



SOCIETY OF ACTUARIES

**Valuation Actuary Symposium  
September 24- 25, 2009**

**Session 16 TS: Asset Modeling Concepts**

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# Asset Modeling Concepts

Presented by  
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September 24, 2009



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## Agenda

- Interest Rate Models
  - Risk-neutral vs. Real-world models
  - Stochastic Deflators
  - Commonly used interest rate models
  - Market calibration
  - Principal Components Analysis
- Equity Models
- Assumption Setting

## Interest Rate Scenarios

### ■ How do we use interest rate scenarios?

- Assets
  - Cash flows may be interest-rate dependent
  - Value fixed income securities, options and derivatives at each valuation date
- Liabilities
  - Embedded Options
  - Interest-rate sensitive policyholder behavior (e.g., lapses)

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## Real world vs. Risk neutral

### ■ Real World Approach

- Basis for traditional actuarial approach
- Discount Rates
  - Based on expected returns (not risk-free rates)
  - Expected return reflects actuary's view of risk
  - Reflect premium for market risk, credit, liquidity, etc.
- Probabilities/Expectations
  - Reflect “real-world” beliefs regarding possible future outcomes

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## Real world vs. Risk neutral

### ▪ Risk-Neutral Approach

- Investor risk-preference is not relevant to pricing of financial instruments
- Discount Rates
  - Based on risk-free rates
  - Actuary's (and market's) view of risk is irrelevant
  - Do not reflect premium for market risk, credit, liquidity, etc.
- Probabilities/Expectations
  - Theoretical construct
  - “Martingale” measure

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## Real-World Approach - Example

### ▪ Simplified Example

- XYZ company sells one-year guaranteed return on the S&P index
- Guaranteed return = 7.5%
- Current price of S&P index = \$100
- At the end of one year, there are two possible states of the S&P index, which we believe are equally likely: Price = \$120 or \$95
- Risk-free rate,  $r = 5\%$

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## Real-World Approach – An Example

- XYZ has effectively sold a put option on the S&P index with  $S = \$100$ ,  $k = \$107.5$ , and  $r = 5.0\%$

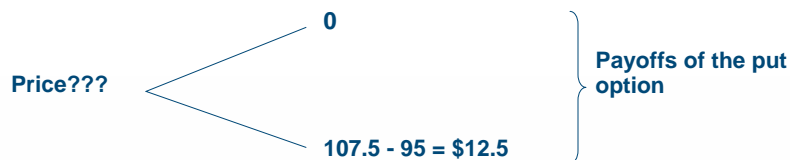


- Discount Rate = Real-world Expected Return on the S&P index =  $(0.5)(20\%) + (0.5)(-5\%) = 7.5\%$

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## Real-World Approach – An Example

- The possible payoffs of the option are illustrated below:



- The price, in the traditional view of the world, is equal to PV(possible future payouts), using real-world expectations and risk-adjusted discount rate:
- $PV(\text{possible future payouts}) = [(\$0)(0.5) + (\$12.5)(0.5)] / (1.075) = \mathbf{\$5.81}$

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## A problem with the Real-World approach

- But, there is a problem...
- The true price of the guarantee XYZ has sold is much higher. Consider the following payoff diagrams that summarize the situation:



- XYZ can replicate the payoff of the guarantee by shorting  $\frac{1}{2}$  share of the S&P index (for income of \$50) and by investing \$57.14 in the risk-free asset, at a net cost of \$7.14 to XYZ
  - This suggests the true price should be \$7.14 (= \$57.14 less \$50.00)

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## The Real-World Approach & Opportunities for Arbitrage

- The problem above can lead to arbitrage
  - Arbitrage exists when:
    - One buys and sells essentially the same thing
    - The trades are simultaneous
    - Overall position is riskless

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## The Real-World Approach & Opportunities for Arbitrage

- Arbitrage opportunities cannot exist in the market for long
  - Buy cheap; Sell expensive
  - Use of replicating portfolios
  - Arbitrarily large positions quickly eliminate arbitrage

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## The Risk-neutral approach & a solution to the arbitrage problem

- Risk-neutral approach solves the problem above...
  - Basic ideas:
    - Use risk-free rate for discounting instead of the risk-adjusted rate
    - Use risk-neutral probability measure (Martingale) instead of actual, real-world expectations



$$q = \frac{1+r+d}{u-d}$$

Where:  
 $r$  = risk-free rate  
 $u = S(1)/S(0)$  in the "up" scenario  
 $d = S(1)/S(0)$  in the "down" scenario

*Note:  $q$  has no intuitive interpretation; it is a theoretical construct that allows us to satisfy the no-arbitrage constraint.*

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## Risk-Neutral vs. Real-World

### ■ Conclusions

- Risk-neutral approaches are preferable to real-world approach because they are arbitrage-free & market-consistent
- Generally, risk-neutral interest rate scenarios are preferable to real-world scenarios for stochastic modeling
  - Certain exceptions apply:
    - Use of stochastic deflators with real-world scenarios
    - P/C companies

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## Risk-Neutral vs. Real-World

### ■ Applications

- Risk-neutral interest rate scenarios used for:
  - Product design and pricing
  - Principles Based Reserving
  - Economic Capital (Life/Health products)

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## Stochastic Deflators

- Serve as link between real-world and risk-free scenarios
  - Deflators:
    - Make real-world scenarios market consistent
    - Are used with real-world scenarios to replicate market-observed prices

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## Stochastic Deflators

- In risk-neutral valuation, expected PV of cash flows is:

$$V_0 = \sum (C_{t,i} * q_i * (1+r)^{-t})$$

- Similarly, in a real-world valuation, expected PV of cash flows is:

$$V_0 = \sum (C_{t,i} * p_i * D_{t,i})$$

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## Stochastic Deflators

- $D_{t,i}$  is the deflator and is found by solving the following equation:

$$D_{t,i} = (q_i / p_i) * (1 + r)^{-t}$$

- $D_{t,i}$  is the implied discount factor that, along with the chosen real-world probabilities, produces a market-consistent price.
- Example available for those interested

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## Interest Rate Generators

- Interest rate models generally fall into two categories:
  - Equilibrium models
  - No-Arbitrage Models

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## Interest Rate Generators – Equilibrium Models

### ▪ Equilibrium models

- Begin with general assumptions regarding economic conditions
- Use assumptions to develop future short rates
- Short rates can be treated as forward rates to develop spot rates and yield curves
- Current yield curve is an output of the model, not an input; resulting yield curve may bear little resemblance to current market conditions
- Drift of the short rate is generally time-independent

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## Interest Rate Generators – No-Arbitrage Models

### ▪ No-Arbitrage Models

- Will produce interest rates exactly consistent with an input yield curve
- Current yield curve is an input to the model
- Drift of the short rate is generally time-dependent (because the shape of the current yield curve guides the movement of future short rates)

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## Commonly used Interest Rate Generators: Equilibrium Models

### Commonly used Interest Rate Generators - Equilibrium Models

Model	Formulaic Description	Comments
Rendleman and Barter Model	<ul style="list-style-type: none"> <li>■ <math>dr = \mu r dt + \sigma r dz</math></li> </ul>	<ul style="list-style-type: none"> <li>■ <math>\mu</math> and <math>\sigma</math> constant</li> <li>■ <math>r</math> follows geometric Brownian motion</li> <li>■ <math>\mu</math> is a drift component. <math>\sigma</math> is a volatility parameter.</li> <li>■ No mean reversion</li> <li>■ Negative interest rates are possible</li> <li>■ Drift is time-independent</li> </ul>
Vasicek Model	<ul style="list-style-type: none"> <li>■ <math>dr = a(b-r) dt + \sigma dz</math></li> </ul>	<ul style="list-style-type: none"> <li>■ Incorporates mean reversion through the term <math>a(b-r)</math></li> <li>■ The <math>\sigma dz</math> term incorporates stochastic element</li> <li>■ Negative interest rates are possible</li> <li>■ Drift is time-independent</li> </ul>
Cox, Ingersoll and Ross Model	<ul style="list-style-type: none"> <li>■ <math>dr = a(b-r) dt + \sigma \sqrt{r} dz</math></li> </ul>	<ul style="list-style-type: none"> <li>■ Very similar to Vasicek model, except for the presence of <math>\sqrt{r}</math> in the second term</li> <li>■ The <math>\sqrt{r}</math> exists in order to preclude negative interest rates</li> <li>■ Drift is time-independent</li> </ul>

Note:  $dz$  is a Wiener process satisfying the following two properties:

- (1)  $dz = Z(dt)^{1/2}$ , where  $Z$  is standard normally distributed
- (2) Values of  $dz$  for separate time intervals are independent

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## Commonly used Interest Rate Generators: No-Arbitrage Models

