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Stochastic Pricing
Session 62 TS

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FINANCIAL SERVICES CONSULTING

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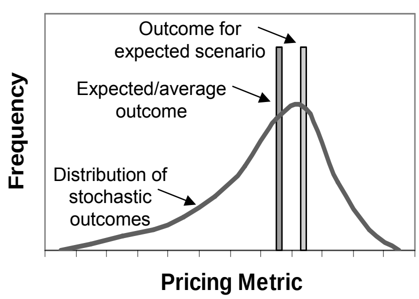
Section 1

Introduction
What is stochastic pricing?

Stochastic pricing requires the projection of outcomes along multiple "scenarios", rather than according to a single "best estimate" or "conservative" forecast



$$E[f(X)] \neq f(E[X])$$

If X represents a set of risk factors, and $f(\cdot)$ represents a cost (or pricing) function, the above reads: **The expected cost *does not always equal* the cost for the expected scenario**

- Definition of a "scenario":
  - A scenario is defined by the simulation of one or more risk factors whose values evolve according to processes that include random components
- Traditional pricing:
  - Puts considerable effort into selecting the best estimate or pricing assumptions
  - Then determines the price such that the target return is earned on this scenario
  - Supplemented with sensitivity testing
- Stochastic pricing:
  - Puts effort into determining a range of potential outcome scenarios
  - Then determines the price such that the target return is earned on average (or with a certain level of confidence) over the range

Section 2

Uses of stochastic pricing

Stochastic pricing can be used for a wide range of products, but is most useful when the loss distribution (risk profile) is asymmetric

Traditional term and whole life insurance

Variable life base product

Variable annuity base product

Fixed annuity product

Variable annuity death benefits

Variable annuity living benefits

Stop loss mortality reinsurance

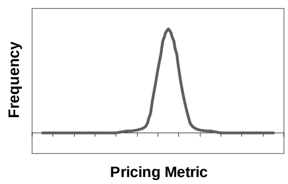
- Individual risks are numerous, but small, independent (non-systematic) and/or diversifiable
- Costs are predictable with high level of confidence
- Volatility of outcomes is small, not a concern to the company, rating agencies or regulators

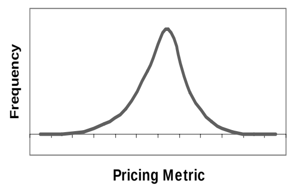
- Individual risks are large, systematic and/or non-diversifiable
- Range of plausible outcomes is wide
- Volatility of outcomes (earnings and/or capital) is significant

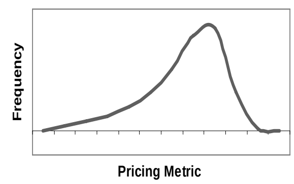
Examples

least useful ⟷ most useful

Traditional term and whole life insurance	Variable annuity base product	Variable annuity living benefits
▪ Individual risks (mostly mortality and lapse) are numerous, but mostly small and diversifiable ▪ Aggregate costs are predictable with high level of confidence	▪ Individual risks are largely non-diversifiable, but not heavily skewed ▪ Material volatility in revenues and costs to company	▪ Individual risks are largely non-diversifiable and heavily skewed: significant downside risk ▪ Volatility and fat "left tail" are significant concerns







Section 3

Methodology

The "scenarios" form the foundation of stochastic modeling. The amount of work to get this right is often underestimated…

- Stochastic elements vary with products and their risk profiles, e.g.

	Variable annuity guarantees	**Fixed annuity products**	**Stop loss mortality reinsurance**
Equity returns	✓		
Interest rates	✓	✓	
Mortality rates			✓

- Random samples typical start with a uniform random number generator: $u_t \sim U[0, 1]$
  - Not all generators are equal; can test robustness using DIEHARD tests
- u_t are transformed into desired distribution using inverse CDF, e.g., to transform into normal random draws: $z_t = \Phi^{-1}(u_t)$, where $\Phi(\cdot)$ is the Normal CDF
  - Be sure to test precision of statistical functions, especially in the extreme tails
- Models and parameters are often selected using Maximum Likelihood Estimation (MLE) techniques, but parameters are usually subjectively adjusted
- Importantly, when multiple random events are modeled concurrently, must ensure consistency across them, e.g. by incorporating a coherent correlation structure

Stochastic scenarios call out for dynamic reaction to evolving situation: most common dynamic element is policyholder behavior

