The C3 Risk—Effective Scenario Sampling Algorithms and Implementation for Stochastic Modelers

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Central Washington University
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Based on the papers by Alastair G. Longley-Cook and Yvonne Chueh

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Applying effective scenario sampling to choose representative scenarios for stochastic models
— Agenda —

- Introduction and background
- Representative scenario theory
- Representative scenario methodology
- RBC C-II application
- Conclusions

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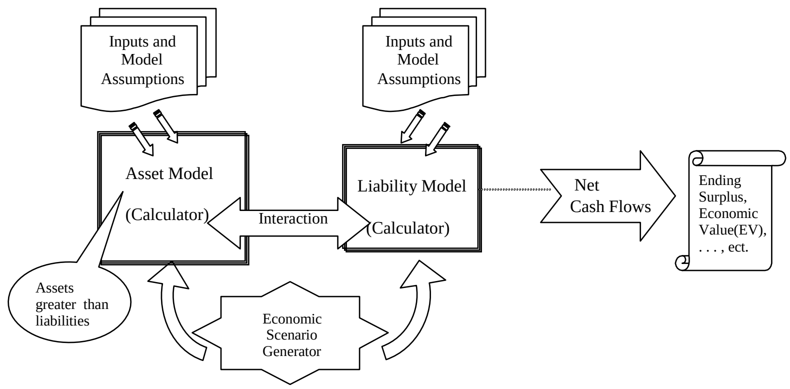
Recap of a stochastic asset-liability cash flow model



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Stochastic asset-liability cash flow models

- Stochastic scenarios are the inputs
  - Composed of randomly generated values for chosen risk parameters
    - – Interest rates
    - – Equity returns
- Cash flows are the outcomes
  - It is the creation of these cash flows that is time-consuming
- Output metrics of model interest includes:
  - PV of accumulated surplus
  - CTE 90 RBC
  - EVA
- Applications includes:
  - Capital determination
  - Reserve setting
  - Risk analysis
  - Pricing

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What is Representative scenario methodology?

- Methodology developed by Longley-Cook (1997)[1] and Chueh (2002)[2]
- To solve stochastic models' run-time issue by reducing number of scenario runs
- To capture cash flow model outcome metric's tail distributions
- Simplified methodology has been Adopted by AAA C3 Phase II RBC scenario picking tool

- Applications includes:
  - Stochastic asset-liability modeling for life industry
  - Dynamic solvency models for property/casualty industry

1 Alastair G. Longley-Cook, "Probabilities of 'Required 7' Scenarios (and a Few More)," *The Financial Reporter* (July 1997)
2 Yvonne Chueh, "Efficient Stochastic Modeling for Large and Consolidated Insurance Business: Interest Rate Sampling Algorithms," *North American Actuarial Journal*, Volume 6, Number 3, July 2002

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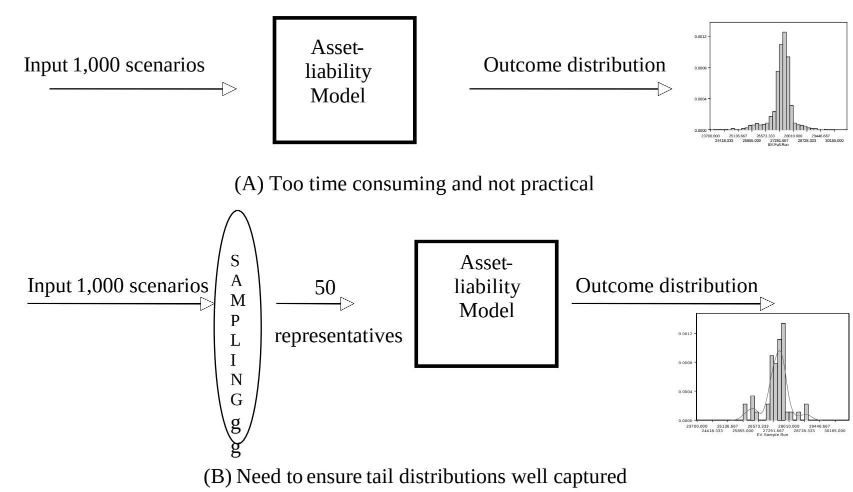
How does representative scenario work for tail distribution?



(B) Need to ensure tail distributions well captured

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What theory is involved for effective sampling? --continuity

- Model output metric is tied to stochastic scenario and is a continuous nonlinear real function of the input risk scenario
  - Uniform continuity of the function is guaranteed as long as there is **(*NEW!*)**
    - – a uniform bound on number of projection periods, and
    - – a uniform bound on cash flow amounts
  - Uniform continuity implies continuity
    - – The closer the two scenarios, the closer their model output metric values
    - – Extreme scenarios produce tail distributions

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What theory is involved for effective sampling? --probability

- The probability of each scenario occurrence is tied to the probability of its resulting model metric value
  - Pivot selection technique is used to Identify the extreme and moderate scenarios
  - Assign appropriate probabilities to moderate and extreme scenarios
  - Number of scenarios assigned to a pivot scenario reflects the frequency of that pivot
  - The probability of extreme scenario will be neither overstated or understated to ensure accurate tail distribution.

