDB rider prices

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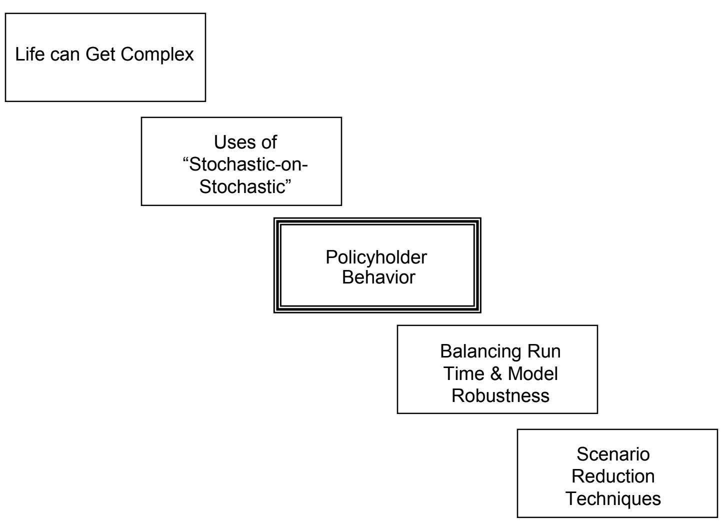
When else could you run into stochastic-on-stochastic

- Primary example – evaluating the effectiveness of a dynamic hedging program
  - Characteristics of the notional you are trying to hedge change over each period
  - Greeks must be reassessed accordingly
  - Then you adjust your asset purchase strategy to rebalance the hedge
- Any other situation where a component of your run changes, and must be reassessed stochastically



What happens in our tail scenarios?

- Typically, the guarantees offered go into the money:
  - SPDA book values are much higher than market
  - SPDA credited rates are higher than market
  - Living benefit or GMDB “shadow fund” values are higher than current fund values
  - GMDB dollar-for-dollar partial withdrawal benefits become extremely valuable, for anti-selection
- What should we expect policyholders to do?
  - It partly depends on the savvy of the policyholder
  - It also depends on agent motivations
  - At least to some extent, the policyholder must be assumed to act in their own best interest

On the VA side, let’s first consider a GMDB, with no reset options

- What decisions does either party have to make?

- Probably, none that will alter your model in any way, even for a fairly complex guarantee. Same is true for *utilization* of a GMAB, if there is a single fixed exercise date. In that case, though, lapse efficiency may be a consideration . . . .
- Clearly, any reset options change the picture, for GMDB or GMAB

But now, consider the GMIB on VAs

- Let’s spend a minute or two discussing features
- Now, in a stochastic scenario environment what policyholder decisions do we need to consider:
  - Most importantly, on the downside?
  - But also, on the upside? (Hint: not necessarily for Cash Flow Testing, but maybe for you guys)
- Are there any company decisions? Yes, but mostly related to investment, if the GMIB provisions are set and guaranteed at VA issue

How can one best model the downside “elective” risk in a GMIB?

- What can we assume to be true?
  - The more “in-the-money” a guarantee is, the more likely a policyholder is to exercise it
  - The presence of a commission on annuitization would make exercise more likely than the absence
- What are some likely assumptions that might in fact not be true?
  - The policyholder has some notion of what kind of SPIA his fund value could buy on a current basis
  - The policyholder has a clue what his guaranteed SPIA would be 1, 2, 3 or more years from now
  - The policyholder realized what he was buying

Downside Risk Example: Set-Up

- A 60-year-old buys a $10,000 single premium VA with a GMIB. This person has a deterministic life span to precisely age 82. (Because I want them to.)
- The guaranteed shadow fund grows at 5% until age 80. The guaranteed purchase rate is 2.5% for a 15-year (certain only) annuity.
- Seven years later, the fund has only grown by a total of 10%.
- On the open market, the policyholder could get a 5% purchase rate for a 15-year certain payout.

