Current Actuarial Modeling Practice and Related Issues and Questions

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1. Introduction

The future of the actuarial profession is a matter of modeling and putting models into practice. One change taking place in the profession is the realization that all actuarial techniques are founded on models and that it is the models, and not the techniques, that are important for the future of the profession. In the U.K., we recently discussed a paper, "The Future of the Profession" (Nowell 1996), that pinpointed, in particular, weaknesses in financial economics and stochastic methods. What this really meant was that actuaries have focused too closely on their techniques—what I call computational tools—and have not kept their eyes above the horizon where progress has, by and large, been made by others. The U.K. has been a little slower than the U.S. and Canada in putting this to rights, but that is now happening. And modeling is the key.

In the first part of this paper I set out a "model of models." It does not attempt to explain everything about models in any philosophical way; it is meant only to give an intuitive guide, so we can take an actuarial tool and recognize where it fits into a modeling framework. Then I discuss at more length two modern actuarial tools—profit tests and model offices—and ask, Are these models at all? And, because I am very interested in model offices and I believe these are just at the beginning of their development, I discuss some of the more down-to-earth practical problems about creating and using them.

2. A Model of Models

2.1 Paradigms

It is impossible to begin a discussion of modeling without referring to Jewell's address at the 1980 International Congress of Actuaries (Jewell 1980). That paper applied Kuhn's idea of paradigms in the discovery and development of fields of thought to the evolution of actuarial science. In general, researchers work within an accepted paradigm, or way of thinking, until a revolutionary breaks the mold and sets up a new paradigm, usually in response to some crisis or anomaly that cannot be patched up in the old way of thinking. A classic example is the discovery, not of the Black-Scholes *formula*, but of the Black-Scholes *model*, with its riskless hedge as a new paradigm. Since then, research has proceeded just as Kuhn would have predicted, embellishing the original and working out details. Meanwhile a new revolution is awaited to solve problems that Black-Scholes cannot, such as incomplete markets.

That image, of cycles of breakthrough and consolidation, is one of those compelling ideas that just feels "right." Kuhn also described the trials and tribulations of the would-be pioneer; breakthroughs are not necessarily accepted without a fight, especially if people have a large investment in the old ideas. Thus, real pioneers might be described as lunatics by the established order; on the other hand, real lunatics can take false comfort from the few examples of genius being ignored.

However, Kuhn's paradigms, applied so vividly to actuarial science by Jewell, mostly help to explain the line that research takes, the emergence and polishing of new ideas. And although they do intersect very fruitfully in actuarial science, the research community and the world of practitioners are not the same and do not respond in the same way to ideas. Thus the acceptance of an idea by research actuaries is not the same as its application by practitioners, and changes in practice are not always the result of new ideas in the research area. (Some die-hard practitioners would probably say that changes in practice are almost never the consequence of any research!)

2.2 A Simple Model

I suggest a model for the relationship between research, new ideas and practice. How new ideas arise lies outside this model.

First, what is a model? In actuarial science we are most interested in statistical or probabilistic models (see Figure 1). What are these? Hoem and Funck-Jensen (1982) described them as follows:

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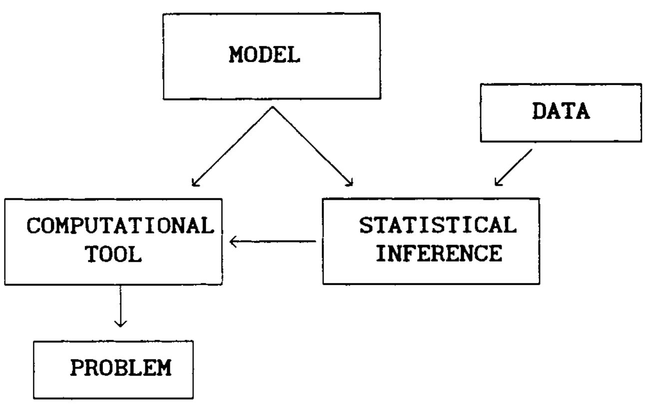
(a) There is a set of assumptions that define the model. The assumptions might usefully be guided by "reality," but "reality" lies outside the model.
(b) By working out the mathematics within the model's rules, we can compute some model quantities from others. For example, in the Markov illness-death model, the fundamental model quantities are the transition intensities. From these we can compute probabilities via the Kolmogorov equations and so gain access to all the actuarial applications based on the calculation of probabilities. Or, in the Black-Scholes model, we start with assumptions of no arbitrage, lognormal stock prices, and a risk-free asset, and from these we can compute hedging strategies for derivatives. These are examples of the construction, within the model, of *computational tools* to solve *practical problems*.
(c) Finally, there are methods of statistical inference used to relate the model to real data. This, and not the models' assumptions, is the link between the model and reality.

FIGURE 1
A MODEL OF A PROBABILISTIC MODEL



One point I want to make is the importance of regarding the model, or the modeling process, *as a whole*, even if our interest lies mainly in one part of it. I think we have tended not to do so in the past, and we have, as a profession, allowed our view to be too narrowly concentrated on that part of the model in which we spend most of our professional lives. It can be difficult to step back and see the whole picture.

That of itself is not surprising, perhaps just human nature. The real danger lies in the education system falling into the same trap, as I fear it has done in the past. I get the feeling that our actuarial education on both sides of the Atlantic is only now clambering out of that particular pit.

To illustrate, I think Figures 2 and 3 help to explain the different attitudes of life insurance actuaries and casualty actuaries to statistics. Figure 2 shows the casualty actuary's world. He or she is dealing with data that change and that always need to be modeled, and the models need to be checked against more data and updated. This world contains both the computational tool and the statistical analysis, and it is plain that both "add value" and are in close touch.

FIGURE 2
THE CASUALTY ACTUARY'S WORLD

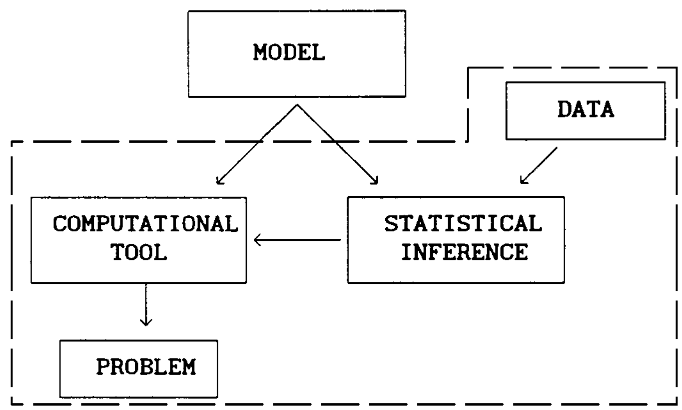
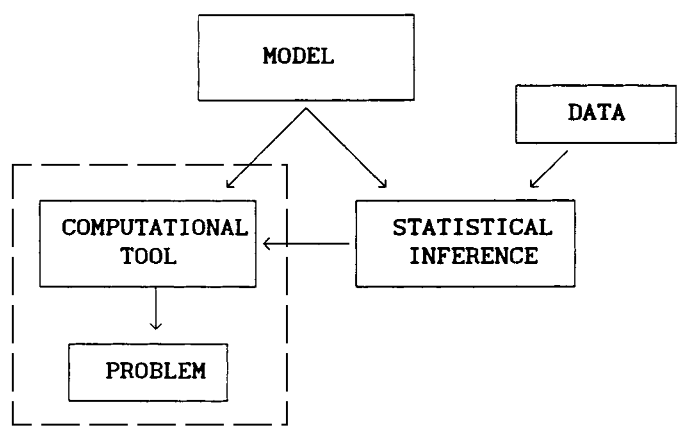