Deterministic models

- Traditionally, everything (including policyholder behavior) was modeled deterministically
  - e.g. lapse rate varies by age and duration, but not explicitly by level of interest rates
- Deterministic rates can still be reasonable if the stochastic elements of the model are not expected to have a significant impact on behavior

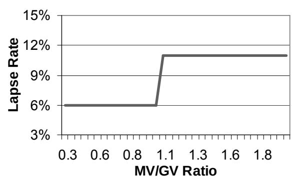
Stochastic models

- Dynamics are sensible in principle, but many would argue the data are insufficient to parameterize a model with any confidence…
- …however, financial (dis)incentives and emerging body of experience confirm dynamic nature of policyholder behavior
- Increasing financial sophistication of policyholders (or their advisors) is likely to increase efficiency in the exercise of options (anti-selection)…
- … but there will always be some non-financially motivated behavior that will appear sub-optimal
- Common to assume dynamic behavior for deferred fixed and variable annuities

Example – variable annuities: dynamic policyholder behavior can take several forms…

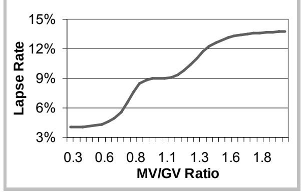
Good

- A first step is to differentiate behavior between broad categories of policyholders
- Example: in-the-money *vs.* out-of-the-money



Better

- Emerging experience suggests that behavior is
  - More gradual than step-wise rates
  - "Sticky" near neutral points
    - e.g. at MV/GV = 1



Best?

- Multi-factor function, such that lapse rates vary with:
  - In-the-moneyness
  - Age, policy duration
  - Time to maturity
  - Level of interest rates
  - Market volatility

$$q^w = f(mv, gv, t, T-t, r, \sigma, ...)$$

- While economically sensible, models might be:
  - More sophisticated than true behavior
  - More computationally intensive (i.e. slower) than needed to gain insight for pricing

After dynamics are built into the model, stochastic pricing is similar to single scenario analysis repeated for a very large number of sensitivities or "trials"

Deterministic pricing

- Project all cash flows, accounting liabilities and required capital amounts along the pricing scenario
- Bring all together into a pricing metric, e.g. IRR, Lifetime ROE, embedded value (EV)
  - EV has the desirable feature that it always produces comprehensible results, but IRR and ROE can "blow up"

Stochastic pricing

$\times$ *N* (number of scenarios)

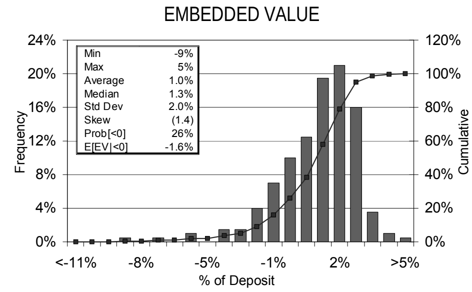
Need to summarize the *distribution* of results

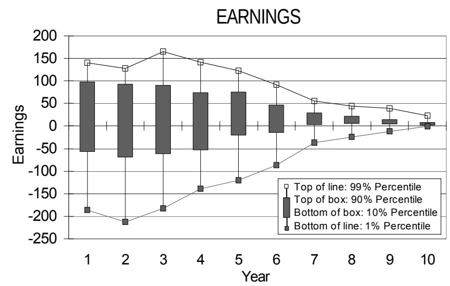
Problem on the horizon?

- What happens when it is difficult to project future accounting provisions (e.g. FAS97 DAC, FAS133, SOP-03-1 and C-3 Phase II RBC for variable annuities) and hence future income and/or required capital? That is, how do we cope when these future accounting provisions themselves require stochastic projections?
- This brings us to the dreaded "stochastic-on-stochastic" (SOS) problem
  More on this later…

In presenting the results from stochastic modeling, it is important to get the right <u>balance</u> of synthesis and richness of information

- Too much detail is overwhelming:
  - Summarize results into a manageable format
- Avoid temptation to over-analyze or give false sense of precision:
  - Communicate the “richness” of information, not the nitty-gritty
- Graphs are ideal
- Don’t over-emphasize nor dismiss too quickly the outlying scenarios
- Ultimately, must decide whether distribution of results meets pricing targets:
  - On average, .e.g. E*[EV]=0*
  - For a given level of confidence, e.g. *Earnings > X 65% of time*
  - *Probability of loss < Y%, etc.*

