

EVALUATING THE RISKS OF MODELING ASSUMPTIONS USED IN RISK MEASUREMENT

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ABSTRACT

Over the past few years, risk measurement has become an important, high-profile responsibility for most firms in the financial services industry. With advances in academic theory and in technology, financial risk modeling has grown increasingly sophisticated. Most firms rely on a number of models to analyze their market risks (for example, sensitivity to changes in interest rates, exchange rates, commodity prices, and so on) for asset/liability management. But it is critical to recognize that even the most sophisticated models must make assumptions about key parameters that affect the results of the analysis. This so-called "model risk" reflects the fact that in the real world risk factors are unstable and the historical data upon which many modeling inputs are based can change. This paper discusses model risk, gives specific examples of how model risk can affect fixed-income portfolio valuation, and explains why risk measurement should involve stress testing of key modeling assumptions. If the results of a valuation or asset/liability analysis change dramatically given a small change in a modeling assumption, management may wish to reduce the firm's exposure to that risk factor, as absolute certainty in financial modeling is an unobtainable goal.

INTRODUCTION

The subject of risk has received much attention in recent years as investors and regulators have become increasingly (and sometimes painfully) familiar with the consequences of failing to adequately measure and manage risks. In responding to the call for papers for the SOA Conference on Integrated Approaches to Risk Measurement in the Financial Services Industry, one would expect that the concept of value at risk (VaR) would be covered extensively, as VaR has gained a good deal of popularity as a way to describe a firm's risk profile. Undoubtedly, one or more presentations would address the different VaR methodologies currently in use (variance/covariance matrix, historical simulation, and Monte Carlo), explaining why these methods can produce markedly different results, and certainly the importance of stress testing in risk management, using specific, "worst-case" scenarios, would be discussed.

This paper approaches the topic of risk measurement from a slightly different angle. It addresses the

fact that, embedded in even the most sophisticated and comprehensive risk measurement systems, there are a variety of assumptions that may or may not be accurate and/or valid over time. These assumptions are necessary and are (I hope) reasonable when implemented. But it is important to remember that the conclusions reached (and actions taken) by using a risk measurement system might change if one or more of the underlying modeling assumptions were to change. This, in and of itself, is a risk that can and should be measured. This concept applies to both the asset and the liability sides of an insurer's risk management process; however, since this author's area of expertise is in fixed-income portfolio analysis, the paper deals primarily with the risk associated with modeling assumptions used in managing the asset side of the business.

The paper first defines model risk in general, giving a specific example of how neglecting to examine modeling assumptions can lead to faulty conclusions. It then discusses what can be done to manage the risks embedded in modeling assumptions by measuring the impact of changes in two of the modeling assumptions required for fixed-income portfolio analysis. Finally, the paper suggests ways to use these measures in the process of managing both risk and return.

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DEFINING MODEL RISK

Model risk is the error in risk measurement or valuation that arises from the fact that modeling assumptions are imprecise and/or become invalid over time. All models that attempt to measure risk have embedded assumptions that can affect the “answer”:

- In asset valuation and portfolio simulations, there are assumptions about modeling the term structure of interest rates, volatility estimates, prepayment speeds, and so on. Each of these affects the calculation of the portfolio’s duration and convexity, total return simulations, security prices, and cash flow projections.
- In VaR analyses, there are assumptions about the correlations (and stability thereof) across risk factors, the time horizon used, the historical period used (in historical VaR calculations), and so on.
- In liability modeling, there are assumptions about policyholders’ responses to declining surrender charges, the sensitivity of annuity holders to changes in interest rates, the amount of new business to be generated over the horizon period, and so on.

THE IMPORTANCE OF MODEL RISK—AN EXAMPLE

A few years ago, a structured security known as a “yield curve note” gained some popularity as an investment for a number of banks and insurance companies. The security’s coupon rate adjusted annually, based on the difference between two points on the Treasury yield curve, plus some fixed amount, with a final maturity typically between seven and ten years. A typical coupon formula was

$$\begin{aligned} &(\text{YTM: 10-year Treasury} - \text{YTM: 2-year Treasury}) \\ &+ 450 \text{ bps.} \end{aligned}$$

At the time, the slope of the yield curve was extremely steep by historical standards. As of year-end 1993, the ten-year Treasury yield was 5.80%, while the two-year Treasury yield was 4.26%, a difference of 154 bps. For comparison, as of September 30, 1997, the ten-year and two-year Treasury yields were 6.16% and 5.83%, respectively, a difference of only 33 bps. So, at the end of 1993, this ten-year/two-year yield curve note would have offered a coupon of $(5.80\% - 4.26\% + 450 \text{ bps}) = 6.04\%$. A naive analysis might suggest this was an attractive investment, as these securities were AAA-rated and offered a substantial yield pick-up versus a Treasury with the same maturity. But it is important to understand the modeling assumptions

that most affect the valuation of the security in order to fully understand its risk profile. Which assumptions are most important to scrutinize?

