Embedded Options in Pension Plans

Valuation of Guarantees in Cash Balance Plans

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Executive Summary

We study three embedded guarantees in cash balance plans. The first two guarantees are relevant for market rate cash balance plans whereas the third guarantee is more applicable for traditional cash balance plans.

The first guarantee is the return of participant pay credits at benefit commencement. We refer to this guarantee as the “money-back guarantee”. The second guarantee is the return of participant pay credits at benefit commencement with a fixed return which might be offered to a participant to protect against inflation. We refer to this guarantee as an “enhanced money-back guarantee”.

The third guarantee is relevant for a traditional cash balance plan. This guarantee provides for a minimum interest rate credit to be earned each year. For example, an interest crediting rate that provides a yield on the 30 year Treasury bond but not less than some maximum, say 4.0%, per annum.

In this paper, we value the guarantees using option pricing fair valuation principles. The first two guarantees are valued using a closed form adaptation of the Black-Scholes formula. The third guarantee is valued via a Monte-Carlo stochastic simulation of the risk-neutral interest rates process using a Hull-White interest rate model.

1. Introduction

This research is the result of a request from the Society of Actuaries (SOA) to create reference material which will address and explore embedded options in defined benefit pension plans. Part I of this research initiative was released in August 2011[1]. That paper accomplished three objectives:

- Defined embedded options using two categories. Broadly speaking, Category 1 options are behavior driven (e.g. retirement, termination) and Category 2 options are based on underlying financial phenomena;
- Drew parallels between embedded options pension plan features and those that exist in the life insurance industry and the financial markets, and
- Provided the results of a survey on the prevalence of embedded options in defined benefit plans.

The objective of Part II is to provide a first step towards valuing these embedded option pension plan features. Embedded options are garnering more attention[2] and we do not expect nor do we intend for this paper to be the last word on the topic. The intended audience of this paper is pension practitioners, although public policy professionals and others may also be interested.

In this paper, we attempt to value embedded options that exist in cash balance pension plans. It is important that plan sponsors understand the value of these guarantees in order to make prudent decisions on plan design, investments, and risk management. Without a thorough understanding of the value of these guarantees, decision making is likely to be impaired.

Subsequent papers will attempt to value the other embedded options that we described in Part I. By breaking the research down into the valuation of each contract type that we catalogued, we hope to make this research more manageable and useable for the reader and practitioner.

We use the terminology “embedded option” and “guarantee” as synonyms. The terminology “embedded option” is used in the actuarial literature (see below). However, other sources (e.g. academic and/or life insurance) tend to use both terms to describe contract provisions like those discussed here.

[1] See http://www.soa.org/research/research-projects/pension/research-catalogue-survey.aspx

[2] For example, Nobel Prize winner Robert Merton and Jeremy Gold discussed embedded options at the 2013 SOA Investment Symposium. http://www.soa.org/Professional-Development/Event-Calendar/2013/investment-smposium/Agenda-Day-1.aspx

2. Background

Under current actuarial practice, many of the embedded options in pension plans are not taken into account as part of funding and accounting valuations[3]. We catalogued these options in Part I of our research. Survey results from Part I of the research showed that embedded options are usually ignored because they are assumed to have zero, or very little value. This conclusion, typically arrived at by rule of thumb or casual inspection, suggests the probability of these options being exercised at a cost to the plan sponsor is quite remote and therefore these options have zero value for the purposes of the valuation being performed[4].

To date, the pension actuarial profession has only briefly touched on embedded options in defined benefit retirement plans. A Public Policy Practice Note - Selecting and Documenting Other Pension Assumptions published in October 2009 by the American Academy of Actuaries briefly describes embedded options. More recently, Actuarial Standard of Practice No. 4 - Measuring Pension Plan Obligations and Determining Pension Plan Costs or Contributions (ASOP 4) Second Exposure Draft includes text on embedded options[5].

Recent developments have increased the focus of pension actuaries on embedded options in defined benefit pension plans. Volatile capital markets[6] have either increased the value of some of these guarantees[7], or at the very least, highlighted to actuaries and plan sponsors alike that these guarantees probably should not be ignored.

The economic landscape, regulatory environment and continued march of plan sponsors de-risking their pension plans is also forcing companies to better understand the risk profile of their pension plan. As a result, actuaries are increasingly being called upon to model adverse economic scenarios and low-probability events. Often times it is under these stressed conditions where the embedded options have the most value and in turn the most potential cost to a plan sponsor.

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[3] This paper focuses on US qualified pension plans. These plans are required to be funded by the IRS. Government pension plans are also required to be funded. Pension plans are also disclosed, for accounting purposes, in company/government financial statements. Funding and accounting valuations constitute the two major valuations types conducted by pension actuaries in the US.

[4] For example, if the valuation is a funding valuation where the liability is generally calculated based on expected outcomes, it may be true that the guarantee indeed has no value under such a scenario.

[5] Specifically, see Section 3.5.3. http://www.actuarialstandardsboard.org/exposure.asp

[6] For example, since the Great Depression, there have been only three occasions where the S&P 500 rose or fell more than 4% in each of four consecutive sessions: October 1987, November 2008, and August 2011 http://www.nytimes.com/interactive/2011/08/12/business/economy/20110812-four-consecutive-days-of-market-volatility-are-rare.html

[7] Either because the guarantees themselves were actually now “in the money” or because valuation assumptions, when used in valuation formulas/approaches, as discussed in this paper, produce a higher value for these guarantees than would have been the case in prior history.

Trends in actuarial standards[8] and accounting standards are increasingly leaning towards reporting of fair values[9] or “market-consistent present values” for pension liabilities. However, these principles are normally applied at the plan level only and often ignore the guarantee. This paper attempts to extend some of these market-consistent valuation principles to the valuation of embedded guarantees[10].

Two cash balance plans with identical contract provisions in every way except for the fact that one plan offers a guarantee and one does not, should not have the same value (unless it is concluded the option value is zero). We attempt to value these guarantees using option pricing techniques. We use this approach because the guarantees can be viewed as equivalent to financial options. To see this, consider how the guarantees work. The money-back and enhanced money-back guarantees protect against a decline in the account balance below the sum of the pay credits. This is equivalent to a put option on the account balance with a strike price of the sum of the pay credits that is owned by the plan participant. The annual interest crediting rate guarantee can be viewed as an interest rate floor with a strike equal to the minimum interest crediting rate that is owned by the plan participant. Therefore, the logic is as follows: if we can value financial options, we can value embedded guarantees in pension plans.

Importantly, our focus in this paper is not on valuing the overall cash balance plan liability but rather valuing just the guarantee portion. This approach, for valuation purposes, of bifurcating the “liability” from the “guarantee” is similar to methods commonly employed for certain life insurance contacts such as guaranteed minimum death benefits and guaranteed minimum accumulation benefit and for financial contracts that contain embedded options. The embedded guarantee values derived in this paper would be added to the overall cash balance plan liability to arrive at the overall liability for the plan[11].

[8] ASOP 4 introduces the concept of “market-consistent present values”. Additionally, ASOP 4, Section 3.5.3c describes, in basic terms, some of the guarantees discussed in this paper.

[9] See FAS 157 – Fair Value Measurements

[10] More nuanced issues like how the financial health of the plan sponsor may impact the valuation of the embedded guarantees is considered out of scope for purposes of this paper. While these issues are important we are focused on the broader issue of acknowledging embedded guarantees exist and putting some value on them using modern valuation principles.

[11] We assume the cash balance plan liability without the embedded guarantee is the “economic liability” that includes the value of any subsidies and costs associated with the uncertainty of hedging the liability. See Risk Management for Cash Balance Plans SOA presentation and Unique Issues for Hybrid Plans for more details.

3. Overview of Cash Balance Plans

A cash balance plan is a defined benefit plan that provides a participant with a periodic pay credit (commonly linked to salary and wages and usually tied to age and/or experience) and prescribes an interest crediting rate (usually tied to a constant maturity bond or a Consumer Price Index) that defines how the accumulation of pay credits will evolve over time. The benefit under the cash balance plan is expressed as a lump sum, known as the cash balance "account" or "current account balance".

Figure 1 illustrates the mechanics of how a cash balance plan works assuming a participant has five years until retirement, grants a pay credit each year until retirement equivalent to 5.0% of pay with constant salary at $100,000, and grants an interest credit equal to 4.0% per annum. We assume pay credits are made at the end of each year.

Figure 1. Cash Balance Plan Mechanics

Year	Pay Credit	Interest Credit	Retirement Benefit
1	$5,000	$0	$5,000
2	$5,000	$200	$10,200
3	$5,000	$408	$15,608
4	$5,000	$624	$21,232
5	$5,000	$849	$27,081

The cash balance plan benefit is a contingent liability of the plan sponsor. Importantly, the contribution of pay credits is not required of the plan sponsor; the plan sponsor has the option of freezing the granting of future pay credits. However, the continuation of future interest credits (i.e. indexing before benefit commencement) is a protected benefit under current U.S. pension law[12]. Moreover, it should be noted that while a cash balance plan has the look and feel of a defined contribution plan (DC) or even a simple bank account, individual accounts are not actually established. Rather, the retirement benefit is a "hypothetical" account in which all the pension assets for all the participants of the plan are pooled together and managed collectively. The participant does not have control over how the plan's assets are invested. Under the terms of the pension plan, the participant is simply promised a benefit at commencement age equivalent to the sum of pay credits granted plus interest credits on those pay credits. Nevertheless, it is sometimes useful to think of a cash

[12] See Internal Revenue Code 411(d)(6) for more details

balance plan in terms of the more familiar DC setup. Because of the apparent similarity in the two designs, cash balances plans are sometimes referred to as "hybrid" plans.

Based on a Kravitz National Cash Balance Plan Report for 2011[13] that analyzed all cash balance plans nationwide from the most recent IRS Form 5500 filings available at the time, December 31, 2009, there were 5,840 such retirement programs representing about 11% of all defined benefit plans and covering approximately 10.5 million participants. The total plan assets held for defined benefit plans was approximately $606 billion at the end of 2009. Among plans currently offered to new hires, 54% are hybrid plans such as cash balance[14]. In the public sector space, according to *Pensions & Investments* magazine more than 15 cities, counties and states have cash balance and other hybrid plans, the majority approved or implemented in the last two years[15].

While cash balance plans have been around since the 1980s[16] IRS regulations passed in October 2010 introduced new interest crediting rate options[17]. Though these new crediting rates introduce opportunities for new cash balance plan design types they also introduce guarantees for the plan participant. These guarantees are of economic value to the plan participant and therefore represent an economic cost to the plan sponsor.

3. 1 Cash Balance Plan Interest Crediting Rates Under October 2010 Regulations

The IRS released final regulations in October 2010 governing cash balance and other hybrid plans under the provisions of the Pension Protection Act of 2006 (PPA)[18]. Prior to the new regulations the IRS allowed a number of "safe-harbor" interest crediting rates. The rates were generally based on bond yields of varying maturities or the Consumer Price Index. The addition of margins was also allowed in some circumstances. IRS Notice 96-8 and Internal Revenue Bulletin (1996-6) provide further details. The 2010 regulations introduced new interest crediting rate options which allow a plan sponsor to choose one of the following:

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[13] http://www.cashbalancedesign.com/articles/documents/NationalCashBalanceResearchReport2011.pdf

[14] http://www.pionline.com/article/20120523/DAILYREG/120529960/2-of-3-firms-with-db-plans-to-keep-them-open-to-new-hires

[15] http://www.pionline.com/article/20120528/PRINTSUB/305289982/cash-balance-plans-gain-favor-as-option-among-public-pension-funds. Public sector information is not included in Form 5500 data as public sector plans are not required to complete the filing.

[16] Bank of America established the first cash balance plan in 1985.

[17] The regulations also cover technical issues regarding whipsaw, age discrimination, plan termination and backloading. While these issues are important in their own right, their relevance in determining the market value of embedded cost of embedded guarantees in cash balance plan contracts is limited.

[18] http://www.dol.gov/ebsa/pensionreform.html. Among other things, PPA is the law governing pension funding requirements in the U.S.

Rates Based on Actual Investment Return

- Actual rate of return on aggregate plan assets which must be diversified so as to minimize volatility. Note that a mix of bonds and equities would be acceptable and a negative return can be credited to a participant account.
- Annuity contract rates could also be credited.
- A rate of return on a Registered Investment Company such as a mutual fund that is not reasonably expected to be significantly more volatile than broad U.S. or international equity markets such as the S&P 500, Russell 2000 or broad based international index funds. Rates of return on industry sector funds, funds that are leveraged or country specific (other than U.S.) funds do not qualify.

Rates Based on Bond Yields

- First, second or third segment corporate bond rates used for minimum funding purposes or minimum lump sum purposes[19].
- A fixed rate of return of up to 5.0%.
- IRS Notice 96-8: yields on Treasury securities of various maturities or CPI (with allowable margins).

[19] U.S. funding rules allow smoothing of high-grade corporate bond yields over a 24-month period. These rates are bucketed into maturity intervals and referred to as “segment” rates using PPA terminology. Interest rates used to calculate lump sum benefits do not allow for smoothing.

4. Cash Balance Plan Guarantees

Guarantees generally fall into one of two categories:

1) guarantee a minimum cumulative rate of return when the crediting rate is based on the actual portfolio return or an equity index only or;

2) guarantee a minimum annual rate of return when the crediting rate is bond based.

Below we describe the guarantees in more detail and provide charts illustrating the guarantee provisions.

4. 1 Guarantee 1a: Return of Pay Credits ("Capital Preservation" or "Money-Back Guarantee") Under Equity and Actual Return Crediting Rate Formulas

Equity and actual return based interest crediting rates must provide a "money-back" guarantee or "return of pay credits". What this means is that participants must receive at least the sum of their pay credits at the time of their benefit commencement. Many actuaries refer to this as a "capital preservation" requirement. See Figure 2 and Figure 3.

Note the asymmetry in Figure 2 in terms of the payoff. At the point where the sum of pay credits equals the account balance on the x-axis there is no payoff for the plan sponsor. As we move to the left in the chart, and the account balance becomes less than the guarantee the plan sponsor's obligation increases.

The money-back guarantee can be viewed as equivalent to a put option that is owned by the plan participant and underwritten by the plan sponsor. The strike or exercise price of the put option is the level of the guarantee at benefit commencement and the account balance basically takes the place of; say a stock, in a traditional equity put option.

Figure 2. Guarantee Value as a Function of Cash Balance Account (at commencement)

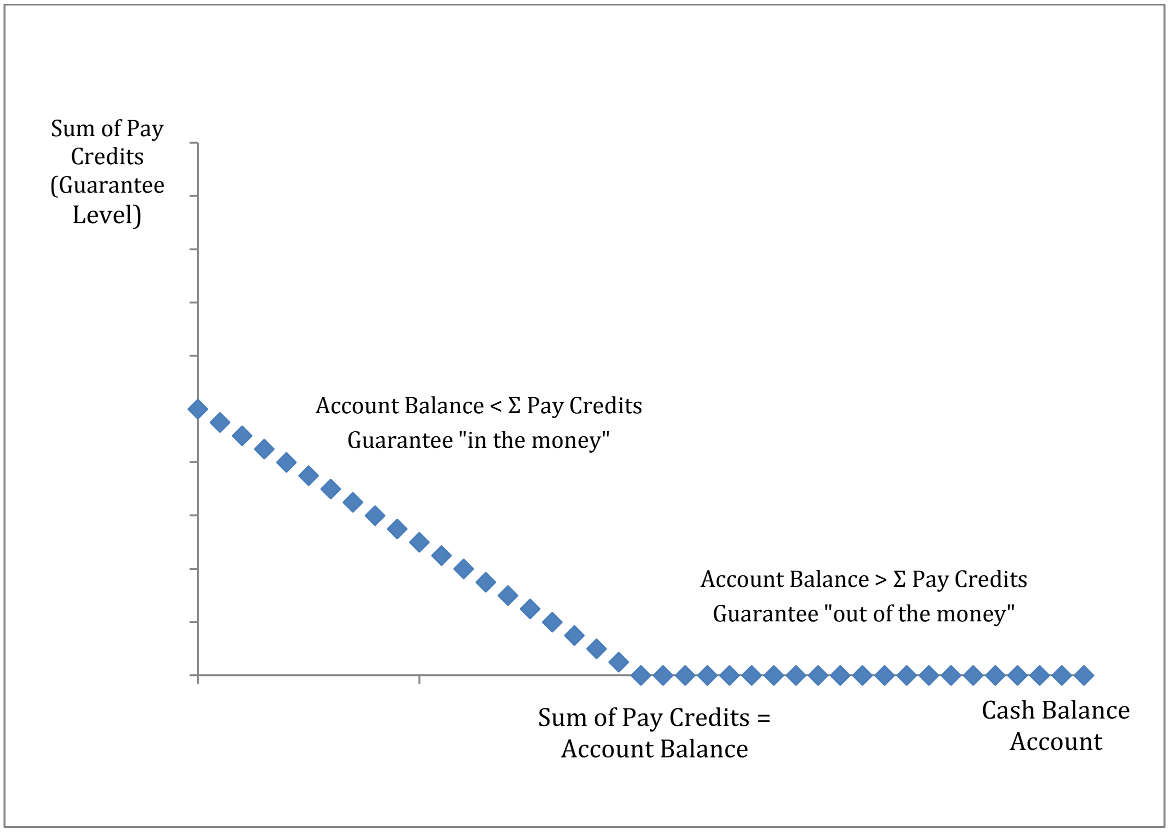
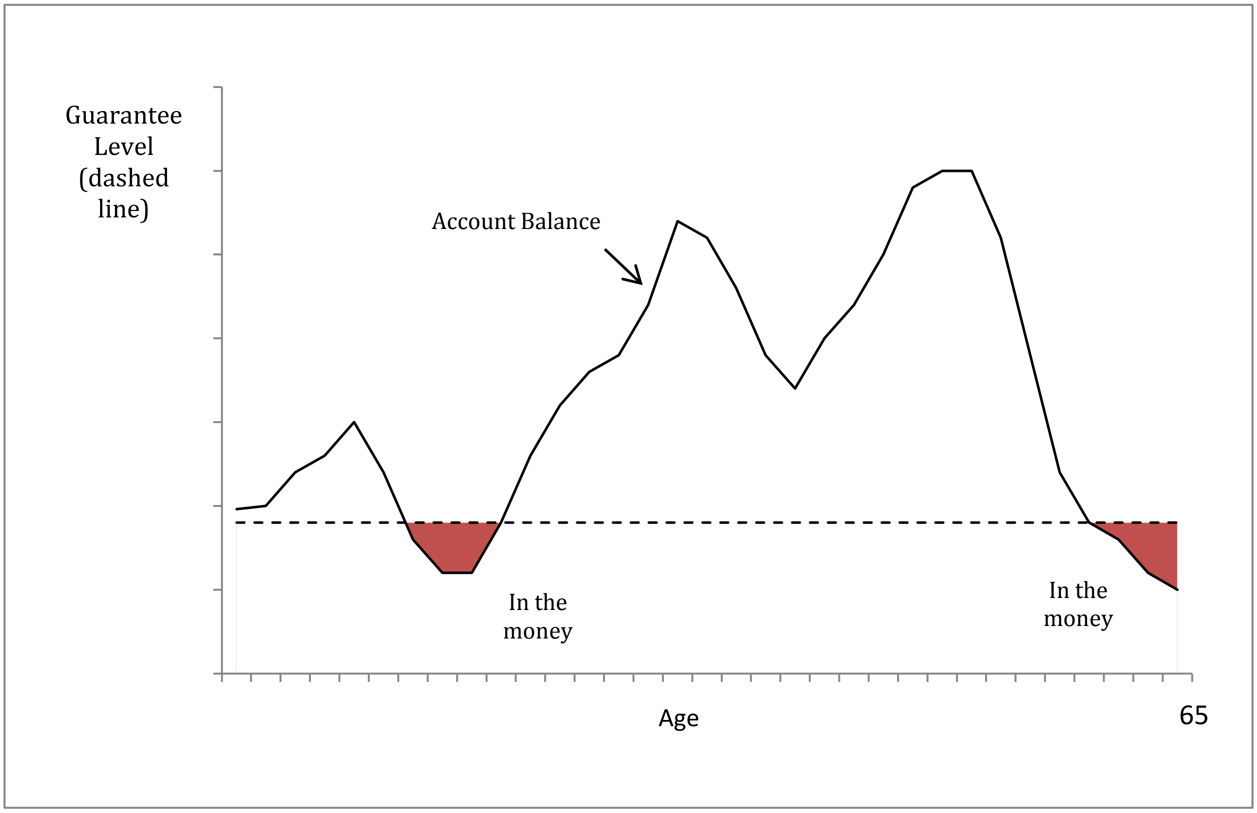


Figure 3. Return of Pay Credits with Money-Back Guarantee



4. 2 Guarantee 1b: Return of Pay Credits with Cumulative Return (Enhanced Money-Back Guarantee) under Equity and Actual return Crediting Rate Formulas

Equity based and actual return based interest crediting rates can apply a cumulative floor of up to 3.0% per year through distribution. If this guarantee is written into the plan, participants must receive at least the sum of their pay credits with up to 3.0% interest applied each year until benefit commencement. This guarantee is essentially identical to the capital preservation guarantee except that the guarantee amount will be higher at benefit commencement given the additional return on pay credits. A plan sponsor might consider such a guarantee if the goal was to provide some level of real principal protection. See Figure 4 and Figure 5. We call this an “enhanced money back” guarantee.

