

Appendix B: Presentations on This Research

New Zealand University

(Presentation given; details to be added.)

CAS Basic DFA Limited Attendance Seminar, Washington, DC, July 2003

A brief reference to the project, along with a description of research highlights, was provided.

Actuarial Research Conference, Ann Arbor, MI, August 2003

The PowerPoint presentation is included in this report as Appendix B-1. An abstract of presentation that was made follows:

The Casualty Actuarial Society and the Society of Actuaries are co-sponsoring research involving the modeling of economic series. This research is important for generating reasonable future economic and financial scenarios, and has critical implications for dynamic financial analysis and cash flow testing. An essential aspect of this research involves determining the interdependencies between the various economic and financial series, especially, but not exclusively, the interrelationships between the different series and interest rates. This session will present a summary of the results and findings of this research, and will provide a description of how the results can be utilized by academics and actuarial practitioners.

American Risk & Insurance Association Annual Meeting, Denver, CO, August 2003

The following session proposal was accepted for presentation at this meeting:

Financial Scenario Generator Project

Introduction

In April, 2001, the Casualty Actuarial Society (CAS) and the Society of Actuaries (SoA) solicited proposals for research involving the modeling of economic series, including inflation, interest rates, equity returns, dividend yields, real estate returns and unemployment rates. We propose to present the results of our work on this project. This research is important for generating reasonable future economic and financial scenarios, and has critical implications for dynamic financial analysis and cash flow testing. One essential aspect of this research involves the interdependencies between the various economic and financial series, especially the interrelationships between the different economic series and interest rates. We will summarize the key work in this area, present the financial scenario generator model, explain the process for determining the model parameters and describe the results of running the model with the selected parameters.

Literature Review

The key prior work in the area of actuarial modeling of financial variables was performed by Wilkie (1986, 1995) and Hibbert, Mowbray and Trunbull (2001). This project builds on their work, and develops a financial scenario generator model that actuaries and researchers can use in dynamic financial analysis, cash flow testing and regulatory applications. The works of Hull-White (1990, 1994), Vasicek (1977) and Hardy (2001) are incorporated into the model.

Financial Scenario Generator

In this model, inflation and real interest rates are each modeled separately, and then combined to determine the nominal interest rate. Equity returns, real estate returns and the unemployment rate are also modeled separately, but are dependent on the nominal interest rate levels determined previously. Inflation is modeled as a one-factor mean reverting process. Real interest rates are modeled as a two factor mean reverting process. Equity returns are modeled by a regime switching process, with two different lognormal distributions used to generate returns. Real estate returns are modeled based on a one factor mean reverting process. Unemployment is modeled as a one factor mean reverting process, with an additional factor representing an inverse relationship to nominal interest rates.

Results

Regardless of the mathematical sophistication of the variables incorporated in a model, the accuracy of the calibration process used to determine the parameters, and the timeliness of the values on which the calibration is based, the most important test of the validity of any model is the reasonability of the results. This section will examine the results of a representative run of the model and compare these results with historical values for interest rates, inflation, stock and real estate returns, dividend yields and unemployment rates.

The model was run using the base parameters to generate 5000 iterations. The results are presented in a variety of different ways to allow individuals wanting to apply the model to understand the usefulness of this approach. Summary statistics, correlation matrices, funnel of doubt graphs, comparison graphs showing model and historical values are all presented to show the reasonability and accuracy the model can provide.

Conclusion

Insurance companies and other financial institutions require models that can generate realistic values for key variables that affect operating results, assets and liabilities. The values must be able to be projected over a long time horizon, be internally consistent and be fully understood by the individuals responsible for applying them within organizations. This research represents a step in this process by providing a model of financial variables that are relevant for insurance companies. This model can be calibrated to any historical data set of inflation, interest rates, equity returns, real estate returns or unemployment patterns, representing different countries or different time horizons. Additional variables can be added to the model as needed. The goal is to foster more accurate financial planning for insurers.

CAS Asset-Liability Management / Finance Limited Attendance Seminar, Chicago, IL, September 2003

A brief reference to the project, along with a description of research highlights, was provided.

Western Risk and Insurance Association, January 2004

The PowerPoint presentation is included in this report as Appendix B-2.

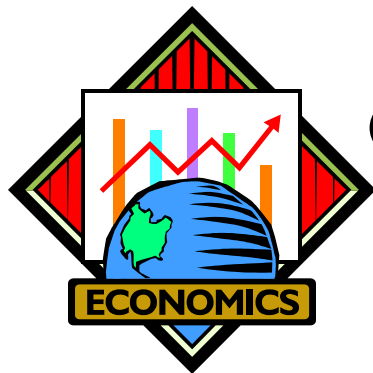
Enterprise Risk Management Symposium, Chicago, IL, April 2004

The PowerPoint presentation is included in this report as Appendix B-3.

Actuarial Research Clearing House 2004

The paper is included in this report as Appendix B-4.

Modeling of Economic Series Coordinated with Interest Rate Scenarios



Research Sponsored by the
Casualty Actuarial Society and the
Society of Actuaries



Investigators:

Kevin Ahlgrim, ASA, PhD, Illinois State University

Steve D'Arcy, FCAS, PhD, University of Illinois

Rick Gorvett, FCAS, ARM, FRM, PhD, University of Illinois

Actuarial Research Conference

August, 2003

Acknowledgements

The investigators wish to thank the Casualty Actuarial Society and the Society of Actuaries for providing financial support for this research, as well as guidance and feedback on the subject matter.

Note: All of the following slides associated with this research project reflect tentative findings and results; these results are currently being reviewed by committees of the CAS and SoA.

Overview of Project

- CAS/SOA Request for Proposals on “**Modeling of Economic Series Coordinated with Interest Rate Scenarios**”
 - A key aspect of *dynamic financial analysis*
 - Also important for regulatory, rating agency, and internal management tests – e.g., *cash flow testing*
- Goal: to provide actuaries with a model for *projecting economic and financial indices, with realistic interdependencies* among the variables.
 - Provides a *foundation* for future efforts

Scope of Project

- *Literature review*
 - From finance, economics, and actuarial science
- *Financial scenario model*
 - Generate scenarios over a 50-year time horizon
- *Document and facilitate use of model*
 - Report includes sections on data & approach, results of simulations, user's guide
 - To be posted on CAS & SOA websites
 - Writing of papers for journal publication

Current Report Structure

Text Sections

- 1) Intro & Overview
- 2) Excerpts from RFP
- 3) Selected Proposal
- 4) Literature Review
- 5) Data & Approach
- 6) Issue Discussion
- 7) Results of Simulations
- 8) Conclusion

Appendices

- A) User's Guide to the Model
- B) Presentations of this Research
- C) Simulated Financial Scenario Data
- D) Financial Scenario Model

Prior Work

- Wilkie, 1986, “A Stochastic Investment Model for Actuarial Use,” *Transactions of the Faculty of Actuaries*
- Wilkie, 1995, “More on a Stochastic Model for Actuarial Use,” *British Actuarial Journal*
- Hibbert, Mowbray, and Turnbull, 2001, “A Stochastic Asset Model & Calibration for Long-Term Financial Planning Purposes,” Technical Report, Barrie & Hibbert Limited

Economic Series Modeled

- Inflation
- Real interest rates
- Nominal interest rates
- Equity returns
 - Large stocks
 - Small stocks
- Equity dividend yields
- Real estate returns
- Unemployment

Inflation (q)

- Modeled as an Ornstein-Uhlenbeck process
 - One-factor, mean-reverting

$$dq_t = \kappa_q (\mu_q - q_t) dt + \sigma_q dB_q$$

- In discrete format, an autoregressive process
- Parametrization
 - Annual regressions on AR process
 - Two time periods: (i) since 1913; (ii) since 1946
 - Base case
 - Speed of reversion: $\kappa_q = 0.40$
 - Mean reversion level: $\mu_q = 4.8\%$
 - Volatility: $\sigma_q = 0.04$

Real Interest Rates (r)

- Two-factor Vasicek term structure model
- Short-term rate (r) and long-term mean (l) are both stochastic variables

$$dr_t = \kappa_r (l_t - r_t) dt + \sigma_r dB_r$$

$$dl_t = \kappa_l (\mu_l - r_t) dt + \sigma_l dB_l$$

Nominal Interest Rates (i)

- Combines inflation and real interest rates

$$i = \{(1+q) \times (1+r)\} - 1$$

where i = nominal interest rate

q = inflation

r = real interest rate

Equity Returns (s)

- Model equity returns as an excess return (x_t) over the nominal interest rate

$$s_t = q_t + r_t + x_t$$

- Empirical “fat tails” issue regarding equity returns distribution
- Thus, modeled using a “regime switching model”
 - Low volatility regime
 - High volatility regime

Equities: Excess Monthly Return Parameters

	<i>Large Stocks (1871-2002)</i>		<i>Small Stocks (1926-1999)</i>	
	<i>Low Volatility Regime</i>	<i>High Volatility Regime</i>	<i>Low Volatility Regime</i>	<i>High Volatility Regime</i>
<i>Mean</i>	0.8%	-1.1%	1.0%	0.3%
<i>Variance</i>	3.9%	11.3%	5.2%	16.6%
<i>Probability of Switching</i>	1.1%	5.9%	2.4%	10.0%

Other Series

- *Equity dividend yields* (y)
 - Modeled such that the log(div. yield) follows an AR process

$$d(\ln y_t) = \kappa_y (\mu_y - \ln y_t) dt + \sigma_y dB_{yt}$$

- *Real estate (property)*
 - Comm. RE transaction data (market appraisals)
 - O-U processes (w/ and w/o inflation)
- *Unemployment* (u)
 - Phillip's curve: inverse relationship between u and q
 - Modeled as an AR(1) process

$$du_t = \kappa_u (\mu_u - u_t) dt + \alpha_u dq_t + \sigma_u \varepsilon_{ut}$$

Some Issues Discussed

- Equilibrium vs. arbitrage-free models of the term structure
- Adequacy of a two-factor model
- Building a model using *@Risk* software
 - We do provide sample output data
- Capability of inputting specific scenarios

Results of Model Simulations

- Tables
 - Key simulated statistics
 - Correlations – actual versus simulated
- Funnel of doubt (“summary graph”) plots
 - Identifies mean and four percentiles (1, 25, 75, 99)
- Histograms of actual values vs. modeled values

Excerpts from Table 1

Iterations: 5000
Parameters: Base Case

	<i>Mean</i>		<i>Range at Year 10</i>	
	<u>First</u>	<u>Last</u>	<u>1%</u>	<u>99%</u>
Nominal 1-year interest rate	0.044	0.068	-0.044	0.171
Large Stocks	0.114	0.121	-0.146	0.315
Small Stocks	0.163	0.141	-0.146	0.417

Figure 1
1 Month Real Interest Rate (B4 to B30)