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What is the current most effective sampling methodology to select representative scenarios?

- The one named “Relative Present Value” Method on Chueh’s paper that
  - Utilitizes distance concept and pivot selection technique
  - Most effective for equity risks besides interest risk among the three methodologies and other less effective techniques
- D is a distance function that
  - measures the distance between given two scenario paths
  - incorporates the present value of $1 annuity in model’s time horizon
  - Captures the relationship between scenario risk parameters and model outcome metrics

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How to choose the most effective representative scenarios using relative present value approach?

Step 1. Reduce each scenario matrix to a key vector such as 1-year short term rates/returns.

$$(i_1^P, i_2^P, i_3^P, \ldots\ldots, i_{30}^P) \qquad (i_1, i_2, i_3, \ldots\ldots, i_{30})$$

Step 2. Calculate the distance between the first pivot scenario and each other scenario by using distance formula D

$$D = \sqrt{\sum_{t=1}^{30}\left(\frac{1}{\prod_{k=1}^{t} 1+i_k} - \frac{1}{\prod_{k=1}^{t} 1+i_k^P}\right)^2}$$

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What's the pivot selection technique?

Step 3. To choose pivots, start with a seed pivot and select the second pivot that maximizes the distance (D) to the seed pivot. The third pivot is the one that is the farthest from the first two pivots

In a mathematical notation, search next pivot scenario x to Maximize

$$\text{dist}(\{\boldsymbol{x}\}, \mathbf{A}) = \inf_{\text{for all } y \text{ in A}} |\,\mathbf{x}\text{-}\mathbf{y}|$$

A is the set of pivot scenarios previously chosen and {x} is one of the rest scenarios

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How to assign probabilities for pivot scenarios?

Step 4. Continue to choose pivot scenarios until reaching a preset goal or desired stability on the tail distribution.

Step 5. Assign a probability to each pivot scenario.

The pivot scenarios are representative scenarios to feed into the stochastic model and their probabilities are

$$\frac{\#\text{of scenarios assigned}}{\#\text{of scenarios in the universe}}$$

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Effective sampling algorithm (relative present value method) -Summary-

- Representative scenarios are chosen by pivot technique such that they are “far apart” from each other.
- Each representative scenario represents a cluster of scenarios by its extremeness and likelihood.
- The probability of each chosen representative is proportional to the size of its scenario cluster.
- The probability of each chosen representative reflects the relative frequency of its cluster (including representative itself) to the scenario universe
- Chueh’s paper (NAAJ 2002) about the required condition for uniform continuity has been successfully removed with a math proof. This makes the methodology extremely powerful.

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- Representative scenario methodology
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Applying representative scenarios to equity risks—the model

- Focus on RBC C3 II
  - New regulation for the equity risk in variable products with guarantees
- A variable annuity (or segregated fund) model was used to calculate capital requirements
  - Recommends/requires stochastic modeling of year-by-year accumulated statutory surplus
  - Complex and path dependent guarantees could require thousands of stochastic runs of the cash flow model
  - Representative scenario methodology can help with his new run-time issue
- Ignore point-of-sale costs
  - Start one month after issue
- Model specifications and assumptions based on typical variable annuities available in the marketplace
  - See next two slides

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Guarantees modeled

Type	Name	Description	Fees
GMDB	**ROP**	Return of Premium	5 bps
GMDB	**Rollup**	5% rollup, capped at 2.5x premium, frozen at age 80	20 bps
GMDB	**Ratchet** or **MAV**	Annual ratchet (maximum anniversary value), frozen at age 80	15 bps
GMDB	**Max** or **High**	Max (Rollup, Ratchet)	25 bps
GMDB	**EDB**	ROP + 40% enhanced death benefit (capped at 40% of deposit), incl. ROP	25 bps (20 EDB, 5 ROP)
GMIB	**Rollup**	5% rollup, capped at 2.5x premium, frozen at age 80, incl. Rollup GMDB	35 bps
GMIB	**Max** or **High**	Max (Rollup, Ratchet), both stop at age 75, incl. ROP GMDB	45 bps

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Other key specifications and assumptions for the variable annuity

- Single $50,000 policy issued to male, aged 65
- 100% invested in S&P 500 Total Return
- M&E risk charge = 1.5% fund value
- Advisory fee = 1.0% fund value
- Fund revenue share = 0.25% fund value
- Surplus earned (discount) rate = 5.77%, 3.75% after-tax
- Annual expenses = $85 per policy, 0.05% fund value
- Surrender charge = 7, 6, 5, 4, 3, 2, 1, & 0% of premium
- Lapse rates = 2, 4.5 , 5, 5.5, 6, 7, 8, 35, 20, & 12.5%
- No front end loads or annual fees

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For GMDBs, CTE 90 values very close to true values with only 50 scenarios

C3-II RBC as % of account value, duration 0 (at the money)

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Accuracy improves as the options become more in the money and payout becomes more likely

- Again, using only 50 scenarios, the representative CTE 90 values are very close to the true values