Like the money-back guarantee, the enhanced money back guarantee can also be viewed as equivalent to a put option except that the strike price is slightly different because the guarantee compounds over time.

Figure 4. Guarantee Value as a Function of Cash Balance Account (at commencement)

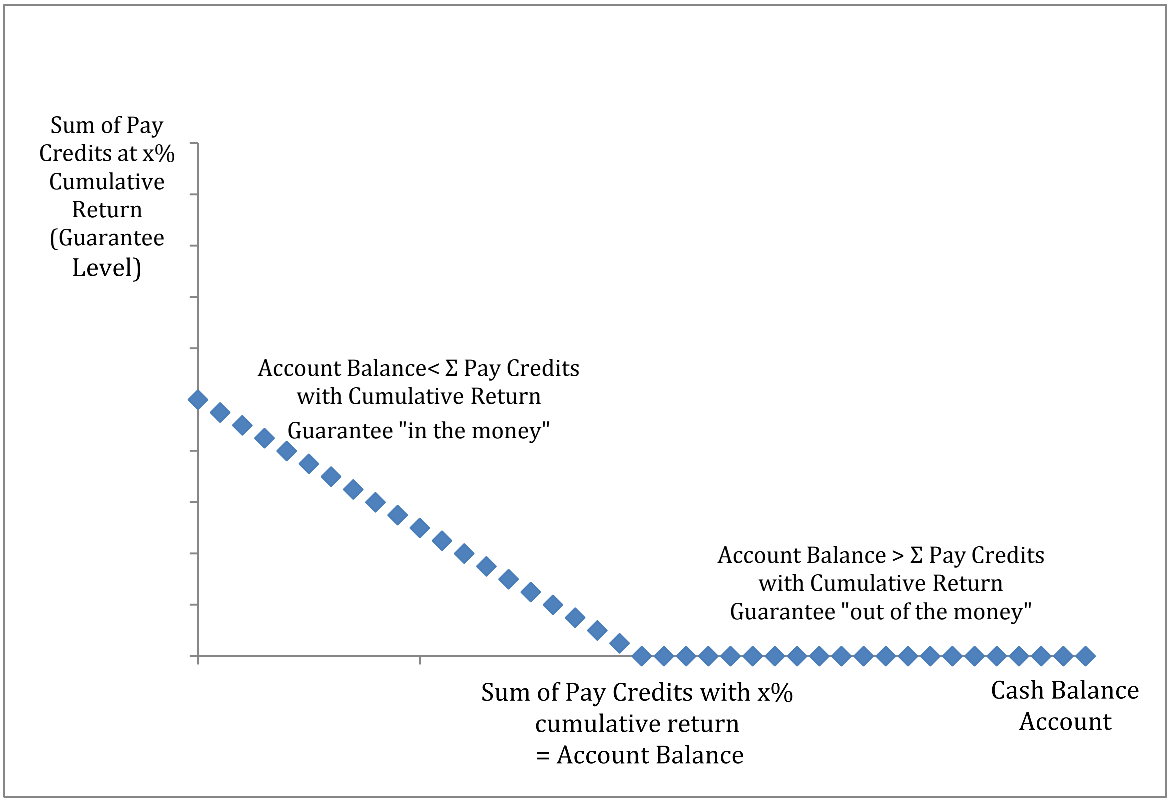
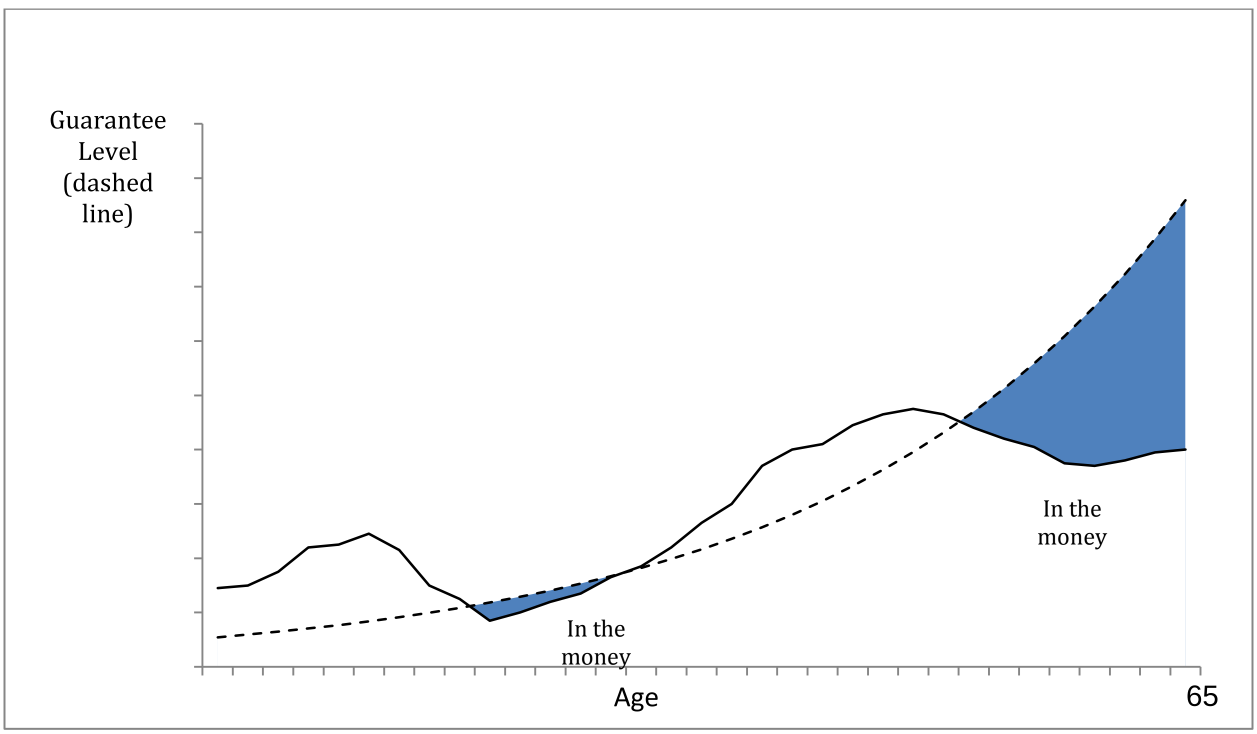


Figure 5. Return of Pay Credits with Enhanced Money-Back Guarantee



4. 3 Guarantee 2: Minimum Annual Interest Rate Guarantee

Bond yield based rates can apply an annual minimum interest crediting rate of up to 4.0%. Each year a participant's account balance would be increased by the greater of the actual yield on the bond based interest crediting rate or the minimum floor rate. See Figure 6 and Figure 7 for a graphical depiction of this guarantee.

In option terms we can say the participant holds a series of put options on the interest crediting rate because the participant has downside protection should the interest rate fall below the strike price and that protection is offered every year. One of the complications with this option type, however, is that the payoff is based upon the balance which is also a stochastic variable.

Figure 6. Illustration of Minimum Interest Credit Rate Applied Annually

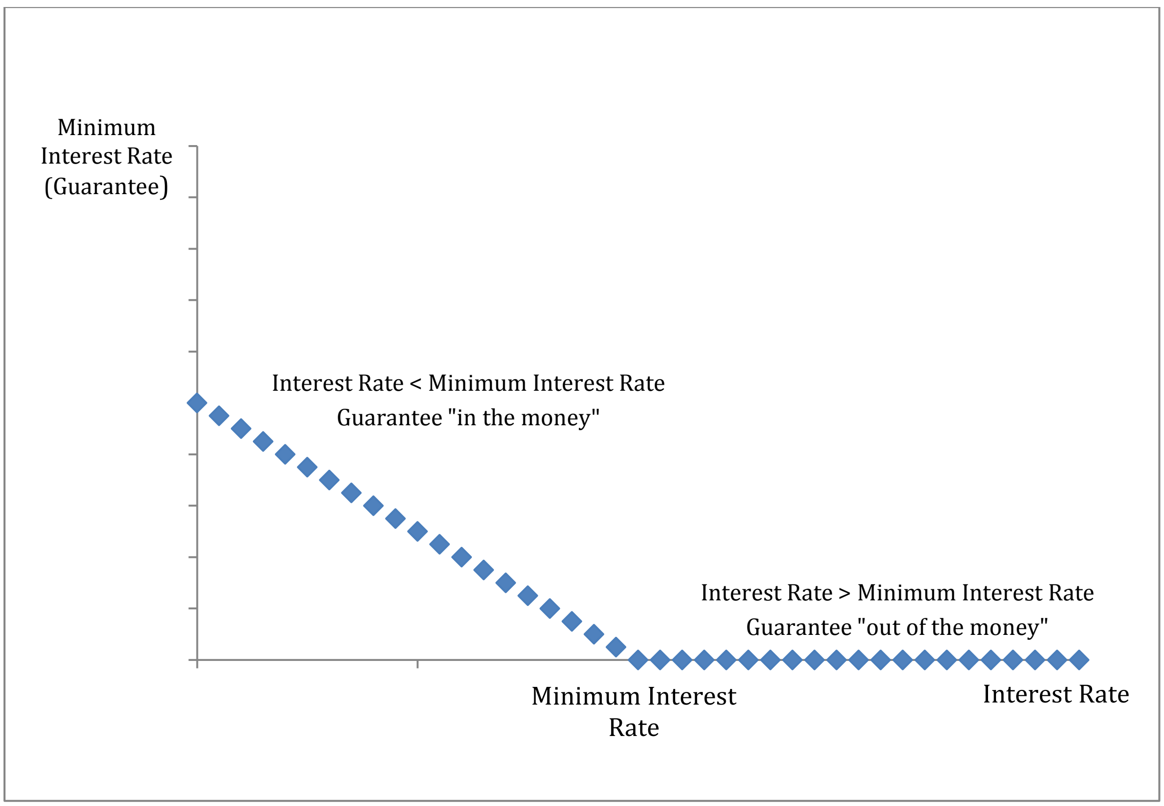
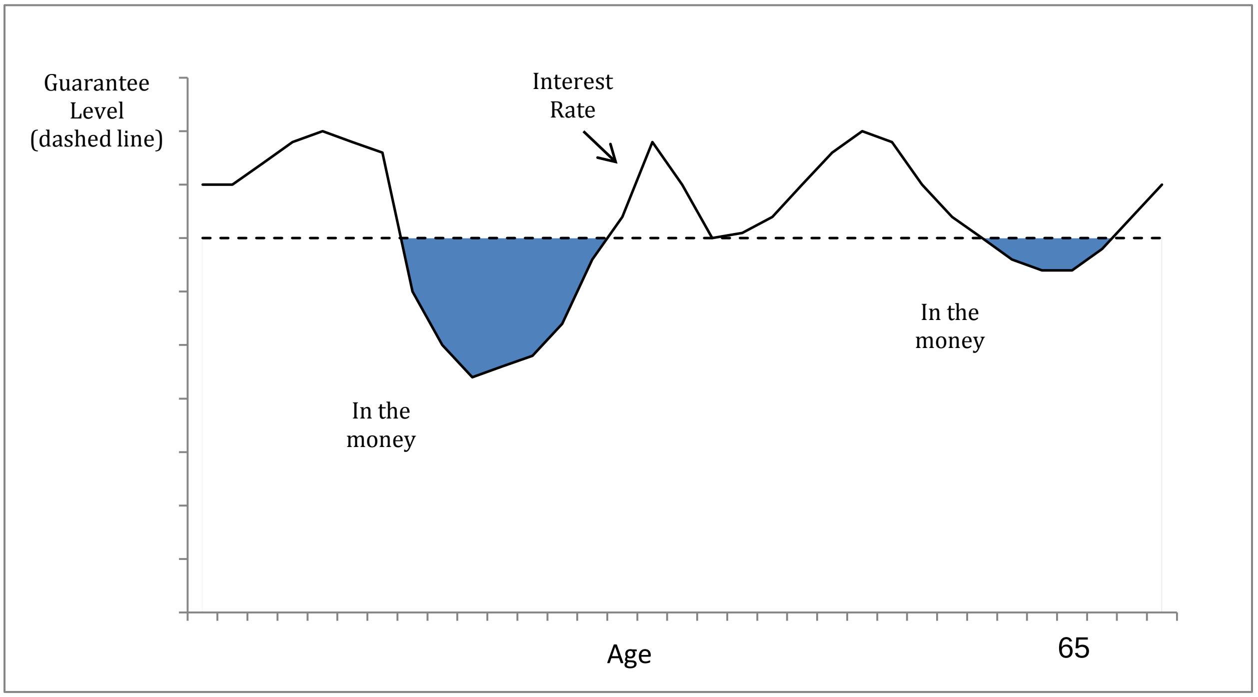


Figure 7. Guarantee Applied Annually



4. 4 Literature Review on Valuation of Guarantees in Pension Plans

The literature on derivatives and option pricing is vast[20]. Applications of option pricing to pension plan liabilities as a whole can be found in the works of Sharpe (1976) and subsequent papers by Willinger (1985, 1992), Steekamp (1999) and Wilkie (1989). Other papers on valuing defined benefit guarantees provided by the Pension Benefit Guaranty Corporation include Marcus (1987), Hsieh, Chen, and Ferris (1994), Pennachi and Lewis (1994), and Lewis and Pennachi (1999). The topic of the valuation of the specific embedded guarantees found in cash balance plans however is much more limited. We have found no prior literature directly on these topics. That said, contracts with embedded guarantees somewhat similar to cash balance plans have been previously studied in relation to defined contribution accounts as proposed under Social Security reform in the US and that already exist in Latin American countries, in German government subsidized pension plans, and in certain life insurance products.

In the US, Social Security reforms discussed in the early 2000s included establishing voluntary individual accounts as a proposed component of a reformed system[21]. Some policymakers proposed having minimum rate of return "guarantees" for defined contribution plan accumulations. Lachance and Mitchell (2002) applied option pricing techniques to value minimum rate of return guarantees and minimum benefit guarantees.

[20] See, as an example, Hull *Options, Futures, and Other Derivatives*
[21] See President's Commission to Strengthen Social Security Final Report (2001; www.csss.gov).

Many Latin American countries have adopted guarantees into their pension systems. Pennacchi (1999) applies option pricing techniques to value a fixed rate of return guarantee (similar to that provided in Uruguay), a rate of return guarantee that is relative to the performance of other pension funds (similar to that provided in Chile), and the guarantee of a minimum pension benefit for a participant in a mandatory defined contribution plan (similar to that offered in Chile). Zarita (1994) and Fischer (1999) also consider pension guarantees in Latin American countries.

In 2002, Germany introduced government-subsidized pension products that required the seller (banks, insurance companies and mutual funds) to provide a "money-back" guarantee to the buyer at the end of the contract term. The buyer is allowed to invest a series of premiums in an underlying pool of assets. While the German pension products are sold on an individual basis and are not provided by a "plan sponsor", the money-back guarantee feature is similar to the return of pay credit feature required for cash balance plans. Grundl (2004) analyzed the value of the money-back guarantee assuming premiums are paid until maturity. Kling, Russ and Schmeiser (2005) extended Grundl's analysis by considering the additional option of the buyer to stop paying premiums at any time during the life of the contract.

Cash balance plan guarantees 1a and 1b show resemblance to Guaranteed Minimum Maturity Benefits (GMMB) and Guaranteed Minimum Death Benefit (GMDB) that are typically added as riders to equity linked insurance contracts. A GMMB guarantees a policyholder a specific monetary amount at the end of the contract. The guarantee can be fixed or dynamic with increases based on equity market performance or some set of regular indexing. A simple GMMB might be the return of premiums, if at the time of maturity of the policy, the equity index has declined below its initial value when the contract was originally initiated. A GMDB guarantees a policyholder a specific monetary amount upon death. Similar to the GMMB, the GMDB can be fixed or dynamic. A simple GMDB might be the return of premiums if, at the time of death, the equity index has declined below its value when the contract was originally initiated. Hardy (2003) covers how to value the equity linked guarantee described herein and a host of others that exist in life insurance products.

Guarantee 2 shares certain characteristics with ratcheted equity indexed annuities, in particular, compound annual ratchet options that are found in the life insurance market[22]. The compound annual ratchet provides a guaranteed annual return tied to participation in an underlying equity index. The major difference between the cash balance plan guarantee and the ratcheted equity index annuity guarantee is the financial variable the guarantee is based on – equities versus interest rates. This difference has implications for the valuation of the two products which we will discuss further in this paper.

[22] See Hardy, M., *Investment Guarantees* Chapter 13 or Hardy, M. "Ratchet Equity Indexed Annuities"

In some respects the modeling of cash balance accounts is also similar to the modeling of retirement accounts (e.g. for a 401(k) plan or for a personal retirement account) to determine safe withdrawal rates and perform other types of retirement analysis. This is because, in many respects, a cash balance plan resembles a DC or personal retirement account. A good overview of the research on this topic is covered in Blanchett, Fink, and Pfau (2013).

5. Historical Analysis of Cash Balance Plan Guarantees

Using historical data we evaluated the size of the money-back guarantee (as a percent of the account balance), enhanced money back guarantee at 3.0% cumulatively applied to the sum of pay credits and the annual minimum interest credit guarantee at 3.0%. Backtesting is a commonly used approach to perform financial analysis. While it is certainly true that the future will not exactly mimic the past, understanding history is an important part of risk management[23]. Providing historical analysis also provides some context for framing our later discussion on valuation. As with any historical backtest these results represent one-trial of history and not a randomization of results but by using rolling periods we are able to build a relatively wide distribution of historical outcomes.

In terms of the money-back guarantee, one of the key reasons this option has not been seriously explored in the past is the belief that a diversified portfolio will earn positive nominal returns over the long-run and therefore over time the guarantee will be of no value. As this line of thinking has been a dominant presumption in dismissing the risks associated with cumulative guarantees it bears looking at the historical record to see if there is support for the conclusion. Certainly the relationship between risk and time horizon is one of the most hotly debated areas in finance and it is beyond the scope of this paper to address all the issues associated with this topic.