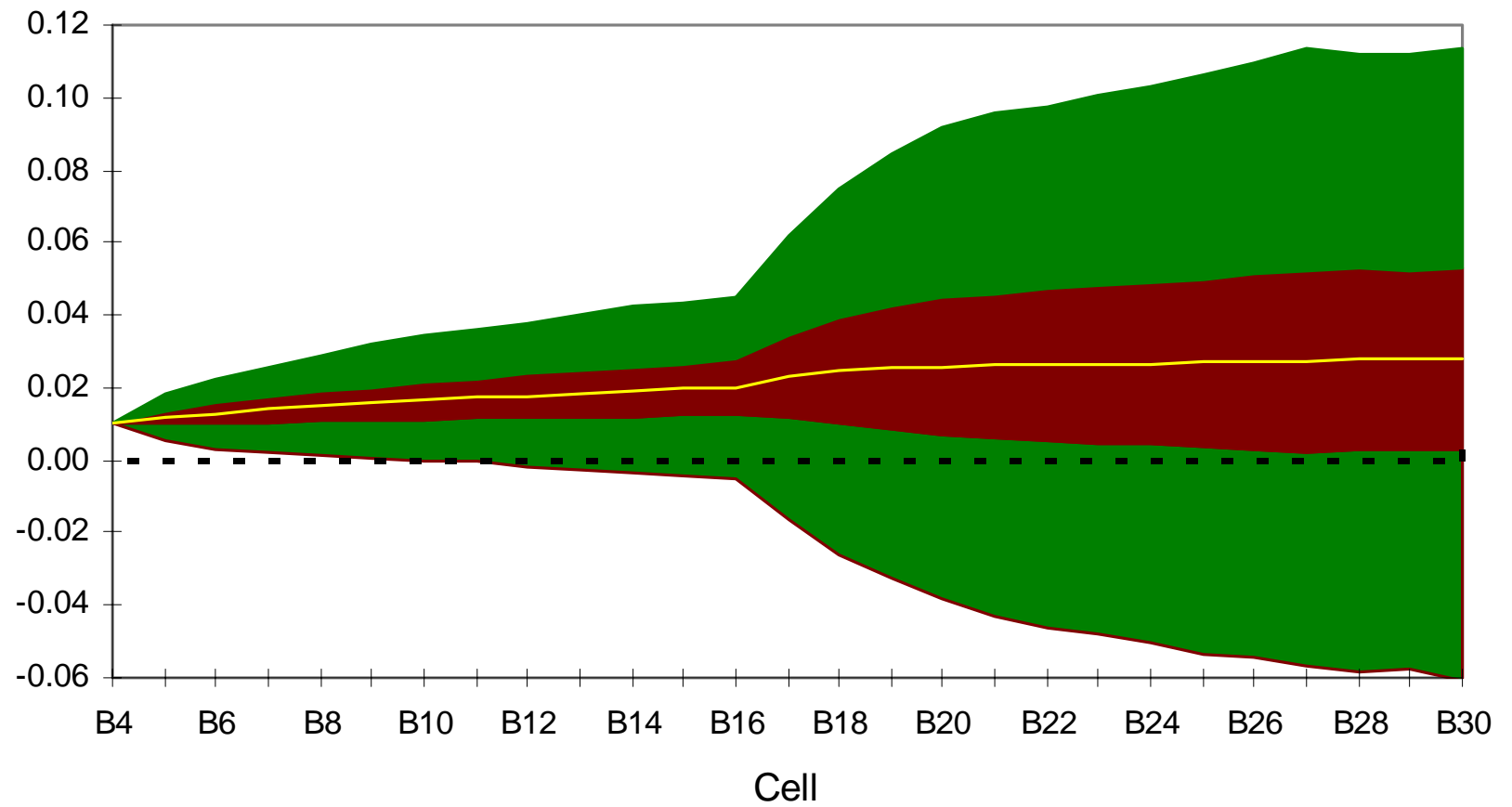


Figure 9
1 Year Nominal Spot Rates (T4 to T30)

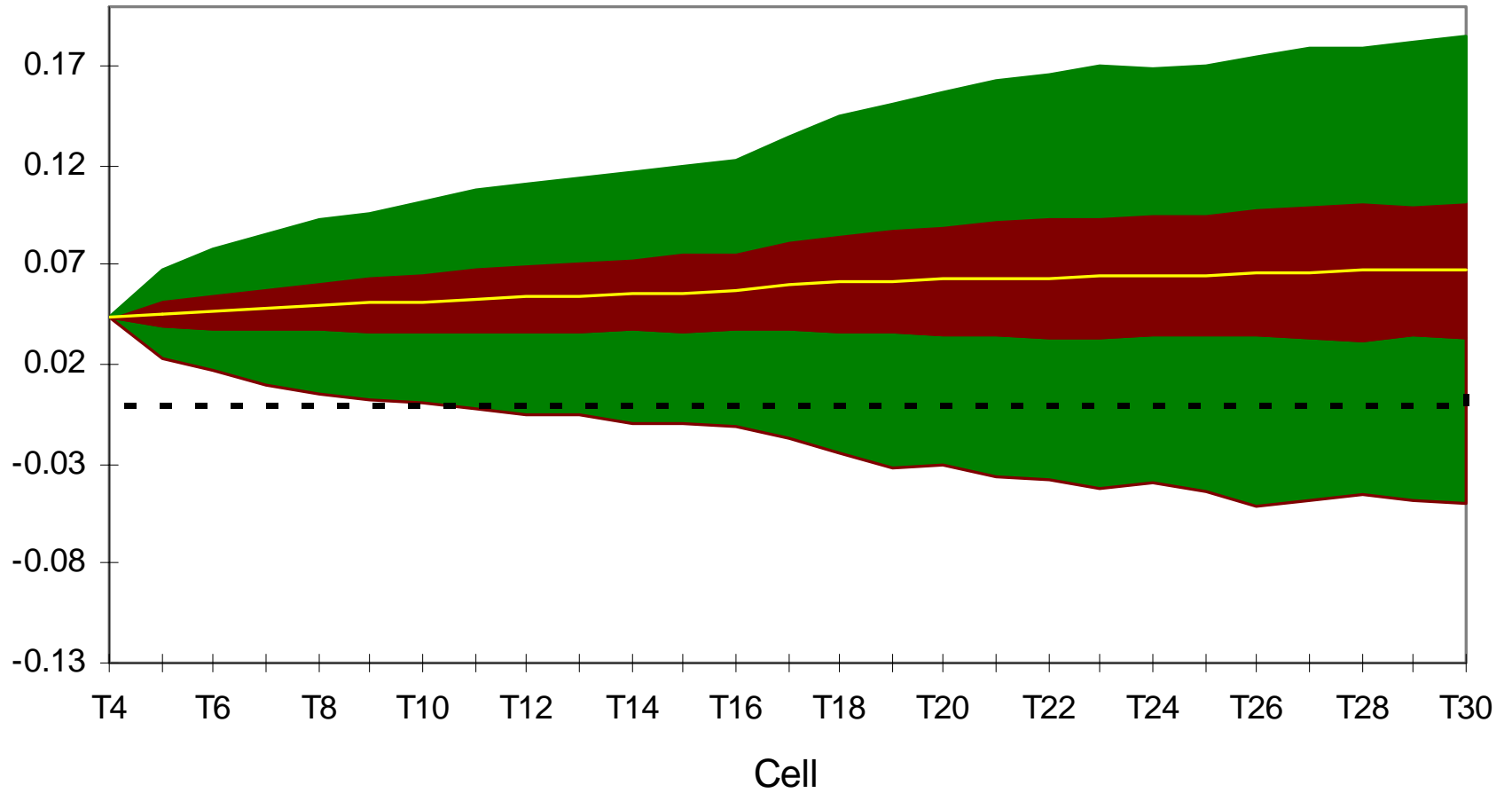


Figure 14
Large Stock Returns (B16 to B30)

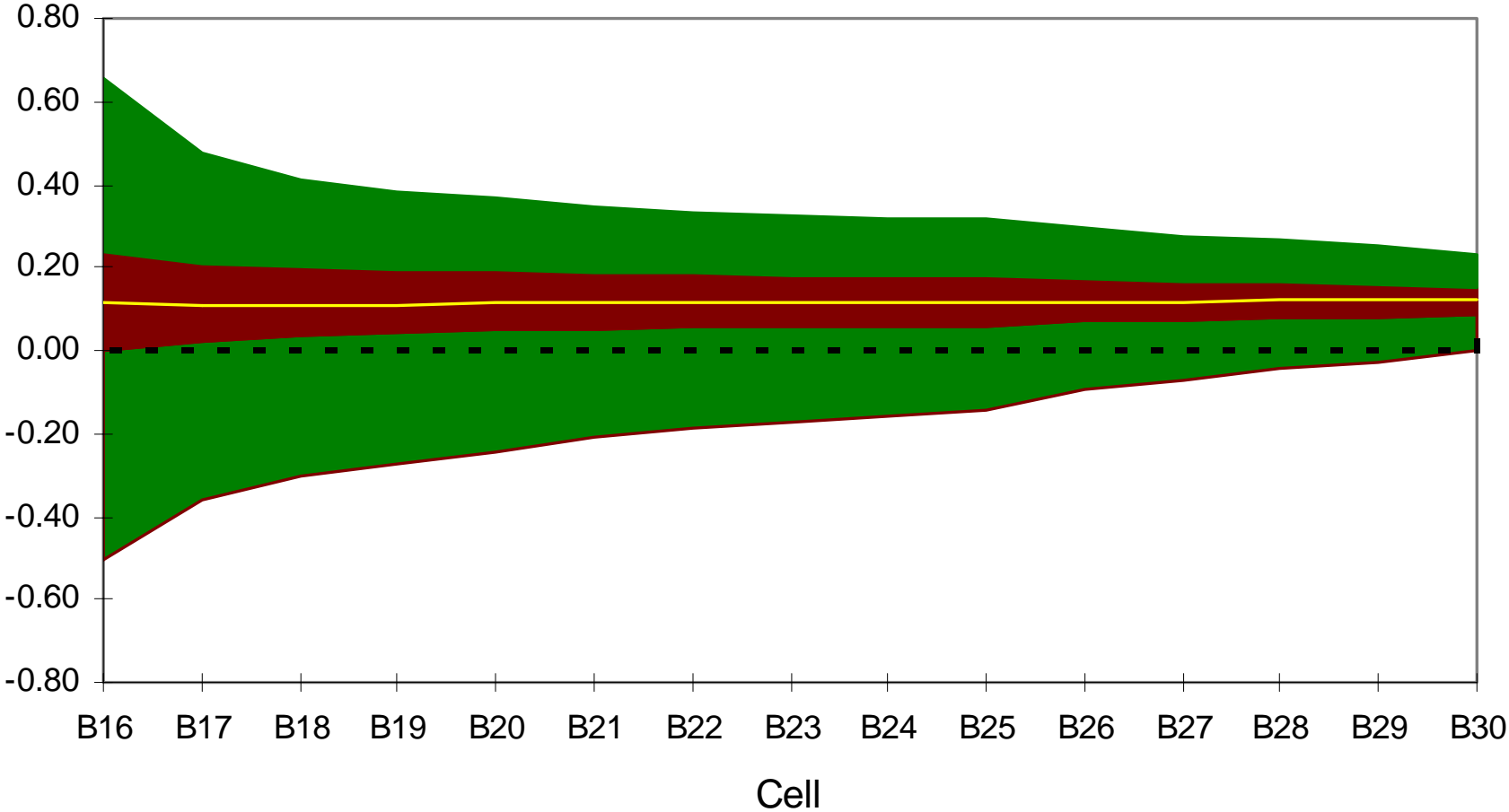


Figure 12
Actual 1 Year Interest Rates (4/53-4/03)
versus Model 1 Year Interest Rates