Notice that the coupon rate drops as the slope of the yield curve flattens. Even if the benchmark 30-year Treasury yield remained unchanged, the coupon on the bond would drop if the steep yield curve reverted back to a more “normal” slope. As of September 30, 1997, the security would have had a coupon rate of $(6.16\% - 5.83\% + 450 \text{ bps}) = 4.83\%$. With a remaining maturity of five to six years, this coupon would be approximately 120 bps below the yield of a Treasury with the same maturity, so its price would have to decline. If the difference between the ten-year and two-year Treasury becomes negative, the coupon rate would drop below 4.50%. While we have not experienced an inverted yield curve in recent years, over the period from 1980 through 1989, the spread between the ten-year and two-year Treasury yields was negative during five of those ten years. An inverted curve could cause the price of the security to drop precipitously.

Note also that the coupon rate must be evaluated in light of the *absolute* level of interest rates, not just the slope of the curve. If interest rates were to rise across all maturities, for example, by 300 bps, the coupon on the yield curve note would be unchanged at 6.04% $(8.80\% - 7.26\% + 450 \text{ bps})$. This would hardly be an attractive rate with the two-year Treasury yielding 7.26%. Therefore, to properly evaluate this type of security would require a term structure model that allows for nonparallel shifts in the yield curve, and that simulates uncorrelated movements in short versus long Treasury rates. A risk model that did not include these features would not accurately capture the price behavior of this security.

QUANTIFYING MODEL RISK

Making assumptions is an unavoidable part of risk measurement; absolute certainty in risk measurement is an unobtainable goal. So what can be done to better understand the importance of these assumptions? *We can quantify the potential error that a change in modeling assumptions would produce.* Investment managers who manage portfolios, actuaries who generate cash flow testing analyses, and others involved in risk measurement can vary the critical modeling assumptions used in their models by some amount and measure the degree to which the “answer” changes. One can then decide whether or not this degree of uncertainty is acceptable and, if not, determine the types of assets and/or liabilities that are contributing most

to the high level of uncertainty and make the appropriate changes.

VEGA—A MEASURE OF VOLATILITY UNCERTAINTY

One of the most important inputs to a risk measurement system are the volatility estimates used within an option model. Many fixed-income securities (and many liabilities) contain embedded options, such as the issuer's option to call the security, the investor's right to put the security to the issuer, a homeowner's option to prepay a mortgage, and so on. Since volatility is what makes an option valuable, the volatility estimates used in an option model can have significant impact on the conclusions reached about the riskiness of a security or portfolio, or even about the entire firm's risk profile.

There are two basic approaches to deriving a reasonable volatility estimate, *historical* versus *implied*. The historical approach uses empirical (historical) changes in the level of Treasury rates; the implied method uses actual option prices and solves for the volatility rate that will cause the model to produce the observed market prices. There have been numerous articles written on the subject, and practitioners may use one method or the other, or some combination of the two, but it is clear that there is no single right answer about the correct level of volatility to use in valuing options. Therefore, we wish to measure the magnitude of the possible error that might be introduced in our risk measurement process by using volatility estimates that do not accurately reflect future market conditions.

The risk measure Vega describes the sensitivity of an asset's price to a change in the level of volatility used in an option model. To derive Vega for a fixed-income security, we shift volatility¹ down and up by some amount (often 1%, for example, moving volatility from 11% down to 10% and up to 12%) and compute two new prices under the lower and higher volatilities (holding the security's option-adjusted spread constant). The different volatility percentages represent one standard deviation of the annual expected change in Treasury yields and control the range of interest rate environments considered when valuing an option. If the model's volatility rate or rates are too low, the

model can fail to sample environments where an option causes important changes in a security's cash flows. If volatility is set too high, a security's sensitivity to changes in interest rates will be overstated. Note that Vega may be computed for all types of securities with embedded options, such as equity derivatives, currency options, and so on, by shifting the volatility of the underlying factors that determine the payoff of the security.