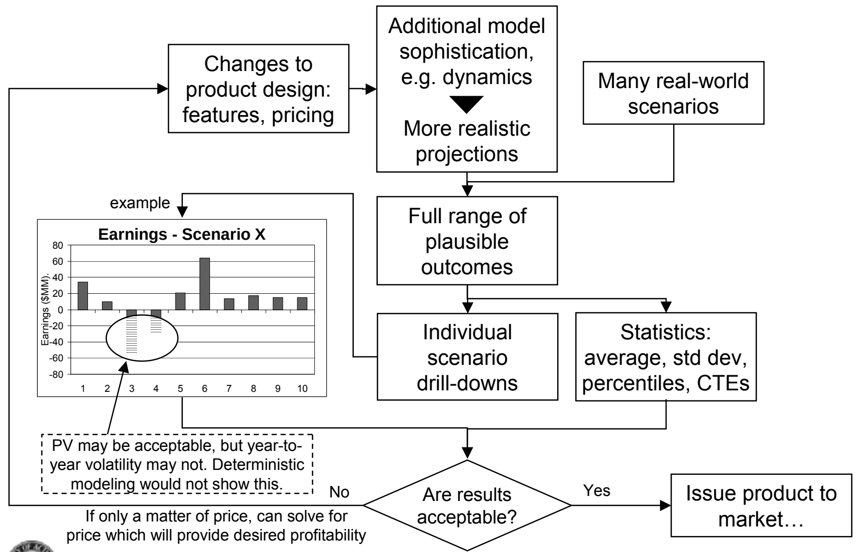




Section 4

Value added by stochastic pricing

Stochastic pricing serves a similar purpose as traditional pricing, but provides a more fulsome understanding of risks (cost/benefit)



Why go through the added work of stochastic modeling?

- For some risks, traditional "best estimate" pricing does not provide a reasonable understanding of cost-benefit tradeoff. In some cases, best estimate assumptions would not lead to any claims…
  - Stop-loss mortality reinsurance
  - Variable annuity death and living benefits
  - Catastrophic coverages

  For many types of products, it is important to model the tail of the distribution; often, this cannot be done analytically

- Stochastic pricing using so-called "real world" scenarios:
  - Provides *N* possible future paths (scenarios), each one being plausible
  - Permits the calculation of profitability measures along each path
    - Each path is realistic and can be the evaluation of specific outcomes (e.g. cashflows)
  - Range of paths provides a distribution of real potential outcomes

  Distributions allow for an assessment of the likelihood and acceptability of profitability (or loss) relative to targets

Section 5

Additional uses of stochastic model

As seen with asset adequacy analysis models, stochastic models can also be used for purposes beyond that for which they were first developed

- In principle, the following applications can benefit from more complete coverage of the potential outcome space that is offered by stochastic modeling:
  - Business planning
  - Earnings forecasts
  - Asset adequacy analysis
  - Setting reserves, regulatory (or economic) risk capital levels – e.g. for variable annuity investment guarantees
    - FAS133
    - SOP-03-1
    - VA CARVM
    - C-3 Phase II RBC
- Often, going from one application to another requires modest changes or additions

Both 'real world' and 'risk neutral' stochastic pricing are useful, but the two probability measures must be interpreted differently

Real world

- Produces realistic distributions of how the future may unfold ("plausible outcomes")
- Allows statements to be made about the likelihood of given outcomes, e.g.
  - 90% probability that earnings will not fall below *X*,
  - 75% probability that embedded value is greater than *Y*
  - 95% probability that reinsurance claims will not exceed *Z*, and if they do, it is on average by an amount *W*
- Allows explicit incorporation of subjective conservatism by choosing results higher than the average
- Parameters based on history, with subjective adjustments for the "market price of risk"

Risk neutral

- Artificial construct designed to determine the *fair value* of a stream of cash flows, consistent with the market prices for financial options and other securities
- Used when closed-form solutions are not available to calculate the fair value:
  - FAS133 liability for certain variable annuity living benefits
  - Liability fair values and sensitivities (the "greeks") to market movements in the context of a dynamic hedging strategy
- While this construct also involves distributions of outcomes, the mean "price" is the only quantity that matters
  - No confidence statements can be made as with the real world projections
- Parameters derived by calibration to observed market prices

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Where 'real world' and 'risk neutral' come together…

Coming together

- Earnings forecast
  - Need real world projections to forecast cash flows (premiums, claims) and to calculate some types of liability provisions (e.g. SOP-03-1)
  - Need risk neutral for other types of liability provisions (e.g. FAS133)
- Pricing
  - Might also need projections of required capital, which in turn could demand the use of stochastic models (e.g. C-3 Phase II RBC)
- Projecting a dynamic hedging strategy
  - Risk neutral valuations (for fair values and the "greeks") embedded in a real world projection

S.O.S.

