A NOTE ON THE MYERS AND READ CAPITAL ALLOCATION FORMULA

Stephen J. Mildenhall*

ABSTRACT

The Myers and Read capital allocation formula is an important new actuarial result. This paper gives an overview of the Myers and Read result, explains its significance to actuaries, and provides a simple proof. Then it explains the assumption that the allocation formula makes on the underlying families of loss distributions as expected losses by line vary. It shows that this assumption does not hold when insurers grow by writing more risks from a discrete group of insureds—as is typically the case. Finally, it shows that this failure has a material impact on the predicted results in a realistically sized portfolio of property casualty risks which will severely limit the practical application of the Myers and Read allocation formula.

1. INTRODUCTION

In an important paper for actuaries, Myers and Read (2001) showed how to allocate the expected policyholder deficit in a multiline insurance company uniquely to each line. Their work can also be used to allocate surplus to each line. Previous work on the allocation problem, including Phillips, Cummins, and Allen (1998) and Merton and Perold (2001), had concluded that such an allocation could be inappropriate and misleading. The Myers and Read result is, therefore, a potentially significant breakthrough of great importance to actuaries.

Myers and Read repeatedly stress that their result is independent of the distribution of losses by line and of any correlations between lines that may exist. They say their "proof requires no assumptions about the joint probability distributions of line-by-line losses and returns on the firm's portfolio of assets" (2001, p. 573). However, while their result makes no assumptions about the *static* distribution of losses with fixed expected loss by line, their derivation does make an important assumption about how the *dynamic* distribution of losses changes shape as expected losses by line change. In this paper, we will explain the significance of the latter assumption. Proposition 1 will show that it is a necessary and sufficient condition for the Myers and Read result to hold. Most importantly, we will show that the assumption does not hold when insurers grow through the assumption of risk from discrete insureds—as they do in the real world. Finally we will show, through three examples extending those in the original paper, that the predicted adds-up result materially fails to hold at the scale where it would be applied in practice.

For the convenience of readers not familiar with Myers and Read's work, we begin with an overview. Consider a simple insurance company that writes two lines of business. The losses from each line are represented by random variables L_1 and L_2, with means l_1 and l_2. Since the company can choose to write more or less of each line, assume that the families $L_1(l_1)$ and $L_2(l_2)$ with varying means l_1 and l_2 are specified. For example, losses from line 1 may be normally distributed with mean l_1 and standard deviation 1000, and for line 2 be normally distributed with mean l_2 and coefficient of variation ν. Assume the company has capital $k = s_1 l_1 + s_2 l_2$ for constants s_1 and s_2, and total assets $l_1 + l_2 + k$. Let $s = k/(l_1 + l_2)$ be the average capital ratio. Also assume that interest rates are zero. (Myers and Read show how to convert from deterministic investment income to stochastic income. This paper focuses on deterministic income and sets it

* Stephen J. Mildenhall, Ph.D., F.C.A.S., A.S.A., M.A.A.A., is a Senior Vice President, Aon Re Services, Aon Center, 200 East Randolph Street, 16th Floor, Chicago, IL 60601, e-mail: steve@mynl.com.

equal to zero for simplicity. Nothing of substance is lost in doing so.) Let

$$D_M(l_1, l_2) = \iint_{x_1+x_2>(1+s)(l_1+l_2)} (x_1 + x_2 - (1+s)(l_1+l_2)) f(x_1, x_2)\, dx_1\, dx_2$$

be the expected default with respect to the joint probability density f of $L_1(l_1)$, and $L_2(l_2)$. It does not matter if the density f is objective, when D_M determines the expected values, or risk adjusted, when D_M determines prices. Then, under certain assumptions on the distributions of the families $L_1(l_1)$ and $L_2(l_2)$ for varying l_1, l_2, but under *no assumptions on the distributions of losses given fixed l_1 and l_2,* Myers and Read prove

$$l_1 \frac{\partial D_M}{\partial l_1} + l_2 \frac{\partial D_M}{\partial l_2} = D_M. \qquad (1)$$

This is obviously a very useful result: it gives a canonical allocation of the default value for the whole company to individual lines of business. It can be used to allocate surplus, and correctly allocate the cost of surplus to individual lines or business units.

2. Notation

We are modeling a multiline insurance company. Losses from each line are modeled by random variables L_i, $i = 1, \ldots, n$, where L_i has mean l_i and distribution function F_i. We will use the notation $F_i(x; l_i) = \Pr(L_i(l_i) < x)$ when necessary to avoid any ambiguity. We often regard l_i as a variable (but not a random variable), so each L_i is really a family of distributions indexed by l_i. Where necessary we emphasize this by writing $L_i(l_i)$. Changes in l_i correspond to increasing or decreasing volume in line i, since l_i is the a priori expected loss. These changes can come about by assuming risks from more insureds, which is typically a discrete change, or by assuming risk from given insureds for a longer period of time, which would be a continuous change.

Assume that the company holds total assets equal to $l_1 + \cdots + l_n + k$, so in a very simplistic sense k is the capital or surplus of the company.

Next, define the probability of insolvency function and the expected policyholder deficit function for a single line i as

$$I_i(l_i, k) = \Pr(L_i > l_i + k) = 1 - F_i(l_i + k)$$

and

$$D_i(l_i, k) = \int_{l_i+k}^{\infty} t - (l_i + k)\, dF_i(t).$$

In both of these equations l_i is performing double duty. It is the mean of L_i, and in $l_i + k$ it determines where F_i is evaluated. To emphasize this we could write

$$I_i(l_i, k) = 1 - F_i(l_i + k; l_i).$$

Similar remarks hold for D_i. D_i is not the expected policyholder deficit for line i within a multiline company; rather, it is the expected policyholder deficit for a monoline company that writes only line i.

Finally, let F be the multivariate distribution of $(L_1(l_1), \ldots, L_n(l_n))$, let $L = L_1 + \cdots + L_n$ be the total losses, and let F_s be the distribution function of L. Both F and F_s depend on $(l_1, \ldots, l_n)$. Define insolvency and expected deficit functions for the whole company by

$$I(l_1, \ldots, l_n, k) = \Pr\left(\sum L_i > k + \sum l_i\right) = 1 - F_s(l_1 + \cdots + l_n + k)$$

and

$$D(l_1, \ldots, l_n, k) = \int \cdots \int_{\sum t_i > k + \sum l_i} t_1 + \cdots + t_n - (l_1 + \cdots + l_n + k)\, dF(t_1, \ldots, t_n). \qquad (2)$$

With this notation $D_i(l_i, k) = D(0, \ldots, l_i, 0, \ldots, 0, k)$.