FIGURE 3
THE LIFE INSURANCE ACTUARY'S WORLD



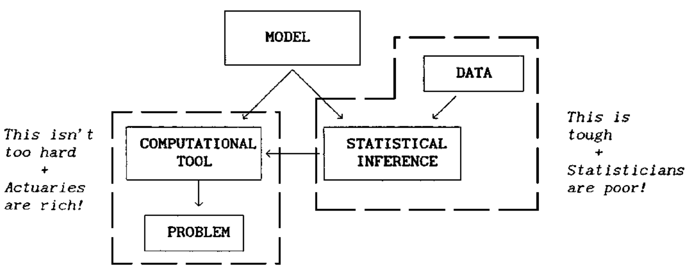
The life actuary, on the other hand, can easily spend a lifetime exercising computational tools (see Figure 3). Such data as he or she sees are as likely to be sales figures as mortality statistics. The profession even contributes to this, by setting up groups of experts to do the statistical work; in the U.K., the Continuous Mortality Investigation Bureau (CMIB) will produce life tables every so often, so the practitioner hardly ever needs to consider the *nature* of the life table that he or she uses.

Investment models might be a little different, since the life insurance actuary has no equivalent of the CMIB to do the statistics, but this is an area in which the notorious statistical method known as "actuarial judgment" has been prominent. I expand on this later in the discussion of profit-testing.

One other illustration—not to be taken entirely seriously!—suggests how our views of the modeling process might be formed in our student days; see Figure 4.

FIGURE 4

THE ACTUARIAL STUDENT'S WORLD



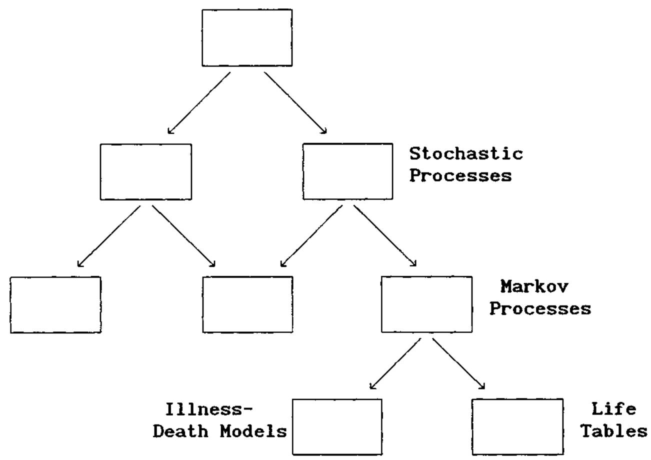
The fact that the *same* computational tool can often be derived within *different* models can be an obstacle to progress, because it can obscure the benefits of a more powerful model. For example, the oldest actuarial tool of all—the life table—can be obtained from a deterministic model of mortality or from several different stochastic models of mortality. It is not at once obvious why we should change from a deterministic to a stochastic viewpoint if the computational tool stays the same, and there has in the past been considerable opposition to just such a change in the education system. The reason is clearer if we regard the model as a whole, recognize it as the link between data and applications, and realize that progress requires an extension of *models* and not *computational tools*. But it can be hard to argue that the whole picture needs to be kept in mind, when for long periods only one part of it is directly remunerative.

2.3 A Hierarchy of Models

This can be illustrated by Figure 5, which arranges models in a hierarchy, the more general ones at the top and the more specialized ones at the bottom. For a concrete example, I have stayed with the life table and other developments in that area. As well as being a hierarchy of models, Figure 5 depicts a hierarchy of *learning* about models.

FIGURE 5

A HIERARCHY OF MODELS



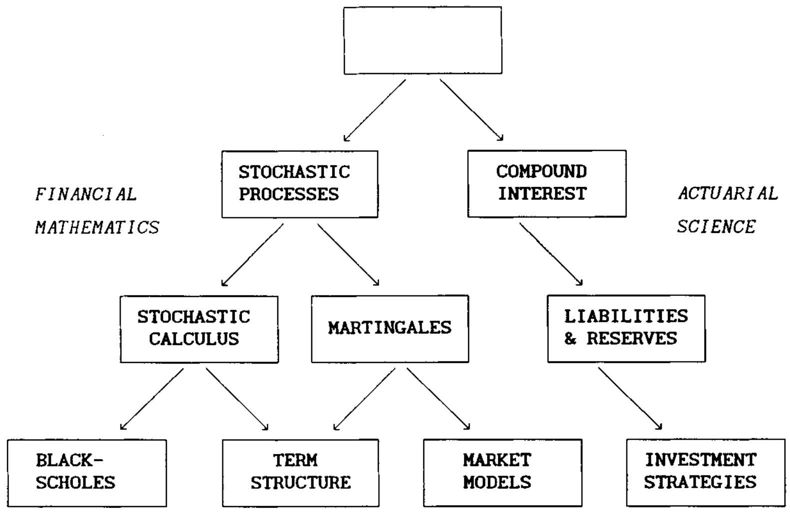
The point about Figure 5, which is meant to be suggestive only and by no means represents a considered taxonomy, is that an expert in some particular area, life tables for example, cannot take a shortcut *across* the triangle to become expert in any other area, like illness-death models. He or she must back up the tree until reaching a suitably general model that contains both areas of expertise.

This is why, for example, efforts to represent sickness by extending life table functions, l_x and so on, adding increments to decrements, are unsatisfying and ultimately not likely to succeed. Such efforts attempt to extend a *computational tool* to new problems, when what is needed is a *more general model.*

I think this is a most compelling reason for basing our education systems on models and not on their computational tools, and that would be my answer to the question posed earlier, Why move from a deterministic to a stochastic approach when the computational tool is the same?

The point is made clear, I think, if we regard actuarial knowledge of financial mathematics as a model somewhere quite far down one branch of such a hierarchy and financial economics as a different branch (see Figure 6). In the U.K., the profession started to feel uneasy about its role and its future in the late 1980s, just about when the recession hit financial services and financial economics really exploded. It responded by seeking "wider fields," to move actuarial expertise into other areas.