### Commonly used Interest Rate Generators - No-Arbitrage Models

Model	Formulaic Description	Comments
Ho-Lee Model	<ul style="list-style-type: none"> <li>■ <math>dr = \theta(t) dt + \sigma dz</math></li> <li>■ <math>\theta(t) = F_t(0,t) + \sigma^2 t</math></li> </ul>	<ul style="list-style-type: none"> <li>■ <math>\sigma</math> constant</li> <li>■ <math>\theta(t)</math> term exists to calibrate model to initial term structure</li> <li>■ Drift is time-dependent</li> <li>■ <math>F_t(0,t)</math> is the partial derivative with respect to <math>t</math> of the forward rate for a maturity <math>t</math>. This term helps ensure that future short rates move in line with initial term structure</li> </ul>
Hull-White Model	<ul style="list-style-type: none"> <li>■ <math>dr = [\theta(t) - ar] dt + \sigma dz</math></li> <li>■ <math>\theta(t) = F_t(0,t) + aF(0,t) + (\sigma^2/2a)(1 - e^{-2at})</math></li> </ul>	<ul style="list-style-type: none"> <li>■ Effectively, a Ho-Lee model with mean-reversion</li> </ul>

Note:  $dz$  is a Wiener process satisfying the following two properties:

- (1)  $dz = Z(dt)^{1/2}$ , where  $Z$  is standard normally distributed
- (2) Values of  $dz$  for separate time intervals are independent

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## Interest Rate Generators – Market Calibration

- How do we calibrate the interest rate models above to market conditions?
- Basic Process:
  - Choose a goodness-of-fit measure
    - $\sum (U_i - V_i)$  for  $i=1$  to  $n$
    - Where  $U_i$  =market price of  $i^{\text{th}}$  calibrating instrument and  $V_i$  = price of  $i^{\text{th}}$  calibrating instrument produced by the model
  - Minimize goodness-of-fit measure
    - Levenberg-Marquardt procedure commonly used

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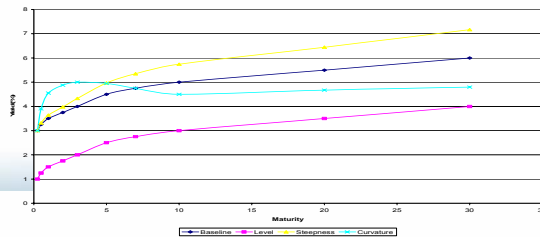
## Principal Components Analysis (PCA)

- Goal is to collapse the infinite number of factors acting on a process into a manageable number of factors so that we can model shocks to the system
- Useful in asset modeling since many economic variables act upon interest rates, asset prices, etc.
- Common applications:
  - Modeling:
    - Interest rates
    - Credit spreads

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## Principal Components Analysis (PCA)

- Key results when modeling interest rates:
  - The majority of variability in interest rate movements can be explained by only three factors:
    - Level of the yield curve
    - Steepness of the yield curve
    - Curvature of the yield curve
  - These results have been confirmed in many currencies, so this does not appear to be a U.S.-specific result



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## Principal Components Analysis (PCA)

- How does this help us model interest rates or assets?
  - Assign a probability distribution to each principal component (level, steepness and curvature)
    - Distribution may be parametric (e.g., Normal distribution) or may be based on historical data
  - Use a random number generator to create values for each of three principal components
  - Use the three factors to construct a yield curve
  - Repeat as many times as necessary – we can quickly and easily generate thousands of stochastic scenarios in this way

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## Principal Components Analysis (PCA)

### ■ Basic steps of a PCA

1. Collect some data to analyze (e.g., monthly yield curve data for the past 20 years)
2. For each dimension in your data (e.g., Treasury yield for 10 year maturity), subtract the mean of the observations from that dimension from each observation
3. Calculate the covariance matrix of the various dimensions
4. Calculate the eigenvectors and eigenvalues of the covariance matrix
5. Rank eigenvalues – the eigenvector associated with the highest eigenvalue is the principal component. In general, the higher the eigenvalue, the more explanatory power is contained in the associated eigenvector
6. Shock eigenvectors using distribution, as described above

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## Equity Models

### ■ Two commonly used models:

- Lognormal return model
  - Underlies the Black-Scholes-Merton equation
- Heston Model

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## Lognormal Equity Returns

$$\ln(S_T) \sim N[\ln(S_0) + (\mu - \sigma^2/2)T, \sigma\sqrt{T}]$$

where

$N[\mu, \sigma]$  is normally distributed with mean =  $\mu$  and standard deviation =  $\sigma$

$S_0$  = stock price at time = 0

$S_T$  = stock price at time = T

***Lognormal equity returns give rise to the familiar Black-Scholes-Merton equation for option pricing and various derivations***

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## Heston Model for Equity Returns

$$d \ln(S_t) = [f(t) - q - \sigma_t^2/2]dt + \sigma_t[\rho dz_1 + \sqrt{1-\rho^2}dz_2]$$

$$d \sigma_t^2 = K(\beta - \sigma_t^2)dt + \sigma_t v dz_2$$

where

$S(t)$  is the index level at time  $t$

$\sigma_t^2$  is the underlying variance of the equity index process at time  $t$

$f(t)$  is the expected return for the current period

$q$  is the continuously-compounded dividend yield over the same time interval

$\rho$  is the correlation between return innovations and variance innovations

$K$  is the strength of mean reversion of volatility

$\beta$  is equilibrium variance

$v$  is the volatility of volatility

$dz_1$  and  $dz_2$  are Brownian motion processes

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## Heston Model for Equity Returns


- Key features/advantages of the Heston model:
  - Volatility is itself a stochastic process, allowing greater flexibility of market consistency:
    - Allows mean reversion of volatility
    - Changes in underlying volatility can be negatively related to index movements over time
    - Can be made consistent with S&P 500 property that distribution of equity price movements exhibit fatter tails and skew relative to lognormal distributions

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## Assumption Setting

- Market consistent approach
- Historically consistent approach

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**Asset Modeling Concepts**

**Fixed Income Asset Categories**

Jason Kehrberg, FSA

Valuation Actuary Symposium  
Session 16 TS  
1:30 - 3:00 PM  
September 24, 2009  
Anaheim, CA



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**Agenda**

- Need for and uses of asset models
- Summary of asset classes and asset class categories
- Yield curves
- Credit spreads
- Defaults
- Bonds
- Mortgages, MBSs and CMOs
- ABSs, CDOs and derivatives
- Validation, reinvestment and negative CF strategies
- Note
  - Focus on the general account of life insurance companies
  - Will not be addressing ESGs

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## Why do life insurance companies need asset models?

- In 2007, 28% of LIC gross income came from assets
  - Second only to premiums (64%)
- Due to this significance, sound actuarial practice prescribes, and many regulations require, detailed asset models:
  - AOM cash flow testing
  - Actuarial appraisals
  - ALM (duration matching, hedging)
  - PBR & RBC C-3
  - Capital and financial planning
  - AICPA SOP 03-1
- Assets are modeled stochastically over many scenarios
  - Can't use a flat net earned rate for most applications
    - LIC net earned rate on GA assets was 6.01% in 2007
  - Most assets are modeled on a seriatim basis

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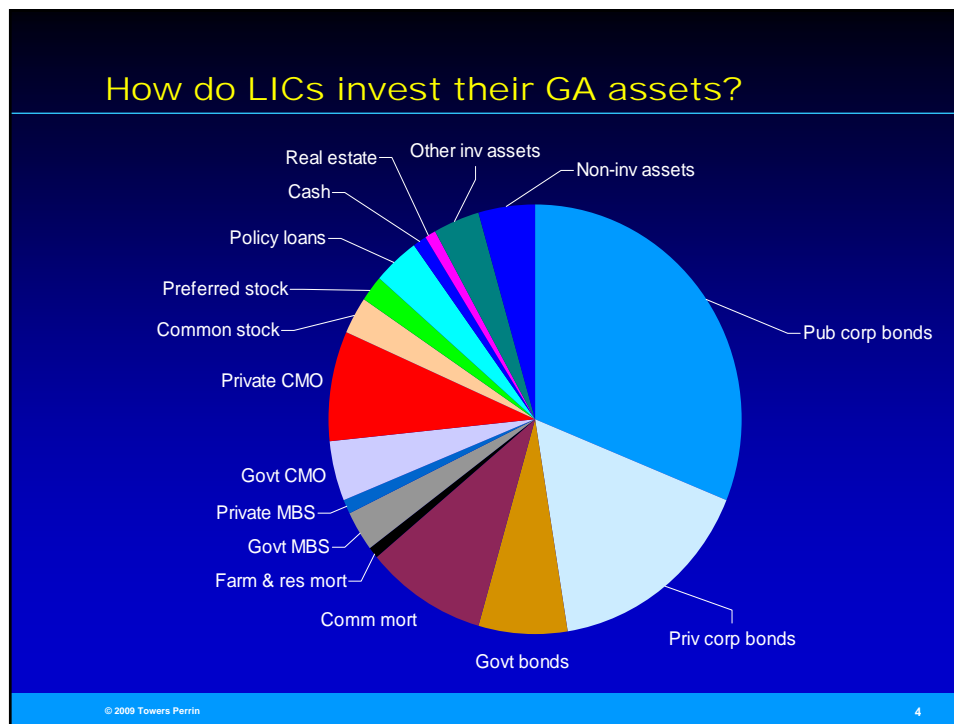
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## Asset classes and asset class categories