C3-II RBC as % of account value, duration 3.5

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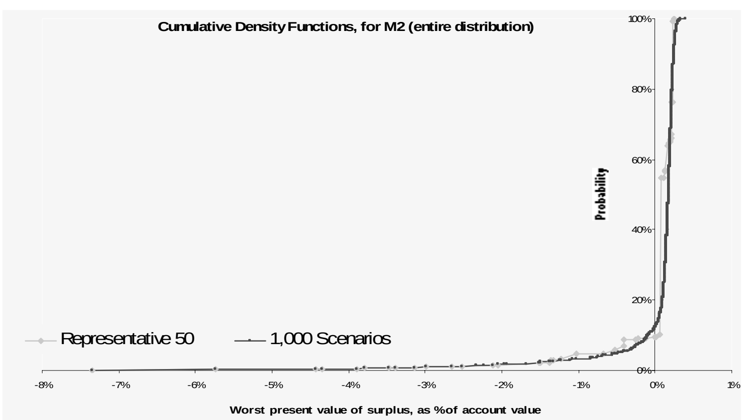
The relative present value method gave the best results

Cumulative Distribution Functions, (entire distribution), ROP GMDB



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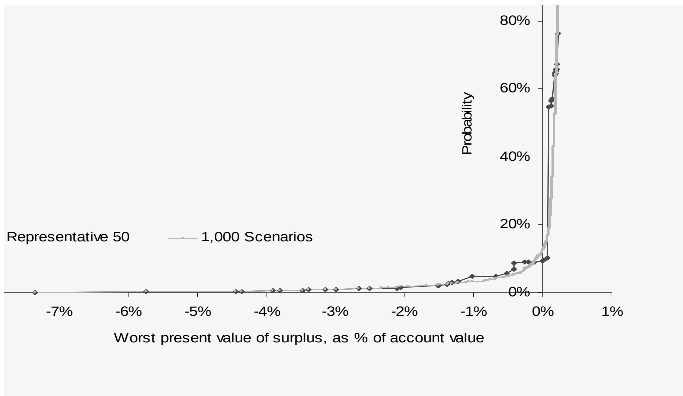
Good fit in the tails is observed since the methodology forces representational scenarios into the tail

Cumulative Distribution Functions, for M2 (left tail only), ROP GMDB



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— Agenda —

Introduction and background

Representative scenario theory

Representative scenario methodology

RBC C-II application

Conclusions

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This methodology appears to be very effective in a very challenging situation

- Accurate CTE 90 RBC C3-II amounts were calculated with far fewer representative scenarios
  - Done for variable annuities with equity guarantees that only payoff in the extreme tail, some path dependent
- The results build on the promising results found in Chueh's paper (2002) with regard to interest risk
- It appears this methodology is both accurate and robust
- The number of scenarios required for stochastic modeling can be dramatically reduced

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Methodology is not limited to CTE 90 RBC calculation

- Other metrics can be calculated for purposes of
  - Economic capital determination
  - Risk Analysis
  - Reserve setting
  - Pricing
- The tail distribution of model metric can be captured with high accuracy by effective representative scenario methodology and promisingly further enhanced by parametric probability curve fitting[1].
- The property/casualty industry faces similar challenges when modeling different outcomes under dynamic solvency analysis

1 Chueh's symposium paper, 2003 stochastic modeling symposium by CIA, SOA

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Acknowledgement!

- Dr. Charles Vinsonhaler (UConn)
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- Jason Kehrberg (Tillinghast-Towers Perrin)
- Dr. Menglin Mark Cao (State Farm Life)
- Many others who were or are still with ING-Aetna (you know who you are!)
- Anonymous paper referees
- …………..

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Thank you!

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Appendix

- Choose the first interest rate path randomly from the 1000 and call it #1
- Measure the distance from #1 to each of the remaining 999 interest rate paths.
- The interest rate path having the largest distance is selected as a representative and called #2.
- Measure the distance from the #2 to each of the remaining 998 interest rate paths.
- Assign each of the 998 interest rate paths to the closer of the 2 representative paths (#1 and #2).
- The path having the greatest distance to its assigned representative will be selected as the next representative.

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Appendix (cont'd)

- In general, when there are k representatives chosen, each of the remaining (1000 - k) paths should be assigned to a closest representative. The k+1 th representative will be the one with the greatest distance from its assigned representative.
- Continue until 100 representatives are obtained.
- The probability assigned to a representative is the ratio of number of its assigned scenarios to 1000.

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TOWERS PERRIN
TILLINGHAST

SOA Annual Meeting
Session 101

Use of cluster analysis for scenario reduction

Steven Craighead

October 17, 2007

SOCIETY OF ACTUARIES

Agenda

- ***Market Requirements***
- Without Scenario Reduction
- Possible Solutions
- Model
- Test Results
- Considerations

Market requirements

- Guaranteed minimum benefits
- Reduction of reinsurance
- Hedge Programs

Agenda

- Market Requirements
- ***<u>Without Scenario Reduction</u>***
- Possible Solutions
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VA hedging application runtime performance

- 171 minutes for 1000 scenarios and 100 model points
- In internal discussions it was decided:
  - That the selected method and scenario set must return results within a 5% error of the full set of scenarios and run at least 8 times faster

Agenda

- Market Requirements
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- ***<u>Possible Solutions</u>***
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Possible solutions

- Revise application for increased runtime improvements
- Use representative scenarios
- Use higher math to obtain the results with no projection.