FIGURE 6
DIFFERENT LINES OF DEVELOPMENT



I think some actuaries imagined that we might be welcomed with cries of wonderment as we emerged from our castles in the sky—life offices and pension funds—to spread enlightenment all round. Indeed, in the U.K. we even invented an "actuarial philosophy" and an "actuarial scientific method," which, as Gilbert and Sullivan might have said, seemed to have been "merely corroborative detail, intended to give artistic verisimilitude to an otherwise bald and unconvincing narrative." I think myself that the "actuarial scientific method" represented a desire to cut across the learning hierarchy without backing up, especially as it was accompanied by attempts to cut back the mathematical parts of the syllabus.

As a profession, we have quite a lot of backing up to do! On the other hand, actuarial work has always dealt with one area in which financial mathematics is undeveloped: incomplete markets. Actuarial expertise in this area is, unfortunately, a mixture of mysticism and guesswork; such terms as "best estimate," or "realistic basis," or "actuarial judgment" can still be found in the textbooks.

It might be a good idea to formulate these ideas more solidly in a modeling framework; for what is experience and judgment if not an unformulated model? You will appreciate, of course, that if I knew how to do that I would be telling you about it, not just suggesting it! But with a few exceptions, actuaries are no longer anywhere close to the level of expertise that is quite general in the mathematics or economics communities, so we should not be surprised, in future, if others do what we cannot. We are way down in one branch of the bigger tree of models.

That said, I find the current developments in education encouraging, and I think you [the U.S. and Canada] are a bit ahead of us [the U.K.].

I am now going to turn to profit tests and model offices. First, I want to try to place these in the framework that I have just discussed, to help us to see the nature of these entities and where they might develop.

3. PROFIT TESTS

Since its introduction by Anderson in 1959, the profit test has been as interesting because of what it *does not do* as because of what it *does*. I do not think there would be much doubt that, in the framework just described, the profit test is a computational tool, and a wonderfully flexible and effective one. Figure 7 illustrates the position before and Figure 8 the position after the invention of the profit test, concentrating on the financial side. I have labeled the output of "actuarial judgment" in Figure 7 "i" to represent the rate of interest, and I have labelled the output of the "econometric analysis" in Figure 8 "r" to represent the risk premium.

FIGURE 7

PRODUCT PRICING BEFORE THE PROFIT TEST

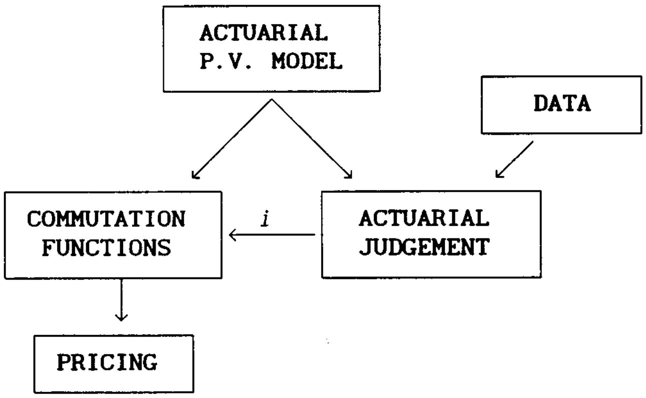
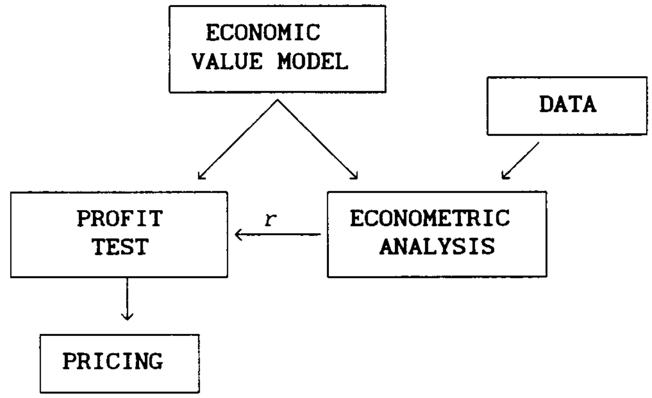


FIGURE 8

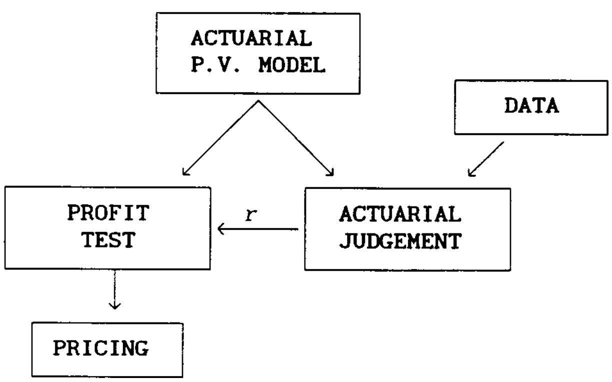
PRODUCT PRICING AFTER THE PROFIT TEST (I)



The interesting point, I think, is that this is *not* what really happened. Instead, the computational tool, the profit test, was more or less grafted onto the old model, as shown in Figure 9. It was some time before the profit test began to be placed more securely in a proper *model* of the valuation of cash flows. Crucial for this was a proper formulation of risk premium and risk discount rate, which needed to draw upon some concepts of financial economics.

FIGURE 9

PRODUCT PRICING AFTER THE PROFIT TEST (II)



I am not saying that financial economics or econometric analysis replaces actuarial judgment—far from it. What I am saying is that actuarial judgment has its hands tied behind its back if it cannot call upon these concepts for guidance, given a computational tool that lives within the model framework of Figure 8.

About 10 years ago, I was a product development actuary in a life office that was just then adopting profit-testing in a wholehearted way. For some years, profit-testing was my day-to-day job. There was no problem in deciding on the financial bases—we just used the old actuarial ways plus sensitivity analysis—but in common with most other actuaries, we were much less sure about the risk discount rate. We knew that it represented a rate of return on capital, allowing for risk, but actuarial science gave us no tools for measuring risk and return. Indeed, at about that time, someone commented on the remarkable number of offices that used a risk discount rate of 15%, which just happened to be the rate used by Anderson (1959) in his numerical examples! In other words, we had the tool, but we did not have the model.

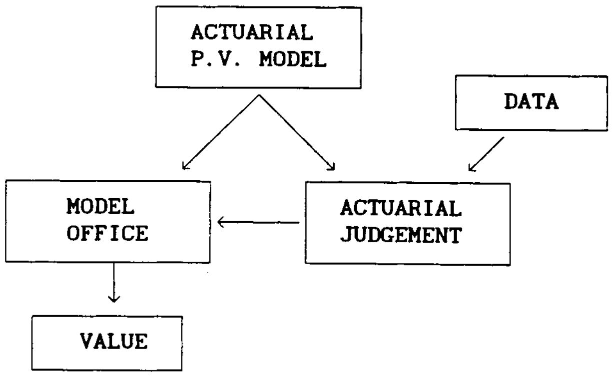
4. MODEL OFFICES

And so to model offices. Where did they come from, and where are they heading? I think that the impulse behind office models comes from several different directions, and this has had consequences for their development, which will continue into the future.