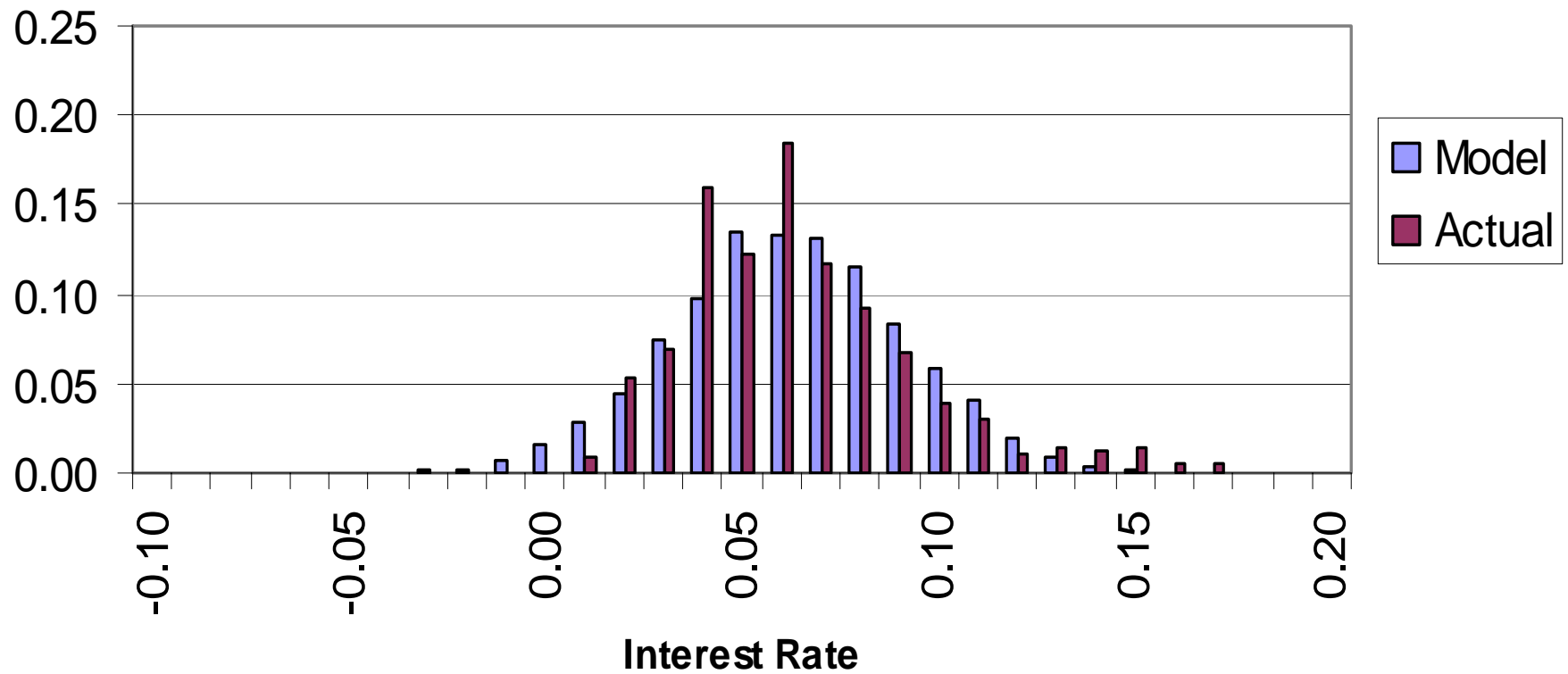


Figure 16
Actual S&P 500 (1871-2002)
versus Model Large Stock Returns

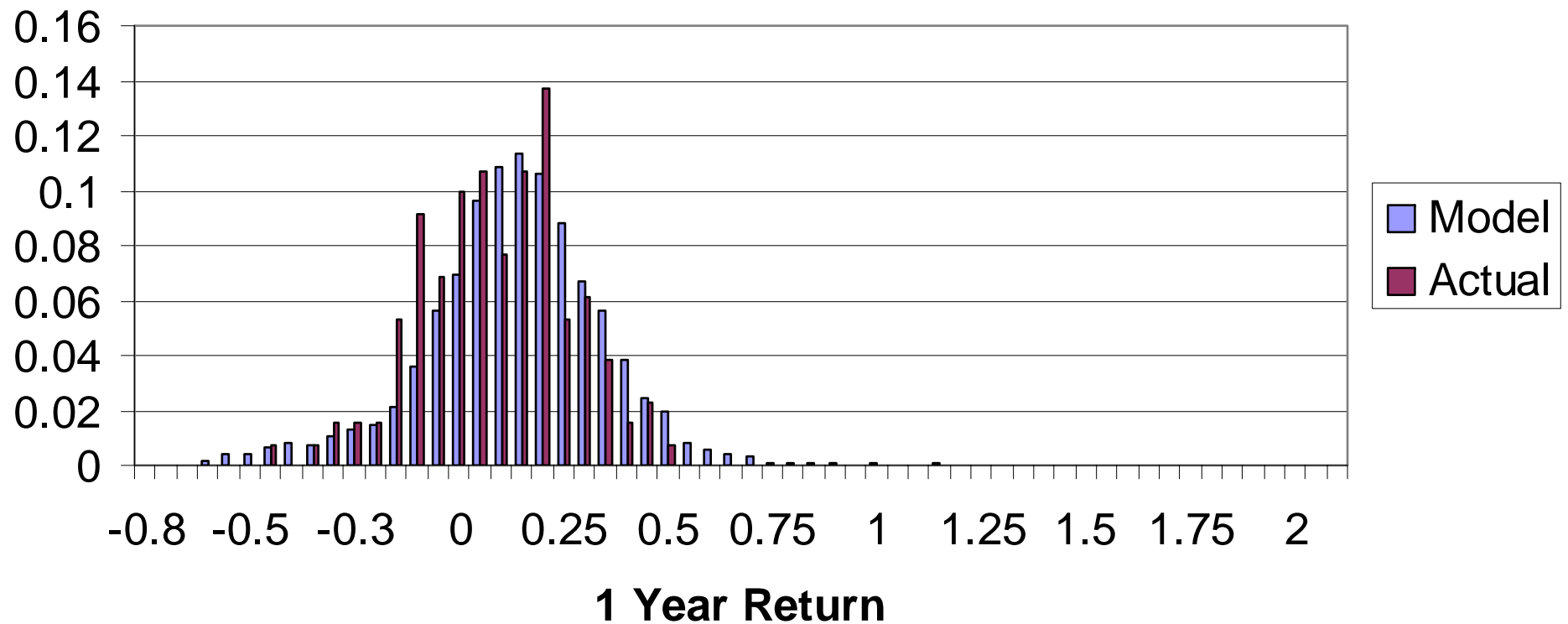
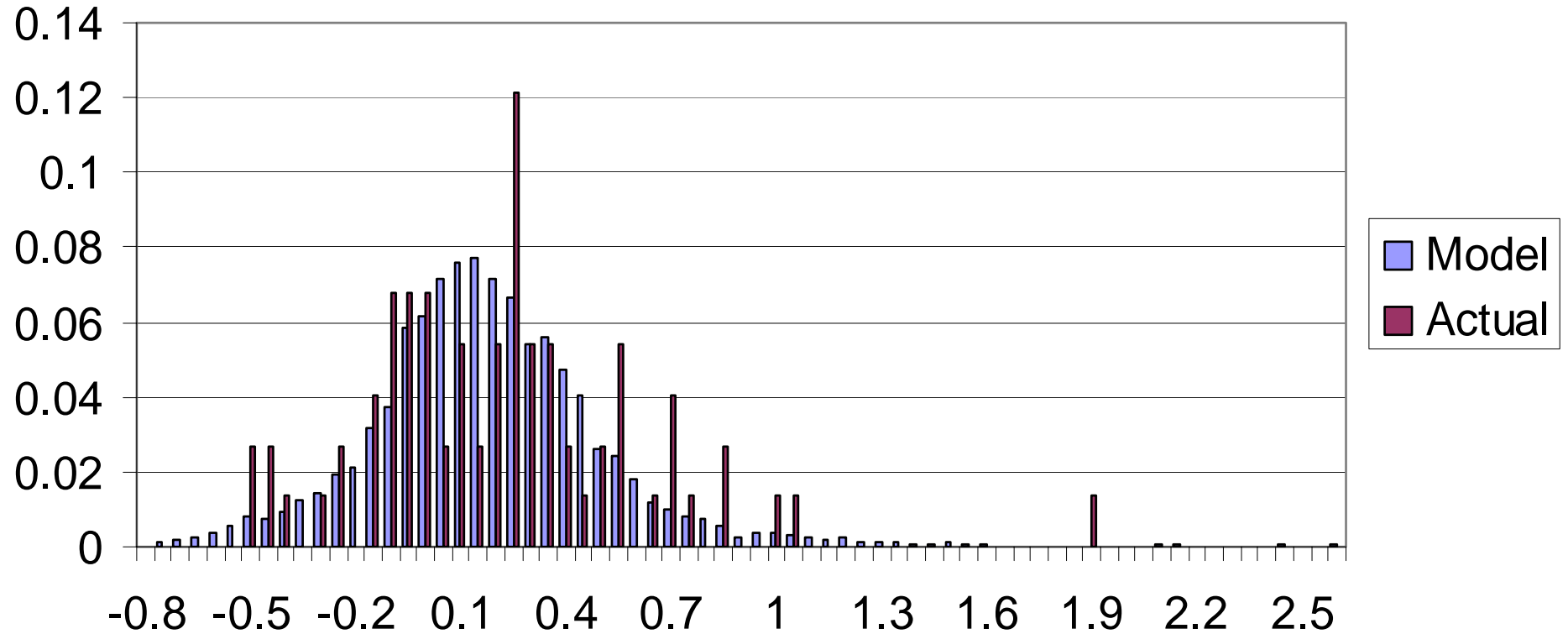


Figure 17
Actual Small Stock Returns (1926-1999) versus
Model Small Stock Returns



Uses of this Financial Scenario Generator

- Insurers
- Regulators
- Pension funds
- Financial planning
- Investment analysis / capital budgeting
- Newly proposed financial risk management solutions

How to Obtain the Model and Associated Documentation

Coming soon to the following sites:

- CAS website
 - <http://www.casact.org>
 - Probably <http://casact.org/research/research.htm>
- SoA website
 - <http://www.soa.org>
 - Probably <http://www.soa.org/research/index.asp>

Gracias

Merci

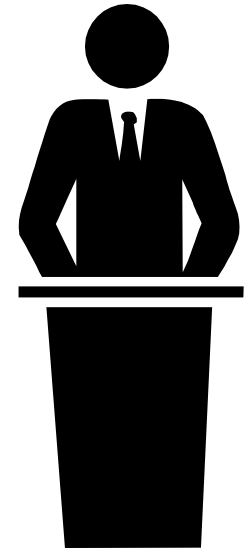
**THANK
YOU**

Vielen
Dank

Grazie

Modeling of Economic Series Coordinated with Interest Rate Scenarios

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Society of Actuaries**



Investigators:

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Rick Gorvett, FCAS, ARM, FRM, PhD, University of Illinois

Western Risk and Insurance Association

January 2004

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Outline of Presentation

- Motivation for Financial Scenario Generator Project
- Short description of included economic variables
- Using the model and motivating this research
- Methodology
- Conclusions

Overview of Project

- CAS/SOA Request for Proposals
 - Stems from Browne, Carson, and Hoyt (2001) and Browne and Hoyt (1995)
- Goal: to provide actuaries with a model for *projecting economic and financial indices, with realistic interdependencies* among the variables.

Prior Work

- Wilkie, 1986 and 1995
 - Widely used internationally
- Hibbert, Mowbray, and Turnbull, 2001
 - Modern financial tool
- CAS/SOA project (a.k.a. the Financial Scenario Generator) applies Wilkie/HMT to U.S.

Economic Series Modeled

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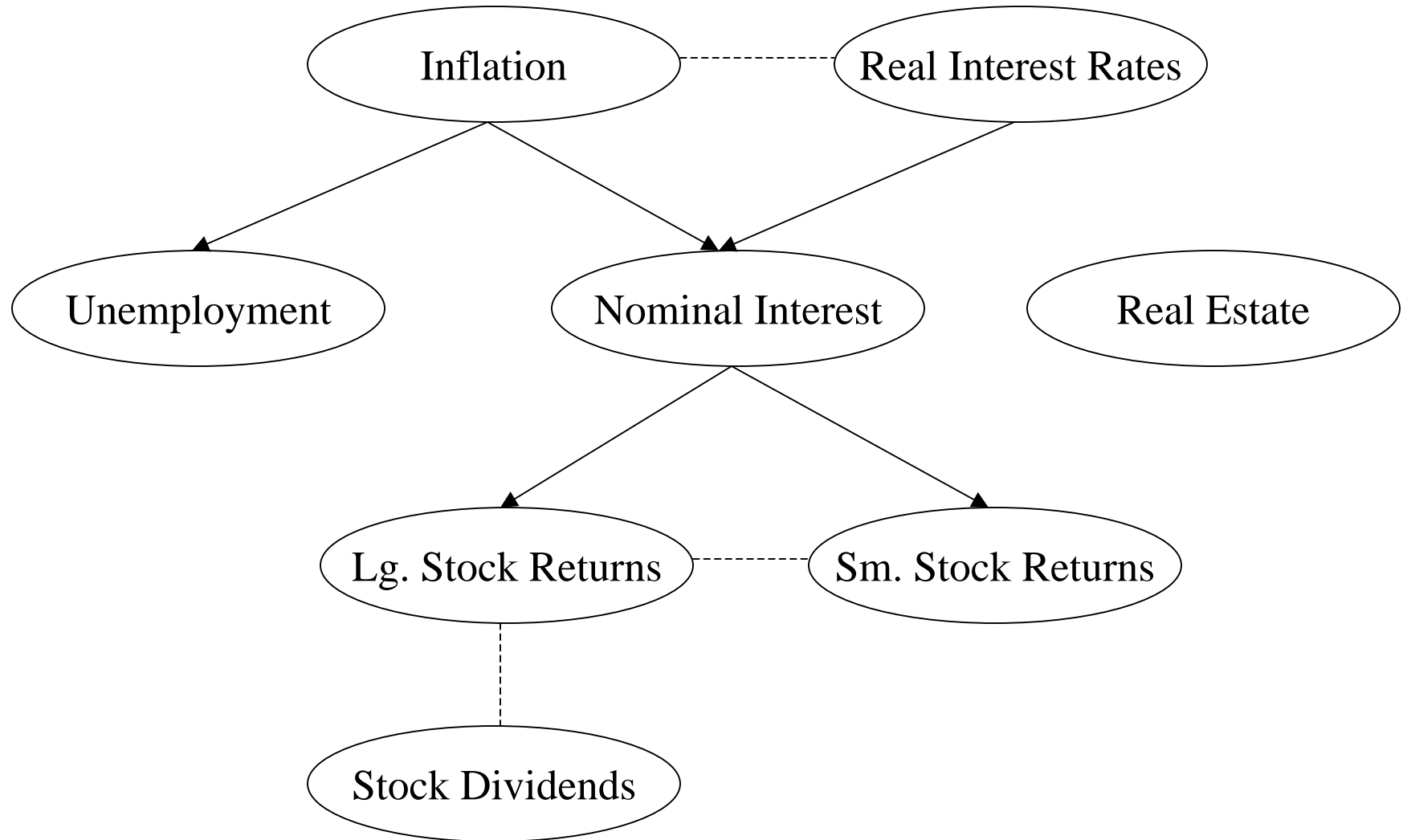
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Other Series

- *Equity dividend yields (y) and real estate*
 - O-U processes
- *Unemployment (u)*
 - Phillip's curve: inverse relationship between u and q

$$du_t = \kappa_u (\mu_u - u_t) dt + \alpha_u dq_t + \sigma_u \varepsilon_{ut}$$

Relationship between Modeled Economic Series



Selecting Parameters

- Model is meant to represent range of outcomes possible for the insurer
- Parameters are chosen from history (as long as possible)
- Of course, different parameters lead to different
 - This research: How much does this affect a life insurer?

