MEASURING FOREIGN EXCHANGE RISK IN INSURANCE TRANSACTIONS

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ABSTRACT

A macroeconomic model of exchange rates is mixed with classical life insurance, annuity and compound Poisson aggregate claim models to create foreign exchange-adjusted insurance models. The resulting models may be used to measure the potential foreign exchange risk of mixed currency products, for example, products for which premiums are collected and benefits are paid in different currencies. Numerical examples illustrate the foreign exchange risks of such products.

1. INTRODUCTION

In its broadest sense, *foreign exchange risk* refers to events abroad that affect the net income of a domestic enterprise. These could include, for example, a recession in a foreign market that adversely affects sales of domestic vendors to foreign buyers. Such a recession can affect not only the domestic vendor, but also the vendor's suppliers who may operate only in the domestic economy. In this sense, foreign exchange risk can affect nearly all enterprises in a global economy. The notion of foreign exchange risk is limited here to financial transactions involving two or more currencies.

Foreign exchange risk arises naturally in insurance products when premiums are remitted in one currency but benefits are denominated in another. For example, a medical policy issued to residents of a developing country may reimburse reasonable and customary catastrophic medical benefits in a developed country. Benefits would be paid in the developed country's currency, but premiums could be paid in the developing country's currency. (For an analysis of the risks inherent in such products see Law 1996). In addition, some developing countries permit local companies to issue life policies denominated in a foreign currency.

Foreign exchange risk affecting insurers may arise in other contexts. For example, the investment department may perceive that additional margins can be earned through investment in foreign securities. Such additional margins can be offset by the change in the relative values of the currencies between the times at which the investment is bought and sold. A prudent manager will consider hedging this exposure. See Derosa (1992) for a discussion of currency options, forwards, futures, swaps, and other hedging techniques that can be used in this context.

A company may also elect to offer insurance products to a foreign insurance market. In the early years of entry, the company will invest heavily, converting its domestic currency into foreign currency. It may, for example, be required to meet minimum capital requirements in order to establish a subsidiary or branch office. In addition, the company will acquire office space, hire and train local staff, and incur considerable marketing costs building local name recognition. The benefits and premiums of the products it sells will usually be denominated in the foreign currency. If it manages its cash flow locally, the company might, at first, appear to avoid foreign exchange risk. Ultimately, though, the company will repatriate its earnings, at which time it will realize a gain or loss due to foreign exchange.

A company might enter a foreign insurance market despite the fact that its investment department has only limited international expertise. If there is a free flow of capital across borders, the company could choose to accept greater foreign exchange risk in order to maximize the return on its investments—for example, it might reinsure the foreign insurance risks with its domestic operation. Premiums would then, in effect, be converted to domestic currency when received, and claims would be converted to foreign currency when payable. This approach might be effective when a company first enters a foreign market, and its

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exposure to foreign exchange risk is dwarfed by its domestic insurance risks. Models of mixed currency insurance products may be used to understand the foreign exchange risk of such a strategy.

This paper describes a stochastic approach to measuring the effect of foreign exchange risk on several different insurance products. I begin with a discussion of different foreign exchange rate regimes. I then develop models of the different regimes and demonstrate how such models may be mixed with models of insurance risks to create foreign exchange-adjusted models of insurance risks. Some examples follow to illustrate the approach.

2. EXCHANGE RATE REGIMES

An *exchange rate regime* refers to the policies a country adopts in order to control movements in its currency's rate of exchange vis à vis other currencies. The policy is often effected through monetary policy but can also be influenced by tariffs, trade policy, and so on.

Exchange rate regimes today may be classified into one of three broad categories: *pegged, managed floating* and *independent floating*.[1] A currency is pegged if it maintains a fixed relationship to another currency or composite currency. For example, the Argentine and Hong Kong dollars are pegged to the U.S. dollar, and many former French colonies peg their exchange rates to the French franc. A handful of countries peg their currency to the SDR (Special Drawing Rights), a composite of the German, French, Japanese, British, and U.S. currencies. Several countries, including Iceland and Hungary, peg their currencies to a weighted average of the currencies of their major trading partners.

Managed floating currencies can be characterized as having either limited flexibility or more flexibility. A currency has limited flexibility when it is allowed to float only within a narrow, well-defined band. For example, several Arabian Gulf states fix their currencies within a narrow range around the U.S. dollar, and countries within the European Monetary System (EMS) allow their currencies to vary within an agreed trading range. To maintain the agreed relationships, the countries of the EMS stand ready to buy or sell domestic or foreign currency.

[1]The International Monetary Fund (IMF) classifies exchange rate regimes as pegged, limited flexibility, or more flexibility. See Derosa 1992 and Krugman 1992 for further explanation of the IMF classification scheme. The classifications used here relate to the models of the exchange rate regimes that follow.

Managed floating currencies with more flexibility vary within a greater range around another currency, a composite currency, or a set of indicators. For example, the Chilean peso is permitted to vary within a 10% band around a reference value that is determined daily based upon the value of the currencies of Chile's major trading partners. Currencies such as the Malaysian ringgit or the Singapore dollar are monitored daily against the values of the currencies of their major trading partners. Their monetary authorities intervene only when deemed prudent, rather than according to a published set of guidelines.

Countries such as the U.S., the Philippines, and Japan, who allow their currencies to float independently, do not have explicit policies intended to maintain their currencies' values vis à vis some standard. The values of their currencies are allowed to vary according to the marketplace. This is not to say, however, that such countries never intervene to influence the value of their currency. Consider, for example, the decision to weaken the U.S. dollar effected by the five major industrial powers at the behest of the Reagan administration on September 22, 1985. This decision directly affected worldwide exchange rates and was a contributing factor to the formation of the Japanese "bubble" economy of the late 1980s (see Fallows 1995 for more detail).

An exchange rate regime may or may not be credible. For example, a country could peg its exchange rate to the U.S. dollar, but if the country has no agreement with the U.S. to maintain the pegged exchange rate and lacks the resources to support it, then its exchange rate regime may not be credible. In time, the marketplace will force the country to adjust its exchange rate. Exchange rates that are not credible are subject to extreme volatility in the short term as pressure for adjustment mounts. The decision to float the Thai baht illustrates this phenomenon.[2] Such volatility is more the subject of technical currency analysis and, therefore, falls outside the scope of this paper.

[2]Until 1997, Thailand's fast-growing economy had been the envy of the world. The Thai baht had been pegged to the currencies of its major trading partners. As its economy faltered and the U.S. Dollar strengthened, the peg became difficult to sustain. Currency speculators attacked, leading to a July 1, 1997 announcement that the peg would never be abandoned followed the next day by an announcement that the baht would henceforth float independently. The baht lost 23% of its value relative to the U.S. Dollar on July 2, 1997. The IMF and Japan led a multibillion dollar bailout of the Thai economy, and ultimately the currencies of several other southeast Asian nations were attacked.

An exchange rate may be subject to secular *drift*—that is, the exchange rate may follow a trend path relative to another currency (see Hull 1997 for a further description of drift). If a country's exchange rate is pegged to another country's currency and yet underlying economic fundamentals cause it to drift, then its exchange rate regime may not be credible. In time, the country may be unable to sustain the peg due to drift. Similarly, if a country's managed floating exchange rate drifts relative to the standard to which it is managed, then the exchange rate regime may only be credible if the standard to which it is managed drifts with the exchange rate. If the standard remains fixed, the managed floating exchange rate will eventually drift into a pegged regime.

3. MODELS OF FOREIGN EXCHANGE

The three categories of exchange rate regimes—pegged, managed float, and independent float—can be modeled within the framework of a monetary model of exchange rates. This model grew out of the work of policy advocates of exchange rate target zones (Krugman and Miller 1992). It was first presented by Paul Krugman, a noted economist, in 1987 at a National Bureau of Economic Research conference on the EMS and has been developed extensively since then[3] (see, for example, Flood and Garber 1992, Krugman 1992, Krugman and Rotemberg 1992, and Smith and Spencer 1992). The model ignores foreign exchange transactions costs.