In the first place, it was inevitable, once profit tests were invented, that someone would think of adding up the cash flows to model an entire portfolio or even an entire office. Indeed, this suggestion is contained in some of the early papers on profit-testing. Given this ancestry, it was not surprising that the early model offices were profit tests writ large. In other words, they were computational tools on a grand scale, as shown in Figure 10.

FIGURE 10

MODEL OFFICES: DESCENDANTS OF PROFIT TESTS



Coming from the actuarial side as they did, these models were driven by modeling of the *liabilities*. Modeling of the assets was typically no more sophisticated than that for a profit test, and modeling of *interactions* between the assets and the liabilities depended on the intervention of the actuary, adjusting the parameters of the liability model to suit the investment scenarios.

This line of development, based on liability pricing and using the profit test and its projected cash flows as building blocks, provided some useful tools and some useful insights. It is quite possible, for example, to use such a tool to project the relatively small number of scenarios needed for dynamic solvency testing in Canada or financial condition reporting in the U.K.

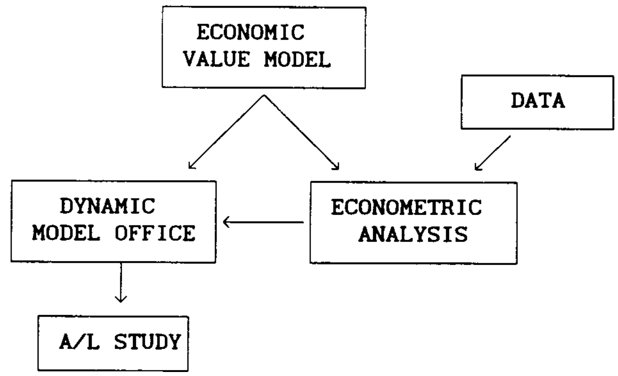
It is adequate for projecting cash flows, for example, in the event of the sale or purchase of a life office. Indeed, that event has probably been the primary cause of many life offices installing modeling systems in the first place. In such cases and in other practical circumstances, the precise pedigree of the model, or its theoretical adequacy, is an irrelevance. What is needed is a set of figures to negotiate with.

Nevertheless, this approach to life office modeling, based on profit tests and rooted in the liabilities, is a dead end.

When we turn to *asset liability* modeling (Figure 11), the limitations of the model office based on adding up profit tests are more clearly seen. If the latter is descended from the models in Figures 7 and 9, an asset-liability model is descended from Figure 8. It belongs to a different branch of the "tree" of models. Notice the addition of the word "dynamic" in the model office box. The essence of an asset-liability model is that it treats the assets and liabilities alike, and a life office's liabilities are certainly not static in the face of extremely dynamic assets.

FIGURE 11

MODEL OFFICES: DESCENDANTS OF ECONOMIC MODELS



Looked at in this way, a model office is still a computational tool, but in a much more comprehensive model framework. I have not said anything about the *form* of such a computational tool. At the moment it is most natural to think in terms of computer programs, algorithms, and associated methods like simulations. An analytical model of a life office's operations, if one should ever be invented, would also be a "dynamic model office" in Figure 11; that is, despite its name, it too would be a computational tool in a wider economic framework.

An important distinction between the models of Figures 10 and 11 that is not immediately apparent is that the dynamic nature of the liabilities in Figure 11 requires decision-making be an intrinsic part of the model, while in Figure 10 it is not necessarily so and, in examples of models that have evolved from profit tests, has not usually been so. That is, the dynamic nature of the assets, an intrinsic feature of the model, drives the decisions made by the life offices' managers, so in an asset-liability model, these also have to be made part of the modeling process. It is this feature, more than any other, that puts distance between the two lines of development of office models shown in Figures 10 and 11.

At a purely practical level, the need to model management decisions creates a need for software much more sophisticated than profit tests.

(1) Decisions are based on aggregated information; for example, asset allocation will depend on the nature and term of the liabilities in a given fund;

bonus distribution will depend on total surplus and so on.

(2) Decisions are applied to aggregates of policies; for example, the asset allocation in a given fund will determine the investment income attributable to any policy within the fund; a uniform bonus will be applied to each policy in a participating fund and so on.

(3) There are global items not related directly to policy cash flows, such as unattributed surplus, free assets (''orphan surplus''), and shareholders' funds. These affect decisions, through solvency and tax considerations, and therefore affect policy cash flows, but they are not part of any policy cash flows.

(4) There are external constraints at a global level such as solvency regulations, risk-based-capital requirements, and limits on asset allocation. These also are outside any policy cash flows.

Therefore, any attempt to model an *office* by projecting individual policy cash flows and adding them up is doomed to failure. It might result in an adequate tool in terms of Figure 10, but that is a dead end.

It is surprising just how many large-scale model office projects have done just that, taken existing profit-testing software and added up the projections. Now that asset-liability modeling is increasingly seen as a requirement, the vendors of these models have been rewriting them to allow for all the dynamic features listed above. This is a good example of the need to backtrack if one goes too far down the wrong branch of the ''tree'' of models!

That is, of course, a purely technical matter relating to modeling software. A much wider issue is the fact that there is relatively little connection between the risk theory of financial mathematics that has developed over the last 30 years and the type of modeling that actuaries do. It would be surprising if this body of theory should give no insights into the intricate web of assets and liabilities that comprise a life office, but it would not be surprising if it was not an actuary who made the connection.

It is interesting, however, that in financial mathematics too, it is necessary to resort to computer methods as soon as decisions are introduced, in moving from European option-pricing to American option-pricing for example. I do not doubt that modeling software will be needed, no matter what advances might be made on the theoretical side!

To sum up where we are today: most practical applications of life office models are deterministic, based on scenarios, and belong to the actuarial branch of the ''tree'' of models, driven by the liabilities where they are quite clearly computational tools. They are doing a useful job there. Most research applications of life office models are stochastic and use the models to estimate quantities of interest by simulation. They are thus computational tools, performing tasks of statistical inference within an asset-liability or economic value modeling framework.

I have taken the time to elaborate on these matters because our ''life office models,'' interesting though they are, are essentially computational tools. That is not a bad thing—we need computational tools—but it does leave us to wonder whether more interesting advances will come from some other direction.

Now, having established that computational tools are good and necessary, but having soothed our consciences by admitting their subsidiary rôle in the modeling process, let us focus on practical detail. What issues face the designers, programmers, and users of computer life office models?

5. Model Offices: Future Needs

I divide future needs into two areas: the purely technical and the professional.