- Asset class – a group of securities that exhibit similar characteristics, behave similarly in the marketplace, and are subject to the same laws and regulations.
  - Four main asset classes:
    - Stocks or equities
    - Fixed income or bonds
    - Money market or cash
    - Real estate
  - Possible others:
    - Hedge funds
    - Commodities
    - Luxury collectibles
    - Derivatives?
- Asset class category – any further breakdown of asset classes, reasons include:
  - Modeling
  - Marketing
  - Diversification
  - Regulation

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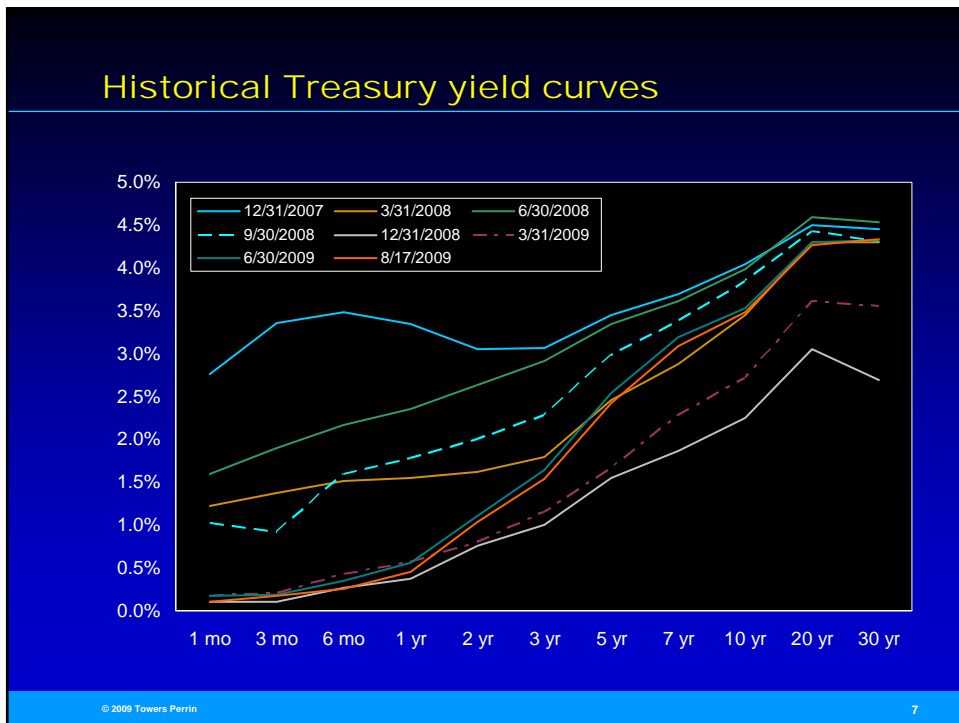
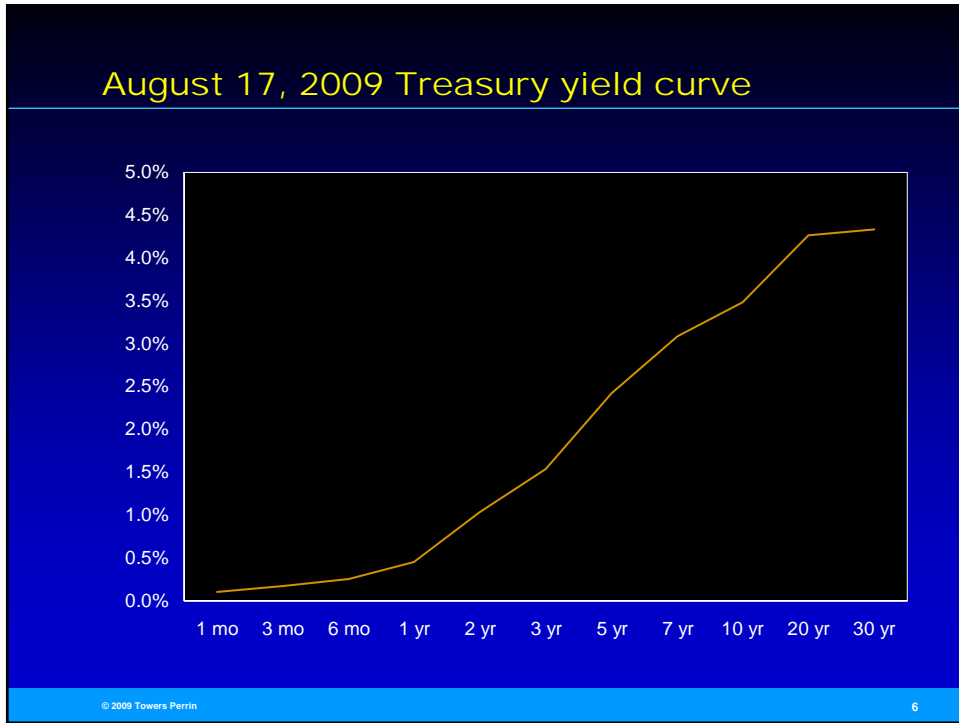
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### Treasury yields and the yield curve

- A bond's market yield, or IRR, is the interest rate at which the present value of future cashflows equals today's price
- A yield curve is simply a graph of the relationship between yield and maturity on bonds of the same credit quality
- Most yield curves are Treasury yield curves
  - All bonds have same default risk → none
  - Largest bond market → liquid, frequent trading
- Uses of Treasury yield curves
  - Graphically summarizes interest rate scenarios
  - Benchmark when looking at other bond prices and yields
  - Leading indicator of economic conditions

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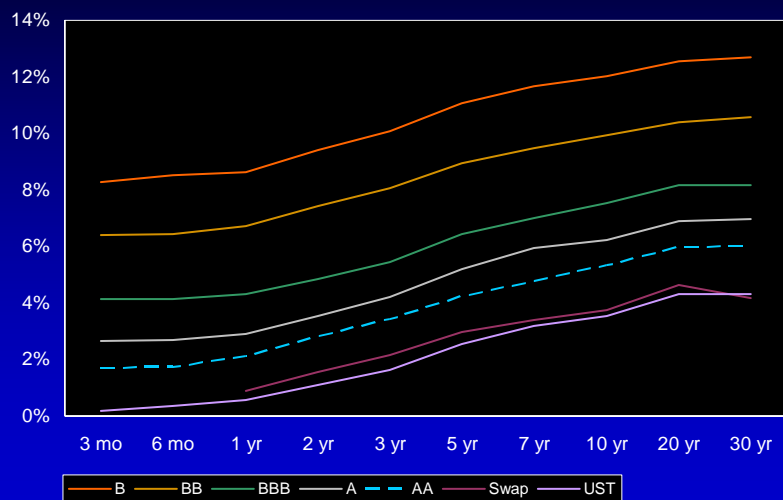
### Credit spreads

- Credit spread is the spread between Treasury and non-Treasury securities that are *identical* in all respects except quality
  - Other factors include liquidity, taxes, margin, local supply/demand shocks, and embedded options
  - Widen in recessions and tighten in expansions
- Harder to determine when embedded options, collateral or prepayment behavior are present
- When modeling, future credit spreads can be flat, graded or stochastic, and are used to determine:
  - Market values
  - Yields on reinvestments
  - Whether financial options are exercised