Consider a monetary model of an exchange rate (Flood and Garber 1992). Let $K(t)$ represent a linear combination of the logarithms of foreign and domestic money supplies, real incomes, money demand disturbances, and real exchange rate movements. Now suppose the logarithm of the spot exchange rate, $S(t)$, at time t is equal to the following differentiable function of $K(t)$:

$$\begin{aligned} S(t) &= g(K(t)) \\ &= K(t) + \alpha E[dS(t)|\phi(t)]/dt \quad \alpha > 0 \qquad (1) \end{aligned}$$

where $E[dS(t)|\phi(t)]/dt$ is the expected value of investors' expectations of the future rate of change in the exchange rate given current information, $\phi(t)$, and α is the semi-elasticity of money demand with respect to an interest rate. The term $K(t)$ is assumed to be subject to random shocks and, therefore, can be considered to follow a random walk. Thus,

$$dK(t) = \eta\, dt + \sigma dZ(t) \qquad t > 0 \qquad (2)$$

where η and σ are constants, dt is an increment of time, and $dZ(t)$ is the increment of a standard Wiener process at time t. The parameters η and σ^2 are, therefore, parameters of an unrestricted Wiener process. (See Hull 1997 for a description of Wiener processes.)

The model defined by Equations (1) and (2) above describes the relationship between an exchange rate and its underlying economic fundamentals at time t, and as t changes, it describes the path the exchange rate follows through time.

The money supplies of the foreign and domestic economies are assumed to be managed by monetary policy. In fact, a country's approach to managing its money supply defines its exchange rate regime. If a country's exchange rate is pegged to another currency, then that country responds to random disturbances by making offsetting adjustments to its money supply. Therefore,

$$dK(t) = 0,$$

and so

$$K(t) = k_0$$

where k_0 is a constant.

Thus, when one currency is pegged to another, model (1) can be simplified to

$$S(t) = g(K(t)) = k_0. \qquad (3)$$

Note that this model is valid only while the pegged exchange rate regime is credible.

The model of a pegged exchange rate is stated relative to the standard to which it is pegged. If a model of the pegged currency relative to a different currency is required, it may be necessary to consider different models of the exchange rate. For example, the Hong Kong dollar is pegged to the U.S. dollar, and the Australian dollar floats independently of the U.S. dollar. If one wishes to model the relationship between the Hong Kong and the Australian dollars, then one would use an independently floating model.

[3]The term *target zone* refers to a range within which a country may choose to manage its foreign exchange rate. A model of a target zone assumes that when an exchange rate is near the center of the zone, it floats independently, but when it reaches the boundaries of the zone, the monetary authorities take action (either buying or selling foreign exchange) to maintain the exchange rate within the zone. This, of course, is a model of a managed floating regime. The target zone model developed by Krugman and presented here is general enough to handle pegged and independent floating regimes.

If a country's exchange rate floats independently, then the money supply would not change as a result of random money demand disturbances. In fact, it is assumed that the money supply remains constant in the independent floating model. Investors' expectation of the rate of change in the exchange rate, $E[dS(t)|\phi(t)]/dt$, would equal the secular drift, η. Therefore, the model of an independently floating exchange rate can be simplified to

$$\begin{aligned} S(t) &= K(t) + \alpha\,\eta \\ &= k_0 + \eta t + \sigma Z(t) + \alpha\,\eta. \end{aligned} \qquad (4)$$

The distribution of $S(t)$ is the normal distribution with mean $k_0 + \eta(t + \alpha)$ and variance $\sigma^2 t$, so the exchange rate is distributed lognormally with parameters $k_0 + \eta(t + \alpha)$ and $\sigma\sqrt{t}$.

The process governing movements of independently floating exchange rates at time t is depicted in Figure 1 for the case of no drift (that is, letting $\eta = 0$ in Equations (2) and (4) above). The logarithm of the exchange rate, $S(t)$, can be thought of as moving randomly up and down the diagonal line, $S(t) = K(t)$.

If a country's exchange rate regime were managed floating, then the money supply would be adjusted in response to random disturbances in money demand as the exchange rate approaches the bounds within which the country chooses to maintain its exchange rate. As noted, such a country stands ready to buy or sell foreign exchange in order to maintain its exchange rate within the bounds it has set. Unlike as in the independent floating regime, then, investors' expectations of the rate of change in the exchange rate, $E[dS(t)|\phi(t)]/dt$, would not equal the secular drift. Following Flood and Garber (1992), if $g(K(t))$ is the solution, $E[dS(t)|\phi(t)]/dt$ may be derived by applying the Ito differential

$$dS(t) = g'(K(t))dK(t) + \tfrac{1}{2}g''(K(t))(dK(t))^2.$$

Taking the expectation of each side conditional on current information,

$$E[dS(t)|\phi(t)] = g'(K(t))\eta + \tfrac{1}{2}g''(K(t))\sigma^2.$$

Substituting into Equation (1), we derive

$$\begin{aligned} S(t) &= g(K(t)) \\ &= K(t) + \alpha[g'(K(t))\eta + \tfrac{1}{2}g''(K(t))\sigma^2]. \end{aligned}$$

The general solution to this differential equation is

$$\begin{aligned} S(t) &= g(K(t)) \\ &= K(t) + \alpha\eta + A_1 \exp[\lambda_1 K(t)] \\ &\quad + A_2 \exp[\lambda_2 K(t)]. \end{aligned} \qquad (5)$$

Unique solutions to the values of A_1 and A_2 are derived by imposing the following conditions:

$g'(\bar{k}) = 0$ where $\bar{k}$ is the value of $K(t)$ at which $S(t)$ attains its upper bound, $\bar{s}$, and

$g'(\underline{k}) = 0$ where $\underline{k}$ is the value of $K(t)$ at which $S(t)$ attains its lower bound, $\underline{s}$.

These conditions are required to eliminate arbitrage from the model (see, for example, Krugman 1992). Following the development in Flood and Garber (1992) and Smith and Spencer (1992), then

$$A_1 = \frac{\alpha\,\sigma^2}{2\Delta}\,[\exp(\lambda_2\underline{k}) - \exp(\lambda_2\bar{k})]\,\lambda_2$$

$$A_2 = \frac{\alpha\,\sigma^2}{2\Delta}\,[\exp(\lambda_1\bar{k}) - \exp(\lambda_1\underline{k})]\lambda_1$$

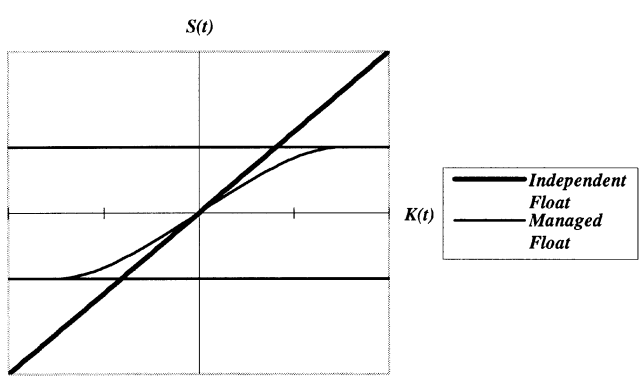
where

$$\Delta = \exp[\lambda_1\bar{k} + \lambda_2 k] - \exp[\lambda_1\underline{k} + \lambda_2\bar{k}]$$

$$\lambda_i = \frac{-\alpha\eta \pm \sqrt{\alpha^2\eta^2 + 2\alpha\sigma^2}}{\alpha\sigma^2}.$$

The first three terms of Equation (5) are equal to the independent floating model above. We can think of these terms as representing a *fundamental* exchange rate. The final two terms represent deviations from the fundamental caused by changes required to remain within the target zone (Krugman 1992). The boundary assumptions here imply that once an exchange rate reaches a boundary it is reflected back into the target zone. Other boundary assumptions are

Figure 1
Independently and Managed Floating Processes *S*(*t*)



possible. For example, once an exchange rate reaches a boundary, it could revert to a pegged regime. (See Smith and Spencer 1992 for an analysis of this boundary assumption.)