Scenario reduction CLARA algorithm

- Developed by Kaufman and Rousseeuw (See Finding Groups in Data: An Introduction to Cluster Analysis,(1990)" published by John Wiley and Sons)
- Publicly available within the cluster library for the R language (See cran.r-project.org)

Agenda

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Assumptions

1. All contracts are single premium variable annuities
2. Six fund accounts associated with each cell (one cash, and five indexed accounts)
3. Each fund has the same annuitization, partial withdrawal, lapse, and mortality assumptions
4. No dynamic annuitization – 20% / year for ages 60-64

Assumptions, continued

5. Free partial withdrawals of 10% of the account value – utilization rates are 0 or 100%
6. Mortality rates follow year 2000 mortality table for males and females, no adjustments
7. Lapse rates

Year 1	2-4	5	6+
Rate 0%	2	3	4

Model points

- 988,086 contracts
  - 50/50 male/female
  - Account value avg. $28,625 with a std dev of $17,875
- Avg. valuation age is 52
- Avg. policy durations ~ 4 years with std dev of 2.8

Model points, continued

Each account categorized as Conservative, Balanced, or Aggressive

	Cons.	Bal.	Agg.
EAFE	20%		
Lehman	40	20%	
Russell 2000	40	40	20%
S&P 400		40	40
S&P 500			40

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Agenda

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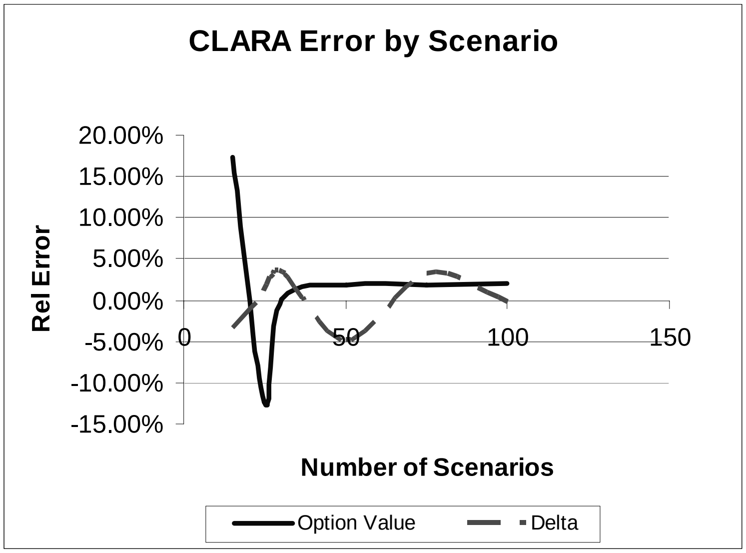

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Fluke of nature?
Pricing a portfolio of European calls

- Simulated 44 different sets of ½ million scenarios
- Priced a portfolio of different European calls, with varying at-the-moneyness and maturity
- Chose different sizes of representative sets from each of the 44 sets of ½ million
- Relative error of the portfolio price vs. the Black-Scholes formula

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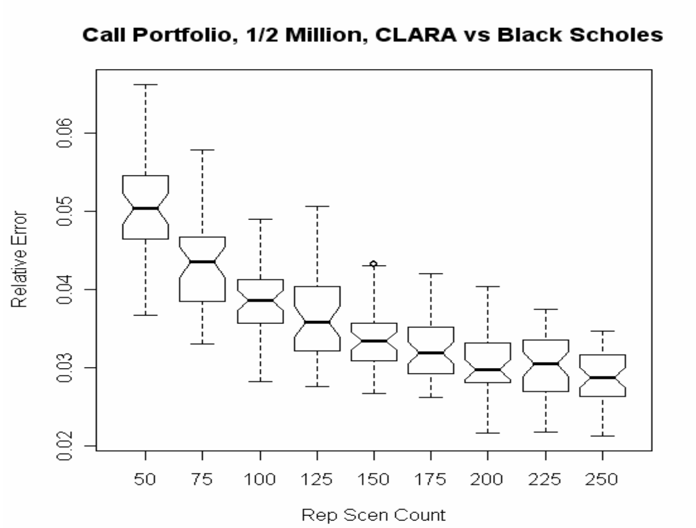
Box plots, error and effectiveness



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Recap

- The use of scenario selection by cluster analysis provides a biased estimate of actual prices
- It is most effective if the set has many scenarios.
- Questions?

AAA Economic Scenario Work Group Reparameterization C-3 Phase 1 Interest Rate Model

Larry M. Gorski, FSA, MAAA
Chair, Economic Scenario Work Group
Claire Thinking Inc

Choosing Scenarios in the PBA Environment

- Use the AAA Life Practice Council Model and required Parameterization – How many scenarios?
- Use pre-packaged scenarios – How many scenarios?
- Use internal model calibrated to AAA LPC Model – How many scenarios?
- Customized pre-selected scenarios – How many scenarios?

What about pre-selected scenario sets (12 – 50 scenarios)?

- The original C-3 Phase 1 (Interest Rate Risk RBC requirements included pre-selected scenarios for cash flow modeling.
- Based on a recommendation from the AAA from the Standards for Stochastic Methods Work Group ("SSMWG"), this option will not be available in the future.