Table 1 shows the Vega of a representative sample of callable corporate bonds². The Vegas for these securities range from 0.01 to 0.60, which means that a 1% misestimate or change in volatility levels could produce as much as a 0.60% error in the valuation of the security. We can see from the table that the level of Vega cannot be determined from any single security characteristic; it depends upon the bond's current price versus its call price, the proximity of the call date, and the final maturity of the bond. All other things being equal, a bond with an embedded option that is "at-the-money" will have a higher Vega than bonds with options that are either deep "in-the-money" or far "out-of-the-money." This reflects the fact that a slight change in volatility estimates can cause enough of a change in an at-the-money option's value to push a bond's price above or below its call price. We can also see that Vega has some correlation with the bond's remaining time to maturity; this is due to the sensitivity of the time value of the embedded options to a change in volatility.

Vega can and should be computed at the portfolio level. Note that there is practically no diversification effect in the case of an error in volatility estimation; almost all fixed-income securities, except bonds with put options, have a positive Vega. This means that if an option model is using a volatility estimate that is too low, all securities with embedded options will be overvalued. Since one goal of risk measurement is to forecast possible changes in the firm's assets relative to that of its liabilities, it is important to understand how the asset/liability match might be affected by a change in this critical modeling assumption. Portfolios that contain callable corporate bonds, mortgage pass-throughs, collateralized mortgage obligations (CMOs), adjustable-rate mortgages, and so on may have substantial Vega risk that may or may not be offset by the Vega of the liabilities.

¹Note that more robust option models will employ some type of term structure of volatility, capturing the fact that the volatility of short-term interest rates is typically greater than the volatility of long-term rates.

²Source: Capital Management Sciences.

Table 1
Vega of Callable Corporate Bonds

Issuer	Call Date	Call Price	Current Price	Vega	Effective Duration	Maturity
Alabama Power	12/01/99	\$106.32	\$107.74	0.44	4.29	12/01/24
American Express	05/15/03	103.475	108.11	0.43	6.25	05/15/22
Anheuser Busch	06/01/02	100.00	100.34	0.13	4.79	06/01/05
Arizona Public Service	05/15/00	104.63	111.39	0.18	2.93	05/15/20
Central Illinois Public Service	07/01/02	100.00	103.01	0.19	4.89	07/01/07
Chase Manhattan	05/15/99	100.00	102.18	0.11	2.21	05/15/04
Duke Power	03/01/98	100.52	97.38	0.13	3.43	03/01/03
Oklahoma Gas & Electric	01/01/98	102.76	103.34	0.03	0.84	01/01/07
Pacific Gas & Electric	06/01/03	103.63	101.04	0.60	7.95	08/01/26
Potomac Edison	12/01/07	103.398	105.76	0.50	6.54	12/01/24
Public Service Co.-Indiana	01/01/98	100.73	100.24	0.10	1.75	01/01/02
Southwestern Bell	09/01/00	100.67	95.63	0.14	4.77	09/01/04
WalMart Stores	12/29/97	102.33	103.05	0.01	0.48	12/29/06

PREPAYMENT UNCERTAINTY— PREPAYMENT MODEL RISK

The prepayment model used to analyze mortgage-backed (and some asset-backed) securities is a critical component of any risk measurement system that encompasses these investments. A prepayment model is typically developed using empirical data of homeowners' mortgage prepayment patterns over time; the goal of the model is to predict future behavior on the basis of the historical data. For a variety of reasons, no two prepayment models will produce the same prepayment forecast, even with the same information about the interest rate environment and the mortgage collateral pertaining to the securities of interest. For example, models may be calibrated to different historical data sets: some use five or even ten years of data, others may use data from only the past three years; some models attach greater weight to more recent data, others attach equal weight to all time periods; and the variables used to explain and forecast prepayment behavior differ across models. Therefore, differences in prepayment modeling across well-respected and capable providers are to be expected.³

In addition to differences in the way models are calibrated and specified, there is some likelihood that the historical data used to fit the model no longer reflect current prepayment behavior. For example, over the past few years mortgage lenders have become

more aggressive in offering low-cost or no-cost refinancing. As a result, a smaller decline in interest rates is now sufficient to entice homeowners to refinance their mortgages compared to five years ago. Further developments in the marketplace (for example, internet-based refinancing or the impact of telecommuting on relocations) will undoubtedly affect future prepayment patterns in ways that the historical data used to fit today's prepayment models do not reflect.