Downside Risk Example

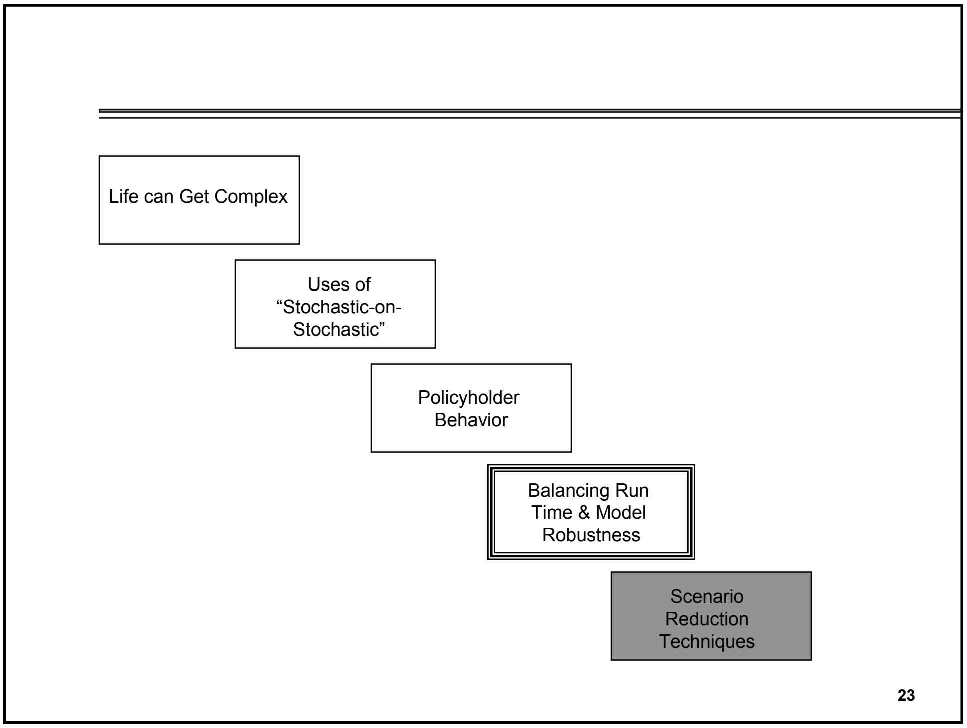
- So, the GMIB shadow fund is at $14,071. The policyholder's fund value is at $11,000. Does the policyholder think they are "in-the-money"?
- The fact is, they are. But if they misguess, and think they could get 6% on the open market, they will not think that they are (assuming they work it out at all).
- Clearly, if the fund were at $6,000, they would be more likely to guess right!

Downside Risk Example

- Okay, so say the policyholder realizes he is “in-the-money.” Will he utilize the benefit right away?
- It depends. This year, the now 67-year-old can get $1,136 per year for life. Next year, the shadow fund will be $14,775 and life expectancy will have dropped a year. They can then get $1,264 per year.
- This trend continues each year. But is the policyholder aware of it at all?
- Did the policyholder take the GMIB intending one day to buy a life income? Or just as a safety net?
- Will any or all of this affect lapse rates?