Recommendation to drop pre-selected Scenario Set (12 & 50 scenario sets)

- Previously done for ease of implementation
- Pre-selected scenarios are based on representative asset and liability portfolios; therefore, use of pre-selected scenarios is inconsistent with PBA.
- Pre-selected scenarios are very time consuming to develop and maintain, requiring a significant resource commitment from the AAA LPC

Economic Scenario Work Group (Members & Interested Persons)

- Bill Carmello
- Steve Craighead
- Mike Davlin
- Doug Doll
- Luke Girard
- Larry Gorski
- Geoffrey Hancock
- Bill Pauling
- Richard Payne
- Link Richardson
- Max Rudolph
- Randall Stevenson
- Steve Strommen

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Charge

- **The AAA LCAS Economic Scenario Work Group (ESWG) was formed to ensure consistency between the interest rate/scenario generators endorsed by the various line of business groups. This will be accomplished by:**
  - Reviewing the current parameterization of the C3 PI Interest Rate/Scenario Generator
  - Modifying, if appropriate, structural features of the interest rate/scenario generator.
  - Developing Calibration Criteria for insurers choosing to develop an internal interest rate model.

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ESWG – First Decision

- Should the C3 P1 Interest Rate Model be retained or replaced?
- The ESWG decided to retain the model but to consider several different models when developing Calibration Criteria.

C-3 Phase I Interest Rate Model

- 3 processes
  - Monthly Log volatility of the change in the log of the 20 year UST rate (long rate)
  - Excess of the short rate (1 year rate) over the long rate (Dif)
  - Log of the long rate
- 11 parameters define the processes
- Processes describe the change of the variable over time
- Each process is stochastic, i.e., includes a random shock

Difference Equations that describe the processes

$$\ln(r_{t+1}) = \ln(r_t) - \beta \ln\left(\frac{r_t}{\bar{r}}\right) + \psi \times \left(d_t + \bar{d}\right) + z_1 \sigma_t^r$$

$$\sigma_t^r = e^{\upsilon_t / 2} = \sqrt{\omega_t}$$

$$d_{t+1} = d_t - \gamma \times (dt + \bar{d}) - \varphi \times \ln\left(\frac{r_t}{\bar{r}}\right) + \sigma^d \times \left(\rho \times z_1 + \sqrt{1-\rho^2} \times z_2\right)$$

$$\upsilon_{t+1} = \upsilon_t + \lambda \times \left(\bar{\upsilon} - \upsilon_t\right) + \sigma^\upsilon \times z_3$$

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Economic Scenario Work Group Update 9

Evolution of Model Parameters

Parameter	Current	SS4	SS5	SS6	SS7	SS8	SS8 (a)	SS8 (b)
$\bar{r}$	6.55%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%
LRSMR	.0048	.00072	.00401	.00265	.00265	.00265	.00265	.00265
$\bar{z}$	-6.92	-7.345	-7.345	-7.345	-7.345	-7.525	-7.525	-7.525
zSMR	.347	.02808	.02808	.02808	.02808	.02808	.02808	.02808
zVOL	.59	.22854	.22854	.22854	.22854	.22854	.22854	.22854
$\bar{d}$	.0105	.01155	.0155	.01155	.01155	.01155	.01271	.01271
DSMR	.042	.02777	.02777	.02777	.02777	.02777	.02777	.02777
v	.003809	.00322	.00322	.00322	.00322	.00322	.00322	.00322
SADJF	.21	.21375	.21375	.21375	.21375	.21375	.21375	.21375
YLF	.00024	.0002	.0002	.0002	.0002	.0002	.0002	.0002
Rho	.16	.12296	.12296	.12296	.12296	.12296	.12296	.12296

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Economic Scenario Work Group Update 10

Parameter Estimation

- Analyze historical data (Maximum Likelihood Estimation techniques)
- Understanding the forces that affect some of the parameters (prospective perspective)
- Reviewing results (generated scenarios) from a common-sense perspective. Do the scenarios make sense?

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Data

- Monthly average interest rates from US Federal Reserve Bank ("FRED") for period April 1953 – April 2006
- Simple methodology for filling in the gaps in the time series of the 20 year interest rates
- Data in the Report

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Process

- Retain the structure of the current C-3 Phase 1 interest rate/scenario generator.
- Change the long rate (20 year US Treasury) Mean Reversion Point from 6.55% to 5.4%
- Determine remaining parameters using Maximum Likelihood Estimation techniques adjusted to satisfy conditions concerning the dispersion of interests rates at the 20 and 30 year time horizons, frequency of yield curve inversions, and steepness of the tail of the distribution.

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Economic Scenario Work Group Update 13

Change in Long Rate Mean Reversion Point

- The basis for this change is a shift in perspective from a completely historical viewpoint to a prospective view driven by an analysis of Federal Reserve Bank behaviors and objectives.
- While the MRP recommendation for today's environment is 5.4%, the ESWG believes if long-term economic and market expectations were to change in the future then the MRP recommendation would have to be reconsidered.
- These expectations include inflation, real growth, market liquidity and other risk preferences.