This Research

- Browne, Carson, and Hoyt (2001) only indicate important variables to consider
- What is the potential model risk, if using the Financial Scenario Generator?
- Specific question: what is the impact of parameter “errors” on projected life insurer results?
- Contribution: Which processes require more attention? Which processes should sensitivity analysis be performed?

Use of the Financial Scenario Generator

- Dynamic financial analysis
- Insurers can project operations under a variety of economic conditions
- Useful for demonstrating solvency to regulators
- May propose financial risk management solutions

Methodology

- Use Financial Scenario Generator to project life insurance product
- Calculate PV of projected surplus/shortfall
- Vary underlying parameters of various processes to determine valuation sensitivity

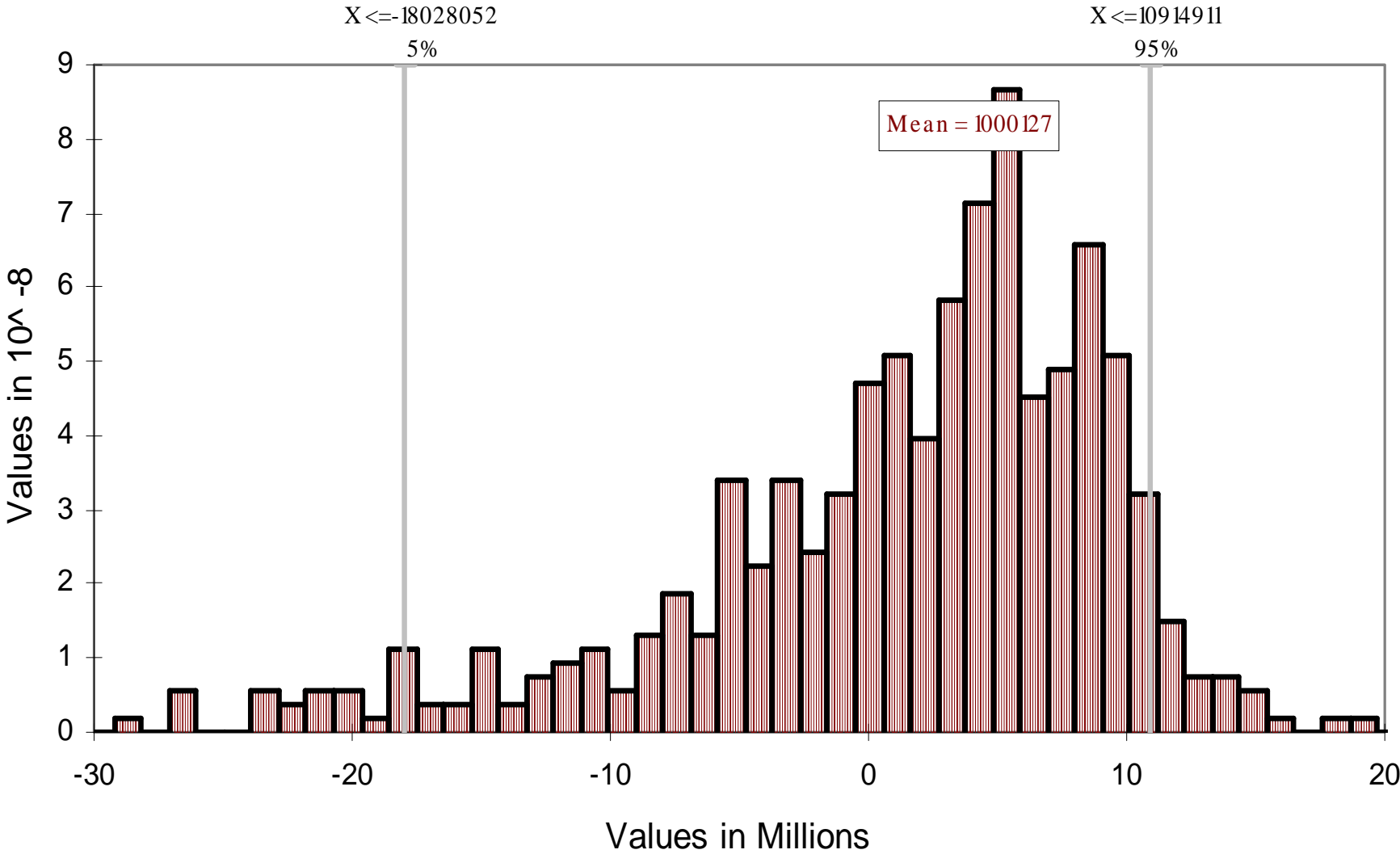
Life Insurance Product Details

- Annual payment whole life product
 - May be interest sensitive whole life
- No expenses
- EOY DB and lapse
- Cash value = required reserve (NLP)
 - Again, may be interest rate sensitive

Cash Flow Projection Details

- 10,000 new policies, issue age 35
- 20 year projection
- NLP
- Lapses: Base and interest sensitive
- Discount any remaining surplus back to time 0

Distribution for PV Surplus/T26



	<i>Mean</i>	<i>Stdev</i>
Base case	1,000,127	9,505,468

Real Interest Rates

Short Rate

Mean Rev Speed	1,007,893	9,413,302
Volatility	975,504	9,537,642

Long Rate

Mean Rev Speed	1,586,248	7,880,753
Volatility	1,004,527	15,076,870
Mean Rev Level	2,374,030	8,803,360

	<i>Mean</i>	<i>Stdev</i>
Base case	1,000,127	9,505,468

Regime Switching Equity Model

Avg Monthly Return	1,517,555	9,464,937
Volatility	1,002,727	9,538,019

Conclusion

- Even with “minor” investments in equities, assumed average return has major impact on profitability
- Reversion of long-term interest rates is crucial
 - Level and speed of reversion

Figure 12
Actual 1 Year Interest Rates (4/53-4/03)
versus Model 1 Year Interest Rates