Wrapping up on GMIB downside risk

- There may be a lot to think about when trying to develop a “reasonable” formula-based assumption about a policyholder behavior
- In this case (GMIB), it is precisely the play in this assumption that makes the benefit difficult if not impossible to hedge exactly
- Your “audience” (new product committee?) will want to be aware of and comfortable with your assumption, and may also like to see various assumptions tested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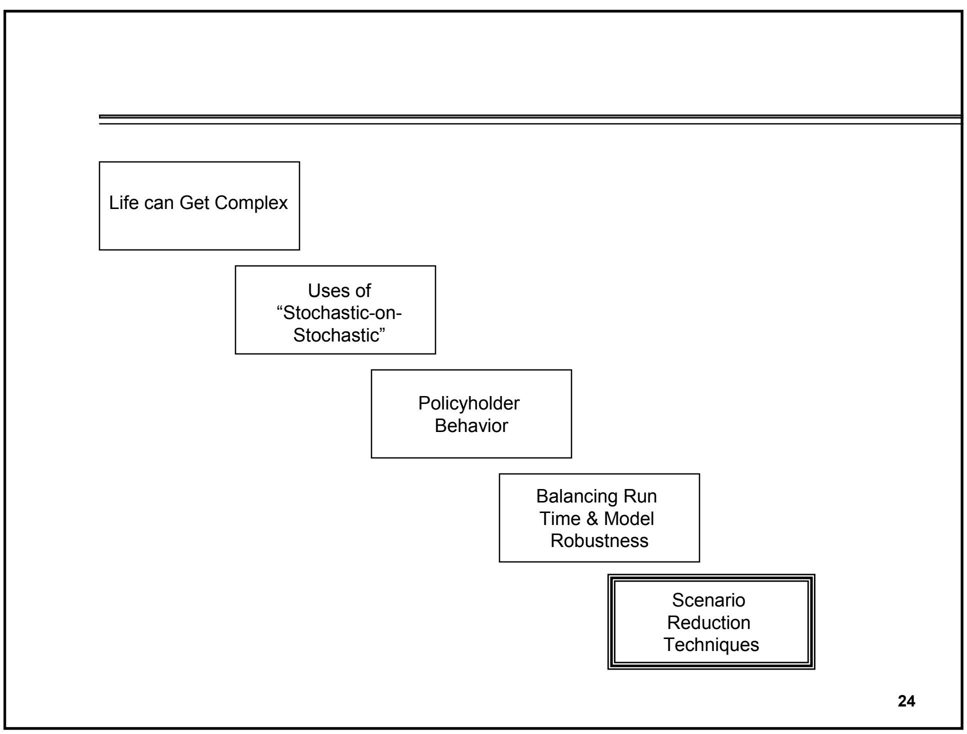
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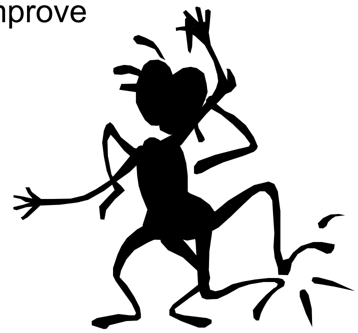
Why the need to reduce scenarios?

- Running thousands of scenarios can debilitate time & resources
- More/faster machines is partial solution
- Smaller/faster models may help a little or a lot
- Reducing scenarios will improve
  - Run-time
  - Flexibility



What is representative scenario methodology?

- Reduces the number of scenarios needed to apply stochastic asset-liability cash flow models
  - Results remain comparable to full scenario run
- Developed by Longley-Cook (1997, 2003)[1] and Chueh (2002)[2]

[1]Alastair G. Longley-Cook, "Probabilities of 'Required 7' Scenarios (and a Few More)," *The Financial Reporter* (July 1997), "Efficient Stochastic Modeling Utilizing Representative Scenarios: Application to Equity Risks" (September 2003), the 2003 Stochastic Modeling Symposium.

[2]Yvonne Chueh, "Efficient Stochastic Modeling for Large and Consolidated Insurance Business: Interest Rate Sampling Algorithms," *North American Actuarial Journal*, Volume 6, Number 3, July 2002

Central to this methodology is the notion of distance (*D*) between scenarios

- *D* is a function of the two scenarios' risk parameters

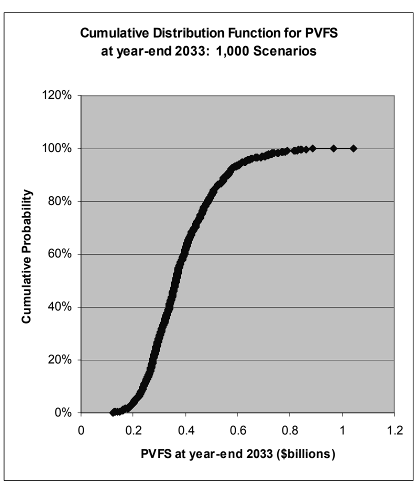
$$D = \sqrt{\sum_{t=1}^{30}\left(\prod_{k=1}^{t}\frac{1}{1+i_k} - \prod_{k=1}^{t}\frac{1}{1+i'_k}\right)^2}$$

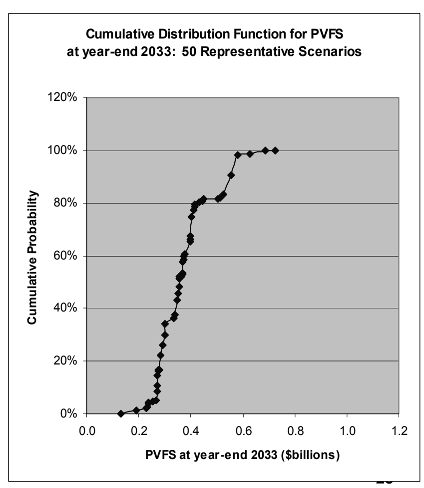
  where i_t, t = 1, 2, … , 30, is an interest rate path consisting of one-year interest rates for 30 years

- Representative scenarios are chosen such that they are "far apart" from one another

The scenario sampling algorithm uses distance (D) to ensure adequate tail representation

- Note how the tail regions are oversampled





Why use representative scenario methodology?