We focus on families of random variables $L(l)$ because in order to compute expressions like $\partial I/\partial l$ we need to know how the distribution $L(l)$ changes shape with changes in l. We need to work with $L(l + \epsilon)$ as well as $L(l)$ because

$$\begin{aligned}\frac{\partial I}{\partial l} &= -\frac{d}{dl} F(l + k; l) \\ &= -\lim_{\epsilon \to 0} \frac{F(l + k + \epsilon; l) - F(l + k; l)}{\epsilon} \\ &\quad - \lim_{\epsilon \to 0} \frac{F(l + k; l + \epsilon) - F(l + k; l)}{\epsilon}.\end{aligned}$$

The partial derivative has a *static* part, where the mean of the underlying variable does not change, and a *dynamic* part, where the point of evaluation is fixed but the mean changes. This shows computing partial derivatives such as $\partial I/\partial l$ is inextricably linked to *families* of random variables.

3. HOMOGENEOUS DISTRIBUTIONS AND HOMOGENEOUS PRICES

Myers and Read make one distributional assumption in their work, here called *homogeneity*. This section explains how homogeneity is a natural assumption to make for assets but not for a portfolio of insurance risks. It also shows how to construct arbitrage-free homogeneous pricing functionals on inhomogeneous outcome distributions.

Consider two portfolios, one consisting of stock in a given company and the other consisting of insurance policies written on identical risks. Assume that the price of the stock is 1 today, and let $X(n)$ be the price of a portfolio of n of these stocks one year from now. If S is the price distribution of the stock one year from now, then the value of the portfolio $X(n)$ has the same distribution as nS. More generally, returns from an investment advisor are likely to be similarly homogeneous; as the advisor gets more funds to invest, he or she will increase holdings in existing positions and not suddenly start to invest in different asset classes.

Next, assume that the expected losses from each insurance policy in the portfolio has a present value of 1 today. Let $L(n)$ be the present value of total losses from a portfolio of n policies. If R is the distribution of losses from one policy, with $\mathrm{E}[R] = 1$, then the distribution of $L(n)$ is the same as the distribution of $R_1 + \cdots + R_n$, but it is not the same as nR, unless the R_i are perfectly correlated. Since diversification is the basis for insurance, we will assume that the R_i are identically distributed, but not perfectly correlated. Realistically the R_i will likely be somewhat correlated.

These two examples highlight an important difference between portfolios of investment- and insurance-type risks. For a stock or other financial asset the meaning of $2X$ is clear: we own two identical, equivalent, and interchangeable stocks with the same price distribution now and at all future times. In contrast, for insurance risks, regulations regarding insurable interest and overinsurance make an interpretation as a policy that pays \$2 for each dollar loss unrealistic. In insurance, when we double expected losses, we write twice as many policies and get a distribution $R_1 + R_2$, with R_i identically distributed, but less than perfectly correlated. A car can have only one driver; it is physically impossible to have two auto policies with perfectly correlated experience! Two auto policies, even if they have identical loss distributions, are subject to different random outcomes, and together they are equivalent to a portfolio of stock in two different companies, not two stocks in one company.

The distinction between these two types of behavior is crucial to the points made in this paper. To make the concept precise here is a formal definition.

DEFINITION 1

A family of random variables $L(l)$ with $\mathrm{E}[L(l)]$ proportional to l is called homogeneous in distribution, or simply homogeneous, if there exists a single random variable U so that $L(l)$ has the same distribution as lU for all l. Families that are not homogeneous are called inhomogeneous.

The requirement that U is independent of l is important since any random variable can be written as $L = \mathrm{E}[L](L/\mathrm{E}[L])$. The future value of multiples of a given stock is clearly homogeneous in distribution. On the other hand, the present value of losses in a portfolio of identically distributed insurance policies is not homogeneous. An exponential variable L with mean l is a parametric homogeneous family, since $L = lU$, where U has an exponential distribution with mean 1. A normal variable with mean l and standard deviation 1 is not homogeneous.

Homogeneity is Myers and Read's only distributional assumption. For it to hold in the way they assume, companies would have to quota share a portion of the entire market in a line, which Butsic (1999) calls a representative insurer approach. There is no major line of U.S. property casualty insurance that operates in this way; it is an unrealistic assumption.

We now turn to the homogeneity of prices. The Fundamental Theorem of Asset Pricing states that the absence of arbitrage in a pricing system is equivalent to the existence of a positive linear pricing rule; see Dybvig and Ross (1989). A pricing rule is a function q that assigns a price $q(X)$ to a random payoff X. The function q is linear if for two random payoffs X and Y we have

$$q(aX + bY) = aq(X) + bq(Y)$$

for all constants a and b. If $q(aX) = aq(X)$, then q is called homogeneous. Homogeneity is a necessary condition for q to be arbitrage free. It is obvious that a sufficiently liquid market cannot be arbitrage free if the pricing functional is not homogeneous.

If $X(x)$ is a homogeneous family of random variables and q is a homogeneous pricing rule, then $q(X(x)) = q(xX(1)) = xq(X(1))$. If the family is not homogeneous, then we need more assumptions in order to make similar statements. If $L(l)$ is an inhomogeneous family of random variables, but it is additive in the sense that $L(l + m) = L(l) + L(m)$, then if q is linear we have $q(L(l + m)) = q(L(l) + L(m)) = q(L(l)) + q(L(m))$. From this it follows by continuity that $q(L(l)) = lq(L(1))$, and we recover a homogeneous pricing rule. In the insurance context, where $L(l)$ is typically modeled as a compound Poisson process or a mixed compound Poisson process, we have additivity (in fact, infinite divisibility). Thus it is possible for an inhomogeneous family to have a homogeneous and arbitrage-free pricing rule.

