SOCIETY OF ACTUARIES

Equity-based Insurance Guarantees Conference
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Impact of Stochastic Interest Rates and Volatilities in Living Benefits

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Moderator

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Rates and Rate Volatility

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Equity Based Investment Guarantee Conference
Boston
1400 – 1500 hours (28 Oct 08)

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Problems for Guarantors

- Complex benefits are exposed to complex risks
- *"The ride matters."*
- If my models don't see the risks, I can't charge for them or hedge them.
- Models that work well for pricing are often unwieldy for hedging.
- Hedging with a simpler model may look good today, but can be dangerous in the long run.

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Two Benefits for Analysis

Stylized GMWB
- 100% invested in equity,
- 10% annual withdrawal,
- ratcheting benefit base,
- payments continued through 30yrs.

Stylized GMIB
- 100% invested in equity,
- ratcheting benefit base,
- 4% rate guarantee,
- 15yr term-certain annuity elected after 15yrs

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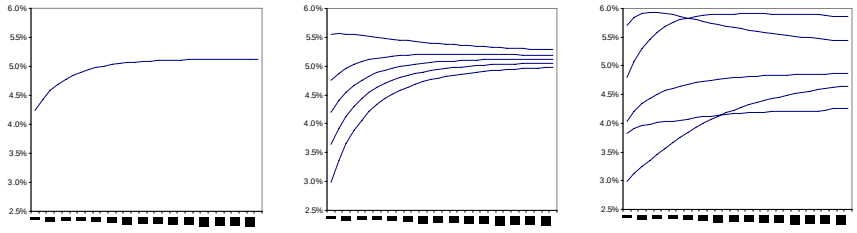
A Spectrum of Rate Models

Representative Yield Curve Movements



Increasing Volatility Characteristics

Static Yield Curve
Market-Consistent Valuation

No-Arb Short-Rate Model (CIR++)
Market-Consistent Valuation And Hedging

Full Forward Rate Market Model (LFM)
Market-Consistent Valuation And Hedging

All models calibrated to same yield curve.
CIR++ and LFM "historically" calibrated to have same average short-rate volatility.

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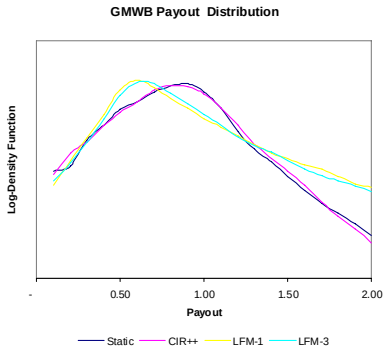
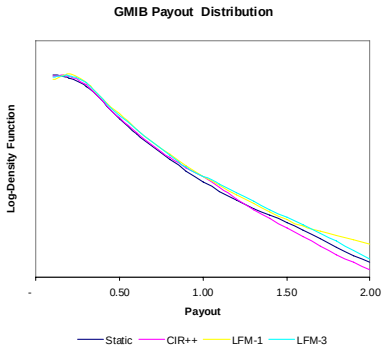
Price and Risk



Model	Probability of Payout	Option Value
Static	0.995	100
CIR++	0.995	100
LFM-1	0.994	103
LFM-3	0.995	104



Model	Probability of Payout	Option Value
Static	0.643	100
CIR++	0.647	104
LFM-1	0.815	145
LFM-3	0.816	142

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Understanding These Results

	GMWB	GMIB
Long-Horizon Discounting	Yes	Yes
Rate Dependent Payout		Yes
Path Dependence	Strong	Weak

- All rate models provide identical discounting.
- Short rate models are heavily constrained and are nearly identical to static yield curves.
- Only LFM models can reasonably value hard rate optionality and other convex exposures to rates (i.e. path dependence).