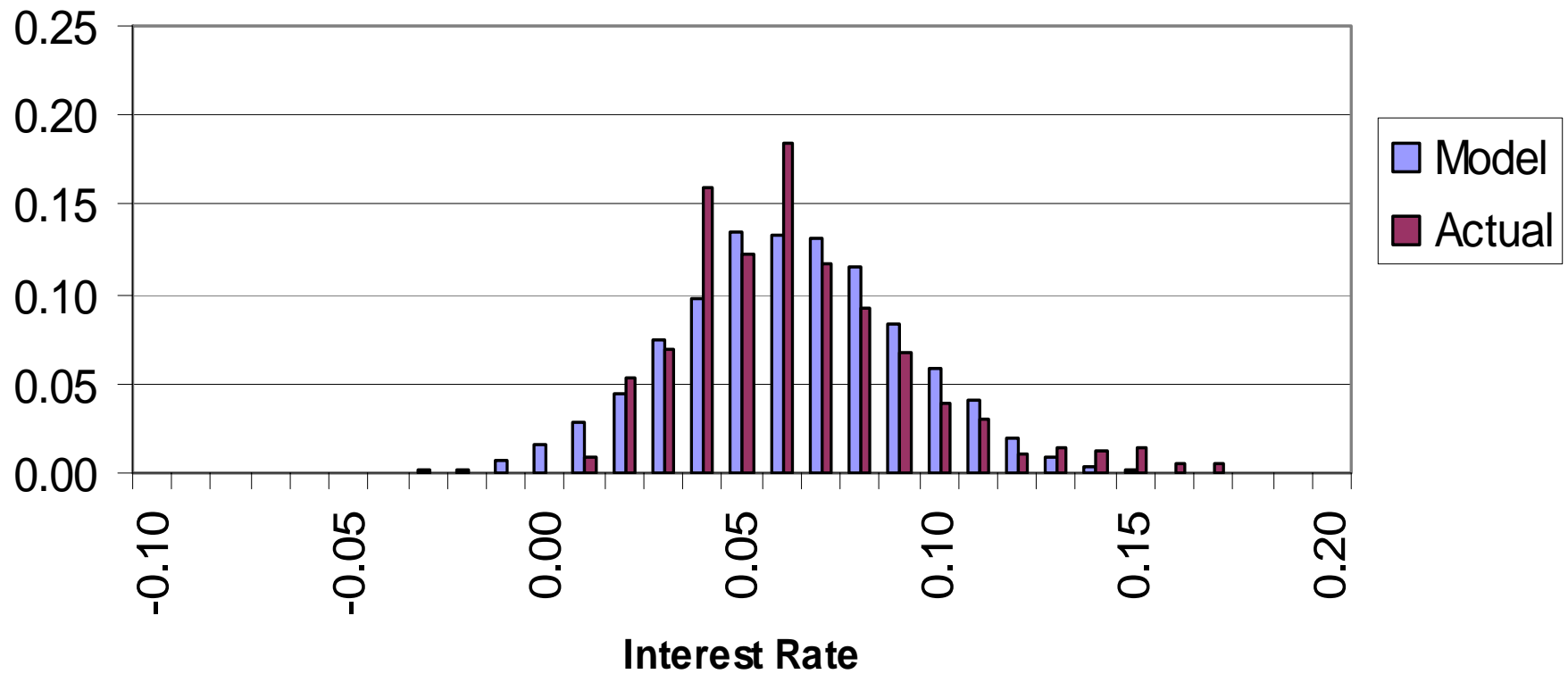


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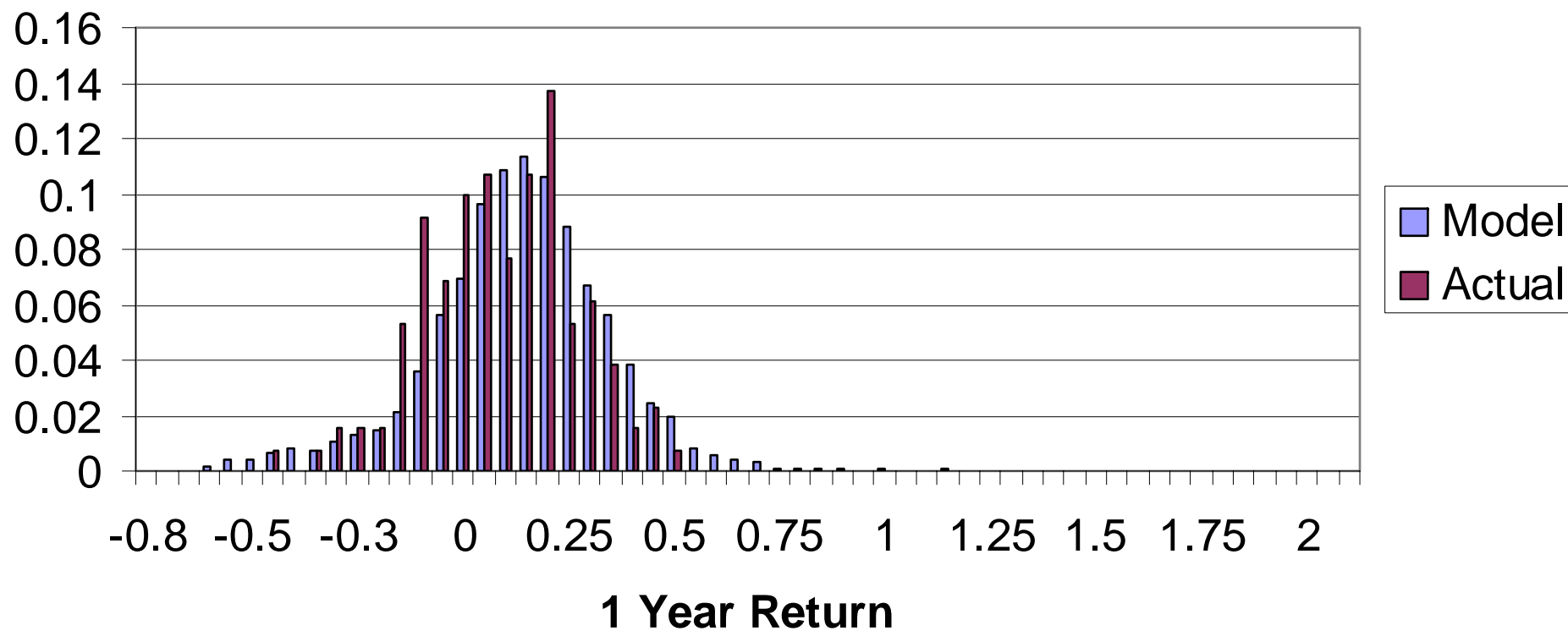
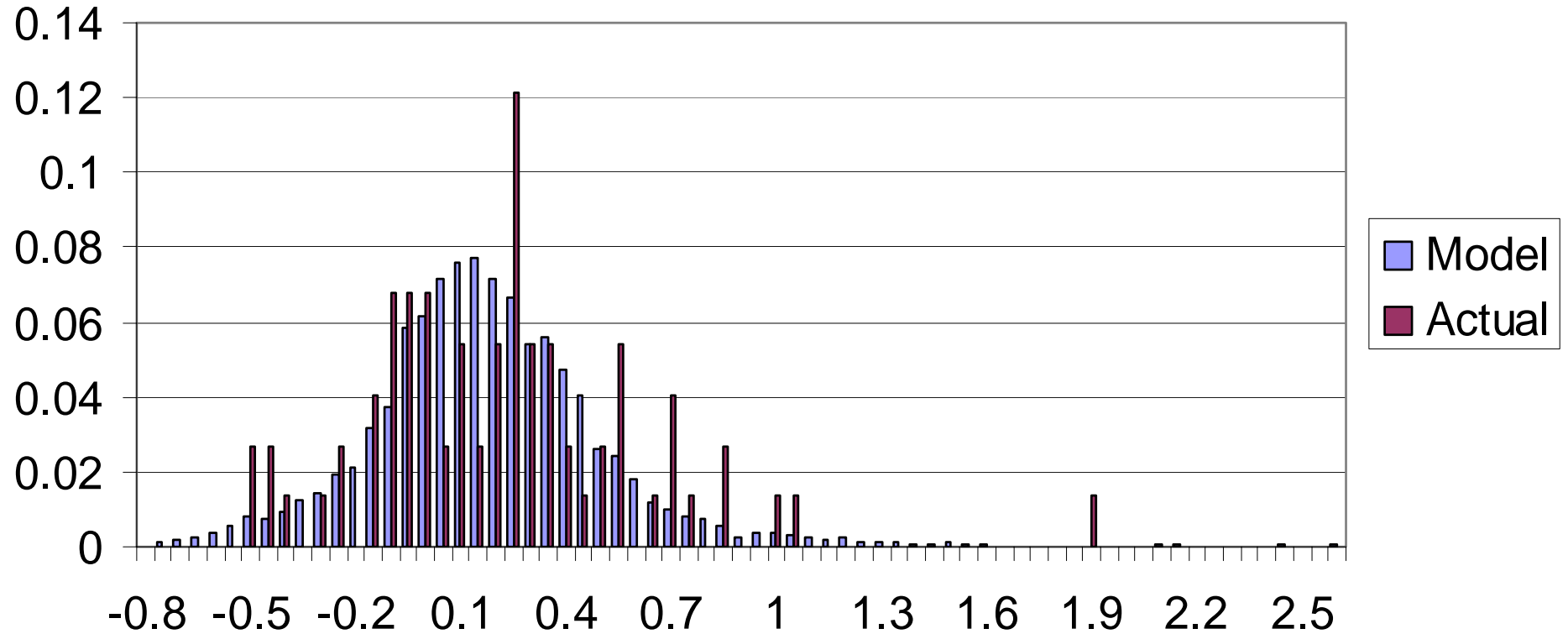


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Modeling of Economic Series Coordinated with Interest Rate Scenarios

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Investigators:

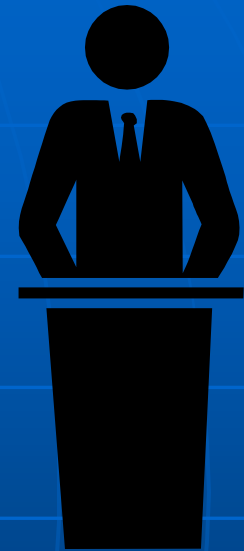
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Enterprise Risk Management Symposium

April 2004



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- Short description of included economic variables
- An overview of the model
- Applications of the model
- Conclusions

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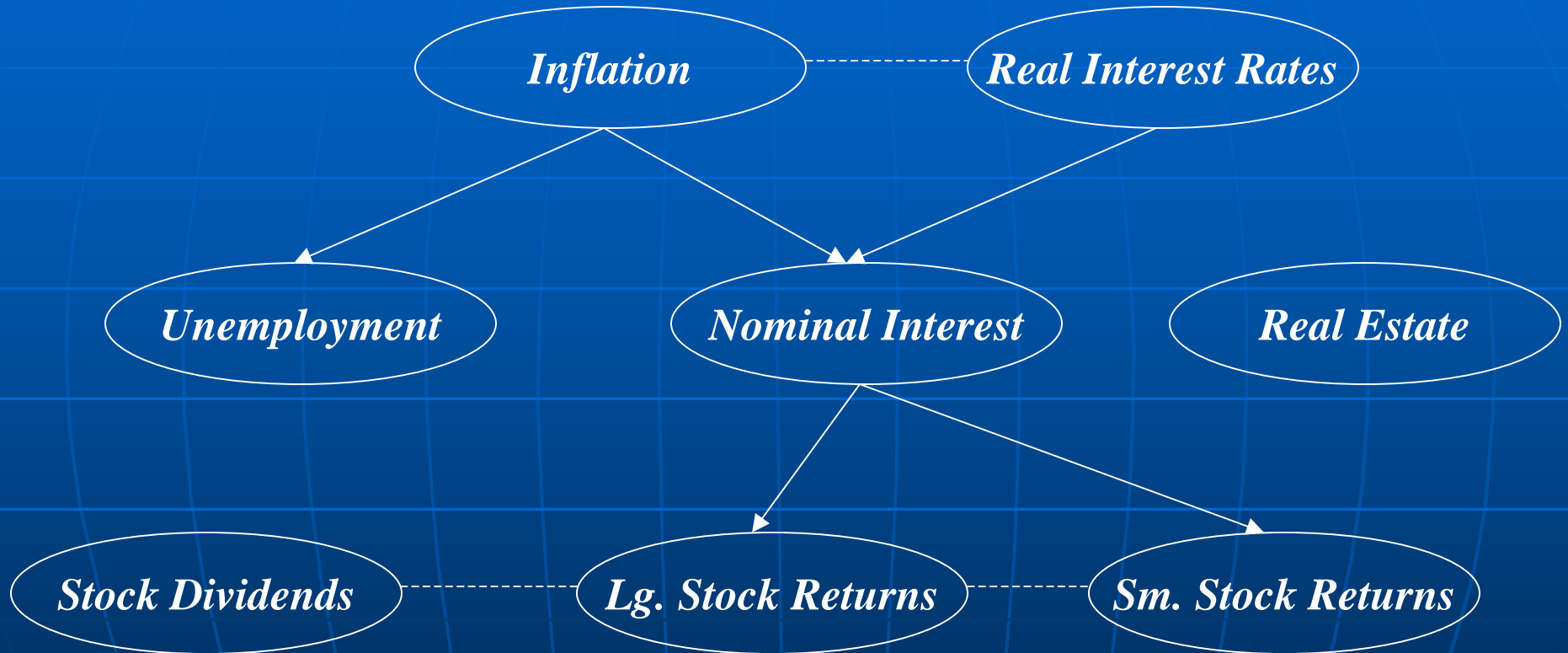
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 - One-factor, mean-reverting

$$dq_t = \kappa_q (\mu_q - q_t) dt + \sigma dB_q$$

- Speed of reversion: $\kappa_q = 0.40$
- Mean reversion level: $\mu_q = 4.8\%$
- Volatility: $\sigma_q = 0.04$

Explanation of the Ornstein-Uhlenbeck process

- Deterministic component

If inflation is below 4.8%, it reverts back toward 4.8% over the next year

Speed of reversion dependent on κ

- Random component

A shock is applied to the inflation rate that is a random distribution with a std. dev. of 4%

- The new inflation rate is last period's inflation rate changed by the combined effects of the deterministic and the random components.

Real Interest Rates (r)

- Problems with one-factor interest rate models
- Two-factor Vasicek term structure model
- Short-term rate (r) and long-term mean (l) are both stochastic variables

$$dr_t = \kappa_r (l_t - r_t) dt + \sigma_r dB_r$$

$$dl_t = \kappa_l (\mu_l - r_t) dt + \sigma_l dB_l$$

Nominal Interest Rates

- Combines inflation and real interest rates

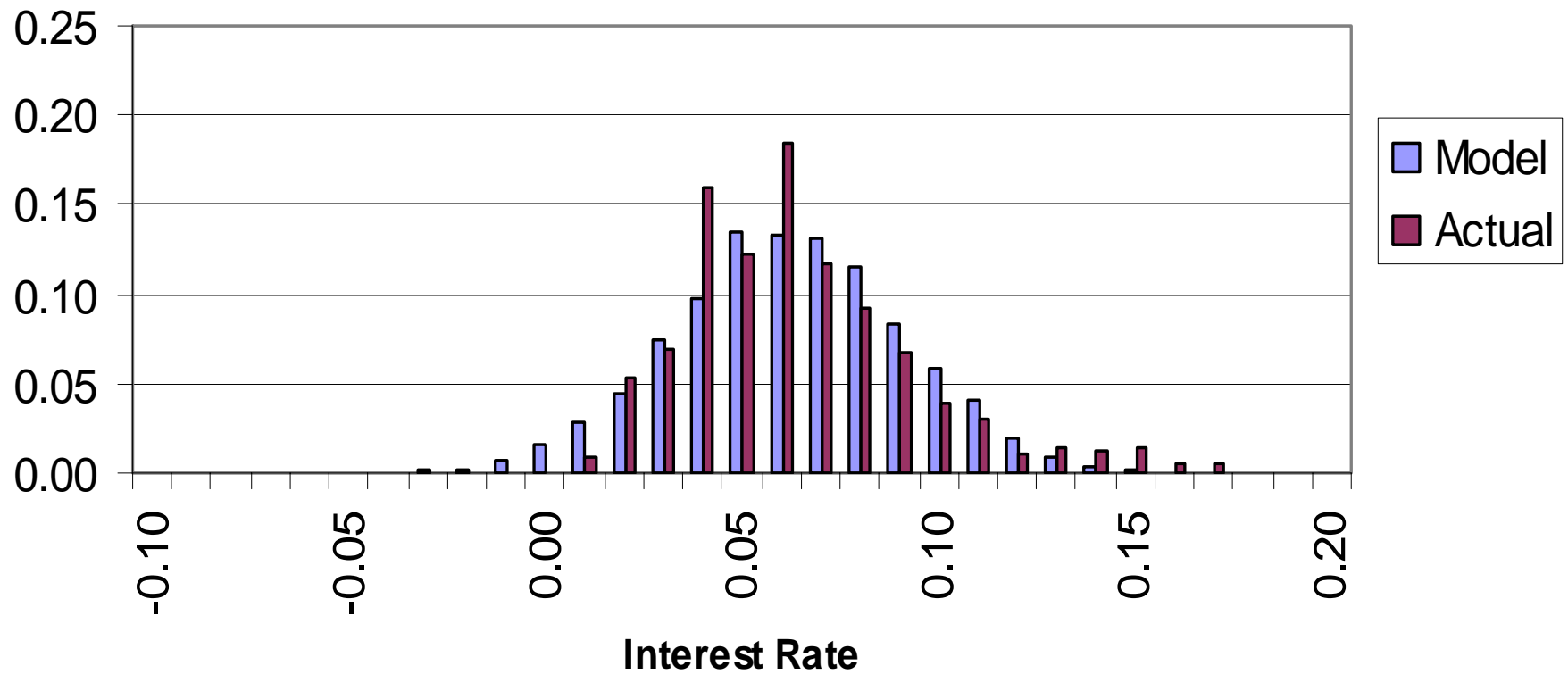
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where i = nominal interest rate