Since modeling future prepayment behavior is an inexact science, all risk measurement systems are subject to some valuation error due to misestimates of future prepayments used in valuing mortgage-backed and asset-backed securities. We can quantify this risk by computing a prepayment uncertainty measure that describes the sensitivity of an asset's price to a change in the level of prepayments predicted by a model. (Rather than describing this as a modeling "error," we can think of it as measuring the sensitivity to a change in a modeling parameter that is impossible to know with certainty.) To derive this prepayment uncertainty measure, we first decrease, then increase, the predicted prepayment speeds generated by the model by 10%, and derive a new price under the slower and faster versions of the model (holding the security's option-adjusted spread constant). Computed this way, mortgage-backed securities priced below par tend to show a negative prepayment uncertainty. This makes intuitive sense, as a slowdown in prepayment speeds means the investor must wait longer to be repaid at par. Conversely, mortgage-backed securities priced above par usually show a positive prepayment uncertainty, because a decline in prepayment speeds allows the investor to receive the above-market coupon rate on the mortgage for a longer period of time than originally forecast.

³For a discussion of how one can assess the accuracy of a prepayment model, see the research paper "Evaluating a Fixed Rate Prepayment Model" by Wesley K. Phoa, Ph.D., and Terrence Nercessian, available from Capital Management Sciences, Los Angeles, California.

One of the most important variables in a prepayment model is the assumed level of refinancing incentive homeowners require to prepay their mortgages when interest rates drop. The required incentive has certainly dropped over the past decade; in the early days of prepayment modeling, it was not unusual to assume that new mortgage rates had to be at least 150 bps lower than a homeowner's mortgage rate before refinancings would occur. Today, a prepayment model may assume that only a 75 bp incentive is necessary to trigger a wave of refinancings. Therefore, we may wish to examine the amount of risk associated with a misestimate in the minimum incentive the model assumes homeowners will require before refinancing their mortgages.

To do so, we dissect the overall prepayment uncertainty measure into two components: refinancing (Refi) uncertainty and relocation (Relo) uncertainty. The Refi measure describes the sensitivity of a valuation to changes in the aforementioned refinancing incentive, and the Relo measure shows the sensitivity to a change in the level of prepayments that occur independent of the level of interest rates (that is, due to demographic factors such as a change in job status or location, growing family, divorce, retirement, and so on). Table 2 shows these measures for a representative sample of 30-year collateral pools. Although the values appear to be small in many cases, we should keep in mind that prepayment estimates from Wall

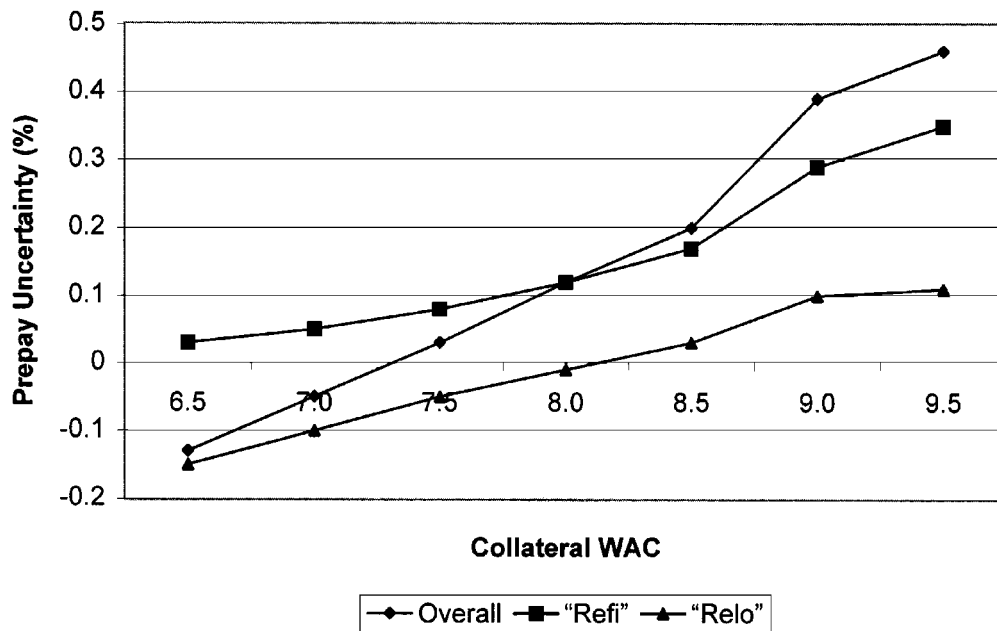
Street firms often differ from each other by much more than 10%. Therefore, even a 20% change in prepayment speeds (for example, from 6% per year to 7.2% per year) would not be unreasonable to contemplate, which would be equivalent to doubling the prepayment uncertainty values.