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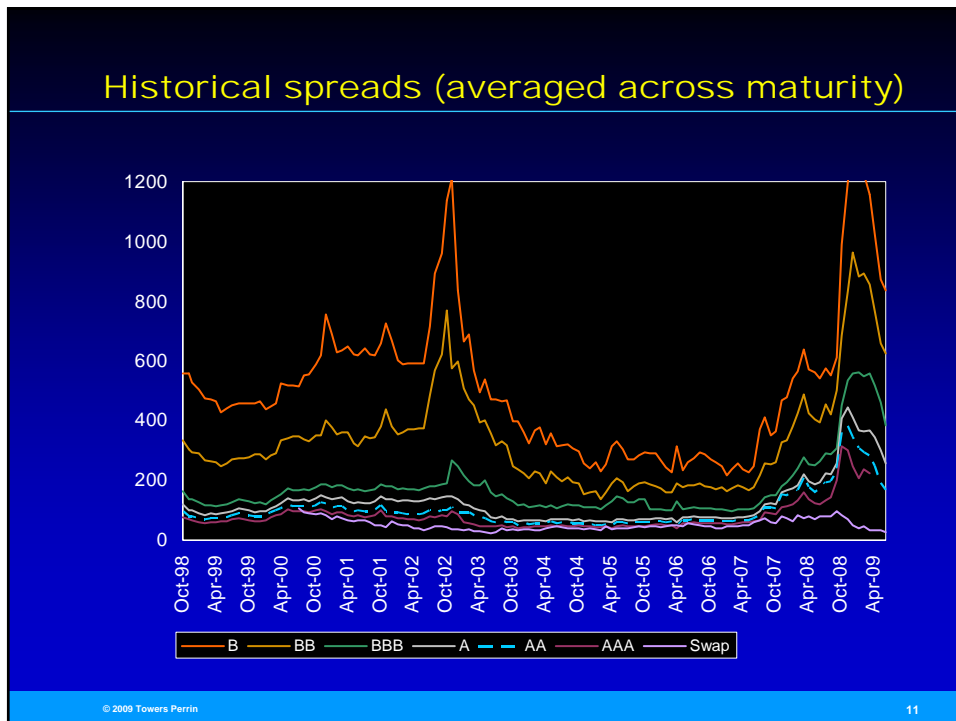
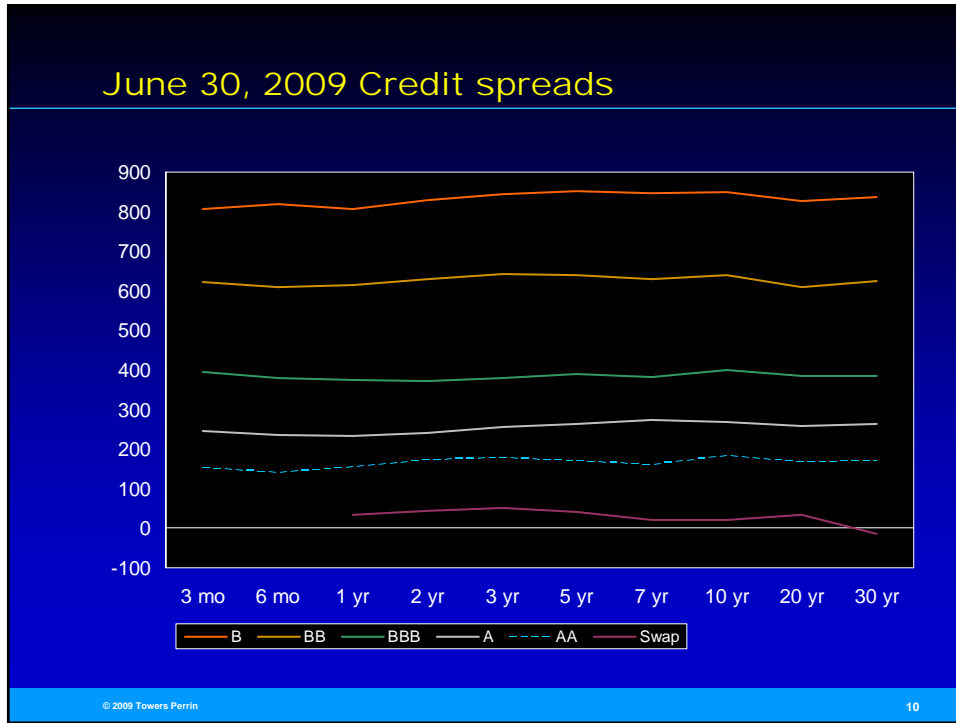
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### June 30, 2009 Corporate yield curves



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## Defaults

- Modeling
  - As a reduction to investment income
  - As a reduction in book value
  - Stochastic
  - As a function of interest rates and credit spreads
- Sources of data for default assumption
  - Rating agencies and Wall Street
  - Company experience and investment advisors

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## Rating agency credit scales

Moody's	S&P / Fitch	NAIC	Quality	
Aaa	AAA	1	Highest Qual.	Investment Grade
Aa1/Aa2/Aa3	AA+/AA/AA-	1	High Quality	
A1/A2/A3	A+/A/A-	1	Strong	
Baa1/Baa2/Baa3	BBB+/BBB/BBB-	2	Medium	
Ba1/Ba2/Ba3	BB+/BB/BB-	3	Speculative	Non-Investment Grade (Junk)
B1/B2/B3	B+/B/B-	4		
Caa/Ca/C	CCC/CC/C	5	Highly Spec.	
C	D	6	In Default	

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### Typical bond models need three types of input

- Future scenarios for Treasury yields and credit spreads
- Certain features of the bond
  - Maturity and perhaps issue date
  - Coupon – rate and mode
  - Values – statutory book, tax book, par and market
  - Call and put provisions
  - Sinking fund schedule
  - Conversion privileges
  - Quality
- Basic assumptions (usually vary by asset class category)
  - Default
  - Investment expense (from AS or investment advisors)
  - Call and put behavior

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### Noncallable bonds

- Cash flows are simple to project:
  - At coupon date cashflow =  $\text{par} \times \text{coupon} / \text{mode}$
  - At maturity date cashflow = par
  - If sinking fund date cashflow = scheduled amount
- Investment income = cashflow + change in investment income due & accrued + accrual of discount – amortization of premium
- Market value = present value of future cash flows
  - Based on asset category yield
  - Yield rate tied to weighted average life

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### Noncallable bonds – sample cash flows

Date	Par – BOY	Coupon	Sinking Fund	Total CF
12/31/2008	\$10,000	\$500	\$2,000	\$2,500
12/31/2009	\$8,000	\$400	\$2,000	\$2,400
12/31/2010	\$6,000	\$300	\$2,000	\$2,300
12/31/2011	\$4,000	\$200	\$2,000	\$2,200
12/30/2012	\$2,000	\$100	\$2,000	\$2,100

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### Callable bonds

- Features
  - Issuer has the option to pay off the bond early
  - Higher coupon than comparable non-callable bond
  - Difference is price of call option sold to issuer
  - Cash flows same as noncallable bonds if not called
- Call behavior is driven by interest rates
  - If  $\text{par} + \text{call premium} + \text{refinancing cost} < \text{PVCF}$  at current rates, there is incentive to call
- Market values are much more difficult to calculate
  - Include the price of the call option
  - No closed form solutions for American calls
  - Multiple scenario / binomial lattice methodology

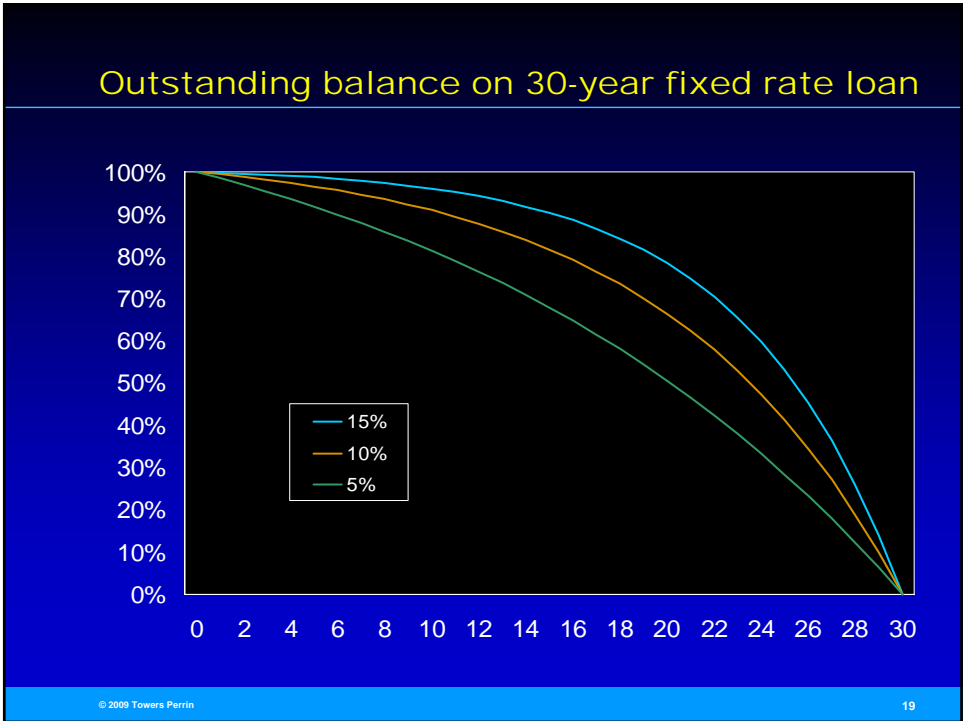
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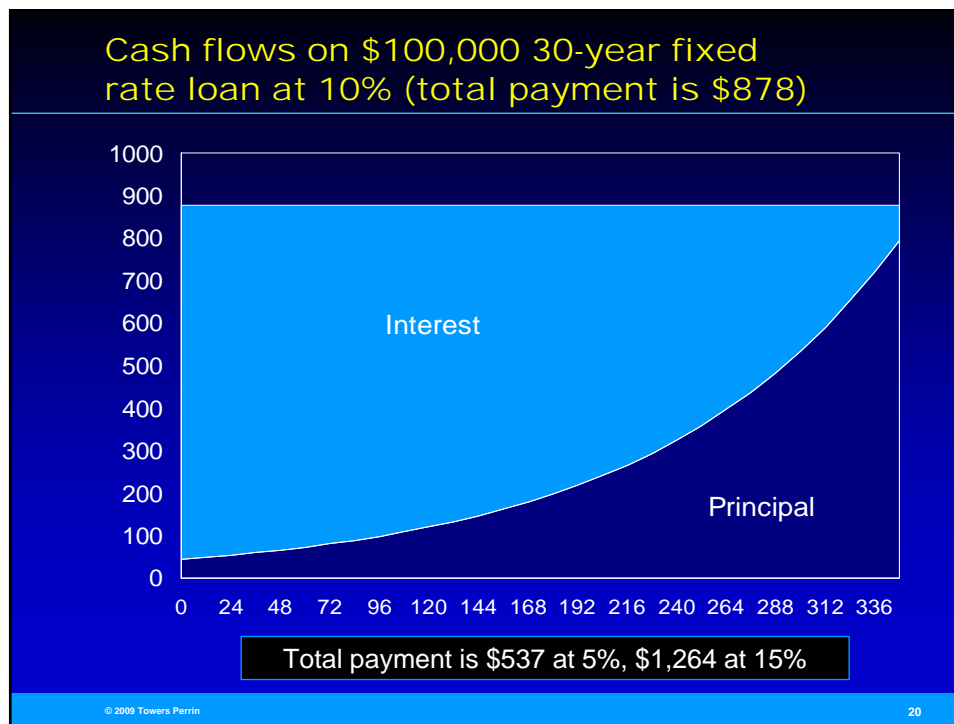
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## Mortgages

