Financial Pricing Models for Property-Casualty Insurance Products: Retrospective Analysis

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Abstract

Insurance companies sell products without knowing what the ultimate costs will be. Moreover, in many cases the time between receipt of premium and the payment of claims could span many years. To manage the risk that companies have assumed, they need special tools to monitor the ongoing profitability of their products.

In this paper we introduce one tool—retrospective analysis—to monitor profitability. Retrospective analysis is mainly concerned with incorporating all available information about a block of business to determine the current estimate of profitability and to attribute any differences to the appropriate sources.

To illustrate the methods used in retrospective analysis we present a simplified but complete model of an insurance policy. We also introduce two economic accounting systems—the net present value system and the internal rate of return system—and analyze how emerging experience alters the profitability of our insurance policy. We pay special attention to quantifying the changes under both accounting systems.

1. Introduction

When an insurance policy is sold, its ultimate profitability is unknown because most of its cash flows will occur in the future. We estimate the ultimate profitability based on prior experience (from similar policies written in the past), policy characteristics, premium charged, amount of capital needed to support the policy, and other parameters. As claims are incurred and settled, the uncertainty surrounding the expected cash flows is resolved. At each point in time the actual cash flows replace projected cash flows. This additional information allows us to revise the assumptions for future periods and to recalculate future expected cash flows. The revised stream of cash flows gives us a new estimate of the profitability of our policy.

There are two sets of cash flows: those between the company and the policyholder and those between the company and its investors/equity holders. We take a broad view of the investors/equity holders of an insurance company to include stock, mutual, and privately owned companies. The relevant cash flows for profitability analysis are the ones between the equity holders and the insurance company. In this paper we refer to these flows of money as the *implied equity flows*.

We use retrospective analysis to quantify the economic value added (EVA) during an accounting period and to provide a revised estimate of ultimate profitability. One can think of EVA as the residual income that remains after all stake holders have been adequately compensated. When EVA is positive we say that the policy has created value, and when it is negative that it has destroyed value. An EVA

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of zero during an accounting period means that all stake holders were adequately compensated for this period and there was neither an increase nor a decrease in net worth.

The EVA during an accounting period depends on the accounting system used. Some accounting systems recognize the EVA in the early part of the policy's cash flows, others spread it evenly, and still others may show a strange pattern. Regardless of the reporting pattern the total EVA (on a present value basis) over the lifetime of the policy is invariant under any reasonable accounting system. In this paper we introduce two economic accounting systems—the net present value (NPV) and the internal rate of return (IRR) systems—and present the methods necessary to quantify the changes in EVA that occur in each accounting period. In this paper we discuss these economic accounting systems only (except where otherwise noted). These two accounting systems are distinct from the conventional statutory accounting (SAP), generally accepted accounting principles (GAAP), or tax (TAX) accounting. In particular, the NPV and IRR accounting systems consider the cost of holding capital, whereas none of the conventional systems (SAP, GAAP, TAX) do. Differences between these systems are documented and illustrated in Feldblum and Thandi (2003).

The main feature of the NPV accounting system is that the expected EVA of the insurance policy is shown at the time the policy is sold. In later periods this estimate is revised to take into account the emerging experience. In contrast, the IRR accounting system spreads the expected EVA for the policy evenly over its expected lifetime. Quantifying changes in this accounting system is more complex.

The paper is organized as follows: Section 2 presents the details of the insurance policy used to illustrate the paper. We describe the assumptions and transactions in detail. The next section ''The Underlying Model'' presents the model used in the main illustrations. All the formulas and input parameters are discussed so the reader can implement the model on a spreadsheet. One must be careful with the implementation of the model. As one replaces expected results with actual results all prior calculations must not be allowed to change.

Section 4 introduces the EVA performance measurement metric. This section also introduces two economic accounting systems: the NPV and IRR systems. We use these systems to illustrate the concept of EVA. In Section 5 we calculate the changes to the EVA as experience in the underlying policy emerges.

Sections 6 and 7 show how the implied equity flows respond to changes in investment returns and incurred losses.[1] Finally, Section 8 shows how to quantify the changes in EVA for the IRR accounting system.