We can see in Figure 1 that the magnitude of the overall prepayment uncertainty is related to the weighted average coupon (WAC), time to maturity (loan age), and current price of the collateral. The absolute magnitude of the prepayment uncertainty increases as the price moves away from par; premium mortgages tend to display the largest positive prepayment uncertainty. This means that a slowdown in prepayments compared to the model's predicted prepayment pattern would cause the largest percentage change in the price of mortgages with high WACs. While risk measurement systems will typically identify the price sensitivity to changes in risk factors such as the level of interest rates (as measured by duration and convexity), we can see here that some high-coupon mortgages have prepayment uncertainty measures more than half the size of their durations. In essence, this means that a 10% misestimate in prepayment modeling can produce as much (or more) of a change in price as a 50 bp shift in interest rates. It is interesting to note that the magnitude of prepayment uncertainty appears to be negatively correlated

Table 2
Prepayment Uncertainty
30-Year Collateral Pools

Collateral Net WAC	Maturity Year	Current Price	Effective Duration	Prepayment Uncertainty		
				Overall	Refi	Relo
6.50%	2026	\$ 97.40	4.89	-0.13	0.03	-0.15
7.00	2026	99.59	4.16	-0.05	0.05	-0.10
7.50	2026	101.61	3.35	0.03	0.08	-0.05
8.00	2026	103.15	2.48	0.12	0.12	-0.01
8.50	2026	104.12	1.75	0.20	0.17	0.03
9.00	2026	107.86	1.02	0.39	0.29	0.10
9.50	2026	108.36	0.63	0.46	0.35	0.11
6.50	2024	97.69	4.58	-0.11	0.03	-0.14
7.00	2024	99.75	3.87	-0.03	0.05	-0.08
7.50	2024	101.61	3.09	0.05	0.08	-0.03
8.00	2024	103.06	2.36	0.14	0.13	0.01
8.50	2024	104.08	1.79	0.22	0.17	0.04
9.00	2024	106.20	0.90	0.39	0.28	0.11
9.50	2024	106.85	0.74	0.46	0.33	0.13
6.50	2021	98.95	4.19	-0.04	0.03	-0.07
7.00	2021	100.56	3.61	0.03	0.05	-0.02
7.50	2021	102.16	2.88	0.10	0.08	0.02
8.00	2021	103.69	2.75	0.18	0.13	0.05
8.50	2021	105.36	2.65	0.28	0.18	0.10
9.00	2021	108.89	2.25	0.48	0.28	0.19
9.50	2021	109.86	2.05	0.57	0.34	0.22

Figure 1
Prepayment



with the level of interest rate sensitivity (duration) of the mortgages.

As one would expect, the refinancing component of the prepayment uncertainty measure is largest for collateral pools with high WACs. If homeowners with above-market mortgage rates begin to refinance at a faster rate than a prepayment model predicts, investors who purchase high-WAC pools face the greatest risk. The relocation component is negative for discount collateral; if the level of housing turnover declines below the model's predicted amount, investors in discount mortgages must wait longer to be repaid. Therefore, prices of discount mortgages would decline. The opposite is true for premiums, where a lower rate of turnover would allow the investor to collect the high mortgage coupon for a longer period of time. We can see from Table 2 that the absolute magnitude of the refinancing uncertainty measure tends to be larger than the relocation uncertainty number (except for deep discount collateral). This is a reminder that the refinancing incentive component of a prepayment model can have a large impact on the projected cash flows for a mortgage-backed security. Table 2 shows only mortgage pass-through pools; CMOs with complicated deal structures can display extremely large prepayment uncertainty measures, two or three times larger than the prepayment uncertainty of the underlying collateral.

CONCLUSION

Risk measurement is an inexact science. It is not possible to know what level of interest rate volatility will be experienced in the future, or to derive a model that predicts prepayment behavior with perfect accuracy. Therefore, the notion of developing risk measures to quantify the potential impact of a modeling error is an important part of the risk management process. This paper has discussed ways to do this for two critical variables that affect fixed-income securities; the same concept can be applied to models that are used to value other investment assets and/or liabilities. These risk measures may be employed in a risk management program to contain the investment portfolio's exposure to changes in volatility and in prepayment behavior within reasonable limits. A risk management program should also monitor the entire investment portfolio's exposure to Vega and prepayment uncertainty over time. Although these risk measures are useful when making relative value assessments for individual security buy/sell decisions, the portfolio level measure is most appropriate for a firm-wide risk management program.

Discussions on this paper can be submitted until October 1, 1999. The author reserves the right to reply to any discussion. See the Submissions Guidelines for Authors for detailed instructions on the submission of discussions.