Clearly, from both an academic and intuitive perspective, it is reasonable to assume that over the long run a stock and bond portfolio will earn positive nominal returns as investors are compensated for bearing exposure to systematic risk exposures. However, while the volatility[24] of *annualized* returns declines over time, the volatility of total *compound* returns actually rises over time[25]. In other words, the potential distribution of terminal wealth (i.e. the cash balance account balance) actually gets wider over time, not narrower, while the annualized dispersion of returns converges to the expected return over time. What this means for the cumulative return guarantee is that risk actually *increases* over time rather than declines; while the probability of a loss may shrink, the magnitude of any such loss increases such that any potential shortfall gets larger over time. Of course, even if the potential distribution of terminal wealth gets wider over longer time horizons, for our purposes, all that matters, in terms of the guarantee, is that positive nominal cumulative returns (or in the case of the enhanced guarantee positive nominal cumulative returns greater than the hurdle rate) are earned.

5.1 Methodology, Data and Assumptions

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[23] Three oft cited quotes sum this up well "history does not repeat itself, but it does rhyme" (Mark Twain), "those who cannot remember the past are condemned to repeat it" (George Santanyana), and "dwell on history and you will lose an eye, ignore history and you will lose both (Russian proverb).

[24] While we recognize there are many ways to try and measure risk, here we measure it as volatility which is equal to the standard deviation of returns around the mean. Throughout the paper we use volatility and risk interchangeably.

[25] See Samuelson (1963), Kritzman(1994), Bodie (1995) and Estrada (2011) for more details

We modeled rolling returns[26] for three different portfolio types and five different discrete exit dates assuming a $100 guarantee and $100 starting account balance. Data is from 1928-2012.

In this analysis we do not assume future benefit accruals are earned each year from date of plan entry through date of benefit commencement. This line of thinking follows that of pension financial economics as described in Enderle et al. (2006). Pension financial economics takes the point of view that plan sponsors should not fund or hedge benefits until they are earned. We discuss this concept more in the valuation section of the paper.

The three different investment portfolios were modeled: 25% equity/75% bond ("conservative"), 60% equity/40% bond ("moderate"), and 75% equity/25% bond ("aggressive"). We represent equity by the S&P 500 Stock Market Index. We represent bonds by the 10 year Treasury bond. Any reference to "stocks" or "bonds" in the following section refers to these two specific asset classes.

These asset classes were chosen for simplicity, because they represent the two core building blocks that a plan sponsor would employ to construct an investment portfolio used for interest crediting purposes in a market rate cash balance plan, and are the traditional foundations for the oft-cited "60/40" portfolio. Moreover, historical data return series are available and the data is readily available. Note, we only modeled U.S. investments due to a well documented "home country" bias[27] and for simplicity[28].

We assumed annual rebalancing to the stated portfolio mix and ignored any transaction costs or other fees that might be an impact on portfolio returns.

All calculations of the money-back guarantee, the enhanced money-back guarantee, and the annual interest rate guarantee were performed annually. We did not measure interim year guarantees levels. Implicit in this approach is the presumption plan participants only enter and exit the plan at the beginning and end of the calendar year and that interest credits are only applied once per annum at the end of the calendar year.

5.2.1 Key Findings for Money-Back and Enhanced Money-Back Guarantee

[26] Returns calculated based on rolling periods, owing to the overlapping investment periods, do not represent truly independent samples. We do not believe this issue is significant for our analysis.

[27] http://www.investopedia.com/terms/h/home-country-bias.asp

[28] We acknowledge that virtually all plan sponsors do indeed diversify their portfolios outside of US-centric investments and outside of the fairly generic stocks and bonds we have presented here. Increased diversification, if implemented properly to achieve diversification in times of stress and to different factor exposures and risk premia, should help to minimize drawdowns and thus help minimize plan sponsor exposure to any guarantee losses that might emerge.

- There were certainly occurrences where the money-back and enhanced money-back guarantees would have been "in-the-money" and thus represented an additional cost to plan sponsors beyond what the plan sponsor would likely have been required to fund under current statute.

- The money-back and enhanced money-back guarantees occurred more frequently the shorter the time horizon. None of the portfolios experienced any guarantee (money-back or enhanced) losses over the 10, 20 or 30 year period. While the range of possible compound returns, and by extension the market rate cash balance plan account, widens as the time horizon lengthens, the minimum compound return, based on the historical record, is never low enough to trigger the guarantee(s) over such a long time horizon, though with the enhanced money back guarantee losses still exist 10 years out.

Figure 9. Frequency of Money-Back Guarantee Loss Expressed as Probability of Event Happening (number of times account balance exceeded sum of pay credits divided by total historical number of rolling periods)

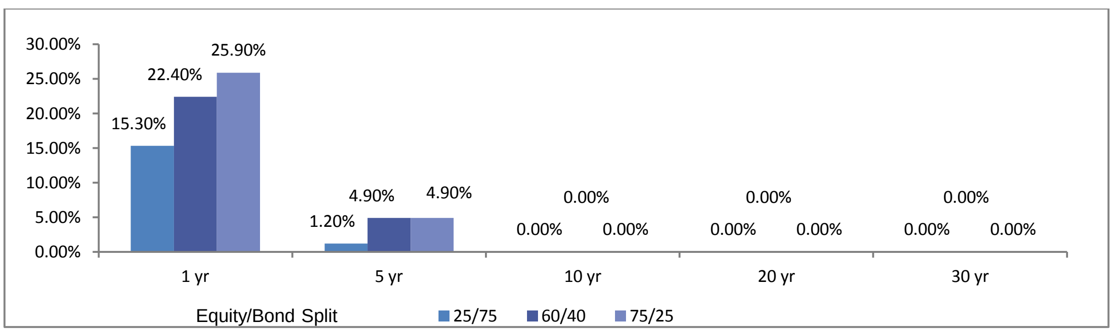
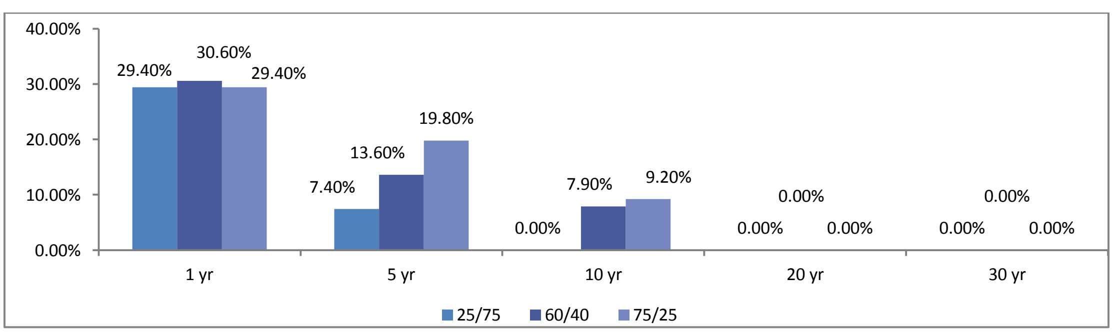


Figure 10. Frequency of Enhanced Money-Back Guarantee Loss Expressed as Probability of Event Happening (number of times account balance exceeded sum of pay credits divided by total historical number of rolling periods)



- The following chart is presented to give plan sponsors a sense for the size of losses generated over the historical period: the largest loss over the exit periods studied based on the historical record are as follows:

Figure 11. Money Back Guarantee - Maximum Loss Amount Expressed as a Percent of Account Balance

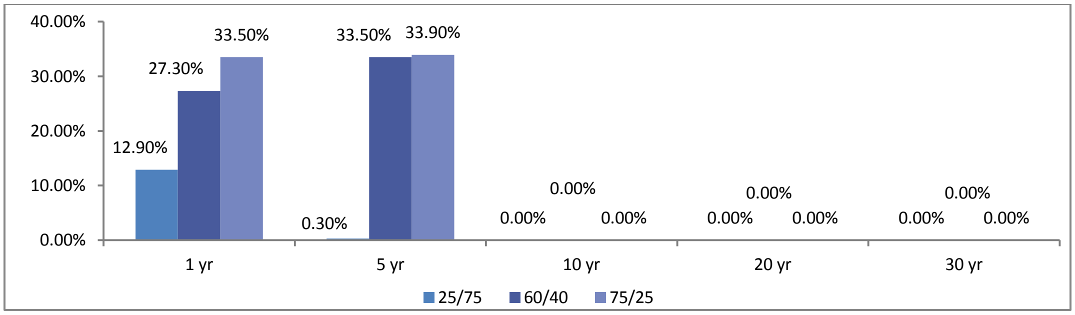


Figure 12. Money-Back Guarantee - Average Loss Amount Expressed as a Percent of Account Balance

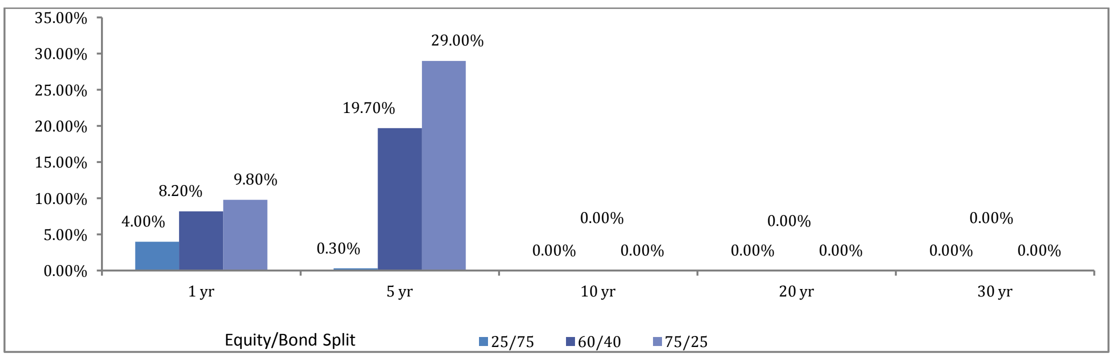


Figure 13. Enhanced Money-Back Guarantee - Maximum Loss Amount Expressed as a Percent of Account Balance

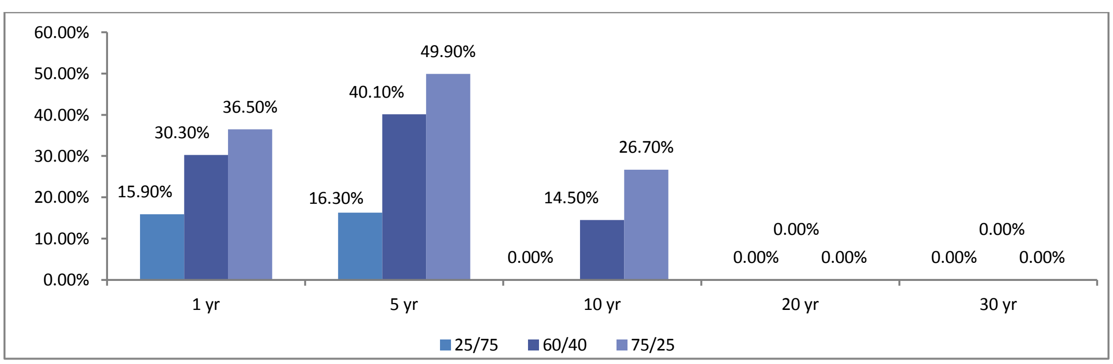
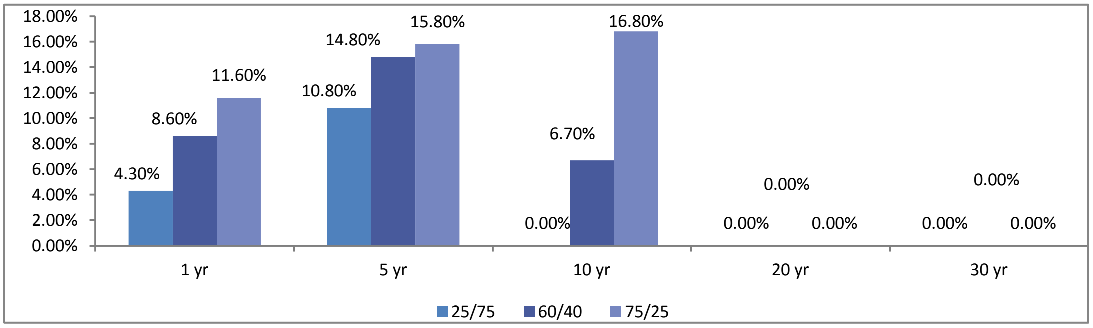


Figure 14. Enhanced Money-Back Guarantee - Average Loss Amount Expressed as a Percent of Account Balance



In the Appendix we show an example for the progression of the $100 account balance for a 60/40 portfolio over rolling 5 year historical periods. To see where the account balance would be "in the money" for each year the line is below the $100 assumed initial guarantee level.

- Not surprisingly, in the charts above, the most aggressive portfolio (defined in terms of volatility), 75% equity/25% bond, produces the greatest number of occurrences, and the highest severity of loss, where the money-back and enhanced money-back guarantee would have been exercised. This owes to stocks being more volatile and subject to greater downside risks and steeper losses than Treasury bonds over the historical testing period.

- The money-back guarantee and enhanced money-back guarantee were evaluated over some market stress periods[29] to better gauge how the guarantee would be affected under these types of scenarios:
    - 1929-1932 – Great Depression
    - 1982 Latin American debt crisis
    - 1987 Black Monday
    - 1989-1991 – US Savings & Loan debt crisis
    - 1992-1993 – European monetary system crisis
    - 1994-1995 – Mexican peso crisis
    - 1997-1998 – Asian financial crisis
    - 1998 – Russian default and collapse of Long Term Capital Management
    - 2001-2002 – Argentina default, dot-com bust, 9/11 attacks
    - 2007-2008 – Subprime mortgage crisis
    - 2008-2009 – Global financial crisis.

[29] We recognize there are different ways to measure these periods. For our analysis, we believe the dates represented are reasonable.

These events all led to significant drawdowns in account balance. In terms of the guarantees however, these events really would impact only relatively new participants who did not have the opportunity to build up their account balance prior to the severe market correction.

5.2.2 Key Findings for Annual Interest Crediting Rate Guarantee

In terms of the annual bond yield based guarantee, of the 85 years worth of closing data, 24 years existed where the 10 year bond yield was below 3.0%, the assumed annual guarantee level. We also assumed the cash balance plan credited the 10-year Treasury rate. This represents approximately 28% of the historical record used in the backtest. Bond yields remained below 3.0% for 21 years straight during the period 1935-1955. More recently, 10 year bond yields were below 3.0% in three of the last five years (2008, 2011, and 2012) and have reached near historic lows.

Many investors believe interest rates will rise in 2013 and subsequent years[30]. In these situations it's important to remember Bob Farrell's[31] Rule 9 "when all the experts and forecasts agree – something else is going to happen". Plan sponsors should likely be aware of the following tailwinds supporting the current, or even lower, bond yields:

- low inflation expectations[32]
- expected slow growth in the economy[33]
- Fed suppressing short-term rates until the unemployment rate improves to 6.5% which, through market expectations, filters into longer-term rates[34]
- Treasuries likely remaining a safe haven in times of financial and economic stress
- baby boomers shifting to fixed income investments as they retire and rebalance their portfolios
- increased investment in fixed income investments by large institutional investors like pensions, insurance companies and banks[35]
- retail investors shifting from equities, or at a minimum maintaining their exposure, to fixed income investments after experiencing flat to negative returns in equity markets over the past decade[36]

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[30] See Bernanke March 1, 2013 speech "Long-Term Interest Rates" page 23; http://www.federalreserve.gov/newsevents/speech/bernanke20130301a.htm
[31] Bob Farrell was the chief market strategist at Merrill Lynch for several decades until he retired in 1992. Bob Farrell has a famous set (among investors) of "10 market rules to remember".
[32] Market based inflation is about 2.6% as of March 2013
[33] See Bernanke speech "Long-Term Interest Rates" March 1, 2013
[34] Ibid
[35] As an example, see: http://www.pionline.com/article/20130304/PRINTSUB/303049996/ldi-bandwagon-rolling-on

- In a balance sheet recession it is common for interest rates to stay low for a long time[37]

This is not meant as a forecast. Interest rates will at some point rise as conditions change. The question is when and by how much. We believe there is certainly the possibility that interest rates will stay low or marginally decline further from current levels. In such a situation the annual interest guarantee could be costly for plan sponsors. On the other hand, a 3.0% annual hurdle rate, even in the context of 10 year Treasury yields below 3.0%, does not present an insurmountable hurdle for plan sponsors.

5.3 – A Brief Look at Forward Looking Returns

From the historical record, it also appears losses are recuperated given enough time. This should not be a surprise given the hurdle rate in our examples is not very aggressive: a zero percent cumulative nominal return in the money-back guarantee scenario and a 3.0% cumulative return in the enhanced money-back guarantee scenario. That said, practitioners should probably at least be cognizant of the fact that for the traditional 60/40 portfolio, the real returns to the portfolio over the past 10 years, were generated primarily in only a few decades (1920s, 1950s, 1980s and 1990s)[38].

However, plan sponsors and plan actuaries should note that while the historical record in the US has shown propensity to recover, in nominal terms, from maximum drawdowns, over a period of less than 10 years, international markets have not always been so kind. For example, the Nikkei 225 hit its all-time high on December 29, 1989 at an intra-day high of 38,915.87[39] and is now, 24 years later, at 13,192.59[40]. As a result, the Nikkei is down about 66% from its all time-high. Guarantees made at the peak of the market would still be "in-the-money" today. Examples similar to this exist in other countries as well[41], which suggests that the U S, to some extent, may be an anomaly in terms of returns and the historical risk premiums. This is all to suggest we ought to learn from history, probably temper our expectations, and recognize there could be more risk to plan sponsors offering investment guarantees than there has been in the past.

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[36] Even though there has been talk of a "Great Rotation" fund flow data shows most of the shift to equities is from cash accounts not bonds. Moreover, for every seller of bonds there is a buyer such that someone must hold the bond, until it is retired.

[37] See Japan as an example

[38] For more statistics on the historical record of a 60% equity/40% bond portfolio see Bradley Jones' "Rethinking Portfolio Construction and Risk Management" or http://gestaltu.blogspot.com/2013/02/balanced-portfolios-keeping-it-real.html

[39] http://en.wikipedia.org/wiki/Nikkei_225

[40] http://www.bloomberg.com/quote/NKY:IND

[41] See Credit Suisse Investment Returns Yearbook 2013 for more examples

Plan sponsors should recognize that current market conditions are likely to result in modest portfolio returns, prospectively. Lower expected returns provide less of a "margin of safety" to protect against the risk of the money-back guarantee being exercised. The chart below[42] displays the real return forecasts to a 60/40 portfolio from a number of reputable advisors. With the Shiller P/E over 2343 and the yield on the Barclays US Aggregate Bond Index at approximately 1.70%[44], it is anticipated that the average forecast for an expected real return for a 60/40 portfolio over the next 7-10 years will be only 1.80%, compared to the long term average of 5.20%. Historically, the approaches used by the practitioners and academics in the below chart have had a high degree of correlation with subsequent market returns[45]. From this, it appears there is less of a "margin of safety" to protect against the guarantees creating potential losses for plan sponsors. While there is no promise that these types of returns will be realized forewarned is forearmed and practitioners may want to consider such outcomes and volatility around these outcomes when analyzing and modeling investment guarantees on a prospective basis.