Several specific examples of arbitrage-free pricing functionals for inhomogeneous insurance distributions have been given in the literature. The first was the fundamental paper of Delbaen and Haezendonck (1989), which was then extended by Meister (1995). The key results are also reviewed in Embrechts and Meister (1995). Delbaen and Haezendonck show that if there are sufficiently many reinsurance markets, then linear pricing functionals transform compound Poisson distributions to compound Poissons distributions. They then characterize the measures equivalent to a given compound Poisson that are themselves compound Poisson and show that these are characterized via a separate adjustment of the frequency and severity. Specifically they show that an aggregate distribution $L_t = R_1 + \cdots + R_{N(t)}$, with R_i independent and identically distributed and $N(t)$ Poisson with mean λt, transforms to a compound Poisson of the form $\tilde{R}_t = \tilde{R}_1 + \cdots + \tilde{R}_{\tilde{N}(t)}$, where $\tilde{N}(t)$ is Poisson with mean $\lambda' t$, and the Radon-Nikodym derivative of $\tilde{R}$ with respect to R is given by

$$\frac{d\tilde{R}}{dR} = \exp(\beta(x))/\mathrm{E}_R[\exp(\beta(R))]$$

for some increasing function β. The transformed distribution can be regarded as risk-adjusted, and prices can be computed as (linear) expected values with respect to the risk-adjusted probabilities, just as pricing is done on the asset side. Meister extends Delbaen and Haezendonck to mixed compound Poisson distributions.

To conclude, this section has defined homogeneous families of random variables and has shown that inhomogeneous families can still be priced using an arbitrage-free positive linear pricing functional. The failure of the adds-up result for inhomogeneous distributions is caused by different assumptions about the shape of the loss distribution rather than the lack of an arbitrage-free pricing functional.

4. STATEMENT AND PROOF OF MAIN RESULT

We can now state our main result. The result depends on two technical lemmas that are stated and proved in Appendix 1.

Proposition 1

For sufficiently differentiable families L_{-i}, the following are equivalent:

1. *For each $i = 1, \ldots, n$, $L_i(l_i)$ is a homogeneous family of random variables.*
2. *For each $i = 1, \ldots, n$,*
$$l_i \frac{\partial I_i}{\partial l_i} + k \frac{\partial I_i}{\partial k} = 0. \tag{3}$$
3. *For each $i = 1, \ldots, n$,*
$$l_i \frac{\partial D_i}{\partial l_i} + k \frac{\partial D_i}{\partial k} = D_i. \tag{4}$$
4. *We have equality*
$$l_1 \frac{\partial I}{\partial l_1} + \cdots + l_n \frac{\partial I}{\partial l_n} + k \frac{\partial I}{\partial k} = 0. \tag{5}$$
5. *We have equality*
$$l_1 \frac{\partial D}{\partial l_1} + \cdots + l_n \frac{\partial D}{\partial l_n} + k \frac{\partial D}{\partial k} = D. \tag{6}$$

The proposition says that each of the five statements holds if and only if all the other four hold.

Put another way, if one of the five fails to hold, then the other four will also fail. This means we can construct simple one-line examples and can use items 2 and 3 to generalize to the multiline case, which simplifies the mathematics of the examples.

PROOF

We shall prove (4) implies (2) implies (1) implies (4), and then (5) implies (3) implies (1) implies (5), which is enough to show all the statements are equivalent.

(4) implies (2): Set $l_j = 0$ for $j \neq i$ in equation (5) to get equation (3). This can also be seen geometrically using Lemma 1, which says I is constant along rays from the origin. Therefore, I_i, which is a restriction of I, is also constant along such rays.

(2) implies (1): Lemma 1 applied to I_i shows that there exists a function $\tilde{I}_i$ so that

$$I_i(l_i, k) = \tilde{I}_i(k/l_i).$$

Let $U_i = L_i/l_i$, then $\Pr(U_i > u) = \tilde{I}_i(u - 1)$ is independent of l_i as required.

(1) implies (4): Assumption (1) implies that I is constant along rays from the origin, so the result follows from Lemma 1.

(5) implies (3): Set $l_j = 0$ for $j \neq i$ in equation (6) to get equation (4).

(3) implies (1): Let $U_i = L_i/l_i$. We have to show $\Pr(U_i > u)$ is independent of l_i. Let $l^+ = \max(l, 0)$. Then notice that

$$\begin{aligned}\frac{\partial D}{\partial k} &= \frac{\partial}{\partial k}\, \mathrm{E}[(l_iU_i - (l_i + k))^+] \\ &= \mathrm{E}\left[\frac{\partial}{\partial k}\,(l_iU_i - (l_i + k))^{+}\right] \\ &= \mathrm{E}[-\mathbf{1}_{\{l_iU_i > l_i + k\}}] \\ &= -\Pr(l_iU_i > l_i + k) \qquad (7)\end{aligned}$$

is minus the probability of default. Next, use Lemma 2 to define $\tilde{D}_i$ so that $D_i(l_i, k) = l_i\tilde{D}_i(k/l_i)$. Therefore,

$$\frac{\partial D_i}{\partial k} = \tilde{D}_i'(k/l_i),$$

and so

$$\Pr(U_i > u) = -\tilde{D}_i'(u - 1)$$

is independent of l_i as required.

(1) implies (5): Assumption (1) shows we can write D as

$$D(l_1, \ldots, l_n, k) = k\tilde{D}(l_1/k, \ldots, l_n/k),$$

so the result follows from Lemma 2. □

The results in Proposition 1 are clearly similar to Myers and Read's results, but they are not exactly the same. Next, we will show how to derive their exact result. For simplicity we shall assume $n = 2$ and work with just l_1 and l_2 in the rest of the section.

Myers and Read's "adds-up" result (their equation A1–3) involves computing the marginal increase in surplus required to hold the default value constant, given a marginal increase in a particular line. We have been taking a slightly different approach: if we hold the surplus and default value constant, what decrease is needed in line 2 to offset an increase in line 1? However, it is easy to reconcile the two approaches. To do this, let s_1 and s_2 be the marginal surplus requirements for each line. Note that s_1 and s_2 are ratios, whereas k is a dollar amount. Myers and Read then use a capital amount $k = s_1l_1 + s_2l_2$ and define the default value D_M (to distinguish from our D) as

$$D_M(l_1, l_2) := D(l_1, l_2, s_1l_1 + s_2l_2).$$

Myers and Read use the following notation in their Appendix 1. They write $\tilde{L}_a = L_a\tilde{R}_a$, where L_a corresponds to our l_1, $\tilde{R}_a$ to U_1, and $\tilde{L}_a$ to L_1. Thus $\tilde{L}_a = L_a\tilde{R}_a$ translates into our $L_1 = l_1U_1$, that is, the homogeneity assumption. The value L_a is the expected value of $\tilde{L}_a$ at time 0. We are ignoring the time value of money here by assuming an interest rate of zero. Myers and Read also work with a fixed interest rate and then integrate over all possible rates—an extra level of sophistication that need not concern us.