2. THE UNDERLYING INSURANCE POLICY

The insurance policy used to illustrate the methods of this paper has the following characteristics:

Premium and expenses: The policy is written on December 31, 20XX. Premium of $1,000 is collected, and acquisition expenses of $275 are paid on that date. General expenses of $150 are paid in full six months later.[2]

Losses and capital requirements: A single accident occurs during 20XX + 1 with a single payment of $650 expected to be made three years from policy inception. For the illustrations of this paper we have chosen to model surplus requirements as 25% of the unearned premium reserve plus 15% of held loss reserves.

Surplus/capital requirements for a company are strongly influenced by outside entities (NAIC RBC requirements to some degree and the rating agencies). The amount of capital allocated to a book of business will have a profound impact on its profitability. We have chosen our surplus rule to model the

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[1] The implied equity flows are the flows of money between the company and its investors. In the literature these flows are also known as distributable earnings or free cash flow.

[2] In practice we rarely collect the full premium or pay the acquisition expenses on the effective date of the policy. For ease of presentation we will assume that there is no difference between cash flows and income statement impacts. For a full discussion of these issues and how they affect the implied equity flows see Feldblum and Thandi (2003).

capital requirements roughly and is not meant as a guideline for actual allocation of company surplus to any book of business.

The determination of total company capital and its allocation to strategic business unit and/or line of business is a complex matter that is outside the scope of this paper. There has been ample discussion in the literature of the issues, and no consensus has been reached. On the one hand, Merton and Perold (2001), Phillips, Cummins, and Allen (1998), and others have concluded that the allocation of the total company capital amount to each line of business could be inappropriate. On the other hand, Myers and Read (2001), and Kreps (2006) show how to allocate the entire capital amount uniquely to each line of business. The reader should consult Venter (2004) and Dhaene, Goovaerts, and Kaas (2003) for excellent surveys on capital allocation methods and the references therein.

Investment yield: The selection of the investment yield depends on the cost of capital. If the selected cost of capital compensates the equity holders for both insurance and investment risk, then the investment yield should be the pretax yield expected over the lifetime of the block of business. If the cost of capital compensates only for insurance risk, then the investment yield should be a risk-free rate.

For our illustrative policy we assume that the pretax yield expected over the lifetime of our policy is equal to 8.00% per annum on a bond equivalent basis. This is 4% each half year, or an 8.16% effective annual return.

Federal income taxes: The marginal tax rate for underwriting and investment income is 35%.[3] The Internal Revenue Service (IRS) loss reserve discount factor for accident year 20XX + 1 is 0.86 at 12 months (Dec. 31, 20XX + 1), 0.88 at 24 months (Dec. 31, 20XX + 2), and 0.90 at 36 months (Dec. 31, 20XX + 3) and thereafter.

The tax basis loss reserves are the held loss reserves times the IRS loss reserve discount factors. The tax basis loss reserves are used to compute taxable underwriting income and the deferred tax asset stemming from loss reserve discounting.

Cost of capital: Cummins and Harrington (1985), Fairley (1979), and Hill (1979) among others have tried to separate the underwriting and investment risks for property-casualty insurance companies. Kozik (1994) has reviewed the literature and concluded that ''reliable estimates of underwriting betas do not exist.''

We assume that the cost of capital reflects all risk to the insurance company, and for purposes of the illustrations in this paper we have selected a rate of 5% per half year, or 10.25% per annum.

ILLUSTRATION 1

Taxable underwriting income—as of December 31, 20XX—is given by[4]

	Item	Amount
	Written Premium	$1,000
less	80% of Change in UEPR	800
less	Acquisition Expenses	275
less	Paid Losses	0
less	Change in Tax Loss Reserves	0
	Taxable Underwriting Income	−$75

[3] In practice, the marginal tax rate for underwriting and investment income might differ, but we have chosen the same rate as a simplifying assumption. Extending the model to use two different (and perhaps time-dependent) rates is straightforward.