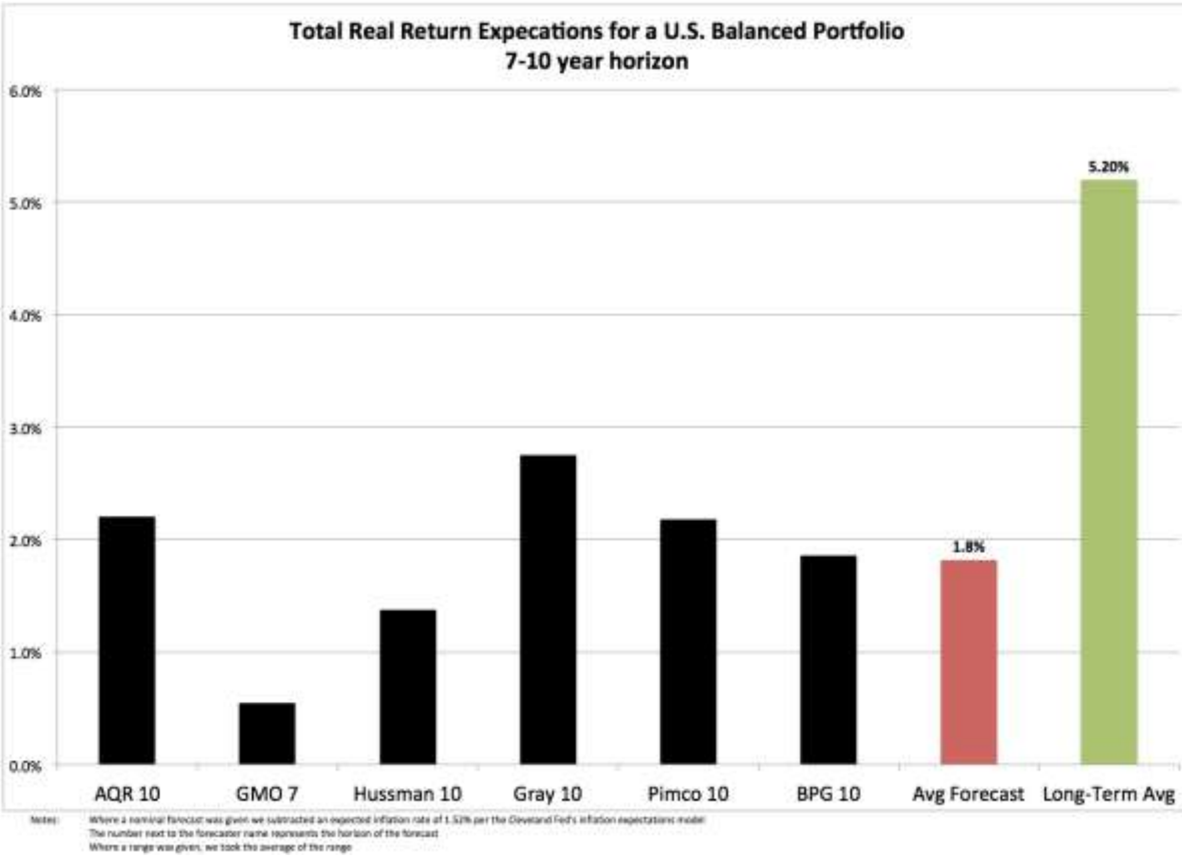
[42] http://gestaltu.blogspot.com/2012/12/dont-take-our-word-for-it.html. Not shown on the chart is an estimate from Bradley Jones at Deutsche Bank of 2.1% real returns over the next 10 years for a 60/40 portfolio, from Andrew Milligan at Standard Life of 2.2% real returns over the next 10 years (http://www.economist.com/news/finance-and-economics/21570702-useful-stab-projecting-investment-returns-over-next-decade-home) and Dimson, Marsh, and Staunton estimate about 3.5% real returns on US equities and .5% real returns on bonds over the next 20 years (see Credit Suisse Global Investment Returns Yearbook 2013)

[43] http://www.multpl.com/shiller-pe/

[44] http://us.ishares.com/product_info/fund/overview/AGG.htm

[45] For example, see http://www.hussmanfunds.com/wmc/wmc130318.htm

Figure 8 – Total Real Return Expectation for US Balanced Portfolio



It should also be noted that although most of the risk in the modeled portfolios is attributed to equities[46], Treasury bonds have exhibited negative correlation to equities over past historical crises, and have thus served to buffer, to an extent, negative market losses in times of severe stress. Given the low historical yields on Treasury bonds (and most fixed income instruments for that matter) and the low levels of inflation, it is questionable how much diversification and negative correlation benefits Treasury bonds will offer, prospectively[47]. What this means is that severity of losses going forward could be higher than the historical record indicates.

[46] A commonly cited statistic is that a typical 60% equity/40% bond portfolio, while relatively balanced in terms of dollar allocation, is not balanced in terms of risk, whereby the portfolio has about 90% contribution to volatility coming from equities and 10% contribution to volatility coming from bonds.

[47] See "The Policy Portfolio and the Next Equity Bear Market" by Bill Hester of Hussman Funds for more details on this line of thinking.

6. Can Traditional Actuarial Approaches Be Improved for Managing Investment Guarantees

Many risks in a pension plan are diversifiable so that with a large enough number of participants, the law of large numbers ensures the total liability is close to the expected value. The central limit theorem can then be used to estimate the likely variation from the expected value.

The key difference is that with investment guarantees much of the risk is actually *undiversifiable.* When an economic or market indicator, like capital market returns or interest rates or inflation becomes unfavorable, it can affect many participants at the same time.

For example, let's take a look at the minimum annual interest crediting rate guarantee to illustrate this concept. If interest rates fall below the minimum guarantee, the guarantee will be exercised for *all participants of the pension plan.* Now, while the chance yields fall below the hurdle may be an unlikely event, if it happens the cost could be significant. In this case, the risk is not diversified over a large number of independent participants. In other words adding more and more participants does nothing to diversify the risk, like it would with traditional risks such as mortality and termination. With this type of low frequency, high severity, asymmetric payoff meaning like we saw in the before charts, undiversifiable risk, expected value is not a very useful metric. What this means is that traditional deterministic approaches that, almost by definition, focus on expected values can probably be improved to properly value and manage the guarantees.

Perhaps an additional example can help explain how using expected values does not capture the cost of an investment guarantee. Say we have two market rate cash balance plans. One that offers capital preservation and one that does not. If we value both plans assuming a deterministic, best guess estimate for the rate of return on the underlying investment portfolio of, say 5%, the value of the guarantee has no worth because we're basically saying returns will always be positive. Without even performing any math or calculations, we should know sort of intuitively that this does not make sense. One would think there should be at least some value to the floor protection. Under the best estimate approach however, the cost is not captured and the option is given away for free.

What is needed in the case of investment guarantees cases is to understand the full distribution of the guarantee liability. This can be accomplished via stochastic modeling or an option-valuation based approach.

Hardy[48] sums the above up well stating "Traditional valuation is based on expected values. This relies on risks being diversifiable, so that when a sufficient number of risks are insured, the law of large numbers applies to ensure that the total liability will be close to the expected value, and the central limit theorem gives the distribution of the sum so that the likely variation from the expected value is known. The key difference with maturity guarantees is that much of the risk is undiversifiable. If equities fall to a 10-year low, payment will become due on a whole cohort of contracts. While this is an unlikely event, if it does occur the sums required to meet the liabilities could be very substantial….The expected value for such contracts is not generally a very meaningful measure. Liabilities will not be close to expected values as they are for diversifiable risks; for most contracts and cohorts the liability will be zero, for a few the liability will be substantial."

The Appendix uses an example to illustrate the concepts discussed in this section.

[48] See Investment Guarantees in Equity-Lined Insurance: The Canadian Approach

7. Generic Valuation Approaches

As this paper has discussed and Part I of this research showed[49], the guarantees written into cash balance plans can be viewed as equivalent to derivative securities based on some underlying asset or economic phenomenon and thus can be valued using option pricing theory. The literature on derivatives and option pricing is extensive[50]. If we can value financial options then we can apply these same valuation approaches to pension plan embedded options given the analogous nature of the contracts. A natural starting place is then to review how financial options are valued. Below we briefly review three common approaches to valuing guarantees consistent with the quantitative finance literature and market practice.

1) Find the equivalent option trading in the market and use that option price as the price of the guarantee: As Emanuel Derman states in *The Boy's Guide to Pricing & Hedging* "If you want to know the value of a security, use the price of another security that's as similar to it as possible. ….. Any two securities with identical payouts, no matter how the future turns out, should have identical current prices."[51] This is also related to the concept of a replicating portfolio (discussed more below). If we can determine the cost to replicate or manufacture the payouts of the guarantee by combining other securities, then the value of the guarantee should be equivalent to the cost to replicate/manufacture the equivalent portfolio.

2) Analytical (i.e. closed-form) techniques: a closed form solution means there is an exact formula that can be used to derive an option price. The best known application for option pricing is the Black-Scholes equation for valuing options.

3) Numerical methods[52]: these methods are used when a closed-form solution does not exist. Two basic numerical methods are used:

    a. Trees - the best known application for option pricing is the binomial option pricing model, which represents the underlying instrument over a period of time rather than at a single point. This method uses a binomial lattice, or tree, for a number of time-steps between the valuation and exercise date. Each node in the lattice represents a possible price of the underlying instrument, and can be used to calculate a value for the option. The option values at the

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[49] See also "Embedded Options in Pension Plans", specifically the Appendix, SOA webcast June 5, 2013.

[50] The current Society of Actuaries syllabus covers option pricing via the coursework on Exam MFE and further on the coursework to earn the FSA credential under the Quantitative Investments Track. We advise the reader who is unfamiliar with derivative pricing theory to consult the appropriate SOA exam syllabus and/or see Hull, J. Options *Futures and Derivatives* or Wilmott, P. *Paul Wilmott Introduces Quantitative Finance.* Boyle (1998) is another good source for actuaries trying to apply option pricing principles to their work.

[51] Derman's essay also covers the difference in "value" and "price". This distinction is relevant for this paper given we are trying to value a guarantee. Derman states "Price is what you pay to acquire a security; value is what it is worth. The price is fair when it is equal to the value."

[52] Other approaches exist, such as finite differences, but in our experience pricing by trees or Monte Carlo are the most familiar to actuaries.

final nodes may be sequentially calculated back to the original node to generate the current option value. This method is often used for American and Bermudian options (options which can be exercised at any time, or at a series of times, respectively). This method is also known as the backward substitution method.

b. Monte Carlo simulation - when the options have several sources of uncertainty, and/or complicated features, the binomial option pricing model is no longer able to handle option pricing effectively. Practitioners will usually switch to Monte Carlo simulations at this point. Monte-Carlo simulations are a class of computational algorithms that rely on repeated random sampling to compute their results. The method also works well when the option payoff depends on the path of the underlying variable. This method generally works as follows[53]:

   1. Simulate a random path for the underlying financial process in a risk-neutral world

   2. Calculate the payoff of the derivative

   3. Repeat steps 1 and 2 to get many sample values for the payoff from the derivative in a risk-neutral world

   4. Calculate the mean of the sample payoffs to get an estimate of the expected payoff in a risk-neutral world

   5. Discount this expected payoff at the risk-free rate to get an estimate of the value of the derivative.

[53] See Hull 2009. The approach is slightly different for interest rate derivatives. See Wilmott Introduces Quantitative Finance Chapter 29.

7. 1 Basic Concepts: Complete Markets, the No-arbitrage Principle and Risk-Neutral Pricing

Three core concepts related to option pricing are complete markets, no-arbitrage, and risk-neutral pricing[54].

1) A complete market (for financial securities) is one in which all possible securities can be constructed with existing assets without friction, i.e. without transaction costs.

   An easy example of a complete market versus an incomplete market is the call-put parity equation used in option trading.

   Call + Cash (equal to PV of the strike) = Put + Underlying Security.

   By re-arranging this equation, you can see how it is possible to synthetically construct a put by buying a call, investing the strike at the risk-free rate, and shorting the stock. But this would only be true in a world where calls, puts, cash, and the underlying security are freely and easily tradable. If calls on a particular underlying security do not trade in the market, then (in this example) this market would be incomplete.

2) Another core concept is the "no-arbitrage" condition. No-arbitrage means that if two cash flows are identical and carry the same level of risk they must have the same value. If the cash flows do not have the same value there is a chance for arbitrage[55] to occur. Option valuation relies on the no-arbitrage principle. If we can replicate by using a set of building blocks (e.g. financial securities) to construct a portfolio that has the identical payoff as the option contract, then the no-arbitrage condition means that the value of the option is equivalent to the value of the replicating portfolio[56]. If this is not the case, the overpriced asset is sold and the underpriced asset is bought leading to an arbitrage profit (because the two assets have the same exact payoff structure and risk profile). In order to replicate an option payoff the

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[54] For a recent example of these concepts applied to actuarial disciplines see Pricing and Hedging Financial and Insurance Products, August 2012 Risk & Rewards. Hardy (2002) gives a good overview of risk-neutral pricing. Many other papers exist on the topic. For example, see Jarvis, Southall and Varnell "Modern Valuation Techniques" for a deeper explanation of many of these topics from an actuarial perspective.

[55] While there are very technical and mathematical definitions of arbitrage here we use the word to mean the opportunity to make a riskless profit with no net upfront investment taking advantage of two or more securities being mispriced relative to each other.

[56] The no-arbitrage condition and the replicating portfolio concept are already used in a pension plan context. In fact, the second exposure draft of ASOP 4 released in January 2012 introduces a "market-consistent present value" that applies these principles to pension plan valuation. All we have done above is extend these principles to a guarantee whose payoff is contingent on the movement of some underlying asset

market has to be complete.

An example commonly used in the pension arena to demonstrate the principle of no-arbitrage and replication is to think about a pension plan with one sum payment of X due 10 years from now. The traditional pension actuary may estimate the expected return on the pension plan assets and discount X back to the present at that expected rate of return. No-arbitrage pricing would look to replicate the cashflow with the identical risk characteristics. A 10-year zero coupon bond would be sufficient. The value of the 10-year bond would be used as the value of the pension obligation as opposed to the discount cash flow at the expected rate of return.[57]

3) Prices of assets depend on their risk. This is normally done by adjusting the discount curve used to bring future cashflows to their present value. However, all investors have different risk preferences, and so the right adjustment to the discount rate is very difficult to quantify and justify.

However, there is another way to approach and solve this problem. In a complete market, with no arbitrage opportunities, it is possible to calculate the probabilities of future outcomes in such a way that they incorporate all investor's risk premia, and then use this new probability distribution, the risk-neutral measure to value securities. Importantly, this is opposed to using actual real-world probabilities, in which every security would require a different adjustment. In short, we can present value cashflows using the risk-neutral probability distribution and discount those probability weighted cashflows by the risk-free rate.

Risk neutral pricing follows from the concept of "no-arbitrage" and complete markets. Ho & Lee (2004) describe risk-neutral pricing "…assets that when a derivative can be replicated (by a dynamic hedging strategy) using the underlying securities, we can value such a derivative by a "risk-neutral" method, assuming the underlying security follows a risk-free drift and the expected cash flow is discounted along the risk-free yield curve. This valuation approach is also called a relative valuation model to emphasize that a derivative can be valued relative to the underlying asset's observed value."

[57] For simplicity, we ignore technical details such as liquidity and taxes so that we can illustrate the concept of no-arbitrage in basic terms.

8. Valuation of Guarantee 1a - "Money-Back" Guarantee

8.1. Model Assumptions and Notations[58]

We value the money-back guarantee for a sample participant who enters the plan at age x and exits the plan at various time horizons.

Following the financial economics approach described in Enderle, Gold, Latter, and Peskin (2006) we value the guarantee assuming no future accruals are earned. This approach considers only benefits accrued through the date of the valuation. We do not project future salaries or future service in calculating the value of embedded derivative. Under this methodology, the benefit is funded as it is accrued. Our approach is most aligned with the Traditional Unit Credit (TUC) method of funding. We use this method for a few reasons:

1) This method is prescribed under current pension funding rules[59]
2) This method is used for Pension Benefit Guaranty Corporation (PBGC) premium liability calculations[60]
3) This method is consistent with the approach an insurance company would employ if the plan were to be transferred to such an institution
4) While the projected unit credit approach (PUC)[61] is required under the current accounting regime for traditional defined benefit plans and cash balance plans, there is debate amongst pension practitioners and accountants whether PUC or TUC is the applicable valuation technique. Given that there are some solid arguments to use TUC for accounting[62], it is another reason we believe it is not unreasonable to use it to value the embedded option.

We introduce some notation to define Guarantee 1a mathematically.

Valuation date: t (assume t=0)
Cash balance account balance at time t: F_t
Guarantee amount at time t: G_t
Benefit commencement date: C

[58] Our approach draws largely from Hardy, M. 2003 "Investment Guarantees - Modeling and Risk Management for Equity Linked Life Insurance" and her description of how to value a GMMB, GMDB, and GMAB in the life-insurance world. We also draw Chen, K. and Hardy, M, 2010 paper titled "The DB Underpin Hybrid Pension Plan: Fair Valuation and Funding".
[59] http://www.dol.gov/ebsa/pensionreform.html; note these rules are applicable for private sector pension funds covered by ERISA. Plans in different sectors may be covered by different rules.
[60] http://www.pbgc.gov/prac/prem.html
[61] PUC takes into account future salary growth but not future service when valuing a pension plan. For more details see, as an example, "Fundamentals of pension accounting and funding" at www.actuary.org/pdf/pension/fundamentals_0704.pdf
[62] See **http://newsletters.soa.org/soap/issues/2009-06-05/3.html** for example

The plan sponsor's liability for the capital preservation guarantee at commencement of the benefit can be defined as:

$$\max(0, G_c - F_c)$$

where

$G_t = \sum$ Pay credits from date of participation through valuation date

As we assume no pay credits are earned after the valuation date:

$G_c = G_t$

$F_c = F_t * (P_c / P_t)$

where P refers to the underlying portfolio index value that the interest crediting rate is based on. We ignore any dividends that might be paid and base the account balance growth on a straight increase in the index price as denoted above.

8.2 Sample Illustration of Guarantee Payoff

Assume the plan participant terminates at year 5 with 100% probability and the crediting portfolio has the following evolution:

<u>Crediting portfolio</u>
Year 1 – 16.0%
Year 2 – 20.0%
Year 3 – (1.0)%
Year 4 – (37.0)%
Year 5 – 10.0%

Cash balance plan account balance at t=0: $100

Figure 15 – Sample Guarantee Payoff Illustration

Year	Portfolio Return	Account Balance	Account Balance with Money-Back Guarantee	Payoff
1	16.0%	116.0		
2	20.0%	139.2		
3	-1.0%	137.8		
4	-37.0%	86.8		
5	10.0%	95.5	100	4.5

The money-back guarantee gives rise to an extra payoff for the plan sponsor beyond what would occur if the guarantee did not exist. The extra payoff for the guarantee is represented in column 4 of the above figure. Note the payoff is only evaluated at the benefit commencement date, time 5.

8.3. Valuation of Money-Back Guarantee Using Closed-Form Adaptation of Black-Scholes Formula

The payoff structure described above is equivalent to a put option with strike price G that the plan participant owns. The plan sponsor is effectively short the put option having "sold" it to the plan participant under the terms of the pension plan contract.

By adding the value of the put option to the account balance or liability[63], the plan sponsor captures the value of the guarantee that is promised the plan participant.

Assume the plan sponsor is providing an interest crediting rate in accordance with the new regulations that is a market rate and based on a diversified portfolio P. Because we recognize the payoff to the plan participant as a European put option with strike of G, a simple and practical first step to value the guarantee would be to apply the Black-Scholes[64] formula.

Let's briefly review the assumptions that underlie the Black-Scholes option pricing formula.