We can now prove their result. In fact, Proposition 1 shows the "adds-up" result holds if and only if the families L_i are homogeneous.

Corollary 1

(Myers and Read) Assume losses L_i form a homogeneous family for each i. Then default values "add up" in that

$$l_1\frac{\partial D_M}{\partial l_1} + l_2\frac{\partial D_M}{\partial l_2} = D_M.$$

Proof

Computing using the chain rule and then applying Proposition 1 item 5 in equation (8) gives

$$
\begin{aligned}
l_1 \frac{\partial D_M}{\partial l_1} + l_2 \frac{\partial D_M}{\partial l_2} &= l_1\left(\frac{\partial D}{\partial l_1} + s_1 \frac{\partial D}{\partial k}\right) \\
&\quad + l_2\left(\frac{\partial D}{\partial l_2} + s_2 \frac{\partial D}{\partial k}\right) \\
&= l_1 \frac{\partial D}{\partial l_1} + l_2 \frac{\partial D}{\partial l_2} \\
&\quad + (s_1 l_1 + s_2 l_2) \frac{\partial D}{\partial k} \\
&= D(l_1, l_2, s_1 l + s_2 l_2) \qquad (8) \\
&= D_M(l_1, l_2)
\end{aligned}
$$

as required. □

Simple Proof

Here is the simple, self-contained proof promised in the introduction. Dividing through by l_1 in the definition of D, equation (2), it is clear that $D_M(l_1, l_2) = l_1 \tilde{D}_M(l_2/l_1)$ for some function $\tilde{D}_M$. Thus

$$
\begin{aligned}
l_1 \frac{\partial D_M}{\partial l_1} + l_2 \frac{\partial D_M}{\partial l_2} &= l_1\left(\tilde{D}_M - \frac{l_2}{l_1} \frac{\partial D_M}{\partial l_1}\right) + l_2 \frac{\partial D_M}{\partial l_2} \\
&= D_M,
\end{aligned}
$$

which completes the proof. □

5. Examples

5.1 Examples of Homogeneity

Homogeneous families can be made from a wide variety of continuous distributions. For example, varying the scale parameter θ and holding all other parameters constant for any of the distributions listed in Appendix A of Klugman, Panjer, and Willmot (1998), which have a scale parameter θ, will produce a homogeneous family. This includes suitable parameterizations of the transformed beta, Burr, generalized Pareto, Pareto, transformed gamma, gamma, Weibull, exponential, and inverse Gaussian. By Proposition 1, sums selected from such families will also be homogeneous. Also, trivially, if U is any distribution with mean 1, then lU is a homogeneous family as l varies.

5.2 Examples of Inhomogeneity

It is easy to construct examples where the homogeneity assumption fails. All members of a homogeneous family have the same coefficient of variation; therefore a family with a nonconstant coefficient of variation will not be homogeneous. For example, let L be normally distributed with mean l and constant standard deviation 1. Then L is not homogeneous. By definition $I(l, k) = 1 - \Phi(k)$ so

$$l \frac{\partial I}{\partial l} + k \frac{\partial I}{\partial k} = -k\phi(k) \neq 0,$$

where Φ and ϕ are the distribution and density for the standard normal. Proposition 1 implies this expression equals zero if L is homogeneous. If the reader is skeptical about using only one variable, he or she will find it easy to construct multivariate distribution examples using normal variables.

It is less simple, but still possible, to construct examples where the coefficient of variation is a constant function of the mean, but that nevertheless fail to satisfy the homogeneity assumption. For example, let $L(l)$ be distributed as a gamma random variable with parameters $\alpha = 4l^2$, $\theta = 1/2$ shifted by $l(1 - 2l)$. Here we are using the Klugman, Panjer, and Willmot parameterization, so $f(t;\ \alpha,\ \theta) = (t/\theta)^\alpha e^{-t/\theta}/t\Gamma(\alpha)$. It is easy to check $L(l)$ has mean l, constant coefficient of variation 1, and skewness $1/l$, since the skewness of a gamma α, θ is $2/\sqrt{\alpha}$. I is given by the incomplete gamma function, $I(l, k) = \Gamma(4l^2, 4l^2 + 2k)$, which does not satisfy the assumptions of Lemma 1, so $L(l)$ is not homogeneous. The reason is clear: the family $L(l)$ changes shape with l and so cannot be homogeneous. Taking this a step further, it is possible to construct a family, all of whose higher cumulants (coefficient of variation, skewness, kurtosis, etc.) are independent of the mean, just as they would be for a homogeneous family, but that nevertheless fails to be homogeneous.

5.3 Aggregate Distributions Are Inhomogeneous

The central distributions of insurance, compound Poisson, and mixed compound Poisson distributions are inhomogeneous, because the coefficient of variation depends on the mean.

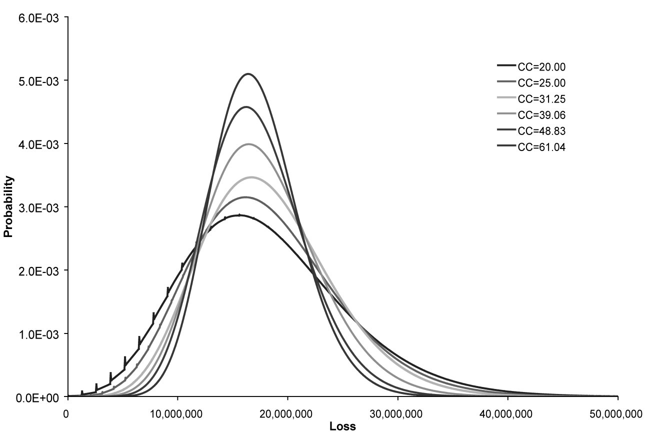
Let $L = R_1 + \cdots R_N$ where the R_i are independent, identically distributed severities and N is a frequency distribution with mean n. Increasing expected losses in this model involves increasing n. Suppose N has contagion c, so, as suggested by Heckman and Meyers (1983), $\text{Var}(N) = n(1 + cn)$. Then

$$\text{CV}(L)^2 = \frac{\text{CV}(R)^2}{n} + \frac{1}{n} + c$$

is clearly not independent of n. Thus L does not satisfy the homogeneity assumption: the aggregate loss distribution changes shape as n increases. This is illustrated in Figure 1, which shows six aggregate loss distributions with the same severity distribution but different claim counts, indicated by "CC = 20" for $n = 20$, and so forth. The individual densities have been scaled so that if the family were homogeneous, then all the densities would be identical, and only one line would appear in the plot.