The process governing movements of managed floating exchange rates is depicted in Figure 1 in a driftless environment. The currency is assumed to be managed to the value k_0. the logarithm of the exchange rate, $S(t)$, can be thought of as moving randomly up and down the s-shaped curve specified by Equation (5). Unlike the independent floating regime, there are limits to the movement of the exchange rate. In the illustration, $S(t)$ has been arbitrarily assumed to be managed such that $K(t)$ falls within 1.5 standard deviations above or below k_0. Interestingly, the s-shaped curve falls between the horizontal axis and the line representing the process governing independent floating regimes. This suggests that a credible managed floating regime exerts a stabilizing force on exchange rates; that is, as exchange rates increase or decrease, the market will anticipate opposite movements caused by intervention of the monetary authorities.[4]

Like pegged exchange rates, the model of managed floating exchange rates is stated relative to the currency and/or standard to which it is managed. If one requires a model of a managed floating currency relative to a third, unrelated currency, then one must mix the model of the managed floating currency with a model of the third currency relative to the managed floating currency's standard. For example, the Malaysian ringgit is a managed floating currency; the standard to which it is managed is only generally known as the value of the ringgit in relation to the currencies of Malaysia's major trading partners, many of whose currencies float independently of the U.S. dollar. If one wishes to model the relationship between the Malaysian ringgit and the U.S. dollar, then one should theoretically mix managed floating models of the Malaysian ringgit to the values of the currencies of its major trading partners with independently floating models relating the currencies of Malaysia's major trading partners to the U.S. dollar.

Finally Equation (5) is not invertible (that is, we cannot solve for $K(t)$ in terms of $S(t)$), so the distribution of $S(t)$ in a managed floating environment cannot be derived directly. We can, nevertheless, through an example contrast the shape of the distribution of a managed floating regime with the shape of the distribution of an independent floating regime. Figure 2 illustrates the shapes of such distributions, again assuming a driftless environment and that $S(t)$ is managed such that $K(t)$ falls within 1.5 standard deviations above or below k_0 in the managed floating model.

The distribution of $S(t)$ in a driftless independent floating regime is the normal distribution with mean k_0 and variance $\sigma^2 t$. The distribution of $S(t)$ in a managed floating regime has probability masses at the bounds of $S(t)$ and is spread out somewhat more uniformly than the independent floating regime between the bounds. Note that the bounds of $S(t)$ are not located at $k_0 \pm 1.5\sigma$. Instead they are located less than one standard deviation from k_0. This is due to the stabilizing property of managed floating regimes previously described.

Commentary

Many researchers have observed that foreign exchange rates appear to follow a random walk but that, nevertheless, technical trading strategies can be used to realize significant risk-adjusted gains in foreign exchange markets. (See, for example, Levick and Thomas 1993 and Rosenberg 1996).

Much of this research shows that a random walk usually models movements in exchange rates effectively, but that occasionally there are pronounced swings in exchange rates that go beyond what a random walk would suggest is likely. These swings are long and sustained enough that gains realized during these periods more than offset losses incurred during normal periods.

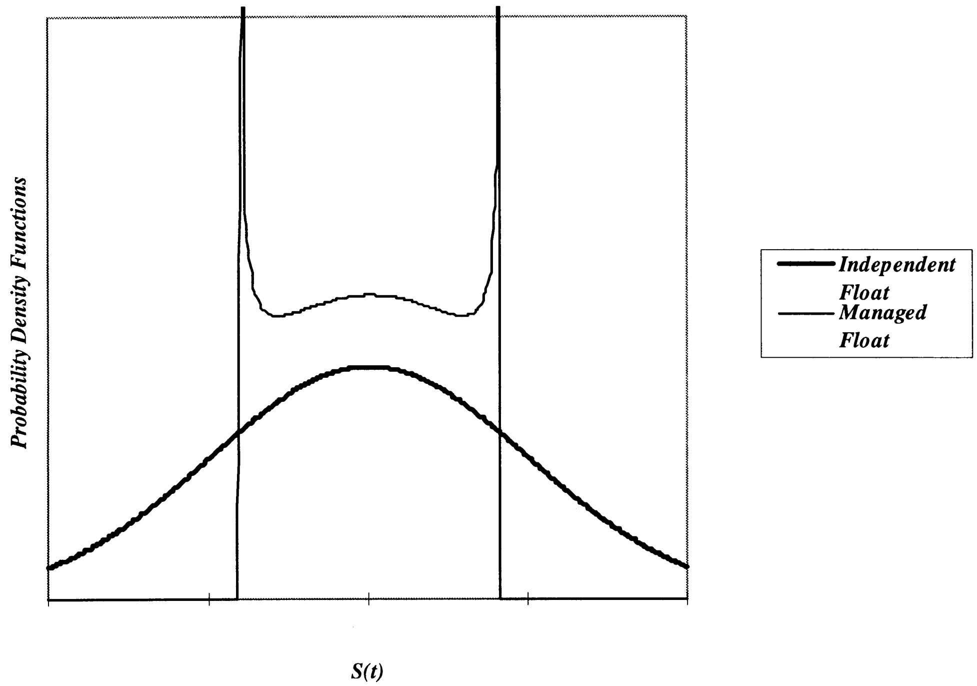
This research suggests that the models of exchange rates discussed thus far are often satisfactory, but may not capture the full extent of a run-up or run-down in exchange rates.

4. INSURANCE MODELS OF FOREIGN EXCHANGE

The models of life insurance and annuities below are constructed by mixing classical actuarial models that assume a fixed interest rate with an exchange rate model. These models provide an illustrative measure of the potential effect of foreign exchange on these products. It should be noted, however, that interest rates and foreign exchange rates are not independent of one another, so complete models of these products would allow interest rates to float and would model

[4] The reduction in the volatility of exchange rates in a managed floating regime is offset by an increase in interest rate volatility. See Flood and Garber (1992).

Figure 2
Independently and Managed Floating Density Functions



the relationship between interest rates and exchange rates.

As noted above, the probability density function of a managed floating regime cannot be derived since the process function (5) is not invertible. Consequently, it cannot be easily mixed with insurance models. Numerical approaches must be used to model mixed currency products in a managed floating regime. The insurance models below are, therefore, mixed with the independently floating exchange rate model defined by Equation (4). The following numerical illustrations give some sense for the effect of managed floating regimes, relative to independent regimes, on insurance models.

4.1 Life Insurance

For a life insurance sold in some countries, premiums are remitted in domestic currency and benefits are payable in foreign currency. The purpose of this product is to shift foreign exchange risk from the insured to the insurance company. This product can be thought of as a life insurance with a fixed foreign currency benefit or, alternatively, with a variable domestic currency benefit.

Consider the net single premium of an n-year continuous term life insurance policy issued at time t to a life aged x, where the benefit is a unit of an independently floating foreign currency payable at the date of death.

Let T be the random variable of the time until death of a life aged x. The value in domestic currency of the benefit payable at time $t + T$ is $e^{S(t+T)}$, and its present value is $e^{S(t+T)-\delta t}$. Its value in foreign currency units at time t is $Z' = e^{S(t+T)-\delta T-S(t)}$, where Z' is the random variable of the present value of the benefit payable at the moment of death. Therefore, the net single premium is $E\,[e^{S(t+T)-\delta T-S(t)}]$. Assuming that the mortality and exchange rate distributions are independent, the expected value can be calculated as follows.

Let $\bar{A}'^{\,1}_{x:\overline{n}|}$ = the net single premium of an n-year continuous term life insurance with benefits payable in foreign currency. Then:

$$\begin{aligned}
\bar{A}'^{\,1}_{x:\overline{n}|} &= E\,[e^{S(t+T)-S(t)-\delta T}] \\
&= E\,[E\,[e^{S(t+T)-S(t)-\delta T}|T]] \quad \text{conditioning on } T \\
&= E\,[e^{-\delta T-S(t)}E\,[e^{S(t+T)}|T]] \\
&= E\,[e^{-\delta T}\exp\{-(k_0 + \eta(t + \alpha) + \tfrac{1}{2}\sigma^2 t)\} \\
&\quad \exp\{k_0 + \eta(t + T + \alpha) \\
&\quad + \tfrac{1}{2}\sigma^2(t + T)\}] \\
&= E\,[e^{-(\delta-\eta-1/2\sigma^2)T}] \\
&= \bar{A}^{1}_{x:\overline{n}|},
\end{aligned} \tag{6}$$

where $\overline{A}^{1}_{x:\overline{n}|}$ is the net single premium of an n year life insurance with benefits payable in domestic currency calculated at force of interest δ-η-$1/2\sigma^2$. The development above is based on the conditioning theorem (2.2.10) from Bowers et al. (1986) and on the formula for the moments of the lognormal distribution also found there.