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versus Model 1 Year Interest Rates



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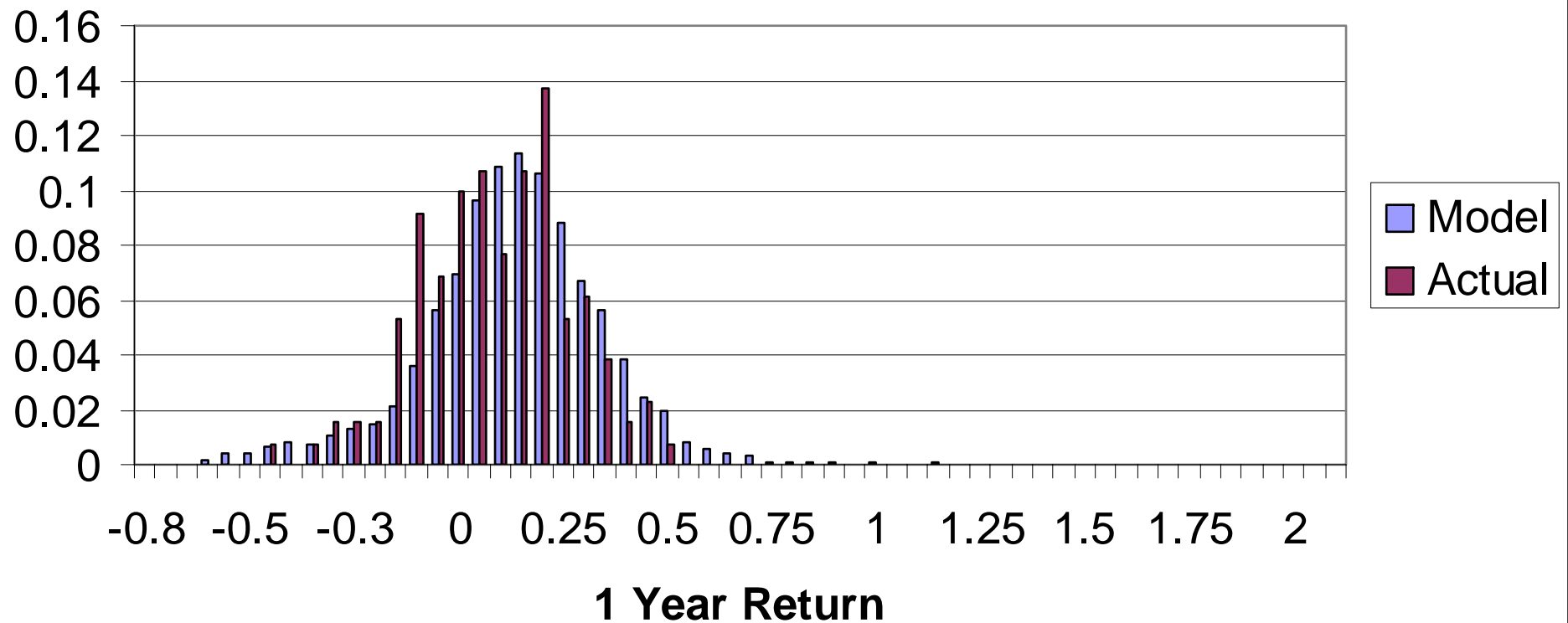
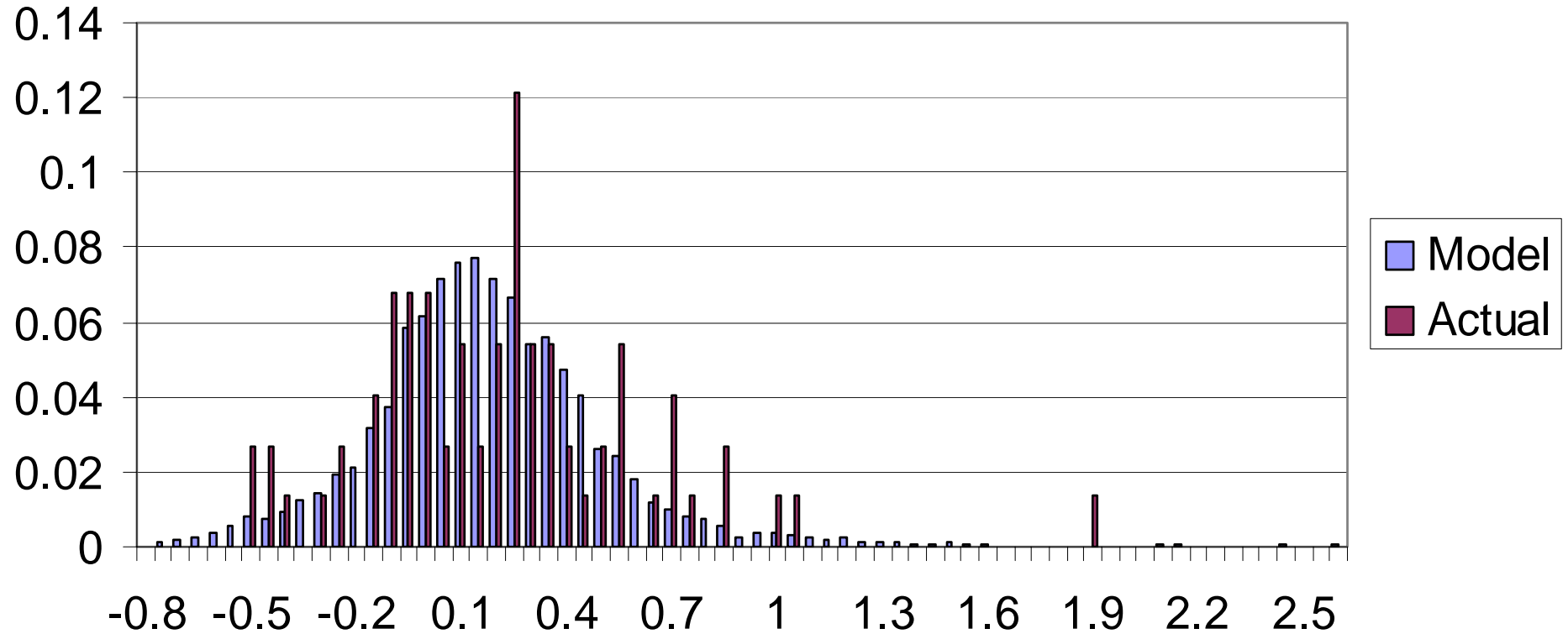


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Other Series

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- Mean-reverting processes

- ***Unemployment (u)***

- Phillip's curve: inverse relationship between u and q

$$du_t = \kappa_u (\mu_u - u_t) dt + \alpha_u dq_t + \sigma_u \varepsilon_{ut}$$

Selecting Parameters

- Historical or calibration with current market prices
- Model is meant to represent range of outcomes possible for the insurer
- Default parameters are chosen from history (as long as possible)
- Of course, different parameters may affect analysis

Model Description

- Excel spreadsheet
- Simulation package - @RISK add-in
- 50 years of projections
- Users can select different parameters and track any variable

Applications of the Financial Scenario Generator

- Financial engine behind many types of analysis
- Insurers can project operations under a variety of economic conditions (Dynamic financial analysis)
- Useful for demonstrating solvency to regulators
- May propose financial risk management solutions

Pension Obligation Bonds of the State of Illinois

- Severe underfunding problem for Illinois' public pension programs
- Severe state budget crisis 2002-?
- Low interest rate environment
- Issue \$10 billion of bonds to meet short-term (interest rate ~ 5.0%)
- Provide \$7.3 billion to state pension funds to invest
- How risky is the strategy?

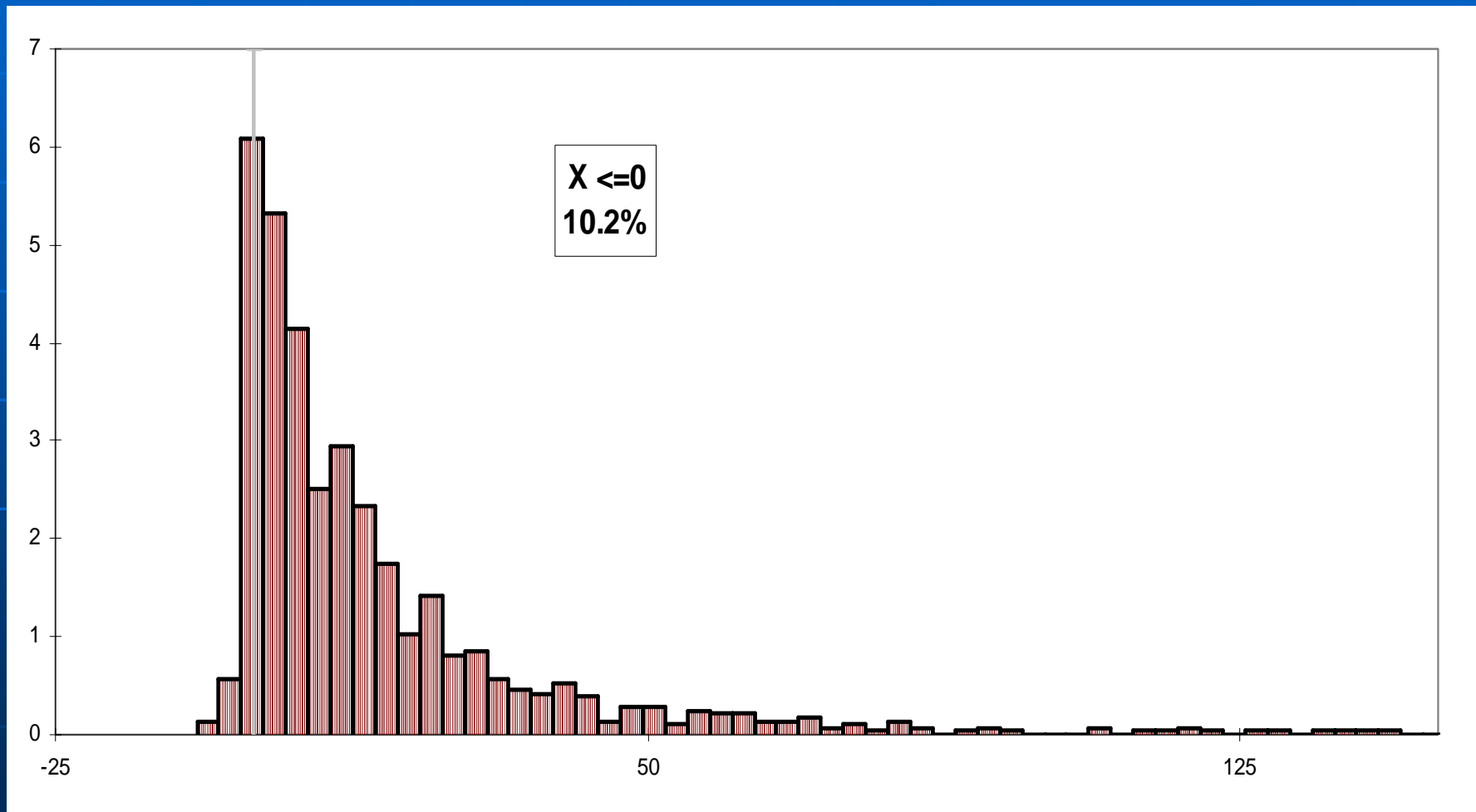
Customizing the Model

- Use the financial scenario generator to develop financial market scenarios
- Add international equities
- Track assets and debt obligations
- Are there funds remaining after the debt is repaid?