If aggregate distributions can be approximated by various families of parametric distributions, and if those families are homogeneous, does this result really matter? The answer is an emphatic "yes." Our example shows that in the real world, where insurers grow by adding discrete insureds, the "adds-up" results do not hold, because the way the aggregate distribution changes shape forces parameters other than the scale parameter

Figure 1
Aggregate Distributions Are Not Homogeneous



to change as the mean increases, and thus homogeneity is lost.

6. Inhomogeneity Is Material

This section will discuss how the Myers-Read formula is likely to be applied in practice, and what we would intuitively expect the formula to show for an insurance portfolio. Then we will extend the examples given in the original paper to allow for inhomogeneity. The extended examples show that the inhomogeneity inherent in a typical portfolio of property casualty risks is large enough to invalidate the Myers and Read allocation formula.

6.1 Use of the Myers and Read Allocation in Pricing

Myers and Read's paper tries to explain how capital should be allocated across a company's different lines of business. They point out that "because surplus is costly, competitive premiums . . . depend on total surplus requirements and on their allocation to lines of insurance" (2001, p. 546). They are expecting capital allocation to be used in the context of measuring profitability and setting targets for divisions or lines within a company. Internal company specific allocations are irrelevant to determining market prices. Knowing how the formula will be applied helps calibrate the scale for our examples of inhomogeneity. Clearly the relevant scale is much smaller than the whole industry; only 7.5% of U.S. property casualty company groups had total gross premium greater than \$1B in 2002, whereas over 84% had total gross premium less than \$300M. An allocation of capital within a company will likely be on a scale of \$10–100M. We will give examples to show that inhomogeneity is very material at this scale.

6.2 Heuristics

Let $L(l)$ be a smooth family of random variables with $\text{E}[L(l)] = l$. Let $F(t, l) = \Pr(L(l) < t)$ be the distribution function of $L(l)$, and $f(t, l) = \partial F/\partial t$ be its density. The expected default value, with capital ratio s, is defined as

$$D(l) = \int_{l(1+s)}^{\infty} (t - l(1 + s))f(t, l)\, dt.$$

Note that $l(1 + s)$ represents total assets: l from the loss and ls from capital. In a more sophisticated model we could consider profit in the premium; here we simply subsume it into the constant s.

By Proposition 1 we know

$$l \frac{\partial D}{\partial l} = D$$

if and only if $L(l)$ is a homogeneous family, which is then equivalent to the Myers-Read adds-up result.

A homogeneous family offers no diversification benefit as the mean increases. Property casualty insurance is based on diversification, and the resulting inhomogeneity in a portfolio of insurance risks means that the relative riskiness of the portfolio decreases as expected losses increase. Since a lower-risk portfolio has a lower expected default, one would expect that

$$l \frac{\partial D}{\partial l} < D \tag{9}$$

for an inhomogeneous insurance portfolio. Instead of "adds-up" we expect to see a "sub-adds-up" result for inhomogeneous distributions that exhibit decreasing coefficient of variation with expected losses, such as aggregate distributions.

Meyers, Klinker, and Lalonde (2003) introduce the heterogeneity multiplier, which is a constant λ defined so that

$$\lambda l \frac{\partial D}{\partial l} = D.$$

They shows that λ is typically greater than 1 (as expected) and find a value close to 1.6 in some empirical examples.

If L is homogeneous, then, for all marginal capital ratios $s > 0$,

$$l \frac{\partial D}{\partial l} = D \geq 0.$$

However, intuitively one would expect that for a large enough capital ratio s it should be possible for the extra capital associated with writing more business to more than offset the extra risk. This would imply that

$$l \frac{\partial D}{\partial l} < 0$$

should be possible for sufficiently large s. This is another difference between homogeneous and inhomogeneous families.

6.3 Extended Myers-Read Examples

The examples given in the Myers and Read paper now will be extended to show the impact of inhomogeneity on the adds-up result. We will focus on the lognormal examples given in Table 2 (p. 560) of the original paper and will follow the same notation as far as possible. There are three lines of insurance L_i, $i = 1, 2, 3$ with expected losses $\mathrm{E}[L_i] = l_i$. Let $l = \sum l_i$, and let $x_i = l_i/l$ be the proportion of losses from line i. Let ρ_{ij} be the correlation between $\log(L_i)$ and $\log(L_j)$. As Myers and Read point out, if the line-by-line loss volatilities are not large, then the volatility of total losses is closely approximated by

$$\sigma_L^2 = \sum_i \sum_j x_i x_j \rho_{ij} \sigma_i \sigma_j,$$

where σ_i is the volatility (coefficient of variation) of line i. If σ_V is the volatility of assets V, then the correlation between log loss and log assets is approximately

$$\sigma_{LV} = \sum_i x_i \rho_{iV} \sigma_i \sigma_V,$$

where ρ_{iV} is the correlation between log assets and log line i losses. Last, let s_i be the marginal level of capital for line i per dollar of losses, and let $s = \sum_i x_i s_i$ be the weighted average capital ratio.