That is, the net single premium of an n-year continuous term life insurance with benefits payable in an independently floating foreign currency can be calculated as the net single premium of an n-year continuous term life insurance with a unit benefit payable in domestic currency calculated at force of interest δ-η-$1/2\sigma^2$.

Following the development above, the ith moment of the distribution of Z' can be found by:

$$\begin{aligned}
E[Z'^i] &= E[e^{iS(t+T)-iS(t)-i\delta T}] \\
&= E[E[e^{iS(t+T)-iS(t)-i\delta T}|T]] \quad \text{conditioning on } T \\
&= E[e^{-i\delta T-iS(t)}E[e^{iS(t+T)}|T]] \\
&= E[e^{-i\delta T} \\
&\quad \exp\{-(ik_0 + i\eta(t+\alpha) + \tfrac{1}{2}i^2\sigma^2 t)\} \\
&\quad \exp\{ik_0 + i\eta(t+T+\alpha) + \tfrac{1}{2}i^2\sigma^2 \\
&\quad (t+T)\}] \\
&= E[e^{-[i(\delta-\eta)-1/2i^2\sigma^2]T}] \\
&= {}^{i}\overline{A}^{1}_{x:\overline{n}|},
\end{aligned} \tag{7}$$

where ${}^{i}\overline{A}^{1}_{x:\overline{n}|}$ is the net single premium of an n-year life insurance with benefits payable in domestic currency calculated at force of interest $i(\delta - \eta) - 1/2i^2\sigma^2$.

That is, the ith moment of the distribution of Z' equals an n-year continuous term life insurance with benefits payable in domestic currency calculated at force of interest $i(\delta - \eta) - 1/2i^2\sigma^2$.

Substituting from Equations (6) and (7), the variance of the distribution of Z' is

$$\text{Var}[Z'] = {}^{2}\overline{A}^{1}_{x:\overline{n}|} - (\overline{A}^{1}_{x:\overline{n}|})^2 \tag{8}$$

where $\overline{A}^{1}_{x:\overline{n}|}$ is calculated at force of interest $\delta - \eta - 1/2\sigma^2$, and ${}^{2}\overline{A}^{1}_{x:\overline{n}|}$ is calculated at force of interest $2(\delta - \eta) - 2\sigma^2$.

4.2 Annuities

Another potential product would be an annuity that accepts premiums in domestic currency but pays benefits in foreign currency. Such a product could be viewed as paying a fixed benefit in foreign currency or a variable benefit in domestic currency.

Consider the net single premium of a continuous whole life annuity issued at time t to a life aged x where the benefit is a unit per annum of an independently floating foreign currency.

Let U be a random variable of the times at which benefits are due, $t < U < T$. The domestic currency value of the benefit payable continuously at time U is $e^{S(U)}$, and its present value at time t is $e^{S(U)-\delta(U-t)}$. The value in foreign currency units at time t is $e^{S(U)-\delta(U-t)-S(t)}$. Following the current benefit payment technique of Bowers et al. (1986), the actuarial present value of a life annuity with benefits payable in an independently floating foreign currency can be calculated as follows.

Let $= \overline{a'}_x$, the net single premium of a continuous whole life annuity with benefits payable in foreign currency. Then:

$$\begin{aligned}
\overline{a'_x} = E[\overline{a'_{\overline{T}|}}] &= \int_0^\infty e^{S(U)-\delta(U-t)-S(t)}{}_{U-t}p_x dU \\
&= \int_0^\infty e^{\eta(U-t)-\delta(U-t)+1/2\sigma^2(U-t)}{}_{U-t}p_x dU \\
&= \int_0^\infty e^{-(\delta-\eta-1/2\sigma^2)(U-t)}{}_{U-t}p_x dU \\
&= \overline{a}_x,
\end{aligned} \tag{9}$$

where $\overline{a}_x$ is calculated at force of interest $\delta - \eta - 1/2\sigma^2$, and $\overline{a'_{\overline{T}|}}$ is the present value of a continuous annuity payable in foreign currency through an insured's future lifetime, T.

A well-known identify from Bowers et al. (1986) relating annuities and life insurance with fixed benefits payable in domestic currency calculated at the same force of interest, $\delta - \eta - 1/2\sigma^2$, is:

$$1 = (\delta - \eta - \tfrac{1}{2}\sigma^2)\overline{a}_x + \overline{A}_x.$$

After substituting from Equation (9) and rearranging,

$$\overline{a'_x} = E[\overline{a'_{\overline{T}|}}] = \frac{1 - \overline{A}_x}{\delta - \eta - \frac{1}{2}\sigma^2}. \tag{10}$$

That is, the value of a life annuity with benefits payable in independently floating currency may be derived from the value of a life insurance with benefits payable in domestic currency with force of interest $\delta - \eta - 1/2\sigma^2$.

The variance of $\overline{a'_{\overline{T}|}}$ may be calculated as:

$$\mathrm{Var}(\overline{a}'_{\overline{T}|}) = \mathrm{Var}\left(\frac{1 - Z'}{\delta - \eta - \frac{1}{2}\sigma^2}\right)$$

$$= \frac{1}{(\delta - \eta - \frac{1}{2}\sigma^2)^2}\,\mathrm{Var}(Z').$$

Substituting from Equation (8),

$$\mathrm{Var}(\overline{a}'_{\overline{T}|}) = \frac{1}{(\delta - \eta - \frac{1}{2}\sigma^2)^2}\,({}^2\overline{A}_x - (\overline{A}_x)^2) \quad (11)$$

where $\overline{A}_x$ is calculated at force of interest $\delta - \eta - 1/2\sigma^2$, and ${}^2\overline{A}_x$ is calculated at force of interest $2(\delta - \eta) - 2\sigma^2$.

Thus, with estimates of the parameters of the distribution of $S(t)$, the actuary can compute net single premiums and variances for many types of life insurance and annuity products where the benefits are payable in an independently floating foreign currency.

4.3 Compound Poisson Aggregate Claims Distributions

A compound Poisson model is often used to model aggregate claim distributions of short term insurance risks. How does the distribution change if benefits are payable in an independently floating currency?

Let the random variable of frequency, N, follow a Poisson distribution with parameter λ, and let the random variable of severity before the effect of foreign exchange, X, be an arbitrary distribution with mean μ_x and variance σ_x^2. From classical risk theory, the mean and variance of the aggregate claims distribution before reflecting foreign exchange risk are $\lambda\mu_x$, and $\lambda(\sigma_x^2 + \mu_x^2)$, respectively.

Assuming that the foreign exchange and severity distributions are independent, the mean and variance of the severity distribution after giving effect to foreign exchange risk are

$$E[Xe^{S(u)-S(t)}] = E[X]E[e^{S(u)-S(t)}]$$
$$= \mu_x e^{\eta(u-t)+1/2\sigma^2(u-t)}$$

and

$$\mathrm{Var}(Xe^{S(u)-S(t)}) = E[X^2 e^{2(S(u)-S(t))}] - E[Xe^{S(u)-S(t)}]^2$$
$$= (\mu_x^2 + \sigma_x^2)e^{2\eta(u-t)+2\sigma^2(u-t)} - \mu_x^2 e^{2\eta(u-t)+\sigma^2(u-t)}$$
$$= \mu_x^2 e^{2\eta(u-t)+\sigma^2(u-t)}(e^{\sigma^2(u-t)} - 1) + \sigma_x^2 e^{2\eta(u-t)+2\sigma^2(u-t)}$$

where t is the time at which the premium is paid and u is the time at which a claim is paid, $t < u$. Because, $S(t)$ is distributed lognormally, $e^{S(u)-S(t)}$ measures the change in the exchange rate from the date of premium payment to the date of claim payment.

Note that the means of the severity distribution before and after giving effect to foreign exchange are not identical, even in the absence of drift. This is due to the fact that the foreign exchange risk is assumed to be distributed lognormally. Unlike the normal distribution, the mean of the lognormal distribution is influenced by both of its parameters.