Asset Allocation

<i>Type of Investment</i>	<i>Allocation</i>	<i>Simulated Avg Ret</i>
Fixed Income Securities	28.3%	6.8%
U. S. Equities:		
Large Stocks	40.6%	13.2%
Small Stocks	10.7%	13.7%
International Equities	18.3%	7.2%
Real Estate	2.1%	9.4%

Projected Distribution of Outcomes



How to Obtain Model

Coming soon to the following sites:

- <http://casact.org/research/research.htm>
- <http://www.soa.org/research/index.asp>

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Modeling of Economic Series Coordinated with Interest Rate Scenarios:

**A progress report on research sponsored by the
Casualty Actuarial Society and the Society of Actuaries¹**

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Abstract

This paper represents a brief progress report on research, being sponsored by the Casualty Actuarial Society (CAS) and the Society of Actuaries (SOA), in the area of modeling of economic series. In particular, research is being undertaken to provide a foundation for the actuarial community's modeling of a variety of economic variables, including interest rates, equity returns, inflation, unemployment rates, and real estate price levels. The specific objectives of this research are to provide a summary of the literature in the area of economic scenario modeling, identify data sources and methodologies in this area useful to the actuarial profession, and produce working software for economic modeling, including appropriate documentation, to be made available to the general actuarial community in order to allow actuaries to use and build upon this research.

Section 1: Introduction

Consider the following activities, each of which is actively engaged in by the 21st-century actuary:

¹ The investigators wish to thank the Casualty Actuarial Society and the Society of Actuaries for providing financial support for this research, as well as guidance and feedback on the subject matter.

- A property-liability insurance company uses dynamic financial analysis (DFA) to compare alternative strategic and operational decisions, and to identify areas of potential opportunity relative to competitors within the insurance industry.
- A life insurer, both for regulatory and internal decision-making purposes, quantifies the impact on its future solvency and profitability of several alternative financial scenarios.
- An insurer evaluates different asset-liability management (ALM) techniques across a variety of possible future economic environments.

All of these activities – activities which every year are the purview of a greater number of actuaries – have at least one component in common: the necessity of systematically and efficiently modeling future values of, and the interrelationships between, economic and financial variables. These and other activities, critical to strategic decision-making and optimization of insurer performance, should be built upon a solid conceptual and practical foundation with respect to modeling the financial and economic environment, which is the broad context within which insurers operate.

Recognizing this, the Casualty Actuarial Society and the Society of Actuaries, in May 2001, issued a Request for Proposals for research in the area of “Modeling of Economic Series Coordinated with Interest Rate Scenarios.” The goal of this solicited research was to provide actuaries with a floor, or foundation, for future work in the modeling of financial scenarios, and to prepare a model for projecting economic and financial indices, incorporating realistic interdependencies among the variables. Specific deliverables from the research project included:

- 1) A *literature review* of work previously done in this area;
- 2) Identification of appropriate *data sources and methodologies* to enhance economic modeling efforts relevant to the actuarial profession; and
- 3) Production of a *working model* of economic series, coordinated with interest rates, that could be made public and used by actuaries via the CAS and SOA websites to project future economic scenarios. Categories of economic series to be modeled included interest rates, equity price levels, inflation rates, unemployment rates, equity dividend yields, and real estate price levels.

As alluded to above, this topic is of considerable value and importance to the actuarial profession and the broader insurance community. For example, a key aspect of the dynamic financial analysis process, which continues to be an area of substantial development and interest among actuaries, is the generation of economic and financial scenarios. These scenarios provide an economic context for the evaluation of an insurer’s alternative operating decisions and their potential impact, across a variety of possible future economic conditions, on future corporate value. Such stochastic simulation efforts are predicated upon the ability to probabilistically express possible future economic and financial environments. In addition, an integrated scenario generation capability is critical to recognizing the interdependencies between the various economic and financial series –

e.g., consistently modeling the relationships between, say, equity returns and interest rate movements.

Similarly, the generation of scenarios is important for regulatory, rating agency, and internal management tests of an insurer's potential future operating conditions. An example is cash flow testing. By testing across scenarios, an insurer's cash position and liquidity can be evaluated over a variety of alternative future economic and financial environments.

This paper represents a status report, as of August 2003, summarizing the authors' progress on this research and the development of a scenario model. The materials underlying this paper, along with a substantial written report, have recently been provided to committees of the CAS and the SOA for comment. These materials represent a draft of the proposed final report and model. It is anticipated that some enhancements, either necessary and/or desirable, will be made in response to forthcoming comments from these committees.

The remainder of this paper is organized as follows. Section 2 provides an overview of the draft report, including the specific deliverables which comprise the report and our research results. Section 3 discusses the specific modeling approaches currently being taken with respect to the various economic series being modeled. Section 4 describes several specific issues that are illustrative of the kinds of important questions asked, and decisions made, in the process of economic modeling. Section 5 concludes, and Section 6 provides a bibliography.

Section 2: Overview of Draft Report

In its current incarnation, the draft report summarizing progress to-date on this research project includes the following sections and attachments:

Section 1:	Introduction and Overview
Section 2:	Excerpts from Original CAS / SOA Request for Proposals
Section 3:	Excerpts from Proposal of Selected Researchers
Section 4:	Literature Review
Section 5:	Descriptions of Data and Approach
Section 6:	Discussions of Issues
Section 7:	Results of Model Simulations
Section 8:	Conclusions and Acknowledgements
Appendix A:	User's Guide to the Financial Scenario Model
Appendix B:	Presentations on This Research
Appendix C:	Simulated Financial Scenario Data
Appendix D:	The Financial Scenario Model

Most of the Section and Appendix titles should be self-explanatory. A few very brief comments on just a few of these components are warranted here.

The *Literature Review* (Section 4) includes brief descriptions of a variety of articles – covering the fields of actuarial science (both life and casualty), finance, and economics – that the researchers believe are relevant, to varying degrees, to this research.² Research on the development of financial scenarios, and the analysis of financial and economic time series, is a continually evolving and growing area. We recommend that efforts be made, at least periodically if not continually (e.g., by a formal charge to appropriate CAS / SOA research committees, or by engagement of other interested persons), to provide an ongoing search for and review of relevant new work in this area, in order that the results from this project might be enhanced and kept current.

Discussions of Issues (Section 6) describes and comments upon some of the specific issues encountered during the course of this research. In some cases, these issues involved decisions with which we as researchers were confronted; our thought processes and the rationales for selected approaches are included here.³

Simulated Financial Scenario Data (Appendix C) is a spreadsheet database of hundreds of scenarios (i.e., simulation paths) of financial and economic variables, generated as output from the Financial Scenario Model. The intent of this data is to provide an alternative to requiring the @Risk simulation package (an add-on to Excel)⁴ in order to run the model. This data can be used directly, in lieu of actually running the model; the “pre-simulated” scenario paths can be used as an input to a DFA model or other analytical program.

The Financial Scenario Model (Appendix D) is an Excel spreadsheet-based program, designed to be run, as mentioned above, through the @Risk simulation add-on. The (ultimately) downloadable version of the model will contain default values of appropriate parameters – however, these can be changed by the user for purposes of updating for new or additional data, sensitivity testing of parameter values, etc.

Please note that our intention is ultimately to post the completed report, along with the financial scenario model and hyperlinks to various presentations and articles emerging from this research, on the websites of both the CAS and the SOA.

Section 3: Progress Report on Research

After an extensive review of the literature in a number of relevant fields, it was found that three articles in particular provided a strong historical foundation for this economic scenario research. Wilkie (1986) used simulation to model future economic scenarios, for a variety of applications. In his article, inflation was modeled as the “driving” variable, with

² Various CAS and SOA members made several suggestions for articles to be included in this literature review. These suggestions are gratefully acknowledged.

³ Often, these issues were either provoked or reinforced by questions or comments from members of the sponsoring actuarial committees. Again, this input was greatly appreciated and valued throughout the project.

⁴ “@Risk” is a software package produced by the Palisade Corporation. For additional information, please see the company’s website at www.palisade.com.

several other economic processes being driven off of inflation in a “cascade”-type fashion. A first-order autoregressive (AR) process was used for inflation, along with certain other variables. Wilkie (1995) followed up on his earlier paper, expanding upon and enhancing the modeling and econometrics. Finally, Hibbert, Mowbray, and Turnbull (2001) examined issues in modeling a number of economic variables. Included in this paper is a comparison of outputs with the results from the Wilkie model. Taken together, these three articles provide an excellent background for understanding our current research.

For this research, the economic series modeled included the following:

- Inflation
- Real interest rates
- Nominal interest rates
- Equity returns
 - Large stocks
 - Small stocks
- Equity dividend yields
- Real estate returns
- Unemployment

The general approach used for each of these processes will be briefly summarized below⁵. In the formulas which follow, standard Brownian motion processes are represented by dB , and subscripts refer either to time (t) or to the relevant economic variable. The descriptions are all provided in a continuous-time framework, as this is the framework in which most financial research of this nature is done. Of course, parameterization and estimation are performed relative to real, actual data, which is discrete in nature. Mathematically, the discrete forms of these processes are analogous to the continuous-time versions provided below.

Inflation

We model inflation (q) as an Ornstein-Uhlenbeck process. Specifically, inflation is assumed to be a one-factor, mean-reverting process of the form

$$dq_t = \kappa_q (\mu_q - q_t) dt + \sigma_q dB_q$$

where q is the short-term inflation rate, κ is the speed of mean reversion, μ is the long-run rate to which the process tends to revert, and σ is a volatility parameter. In discrete format, this amounts to an autoregressive process. The model was parameterized using U.S. Bureau of Labor Statistics Consumer Price Index (CPI) data. Two time periods were examined, in order to test the sensitivity of the parameters to different economic epochs: 1913 to 2001, and 1946 to 2001. Because of the noise in monthly CPI series, annual CPI values and regressions were employed. Using an approach similar to Vasicek (1977) for

⁵ Much more detail regarding the modeling, mathematics, parameterizations, etc., is provided in the forthcoming formal report of this research, as well as in one or more planned articles.

interest rates, we then produced a term structure of inflation rates (necessary for modeling nominal interest rates – see below).

Real Interest Rates

Real interest rates (r) are modeled according to a two-factor Vasicek model⁶. This structure is similar to the one-factor Vasicek model (Vasicek, 1977), in that it is a mean-reverting process with the short-term real interest rate being a stochastic variable. However, in addition, there is a second stochastic factor: the long-run mean (l , below) to which the short-term rate tends to revert. The specific formulas used are

$$\begin{aligned}dr_t &= \kappa_r (l_t - r_t) dt + \sigma_r dB_r \\dl_t &= \kappa_l (\mu_l - r_t) dt + \sigma_l dB_l\end{aligned}$$

These equations were parameterized with monthly Federal Reserve data from 1982 to 2001, using two-stage least-squares estimation techniques. Real interest rates, while not directly observable, were estimated *ex post* by determining the differences between nominal interest rates and inflation rates.

Nominal Interest Rates

Once inflation and real interest rates have been modeled, nominal interest rates (i) are determined from them, based on the standard Fisher (1930) relationship:

$$i = \{(1 + q) \times (1 + r)\} - 1$$

The model software developed in coordination with this research will allow the user, if s/he so desires, to toggle a “non-negativity” switch, thereby preventing future modeled nominal interest rates from falling below zero.

Equity Returns

A great deal of attention has been paid to modeling equity returns in the financial, economic, and even physics (under the guise of “econophysics”) literatures⁷. One of the empirical observations that is frequently noted regarding historical equity returns is the “fat tails” issue – that actual equity returns, when examined as a historical distribution of returns, tend to have fatter tails than typical theoretical distributions would suggest. To that end, we have used (along the lines of Hardy (2001)) a “regime-switching” model for equity returns, with the two regimes having, respectively, low and high volatility. A Markov Chain framework represents the probabilities of switching between regimes from month to month.

We have modeled equity returns (s_t) by specifying an excess return (x_t) over and above the modeled nominal interest rate:

$$s_t = q_t + r_t + x_t$$

⁶ This is also a simple case of the two-factor Hull-White interest rate model. See Hull and White (1994).