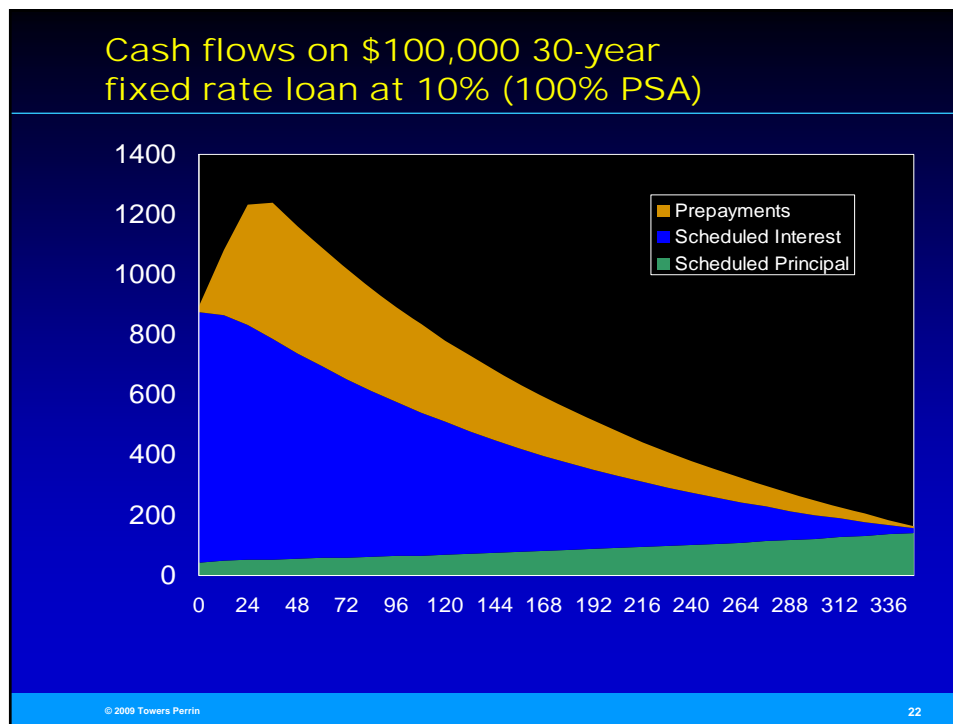
- Property (collateral)
  - Residential (1-4 family homes)
  - Commercial (office, retail, industrial, multifamily, hotels)
- Amortization pattern (e.g., normal, IO, balloon)
- Interest (contract) rate – fixed, adjustable, hybrid (e.g. 5/25)
- Underwriting
  - Conforming – full doc, high credit score, low PTI/LTV/amt
  - Nonconforming
    - Jumbo
    - Alt-A – partial doc, 2nd home, above max PTI or LTV
    - Subprime – weak credit score, no doc

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- Mortgages are more difficult to model than bonds primarily because of their prepayments
- Prepayments
    - Homeowners – no-fee prepay anytime (refis, move)
    - Commercial – prepayment lockouts/penalties exist
  - Factors influencing prepayments
    - Refinancing incentive – contract rate vs. current rate
    - Age of the mortgage (seasoning)
    - Seasonality
    - Burnout
    - Macroeconomic factors?
  - Public Securities Association (PSA) prepayment standard
    - Prepayments are 0.2% in 1st month, increasing linearly to 6% in month 30, level thereafter
  - Extension risk = prepayments slower than expected
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## Mortgage pass-through securities

- An security whose holders share equally in the cash flows generated from the pool of mortgages backing the security.
- Two main types – residential and commercial
- Important characteristics that help determine prepayments
  - Issuer or guarantor – agency or nonagency
  - Gross WAC = Passthrough rate + servicing fee
    - Decreases over time as higher rates prepay
  - Weighted Average Maturity (WAM)
- Modeling
  - Similar to modeling regular mortgages
  - Use gross WAC, not passthrough rate, to model prepays
  - Usually only model defaults on non-agency backed pools, on underlying collateral only

## Collateralize Mortgage Obligations (CMOs)

- A CMO creates many different bond classes (tranches) from the same mortgage pool
  - Payment rules (principal and/or interest) to these tranches are specified in prospectus
- Tranches can vary by:
  - Maturity
  - Quality
  - Prepayment risk
  - Interest rate risk
- Types of tranches:
  - Sequentials, PACs/TACs, support, accrual (or Z) bonds, IOs/POs, floaters/inverse floaters, VADM, equity/residual

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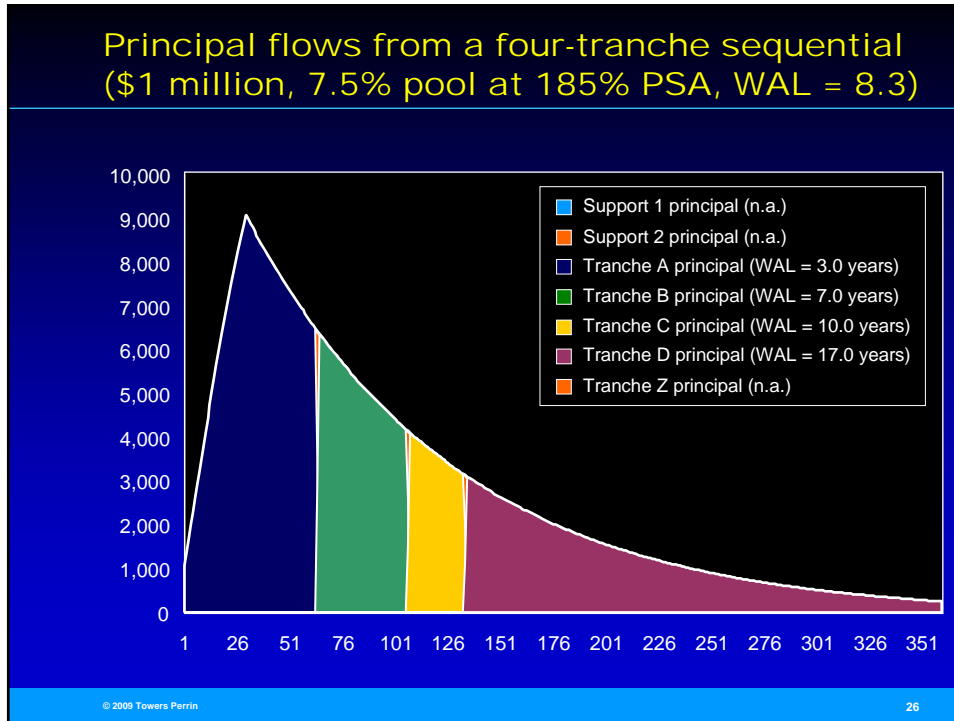
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## Sequential-pay classes