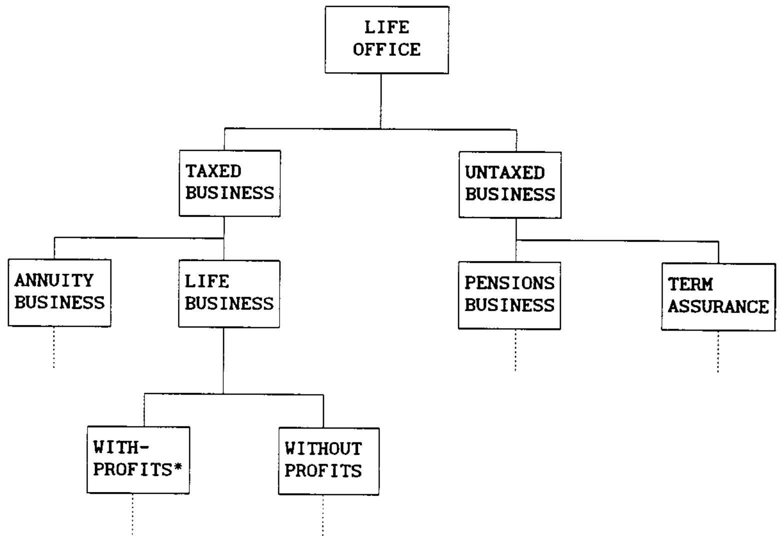
5.1 Technical Needs

I have already outlined the need for a model office program to support the modeling of decisions at a high level in the office, that is, at a level above that of individual policies. Even if it is intended only to carry out large-scale, deterministic projections, this is a sign of a usable tool, rather than a cobbled-together collection of lower level functions.

To support that sort of decision-making, a model office has to reflect the decision-making structure of an office. A life office is a complicated thing. Roughly speaking, any life office can be mapped by its organizational structure, which can take many forms. Figure 12 is an example. The office could be organized by function within line of business, or the other way round. And every time there is a change of management, the structure flips over.

FIGURE 12

PART OF A LIFE OFFICE STRUCTURE



The location of the boxes in this organizational chart usually makes it plain where decisions are being made; for example, in the with-profits class marked * there will be bonus rates to be declared, and (in the U.K.) a terminal bonus policy to be decided, and so on. So a model office program ought to locate these decisions in that part of the model office.

That raises two questions: (1) How to do that in the first place, and (2) how to do it flexibly enough that the model can evolve as the office evolves?

The answer to both questions, I think, depends on the model office software being designed primarily to reflect organizational structure, with computation and detailed cash-flow engines being subsidiary, organized within that structure. I can give you a concrete example, from my own model office program called MO.

MO has two kind of objects, classes and parameter sets. The word "classes" suggests an object-oriented approach, and there are elements of object-oriented design in MO, although it is not, in fact, written in an object-oriented language.

5.1.1 Classes

A "class," in MO, is essentially one of the boxes in Figure 12, but these are anonymous boxes, boxes without identity. The first step in constructing a model of a particular office is to bolt together a series of boxes into the right structure and to give them appropriate names.

The *names* of the classes mean absolutely nothing to MO; they are only labels to help the user. At this stage, the model is nothing more than a structure of empty boxes that cannot do anything.

In fact, there are two kinds of classes: classes with policy data and classes without policy data. Classes with policy data live at the bottom of the hierarchy and do the nitty-gritty work of crunching policy cash flows. They are the nearest thing in MO to profit tests. Classes without policy data live higher up the hierarchy. Their purpose is to represent aggregates of business and to give the office its structure. But at this stage, I emphasize, all these classes are just empty, formless boxes.

It is worth mentioning one key feature: the classes are all empty, formless, *identical* boxes, apart from the policy data/no policy data dichotomy and the connections that have been made between them. MO treats all alike, which means that, when it comes to computation, MO has only one task to perform, processing a class. At run time, MO moves around the hierarchy in a defined manner, processing each class it meets without knowing or caring *what* that class represents. This is a tremendous benefit when the model is extended, new features added, or the structure changed.

5.1.2 Parameter Sets

What gives MO its function is the *parameter sets*. To control the office and define its features, we need to specify parameters for investment conditions, new business, valuation and pricing bases, expenses, bonus, tax, and many other things. We can think of some desirable features of such "parameter sets":

- They should be changeable at arbitrary times. For example, we should be able to change investment conditions every year if we wish, and everything else too.
- Some of these parameter sets model external forces, such as the stock market. We might want the "parameters" that are the inputs to the model office to be the outputs from some other model.
- Some of these parameter sets model management decisions, which must be responsive to external forces. In a dynamic model office, these "parameters" are not preordained numbers but algorithms, such as dynamic asset allocation strategies.
- An important feature of these "algorithms" (parameter sets) is that sometimes an asset allocation (say) is an input and sometimes it is an output; the same is true of any decision that might be modeled dynamically. So the distinction between inputs and outputs is blurred.

Here is what MO does. The various parameters needed to define the business and control the projections are packaged into well-defined units, such as market values of assets, tax rates, bonus rates, asset allocation, and so on. Those which relate to dynamic decision-making will include a choice of algorithms, with all the appropriate parameters for the user to set up.

Each "package" looks after all its own inputs and outputs. Any parameter set can be changed at any time, so the package maintains a time-stamped list of input parameters. Some parameter sets (the dynamic ones) also produce outputs, so each package also maintains a time-stamped list of outputs, as shown in Figure 13. In fact, the box marked "parameter set" contains little more than housekeeping routines to maintain the list of inputs and outputs.

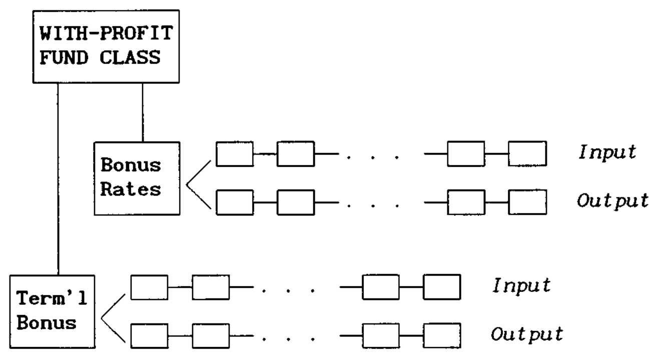
5.1.3 Building the Model

The real work of building the model is attaching parameter sets to classes. MO allows the user to create a new parameter set and hang it on to any class in the hierarchy; that class then has the function bestowed by the parameter set.

For example, to turn an empty class labeled "with-profit fund" into a functional with-profit fund, we create the appropriate parameter sets and attach them to that class (see Figure 14). Like the classes, the parameter sets are identical software objects, except in what they contain. MO does not know or care what kind of parameter set it is dealing with, until a calculation is needed.

FIGURE 14

WHAT MAKES A WITH-PROFIT FUND A WITH-PROFIT FUND



The software "units" defined here—classes and parameter sets—suffice to define the entire structure and are flexible enough that they can fulfill other purposes too. For example, MO has to keep track of all its output: assets, liabilities, profit, and so on. This is simply done by a parameter set with no inputs (see Figure 15), so there is no functional difference between inputs and outputs in MO.