⁷ For econophysics, see, for example, Sornette (2003).

The excess return represents a risk premium attributable to capital appreciation (since the dividend yield of equities is modeled separately – see below). We estimated the processes underlying small stocks and large stocks separately.

Equity Dividend Yields

The equity dividend yield (y) is modeled such that the natural logarithm of the dividend yield follows an AR process:

$$d(\ln y_t) = \kappa_y (\mu_y - \ln y_t) dt + \sigma_y dB_{y,t}$$

The parameterization process for this model is similar to that involved for the inflation model, described above.

Real Estate Returns

We estimated two versions of an Ornstein-Uhlenbeck model for real estate returns: including and excluding inflation.⁸ We used quarterly National Council of Real Estate Investment Fiduciaries (NCREIF) pricing indices to capture returns on commercial properties. The NCREIF data is generated from market appraisals of various property types, including apartment, industrial, office, and retail.

Unemployment

With respect to the unemployment process, a well-known tool, the Phillip's Curve, posits an inverse relationship between the unemployment rate (u) and inflation (q).⁹ We have chosen to build upon that relationship, but to include an AR(1) process:

$$du_t = \kappa_u (\mu_u - u_t) dt + \alpha_u dq_t + \sigma_u \varepsilon_{ut}$$

Unemployment data, from 1948 to 2001, from the Bureau of Labor Statistics was utilized.

Section 4: Discussion of Several Issues

This section highlights several of the key issues and questions encountered during this research project. This section is not meant to be a comprehensive survey of all critical issues, but rather illustrative of the kinds of problems encountered, and the thought process of the researchers with regard to their resolution.

Inflation versus Interest Rates

In any effort to model key economic variables, a question which must be resolved early in the modeling process involves which variables come “first” – i.e., which variable(s) is(are) key and causal, and which other variables are functions of those. This is a typical question when dealing with interest rates and inflation. Interest rates are probably more frequently

⁸ While we expected inflation to be a driver of real estate returns, the results to-date have not indicated significance.

⁹ While this relationship seems plausible in many ways, it should be noted that more recent data (since the original publication of the Phillips Curve in 1958) has not fit this inverse relationship as well.

directly modeled, and a potential disadvantage of varying from that approach can be an inability to take advantage of, and build upon, extensive prior work of numerous authors in interest rate modeling. On the other hand, in the U.S., the Federal Reserve has a large role in impacting the levels of interest rates, in response to inflationary pressures and general economic conditions. This decision, therefore, is not a trivial one.

We chose to model both inflation and real interest rates as the “driver” variables. This decision led to the modeling structures for those two variables, along with nominal interest rates, mentioned in the preceding section. Under our approach, is it possible for the user of the model to correlate the shock (dB) terms of the inflation and real interest rate processes, allowing for a direct, partial connection between these series.

Equilibrium vs. Arbitrage Free Models

One of the primary processes in a financial scenario model is a term structure process. A tremendous variety of term structure models is available for both practitioners and researchers. (For a discussion of many of the available models, see Yan (2001)). No single term structure model has yet proven itself worthy for all possible applications (see the discussion in Chapman and Pearson (2001)). In virtually all cases, the user of a term structure model has one or more tradeoffs to consider – e.g., complexity of the model vs. accuracy. The nature of these tradeoffs depend on the specific application of the term structure model.

There are several important issues to consider when choosing among term structure models. One consideration is related to the theoretical background of the model. Specifically, term structure models are typically categorized as “equilibrium” models and “arbitrage-free” models. Equilibrium models typically begin with an assumption for short-term interest rates, which are usually derived from more general assumptions about the state variables that describe the overall economy. Using the assumed process for short-term rates, one can determine the yield on longer-term bonds by looking at the expected path of interest rates until the bond’s maturity. One of the primary advantages of equilibrium models is that the prices of many popular securities have closed-form analytic solutions. Another advantage is that equilibrium models are fairly easy to use. On the negative side, equilibrium term structure models generate yield curves which are inconsistent with current market prices. While the parameters of these models may be selected carefully, there is no guarantee that the resulting term structure will generate observed market prices.

Contrary to equilibrium models, arbitrage-free term structure models project future interest rate paths that emanate from the existing yield curve. For applications using arbitrage-free term structure models, resulting prices will be based on the concept of arbitrage. Unfortunately, arbitrage-free term structure models are frequently more difficult to use than their equilibrium counterparts.

Although we will not elaborate here, other considerations in selecting a term structure model include pricing accuracy, internal consistency, data issues, intended use of the model, and the time horizon over which simulations will be performed. With regard to the last two considerations mentioned above, it is important to bear in mind that insurance and

actuarial applications of such models, such as in DFA models, generally involve long time horizons (e.g., five years of simulation projections), with generally fairly course time intervals (monthly, quarterly, or even annual). This is a very different framework from a short-term (hourly, daily, or weekly) trading horizon. It may well be that different models are justifiable for such different analytical frameworks.

Adequacy of a Two-Factor Interest Rate / Inflation Model

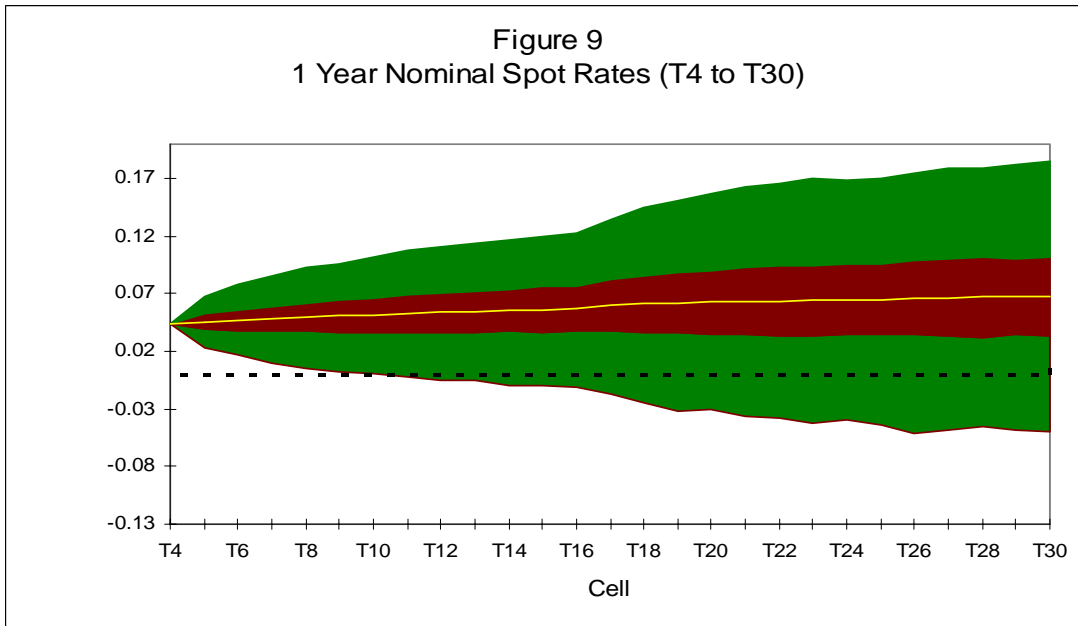
The number of factors to use in modeling interest rates is a decision which frequently elicits passionate debate. Again, as mentioned above, it is important to keep in mind the purpose toward which this research is working: to produce reasonable distributions of future economic values. Our work is *not* intended for security-trading purposes. This is a hugely important context to keep in mind – it has implications for the type of interest rate model used, the number of parameters employed, etc. Furthermore, there is often a misunderstanding as to the types and movements of yield curves that are available from two-factor (and with respect to some issues, even one-factor) models. For example, humped curves are indeed possible. (A good paper for considering the types of yield curve movements that predominate historically is Litterman and Schenkman (1991).) We believe that the two-factor model we have employed is a reasonable selection in view of both historical and parsimony considerations.

Section 5: Types of Results

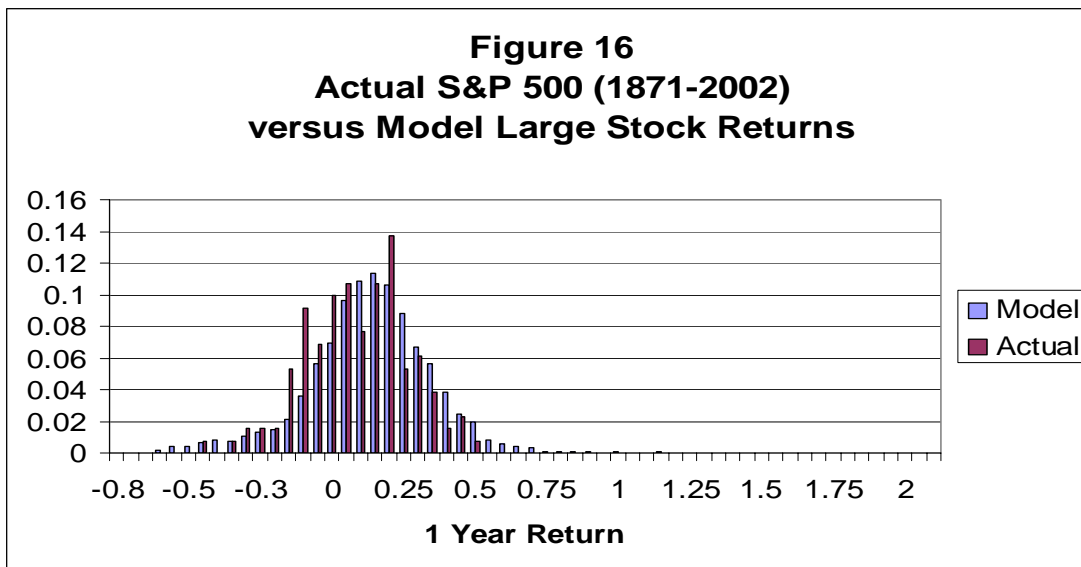
In addition to documentation regarding the analytical infrastructure of this research, our work has provided a number of results in terms of

- *Parameters* for the various economic processes;
- Indications of future *volatilities* for the modeled processes; and
- *Comparisons* of model fits with historical data.

Since our formal report is, as of this writing, currently being reviewed by the sponsoring societies, we have refrained from providing specific quantitative results. These will be provided on the CAS and SOA websites, as part of our formal report, upon completion of this project. However, below are two samples of exhibits from preliminary work. These are included for illustrative purposes only. The first exhibit, immediately below, is a “funnel of doubt” chart, which shows the volatility of simulated future values, over time, of a stochastic variable (in this case, the 1-year nominal spot interest rate). The solid relatively-horizontal line in the middle shows the mean value over time of the (in this case, 5,000) simulations of that stochastic variable. The two shaded areas surrounding the mean line are the 25th to 75th percentile values (the dark shaded section), and the 1st to 99th percentile values (lighter shaded section).



The second illustrative exhibit, immediately below, needs little explanation. This type of exhibit graphically represents the nature of the fit between actual and modeled values (in this case, of the distribution of large stock returns).



Section 6: Conclusion

At the time of writing this article, and the presentation of this material at the Actuarial Research Conference (August 2003, in Ann Arbor, MI), an initial draft of the final report of this research had been submitted to the oversight committees of the CAS and the SOA. While comments from these committees will undoubtedly improve and enhance this research, the current key elements and deliverables of this research include:

- *A literature review* summarizing the relevant literature from several fields, including actuarial science, economics, and finance;
- *A summary of data sources and methodology* providing the analytical underpinnings of our work;
- *A Financial Scenario Model*, programmed in Excel and @Risk, and designed to be available for use by actuaries by downloading from the CAS or SOA website;
- *Simulated financial scenario data*, based on default model parameter assumptions, which can be used in lieu of the model if @Risk software is not available;
- *A User's Guide* to the model; and
- *A discussion of results* from sample implementation of the model.

This paper has presented a survey and highlights of our progress on this actuarial research. Hopefully, it has provided at least a flavor of this work. Significantly more detail, along with specific parameter values and quantitative results, is included in our formal report. The formal report will be available through the CAS and the SOA websites in the near future.

This research, and the accompanying model, should be of wide interest, including to such insurance and financial professionals as insurers, regulators, and pension funds. Specific applications of this research might include parts of dynamic financial analysis, cash flow testing, financial planning, investment analysis, capital budgeting, and analysis of alternative financial risk management solutions.

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