These assumptions are as follows:

1) The underlying economic phenomenon follows a geometric Brownian motion with constant variance σ^2. In discrete time, the phenomenon follows a lognormal distribution with returns independent and identically distributed. In the case of valuing the capital preservation guarantee, this means the diversified portfolio (with which the interest crediting rate is based upon) is assumed to be lognormally distributed with constant volatility
2) Interest rates are constant
3) No transaction costs
4) No taxes
5) Securities are infinitely divisible
6) Short selling is allowed and borrowing and lending rates are the same

We assume that all assumptions of the Black-Scholes option pricing model hold true. In this example, the value of the guarantee, V, is calculated as:

$$V = e^{-rC} E^q[G-F_c]$$

where E^q is the expectation under the risk-neutral measure, and G and F_c are as we have described them previously and r is the risk-free interest rate.

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[63] For a market rate cash balance plan that has an interest credit that can be replicated by the plan participant, the liability, without the guarantee, should equal the account balance. We assume the plan actuary has already valued the cash balance liability, without the embedded option, appropriately.

[64] Black, F. & Scholes, M. 1973 "The pricing of options and corporate liabilities" *Journal of Political Economy* 81 (1973), 637-659

Under the Black-Scholes option pricing formula the expectation noted above can be expressed as follows:

$V = Ge^{-r(C)}[\ (1-N(d_2) - F_t(1-N(d_1)]$
$d_1 = [\ln(F_t/G) + (r+\sigma^2/2)(C)]/\ [\sigma \text{sqrt}(C)]$
$d_2 = d_1 - \sigma \text{sqrt}(C)$

N refers to the normal distribution.

Note, this formula does not factor in any decrements. The cash balance benefit is commenced upon the retirement decrement; the guarantee stays in place even if the employee decrements due to termination and defers the receipt of the benefit for a period of time or if the participant dies while working whereby we assume the spouse is entitled to the participant's full benefit at commencement.

We model time to retirement as a deterministic variable, as opposed to a stochastic variable that relates to the guarantee level. This approach to modeling decrements, as diversifiable risks, bears significant resemblance to current practice employed by pension actuaries for valuing overall pension plan obligations. We do not consider any relationships between the level of the guarantee and the decision to retire.

Let's consider decrements in the valuation equation. Let ${}_tp_x$ be the chance of survival from time 0 to time t. Let r_t be the chance of retirement at time t. Let n be the latest the participant can retire. To get the value of the capital preservation guarantee over the different ages the participant can retire, we can take the expected value over the various retirement ages. Expressed mathematically in discrete terms:

$$V = \sum_{T=1}^{n} V_t * {}_tp_x * r_t$$

8.4 Numerical Analysis

The table below shows the value of the money-back guarantee assuming a 1, 5, 10, 20 and 30 year time horizons. We assumed 100% chance of survival and retirement at each respective time horizon.

We show the value of the guarantee under six different investment portfolios. Stocks are represented as the S&P500 index. Bonds are represented by the Barclays Aggregate index. These portfolios were intended to mimic the typical split, albeit in a simplified manner, the way pension plan sponsors might structure the market rate cash balance plan interest crediting rate portfolio. We realize plan sponsors will take different approaches to their crediting rate with some leaning more towards equities, others preferring a more balanced approach, and some desiring a more bond weighted portfolio. We also wanted to stress-test the value of the guarantee under different portfolio allocations to better understand how asset allocation decisions impact the guarantee valuation.

Figure 16: Example Costs, Percentage of Starting Account Balance, Account Balance is Equal to Pay Credits at Valuation Date

		Time Until Benefit Commencement				
Equity/Bond Split	Volatility	1	5	10	20	30
100/0	15.00%	5.87%	11.19%	9.44%	4.87%	2.63%
75/25	11.00%	4.28%	7.76%	5.48%	1.92%	0.74%
60/40	9.00%	3.49%	6.05%	3.64%	0.88%	0.24%
50/50	8.00%	3.09%	5.20%	2.78%	0.51%	0.11%
25/75	5.00%	1.89%	2.69%	0.70%	0.02%	0.00%
0/100	4.00%	1.50%	1.88%	0.28%	0.00%	0.00%

The volatilities used in the table were all calculated from historical data and are what is used in the Black-Scholes formula for sigma[65]. We assumed a flat volatility structure for each asset allocation i.e. we did not have the volatility surface change based on time horizon or any other factor. We used risk free rates of .20%, .40%, .80%, 2.0%, 3.0% and 3.3%, for time horizon 1 through 30, respectively. These risk free rates are based on the yields for Treasury STRIPS at the respective maturity date, and rounded to the nearest 10 basis points. Sources for the data used to determine the volatilities and risk-free rates are provided in the Appendix.

The results from Figure 16 show what a plan sponsor would have to add to their pension liability to account for the value of the guarantee.

We also retested the "money-back" guarantee assuming different levels of initial funding, for the 60% equity / 40% bond portfolio. What we mean here is that at the valuation date the guarantee and the account balance are not equal but (e.g. $100 of guarantee and $100 of account balance) but rather the guarantee is either lower or higher than the account balance. Deviation between the account balance and the guarantee will be the norm because it would be rare for the two variables to track one another in tandem (i.e. the guarantee is increasing each year with an additional pay credit whereas the account balance is increasing each year with an additional pay credit but is also subject to investment return).

Figure 17: Example Costs, Percentage of Starting Account Balance, 60/40 Portfolio

Guarantee % of	Time Until Benefit Commencement				
Account Balance	1	5	10	20	30
60%	39.70%	35.20%	20.80%	5.40%	1.50%
80%	19.80%	18.30%	10.40%	2.50%	0.70%
100%	3.49%	6.05%	3.64%	0.88%	0.24%
120%	0.00%	0.80%	0.70%	0.20%	0.10%
140%	0.00%	0.00%	0.00%	0.00%	0.00%

8.5 Observations

[65] Market-implied volatilities could also be used if options exist on the underlying portfolio types used in the examples. The authors are not aware of these types of options being in existence. Using market-implied volatilities would give a more "market-consistent" value than what we have estimated.

- Consistent with the underlying option pricing theory, the more volatile the asset allocation the more expensive the put option offered to the plan participant.

- Option prices increase up through time 5 and then decline over time.

- Lower levels of funding have higher option costs, and vice-versa. Longer time horizons mitigate the option cost, similar to the examples where the option was "at-the-money" at the valuation date.

- With the account balance being less than the sum of the pay credits at the valuation date (Figure 17), the guarantee costs are significantly higher than when the contract is at the money to begin with. For account balances that are higher than the sum of the pay credits at the valuation date, the guarantee is virtually worthless.

  From an investment perspective, and considering just the risk management of the guarantee, this result might suggest investing conservatively at inception of a market rate cash balance so that the portfolio builds enough cushion over the guarantee to essentially render the guarantee valueless. Then, the portfolio can be shifted to more risky investments.

  On the other hand, it's arguable that more risk should be taken at the start of the plan because even if a loss does occur it is likely to be of small magnitude. Thinking about the investments in this manner is similar to the logic employed for many target date funds that allow for more risky investments the further a participant is from retirement because it is conventionally assumed that the further you are from retirement the longer a participant has to make up for drawdowns and the lower the severity of any such drawdowns. In other words, even a big drawdown in percent terms is small in absolute terms. This line of thinking is also very much linked to the topic of sequence-risk.

8.6. Monte-Carlo Simulation Analysis

In this section we simulate the liability distribution for the money-back guarantee over a 10 year time horizon by performing Monte-Carlo simulations. Practitioners could certainly choose a different time horizon, express their own views on the capital markets and take different approaches to economic scenario generation and the modeling exercise.

We performed stochastic analysis including the following steps:

- Conduct 10,000 simulated iterations of returns for one set of capital market assumptions using a lognormal distribution over a 10 year time horizon. Other asset models (e.g. regime switching lognormal) could be used especially those that incorporate fatter-tails. It is in the left tail of the distribution where the guarantees will show the most value.

- Project sample participant's account balance with simulated returns and assumption of no future pay credits
- Graph the distribution function for the net present value of the guarantee distribution
- Graph the probability density function for the net present value of the guarantee

Assumptions:

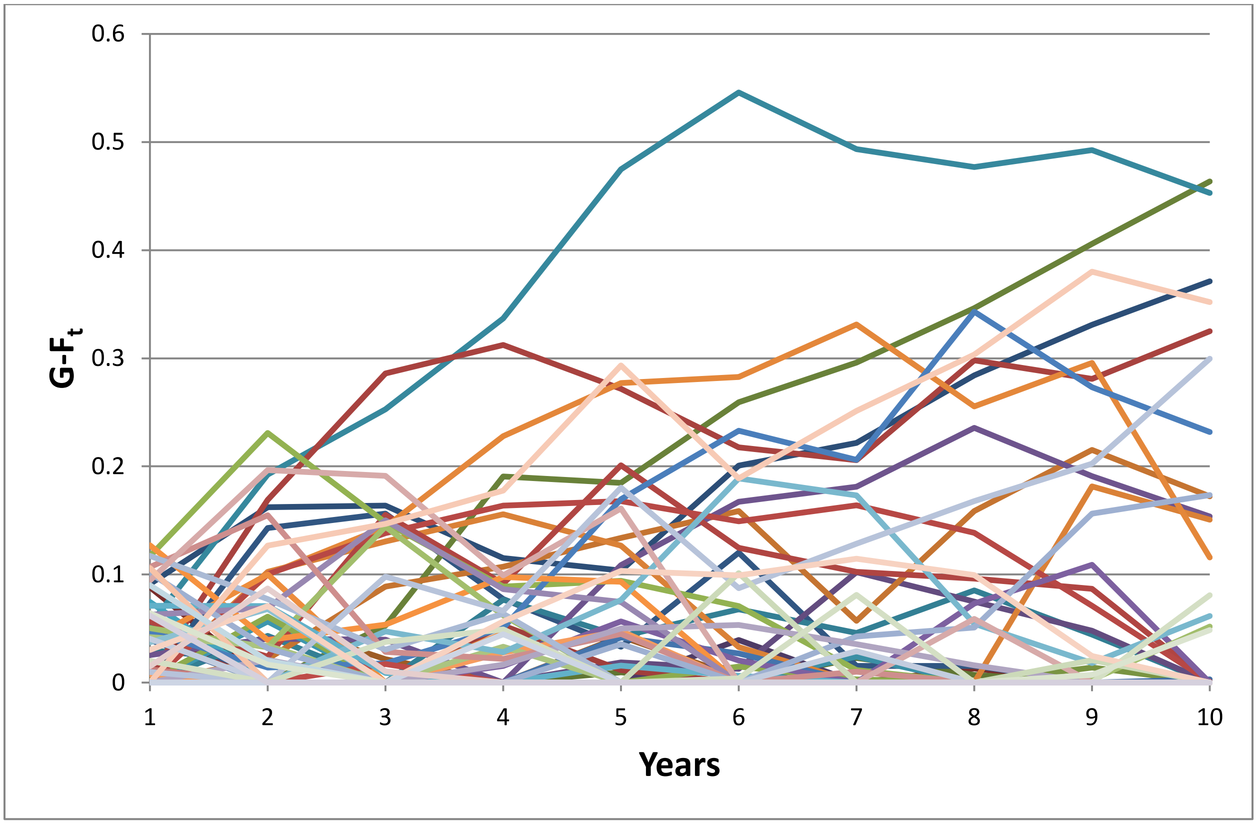
- asset mix is static and based on the 60/40 equity/bond splits used for the option pricing examples. No mortality, termination or death over the projection horizon
- Assume no taxes, transaction costs, and that portfolios are rebalanced yearly
- current capital market conditions as indicators of future expected market returns:
    - significant evidence that starting government bond yields are very good predictors of subsequent annualized bond returns, to the extent the bonds are held for a period of time approximately equal to their duration. For example, Finke, Pfau, and Blanchett[66] show an R^2 of 88% between the starting yield on the Ibbotson Intermediate-Term Government Bond and subsequent 5-year returns and an R^2 of 92% for subsequent 10-year returns. Similar results are found for high-grade corporate bond yields. We use the yield on the Barclays Aggregate index of 1.70% as an estimate of expected returns on the fixed income portion of the portfolio over the next decade[67].
    - We have already discussed prospective expected US equity market returns. We use the average equity return assumption from Figure 8, 1.80% real returns over the next 10 years, and add 2.50% inflation to arrive at an expected arithmetic nominal return of 4.3% for equities.
- Standard deviation of the portfolio is equal to the historical average we calculated for a 60/40 portfolio of 9.3%.
- Discount rate of 2.0% (a stochastic discount rate or term structure of discount rates could also be used. A constant interest rate is used for simplicity).

[66] See "The 4% Rule is Not Safe in a Low Yield World" (Finke, Pfau, and Blanchett)
[67] For simplicity, we do not make any adjustments here for defaults or recovery rates or any other adjustments. See Research Affiliates Fundamentals October 2010 "Hope is Not a Strategy" for more details

8.7. Results

In figure 18 we show a sample of 100 simulated portfolio returns and subtract the value of the guarantee. Recall, the guarantee is deterministic and set at the start of the projection period equal to the sum of the pay credits to date. All simulations were run assuming the starting cash balance account is equal to the guarantee.

Figure 18: 100 Simulated Portfolio Paths Less Guarantee; Starting Account Balance of $1, Guarantee of $1



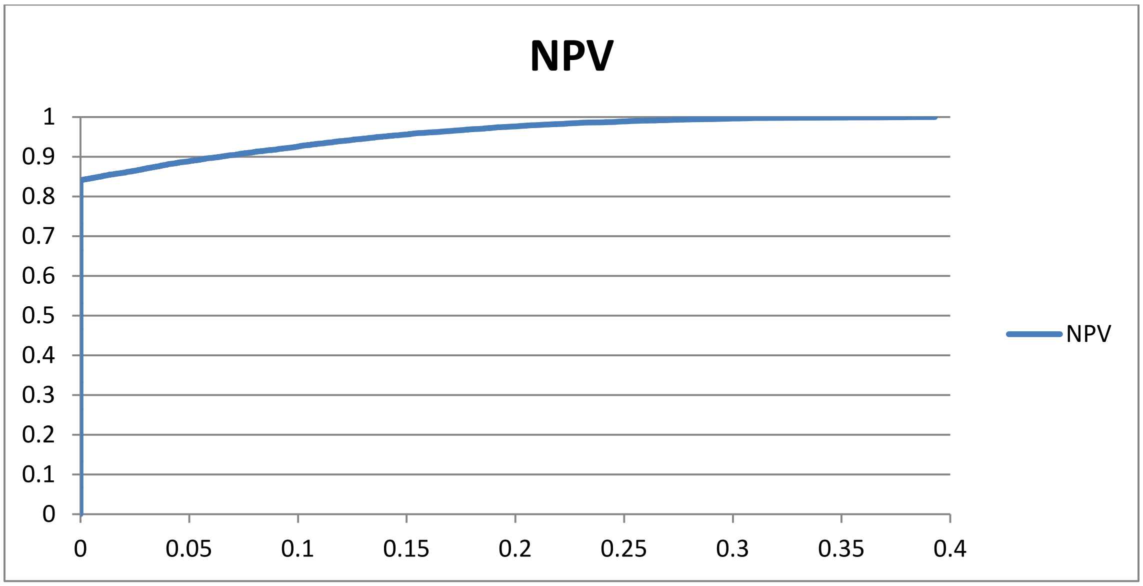
In figure 19 we show fund cash flows for one sample scenario. We can see that at year 10 the guarantee is greater than the fund value by .011. On a discounted basis, the value is .009.

Figure 19: Present Value of Guarantee Distribution for One Scenario

t-1 to t	F_{t-1}	I_t	F_t	G	$G-F_t$	Prob. of Commencement	Prob. of Death	Cashflow	Discount Factor	Present Value
0-1	1.000	-3.01%	0.970	1	0.030	0	0	0		
1-2	0.970	1.06%	0.980	1	0.020	0	0	0		
2-3	0.980	5.28%	1.032	1	-0.032	0	0	0		
3-4	1.032	9.73%	1.132	1	-0.132	0	0	0		
4-5	1.132	6.18%	1.202	1	-0.202	0	0	0		
5-6	1.202	-15.87%	1.012	1	-0.012	0	0	0		
6-7	1.012	4.39%	1.056	1	-0.056	0	0	0		
7-8	1.056	7.91%	1.139	1	-0.139	0	0	0		
8-9	1.139	-4.52%	1.088	1	-0.088	0	0	0		
9-10	1.088	-9.05%	0.989	1	0.011	1	0	0.011	0.82	0.009

We ran the scenarios 10,000 times and recorded the net present value. We assumed a discount rate of 2%, consistent with the yield on the 10 year Treasury bond and the yield we used for the Black-Scholes valuation above. The NPV ranges from $0 to .39. The average NPV is .017. There are 1,591 occurrences of a guarantee, or 15.91% of the time a guarantee amount emerged. It's worth noting that in the historical record, we showed earlier, there were 0 times where a guarantee existed over a 10-year time horizon. Our probability is significantly higher owing to the conservative, though in our estimation, not all that unrealistic, capital market return assumptions and the fact that we ran 10,000 scenarios compared to the much fewer number of rolling 10 year periods available in the historical record.

Figure 20: Simulated Distribution Function for Money-back Guarantee NPV



Decrements, more complex asset return dynamics, and/or different plan design features can all be built into the stochastic model, the key being to get an approximation for the loss distribution function so that proper planning can be performed to manage the exposure.

8.7. Possible Hedging and Risk Management Strategies

Up to this point, we have described a valuation approach for the money-back guarantee, performed numerical analysis, using some stylized assumptions and scenarios, to estimate the guarantees fair value, and have provided a framework and some Monte Carlo analysis to illustrate the plan sponsor's potential liability exposure for providing the guarantee.

The plan sponsor who offers a cash balance plan with a money-back guarantee will likely be looking for ways to manage the risks associated with such a plan. In this section, we provide some ideas for how a plan sponsor might want to go about this task. As each plan sponsor has different goals, risk tolerances/appetite, time horizons, etc. the ultimate decision on how to manage the plan will likely need to be decided on a case by case basis.

The Plan sponsor who wants to hedge the money-back guarantee may want to consider the following, either in isolation or in some type of combination:

- purchase a put option that matches the terms of what has been promised in the plan. The number of options needed to hedge the guarantee would need to be re-determined periodically, for example, at each valuation date.

- synthetically, purchase the replicating portfolio which is a portfolio that combines a a long position of $Ge^{-r(C)}(1\text{-}N(d_2)$ zero coupon bonds maturing at time C and a short position of $F_t(1\text{-}N(d_1)]$ in the underlying portfolio that the interest crediting rate is based upon. As was the case above, the replicating portfolio would need to be updated to reflect items such as plan experience and actual experience of the diversified underlying interest crediting rate portfolio.

- Another way to think about the pension promise is similar to that of an equity linked certificate of deposit (CD), although in our case the interest crediting rate is tied to a diversified portfolio and not solely to an equity market, like a traditional CD of this nature would normally be structured as. A typical equity linked CD provides the CD holder with the return of his/her principal and upside participation in an underlying equity index. A way to hedge this product, and by analogy the cash balance plan money-back guarantee, would be to invest in zero coupon bonds, of face value equal to the current guarantee value, maturing at the time the participant is expected to commence their benefit combined with the purchase of call options on the underlying diversified portfolio at a strike price equal to the starting account balance. This hedging strategy works because the zero coupon bonds provide the guarantee amount and because such fixed income investments are sold at a discount, the proceeds can be invested in call options that payoff to the extent the underlying indices rise to a value greater than the guarantee amount.

- Another alternative a plan sponsor could consider is the purchase of insurance policies to support the pension plan liabilities. A premium would be paid to an insurance company that protects against losses on the portfolio very similar to paying a premium on a home to protect against harmful events occurring. It is likely this option will be more expensive than purchasing investment securities noted below because of profit loads charged by insurance companies.