FIGURE 13

A PARAMETER SET IN MO

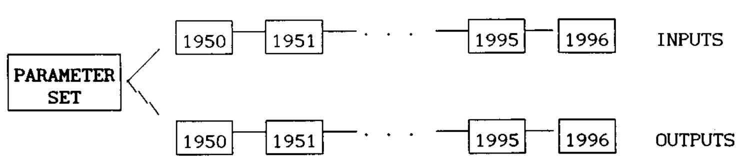
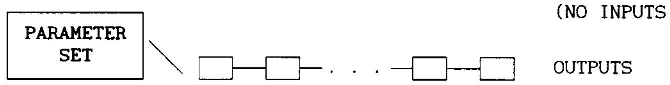


FIGURE 15
HOW MO STORES OUTPUTS



This is only one example; the benefits of this "black box" approach are enormous. What MO actually does, then, is to step through the sequence of events at the end of each year, in order:

- Update assets
- Pay claims (plus expenses)
- Write new premiums
- Pay expenses
- Reinvest assets
- Valuation
- Bonus distribution.

To process each event, MO traverses the hierarchy and interrogates each class to determine whether it has the relevant parameters. For instance, if MO is engaged in the final step, bonus distribution, it will ask each class it meets, "Do you possess bonus rate parameters?" If not, MO just moves on. If yes, MO calls up the bonus routines and starts processing the business contained in that class. If, at any stage in a calculation, MO needs to find some other information (for example, to compute a moving average of yields in the middle of a bonus calculation), the hierarchical nature of the model is helpful; MO just searches upwards until it locates an "ancestor" class that has what it wants. In this way, the properties of each class "flow down" to the subclasses and ultimately the policies that they contain.

5.1.4 Where Does This Take Us?

I think this illustrates the key feature of second-generation model office software. First-generation was the "sum of profit tests" idea; a lot of life office modelers made that mistake and have had to backtrack. Second-generation models are based on the functional, decision-making structure of a company.

It might have been the impulse towards stochastic asset-liability modeling that drove home the need for second-generation models, but it is worth emphasizing that they can do all that first-generation models can, better and more easily, with less trouble for the user.

The issue for designers and users of model office software therefore is this. How well does the model map the functional, decision-making structure of an office? And just as important, how easy is it to change when a new management comes in and the structure turns around? Or when a major new line of business must be added?

I do not think existing software is very close to meeting this criterion. I should add that MO is a small-scale research tool—a testbed for ideas about modeling—I have not seen the same techniques fully reflected in industrial-scale models.

There are a lot of first-time users of model office software out there. Most of them have bought a package from a vendor such as a consultant and have discovered the overhead of allocating highly skilled staff to the task practically full time or else forking out consultancy fees. If they discover, over the next 10 years or so, that their models do not evolve in step with their businesses, without expensive rewriting, redesigning, and reimplementing, they are going to regard their software as part of the problem, not part of the solution. All this at a time when changes in regulations, in the direction of dynamic solvency testing, mean that an office without an adequate model is more and more like a ship without a radar.

Second-generation models will undoubtedly come, I know more about the U.K. than the U.S.; perhaps you have them already. But as things stand, they will emerge piecemeal. Some vendors will write good software; some, not-so-good software, and nowadays the effect on their clients could be non-ignorable. That brings me to the second matter: professional issues.

5.2 Professional Issues

At some point, I think the profession will have to interest itself in the qualities and capabilities of office-modeling systems. Partly this is because such systems are often sold by consultancies as part of an actuarial service; they thus fall indirectly under the profession's jurisdiction. Partly this is because of the likelihood that they will play an increasing role in regulation. Even if they do not displace the actuarial certificate as the arbiter of solvency, they will in one way or another influence the attention that the supervisors pay to an office's business.

As yet, there is no accepted wisdom about what a model office system should be able to do, or how well it must do it. Different systems from different vendors are in competition, and differences between them are generally presented as marketing points, rather than raising questions in the minds of the professional onlooker. There is a good deal of "trade secrecy" about the workings of model office systems, and often a good deal of prior commitment to purchase is needed before much can be learned about any system. I have known cases in which a model office system has been sold to top management, and it has been some time before the more junior staff detailed to operate it have discovered what it can and cannot do.

Some of the issues facing a potential purchaser of model office software are as follows:

- *Transparency*. How easy is it for the purchaser to identify the algorithms that the model uses? This can be summed up as, what are the possible inputs, what are the possible outputs, and what happens in between? It is important to be able to form a judgment about this in deciding whether any system is suitable for a particular purpose in a particular company.
- *Flexibility*. How easy is it to update a model as the office evolves, management changes, and new products emerge? Can this be done with a moderate commitment from the office's own resources or will the model really remain the property of the vendor?
- *Verifiability*. Any model will make assumptions and approximations. High on the list for most model offices is some form of condensation of the business into model points and discretization of cash flows. How accurate are these aspects of the model? In a dynamic model, with decision-making included, there are many more questions of this nature. Even if the model offers the capability of doing what the purchaser wants, there is still a question over how accurately it can do it.
- *Comparability*. How do different models from different vendors compare when posed the same task? Benchmarking is universal elsewhere in the retail software industry, why not in the actuarial software industry? This one area that ought to be of concern to the profession and to supervisors alike.

All these issues take on a more serious aspect the more the commitment of a business to one system from one vendor influences its financial future and its standing with the regulators.

Some of these decisions can reasonably be left to the individual actuary to make, provided the individual actuary has enough information. This is by no means ensured at present. However, vendors are not restricted from selling their wares in territories that do not have the actuarial systems of scrutiny and control that we have in the U.K. and the U.S. and Canada, so there might still be an issue here.

When the vendor is an actuarial consultancy, it might be assumed that professional control will be exerted at the design and marketing stage, and I am sure that actuaries concerned with the design and installation of model office systems do approach it in a professional manner. Professional standards must confer a significant benefit here; however, they place a duty on the profession as a whole as well as on individual members.

Also, asset-liability modeling for long-term institutions, in which model office-type software and simulation is currently a key tool in the actuarial world, may converge with asset-liability modeling for short-term institutions, in which modern financial mathematics is a key tool in the nonactuarial world. It might therefore be helpful if the actuarial profession were to develop a clear view of the methodology with which it might make a major contribution to such a development.

So although the profession will probably prefer to keep watching and see how matters develop, I think it should be making some modest plans to contribute its views as regulatory changes move towards a position in which (1) greater reliance is placed upon modeling and (2) the choice and quality of a model begin to have direct financial consequences, much as the choice of valuation basis has today.

One possible way forward might be for a task force (or in the U.K., a working party) to look at the following areas:

(1) The current use of modeling software
(2) The likely future use of modeling software, especially in connection with regulations
(3) Possible guidelines for actuaries in the position of assessing modeling software for use in a given institution
(4) Possible benchmark standards for key projections over given periods of time.

The real meat of these suggestions would be in (3) and (4). In the U.K., the result of (3) could be given some real force by including guidance in the duties of the appointed actuary; otherwise its effectiveness would, of course, depend on the position of the actuary. In any case, these suggestions are meant to be by way of a helpful push in the right direction, not strong prescription.

There is one final matter of some professional concern, although secondary. Many research papers are beginning to use stochastic model offices as estimation tools, in studies of solvency and so on. In cases in which the research is carried out by using a very complicated piece of software that has not been subject to professional peer review, I think that the academic standards of the journals that carry these papers are undermined. Perhaps the publishers and editors of such journals should consider the use of a different category of paper, such as "workshop" or "non-refereed," unless the author is willing to subject the software to the same peer review as the article.