- Representative scenarios reduce run-time by reducing the number of scenarios, allowing greater flexibility
- Actuarial knowledge of the relationship between scenarios and cash flows
- Chueh (2002) has proven this to be more effective than simply generating fewer scenarios
  - Results are often very similar to those using all scenarios

1. Create risk parameter scenarios

- 1,000 (*N*) equity scenarios based on the RBC C3-II version of the regime-switching lognormal model[1]
  - Calibrated to the S&P 500
  - 360 monthly returns (30 years)
- Interest and inflation were left deterministic, as were credit spreads and default rates

[1]Mary R. Hardy, “A Regime-Switching Model of Long-Term Stock Returns,” *North American Actuarial Journal*, Volume 5, Number 2, April 2001

2. Choose the representative scenarios

- 50 (n) representative scenarios were selected using algorithms described in Chueh's paper (2002)
- Relative present value distance method
  - $D_1 = \sqrt{\sum_{t=1}^{30} \left(r_t - r_t'\right)^2 \cdot V^t}$ (method 1, M1)
  - $D_2 = \sqrt{\sum_{t=1}^{30} \left(\prod_{k=1}^{t} \frac{1}{1+r_k} - \prod_{k=1}^{t} \frac{1}{1+r_k'} \right)^2}$ (method 2, M2)

  Significance method*
  - $S = \sqrt{\sum_{t=1}^{30} \left(\prod_{k=1}^{t} \frac{1}{1+r_k} \right)^2}$ (method 3, M3)

*S refers to the significance of a scenario, as defined by Chueh (2002), and is used in a slightly different method in which each representative scenario has an equal probability of occurrence

3. Assign probabilities to the representative scenarios

- The probability, p_j, for a representative scenario is the number of scenarios assigned to it divided by *N*
- p_j is the probability of occurrence of *any* scenario assigned to the representative scenario
- p_j is based on the risk parameter distribution (inputs), but applied to the representative cash flows (outputs)
  - This stresses the importance of a close relationship between inputs and outputs

Scenario Sampling Algorithm

Step 1 choose an arbitrary scenario out of the 1000 scenarios and call it Pivot #1

Step 2 calculate the distances from Pivot #1 to the remaining 999 paths

Step 3 name the scenario with the largest distance to Pivot #1 as Pivot #2

Step 4 calculate the distances of the 998 remaining scenarios to Pivot #1 and Pivot #2

Step 5 assign each of the 998 scenarios to the closest of Pivot #1 or Pivot #2

Step 6 rank these 998 distances in descending order. The scenario with the largest distance is called Pivot #3.

Step 7 follow the above procedure to select the remaining pivot 47 scenarios

Step 8 if the number of scenarios associated to a pivot scenario is x, then assign a probability of x/1000 to this pivot scenario