Let D be the value of the default option, $d = D/l$, and

$$d_i = \frac{\partial D}{\partial l_i}.$$

An easy computation shows

$$d_i = d + \frac{\partial d}{\partial x_i}.$$

Myers and Read's Appendix 2 shows that

$$d = N(z) - (1 + s)N(z - \sigma), \tag{10}$$

where

$$z = \frac{-\log(1 + s) + \sigma^2/2}{\sigma}$$

and

$$\sigma^2 = \sigma_L^2 + \sigma_V^2 - 2\sigma_{LV}. \tag{11}$$

They also compute

$$\frac{\partial d}{\partial x_i} = \frac{\partial d}{\partial s}\frac{\partial s}{\partial x_i} + \frac{\partial d}{\partial \sigma}\frac{\partial \sigma}{\partial x_i}.$$

Both $\partial d/\partial s$ and $\partial d/\partial\sigma$ can be computed from equation (10). Next, since $l_i = x_i l$,

$$\frac{\partial s}{\partial x_i} = \frac{\partial s}{\partial l_i}\frac{\partial l_i}{\partial x_i} = \left(-\frac{1}{l^2}\sum_i l_i s_i + \frac{s_i}{l}\right) l = -s + s_i.$$

Finally, Myers and Read compute $\partial\sigma/\partial x_i$ by noting

$$\frac{\partial \sigma}{\partial x_i} = \frac{\partial \sigma}{\partial l_i}\frac{\partial l_i}{\partial x_i} = l\,\frac{\partial \sigma}{\partial l_i},$$

and then differentiating equation (11) with respect to l_i to get

$$\begin{aligned}\sigma\frac{\partial \sigma}{\partial l_i} &= \sigma_L\frac{\partial \sigma_L}{\partial l_i} - \frac{\partial \sigma_{LV}}{\partial l_i}\\ &= \sum_k \frac{l_k}{l^2}\rho_{ik}\sigma_k\sigma_i - \frac{1}{l}\sum_{jk}\frac{l_j l_k}{l^2}\rho_{ij}\sigma_j\sigma_k\\ &\quad - \left(\frac{\rho_{iV}\sigma_i\sigma_V}{l} - \sum_j \frac{l_j}{l^2}\rho_{iV}\sigma_i\sigma_V\right)\\ &= (\sigma_{iL} - \sigma_L^2 - (\sigma_{iV} - \sigma_{LV}))/l. \end{aligned}\tag{12}$$

Here the first term in the middle line defines σ_{iL}, and the last two terms define σ_{iV} and σ_{LV}, respectively. This derivation has used the fact that $\partial\sigma_i/\partial l_i = 0$ for each i. Combining these equations gives

$$d_i = d + \frac{\partial d}{\partial s}(s_i - s) + \frac{\partial d}{\partial \sigma}\frac{1}{\sigma}((\sigma_{iL} - \sigma_L^2) - (\sigma_{iV} - \sigma_{LV})). \tag{13}$$

We have now defined all the expressions needed to understand Myers and Read's Table 2, which is reproduced here in Tables 1, 2, and 3.

Table 1 summarizes the input parameters; Table 2 shows the resulting values for σ and d. The entries in Table 3 are determined using equation (13) by setting $s_i = s$ and solving for d_i in the first column, and setting $d_i = d$ and solving for s_i in the second. Weighting the individual line values by x_i recovers the adds-up result; the total in the first column is exactly d and in the second is exactly s.

In an inhomogeneous family the volatility varies with expected losses, so $\sigma_i = \sigma_i(l_i)$, and we need to add $\partial\sigma_i/\partial l_i$ terms to equation (12). The additional terms are

$$\sum_j \frac{l_i l_j}{l^2}\rho_{ij}\sigma_j\frac{\partial \sigma_i}{\partial l_i} - \frac{l_i}{l}\rho_{iV}\sigma_V\frac{\partial \sigma_i}{\partial l_i}.$$

Combining these terms with equation (13) we get the following equation for d_i:

$$\begin{aligned} d_i = d &+ \frac{\partial d}{\partial s}(s_i - s) + \frac{\partial d}{\partial \sigma}\frac{1}{\sigma}((\sigma_{iL} - \sigma_L^2)\\ &- (\sigma_{iV} - \sigma_{LV})) + \frac{\partial d}{\partial \sigma}\frac{\partial \sigma_i}{\partial l_i}\frac{1}{\sigma}\\ &\times\left(\sum_j \frac{l_i l_j}{l}\rho_{ij}\sigma_j - l_i\rho_{iV}\sigma_V\right). \end{aligned}\tag{14}$$

We now have to determine how σ_i is likely to vary with l_i in a real-world portfolio. To do this, we look at a realistic aggregate loss distribution in

Table 1
Base Case Parameters

Item	Amount	x_i	σ	Correlations: Line 1	Line 2	Line 3	Cov/L	Cov/V
Line 1	100	33.3%	10.00%	1.000	0.500	0.500	0.0092	−0.0030
Line 2	100	33.3	15.00	0.500	1.000	0.500	0.0150	−0.0045
Line 3	100	33.3	20.00	0.500	0.500	1.000	0.0217	−0.0060
Liability L	300	100.0	12.36	0.742	0.809	0.876	0.0153	−0.0045
Assets	450	150.0	15.00	−0.200	−0.200	−0.200		0.0225
Surplus	150	50.0						

Table 2
Base Case σ and *d*

σ	21.62817%
d	0.311220%
Delta	−0.0237
Vega	0.0838

order to determine $\sigma_i(l_i)$ and then to approximate the actual distributions with a family of lognormals. If we use a compound Poisson model where $L = R_1 + \cdots + R_N$, N Poisson with mean n, then the volatility (coefficient of variation) of L is

$$\sigma(l) = \sqrt{\frac{x(\gamma^2 + 1)}{l}},$$

where $x = \mathrm{E}[R]$ is severity, γ is the coefficient of variation of R, and $l = \mathrm{E}[L] = nx$. Hence

$$\frac{\partial\sigma}{\partial l} = -\frac{1}{2}\sqrt{\frac{x(\gamma^2 + 1)}{l^3}}.$$

More generally, if N is a negative binomial (gamma mixture of Poisson frequencies) with $\mathrm{Var}(N) = n(1 + cn)$, then the volatility of L is

$$\sigma(l) = \sqrt{\frac{x(\gamma^2 + 1)}{l} + c},$$

and so

$$\frac{\partial\sigma}{\partial l} = -\frac{x(\gamma^2 + 1)}{2l^2\sigma(l)}.$$

Before presenting examples with inhomogeneous losses we have to determine reasonable values for x and γ. Since the examples use expected losses of 100, the expected claim count for each line will be $100/x$. We will consider three cases. The first example calibrates expected claim counts to correspond to a book of approximately \$300M casualty business with an average severity of \$4,688 per claim. This is a scale appropriate to a large division or whole medium-sized company. The second example uses a similar overall scale but a higher severity and hence greater inhomogeneity. The third example calibrates to a \$3B total loss; this corresponds to the largest of companies, and we expect the effect of inhomogeneity to be less material. Only 3% of U.S. property casualty companies have one or more lines with more than \$1B of gross premium.