6. CONCLUSION

I think that modeling, which in much of the profession means software, is approaching the point at which some involvement on the part of the profession is indicated. The key issue that I think will emerge with second-generation software is how to design models that are flexible enough to evolve with the institutions they represent. I hope I have indicated, by example, the sort of approach that I think would be useful.

But we must not forget that cash-flow modeling is part of a wider universe, and we should keep firmly in mind its place as a computational tool in the model of models, and ensure that the profession is expert in modeling in the wider sense, as well as in the highly specific applications that make up the current source of the actuary's remuneration!

ACKNOWLEDGMENTS

I am fortunate to work at Heriot-Watt University, where it is rare for a day to pass without there being a discussion on the application of mathematical models to insurance and financial problems, and I would like to mention David Bowie, Andrew Cairns, Mary Hardy, Cliff Speed, Howard Waters, and David Wilkie. The model of models and the hierarchy of models used in this paper have appeared in an article in *The Actuary* magazine (Bowie et al. 1996).

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Discussion

STEPHEN J. STROMMEN*

Professor Macdonald has provided an excellent introduction to many of the issues facing the actuarial practice of modeling. I would like to highlight several of his points from a practitioner's point of view.

When I have mentioned a conference on modeling to my colleagues, most have assumed that the main emphasis would be on model offices and other "computational tools." Professor Macdonald correctly points out that such a focus would be far too narrow. Modeling, in a more abstract sense, means the development of theoretical frameworks for problem analysis. And much work needs be done to add breadth to the variety of theoretical frameworks employed by actuaries.

Nevertheless, there are far more actuaries applying computational tools to real world problems than there are developing new theoretical frameworks. To most practicing actuaries, "modeling" means developing or applying the computational tool. And as these tools get more and more complex, issues of education, training, and professional standards arise.

We need to respect both views of modeling as equally valid and not subjugate one as "mere computational tools." Particularly when the large body of practicing actuaries works more closely with those tools than with abstract theory, we need to be sure that the profession does not fail to focus on tool-related issues.

I would like to put the discussion of computational tools in a slightly different perspective. As I see it, computational tools are the practical implementation of theoretical models. They are the technology connected with the pure science of theoretical models.

Technology is important, not just in actuarial practice, but in other professions as well. Physicians do

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not simply learn biochemistry; they learn to use the x-ray machine and CAT scan for diagnosis. They learn this in school, not just on the job. Engineers do not simply learn physics; they learn about the materials that are available to them for construction of practical devices. They learn this in school, not just on the job. In the same way, actuaries should not just learn the mathematical theory of actuarial science; they should learn about the computational tools that allow them to apply that theory to real world problems.

In my opinion, our educational system has been weak in that regard. An important issue is how to go about removing that deficiency.

Here's another look at the relationship between theoretical models and computational tools. Everything begins with a practical problem that faces the practicing actuary. Consider the need to understand the distribution of aggregate claims for a line of business in an insurance company. The actuary must make financial arrangements to protect the company from the upper tail of the distribution where too many claims could put the company in financial difficulty. But how does one estimate the upper tail? For many decades there have been theoretical frameworks rooted in risk theory for evaluation of this problem. However, it has only been in the last couple decades that work involving ordinary generating functions and the Laplace transform has led to much more practical computational tools to compute the tails of the distribution of aggregate claims.

Or consider the need to put a value on interest-sensitive options like the option to prepay a mortgage with no penalty. Without theoretical models of the term structure of interest, stochastic calculus, and the theory of low-discrepancy sequences, we could not have the computational tools we have today to put a value on such options.

The point here is that these three things must always come in order. First, there must be a practical problem, then a theoretical framework, and then a computational tool. The computational tools of the future will never evolve without new or more developed theoretical frameworks. This, I believe, is one of Professor Macdonald's central messages, and I concur.

Now let's turn to model offices. Professor Macdonald spent a fair amount of time discussing the difference between first-generation and second-generation model offices. The key difference is the dynamic nature of the calculations in a second-generation model office, and the interdependence of assets and liabilities in the simulation that is carried out.

I prefer to think of this as an implementation of control theory. A life company, or any other financial institution for that matter, must operate within certain bounds and is subject to various external shocks. When the institution is represented as a model office system, the problem at hand is to create a control mechanism that will keep the system within stated bounds, given a set of initial conditions and possible external shocks. Can the company remain viable under reasonably adverse sets of conditions?

The point is that, while most actuaries agree that dynamic models are essential, we tend to focus on refining our models without reframing the problem as one of control theory. That is why profit-testing systems continue to be used as the basis for model offices when a much different architecture is needed to implement a control mechanism. Second-generation models, as defined by Professor Macdonald, are models that implement that different architecture.

Professor Macdonald describes a design for model offices that has a great deal in common with one that I describe in my paper "An Object-Oriented Design for Dynamic Simulation Models" (p. 38). The high points, as I see them, are as follows:

First, the system implements a controller, which simulates management decisions based on the state of the enterprise at points in time. This is the basic definition of a second-generation model.

Second, the system uses the concepts of an object-oriented design by making use of "polymorphism." This refers to the way various blocks of business such as life insurance and pensions can be plugged in to the model as modules, each with exactly the same software interface to the rest of the system.

As I said, I describe a model design approach that shares these traits in more detail in my paper. Leaving that aside for the moment, then, there are some practical issues Professor Macdonald raises that deserve repeating.

First, commercially available model office systems are very complex pieces of software. While vendors have lately been working to improve their system documentation, there are still areas in which some techniques are not clearly disclosed. And sometimes users don't even know enough to be able to ask the appropriate questions.

For example, the two most widely used model office systems in North America use fundamentally different algorithms to implement some kinds of dynamic behavior, such as when interest credited on the liabilities depends upon the earnings of an associated investment portfolio. Both systems will arrive at what is for practical purposes the same answer, but get there in very different ways. I believe it is important for users of such models to understand the difference, because it affects the way other kinds of dynamic behavior can be added to the model.

How should the profession address this issue? Is it an issue at all? The American Academy of Actuaries sets standards for areas of practice that use these models. Are the existing standards sufficient?

I believe that the existing standards are sufficient. The burden is on the practicing actuary to understand the tools he or she uses. Vendors who fail to disclose their methodology will find it difficult to sell their software. I think we can trust the integrity of the practitioner to keep the vendors in check here. At the same time, there are some trade secrets that simply speed up an approximation process without changing its fundamental nature. I am perfectly comfortable leaving such secrets in the hands of vendors, thereby encouraging them with the profit motive to find new ones.

Another issue is the possibility of involvement by the profession in development, testing, and measurement of performance standards for model offices and other computational tools. Any actuary who has had to choose a third-party modeling system for the company understands how much work is involved in comparing systems. Simply evaluating one system can require months of testing. Having some sort of impartial third-party review of such systems would be of value.