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Economic Scenario Work Group Update 14

Benchmark – Frequency of Yield Curve Inversions

- Yield Curve Inversions (Observable)
  - A yield curve inversion occurs when the short rate (1 yr UST interest rate) exceeds the long rate (20 yr UST interest rate).
  - The definition of yield curve inversion used when the current parameterization was developed is unknown.
  - Frequency sensitive to definition
  - Frequency sensitive to historical period used for data
  - Recommended Parameter Set SS8 (b) – 19%
  - Frequency of yield curve inversions – Current parameters – 22.8% (12/31/05 yield curve)
  - Historical - The frequency of yield curve inversions calculated from the time series of GS 1 year and 20 year rates was 20.9%.

Benchmark – Dispersion Statistic

- Dispersion Statistic (Not Observable)
  - (95 ptile value – 5 ptile value)/(50 ptile value)
  - ESWG Target (1.4 – 1.6) @ 30 years
  - Dispersion Statistic – Current Parameters – 1.35
  - Recommended Parameter Set (SS8 (b)) – 1.52

Information used to determine Model Parameters SS4 (MLE with Long Rate MRP = 5.4%

				2.183668595
Outputs	**r12**	**r180**	**r240**	**r360**
Simulation	**1**	**1**	**1**	**1**
Statistics / Cell	**D17**	**D185**	**D245**	**D365**
Minimum	0.036460672	0.005987468	0.003361304	0.003892351
Maximum	0.082191862	0.593731701	0.596752226	0.643189907
Mean	0.052568997	0.061312277	0.062489115	0.064465133
Standard Deviation	0.005040019	0.032024332	0.035923612	0.042214481
Variance	2.54018E-05	0.001025558	0.001290506	0.001782062
Skewness	0.344282866	3.538013344	2.8755806	3.08092358
Kurtosis	3.691484487	39.7532442	23.27939132	24.58186767
Number of Errors	0	0	0	0
Mode	0.05183192	0.047141355	0.041993382	0.038530903
5.0%	0.044951271	0.025886957	0.024036279	0.021835055
10.0%	0.046519887	0.031107729	0.028984325	0.026736639
15.0%	0.047523126	0.034820322	0.032923128	0.030616449
20.0%	0.048397433	0.038233582	0.036391348	0.034133442
25.0%	0.04920746	0.041088607	0.039752923	0.037685443
30.0%	0.04988008	0.044099409	0.042888347	0.040710963
35.0%	0.050517142	0.046907101	0.04576876	0.043823853
40.0%	0.051128022	0.04951233	0.04870199	0.04727485
45.0%	0.05175028	0.052001603	0.051838275	0.050684109
50.0%	0.052285172	0.054947298	0.054944176	0.054064665
55.0%	0.05289863	0.058228392	0.058348134	0.058262467
60.0%	0.053524755	0.061642185	0.06156737	0.062822208
65.0%	0.054203257	0.064981528	0.065378197	0.067497171
70.0%	0.054878376	0.069183879	0.069933176	0.073094398
75.0%	0.055652235	0.073944651	0.074795127	0.079647809
80.0%	0.056556724	0.078880817	0.081804201	0.087246209
85.0%	0.057651926	0.086480632	0.089965776	0.097056247
90.0%	0.059083126	0.097117379	0.10317225	0.112013154
95.0%	0.061099075	0.115528673	0.126728237	0.139894366

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Information used to determine Model Parameters SS8 (b) (Recommended Parameterization)

Outputs	**r12**	**r60**	**r120**	**r180**	**r240**	**r360**	**Count**
Simulation	**1**	**1**	**1**	**1**	**1**	**1**	**1**
Statistics / Cell	**D17**	**D65**	**D125**	**D185**	**D245**	**D365**	**N367**
Minimum	0.026485126	0.010382106	0.011189969	0.007703606	0.005582088	0.007666491	0
Maximum	0.101632059	0.185539126	0.174468577	0.172911748	0.180015147	0.181762934	253
Mean	0.052997576	0.057192974	0.058697889	0.05917558	0.059192833	0.059203824	68.421
Standard Deviation	0.008196121	0.017764168	0.021977848	0.023776574	0.024843843	0.025505028	41.28941802
Variance	6.71764E-05	0.000315566	0.000483026	0.000565325	0.000617217	0.000650506	1704.816041
Skewness	0.516978318	1.326361707	1.229317484	1.190048834	1.209275793	1.157387438	0.722115823
Kurtosis	3.90504046	6.965510716	5.563685243	5.205673465	5.064366788	4.740555907	3.35235544
Number of Errors	0	0	0	0	0	0	0
Mode	0.051400064	0.053000818	0.051317261	0.051034753	0.049912112	0.041845451	63
5.0%	0.040815629	0.03352952	0.03020799	0.02884252	0.028066471	0.026985127	12
10.0%	0.043290675	0.03774225	0.034852181	0.033528574	0.032391597	0.03195126	19
15.0%	0.044852998	0.040673137	0.038219776	0.036620196	0.035759185	0.035230398	26
20.0%	0.046224069	0.043108728	0.040964168	0.039712824	0.038631245	0.038294669	31
25.0%	0.047442112	0.045411274	0.043501318	0.042423014	0.041599073	0.041178424	37
30.0%	0.048479047	0.047366407	0.045876257	0.045128152	0.044425461	0.043575466	42
35.0%	0.049481932	0.04919615	0.048263155	0.04761954	0.046941776	0.046232253	47
40.0%	0.05053841	0.051070135	0.05067813	0.049871169	0.049685862	0.048917305	53
45.0%	0.051482335	0.0528448	0.052804835	0.05231116	0.052165318	0.05135832	58
50.0%	0.052398406	0.054801747	0.055153985	0.054978024	0.054622553	0.05421621	63
55.0%	0.05338214	0.056616731	0.057543948	0.057708185	0.057652894	0.057119165	69
60.0%	0.054365512	0.058691304	0.060015816	0.060818225	0.060520988	0.060397942	74
65.0%	0.055490211	0.060845435	0.062860988	0.063784167	0.063529208	0.063826099	80
70.0%	0.056584373	0.063288428	0.065990306	0.067243181	0.066865027	0.067563243	87
75.0%	0.057811439	0.06597814	0.069573931	0.071414679	0.07111045	0.07202296	93
80.0%	0.059327692	0.069015324	0.07385537	0.075783707	0.076027036	0.077455193	102
85.0%	0.061279722	0.073134251	0.078888699	0.081424184	0.082482487	0.083566956	112
90.0%	0.063507497	0.078807905	0.086114869	0.089721903	0.091405511	0.092751876	125
95.0%	0.067179441	0.08862821	0.100029662	0.103937127	0.108151361	0.109547779	144
Filter Minimum							
Filter Maximum						1.522840711	