- Stochastic-on-stochastic
  - Occurs in all situations described at left
  - Typically represent a significant hurdle in terms of run-time, e.g.
    - Base stochastic valuation consists of 1000 quarterly scenarios projecting for 30 years ≈ 120,000 nodes
    - If you need one (or more!) stochastic valuations at each of these nodes…
  - Approximations which save enormous amounts of run-time without severely compromising accuracy are tremendously valuable (e.g. analytic closed-forms, etc.)

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What are the key lessons to be learned?

- Complexity is fine when needed, but simpler is usually better:
  - Often, the *change* in value is more important than absolute $
- Uncertainty (sampling error, model risk) is important:
  - Explore ways to minimize (e.g., variance reduction, control variates)
  - Give ranges instead of single 'point estimates'
  - Avoid false impression of precision
  - Don't over-emphasize the stochastic components to the neglect of the liability models ~ compromises can mask or distort exposure
- Models are powerful, but only tools ~ use them wisely:
  - Don't under-estimate the time and effort needed for successful implementation and wider acceptance within the company
  - Not an exercise in rocket science ~ the models have a business context
  - Scenario stress testing is always a good idea (reality check)

Session 62: Stochastic Pricing A Variable Annuity Case Study

SOA Annual Meeting
November 15, 2005

Craig Ryan, FSA, MAAA
Nationwide Financial

Outline

- Variable Annuity
- Base Product Risk First
- Death Benefit Risk
- Living Benefit Risk

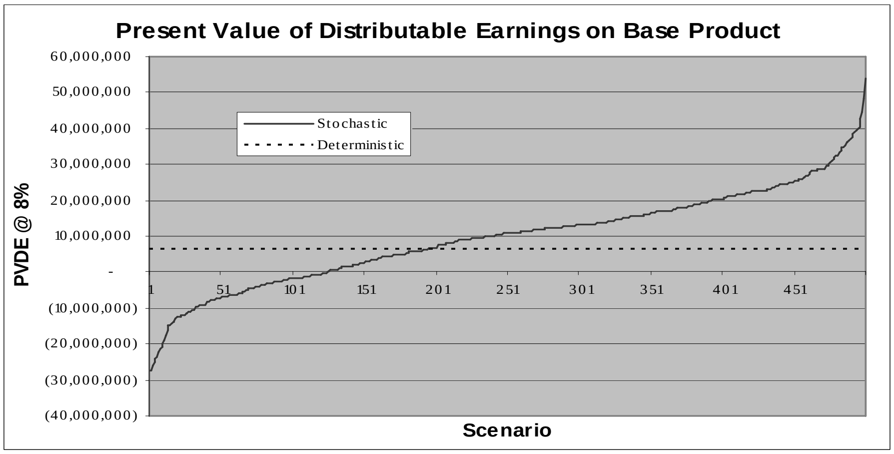
Assumptions

- **Upfront Commission = 6%**
- **Only Income is 125 bp on assets**
- **Expenses**
  - Maintenance - $50, Asset Based 20 bp
  - Initial – 100 bp to sell, $100 to put on system, mail prospectus, etc…
- **Fund Fee drag of 75 bp**
- **Annuity 2000 Mortality w/o Improvements**
- **7 Year CDSC of 7,6,5,4,3,2,1,0%**
- **Base Lapses – 1,2,3,4,5,6,7,25,25 15% ultimate**
- **Partial Withdrawals = 3%/ Year**
- **No recurring premium, average size 80,000**
- **CARVM Reserves & Current Risk Capital Requirements (not C3 Phase II)**
- **65 Year Old Male**
- **Statutory, not GAAP Basis**