The mean and variance of the compound Poisson aggregate claims distribution after reflecting foreign exchange risk are:

$$\text{Mean} = \lambda\mu_x e^{\eta(u-t)+1/2\sigma^2(u-t)} \quad (12)$$

and

$$\text{Variance} = \lambda e^{2\eta(u-t)+2\sigma^2(u-t)}(\mu_x^2 + \sigma_x^2). \quad (13)$$

Thus, with estimates of the parameters of the distribution of $S(t)$, the actuary can estimate the effect of independently floating foreign exchange on a compound Poisson distribution with benefits payable in foreign currency. Note, though, that in most applications the difference between the date of claim payment and the date of premium payment, $u - t$, is subject to random fluctuations. That is, this difference is actually a random difference, $U - T$. In order to fully measure the effect of foreign exchange on the compound Poisson model then, the compound Poisson model should be mixed with the distribution of $U - T$, which is outside the scope of this paper.

5. NUMERICAL EXAMPLES

Following are examples of the incremental risk due to foreign exchange in both managed and independently floating environments in the context of life insurance, annuities, and aggregate claims. The examples are intended to contrast conditions under which foreign exchange risk may be more or less significant. For simplicity, the life insurance and annuity examples are calculated at two different but constant forces of interest.

A common measure of risk is the ratio of the standard deviation to the mean. To illustrate why, suppose a standard deviation principle is used to develop risk charges for insurance products. Assuming that a normal distribution with mean μ and variance σ^2 is a good approximation to an aggregate claim distribution, the risk charge relative to the pure premium required to assure that the gross premium is sufficient 97.5% of the time is

$$\frac{\mu + 1.96\sigma}{\mu} - 1 = 1.96\,\frac{\sigma}{\mu}.$$

Thus, the risk charge is a function of the ratio of the standard deviation to the mean. This ratio is used in the following subsection as a relative measure of the incremental effect of foreign exchange in the context of insurance models.

5.1 Life Insurance

The risk of a continuous whole life insurance as measured by the ratio of the standard deviation to the mean can be calculated as follows:

$$Risk = \frac{\sqrt{{}^2\bar{A}_x - \bar{A}_x^2}}{\bar{A}_x}$$

where $\bar{A}_x$ is calculated at force of interest δ, and ${}^2\bar{A}_x$ is calculated at force of interest 2δ if premiums and benefits are paid in domestic currency. $\bar{A}_x$ is calculated at force of interest $\delta - \eta - 1/2\sigma^2$, and ${}^2\bar{A}_x$ is calculated at force of interest $2(\delta - \eta) - 2\sigma^2$ if premiums are paid in domestic currency and benefits are paid in an independently floating foreign currency.

The ratio of *Risk* when benefits are payable in foreign currency to *Risk* when benefits are payable in domestic currency is a measure of the incremental risk of offering benefits payable in foreign currency. When this ratio is greater than 1, foreign exchange risk increases the overall risk of the product. When it is less than 1, then foreign exchange risk partially offsets other risks of the product and, therefore, reduces the overall risk.

Table 1
Independent Floating Risk Factors Male Whole Life Insurance (δ = 7.0%, 1970–75 Basic Table)

Country	η	σ	Issue Age 35	45	55	65
Australia	−0.067	0.037	2.73	2.33	2.06	1.89
Canada	0.059	0.029	0.11	0.13	0.14	0.16
Italy	0.119	0.151	0.50	0.63	0.78	0.94
Japan	−0.096	0.056	3.52	2.91	2.50	2.25
Sweden	0.127	0.188	0.61	0.77	0.96	1.17
Switzerland	0.015	0.090	0.64	0.67	0.71	0.73
United Kingdom	−0.081	0.125	2.90	2.45	2.16	1.97

In Table 1, this ratio has been calculated at force of interest δ = 7.0% on the 1970–75 Basic Life Table for several issue ages and countries whose currencies float independently of the U.S. dollar.

Because economic data, such as the velocity of money, are difficult to measure or even unobservable, the method of moments has been applied to the daily 12-month changes in historical exchange rates during the period January 2, 1991 through December 31, 1993 to determine the parameters of the distribution of $S(t)$. The exchange rates were downloaded from the Federal Reserve Board of Chicago site on the World Wide Web (http://www.frbchi.org/econinfo/finance/for-exchange/welcome.html). These exchange rates represent the amount of foreign currency that could be purchased with one U.S. dollar on each day. If the drift, η, is less than 0, then the U.S. dollar weakened relative to the foreign currency during the observation period.

Using Japan as an example, the product we are analyzing is one in which the buyer pays a single premium in Japanese yen for a whole life policy that pays a fixed benefit expressed in U.S. dollars. Since the U.S. dollar weakened against the yen during the observation period, the net single premium of this policy is less than the net single premium of a fixed benefit expressed in yen based upon the exchange rate at issue. However, Table 1 shows that the risk of the U.S.-dollar denominated policy is much greater than the risk of a yen-denominated policy.

In general, Table 1 suggests that a policy denominated in a foreign currency that is weakening relative to the domestic currency is riskier than a policy denominated in the domestic currency. In some cases, it is much riskier. Following the normal approximation described above, if the risk charge required to assure premiums are sufficient 97.5% of the time is 20% before reflecting foreign exchange risk, then the risk charge needed for a U.S. dollar-denominated policy issued to a 35-year-old Japanese citizen is 70.4% (20% × 3.52) after reflecting foreign exchange risk.

Table 1 also suggests that a policy denominated in a currency that is strengthening relative to the domestic currency, though more expensive, is less risky than the policy denominated in the domestic currency. It is also notable that the risk factor tends toward 1.00 as issue age increases, lessening foreign exchange risk. This is due to the fact that exposure to foreign exchange risk is shorter as issue age increases.

Table 2 is identical to Table 1 except that the force of interest, δ, is 5.0%.

A comparison of Table 2 to Table 1 suggests that:

Table 2
Independent Floating Risk Factors Male Whole Life Insurance (δ = 5.0%, 1970–75 Basic Table)

Country	η	σ	Issue Age			
			35	45	55	65
Australia	−0.067	0.037	3.44	2.87	2.49	2.26
Canada	0.059	0.029	0.14	0.15	0.17	0.19
Italy	0.119	0.151	1.05	1.25	1.49	1.77
Japan	−0.096	0.056	4.70	3.75	3.13	2.77
Sweden	0.127	0.188	1.24	1.47	1.77	2.11
Switzerland	0.015	0.090	0.54	0.57	0.60	0.62
United Kingdom	−0.081	0.125	3.71	3.05	2.63	2.37

- Risk increases as the force of interest decreases when the foreign currency is weakening relative to the domestic currency.
- Risk decreases as the force of interest decreases when the foreign currency is strengthening relative to the domestic currency and the rate of drift is less than the force of interest.
- Risk increases as the force of interest decreases when the foreign currency is strengthening relative to the domestic currency and the rate of drift is greater than the force of interest.

Clearly, incremental independently floating foreign exchange risk varies dramatically by country, age, and interest rate for long-term products such as whole life insurance. Before offering a whole life policy denominated in a foreign currency, the insurer should carefully examine its approach to managing the inherent foreign exchange risk. One approach would be to invest the cash flow in instruments denominated in the foreign currency. This way the product would be largely insulated against movements in exchange rates. Another approach would be to hedge the exposure against unfavorable changes in exchange rates using, for example, American-style options that could be exercised whenever benefits become payable. If the latter approach is adopted, the cost of the hedge must be factored into the pricing of the product.

Suppose that at January 1, 1994, the seven countries in the examples above adopted monetary policies that were intended to effect a managed floating regime relative to the U.S. dollar. Further suppose that the policy adopted by each country establishes a ±20% zone around the December 31, 1993 exchange rates and that the markets would have judged the managed floating regimes to be credible. Assume that the targeted exchange rates and the ±20% zone are fixed despite the fact that the exchange rates have exhibited significant drift relative to the U.S. dollar. In effect, this means that their exchange rates move over time from a managed floating regime to a regime pegged to the U.S. dollar. The incremental risk factors that arise based on these assumptions are shown in Table 3.