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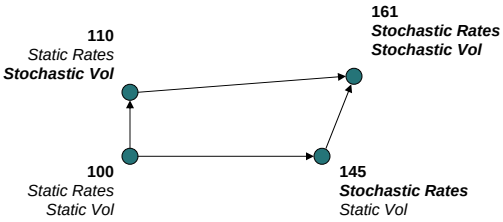
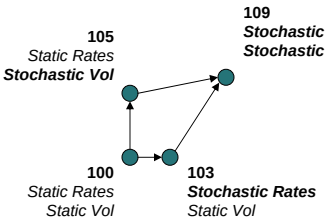
Stochastic Equity Volatility

What happens when we add Heston Volatility with same average volatility level?

(Heston is mild, results would be even more dramatic with SABR volatility.)



GMIB



GMWB

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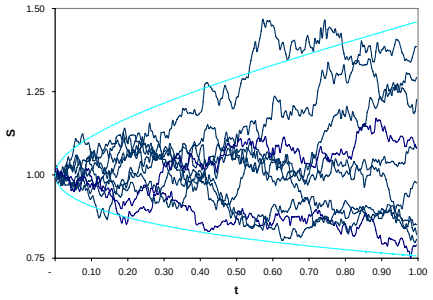
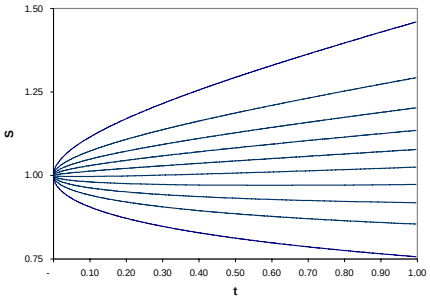
A Useful Touchstone

Model A



Model B



Both scenario sets have identical return distibutions at every maturity.

How would the value of your benefit be different?

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First Insights

- Know the value of the hard and soft option exposure in your benefit.
- Don't underestimate the risk or cost of interest rate volatility in your products.

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LIBOR Forward Model

- LFM is a much different proposition than short-rate models – especially for insurance applications.
  - You are modeling all forward rates, not just the front-end money market rate.
  - Insurance liabilities don't line up with commonly quoted LIBOR forward periods.
  - LFM is a discrete forward rate model. Naïve approaches to insurance liabilities blow up.
  - You have new modeling choices around issues like "natural tenor". Naïve approaches to insurance liabilities blow up.
  - Calibration is now focused on rate volatility rather than yields.

References

Rebonato, "*Modern Pricing of Interest Rate Derivatives*" (theory)
Brigo & Mercurio, "*Interest Rate Models – Theory and Practice*" (praxis)
Rebonato, "*Volatility and Correlation*" (calibration)

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Some Thoughts on Calibration

- For **static yield curve**, forward rates are directly observable. But which forward rates: Treasury, LIBOR, OIS?
- For **CIR++** there is a short-rate volatility parameter. Usually set by assumption.
- For **LFM** there is a term structure of volatility and correlation parameters!
  - Is only visible in complex, illiquid, short-duration instruments.
  - Choice of instruments matters. Natural instrument?
  - What principals drive calibration choices?
- Do you mix market and historical calibration?

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Hedging Changes in Rates

- Classic textbook example:
  - Get out your Black-Scholes formula.
  - Compute Delta and Rho (even Vega if you like)
  - Dynamically Hedge
  - Come out close at the end.
- What is wrong with this setup!
- Most static yield curve and short-rate hedgers do something similar with “key rate durations”.
  - Key rates in static forward curve.
  - “Shift Factors” in CIR++ model.
- Looks fine in the short-run, but…

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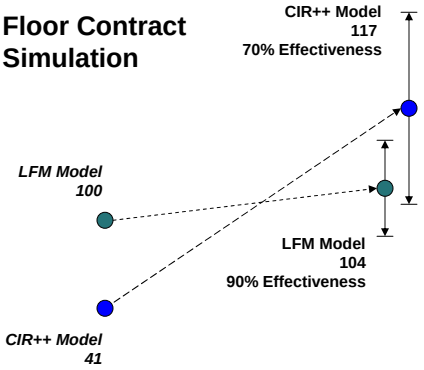
Extra-Model Hedging
Both Known and Unknown Danger

Floor Contract Simulation



- Simulated dynamic hedging of a floor contract through historical yield curves.
- Both models result in a range of emerged costs (initial cost plus cumulative hedge error).
- CIR++ model suffers from heavy bias and modest effectiveness (proportion of real-world fluctuation covered by hedge).