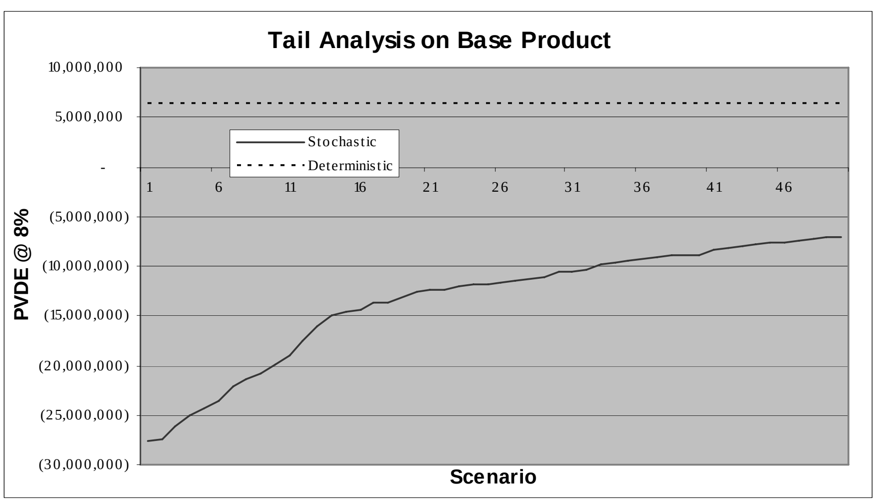
- **Ways to Analyze**
  - Distributable Earnings (income distributed after holding capital) summed up
    - Lack of discounting doesn't show true nature of income stream
  - Present Value of Distributable Earnings at a determined interest rate
    - What is appropriate interest rate (cost of capital, which is?)
    - Not always easy to explain to Sr Mgmt
  - Overall Internal Rate of Return
    - Just one number, never realized
    - Misses tail risk
    - Not always easy to explain why miss it to Sr Mgmt
  - CTE measures
    - Measure tail risk of something like a PVDE
  - Some combination of the above
  - Many others – I'll focus on PVDE since I'm looking more at volatility of results by scenario

Stochastic vs Deterministic

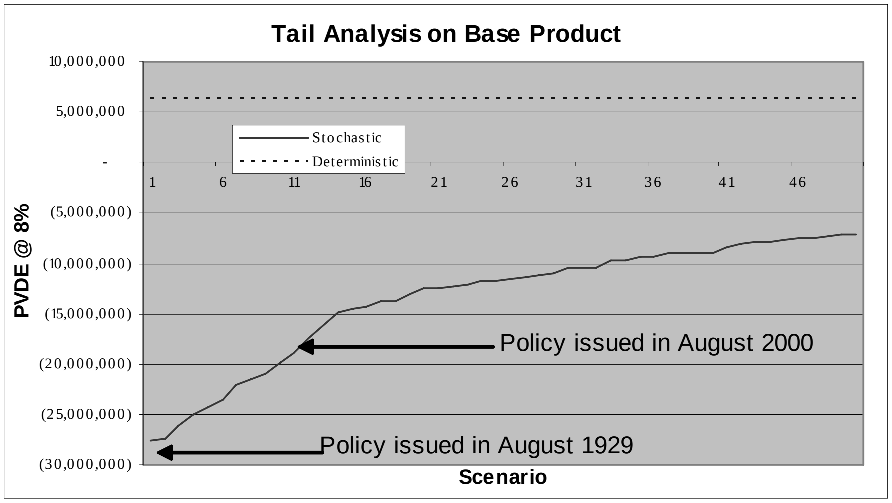
- Assume 10% Gross Growth (approx 8% net growth) on Deterministic, 100% investment in S&P Fund



Tail Risk



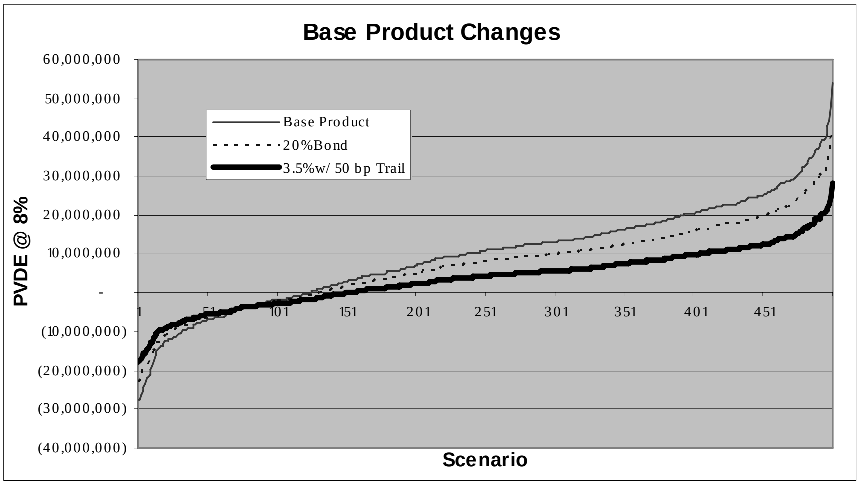
Tail Risk – Real?