A numerical approach was applied to estimate the cost of life insurance with benefits payable in foreign currency in a managed floating regime. The boundaries of $K(t)$ for each country were established such that the boundaries of $S(t)$ were 20% above or below the December 31, 1993 exchange rates. The values of η and σ were set to the values indicated in Tables 1 and 2. The value of α was arbitrarily set to 1.00 to assure an observable contrast between the independently floating and managed floating regimes.

A comparison of Table 3 to Table 2 suggests the following:

- The absolute value of the incremental risk of foreign exchange in a managed floating regime can be significantly less than that in an independently floating regime. For example, in the case of Sweden, what had been a significant additional risk becomes a slight risk offset due to the change in regime.
- In contrast to the independently floating examples, the incremental risk tends to move away from 1.00 with age. This is due to the assumption that the managed floating regime tends toward a pegged regime over time. The period during which exchange rates float has a larger effect at older ages than at younger ages.

These examples illustrate how important it is to assess the credibility of nonindependently floating regimes in the context of long-term products such as whole life insurance.

Table 3
Independent Floating Risk Factors Male Whole Life Insurance (δ = 5.0%, 1970–75 Basic Table)

Country	η	σ	Issue Age			
			35	45	55	65
Australia	−0.067	0.037	1.02	1.03	1.04	1.08
Canada	0.059	0.029	0.93	0.91	0.86	0.78
Italy	0.119	0.151	1.00	1.00	1.00	1.00
Japan	−0.096	0.056	1.00	1.00	1.00	1.00
Sweden	0.127	0.188	0.99	0.98	0.97	0.96
Switzerland	0.015	0.090	0.97	0.97	0.98	0.99
United Kingdom	−0.081	0.125	1.04	1.05	1.07	1.11

5.2 Annuities

The ratio of the standard deviation to the mean of an annuity with benefits payable in domestic currency is:

$$Risk = \frac{\sqrt{({}^2\bar{A}_x - \bar{A}_x^2)}}{\delta \bar{a}_x}$$

where $\bar{a}_x$ and $\bar{A}_x$ are calculated at force of interest δ, and ${}^2\bar{A}_x$ is calculated at force of interest 2δ. From Equations (10) and (11), the risk factor after accounting for foreign exchange risk is

$$Risk' = \frac{\sqrt{({}^2\bar{A}_x - \bar{A}_x^2)}}{(\delta - \eta - \frac{1}{2}\sigma^2)\bar{a}_x}$$

where $\bar{a}_x$ and $\bar{A}_x$ are calculated at force of interest $\delta - \eta - 1/2\sigma^2$, and ${}^2\bar{A}_x$ is calculated at force of interest $2(\delta - \eta) - 2\sigma^2$.

A measure of the incremental foreign exchange risk of a continuous life annuity with benefits payable in foreign currency can be calculated by dividing *Risk* into *Risk′*. Again, when this ratio is greater than 1, foreign exchange risk increases the overall risk of the product, and when it is less than 1, foreign exchange risk reduces the overall risk.

Table 4 is analogous to Table 1 but for continuous life annuities. The values of the annuities were determined at force of interest $\delta = 7.0\%$ on the 1983 Group Annuity Mortality table.

Using Japan as an example again, the product considered here involves a single premium payable in Japanese yen with a continuous annuity payable in U.S. dollars. Since the U.S. dollar weakened relative to the yen during this period, the cost of this annuity would be less than a benefit payable in yen based upon the exchange rate at issue. Unlike with life insurance, however, foreign exchange risk reduces the overall risk of the annuity.

Table 4 suggests, in fact, that foreign exchange risk reduces the overall risk of annuities payable in foreign currency when the foreign currency is weakening relative to the domestic currency, and that foreign exchange risk increases overall risk when the foreign currency is strengthening relative to the domestic currency—just the opposite result of its effect on life insurance. This is because, relatively speaking, the mean of a life insurance is more affected by drift than the mean of an annuity due to the longer duration of the life insurance. Thus, when foreign currency is weakening relative to domestic currency, the denominator of the risk factor for life insurance decreases relatively more than that of an annuity, thereby increasing the relative risk of the life insurance.

As with life insurance, the incremental risk of a foreign-denominated annuity can be very large and should be managed carefully, and the risk factor tends toward 1 as issue age increases. Again, this is because the benefit period is shorter as issue age increases.

Table 5 is analogous to Table 2. It shows the values of the incremental foreign exchange risk factor for annuities calculated at force of interest 5.0%.

From this table, we can see that:

- Risk decreases as the force of interest decreases when the foreign currency is weakening relative to the domestic currency.
- Risk increases as the force of interest decreases when the foreign currency is strengthening relative to the domestic currency.

Table 6 illustrates the incremental risk factors that develop from a managed floating regime in the context of annuities. These factors are based on the same assumptions as Table 3.

A comparison of Tables 5 and 6 suggests again that a managed floating regime may significantly alter the incremental risk of foreign exchange compared with

Table 4
Independent Floating Risk Factors Male Whole Life Insurance (δ = 7.0%, 1983 Group Annuity Table)

Country	η	σ	Issue Age 35	45	55	65
Australia	−0.067	0.037	0.61	0.66	0.71	0.76
Canada	0.059	0.029	2.19	1.81	1.58	1.40
Italy	0.119	0.151	6.30	4.28	3.13	2.34
Japan	−0.096	0.056	0.53	0.58	0.64	0.69
Sweden	0.127	0.188	7.54	5.04	3.61	2.62
Switzerland	0.015	0.090	1.25	1.19	1.14	1.10
United Kingdom	−0.081	0.125	0.59	0.64	0.69	0.74

Table 5
Independent Floating Risk Factors Male Life Annuity (δ = 5.0%, 1983 Group Annuity Table)

Country	η	σ	Issue Age 35	45	55	65
Australia	−0.067	0.037	0.54	0.61	0.67	0.74
Canada	0.059	0.029	2.39	1.93	1.65	1.44
Italy	0.119	0.151	6.43	4.51	3.33	2.48
Japan	−0.096	0.056	0.46	0.53	0.60	0.67
Sweden	0.127	0.188	7.56	5.26	3.83	2.79
Switzerland	0.015	0.090	1.30	1.22	1.16	1.12
United Kingdom	−0.081	0.125	0.52	0.59	0.65	0.72

Table 6
Independent Floating Risk Factors Male Life Annunuity (δ = 5.0%, 1983 Group Annuity Table)

Country	η	σ	Issue Age			
			35	45	55	65
Australia	−0.067	0.037	0.99	0.98	0.98	0.97
Canada	0.059	0.029	1.08	1.08	1.09	1.10
Italy	0.119	0.151	1.00	1.00	1.00	1.01
Japan	−0.096	0.056	1.01	1.00	1.00	0.99
Sweden	0.127	0.188	1.02	1.02	1.02	1.02
Switzerland	0.015	0.090	1.04	1.03	1.03	1.03
United Kingdom	−0.081	0.125	0.99	0.98	0.97	0.97

an independently floating regime. As in the life insurance examples, the incremental risk factors may tend away from 1.00 as age increases due to the assumption that the managed floating regime will tend toward a pegged regime over time. In some situations, the incremental risk of foreign exchange appears to be negligible. This conclusion, however, depends on the credibility of the managed floating regime.

5.3 Compound Poisson Aggregate Claims Distribution

From classic risk theory, the risk inherent in the compound Poisson distribution before reflecting foreign exchange risk as measured by the ratio of the standard deviation to the mean is

$$Risk = \frac{\sqrt{\lambda(\sigma_x^2 + \mu_x^2)}}{\lambda\mu_x}.$$

From Equations (12) and (13), risk after accounting for the effect of foreign exchange is

$$Risk' = \frac{\sqrt{\lambda(\sigma_x^2 + \mu_x^2)}}{\lambda\mu_x} e^{1/2\sigma^2(u-t)}.$$

Comparing these equations, it is clear that foreign exchange risk increases the aggregate risk of an insurance portfolio under a compound Poisson model by a factor of $e^{1/2\sigma^2(u-t)}$. Interestingly, the drift parameter, η, does not appear in this measure of incremental foreign exchange risk. This is because the difference $u - t$ is treated as though it is nonrandom. Therefore, the examples below only apply in a driftless environment.

To illustrate the general order of magnitude of foreign exchange risk under a compound Poisson model with no drift, the risk factor, $e^{1/2\sigma^2(u-t)}$, has been calculated and is shown in Table 7.