- Another option is described by Hardy in her book *Investment Guarantees.* Hardy writes on page 12, "…using a stochastic simulation to determine an approximate distribution for the guarantee liabilities, and then using a quantile reserving to convert the distribution to a capital requirement…. To calculate the quantile reserve, the insurer assesses an appropriate quantile of the loss distribution, for example, 99 percent. The present value of the quantile is held in risk-free bonds, so that the office can be 99 percent certain that the liability will be met. This principle is identical to the *value-at-risk* (VaR) concept of finance, though generally applied over longer time periods by the insurance companies than by the banks."

A similar approach could be used by pension funds. Pension funds could perform Monte-Carlo simulations similar to those run in the previous section. The plan sponsor could choose a quantile that aligns with their goals and objectives. Then, the plan sponsor could hold an amount of assets in risk-free bonds that would match the net present value of the guarantee distribution at the quantile level over the specified time horizon.

Plan sponsors could also use the Conditional Tail Expectation (CTE) instead of a quantile measure as CTE is a more robust risk measure[68].

For example, from Figure 20 we can see the 95th percentile NPV is approximately .14 (or 14% of the opening account balance/liability). If the plan sponsor wanted to be 95% certain they would have enough funds to ensure payment of the guarantee 95% of the time, the plan sponsor could invest .14 in risk free assets (e.g. 10 year zero coupon bonds) earning 2.0%.

Evaluating other risk measures of the guarantee distribution, like kurtosis and skewness, to better analyze the behavior of the guarantee is something plan sponsors may also wish to consider to inform decision makers with respect to portfolio allocation and other plan design features.

Other alternatives the plan sponsor could consider in terms of managing the risk of the guarantee, either solo or in conjunction with the approaches described above, are as follows:

- Increase funding to the plan to account for the cost of the money-back guarantee provision. The plan sponsor is recognizing a guarantee has been granted, is assigning a value to that guarantee, and is funding the cost of that extra promise. This can be thought of as similar to self-insurance. Each year the plan sponsor could evaluate the option cost for each plan participant and determine if additional funding is needed.
- Design an investment strategy that minimizes the risk of capital losses. Investment strategies such as real-return and absolute return may be appealing in this regard. Tactical/adaptive asset allocation may also make sense.

[68] See Artzner – Coherent measures of risk

9. Valuation of Guarantee 1b – "Enhanced Money-Back" Guarantee

The plan sponsor's liability for the enhanced money back guarantee is defined as:

$$\max(0, (G_t*(1+return)^\wedge(C-t))- F_T))$$

where t, T, and F_T have been defined previously

Everything is the same between Guarantee 1a and 1b except that the strike price compounds over the expected time horizon. Therefore, as we did with Guarantee 1a, we can again apply the Black-Scholes formula as a practical closed form solution to value the enhanced money back guarantee. However, to make the formula work we have to make one change. We have to change the strike price from the pay credits, G, to the pay credits with a compound, deterministic return, G*(1+return)^(C-t).

Figure 21: Example Costs, Percentage of Starting Account Balance, 60/40 portfolio, With Guarantee Increasing at Varying Enhanced Percentages

Enhanced	Time Until Benefit Commencement				
Percentage	1	5	10	20	30
0%	3.49%	6.05%	3.64%	0.88%	0.24%
1%	4.02%	8.55%	6.65%	2.73%	1.24%
2%	4.59%	11.65%	11.21%	7.04%	4.75%
3%	5.21%	15.34%	17.56%	15.44%	14.01%

We can perform similar Monte-Carlo analysis and apply similar risk management and investment ideas to the enhanced money-back guarantee as well. Because of the similarity in approaches we do not present detailed results for Guarantee 1b.

10. Valuation of Guarantee 2

10.1. Model Assumptions and Notations

We now consider the valuation of an annual minimum interest rate credit guarantee, Guarantee 2. Guarantee 2 is similar to Guarantee 1a and Guarantee 1b except that Guarantee 2:

- is in effect *every year* whereas guarantee 1a and 1b can only be exercised at *distribution* of the account balance. What this means is that the path the underlying interest crediting rate takes is relevant to the guarantee valuation whereas with Guarantee 1a and 1b the path did not matter; all that matters with Guarantee 1a and Guarantee 1b is the ultimate level of the account balance at commencement;
- has an underlying interest crediting rate based on the yield of a fixed income security instead of based on the return of a diversified portfolio. What this means is that we need a model to simulate interest rate yields.

We follow the same approach as we did for Guarantee 1a and Guarantee 1b and assume no further pay credits, beyond those already credited to the plan participant as of the valuation date, are granted to the plan participant by the plan sponsor[69].

While the analysis that follows is specifically for a floor interest crediting rate, the valuation approach could be extended to value caps on the interest crediting rate and caps and floors, in combination, on the interest crediting rate. We use the word "floor" to be synonymous with the annual minimum interest crediting rate.

We again introduce some notation and formulas to define Guarantee 2 mathematically: we denote with $\mathrm{F_t}$ the current cash balance plan account balance at time t, R_t^M the M year US Treasury bond yield at time t, k the annual minimum interest crediting rate guarantee, and C the commencement date.

The cash balance plan account balance at time t=1,…,C and with t measured in years grows according to the following expression:

$$(1)\ F_t = F_{t-1}(1 + \max(k, R_{t-1}^M)).$$

From the above equation, we see that the evolution of $\mathrm{F_t}$ is stochastic given that the yield, R_t^M, is not known for any t>0. Furthermore $\mathrm{F_t}$ is path-dependent because the balance at the end of the simulation horizon (e.g. 5, 10 or 30 years) cannot be calculated in terms of the distribution of the Treasury yield at C years, but depends on the actual path the Treasury rates follow. In the above equation, the term R_{t-1}^M indicates that the pension plan sponsor

[69] This assumption does not affect the generality of the model. While modeling additional pay credits would introduce more complexity, especially if those pay credits were to be modeled stochastically, the principles discussed here would be the same. The guarantee values expressed in this paper would be greater given the account balance would grow larger in size due to the additional pay credits. That said, expressing the guarantee as a percent of the account balance plus pay credits would likely lead to roughly the same percentage as expressing the guarantee as a percentage of the initial account balance without additional pay credits.

pays the M year Treasury yield for the previous year, for example, the yield at 12/31 of the prior year.

The theoretical net present value of the entire cash balance plan account, i.e. the "economic liability" or "discounted expected account balance", is given by the following expression:

$$(2)\ L = F_0\, E^q\left[e^{-\int_0^C r_s ds}\prod_{t=1}^{C}(1+\max(k, R_{t-1}^M))\right],$$

where the expression $e^{-\int_0^C r_s ds}$ represents the discount factor from year C to zero[70] and $E^q[*]$ represents the expectation (under the risk neutral probability), in this case applied to the sum of future cashflows from the pension plan, assuming the account balance is paid out as a lump sum at commencement, C.

The above equation is saying that the liability for the cash balance plan is the expected value of the present value of the final plan balance. While this concept of discounting cashflows is familiar to pension actuaries, the part that might not be so familiar is that the discounting is done under what is called the "risk-neutral" measure. We discussed this concept briefly earlier in the paper. While an in-depth discussion of the mathematical complexities and the theory behind risk neutral pricing (aside from what was described earlier) is outside the scope of this paper, the important take-away is that risk neutral pricing is appropriate for producing a market consistent, i.e. fair value, for the cash balance plan liability.

Note, similar to the approach for Guarantee 1a and 1b, we have ignored decrements for illustrative purposes, but decrements could fairly easily be built into the equation.

The plan sponsor's liability for the annual guarantee as the valuation date can be defined as:

$$(3)\ V = F_0 E^q\left[e^{-\int_0^C r_s ds}\left(\prod_{t=1}^{C}(1+\max(k, R_{t-1}^M)) - \prod_{t=1}^{C}(1+R_{t-1}^M)\right)\right].$$

This equation moves away from valuation of the whole cash balance plan liability and focuses solely on the discounted cashflows (under the same risk-neutral measure) that arise due to the annual minimum interest crediting rate guarantee. Equation 3 states that we can 1) present value the cash balance plan with the annual interest crediting rate floor and, 2) present value the cash balance plan without the annual interest crediting rate floor

[70] The discount rate cannot be extracted from the expectation because the discount rate is correlated with the Treasury rate and the Treasury rate drives the payoff of the guarantee. In other words, the discount rate itself is stochastic. In the short rate models the Treasury rate is a function of the short rate. We describe short rate models in more detail later in the paper.

and, 3) take the difference in those two values in order to calculate the value of providing the annual minimum interest credit.

10.2. Sample Illustration of Guarantee Payoff

Before moving on to the valuation exercise, it is instructive to go through an example showing how the guarantee actually works.

Assume the plan participant terminates at year 5 with 100% probability and the interest crediting rate is based on the 30 year Treasury bond yield which has the following evolution:

30 year Treasury bond yield
Year 1 – 6.0%
Year 2 – 2.0%
Year 3 – 1.0%
Year 4 – 7.0%
Year 5 – 10.0%

Cash balance plan account balance at t=0: $1000

Figure 22: Sample Illustration of Guarantee Payoff

Year	**0**	**1**	**2**	**3**	**4**	**5**
30 Year Treasury Yield	6.0%	2.0%	1.0%	7.0%	10.0%	
Account Balance without Guarantee	1000.00	1060.00	1081.20	1092.01	1168.45	1285.30
Annual Interest Rate Guarantee	6.0%	3.0%	3.0%	7.0%	10.0%	
Account Balance with Guarantee	1000.00	1060.00	1091.80	1124.55	1203.27	1323.60
Account Balances Difference						38.30

The annual minimum interest rate guarantee gives rise to an extra payoff for the plan sponsor beyond what would occur if the minimum floor did not exist. The extra payoff for the guarantee is represented in the fifth row of the above figure and is a function of the plan balance evolution which is driven by the evolution of the 30-year interest crediting rate.

Although the annual minimum interest guarantee is in effect in year 2 and year 3, as we are assuming 100% chance of commencement at year 5, we show only one payoff, $38.30. In short, the plan sponsor owes the plan participant $1323.60 at commencement, an increase of $38.30 over what would be owed to the participant without the annual minimum interest guarantee.

10.3. Valuation of the Annual Minimum Interest Rate Guarantee Using Monte Carlo Simulation

As discussed previously in the paper, path dependent options are well suited for valuation using Monte-Carlo techniques. Because the annual minimum interest rate guarantee is path dependent, and because the intended audience of this paper is quite familiar with Monte-Carlo simulations, we use this approach to value the guarantee[71,72].

As we noted in Part I of this paper, the annual minimum interest rate guarantee does share certain characteristics with equity indexed annuities, in particular, compound annual ratchet options that are found in the life insurance market[73]. Hardy shows a closed-form solution for the compound annual ratchet in her paper "Ratchet Equity Indexed Annuities". It would be useful if the formula provided in that paper could be adapted to the annual minimum interest guarantee for cash balance plans. This would forgo complex Monte-Carlo simulations and follow an approach similar to what was used for Guarantee 1a and 1b where closed form and simpler approximations were possible.

The problem in adapting Hardy's closed form formula is that interest rates exhibit mean reverting properties and are time dependent. With these properties, no closed form pricing expression for the value of the guarantee is available. In Hardy's paper, the closed form formulation, is appropriate given the underlying process (equity prices) follows a geometric Brownian motion (GBM) and that interest rates are held constant, i.e. interest rates are time invariant. This keeps the guarantee payoff distribution in the compound ratchet premium example log-normal and thus a natural fit for an application of the Black-Scholes formula. Unfortunately, the cash balance plan minimum annual interest rate guarantee makes it inconvenient to use the GBM model[74] and to simplify the valuation of the guarantee using a closed form solution.

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[71] Readers who are more interested in the suitability of each valuation technique to different option types are encouraged to reference the derivatives textbooks previously cited.

[72] Trees could also be used to do the valuation but we avoid this approach because the intended readers of this paper are likely less familiar with this technique. Moreover, given the path-dependency, recombining lattices/trees cannot be used, making trees a less attractive valuation technique.

[73] See Hardy, M., *Investment Guarantees* Chapter 13 or Hardy, M. "Ratchet Equity Indexed Annuities" for more details.

[74] In the pricing of interest rate linked contracts, it is necessary to distinguish between what is simulated by the stochastic model (the short rate in our case) and what is the underlying for the contract (the M year Treasury bond yield). These two variables are connected but are logically different things. Given a certain dynamic for the short rate, it is usually possible to calculate an expression for the interest yield at a given maturity. For example, in the Rendleman and Bartter model (also known as Dothan model) although the short rate is modeled by a GBM, the expression used to calculate bond prices and other interest rates (i.e.. the 30 year yield) is computationally very challenging involving trigonometric and hyperbolic functions, gamma function and modified Bessel function of the

Now that we see that an adapted close form solution is not readily available, we move on to value the minimum interest guarantee using a Monte-Carlo simulation approach.

To perform the valuation via Monte-Carlo simulation, we do the following[75]:

1) simulate the short rate evolution under the risk neutral distribution up to the commencement year C using the Hull-White model (See Appendix for a more in-depth discussion of the short rate and the model);

2) once we simulate the short rate dynamic, calculate the M year Treasury bond yields at every year 1,…,C;

3) determine the evolution of the account balance with the annual minimum interest guarantee and the evolution of the account balance without the annual minimum interest rate guarantee;

4) determine the difference in account balances with and without the guarantee at assumed commencement date; this is the extra amount the plan sponsor would be responsible for at benefit commencement owing to the existence of the guarantee;

5) calculate the average interest rate (the short rate) realized up until the commencement date

6) perform many more realizations of 1-5

7) for each simulation discount the payoff by the average rate in the simulation to determine the present value of the payoff

8) Calculate the average present value of the payoff over all simulations

To put some numbers around this, refer back to the sample illustration discussed above, Figure 22. The 30 year Treasury yields in that example would have been derived from the simulated short rates and would represent one path in our simulation. The payoff of $38.30 in year 5 would be discounted back at the average rate along the simulated path. This would give us the present value of the payoff. If the short-rate average of the simulated path is 3.0% we would calculate a present value of $33.03. We would run a series of these simulations and compute the present value for each simulation. We would then average those present values. The resulting average would be the estimated price of the guarantee.

Using the simulation methodology, we can express mathematically the theoretical net present value of the entire cash balance plan account as follows:

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second kind. This fact, in addition to the observation that interest rates do not really behave like a GBM, makes us consider other interest rate models than Rendleman and Bartter.

[75] See Wilmott – Introduction to Quantitative Finance Chapter 29

$$(4)\ L = F_0 \frac{1}{W} \sum_{\omega=1}^{W} e^{-\int_0^C r_s ds} \prod_{t=1}^{C} (1 + \max(k, R_{t-1}^M)).$$

Where W is the number of simulated scenarios and ω represents the ω-th scenario. This is the same formula as equation (2) where we have simply used the average of the simulation runs as an approximation of the expectation, E^q . In the Monte Carlo simulation, the integral $\int_0^t r_s ds$ is calculated numerically as opposed to integrating directly.

With the same procedure, the liability arising from the annual minimum interest crediting guarantee can be calculated by differencing the present value of the cash balance plan with and without the guarantee as follows:

$$(5)\ V = F_0 \frac{1}{W} \sum_{w=1}^{W} e^{-\int_0^C r_s ds} \left(\prod_{t=1}^{C} (1 + \max(k, R_{t-1}^M)) - \prod_{t=1}^{C} (1 + R_{t-1}^M) \right)$$

Expression (5) represents the numerical counterparty of expression (3) and describes, in mathematical terms, what we have written in Steps 1-8.

10.4. Numerical Analysis

Now that we have a valuation map, we perform some Monte-Carlo simulations to estimate the value of the guarantee.

We assumed a valuation date of January 1, 2013 and we used February 2, 2013 market data to calibrate the simulation model. We assume the short rate follows the Hull and White process described in more detail in the Appendix.

The parameters used in the model were calibrated against swaptions. The results of the calibration are as follows: alpha=0.022 and sigma=0.0085. We discuss calibration in more detail in the Appendix.

The important point here is that we have generated a risk neutral, arbitrage-free interest rate model that is appropriate for the valuation of interest sensitive guarantees and that is commonly used in financial engineering. Commercial providers exist that can provide these scenarios to pension practitioners who cannot create the models on their own.

We ran different simulations for every combination of the following parameters. This allows us to evaluate the floor under a multitude of conditions and to test the robustness of results.

- Interest crediting rate:
  - discount rate on 3 month Treasury bill
  - yield on 1 year Treasury Constant Maturities
  - yield on 10 year Treasury Constant Maturities
  - yield on 30 year Treasury rate maturities
- Floor (k):

        - 1.0%
        - 2.0%
        - 3.0%
        - 4.0%.
    - Commencement date, C, in years:
        - 5
        - 10
        - 30

As was the case with Guarantee 1a and 1b we assumed a 100% chance of survival to point C.

We assumed an initial starting account balance at the valuation date of F_0=$1,000.

Results are shown in the tables below where we calculate the net present value of the cash balance plan contract under the risk neutral probability measure both with and without the floor guarantee.

The value of the minimum annual interest rate guarantee can be calculated by subtracting the column indicated showing a 0% floor from one of the other columns showing a floor ranging from 1% to 4%. For example, the additional cost of the plan sponsor providing a floor guarantee of 3% for 5 years, using a 1 year Treasury rate as the underlying interest crediting rate is equal to $1,113-$1,000 or $113.

Figure 20: Estimated Valuation of Guarantee for a Simulation Horizon of 5 Years for Several Treasury Rate Maturities (rows), and Several Guarantee floors (columns).

M \ K	0%	1%	2%	3%	4%
3 month	993.8308	1026.211	1065.431	1113.427	1166.737
1 year	1000.228	1028.26	1066.644	1113.98	1166.934
10 year	1069.379	1071.95	1085.81	1121.256	1169.073
30 year	1090.186	1090.826	1095.737	1122.353	1168.79

Figure 21: Estimated Valuation of Guarantee for a Simulation Horizon of 10 Years for Several Treasury Rate Maturities (rows), and Several Guarantee Floors (columns).

M \ K	0%	1%	2%	3%	4%
3 month	985.3626	1032.485	1089.961	1168.727	1267.044
1 year	1000.585	1041.864	1097.34	1174.017	1270.51
10 year	1096.409	1105.097	1130.352	1189.246	1276.008
30 year	1118.878	1122.578	1136.391	1184.869	1270.727

Figure 22: Estimated Valuation of Guarantee for a Simulation Horizon of 30 Years for Several Treasury Rate Maturities (rows), and Several Guarantee Floors (columns).

M \ K	0%	1%	2%	3%	4%
3 month	963.4082	1092.09	1230.00	1447.92	1772.68
1 year	1000.79	1125.15	1260.53	1475.46	1796.92
10 year	1130.404	1202.14	1297.89	1486.32	1791.10
30 year	1243.65	1276.30	1341.46	1500.89	1791.03

Figure 23: Value of the Annual Minimum Interest Guarantee Expressed as a Percent of Liability (without the guarantee, i.e. 0% floor).