In all three examples x and γ are chosen to approximate a real line of business, and then c is determined so that the volatility of each line is the same as shown in Table 1. This means the values of d, Delta, and Vega are as shown in Table 2 in all three cases, because these quantities do not depend on $\partial\sigma_i/\partial l_i$. Only d_i and s_i, which are shown in Table 3, vary between the examples. The values for the first example are shown in Table 4. Line 1 roughly corresponds to \$100M expected loss for workers compensation. It has an average of 50,000 claims, each with average severity \$2,000 and severity volatility of 20. Line 2 corresponds to a book of general liability policies with \$1M limits, and Line 3 approximates a book of medium-sized property risks. The contagion values c have been chosen so that the implied line volatilities are as shown in Table 1, viz., 0.10, 0.15, and 0.2. (These volatilities are lower than one would expect to see in real portfolios, but are used to facilitate comparison with the original paper.) The by-line defaults are shown under "Default Value"; the weighted average default is 0.1667% less than one-half $d = 0.311\%$. The last column shows the individual line surplus allocations; again, the weighted average allocated capital is 43.9% less than the actual 50%. This shows that we recover subadditivity, as expected from Equation (9).

To gauge the magnitude of the differences between Tables 3 and 4 on pricing, suppose the company desired a 10% return on allocated surplus from underwriting cash flows. Table 3 would give profit targets of 3.8%, 5.0%, and 6.3% for lines 1, 2, and 3, whereas Table 4 would give 3.0%, 4.5%, and 5.7%. These differences are material relative to the interline differences. As we have already observed, the total target in Table 4 would fall short because of the failure of targets to add up. The targets in Table 4 could be rescaled, but that would introduce an arbitrary choice that the canonical Myers Read decomposition specifically sought to remove.

Table 3
Homogeneous Case: "Adds-Up" Holds

Line	Default Value/Liability	Surplus/Liability
1	0.016%	37.55%
2	0.300	49.55
3	0.617	62.90
Total	0.311	50.00

Table 4
Average Severity Lines

Line	E[N]	x = E[X]	c	γ	Default Value/ Liability	Surplus/ Liability
1	50,000.0	2,000.0	0.002	20.000	−0.1727%	29.5740%
2	4,000.0	25,000.0	0.016	5.000	0.1913	44.9394
3	10,000.0	10,000.0	0.030	10.000	0.4815	57.1892
Total	64,000.0	4,687.5			0.1667	43.9008

Table 5
Higher Severity Lines

Line	E[N]	x = E[X]	c	γ	Default Value/ Liability	Surplus/ Liability
1	5,000.0	20,000.0	0.005	5.000	−0.1062%	32.3795%
2	2,000.0	50,000.0	0.010	5.000	0.0822	40.3330
3	10,000.0	10,000.0	0.004	19.000	0.1318	42.4277
Total	17,000.0	17,647.1			0.0359	38.3801

Table 6
Large Company Example

Line	E[N]	x = E[X]	c	γ	Default Value/ Liability	Surplus/ Liability
1	1,000,000.0	1000.0	0.010	10.000	0.0139%	37.4523%
2	1,000,000.0	1000.0	0.022	15.000	0.2967	49.3856
3	1,000,000.0	1000.0	0.040	20.000	0.6115	62.6747
Total	3,000,000.0	1000.0			0.3074	49.8375

Table 5 also uses $100M expected losses by line, but increases the severities by line—thereby increasing the inhomogeneity. This type of book, while more extreme than the first example, is still a realistic example. The failure of the adds-up theorem is more pronounced in this case, with the surplus allocation total being 24% lower than required. Line 1 is closer to homogeneous because γ is lower; line 3 is less homogeneous because γ is higher. These observations are reflected in the differences in surplus allocation.

Table 6 uses $1B expected losses by line, generating over 3M claim counts. Here the results are very close to homogeneous, as expected. The default value is 0.307%, very close to the homogeneous 0.311%, and the surplus total is 49.8% versus 50%. According to 2002 statutory annual statements, only 3% companies reported one or more line of business with more than $1B gross earned premium. Thus such large company/lines are the exception, and to the extent Myers Read is used for internal capital allocation, the scale will generally be at or below that used in Tables 4 and 5.

7. CONCLUSIONS

This paper has explained the importance of the homogeneity assumption in the derivation of Myers and Read's "adds-up" result. Proposition 1 shows that the assumption is necessary as well as sufficient. We have shown that aggregate loss distributions are not homogeneous, and have given examples to show that the inhomogeneity in a realistically sized loss portfolio will cause the adds-up result to materially fail. Thus the Myers Read allocation formula is not the panacea it seemed, and it will find little practical application in insurance companies. The methods introduced by Myers and Read can, however, be usefully applied to manage a company using constrained optimization, and maximizing return on marginal surplus. This is a more fruitful approach than trying to allocate capital, and it is discussed further in Meyers, Klinker, and Lalonde (2003).

APPENDIX 1

TWO TECHNICAL LEMMAS

Lemma 1

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a differentiable function of n variables. Then

$$x_1 \frac{\partial f}{\partial x_1} + x_2 \frac{\partial f}{\partial x_2} + \cdots + x_n \frac{\partial f}{\partial x_n} = 0$$

if and only if f is constant along rays from the origin.

Note: If f is constant on lines through the origin then f is called *homogeneous.* The lemma requires only that f be constant along rays from the origin; along a line f can change as the line passes through the origin. The function $x \mapsto x/|x|$ is a good example of what can occur: it changes value from $+1$ to -1 at zero. If f is constant along rays from the origin, then in half spaces through the origin f can be expressed as a function of x_i/x_j, $i = 1, \ldots, n$ when $x_j \neq 0$, for each j. In our applications of this lemma, the domain of f is the positive quadrant, and hence there is no difference between lines through the origin and rays from the origin in the domain. I would like to thank Christopher Monsour for pointing this out to me.