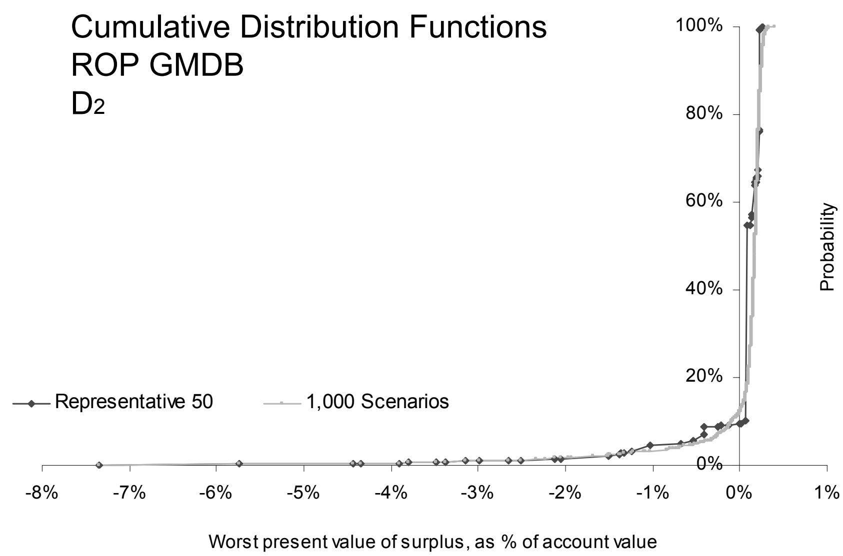
4. Run the cash flow model

- The cash flow model was run for the representative scenarios, generating *n* outcomes
- The cash flow modeling software used was the Variable Annuity application for the ***MoSes*** software system
- Each outcome has an associated probability p_j
- The outcome used for RBC C3-II is the worst present value of year-end statutory surplus

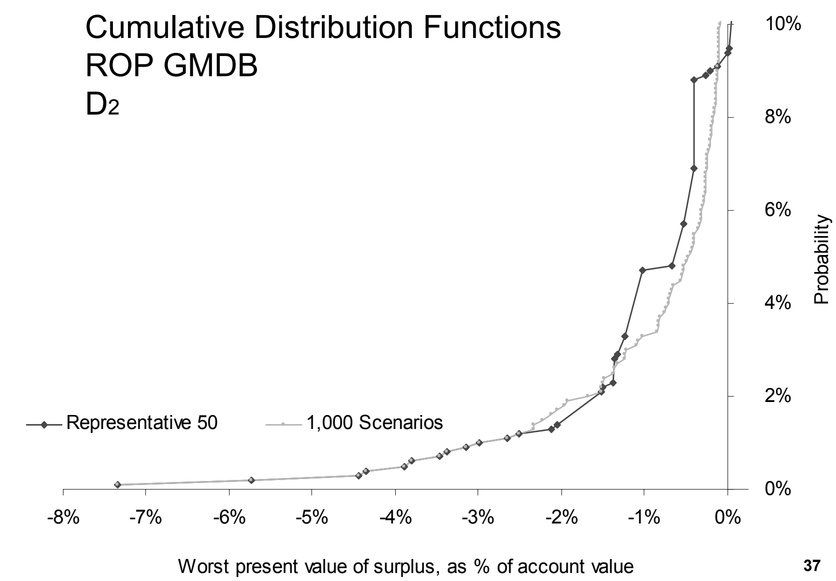
5. Analyze results

- Metrics such as required capital can be calculated directly from the set of *n* outcomes and probabilities

The relative present value method with distance measure D_2 gave the best results



Good fit in the tails is observed since D2 forces representational scenarios into the tail



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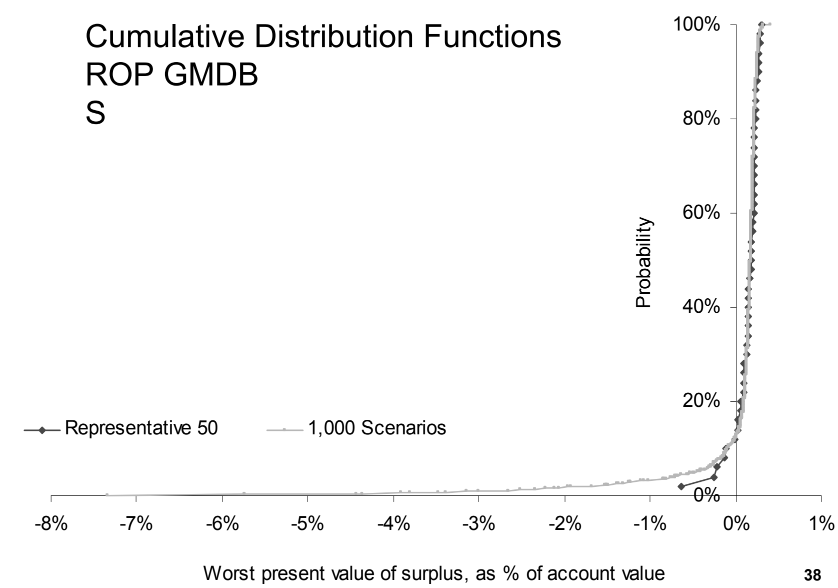
S representational scenarios are evenly spaced — Fit is good in the middle, but poor in the tails