Table 7
Independent Floating Risk Factors Compound Poisson Model exp{1/2σ^2(u − t)}

Country	During Periods Ending 12/31/93		
	6 Months	1 Year	2 Years
Australia	1.0004	1.0007	1.0013
Canada	1.0002	1.0004	1.0009
Italy	1.0040	1.0114	1.0056
Japan	1.0007	1.0016	1.0045
Sweden	1.0053	1.0178	1.0046
Switzerland	1.0023	1.0040	1.0057
United Kingdom	1.0027	1.0079	1.0044

These factors were calculated based upon the Federal Reserve Board of Chicago data described earlier. The parameter σ was derived by applying the method of moments to the 6-month, 1-year, and 2-year differences occurring during the three-year period ending December 31, 1993. Because these currencies drifted relative to the U.S. dollar during the observation period, these factors are valid only if drift is not expected in the future.

It is notable that the largest risk factor observed is 1.0178. This suggests that foreign exchange risk may not significantly affect shorter-term insurance risks in a driftless environment. If true, a company entering or expanding its presence in a foreign market and focused on shorter-term risks might be willing to accept foreign exchange risk "naked" rather than devote considerable resources to the task of managing foreign exchange exposure.

Commentary

Before drawing broad conclusions about the effect of foreign exchange risk on insurance products, the reader should keep the following caveats in mind:

- The independently floating examples apply only to the specific countries identified.
- The hypothetical managed floating examples depend on the assumption that the boundaries are fixed, and that therefore managed floating regimes will tend toward pegged regimes over time, due to drift. Other managed floating regimes are conceivable and could lead to different observations.
- The models of foreign exchange developed here do not handle changes in regimes.
- The compound Poisson risk factor, $e^{1/2\sigma^2(u-t)}$, did not take into account the distribution of the difference between the date claims are paid and the date premiums are paid. Foreign exchange risk may have a

greater effect on other aggregate claim models than shown above.
- As noted, random walk models capture some aspects of exchange rate movements well, but they do not capture the full extent of run-ups or run-downs in exchange rates.
- Finally, the period examined in the illustrations may not have been long enough to capture the full volatility of exchange rates.

Nevertheless, the examples do suggest that foreign exchange risk is most significant with respect to longer-term risks. Companies contemplating entry into foreign markets with longer-term products would do well to devote considerable attention to management of foreign exchange risk.

6. CONCLUSION

Seeing significant growth opportunities, many insurers are considering entering and/or expanding their presence in foreign markets. They may have much to offer their markets in terms of products and services, but may be reluctant to proceed due to unknown factors such as foreign exchange risk. The models developed here could help them quantify the magnitude of their potential foreign exchange exposure, better enabling these insurers to judge their approach to their chosen markets.

The models developed here do not reflect the risk associated with exchange rate regimes that are not credible. Lack of credibility exposes the insurer to a potentially highly volatile exchange rate environment, in which timing, and not spread of risk, would be the key to success. If an insurer chooses to operate in such an environment, it should establish processes to closely monitor and manage foreign exchange exposures in order to limit losses and/or capitalize on opportunities presented.

The numerical examples suggest that incremental foreign exchange risk may be small in the context of shorter-term insurance risks and very large in relation to longer-term products such as whole life and annuities. Caution should be exercised before applying these results directly to specific business situations.

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Discussions on this paper can be submitted until October 1, 2000. The author reserves the right to reply to any discussion. Please see the Submission Guidelines for Authors on the inside back cover for instructions on the submission of discussions.

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"Measuring Foreign Exchange Risk in Insurance Transactions," John I. Mange, April 2000

HANGSUCK LEE*

Mr. Mange has written an interesting and timely paper. My comments concern Section 4 on "Insurance Models of Foreign Exchange." The author developed several formulas related to continuous term life insurance and whole life annuity with benefits payable in a foreign currency. The models of life insurance and annuities are constructed by mixing the traditional actuarial models that assume a constant force of interest with an exchange rate model. As pointed out by the author, these models need to be generalized by using stochastic force of interest models. In this discussion, I shall give alternative proofs and corrections of Equations (6), (7), (9) and (11). I shall also point out a paradox arising from the model.

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Let $T = T(x)$ be the random variable of the future lifetime of (x). Let t be the current time. Consider an n-year term life insurance issued to (x) with the death benefit being one unit in foreign currency. The value in domestic currency of the death benefit payable at the time of death, $t + T$, is $e^{S(t+T)}$. An n-year term life insurance provides for a payment only if the insured dies within n years. Let $I(\cdot)$ denote the indicator function. Then $I(T < n) = 1$ if death occurs within n years, and $I(T < n) = 0$ otherwise. Hence, $e^{S(t+T)-S(t)-\delta T}I(T < n)$ is the random variable of the time-t value in foreign currency of the death benefit. In Equation (6) of the paper, the indicator random variable, $I(T < n)$, is missing. Also, $S(t)$ is known, not random at time t. Below is a derivation of Equation (6). From Equation (4) of the paper, it follows that

$$S(T + t) - S(t) = \eta T + \sigma[Z(T + t) - Z(t)], \quad \text{(D1)}$$

which has the same distribution as $\eta T + \sigma Z(T)$. Here, $\{Z(s), s \geq 0\}$ is a standard Brownian motion. Under the assumption that T and $\{S(\tau), \tau \geq 0\}$ are independent, the expected present value in foreign currency of the term insurance can be derived as follows.

$$\begin{aligned}\overline{A'}^{\,1}_{x:\overline{n}|} &= \mathrm{E}[e^{S(t+T)-S(t)-\delta T}I(T < n)] \\ &= \mathrm{E}[\mathrm{E}[e^{S(t+T)-S(t)-\delta T}I(T < n)|T]] \\ &= \mathrm{E}[e^{-\delta T}I(T < n)\mathrm{E}[e^{S(t+T)-S(t)}|T]] \\ &= \mathrm{E}[e^{-\delta T}I(T < n)\ \exp(\eta T + \tfrac{1}{2}\sigma^2 T)] \\ &= \mathrm{E}[\exp\{-(\delta - \eta - \tfrac{1}{2}\sigma^2)T\}I(T < n)] \\ &= \bar{A}^{\,1}_{x:\overline{n}|} \text{ @ force of interest } \delta - \eta - \tfrac{1}{2}\sigma^2.\end{aligned} \quad \text{(D2)}$$

If we replace n in Equation (D2) by ∞, we obtain the expectation of the present value in foreign currency of the death benefit payable in foreign currency at the moment of death,

$$\begin{aligned}\overline{A'}_x &= \mathrm{E}[e^{S(t+T)-S(t)-\delta T}] \\ &= \bar{A}_x \text{ @ force of interest } \delta - \eta - \tfrac{1}{2}\sigma^2. \end{aligned} \quad \text{(D3)}$$

The j^{th} moment of the random variable $e^{S(t+T)-S(t)-\delta T}I(T < n)$ can be calculated as follows.

$${}^{j}\overline{A'}^{\,1}_{x:\overline{n}|} = \mathrm{E}[e^{j[S(t+T)-S(t)-\delta T]}I(T < n)]$$

$$= \mathrm{E}[\mathrm{E}[e^{j[S(t+T)-S(t)-\delta T]}I(T<n)|T]]$$
$$= \mathrm{E}[e^{-j\delta T}I(T<n)\mathrm{E}[e^{j[S(t+T)-S(t)]}|T]]$$
$$= \mathrm{E}[e^{-j\delta T}I(T<n)\ \exp\{j\eta T + \tfrac{1}{2}j^2\sigma^2 T\}]$$
$$= \mathrm{E}[\exp\{-(j(\delta-\eta) - \tfrac{1}{2}j^2\sigma^2)T\}I(T<n)]$$
$$= \bar{A}^{\,1}_{x:\overline{n}|}\ @\ \text{force of interest}\ j(\delta-\eta) - \tfrac{1}{2}j^2\sigma^2. \tag{D4}$$

Note that the force of interest is not $j(\delta - \eta - \frac{1}{2}\sigma^2)$.