	C=5				C=10				C=30			
M \ K	**1%**	**2%**	**3%**	**4%**	**1%**	**2%**	**3%**	**4%**	**1%**	**2%**	**3%**	**4%**
3 month	3.3%	7.2%	12.0%	17.4%	4.8%	10.6%	18.6%	28.6%	13.4%	27.7%	50.3%	84.0%
1 year	2.8%	6.6%	11.4%	16.7%	4.1%	9.7%	17.3%	27.0%	12.4%	26.0%	47.4%	79.6%
10 year	0.2%	1.5%	4.9%	9.3%	0.8%	3.1%	8.5%	16.4%	6.3%	14.8%	31.5%	58.4%
30 year	0.1%	0.5%	3.0%	7.2%	0.3%	1.6%	5.9%	13.6%	2.6%	7.9%	20.7%	44.0%

10.5. Observations

- Before even considering the annual minimum interest rate guarantee, note the liability associated with the starting account balance of $1,000 that provides an interest crediting rate of the 10 year or 30 year Treasury bond does not equal $1,000. This is because the plan sponsor is already providing a *subsidy* to the plan participant. It is a subsidy because it is virtually impossible to guarantee the yield associated with the 10 year or 30 year Treasury bond, per annum, yet that is what is being promised to the plan participant.

  If the plan sponsor were to invest in a 10 year or 30 year Treasury instrument thinking they will earn just the yield, the plan sponsor would be ignoring the significant duration risk associated with these types of fixed income investments.

  Therefore, an inherent subsidy is being provided to the plan sponsor to the plan participant as the plan participant is receiving a benefit that he/she cannot replicate on his/her own on a costless basis. A subsidy to the plan participant is thus a cost to the plan sponsor.

  Using the same argument, albeit in a bit subtler of fashion, by providing an interest crediting rate shorter than 1 year, without any floor, the resulting liability is actually slightly less than $1000. This is because the interest credit is only being applied annually to the account but the interest is actually being earned quarterly. In effect, the plan participant is providing a slight subsidy here to the plan sponsor due to the timing (maturity) difference.

Consider how different the valuation results above are from those derived under current actuarial practice. Under current actuarial rules, the plan actuary projects the starting account balance to C using an assumption for the interest crediting rate and then discounts the projected balance back to the valuation date using the yields on high grade corporate bonds, as prescribed under PPA or FASB. Financial theory states that corporate bond yields can be expressed as the sum of Treasury yields plus a spread for credit risk. Therefore, corporate bond yields should always be higher than Treasury bond yields. This has been the case historically in the US[76]. Under current valuation methodology, and using the fact we just established that corporate bond yields are higher than Treasury bond yields, the liability for a plan that credits the 10 year (or 30 year) interest crediting rate will actually have a *lower* liability than the starting account balance. This is the polar opposite of the result we derive which shows the liability to be *higher* than the account balance owing to the inherent subsidy provided by the plan sponsor.

While the general valuation approach (project the account balance to decrement date and then discount that balance back to the valuation date) is broadly the same between what we have presented here and current actuarial practice, the liability is quite different. This owes primarily to the use of the risk-neutral methodology.

- The higher the annual minimum interest guarantee the higher the value of the embedded derivative. This makes intuitive sense as a richer benefit is being provided to the plan participant and thus should have a higher valuation.

- The longer the duration of the Treasury interest crediting rate the higher the value of the embedded derivative. This is because the yield curve is normally upward sloping and thus shorter duration Treasuries have lower yields than longer duration Treasuries. Lower yields mean there is a greater chance the interest rate guarantee will be in the money and create a cost to the plan sponsor.

  Additionally, shorter maturity yields historically have exhibited more volatility than longer maturity yields[77]. Higher volatility should increase the cost of the guarantee, all else equal. This is reflected in the simulations: for all commencements C, it can be expected that the funds value grows according to the increase of the credited Treasury maturities. This happens for lower values of the fixed rates K. For K equal to 3% or to 4%, funds with shorter credited interest rate maturities have a higher value. This behavior, can be explained by the fact that the rates with shorter maturity have a lower expected value but they are more volatile (the 3 month and one year Treasury bills are different instruments than the 10 and 30 Year Treasury Notes), and hence shorter rates have an greater probability to be higher of the fixed rate K when K is high.

[76] See http://research.stlouisfed.org/fred2/categories/22
[77] See Section 5.1 of Ho & Lee “The Oxford Guide to Financial Modeling”

- As C increases the value of the guarantee increases. This is mostly because the account balance is growing over time such that if the Treasury yield falls below the strike later in the contract life as opposed to earlier the payoff will be larger simply owing to the fact that the balance grows exponentially over time, as shown in Equation 1.

10.6. Monte-Carlo Simulation Analysis

The valuation process described above uses risk-neutral interest rate models and valuation techniques, which preserve the prices of observed instrument in the market, the no-arbitrage principle and assumes complete markets.

For risk management purposes, for example for simulating the growth in the account balances over some time horizon or evaluating the distribution of guarantee exposure at various time horizons, pension sponsors should calibrate the interest rate models under the real world method in order to obtain reasonable scenarios [78]. This approach is likely very familiar to pension plan practitioners who commonly perform real world method projections for plan sponsors using a variety of software packages.

To show an illustration of how this would work, we provide detailed results for the following scenario:

- Interest crediting rate equal to yield on 30 year Treasury rate
- Floor (k) equal to 3.0%
- Commencement date, C, of 5 years

We selected the 30 year US Treasury bond yield for the interest crediting rate because it is a commonly used interest crediting rate by US pension fund sponsors of cash balance plans. While Lowman (2000) cites 4.0% as the most common interest crediting rate floor, 3.0% in the current market environment does not seem unreasonable. Our general approach here would hold for a different floor rate and we recognize each plan is going to be different. Our results here are geared more towards highlighting a process rather than being prescriptive. We choose a 5 year time horizon to compliment the 10 year forecasts we did previously for the market rate cash balance plan guarantee.

We assumed a 100% chance of survival to point C. We also assumed an initial starting account balance at the valuation date of F_0=$1,000.

[78] We provide details in the Appendix on how to do the simulation for the real world model. All the derivative textbooks cited previously provide information on this distinction between risk neutral and real world measures and the appropriate uses of each technique. These concepts have also been discussed in various SOA venues (e.g. webcasts, conferences, etc.) and are covered extensively on the Quantitative Finance track of the FSA curriculum.

Under the real world distribution method, obtained by adjusting the drift of the Hull-White interest rate model, we can calculate several statistical pieces of information about the risk profile exposure of the plan sponsor. We model the account balance based on Equation 1.

In Figure 24 we provide an example of the cash balance trajectory and the descriptive statistics for the distribution of the account balance at commencement.

In Figure 25, we provide descriptive statistics for the guarantee payoff at commencement where we assume commencement is at year 5. In essence, we are calculating the payoff, as in Figure 22, many times and providing statistics of the resulting distribution function under the real world measure.

Figure 24: 10 Simulated Account Balance Paths of the Plan Which Receives the Maximum Between 3% and the 30 Year Treasury Rate; Starting Account Balance of $1.

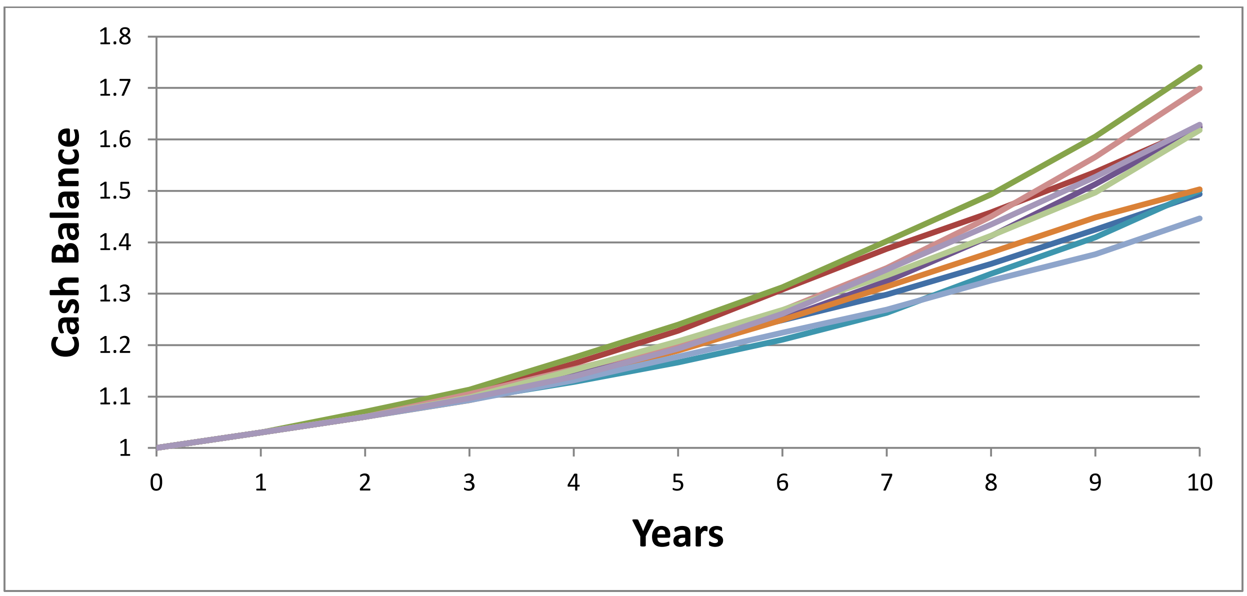

Figure 25: Descriptive Statistics for the Distribution of the Cash Balance Account at 5 Year Commencement . The Plan that Guarantees the Maximum Between 3% and the 3 month, 1, 10 and 30 year Treasury Rates, Annually. Balance Projection Calculated Under Real World Measure.

M	Mean	Std.Dev.	Skeewness	Kurtosis	1%	5%	95%	99%
3 month	1,170.22	17.79	2.27	6.19	1,159.27	1,159.27	1,208.56	1,237.52
1 year	1,172.12	19.61	2.11	5.29	1,159.27	1,159.27	1,214.10	1,244.46
10 year	1,189.67	30.38	1.15	1.12	1,159.27	1,159.27	1,249.43	1,281.31
30 year	1,190.71	29.44	1.04	0.81	1,159.27	1,159.27	1,248.04	1,277.02

10.7. Possible Hedging and Risk Management Strategies

Up to this point, we have described a valuation approach for the floor guarantee, performed numerical analysis, using some stylized assumptions and scenarios, to estimate the guarantees fair value, and have provided a framework and some real-world simulations to illustrate the plan sponsor's potential liability exposure for providing the guarantee.

The plan sponsor who offers a cash balance plan with a floor guarantee will likely be looking for ways to manage the risks associated with such a plan. In this section, we provide some brief ideas for how a plan sponsor might want to go about this task. As each plan sponsor has different goals, risk tolerances/appetite, time horizons, etc. the ultimate decision on how to manage the plan will likely need to be decided on a case by case basis. The Plan sponsor who wants to hedge the floor guarantee may want to consider the below approaches. Note, we are assuming:

- no further pay credits
- deterministic date of benefit commencement, C, equal to 5 years
- account balance at time 0 of $1
- 5-year zero coupon Treasury bond yields 1.0% per year.
- Annual interest crediting guarantee equal to max (3.0%, 30 year Treasury yield)
- K is set deterministically at 3.0%

10.7.1 Possible Hedging strategy 1:

Dynamic moving notional, interest rate call options entered into at t=1, t=2, etc. This would work as follows:

The account will grow, at a minimum, to 1.16 at the end of year 5. This can be seen by taking $1*(1.03)^5.

So, we need 1.16 at the end of year 5. How can we accomplish this? We the plan sponsor can buy a 5 yr zero coupon treasury. How much capital would we need to invest in a 5 yr zero to get us 1.16 at the end of year 5? That can be computed by taking 1.16*(1.01)^-5 = 1.10

Now, the pension plan only had $1 of initial seed capital and so it will need to make up for the 10 cent difference in order to finance the minimum interest guarantee.

If the first year's interest crediting rate is known to exceed K at T=0 the bond purchased should be sufficiently increased to cover one year at the 30 year treasury rate and the remaining years at the K minimum.

The plan sponsor also needs to buy protection against the possibility the account balance grows to more than 1.16 at the end of year 5. How can this be accomplished? For example, say the 30 year treasury rates evolved as such: 2%, 6%, 1%, 7%, 10%. The account would grow, without the guarantee, to 1.29 and with the guarantee to 1.32. The plan sponsor needs to make up the difference in 1.32 and 1.16 = .16. How does the plan sponsor get such a payoff? The plan sponsor needs to buy an interest rate call option.

The plan sponsor enters into an interest rate call option with a counterparty. The Plan sponsor pays an option premium. The counterparty to the interest rate call option (the party that is short the contract) pays max(30 yr treasury rate - K, 0).

This hedging strategy takes the account balance at the start of the year and essentially says to the counterparty, "If the 30 year treasury yield is above K, I want you pay the difference of 30 yr rate minus K rate." For example, using the sample Treasury evolution that we evaluated earlier, assume at T=0 it is known that the 30 year is yielding 2%. At the start of yr 1, the account balance would equal 1.03 (1*(1+max (3%,2%)). An interest rate option would be purchased at T=0 that pays off 30 yr rate – K on a notional of 1.03. With the 30 yr rate coming in at 6% for year 2, and k=3%, the option would have a payoff equal to .0309 = 1.03 (.06-.03). Of course, at that time, a bond will need to be purchased that pays off .0337 = .0309*1.03^3 in three years.

The dynamic moving notional, interest rate call option approach suffers from two shortcomings. First, because the call options are bought annually, there can be risk associated with the variability in their costs. Second, the payoffs from the options must be reinvested to pay off a minimum of K%. To the extent, that the option pays off when interest rates are high, it should mitigate the impact of item 2. In fact, if interest rates are sufficiently large, the bond yielding k% may cost less than the option payoff. The excess assets can be used to purchase the next year's call which will be more expensive due to the high interest rate which both increases the notional and the cost of the option due to the increase in expected payoff of 30 yr –k, offset by the higher risk free rate. These risks can be mitigated by purchasing upfront bonds yielding higher than k%, let's say 4%. This would be a greater cost upfront but future costs would be less as the calls would only have a payoff of max (30 yr treasury -4%, 0). In addition, the plan sponsor can make additional earnings by selling a put that pays off up to 1% to the extent the 30yr Treasury is below 4%.

10.7.2 Possible Hedging strategy 2

Static moving notional, interest rate hedge entered into at time zero. This would work as follows:

- The Plan sponsor enters into an interest rate swap
- Plan sponsor pays a fixed rate
- Counterparty pays max(k, 30 yr treasury rate).
- Swap payments are exchanged each year in cash

The notional is a stochastic variable. This is because its value depends on the evolution of the underlying interest crediting rate.

If we want to set the hedge at time zero we can make a guess as to what the notional might be at each successive measurement date. We run our interest rate model and determine the expected value of the account balance at each time 1 to 5. We find those values to be 1.0 at t=1, 1.03 at t=2, 1.067 at t=3, 1.11 at t=4, and 1.17 at t=5. We use these values as the notionals in the interest rate swap contract.

Using our interest rate model, we also calculate the fixed rate the plan sponsor will need to pay to enter into this swap and set the starting value of the contract to zero. We determine that fixed rate to be 3.55% for a K=3%.

As the option payoff will be sufficient to cover the needed interest on participant's accounts, however, the Plan sponsor will also need to purchase a bond paying off the account balance without interest at retirement and a series of bonds paying the fixed rate. These amounts would be known at time zero.

This method will not be a perfect hedge because the notional can only be guessed at. This strategy reduces the first risk associated with Hedging Strategy 1 as the options are purchased at time zero and new options will only be needed when the account balance is greater than the stochastic amount predicted. Further, this strategy eliminates the second shortcoming of Hedging Strategy 1 as the floating rate paid is the amount of interest needed at retirement.

When additional options are needed, under a higher than predicted interest rate environment, the plan sponsor is still hit twice by both the increased cost of options and an increasing notional greater than the stochastic estimate. Interest earned on the floating rate payoffs can offset these costs.

Hedging Strategy 2 also suffers from the risk of overfunding in a low interest rate environment as the notional on the swap will exceed the account balance.

It should be clear at this point that whether using Hedging Strategy 1, which calls for annual option purchasing, or Hedging Strategy 2, we have failed to implement a perfect hedge. Firstly, a perfect hedge requires constant rebalancing and we have only considered at most annual strategies. Moreover, it is difficult, if not impossible, to build a hedge when the underlying is not investable like a stock, future, or even the 30 yr treasury bond, but rather the non-investable 30 yr treasury rate.

To our knowledge there is no financial instrument that will allow the plan sponsor to earn the 30 year rate per annum, at least not without principal risk (for example, investing in 30 year bonds exposes the plan sponsor to enormous principal risk given the long duration nature (roughly 15) of the instrument). The plan sponsor can invest in a one year treasury and by floating for floating swap (one year treasury rate for 30 yr treasury rate), however, this approach would not address the issue of the moving notional. The actuary should make the plan sponsor aware of these risks when designing a Plan.

Other alternatives the plan sponsor could consider in terms of managing the risk of the guarantee, either solo or in conjunction with the approaches described above, are as follows:

- Change the design of the plan to remove or lower the guarantee level. This would only impact interest credits on future pay credits as those pay credits already accrued as of the plan change date, by law, must continue to be credited with the historical interest crediting rate and guarantee. That said, the sooner the guarantee is removed the less risk and cost the plan sponsor faces.

- Increase funding to the plan to account for the cost of the minimum interest credit guarantee plan provision. The plan sponsor is recognizing a guarantee has been granted, is assigning a value to that guarantee, and is funding the cost of that extra promise. This can be thought of as similar to self-insurance. Each year the plan sponsor could evaluate the option cost for each plan participant and determine if additional funding is needed.

With or without the guarantee, a carefully designed investment strategy that aligns with the plan sponsors goals, objectives, risk tolerance, current market conditions and many other criteria should also be considered.

11. Conclusions

In this paper we identified three different guarantees embedded in cash balance plans. We also provided some initial insights into methodologies that could be used to value these guarantees. These methodologies stem primarily from the financial engineering literature and are commonly used to value derivatives in the capital markets. We adapted these approaches and applied them to valuation of guarantees in the pension world.

Plan sponsors should recognize when they grant subsidies and guarantees to plan participants. Further, they should value these guarantees and devise funding, investment, and plan design strategies to properly manage the risks associated with such promises.