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This methodology appears to be both accurate and robust

- Accurate CTE 90 RBC C3-II values were calculated with a 95% reduction in scenarios
- Other metrics can be calculated for purposes of:
  - Economic capital determination
  - Reserve setting
  - Risk analysis/management
  - ALM
  - Pricing
  - Optimizing investment strategies
  - Hedging strategy

Stochastic Modeling Techniques

Balancing Run Time and Model Robustness

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Friday, May 7, 2004

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Overview

Modeling Requirements

Modeling Techniques & Considerations

Case Study

Final Thoughts

Modeling Requirements

Product complexity continues to increase

~ new VA riders with additional options

Need to capture/quantify all options and guarantees inherent in the product

~ if you think it's cheap, prove it!

Even more variables and parameters to consider

~ dynamic p/h behavior, optional resets, correlations, etc.

Modeling Requirements

New reserving and capital requirements

Traditional pricing tools or "black boxes" may not be up to the challenge

~ need programming skills and creativity

After all of the product's functionality is modeled, you will also need to consider the 'run time'

~ availability of better and faster computers have allowed for more robust actuarial models

Minimizing Run Time

Reducing the number of scenarios (more on this later) OR length of projection

Pre-generate scenario independent cash flows

Simplify or carve-out formulas/variables not used or immaterial to the results of the run

Software/vender specific optimizing techniques

Group data into model points (discussed here)

~ for pricing, assumes we use in-force data to model new business

⑦ ***Always test accuracy of the revised model!***

Grouping Considerations

Need to be very careful how data is aggregated

~ mentioned in appendix of C3 Phase II report (Methodology Note C3-02)

~ characteristics to consider shown on next slide

Number of model points will also depend on:

~ complexity of the model

~ product being modeled

~ size and nature of the seriatim data

~ how relevant the underlying data is to the business being modeled (pricing or valuation)

Grouping Considerations

Grouping criteria should consider the following:

~ in-the-moneyness

~ product variations (including rider variations)

~ annuitant and/or owner age

~ gender

~ duration of contract

~ market

~ distribution channel

~ other factors

Grouping Techniques

Make sure models points are a good representation of the seriatim data

~ validate results of the ‘condensed’ model with the seriatim model before finalizing grouping conditions

~ adjust groupings/model points until sufficient

~ model points do not need be be equally weighed

~ quality of fit will depend on the purpose of the model

If a seriatim run cannot be performed, consider using a ‘control’ model to determine optimal grouping

Case Study

This exercise would be similar to using a 'control' model to determine our grouping criteria

Business to be modeled

- VA with GMDB riders
- 3 different products (B, C, and L-share)
- 10,000 contracts issued over a period of 2 years

Model details

- 1,000 scenarios over 20 years; dynamic assumptions

Case Study

Seriatim Data Fields

- Product type
- Issue month and year
- Attained Age
- Gender
- Asset allocation
- Moneyness

Validation

- compare the distribution of the PV of GMDB claims

Case Study - Results

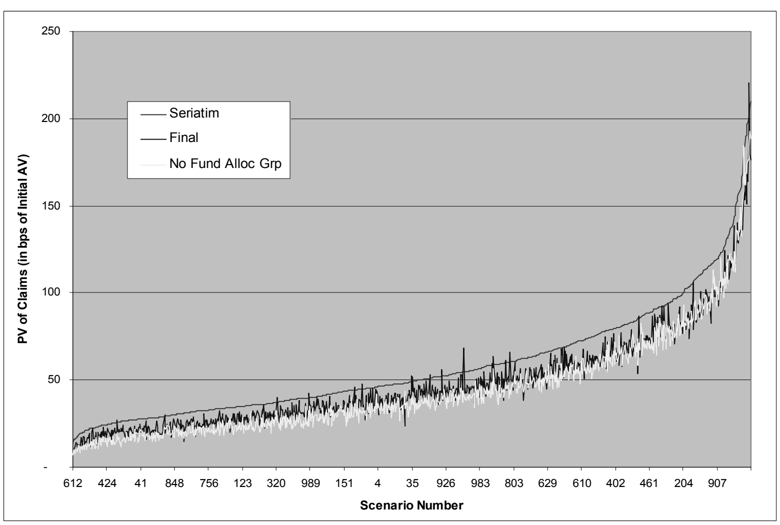
Results using various grouping criteria:

