SOCIETY OF ACTUARIES

ERM Symposium
April 2009

R3 - Research Paper Session: Impact of Correlation Crises in Risk Theory

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Moderator/Reviewer

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Impact of Correlation Crisis in Risk Theory
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Discussion by
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The Stochastic Process

$$R(t) = u + c \cdot t - S(t)$$

$R(t) =$ Net worth (Reserve) at time t

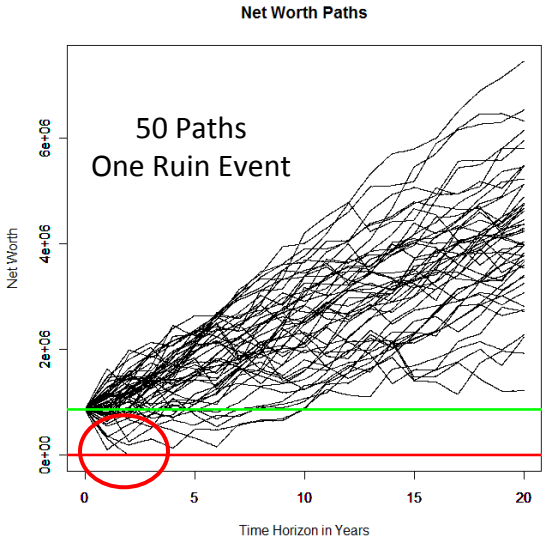
$u =$ Initial capital - $R(0)$

$c =$ Premium income rate

$S(t) = \sum_{i=1}^{N(t)} X_i =$ Total loss during period t

The Probability of Ruin

$$\Psi(u,t) = \Pr\{R(t) < 0 \text{ at any time in } (0,t)\}$$



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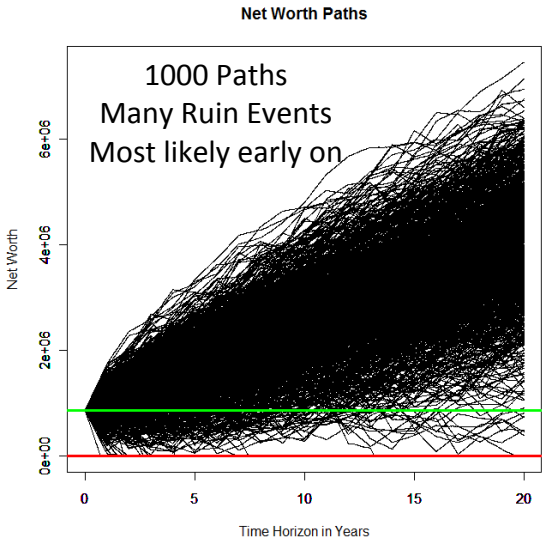


Time	Ruin Events
1	1
2	8
3	12
4	12
5	13
6	13
7	14
8	15
9	15
10	15
11	15
12	15
13	15
14	16
15	16
16	16
17	16
18	16
19	16
20	17

Evolution of Ruin Theory

- Infinite time ruin theory – $\psi(u,\infty)$
- Finite time ruin theory – $\psi(u,t)$
- Multi-line ruin theory
  - Correlation between lines
- Analytic solutions and simulation solutions

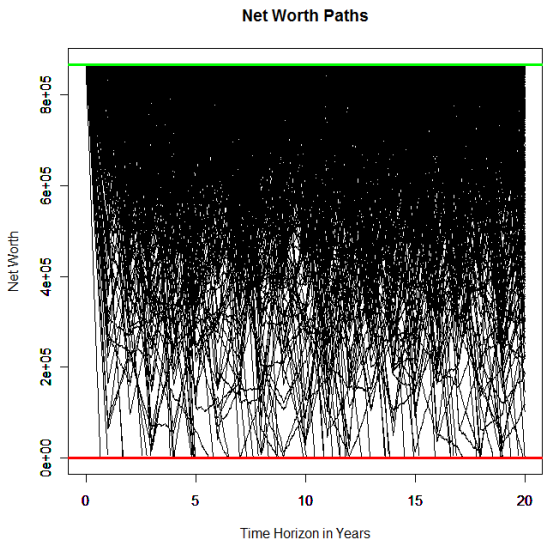
Example of Simulation Solution

- Three lines of insurance
- Pareto claim severity distributions
- Correlated Poisson claim count distributions
  - Grand mean, (λ_1, λ_2, λ_3) based on exposure
  - Randomly select percentiles (p_1, p_2, p_3) from a t-copula with off-diagonal coefficient of correlation, ρ . 1 per year.
  - Multiply each λ_i by p_i^{th} percentile of a gamma distribution
- Simulate count, severity and add up the losses
  - 100 periods per year, 20 years, 1000 loss paths