- ***Over a quarter, normal market volatility swamps bias. But over long horizons bias can be large.***

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Understanding These Results

- CIR++ says long-dated forward rates don't move much, so the price appears cheap.
- When you hedge you will pay for actual volatility – usually at the most inopportune time which results in a loss.
- When you hedge each of the forward periods you are over-hedging. Trading incidental noise increases cost and decreases effectiveness.

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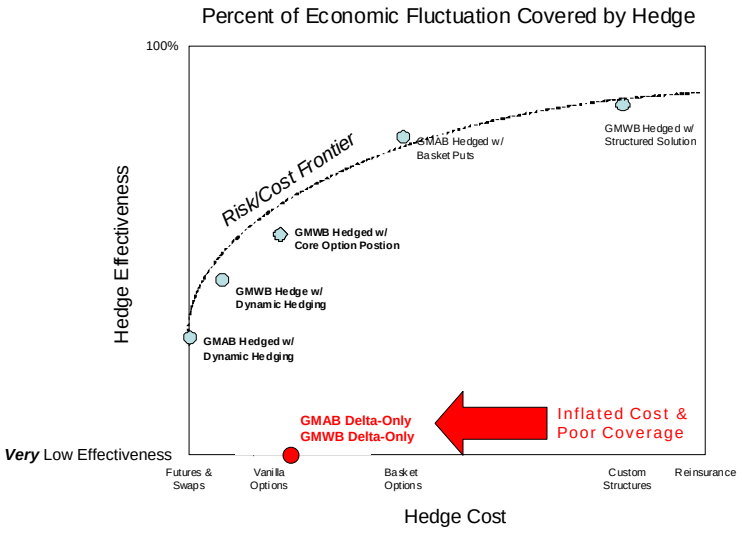
Another Example
"Dumbing Down" is Dangerous Too



Understanding These Results

- In a "combined" valuation model, rates and equity are not independent.
- The hedge income components are not independent either. Some of the equity exposure is funded by the rate hedge!
- Similarly, volatility exposure comes from both sources and in different flavors.
- Hedge strategy may necessitate different models for valuation and hedging.

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More Insights

- Know the value of the hard and soft option exposure in your benefit.
- Don't underestimate the risk or cost of interest rate volatility.
- Using the wrong model can be both effective and costly.
- Misusing the right model can be worse.
- The feedback process is generally slow.

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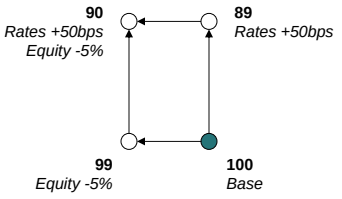
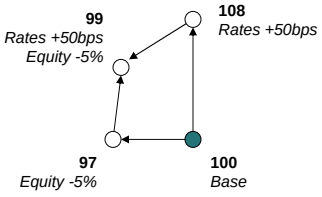
GMWB Cross Exposure



CIR++



LFM

- Sum of the parts does not equal the whole.
- Problem is particularly pronounced when your benefit is vol exposed and/or long duration.
- Without the right model, you will be surprised later.

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More Insights

- Know the value of the hard/soft option exposure.
- Don't underestimate the risk or cost of interest rate volatility.
- Using the wrong model can be costly.
- Misusing the right model can be worse.
- The feedback process is generally slow.
- Volatility exposure is complex and uncontained.

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Practical Implications

- Make sure you understand the risks you are selling. (*A model suite comes in handy.)*
- Be explicit about the risks you are hedging and the risks you are retaining. (*Make sure your hedge models are consistent with your view.)*
- When your valuation model "sees all" and your hedge model "sees some", hedge tracking and effectiveness reports are crucial.
- Hedge breakdowns manifest slowly. A deep, detailed hedge attribution system is crucial.

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