PROOF

Sufficiency: If f is constant along rays through the origin, then by the note we can assume locally that $f(x_1, \ldots, x_n) = \tilde{f}(x_1/x_n, \ldots, x_{n-1}/x_n)$ for some function $\tilde{f}$ of $n - 1$ variables. An easy calculation shows

$$x_1 \frac{\partial f}{\partial x_1} + \cdots + x_n \frac{\partial f}{\partial x_n} = \frac{x_1}{x_n}\tilde{f}_1 + \cdots + \frac{x_{n-1}}{x_n}\tilde{f}_{n-1} - x_n\left(\frac{x_1}{x_n^2}\tilde{f}_1 + \cdots + \frac{x_{n-1}}{x_n^2}\tilde{f}_{n-1}\right) = 0,$$

where $\tilde{f}_i = \partial\tilde{f}(x_1, \ldots, x_{n-1})/\partial x_i$.

Necessity: Let $\mathbf{v} = (x_1, \ldots, x_n)$ be a differentiable curve, so $\mathbf{v} = \mathbf{v}(t) : \mathbb{R} \to \mathbb{R}^n$, with $d\mathbf{v}/dt = \mathbf{v}$. This means $\mathbf{v}$ is equal to its own tangent vector for each t. By separating variables it is easy to see that $\mathbf{v}$ is a line through the origin. (It has the form $e^t(k_1, \ldots, k_n)$ for constants of integration k_i.) Then, by the chain rule

$$\frac{d}{dt} f(\mathbf{v}(t)) = x_1 \frac{\partial f}{\partial x_1} + \cdots + x_n \frac{\partial f}{\partial x_n} = 0,$$

by assumption, so the directional derivative of f along each half of any such line $\mathbf{v}$ is constant, that is, f is constant along rays from the origin, as required. Since $\mathbf{v}$ never reaches the origin, we cannot assert that f is constant along lines through the origin. □

Lemma 2

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a differentiable function of n variables. Then

$$x_1 \frac{\partial f}{\partial x_1} + \cdots + x_n \frac{\partial f}{\partial x_n} = f \qquad (15)$$

on a half space where $x_1 > 0$ (resp. $x_1 < 0$) if and only if there exists a differentiable function $\tilde{f}$ so that $f(x_1, \ldots, x_n) = x_1\tilde{f}(x_2/x_1, \ldots, x_n/x_1)$ on that half space, and similarly for $x_2, \ldots, x_n$.

PROOF

If $f(x_1, \ldots, x_n) = x_1\tilde{f}(x_2/x_1, \ldots, x_n/x_1)$, then, using subscripts on $\tilde{f}$ to denote partial derivatives,

$$x_1 \frac{\partial f}{\partial x_1} + \cdots + x_n \frac{\partial f}{\partial x_n} = \left(x_1\tilde{f} - \sum_{j=2}^{n} x_j\tilde{f}_{j-1}\right) + \sum_{j=2}^{n} x_j\tilde{f}_{j-1} = f.$$

The first sum comes from the partial derivative with respect to x_1 and the second sum comes from all the remaining partials.

On the other hand, suppose f satisfies equation (15) and let $\tilde{f}(t, s_2, \ldots, s_n) = f(t, s_2t, \ldots, s_nt)/t$ where $t > 0$ (resp. $t < 0$). We must show $\tilde{f}$ is independent of t. Differentiating

$$\frac{\partial}{\partial t}\left(\frac{f(t, s_2t, \ldots, s_nt)}{t}\right) = -\frac{1}{t^2}f + \frac{1}{t}\left(\frac{\partial f}{\partial x_1} + \sum_{j=2}^{n} s_j \frac{\partial f}{\partial x_j}\right) = 0$$

and the result follows. □

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Discussions on this paper can be submitted until October 1, 2004. The author reserves the right to reply to any discussion. Please see the Submission Guidelines for Authors on the inside back cover for instructions on the submission of discussions.

"A Note on the Myers and Read Capital Allocation Formula" Stephen J. Mildenhall, April 2004

HANS U. GERBER*

The purpose of this discussion is to point out some credit to my compatriot Leonhard Euler (1707–1783). In fact, the two Technical Lemmas in the Appendix of the paper are special cases of Euler's Homogeneous Function Theorem, which is available at http://mathworld.wolfram.com/EulersHomogeneousFunctionTheorem.html, as well as in several advanced calculus texts. Furthermore, note the usefulness of Euler's formula, for homogeneous functions, is explained by Cecil J. Nesbitt and Donald A. Jones, in their discussion of John C. Fraser (1962), and by Graham R. McDonald, in his discussion of Raymond L. Whaley (1974).

* Hans U. Gerber, ASA, PhD, is Professor of Actuarial Science, Ecole des hautes études commerciales, Université de Lausanne, CH-1015 Lausanne, Switzerland, e-mail: hgerber@hec.unil.ch.

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"Disruption of a Managed Competition Environment by Low-Ball Premium Bids: The Minnesota State Employees Group Insurance Program," Harry Sutton, Roger Feldman, and Bryan Dowd, April 2004

TIMOTHY M. ROSS*

Market characteristics will affect the degree of relevance to other markets. First, the Minnesota health plan market is a highly concentrated, near-oligopoly. Second, Minnesota health plans are statutorily non-profit, with relatively narrow limits on accumulated surplus. Third, the state health plan constitutes a significant share of the commercially insured market. Readers may find it useful if the authors could provide the market share of the state health plan.

It would be useful to solicit comments from the plan actuary.

The presumed goal of the managed competition was to control state health plan costs. A comparison of aggregated plan costs, with adjustment for plan design, to state or national trends over the same time period would be useful in understanding the effect of managed competition and the disruption.

On a broader note, the authors use "age/gender" factors taken from Milliman USA. This highlights the absence of SOA-sponsored experience studies providing demographic factors in the published literature. The SOA and the Health Section are to be commended for the research sponsored and published in recent years, but much work remains to be done.

As an editorial note, the word "gender" carries a cultural connotation, while "sex" refers to the purely biological (The American Heritage Dictionary 1997). Without enrollment data as to "gender," the actuary can only develop and use "age/sex" factors.

* Timothy M. Ross, ASA, MAAA, is proprietor of Ross Health Actuarial, 719 Crosby Drive, Hudson, WI 54016, e-mail: timross@rosshealthactuarial.com.

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