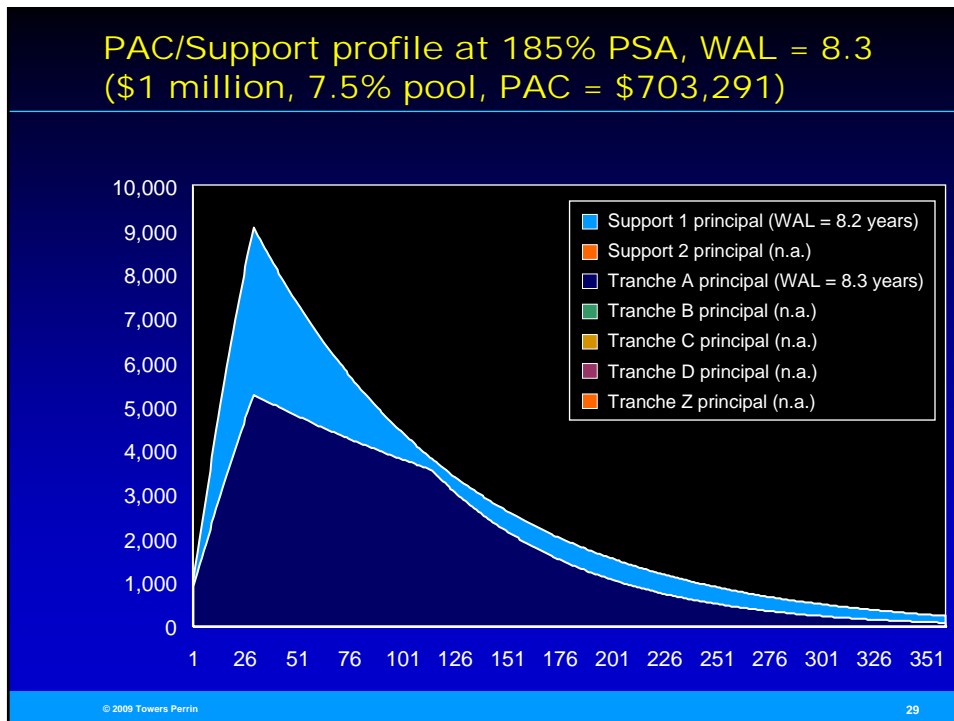
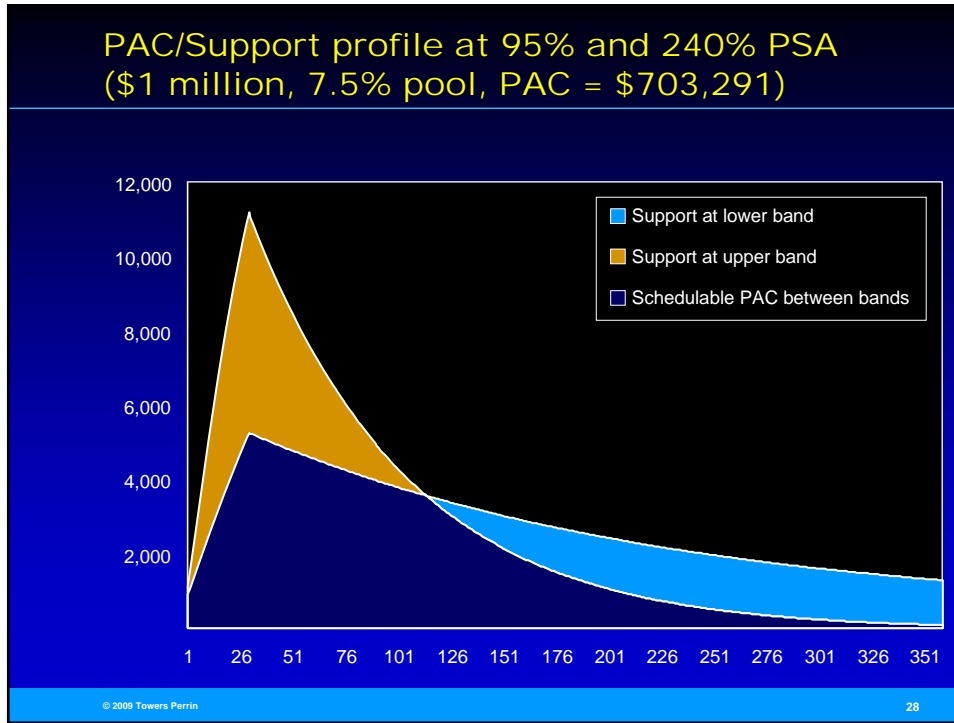
- CMO's initially brought to market to broaden range of maturities available
- Pays principal sequentially from shortest to longest tranche
- Modeling
  - Model cash flows of underlying mortgage pool
  - Allocate principal cash flows to tranches based on CMO structure
  - Projected principal payments equal proportional share of tranche
  - Project coupon payment based on tranche coupon rate applied to outstanding principal balance

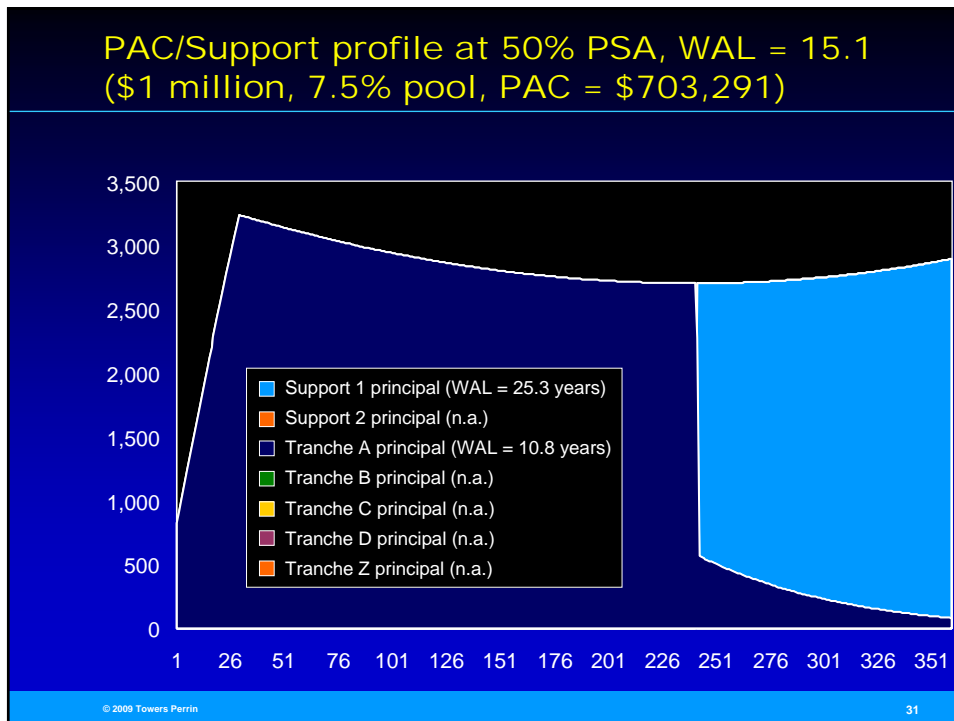
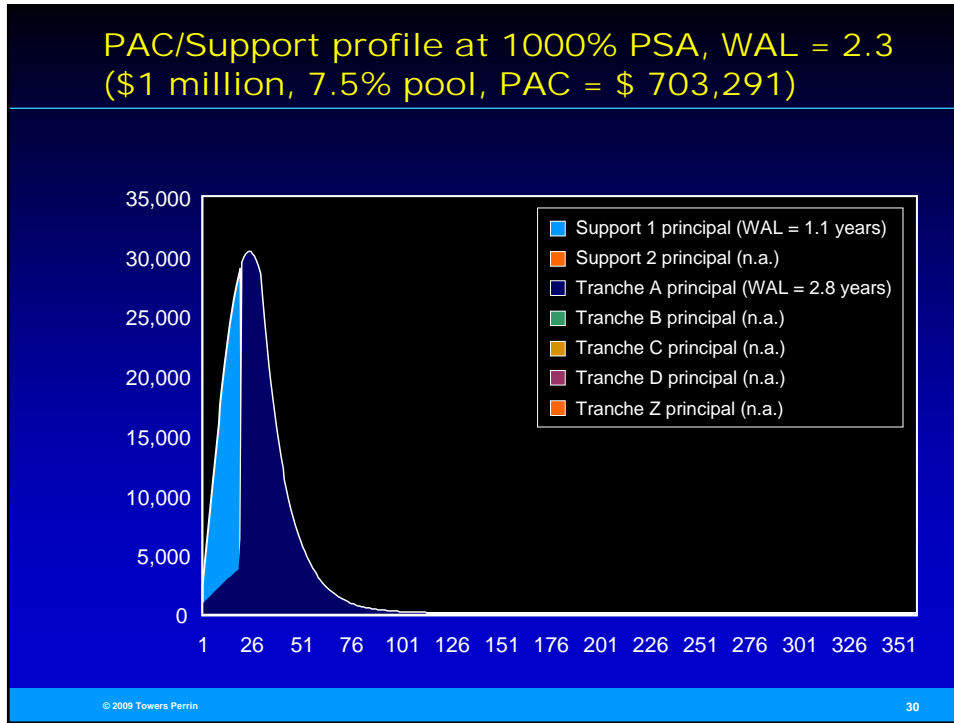
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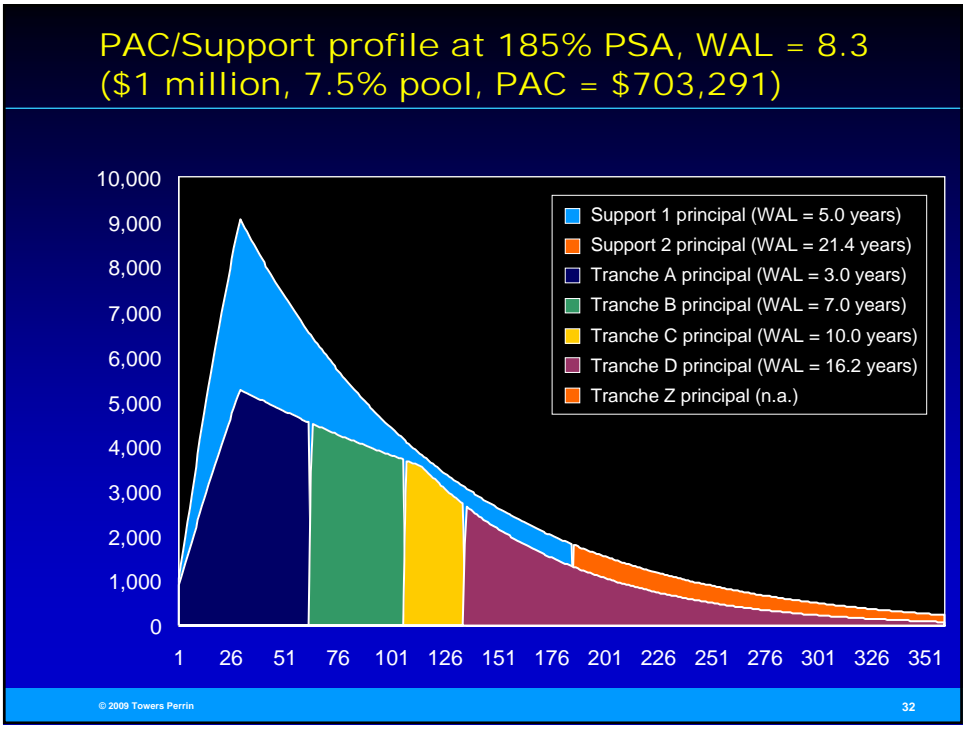
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- ### Redirection of principal payments
- Planned Amortization Class (PAC) bonds
    - Deterministic CFs within a band (e.g., 100 – 300 PSA)
    - Prepayments within the band of protection are redirected to a companion (support) class
  - Variations
    - Divide both sequentially
    - Divide companions into Type II/III PACs & companions
    - Divide PACs into super/subordinate PACs
    - PACs with lower band set at pricing speed are TACs
  - Typical spread to Treasury
    - Companion > Type II/III PAC > Sequential > TAC > Sub-PAC > PAC > Super-PAC
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**WALs of CMO structures on 30-year, 7.5% pool**  
 185% PSA pricing speed, PAC bands of 95-240