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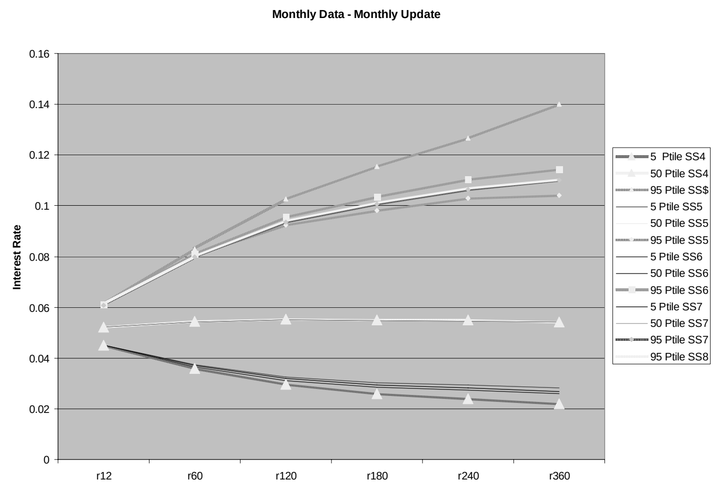
20Yr US Treasury Rate Scenarios under various Parameter Sets
Starting Long Rate 5.11%
Starting Short Rate 4.88%



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Economic Scenario Work Group Update 19

Evolution of Model Parameters

Parameter	Current	SS4	SS5	SS6	SS7	SS8	SS8 (a)	SS8 (b)
$\bar{r}$	6.55%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%
LRSMR	.0048	.00072	.00401	.00265	.00265	.00265	.00265	.00265
$\bar{z}$	-6.92	-7.345	-7.345	-7.345	-7.345	-7.525	-7.525	-7.525
zSMR	.347	.02808	.02808	.02808	.02808	.02808	.02808	.02808
zVOL	.59	.22854	.22854	.22854	.22854	.22854	.22854	.22854
$\bar{d}$	.0105	.01155	.0155	.01155	.01155	.01155	.01271	.01271
DSMR	.042	.02777	.02777	.02777	.02777	.02777	.02777	.02777
v	.003809	.00322	.00322	.00322	.00322	.00322	.00322	.00322
SADJF	.21	.21375	.21375	.21375	.21375	.21375	.21375	.21375
YLF	.00024	.0002	.0002	.0002	.0002	.0002	.0002	.0002
Rho	.16	.12296	.12296	.12296	.12296	.12296	.12296	.12296

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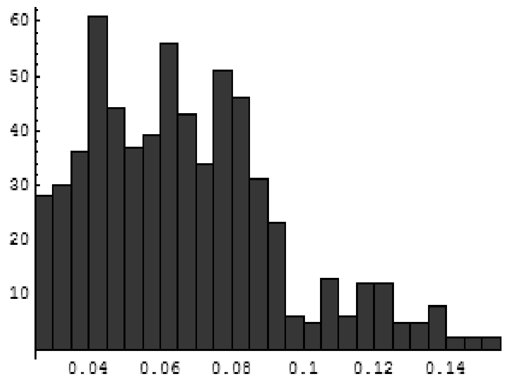
Economic Scenario Work Group Update 20

Distribution of Monthly Average 20 YR UST Interest Rates
GS20 -- April 53 - April 2006



	Historical GS20 -- April 53 - April 2006
0.05	0.0306
0.1	0.0374
0.15	0.04
0.2	0.0419
0.25	0.0452
0.3	0.049
0.35	0.0536
0.4	0.0578
0.45	0.0604
0.5	0.0636
0.55	0.0674
0.6	0.0714
0.65	0.0757
0.7	0.0787
0.75	0.0814
0.8	0.0856
0.85	0.0908
0.9	0.1063
0.95	0.1216