Control Parameters

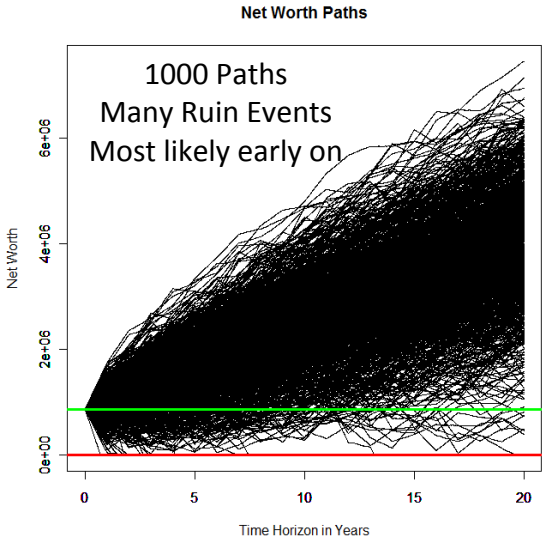
- Profit loading in premium – 10%
  - Premium = E[Count]·E[Severity]·(1+Profit)
- Initial capital = One half of Premium
- Return money in excess of initial capital to investors

Effect of Returning Profit to Investors

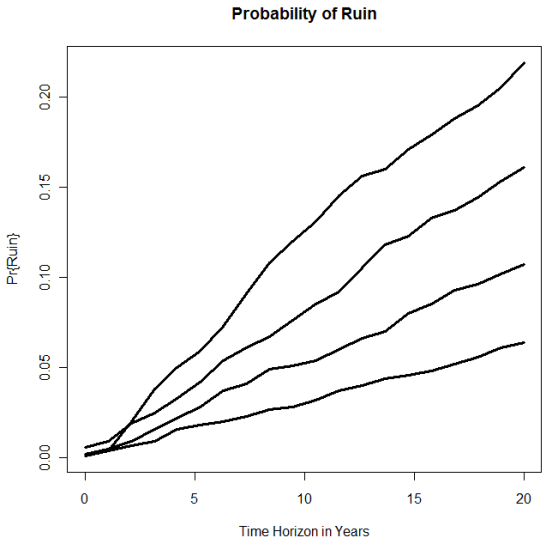


The Probability of Ruin

$$\Psi(u,t) = \Pr\{R(t) < 0 \text{ at any time in } (0,t)\}$$



Effect of Changing ρ in t-Copula
Big Insurer



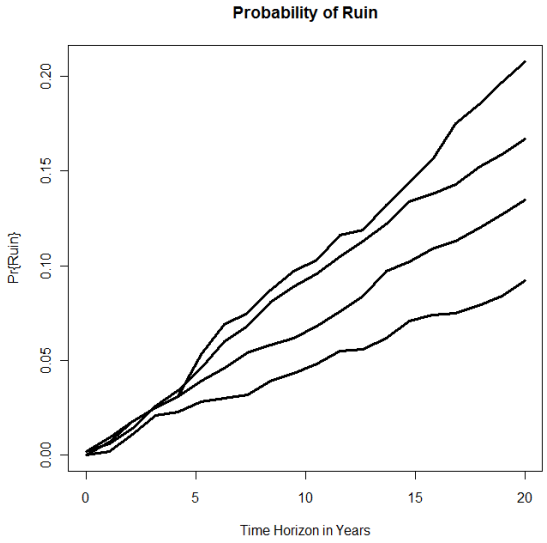
ρ = 0.9

ρ = 0.7

ρ = 0.5

ρ = 0.3

Effect of Changing ρ in t-Copula Small Insurer



ρ = 0.9

ρ = 0.7

ρ = 0.5

ρ = 0.3

Explaining the Effect by Size of Insurer

Parameter Risk

Important for Large Insurers

- Correlated Poisson claim count distributions
  - Grand mean, (λ_1, λ_2, λ_3) based on exposure
  - Randomly select percentiles (p_1, p_2, p_3) from a t-copula with off-diagonal coefficient of correlation, ρ . 1 per year.
  - Multiply each λ_i by p_i^{th} percentile of a gamma distribution

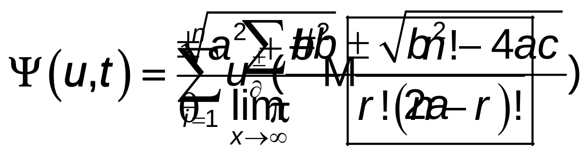
Process Risk

More Important for Small Insurers

- Simulate count, severity and add up the losses
  - 100 periods per year, 20 years, 1000 loss paths

Two Cultures in Ruin Theory
Analytic and Simulation

- Analytic view of Simulation Culture
  - A series of one damn result after another
  - Does not reveal insight into process
- Simulation view of Analytic Culture



- Each culture can make contributions

Analytic ⟷ Simulation
Meyers

- Problems come in faster than analytic solutions
  - Multiple risks (including asset risk) and correlations
    - Some progress
  - Uncertainty over time – Loss Reserves
  - Changing economic conditions
  - Parameterization of models
  - Risk Measures - TVaR
  - Accounting conventions
- Can capital requirements be set with a handful of stress tests?