Let us now consider a continuous whole life annuity issued at time t to (x) for one unit of the foreign currency per annum, as in Section 4.2 of Mange's paper. Then

$$\overline{a'}_{\overline{T}|} = \int_0^T e^{S(t+u)-S(t)-\delta u}du \tag{D5}$$

is the present value in foreign currency of the whole life annuity. Its expected value can be calculated as follows:

$$\overline{a'}_x = \mathrm{E}[\overline{a'}_{\overline{T}|}]$$
$$= \mathrm{E}\left[\int_0^T e^{S(t+u)-S(t)-\delta u}du\right]$$
$$= \mathrm{E}\left[\int_0^\infty e^{S(t+u)-S(t)-\delta u}I(u<T)du\right]$$
$$= \int_0^\infty \mathrm{E}[e^{S(t+u)-S(t)-\delta u}I(u<T)]du$$
$$= \int_0^\infty \mathrm{E}[e^{S(t+u)-S(t)-\delta u}]\mathrm{E}[I(u<T)]du$$
$$= \int_0^\infty \exp[-(\delta-\eta-\tfrac{1}{2}\sigma^2)u]\,{}_up_x du$$
$$= \bar{a}_x\ @\ \text{force of interest}\ \delta-\eta-\tfrac{1}{2}\sigma^2 \tag{D6}$$

In Equation (9) of the paper, the lower limit of the integral should be t.

The formula for the variance of $\overline{a'}_{\overline{T}|}$, Equation (11), in the paper is incorrect. Although Equation (10),

$$\mathrm{E}[\overline{a'}_{\overline{T}|}] = \mathrm{E}[(1-e^{S(t+T)-S(t)-\delta T})/(\delta-\eta-\tfrac{1}{2}\sigma^2)], \tag{D7}$$

is correct, it does not follow that

$$\overline{a'}_{\overline{T}|} = (1-e^{S(t+T)-S(t)-\delta T})/(\delta-\eta-\tfrac{1}{2}\sigma^2) \tag{D8}$$

is true. In particular, the formula

$$\mathrm{Var}(\overline{a'}_{\overline{T}|}) = \mathrm{Var}[(1-e^{S(t+T)-S(t)-\delta T})/(\delta-\eta-\tfrac{1}{2}\sigma^2)], \tag{D9}$$

is invalid. Therefore, Equation (11) is incorrect. Below is a derivation of the variance of $\overline{a'}_{\overline{T}|}$. The second moment of $\overline{a'}_{\overline{T}|}$ can be calculated as follows.

$$\mathrm{E}[(\overline{a'}_{\overline{T}|})^2] \tag{D10}$$
$$= \mathrm{E}\left[\left\{\int_0^T e^{S(t+u)-S(t)-\delta u}du\right\}^2\right]$$
$$= \mathrm{E}\left[\left\{\int_0^\infty e^{S(t+u)-S(t)-\delta u}I(u<T)du\right\}^2\right]$$
$$= \mathrm{E}\left[\int_0^\infty e^{(\eta-\delta)u+\sigma Z(u)}I(u<T)du \times \int_0^\infty e^{(\eta-\delta)v+\sigma Z(v)}I(v<T)dv\right]$$
$$= \mathrm{E}\left[\int_0^\infty\int_0^\infty e^{(\eta-\delta)(u+v)+\sigma[Z(u)+Z(v)]} \times I(u<T,\, v<T)dudv\right]$$
$$= \mathrm{E}\left[2\int_0^\infty\int_0^v e^{(\eta-\delta)(u+v)+\sigma[2Z(u)+Z(v)-Z(u)]} \times I(v<T)dudv\right]$$
$$= 2\int_0^\infty\int_0^v \mathrm{E}[e^{(\eta-\delta)(u+v)+\sigma[2Z(u)+Z(v)-Z(u)]} \times I(v<T)]dudv$$

$$=2\int_0^\infty\int_0^v E[e^{(\eta-\delta)(u+v)+\sigma[2Z(u)+Z(v)-Z(u)]}]$$

$$\times E[I(v<T)]dudv$$

$$=2\int_0^\infty\int_0^v e^{(\eta-\delta)(u+v)} E[e^{2\sigma Z(u)}]E[e^{\sigma[Z(v)-Z(u)]}]$$

$$\times\ E[I(v<T)]dudv$$

$$=2\int_0^\infty\int_0^v e^{(\eta-\delta)(u+v)}\exp(2\sigma^2 u)$$

$$\times\exp[\tfrac{1}{2}\sigma^2(v-u)]_v p_x dudv$$

$$=2\int_0^\infty \exp[-(\delta-\eta-\tfrac{1}{2}\sigma^2)v]_v p_x$$

$$\times\left\{\int_0^v \exp[-(\delta-\eta-\tfrac{3}{2}\sigma^2)u]du\right\}dv$$

$$=\frac{2}{\delta-\eta-\frac{3}{2}\sigma^2}\int_0^\infty\{\exp[-(\delta-\eta-\tfrac{1}{2}\sigma^2)v]$$

$$-\exp[-2(\delta-\eta-\sigma^2)v]\}_v p_x dv$$

$$=\frac{2}{\delta-\eta-\frac{3}{2}\sigma^2}[{}^{(1)}\bar{a}_x-{}^{(2)}\bar{a}_x]$$

where ${}^{(1)}\bar{a}_x$ denotes $\bar{a}_x$ evaluated with the force of interest $\beta-\eta-\frac{1}{2}\sigma^2$ and ${}^{(2)}\bar{a}_x$ denotes $\bar{a}_x$ evaluated with the force of interest $2(\delta-\eta-\sigma^2)$. Thus $Var(\overline{a'}_{\overline{T|}})$ needs to be modified by Equations (D6) and (D10) as follows.

$$\text{Var}(\overline{a'}_{\overline{T|}})=E[(\overline{a'}_{\overline{T|}})^2]-E[\overline{a'}_{\overline{T|}}]^2$$

$$=\frac{2}{\delta-\eta-\frac{3}{2}\sigma^2}[{}^{(1)}\bar{a}_x-{}^{(2)}\bar{a}_x]-[{}^{(1)}\bar{a}_x]^2. \quad \text{(D11)}$$

Let me conclude this discussion with a paradox. Let $P_f(t, t+\tau)$ denote the time t value in foreign currency of a zero-coupon bond that pays one unit in foreign currency at time $t+\tau$. Then

$$P_f(t, t+\tau)=\exp[-(\delta-\eta-\tfrac{1}{2}\sigma^2)\tau]. \quad \text{(D12)}$$

Formula (D12) can be seen as a special case of (D3) with $T(x)$ being the degenerate random variable that always takes the value τ. Since τ is an arbitrary positive number, the foreign force of interest is the constant value $\delta-\eta-\frac{1}{2}\sigma^2$. Now let us consider a domestic zero-coupon bond maturing for 1 at time $t+\tau$. Then its payment at time $t+\tau$ in foreign currency is $e^{-S(t+\tau)}$, the present value of which at time t in foreign currency is $\exp[-S(t+\tau)-(\delta-\eta-\frac{1}{2}\sigma^2)\tau]$. Hence, the time-$t$ value in domestic currency of the domestic zero-coupon bond is $\exp[-S(t+\tau)-(\delta-\eta-\frac{1}{2}\sigma^2)\tau+S(t)]$. Thus, its expected value is

$$E[\exp\{-[S(t+\tau)-S(t)]-(\delta-\eta-\tfrac{1}{2}\sigma^2)\tau\}]$$

$$=\exp[-(\delta-\sigma^2)\tau]. \quad \text{(D13)}$$

Formula (D13) implies that the domestic force of interest is $\delta-\sigma^2$, which is not δ unless $\sigma=0$. The paradox occurs because of the assumption that the exchange rate is a diffusion process. For further discussion on the exchange rate and paradox, see Chapter 1, "Siegel's Paradox," in Kritzman (2000). Finally, in Section 5 of Mange's paper, we find $\delta=5.0$ or 7.0%, $\eta=0.127$, and $\sigma=0.188$ for Swedish currency rate. It follows that the Swedish force of interest is negative!

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"Calculating Funding Premiums for Universal Life Insurance," Calvin Cherry, April 2000

DENNIS RADLIFF*

The author gave a thoughtful discussion of the methods used for calculating the premium required to reach a funding target under a universal life prod-

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