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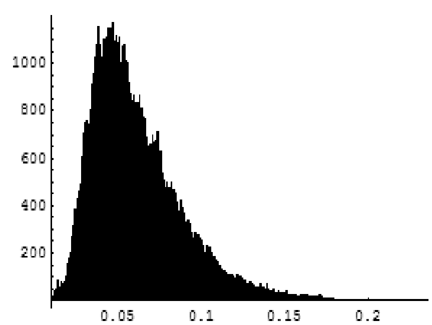
Distribution of Modeled 20 YR UST Interest Rates
SS 8 (b)



	Modeled SS 8 (b)
0.05	0.0272282
0.1	0.0317501
0.15	0.0354157
0.2	0.0382433
0.25	0.0411524
0.3	0.0439318
0.35	0.0465703
0.4	0.0492575
0.45	0.0521246
0.5	0.0549978
0.55	0.0583557
0.6	0.0619866
0.65	0.0657482
0.7	0.0702388
0.75	0.0747485
0.8	0.0805931
0.85	0.0876208
0.9	0.0973099
0.95	0.112576

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Impact of changes to the model 1 and 30 year time horizons

	1 Year Time Horizon		30 Year Time Horizon	
	Before	After	Before	After
0.05	0.043704	0.0412492	0.03431	0.028554
0.1	0.045755	0.0435666	0.039191	0.0331312
0.15	0.047067	0.0451976	0.043228	0.0365567
0.2	0.048127	0.046487	0.046626	0.0395593
0.25	0.049075	0.047623	0.049533	0.0423447
0.3	0.049844	0.0486485	0.052694	0.0450467
0.35	0.050578	0.0495961	0.055366	0.0477778
0.4	0.051286	0.0505795	0.058127	0.0502675
0.45	0.051969	0.051537	0.06095	0.0529133
0.5	0.05268	0.0524929	0.063825	0.0559763
0.55	0.053368	0.0535012	0.066916	0.0587876
0.6	0.054041	0.054443	0.070311	0.0620465
0.65	0.054737	0.0554938	0.074004	0.0653173
0.7	0.055536	0.0566722	0.07782	0.0696313
0.75	0.056404	0.0579639	0.082507	0.0741629
0.8	0.057446	0.05946	0.088022	0.0798343
0.85	0.058695	0.0613163	0.094348	0.0877749
0.9	0.060429	0.0638799	0.104311	0.098488
0.95	0.063117	0.0680827	0.120511	0.114737

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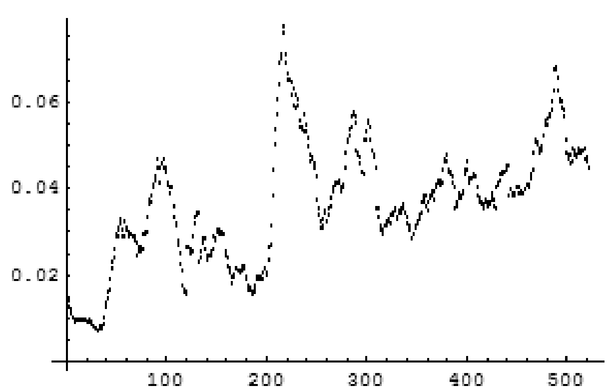
Economic Scenario Work Group Update 23

Determining the "seed" for the stochastic log variance process

Monthly Volatility Delta Log Long Rate after applying 12 month Moving Average filter



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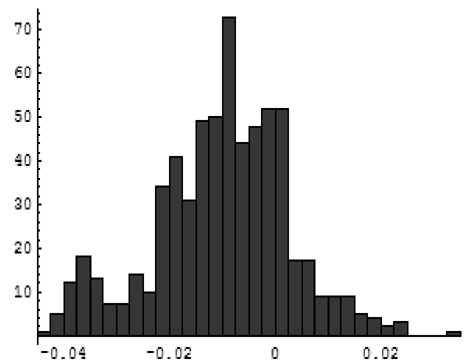
Economic Scenario Work Group Update 24

Distribution of the Excess of the short rate over the long rate- Historical



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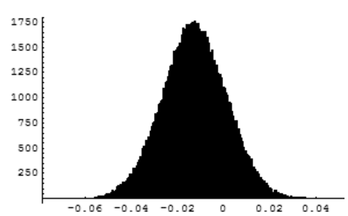
Distribution of the Excess of the short rate over the long rate

Distribution of the Excess of the Short Rate over the Long Rate (d_t)
Parameter Set 8 (b) – 10,000 years



	Modeled SSb(b)	Historical 1953- 2006
Maximum	.05183	.0333
Minimum	-.07868	-.0432
Mean	-.01266	-.001

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Next steps

- Automatic updating of the mean reversion point of the long rate process.
- Develop calibration criteria.
- Coordinate equity scenarios with interest rate scenarios.
- Change interest rate model?

Selecting Scenarios

- Representative Scenarios
- Variance Reduction Techniques
  - Importance Sampling
  - Control variate methodology
  - Neural Networks