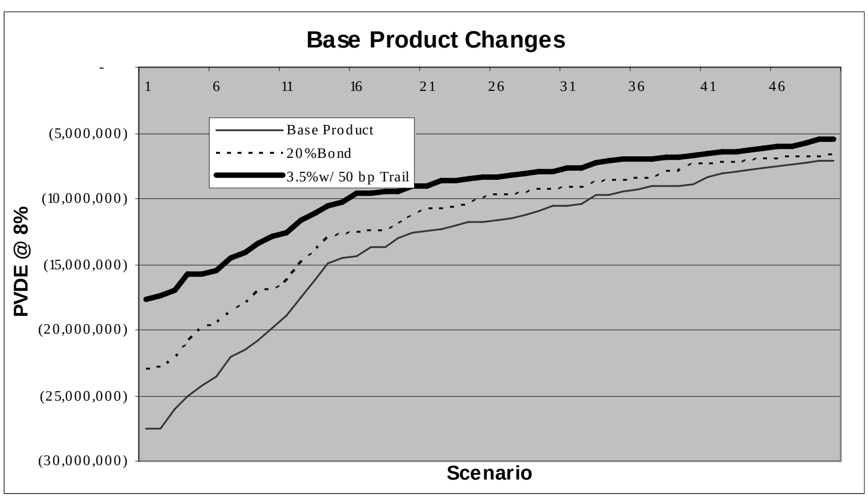
What happened & What does this mean?

- **Commissions and expenses**
  - Variable fee collection on a fixed expense
  - Pay out over 7% upfront, income is coming on a declining balance
- **Early down scenarios – 2000!**
- **So what do you do about it?**
  - Product changes
    - Trails
    - Asset Allocation
    - Hedge fee risk
    - Basis for fees
  - Education of internal and external audiences