As Professor Macdonald mentioned, a "working party" or "task force" could set up a battery of standard problems and publish evaluations of how each commercial model addresses them and the results it provides. Elsewhere in the software industry it is common for competing packages to be compared by checklists of features; perhaps that could be done here as well. Creating and staffing such a testing body poses a problem, however. First, the amount of time required would be substantial, making it difficult to carry out as a purely volunteer effort. Second, many of the most qualified people are employed by the vendors themselves and thus could not be counted upon to be impartial.

One could take this a step further and commission the development of some sort of standard software modeling framework. This would of course step on the toes of the vendors and tends to imply that they are not adequately meeting the profession's needs by themselves.

Is that the case? Is there a need for involvement by the profession here? I think the answer varies by practice area. In North America the property/casualty insurance area has perhaps been less well served than the life insurance and pension practice area. Whatever the case, this deserves more discussion.

A final issue is the use of computational tools for research. When analysis in a peer-reviewed paper is done using software that is not fully and completely documented in the paper, the "black box" syndrome results. This kind of situation can be handled by peer reviewers simply pointing out the lack of documentation of the technique being used.

Sometimes, however, the technique in use is fully documented but is very complex. In such cases it could be of value for the peer reviewers to have access to the software used by the author of the paper to carry out the analysis. In fact, it could be of value for all readers of such papers to have access to such software. This is true any time the effort needed to implement the software is nontrivial (that is, takes much more time than reading the paper).

Should authors be required to make their computational tools available to reviewers and/or readers of their published papers? As a practitioner, I would find this a very valuable service.

To close, I once again thank Professor Macdonald for his excellent summary of the state of actuarial modeling practice. My main thought is that the use of model offices has risen to a position of great importance among practicing actuaries. Yet there is very little literature on the subject of the appropriate design and construction of such models. This is a very fertile area for research. I hope that the papers from this conference will stimulate more work in this area.

Additional discussions on this paper will be accepted until January 1, 1998. The author reserves the right to reply to any discussion. See the Table of Contents page for detailed instructions on the preparation of discussions.

It would be very interesting to learn whether one could develop statistical estimation procedures for finite truncations of the series expansion of the option price, (2).

Incomplete Markets

In the earlier sections of the paper, the *Esscher* transform has not arisen because the martingale-determined equation has a unique solution, in this case determined by λ^*. But this situation changes when an *incomplete* market is encountered, for example, in the earlier paper of Gerber and Shiu (1994, Sec 4), which includes the important example distributions of the shifted Gamma and shifted inverse Gaussian processes. That paper and the published discussions of it provide important insights about how to respond to the nonuniqueness that characterizes incomplete markets

In the present paper the authors achieve a fruitful "universality" of their approximation formula for incomplete market models, provided that option prices are calculated by the Esscher method. This result adds a further strong justification for using the Esscher approach in incomplete markets.

In other non-binomial discrete stock price distributions, a similar equation on martingale probabilities arises without having a unique solution, see Chan and van der Hoek [1996, (2 5)], where the Esscher as well as other approaches are applied. In particular, it is shown in Chan and van der Hoek that the Esscher transform distribution delivers an option price close to one determined by the minimum relative entropy distribution when the time steps of the underlying distribution, here $X(1)$, are sufficiently small.

It would be interesting to investigate what kinds of approximations would be forthcoming from other approaches to incomplete markets, for example, the entropy approach; see also Buchen and Kelly (1996)

The Gerber and Landry paper opens up new areas of both applied and basic research in nonstandard financial markets.

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AUTHORS' REPLY

HANS U. GERBER AND BRUNO LANDRY

We thank Dr. Kortanek and Dr Medvedev for their discussion, which captures well the ideas of our paper. Since the publication of the paper, the linear approximation and other formulas have been used and tested with real data; see Pafumi (1997) We would also like to point out papers by Chan (1997) and Eberlein and Keller (1995), which contain additional aspects of the method of Esscher transforms.

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"Current Actuarial Modeling Practice and Related Issues and Questions," Angus Macdonald, July 1997

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Angus Macdonald is to be congratulated for recognizing the importance of articulating a broader

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framework within which to consider actuarial models The "Model of Models" that he describes is a bold attempt in this direction.

Enormous progress has been made within the methodology of science over the past two thousand years in understanding the nature of models and their relationship to reality The methodology of economics is a more recent discipline, which has emerged as a major area of professional academic study only during the past two decades (Backhouse 1994). Actuaries are not alone in struggling with the difficult questions concerning the nature of models; there is a vast body of highly respected research and thought on which to draw within these related methodological studies

Angus proposes a hierarchy of models in which, as I understand it, models at a higher level are more general or abstract and, in some sense, license models at a lower level This approach would appear closely related to that of the covering law model of science, which is perhaps best expounded in the classic 1960s works of Carl Hempel and Ernest Nagel (Hempel 1965, Nagel 1979) The philosophy of science has been through two revolutions since then: that driven by the rise of the sociology of science (Kuhn 1962), and that driven by the causalist writers such as John Mackie, Nancy Cartwright, Ian Hacking, and Wesley Salmon Positivism, the philosophy closely associated with the covering law model, collapsed in the 1960s. Covering law type accounts have generally foundered on the problem of establishing a sufficient definition of each level in the hierarchy (for example, what is a "computational tool" and what is a "model") and of providing an account of how a higher level model licenses one at a lower level. Cartwright's discussion of the issues is particularly illuminating (Cartwright 1983)

In contrast to Professor Macdonald's view, I am persuaded by recent methodological advances that seeking to root actuarial models in more abstract covering models is a mistake. In a forthcoming paper, I argue that actuarial science has developed a robust empirical method of linking its models to reality, working predominantly with low-level generalizations (Pemberton 1998) I propose that we should be celebrating the firm roots of our models within the bedrock of empirical facts, and using the understanding of our methods afforded by recent developments in methodology to strengthen this approach yet further

Professor Macdonald is to be applauded for provoking an essential debate on these topics within our profession In my view it would be unwise for the actuarial profession to seek to reinvent the complex wheels of understanding in this area that have been painstakingly constructed by the philosophers.

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"Complex Dynamics, Market Mediation and Stock Price Behavior," Richard H. Day, July 1997

JOSEPH K. WANG*

Professor Day's model successfully reproduces actual price patterns of the market and, importantly, price patterns considered by market technical analysts to be predictive of future price movements In fact, the appearance of price patterns in the model even when fundamental values are unchanged is consistent with concepts in technical analysis of "overextended" markets and "overbought" and "oversold" conditions, where prices deviate far from fundamental values The α-investor parameters m and M correspond to "support" and "resistance" price levels in technical analysis, which often remain constant for years independent of external news As noted by Professor Day, his model is simplified and does not account for various factors influencing real market activity Inclusion of some factors would probably complicate the model with little improvement in explanatory power, but consideration of the relationship between price and traded volume, or number of shares of stock sold at given price and time, would improve the model, as I discuss below.

In technical analysis, broadly speaking, decreasing volume indicates impending change in the current

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