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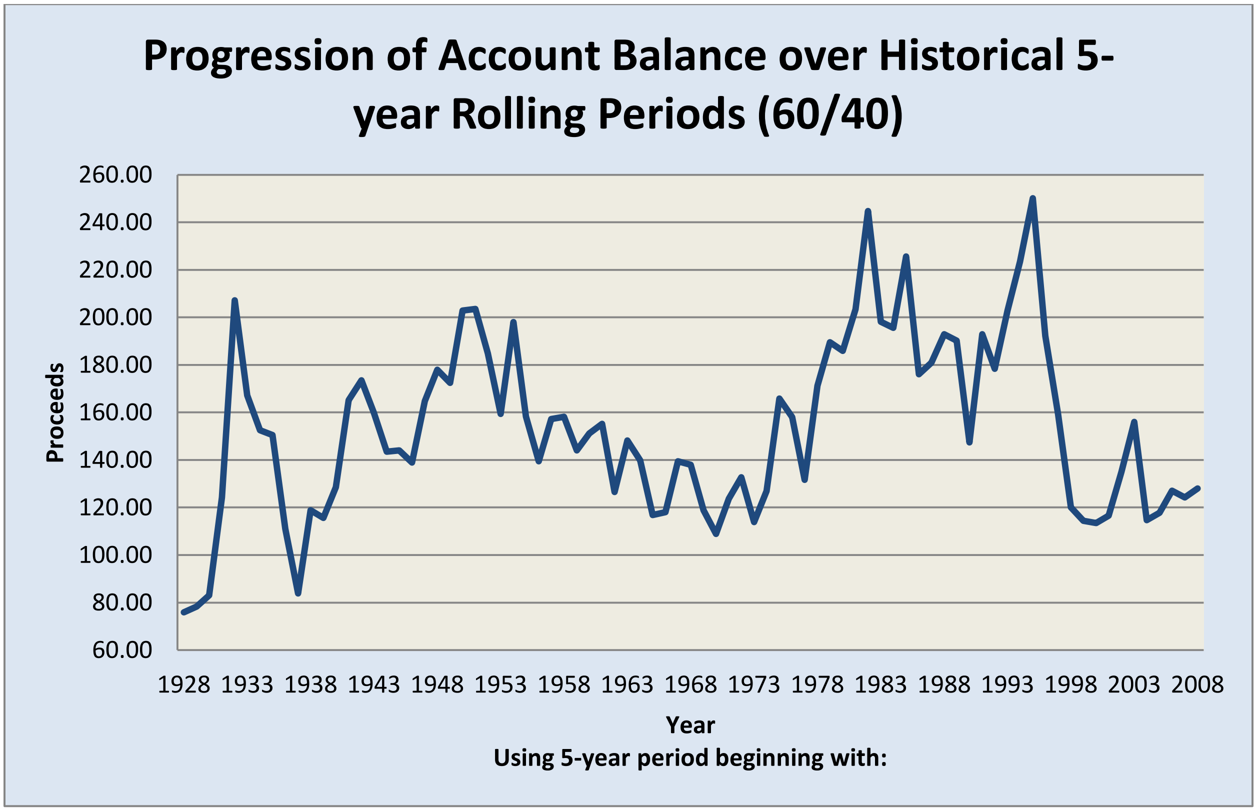
Appendix

I. Sample Progression of Account Balance



II. Traditional Actuarial Methods for Investment Guarantees Can be Improved

Assume the following[79]

- A plan has one participant with an account balance at time 0 of $1,000.
- The participant retires with 100% probability in 20 years with no decrements prior to retirement.
- The plan guarantees a cumulative return on the $1,000 of 3.0% per annum.
- The plan actuary estimates there is a 4.0% chance the diversified portfolio will not achieve a 3.0% annual compounded rate of return over the 20 year time horizon.
- If the portfolio underperforms the 3.0% compounded annual growth rate, assume the portfolio value at the end of 20 years is $0. This would mean that the value of the plan sponsor guarantee at the end of 20 years would be $1,806 ($1,000*(1.03^20)).
- The 20 year zero coupon rate is 3.0%

The expected value of the payout is $72 [(probability of loss)*(loss amount) or (.04)*($1,000*(1.03^20))]. Discounting the $72 at the 20 year zero coupon rate gives a present value to the payoff of $40 ($72 / (1.03^20)). This means the plan sponsor could then invest $40 today in the 20 year zero coupon bond to guarantee he would have the required $72 at the end of the 20 year period. This is a simplistic representation of the traditional actuarial approach to determine the valuation of a contingent cashflow. It is important to note that with only *one* plan participant, the plan sponsor would actually need to hold $1,000 ($1,806*(1.03^-20)) to be 100% certain of covering the payout to the plan participant. This is due to the fact that the account value has the potential to suffer a complete loss (4% of the time).

This example illustrates an important departure from the traditional concept for managing most risks in a defined benefit pension plan. Most risks in a pension plan are diversifiable owing to a large number of independent participants. Given enough participants in a plan, the probability of a risk event (e.g. mortality, termination, etc.) happening can be estimated with a high degree of confidence. For example, with a large enough cohort, or group of participants, expected values (i.e. deterministic techniques that use "best estimate" values) are adequate for risks such as mortality as the risk is diversifiable (participant deaths are generally independent of one another) and the central limit theorem and the law of large numbers applies.

[79] This section follows very closely Richard Q. Wendt's 1999 article in Risk & Rewards titled "An Actuary Looks at Financial Insurance"

The problem with this traditional analysis is that the risk exposure in a cash balance plan, like the one in the example above, is *not* diversifiable and is conflated by the fact that investment periods overlap and are not independent. Adding more and more participants in a given year does not reduce risk or garner a more refined estimate of the probability of the event happening.

To demonstrate this concept, let's assume we have a typical open plan, which has participants enter each year. We may have a plan, identical to the one participant plan outlined above, however it has 100 participants (a "cohort") entering *each year*. Over a 20-year period of time the plan will contain 2,000 participants[80]. While more plan participants would generally mean plan risks (e.g. mortality) are being spread amongst a larger group of independent participants, this is not necessarily the case with the market rate cash balance plan guarantees. For the cumulative guarantees, instead of having 2,000 independent participants with whom the guarantee can apply, instead the plan sponsor really has a much smaller number, 20 cohorts. This is because each of the 20 cohorts is exposed to the same capital market experiences. Thus, like the numerical example presented previously, if the guarantee is in the money, it affects the *entire cohort* not just one participant. Moreover, because investment periods overlap, negative (or positive for that matter) market events that impact one cohort will also impact other cohorts. In other words, if there is a big capital market loss in year 18, all 18 cohorts will be affected in some way (granted, it does not mean that all cohorts will be "in-the-money"; there will be some smoothing of losses given the different historical experiences of the cohorts, but nonetheless all will be affected). The law of large numbers of independent events used by pension plans to manage many risks does not apply with cash balance plan guarantees.

This example helps to illustrate that a traditional deterministic approach will not be adequate to ensure proper risk management for the cash balance plan with cumulative guarantees. The same logic applies for a plan that guarantees an annual rate of return – *each* plan participant will be eligible for the floor crediting rate. If the guarantee is triggered, even though this trigger may have a low probability, based on the performance of the underlying financial variable, all of the participants or cohorts may be affected. Having a "best guess" estimate for the expected number of claims and the exposure in those cases is not very useful in this case. For risk behavior of the sort described here stochastic modeling and/or an options based approach is recommended.

Looking back to our example can illustrate the problem with the traditional actuarial analysis numerically. The only way $72, the expected value under traditional actuarial techniques, can support a potential liability of $1,806, the total loss for a participant 4.0%

[80] This assumes no participants decrement. The argument set forth in the text is not impacted if decrements are considered. The example is clearer assuming no decrements.

of the time, is if the plan has, for example, 100 participants and only 4 or less of them experience underperformance in their plan assets at the end of the time horizon.[81] In a cash balance plan that guarantees a cumulative return however, *all* 100 plan participants would be impacted at year 20 even though there is only a 4% chance of the event actually occurring. This type of risk is characterized as having a low frequency and high severity. In our example, the plan sponsor would remain solvent 96% of the time but would be insolvent (or need to fund additional amounts to the plan) in 4% of scenarios. To be 100% certain the plan sponsor had enough capital to pay the guarantee, $173,376 ($1,806*100 - $7,224) would be required at year 20; $95,995 at time 0. This is a significantly higher amount than the $4,000 determined under the traditional actuarial approach.

We acknowledge that the plan sponsor does not need to fund to ensure 100% solvency under all scenarios. This example is meant to illustrate the potential hidden risks contained within a market rate cash balance guarantee. The decision on how much to fund the plan to cover the guarantee should be made by senior management with considerations of their pension risk as part of their risk appetite and in accordance with statutory funding rules. Discussions with the plan actuary and senior management should be the starting point of understanding the potential costs and funding considerations for each embedded option.

[81] This works as follows: 100 * $40 = $4,000. The $4,000 grows to $7,224 at the end of the 20 year time horizon. If 4 people receive benefits they will receive a total of $7,224 ($1,806*4).

III. III. Interest Rate Models

In this Appendix we describe in details the models and the numerical methods used to calibrate our models and to perform our simulations. First, we give a brief introduction on stochastic differential equations, then we introduce a general class of interest rate models called short rate models and finally describe the Hull-White model which is a particular short rate model and is the chosen model for our simulations.

Stochastic differential equations

Evolution of indexes and rates in the financial markets is not deterministic: even if we know the values of all the variables at present time, we are not able to predict their values at future times. Therefore, we need a model to predict the future path of interest rates.

We can make a few observations before providing model formulas. First, it is natural to suppose that the value of a variable cannot be too different from its value at a slightly preceding time. Second, we know that different kind of assets behave differently: index and equities can grow indefinitely while interest rates tend to assume values within a certain range.

We can express these observations in the form of stochastic differential equations. Stochastic differential equations are usually written in the form

$$dX(t) = a(t,x)dt + b(t,x)dW$$

where the two functions a and b are called drift and diffusion and where W is a Wiener process (also known as Brownian motion). The meaning of this equation is that it is possible to simulate a realization of a process described by it starting from a given value and using the following formula

$$X(t+\Delta t) = X(t) + a(t,x)\Delta t + b(t,x)\sqrt{\Delta t}N$$

where N is a draw from a standard normal distribution. We can see that the value of X at time $t + \Delta t$ is given by its value in t plus a deterministic part $a\Delta t$ and a probabilistic part $b\sqrt{\Delta t}N$.

Given different functions, a and b, the process will have different properties, for example a different mean and variance at future times[82].

Short-rate models

The more tractable[83] models for interest rates are the so called short-rate models[84]. In this kind of model it is assumed the existence of an instantaneous rate described by a certain

[82] For a more formal definition of stochastic differential equation see Tomas Björk "Arbitrage Theory in Continuous Time".

stochastic differential equation. This instantaneous rate can be thought of as the rate with the shortest maturity present in the market, for example an overnight rate. For each model there exist formulas for calculating the whole interest rate curve for any future date given the value of the short rate at a given date. Short-rate models are generally divided into two categories: equilibrium models and arbitrage-free models. Below we briefly describe the differences in these categories. As we will be using a specific arbitrage-free model to perform our simulations we leave the more specific details of interest rate models to the reader.

Equilibrium models are defined by only a few scalar parameters and cannot exactly fit the interest rate curve at a given time. From the fact that the market interest rate curve is different from the model interest rate curve, it follows that from the point of view of the model, investing in market bonds could cause arbitrage opportunities. Although these models don't fit the market interest rates curve it is to be said that the evolution of equilibrium models is completely arbitrage-free, in that it is not possible to make up an arbitrage portfolio buying bonds or bond options present within the model. Arbitrage arises when considering both model bond and market bonds, but the model is internally arbitrage-free.

Arbitrage-free short rate models are usually defined in terms of some scalar parameters and also in terms of a function of time which enters directly in the dynamics of the stochastic differential equation. This function is chosen in a way to force the model to be consistent with the interest rate curve observed in the market.

Two examples of commonly used short-rate equilibrium models are the Vasicek[85] model and the Cox, Ingersoll and Ross (CIR) model[86].

The Vasicek model is defined by the following dynamic:

$$dr(t) = a\big(b - r(t)\big)dt + \sigma dW$$

while the CIR model is defined as:

$$dr(t) = a\big(b - r(t)\big)dt + \sigma\sqrt{r}dW$$

In both formulations r represents the short rate, a is the mean reversion rate for the short term rate, b represent the long term rate, and σ is the short rate volatility. One of the

[83] Short rate models are considered tractable because usually their evolution is usually expressed in terms of one or two factors. There are other class of less tractable models which for example simulate directly the joint evolution of forward rates observed in the market. An example is the Libor Market Model also called BGM Model (Brace Gatarek Musiela Model).
[84] The derivative models texts noted above cover all short rate models in detail.
[85] O.A. Vasicek "An Equilibrium Characterization of the Term Structure," *Journal of Financial Economics*, 5 (1977)
[86] J.C. Cox, J.E. Ingersoll, and S.A. Ross, "A Theory of the Term Structure of Interest Rates," *Econometrica*, 53 (1985)

drawbacks of the Vasicek model is that the short-rate can go negative. The CIR model corrects for this by not allowing negative rates.

The Hull-White model

The most important (for its wide adoption) arbitrage-free model is the Hull-White (HW) model which is defined by the following equation:

$$dr(t) = [\theta(t) - ar(t)]dt + \sigma dW$$

where a, σ are two real positive numbers and $\theta(t)$ is a function of time which is fixed by requesting consistency with the market rate curve. In particular if $f(t)$ is the function of instantaneous forward rate, that is

$$f(t) = -\frac{\partial}{\partial t}\ln[P(0,t)]$$

where $P(0,t)$ is the price at time 0 of the zero coupon bond with maturity t, then the function $\theta(t)$ is given by

$$\theta(t) = af(t) + \frac{\partial f(t)}{\partial t} + \frac{\sigma^2}{2a}[1 - e^{-2at}]$$

Moreover it can be shown that in the Hull-White model the price in t of a zero coupon bond with maturity T is given by

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

where the two functions A, B are

$$B(t,T) = \frac{1}{a}\left[1 - e^{-a(T-t)}\right]$$

$$A(t,T) = \frac{P(0,T)}{P(0,t)}exp\left[-B(t,T)\frac{\partial \ln[P(0,t)]}{\partial t} - \frac{\sigma^2}{4a^3}(e^{-aT} - e^{-at})(e^{-at} - 1)\right]$$

Knowing the function $P(t,T)$the rates associated to treasury bills, with maturities lower or equal one year, are given by

$$R(t,T) = \frac{1}{T-t}\left[\frac{1}{P(t,T)} - 1\right],$$

while rates with longer maturities, associated to treasury notes and treasury bonds, are given by

$$R(t,T) = \frac{1 - P(t,T)}{\frac{1}{m}\sum_{i=1}^{n} P(t,T_i)},$$

where m are the bonds coupons per year $(m = 2)$ and $T_1, \dots, T_n$ are its payment dates.

The major drawback of the Hull-White model is that it can lead to negative realizations of the short rate. Similar models which avoid this issue exist but are more complicated[87]. We focus on the Hull-White model which represents a good compromise between mathematical tractability and economic meaningfulness.

The Hull-White model has closed form formulas for swaptions[88] and interest rate caps[89]. This fact simplifies the (risk-neutral) calibration of the model parameters against market prices for these kind of instruments[90].

The idea is to choose model parameters that make the model predict prices for that instruments which are as close as possible to that seen in the market. To do that is possible to proceed as follows: given that swaptions with different option maturities and whose underlying swaps have different durations are traded in the market, market prices for all those swaptions can be observed and grouped into a matrix of swaptions market prices indicated with $\mathrm{SP}_{\mathrm{ij}}^{\mathrm{mkt}}$ where the indices represent different option maturities and interest rate swap durations. Prices of the same swaptions can be calculated within the Hull-White model for a given choice of the parameters a, σ. We indicate the (i,j)th element of the predicted matrix with $\mathrm{SP}_{\mathrm{ij}}^{\mathrm{hw}}(\mathrm{a}, \sigma)$.

The a, σ which best match the prices observed in the market can be calculated by a non-linear regression, minimizing a function $\mathrm{f}(\mathrm{a}, \sigma)$ which represents the distance between model prices and observed market prices defined as follows:

$$f(a,\sigma) = \sum_{ij} \left[SP_{ij}^{mkt} - SP_{ij}^{hw}(a,\sigma)\right]^2$$

Using the same strategy it is possible to calibrate the model against cap prices at different maturities and with different strike rates.

In order to calibrate the Hull-White model it is thus necessary to observe from the market the following data:

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[87] See for example the squared Gaussian model in Antoon Pelsser "Efficient methods for valuing interest rate derivative", Springer 2010.

[88] These are options to enter into interest rate swaps. An interest rate swap allows two parties to exchange different interest rate driven cash flows, most commonly, a floating rate interest payment for a fixed rate interest rate payment.

[89] An interest rate cap provides a payoff to the holder of the contract if interest rates rise above a certain threshold.

[90] Calibration of interest rate is quite sensitive to the category of instruments used: for example the calibration of a model against swap can provide significantly different model parameters w.r.t the calibration of the same model again caps.

- The zero rate curve in order to determine the function $\theta(t)$ which enters in the dynamic.
- The swaption or cap prices to fix the two parameters using the procedure described above.

Real World Simulations

Given the short-rate dynamic in the real-world measure

$$dr(t) = \mu\big(t, r(t)\big)dt + \sigma\big(t, r(t)\big)dW^P$$

and indicating with $\mathrm{F}^{\mathrm{T}}(\mathrm{r}(\mathrm{t}), \mathrm{t})$ the price in t of a zero coupon bond with maturity T from Ito's lemma we can derive the price process as

$$dF^T = F^T \alpha^T dt + F^T \sigma^T dW^P$$

where

$$\alpha^T = \frac{\left(F_t^T + \mu F_t^T + \frac{1}{2}\sigma^2 F_{rr}^T\right)}{F^T}$$

$$\sigma^{\mathrm{T}} = \sigma\, \mathrm{F}_{\mathrm{t}}^{\mathrm{T}} / \mathrm{F}^{\mathrm{t}}$$

(subscript indicates derivation).

Imposing no arbitrage it's possible to find that two bonds with maturities T and S satisfy

$$[\alpha^S(t) - r\,(t)]/\sigma^S\,(t) = [\alpha^T(t) - r\,(t)]/\sigma^T\,(t)$$

From the fact that S appears only in the left term and that T appears only in the right one, it follows that they both are independent of the bond maturity letting us to define the market price of risk as

$$\lambda(t) = [\alpha^T(t) - r\,(t)]/\sigma^T\,(t)$$

It can be shown that in the risk-neutral measure the dynamic of the short-rate is

$$dr(t) = [\mu - \lambda\sigma]dt + \sigma dW^Q$$

With the standard calibration of Hull-White model we fix the parameters entering in the risk-neutral dynamic

$$dr(t) = [\theta(t) - ar(t)]dt + \sigma dW^Q$$

so knowing λ we can write the real-world dynamic as

$$dr(t) = [\theta(t) - ar(t) + \lambda\sigma]dt + \sigma dW^P$$

The problem of simulating the short-rate in the real-world measure is thus reduced to that of estimating λ.[91]

Estimating the market price of risk

The simplest possible assumption on λ is clearly to assume it is constant. Its value can then be estimated from the historical series of zero rate curve after having estimated α^T and σ^T from the observed dynamic of bond prices.

Given a historical series of zero rate curves it's possible to calculate the series of prices of a zero coupon bond maturing at a certain time. Let's indicate this series with $F^T(i)$. After calculating the series of returns $R_i^T = \frac{F^T(i+1)}{F^T(i)} - 1$, the two parameters α^T and σ^T can be estimated from its mean and standard deviation as follows:$\alpha^T = Mean(R_i)/dt$

$$\sigma^T = StDev(R_i)/\sqrt{dt}$$

where dt represents the interval between observations. From the zero rate series it's also possible to calculate a proxy for the historical series of short rate by extrapolating the curve at zero for each observation. Taking the mean value for the short rate, let's call it r, a rough estimate for lambda is given by

$$\lambda = \frac{\alpha^T - r}{\sigma^T}$$

This procedure can be carried on for different bond maturities returning different estimates for lambda.

In order to obtain an estimate for the risk premium we observed the daily US zero rate time series from 2/01/2001 to 2/01/2013 and calculated lambda using bonds with maturities between fifty and twelve years (at the start of the series) obtaining estimates for lambda between 0.46 and 0.55 and thus giving a drift adjustment $\lambda\sigma$ (with sigma = 0.0085) between 0.004 and 0.0047. We hence assume that 0.0045 is a reasonable value for the drift adjustment.

[91] For more details on real world calibration see Tomas Björk "ArbitrageTheory in Continuous Time", *Oxford Finance Series* (2009). Short rate models chapter.

IV. Data Sources

S&P 500 Stock Market Index – obtained from Robert Shiller's website; http://www.econ.yale.edu/~shiller/data.htm and from Aswath Damordoran's website; http://pages.stern.nyu.edu/~adamodar/

Bond data – obtained from Robert Shiller's website; http://www.econ.yale.edu/~shiller/data.htm and from Aswath Damordoran's website; http://pages.stern.nyu.edu/~adamodar/; and St. Louis Federal Reserve website; http://research.stlouisfed.org/fred2/

Treasury STRIP data - Bloomberg