PSA	50%	95%	185%	240%	300%	185% (Z)
Pass-through	15.4	12.2	8.3	6.9	5.9	8.3
<b>Sequential-pay</b>						
A	6.6	4.5	3.0	2.6	2.3	2.4
B	16.0	11.3	7.0	5.7	4.8	5.2
C	20.9	15.9	10.0	8.1	6.7	6.8
D (Z)	26.3	23.3	17.1	14.2	11.9	20.7
<b>PAC/Companion</b>						
A	4.3	3.0	3.0	3.0	2.9	2.4
B	10.4	7.0	7.0	7.0	6.1	5.3
C	14.4	10.0	10.0	10.0	8.3	7.2
D (Z)	19.1	16.2	16.2	16.2	13.6	20.2
Support 1	24.6	19.2	5.0	2.8	2.2	5.0
Support 2	29.2	28.2	21.5	6.6	4.2	21.4

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## Modeling PACs

- Cash flow priorities:
  - Model cash flows of underlying mortgage pool:
  - Allocate principal payments to “most protected” followed by “less protected” PAC
  - If principal remaining, allocate to support tranches followed by “less protected” followed by “most protected” PAC
- If a tranche is less protected than the modeled tranche, it is a support tranche
- Need principal payment schedules for modeled and more protected tranches and equally protected tranches scheduled before modeled tranche
- Need balances for support tranches and equally protected tranches scheduled after modeled tranche

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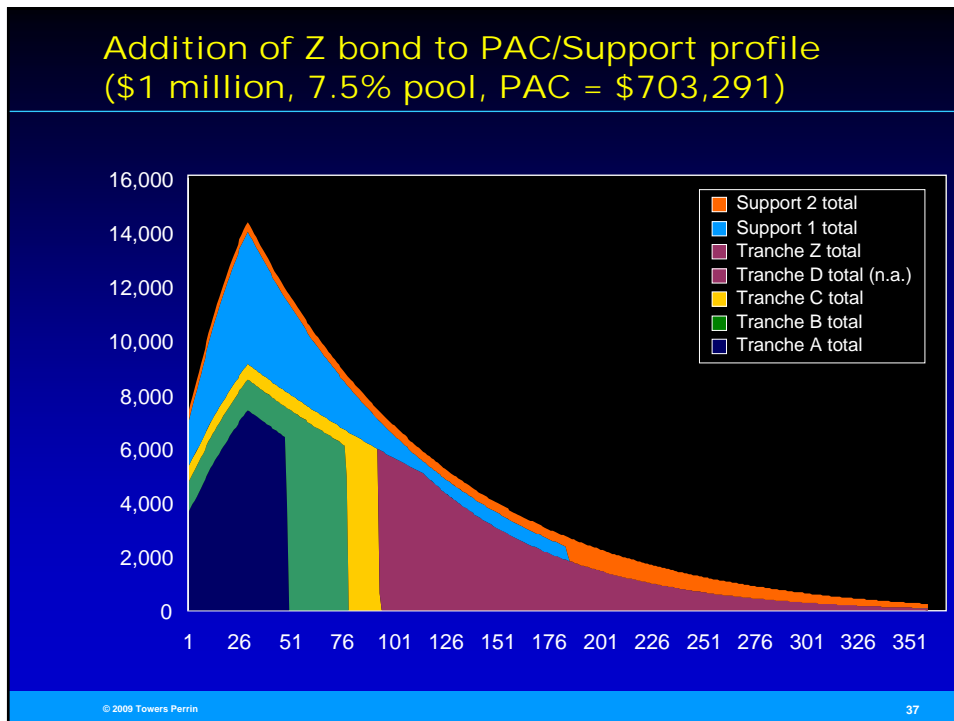
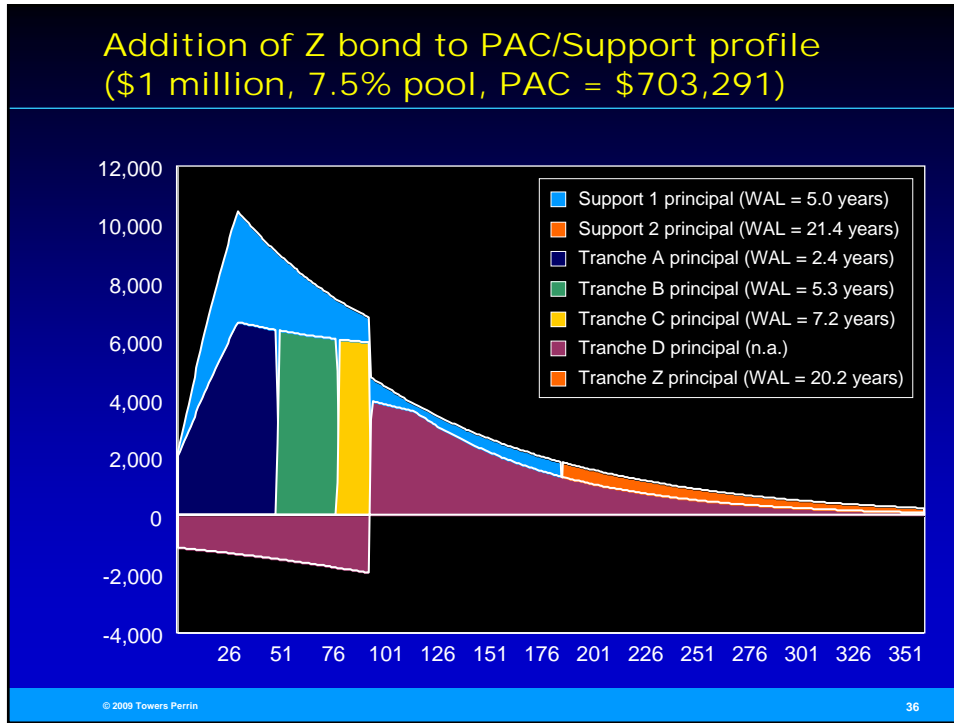
34

## Redirection of interest payments

- Zero coupon (Z) or accrual bond class
  - Current interest on Z tranche is capitalized and used to speed up the pay down of earlier bond classes
  - Must add Z accrual before allocating principal CFs
  - VADM bonds – use Z bonds as support to pay interest and principal
- Floaters and inverse floaters
- Principal only (PO) and interest only (IO) classes
- Modeling:
  - Model the principal (even if notional)
  - Determine coupon rate
  - Apply to Principal

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## MBS market value calculations are difficult because of the prepayment option

- But necessary for sales, reinvestment and/or accounting
- Can assume constant PSA regardless of interest rate
  - Quick, but likely an oversimplification
- Can run a Monte Carlo simulation of interest rate paths and resultant cash flows
  - More robust, but time consuming (SoS issues)
  - Need prepayment assumption
    - Proprietary or vended (e.g. ADCO, BondEdge)
  - To speed things up, run full blown model on a few mortgages to develop a market to book ratios, then interpolate for other mortgages

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## Credit enhancement

- Usually assume no defaults on agency pools (move)
- For non-agency model defaults on underlying collateral (move)
- Nonagency (whole loan) deals usually have some form of credit enhancement
  - Internal
    - Reserve funds (cash or excess spread)
    - Overcollateralization
    - Senior/subordinated structure
  - External (i.e., third party guarantees)
    - Pool insurance (only covers defaults and foreclosures)
    - Wrapper (insured or guaranteed by 3<sup>rd</sup> party)
    - Bank letters of credit

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### A current issue – capital on RMBS

- As of year-end 2008 life & health insurers held \$145B non-agency RMBS with associated C-1 capital of \$2.0B
- During the first half of this year 64% of all AAA rated non-agency RMBS were downgraded to BIG
  - Rating corresponds to likelihood, not severity, of loss
- Associated C-1 capital has climbed 440% to \$11.0B
- ACLI has proposed NAIC make changes to RBC calc that recognize both likelihood and severity of loss
- Alternative capital markets solution is re-REMICs (aka re-CMOs, structured collateral, or kitchen sink bonds)