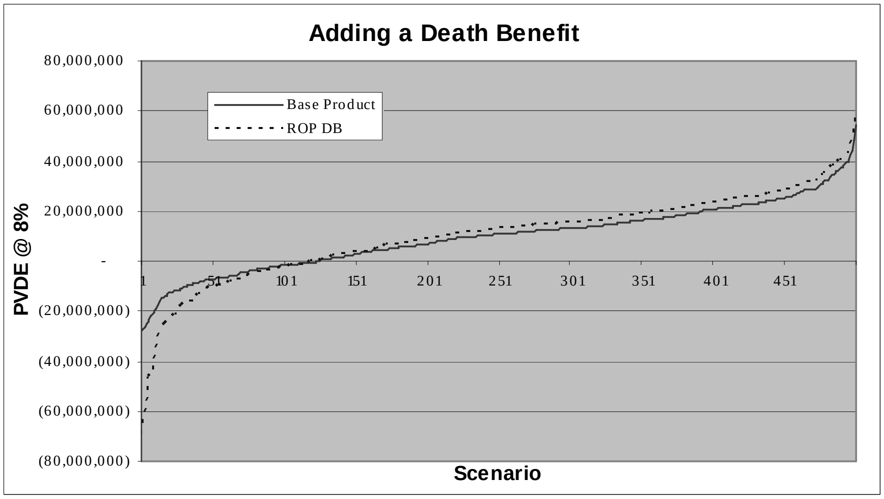
Product Changes



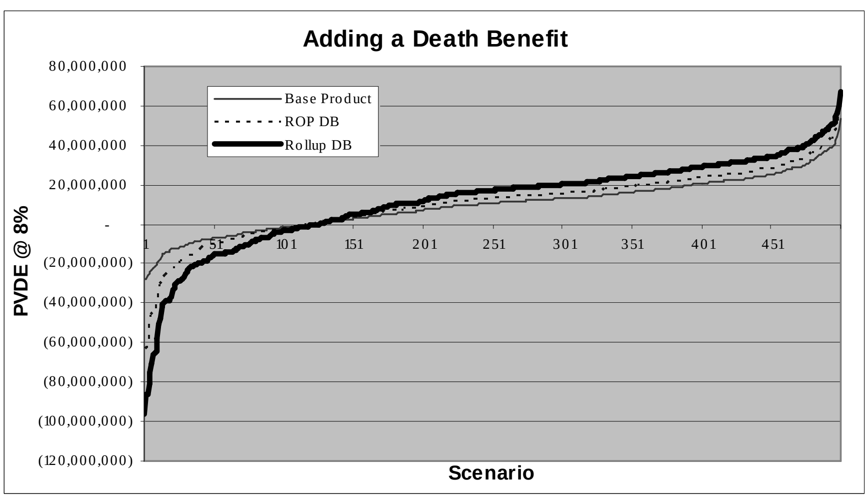
Product Changes



Adding A Death Benefit



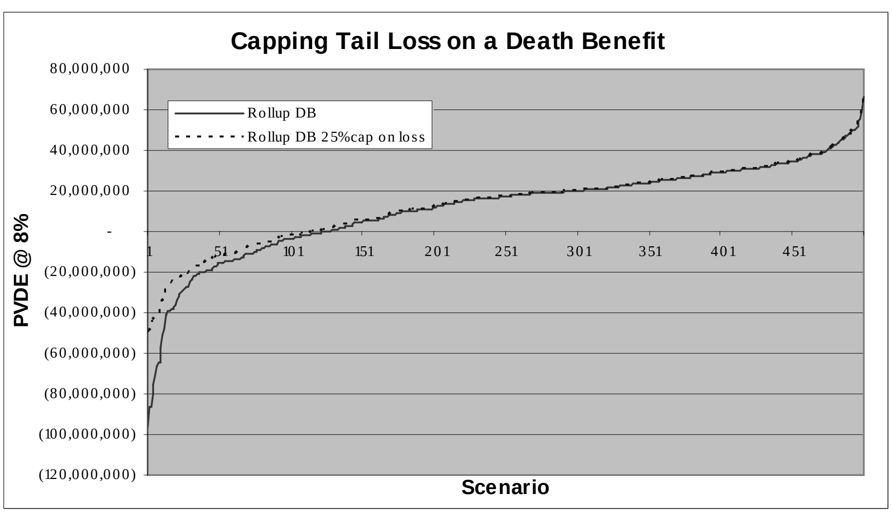
Adding a Death Benefit



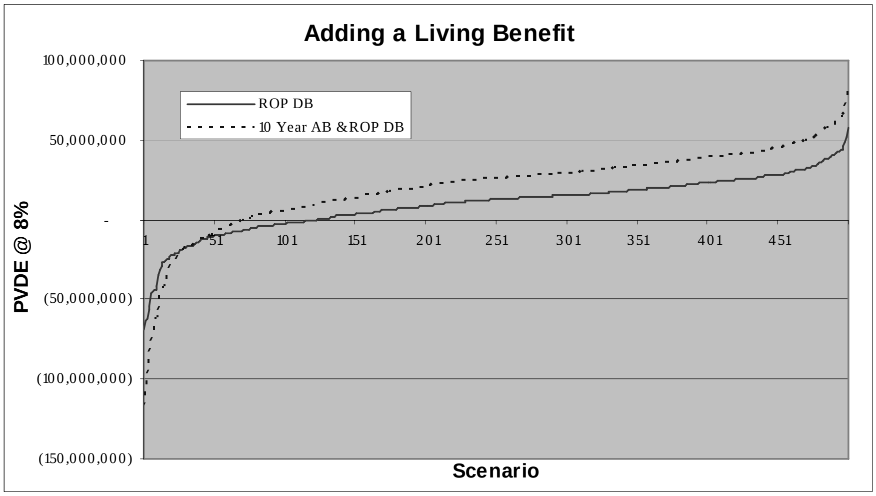
Any Way to manage DB Risk

- **Charging on Benefit Basis**
- **Limit older age risk**
  - Lock in earlier
  - Maximum issue ages for DBs
- **Cut off tail risk – Large VA player has done this on their benefit**

Adding a Death Benefit Capping Tail Loss



Adding a Living Benefit



Other Things to consider

- **C3 Phase II**
  - Stochastic Models used here also?
  - Implementing for pricing means $Stochastic^2$
- **Dynamic Behavior**
  - Will lapses change with benefits being in the money?
  - Will use of an elective benefit (such as GMIB and GMWB) increase with market drops?