Run	# Model Pts	Mean	10th %-tile	90th %-tile	90 CTE
Seriatim	10,000	100	47	171	225
Final - All Groupings	800	83%	76%	85%	85%
No Fund Allocation Groups	500	76%	63%	83%	85%
No Moneyness Groups	400	75%	62%	82%	85%
Annual Issues Only	250	77%	58%	87%	89%
No Product Groups	350	82%	75%	84%	86%

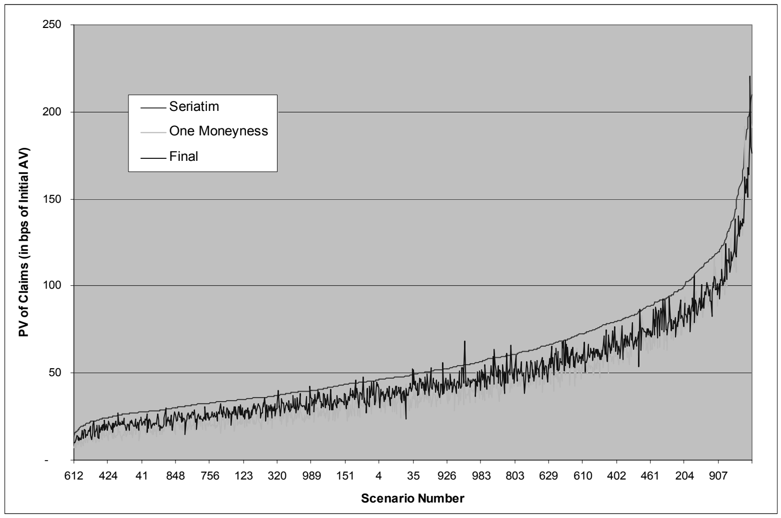
* Seriatim results have been adjusted with mean equal to 100

* Results for various groupings reflect 'accuracy' of match

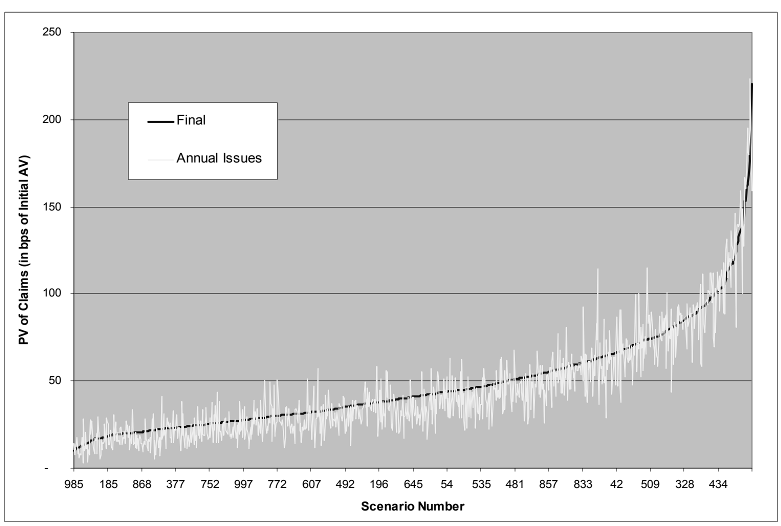
Case Study - Results



Case Study - Results



Case Study - Results



Case Study - Results

From our data sample:

- Model points should reflect differences in asset allocation (aggressive vs conservative investors) -- diversification benefits should not be built in
- Model different issue months to capture diversification over time
- Different product types did not have a significant effect on GMDB claims (may not always be true)
- Number of groups for a given category will depend on variability of underlying data or business
- Make sure nothing is "lost" when grouping data points

Final Thoughts

Compare all relevant outcomes/results

Be sure to look at the entire distribution of results

- comparing the mean is not sufficient

Grouping criteria or methods should be monitored for reasonableness

- significant changes in data, change in purpose of model, etc.

The most optimal set of model points will depend on the data and/or the business being modeled