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### Asset Backed Securities (ABSs)

- Securities collateralized by assets that are not mortgage loans, but using similar structures as MBSs
- Amortization
  - Home equity, manufactured housing and auto loans use a fully amortizing structure
  - Credit card receivables use a controlled amort (soft) / accum (hard bullet) structure with a revolving period
- Tranching – CMOs tranche prepayment risk, whereas ABSs tranche default risk using senior/sub structure
- Prepayments
  - MPR – CC
  - ABS – Auto
  - CPR – HELs, Mfd housing
- Spreads:
  - ▲ HELs
  - Mfd housing
  - Auto
  - CC

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### Collateralized Debt Obligations (CDOs)

- Similar to ABS, but collateral is a *wide* variety of financial instruments; even other CDOs
  - Synthetic CDO – underlying credit exposure is gained by selling CDSs on pools of 100+ companies
- Main appeal to investor is (was) yield
- Complexity risk – rather than reduce risk through diversification, these may spread risk and uncertainty about the value of the underlying assets more widely
  - Many CDOs have had substantial write-downs
  - CDO issuance decreased 90% from 2007 to 2008
- Modeling strategy is the same as for other ABS:
  - Project cash flows of underlying collateral
  - Use CDO structure to parse CFs amongst tranches

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### Derivatives

- Futures/forwards, options, and swaps are primarily used by insurance companies for hedging purposes
  - Underlying is usually equity or interest rate, or credit
- Modeling cash flows is usually straightforward
- Pricing/valuation is more difficult, but necessary for:
  - Modeling future hedging transactions
  - Measuring hedge effectiveness for accounting, etc.

Analytical formula	Numerical tree	Monte Carlo
← Valuation techniques →		
Fast		Flexible

- Criticisms: often highly leveraged, counterparty risk, complexity, systemic risk issues

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## Validation of asset models

- Starting values
  - Book
  - Par
  - Market
    - How to handle model to actual differences?
      - Hold “market spread” level, grade to 0, set to 0
- Portfolio yield (i.e., book yield at time zero)
- Cash flow analysis
  - Principal and interest payments
  - Calls and prepayments
  - Defaults

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## Reinvestment strategies

- Static
- Function of economic scenario
- Function of time period
- Function of asset/liability duration mismatch
- Usually quite simple
- Market values needed for reinvestment
- How much “alpha” to include?
  - Uncertainty premium (credit spread)
  - Liquidity premium
  - True excess spread (probably zero)

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### Negative cashflow disinvestment strategies

- Can simply borrowing or “buy” negative assets (i.e., borrow from another line of business)
- Or can sell assets
  - Requires market value calculations
  - Requires asset group sales priority
    - May want to sell “model purchased” assets first
    - May not want to sell difficult classes to value
- Requires individual asset selection criteria
  - Pro rata
  - Maximize capital gain
  - Longest or shortest duration



# Asset Modeling Concepts

September 24, 2009  
Presented by Kevin Denyer, FIA, FSA  
Actuarial Resources Corporation



## Agenda

- 1 Modeling Errors
- 2 Data and Assumptions
- 3 Methodology
- 4 Validation
- 5 Investment Strategy
- 6 Performance

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## Modeling Errors



### ■ Possible Sources of Errors

- ▶ Bad data
- ▶ Misunderstood data
- ▶ Misunderstood assets
- ▶ Treatment of accrued interest
- ▶ Timing of asset cash flows
- ▶ Approximate or unknown calculation methods
- ▶ Unknown 'black box' methods
- ▶ Assuming last year's model was correct

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## Know Your Data



### ■ Yield Curve

- ▶ Spot rates, par yields
- ▶ Implied coupon frequency

### ■ What date?

- ▶ What month does annual data apply to?
- ▶ What method to use for intermediate values?

### ■ Source Data

- ▶ Is it all factual or are there any built in assumptions?
- ▶ Clean or dirty values?
- ▶ Are the returns consistently annual or monthly for all asset classes
- ▶ Are historic values needed for averaging?

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## Sufficient Data



- **Short Term Yield Curve**
  - ▶ Is annual data sufficient?
  - ▶ Do you need 1 month or 3 month rates or is interpolation adequate
  - ▶ What interpolation method?
  
- **Long Term Yield Curve**
  - ▶ How are values derived beyond the available data
    - Longer terms to maturity
    - Future projection years

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## Assumptions



- **Credit Spreads**
  - ▶ Initial values implied by Market value
  - ▶ How should they vary in future years?
  
- **Default Risk**
  - ▶ What is the risk of future defaults?
  - ▶ How much is recovered when a default occurs?
  
- **Mortgage Prepayment Rates**
  - ▶ What model to use?
  - ▶ Is it suitable for all mortgages?
  
- **Scenario Generator**
  - ▶ Does it produce values for all asset classes?
  - ▶ Can you use proxies instead? What adjustments?

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## Modeling Methodology



- **Seriatim**
  - ▶ Each asset modeled individually
  - ▶ Is all the data readily available?
  - ▶ Takes longer to run
  - ▶ Easier to validate
  
- **Aggregated**
  - ▶ Group similar securities by maturity, quality etc
  - ▶ Faster to run
  - ▶ Same grouping applicable for all purposes?
  - ▶ Extra process to validate

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## Model Validation



- **What to Validate?**
  - ▶ Income and Balance Sheet values
  - ▶ Initial or projected amounts
  - ▶ By portfolio, asset class or individual asset
  
  - ▶ Threshold depends on purpose of model
  - ▶ Document assumptions and limitations
    - Results should only used for intended purpose

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## Validation – Individual Assets



- **Bonds**
  - ▶ Spreads
    - Movement compared to previous period
    - Investigate anything over x%
  - ▶ Effective duration
    - Summarize values by asset
    - Movement compared to previous period
- **Independent Calculation**
  - ▶ 'Back of envelope' or separate spreadsheet
    - Excel useful for valuing options
  - ▶ Broad reasonableness checks
    - e.g. above or below par

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## Validation - Individual Assets



- **Can you explain the results and changes from last period?**
  - ▶ Movement in bond yields v. market values
  - ▶ Movement in Amortized Book Value v. amortization rate
  - ▶ Mortgage prepayment rates v. change in outstanding debt
    - Financial
    - Non-Financial
    - Seasonal?
  - ▶ Volatility for options: implied v. assumed
  - ▶ Which bonds have been called?

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## Validation - Returns



- **Book Yield**

- ▶ Simple formula based on opening and closing values, and net cash-flow
- ▶ Can be applied to all assets or a subset
- ▶ Unlikely to match exactly

- **Cash-Flow Roll forward**

- ▶ Book value consistent with previous value adjusted for
  - Investment income
  - Change in accrued interest
  - Maturity proceeds
  - Purchases and sales

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## Validation - High Level



- **Independent source**

- ▶ Annual statement
- ▶ Investment accounting system

- **Summarize key results v. previous period**

- ▶ Total value
- ▶ Total return
- ▶ Income and gains
- ▶ Distribution by asset type
- ▶ Variance from benchmarks: distribution and returns

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## Validation - Sensitivities



- **Sensitivities**
  - ▶ Run one at a time to understand impacts
    - It is easier to isolate modeling issues, interpret results and learn
  - ▶ Run together to understand correlations
  - ▶ Identify key drivers of results
    - Bond yields: what term?
    - Mortgage Prepayment rates
    - Option volatility
  - ▶ Make sure you can explain all the changes in results

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## Validation – Stochastic Model



- **Stochastic Generator**
  - ▶ Verify the means and standard deviations before running
  - ▶ Check it is calibrated to the initial conditions
- **Projection Model**
  - ▶ Check output of individual simulations
    - Not the simple ones!

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## Investment Strategy



- **What Strategy to use?**
  - ▶ Consider purpose of the projection
  - ▶ Covers both purchase and sale strategies
  - ▶ Any realignment of assets?
  - ▶ Constant v. dynamic
  - ▶ Scenario dependent?
  - ▶ What can your system do?

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## Investment Strategy



- **Selling Strategies**
  - ▶ Sell in all in equal proportion
  - ▶ Prioritize asset classes in order
  - ▶ Sell longest/shortest duration first
    - Or some other characteristic: term to maturity, unrealized gains
  - ▶ Match duration of liabilities
  - ▶ Cash balance/borrowing
  - ▶ Negative assets
  - ▶ Exclude assets held to maturity
  
  - ▶ Or any combination of the above!

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## Investment Strategy



- **Buying Strategies**
  - ▶ Invest in target distribution
  - ▶ Deviate from current proportions
  - ▶ Profile of new assets
  - ▶ Purchase to match liability duration
  - ▶ Exclude some asset classes
  - ▶ Prioritize order, with threshold limits
  - ▶ Cash balance/minimum purchase

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## Performance



- **Runtimes**
  - ▶ Generally far more individual liabilities
  - ▶ So assets can be forgotten
  - ▶ But they still take time to run
    - Particularly with complex hedging strategies
- **Timetable**
  - ▶ Asset data from different system to liabilities
  - ▶ When will information be available?

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???

**Any Questions?**

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