[4] In this illustration we are assuming that acquisition expenses are fully incurred for tax purposes on December 31, 20XX.

At policy inception no premium has been earned. The unearned premium reserve (UEPR) is \$1,000, and the tax basis loss reserves are zero. The factor of 80% for the change in the UEPR is the revenue offset provision of the 1986 Tax Reform Act.

The deferred tax asset due to loss reserve discounting equals the tax rate times the amount of the loss reserve discount that reverses within the next 12 months. Algebraically we have

$$\text{DTA}_t^{\text{RD}} = \text{Tax Rate} \times \{\underbrace{(\text{HR}_t - \text{TaxRes}_t)}_{\text{discount at time } t} - \underbrace{(\text{HR}_{t+1} - \text{TaxRes}_{t+1})}_{\text{discount at time } t+1}\},$$

where DTA_t^{RD} stands for the deferred tax asset due to loss reserve discounting at valuation date t, HR is the held loss reserve, and TaxRes is the IRS tax basis loss reserve.[5] The subscript $t + 1$ represents 12 months after time t.

For example, at the December 31, 20XX+1 valuation the held loss reserves are \$650. The expected loss reserves one year later are still at \$650 because we expect to pay the entire loss at year end 20XX + 3. The tax basis loss reserves are

Date	Held Loss Reserves	Tax Loss Reserves
12/31/20XX + 1	\$650	\$650 × 0.86 = \$559
12/31/20XX + 2	\$650	\$650 × 0.88 = \$572

The deferred tax asset due to loss reserve discounting on December 31, 20XX + 1 is

$$35\% \times \{(650 - 559) - (650 - 572)\} = 4.55. \qquad \square$$

The return on capital provided by the insurance policy is determined from the implied equity flows. These flows represent the capital contributions necessary from the suppliers of capital and the capital distributions the company can make to them. These implied equity flows are analogous to the concept of *free cash flow* in the general financial literature or *distributable earnings* in the life insurance industry (Atkinson and Dallas 2000, p. 972).

To determine the implied equity flows we model underwriting and investment income, asset requirements, taxes, and deferred tax assets. For the illustration in this paper we use a discrete model with semiannual valuations. Period 1 spans the interval between time 0 and time 1/2; period 2 spans the interval between time 1/2 and time 1; and so forth. The valuation period denoted by an asterisk is an infinitesimally small period right before the start of period 1. This period shows the amount required to fund the policy.

Premiums and acquisition expenses are paid at the start of the period. General expenses, which are incurred evenly over the policy term, are modeled as a single payment six months after policy inception. Investment income is earned at the end of each period based on the income-producing assets at the start of the period. Loss reserves are set at the end of each period.

Taxable underwriting income is calculated at the end of each calendar year. Corporate taxpayers pay taxes periodically over the course of the tax year. For the discrete illustration here, taxes on underwriting income are paid at the middle of the year and at the end of the year. The midyear tax payment is half the estimated annual taxes on underwriting income. The end-of-year-tax payment is equal to the reestimated tax liability minus the payment made six months earlier. Taxable investment income is computed every six months, and taxes on this income are paid each period.

[5] For a complete explanation of the deferred tax assets and liabilities related to property-casualty insurance contracts, see Feldblum and Thandi (2003).

One may view the implied equity flows from either a *balance sheet* or an *income statement* perspective. The balance sheet perspective determines the implied equity flows as the difference between available assets and required assets at a valuation date. The available assets are equal to the previous valuation's required assets plus the net cash flows during the period plus the change in the deferred tax asset, that is,

$$\text{Available Assets}_t = \text{Required Assets}_{t-1} + \text{Net Cash Flow}_t + \Delta\text{DTA}_t,$$

where Net Cash Flow$_t$ represents the cash flows between the valuation dates $t - 1$ and t, and DTA stands for the deferred tax asset. The required assets are equal to the sum of the unearned premium reserve, held loss reserve, and surplus required at the valuation date t.

From an income statement perspective the implied equity flows at a valuation date t are equal to the difference between the income during the period from $t - 1$ to t and the change in the required surplus during the period. Note that a direct charge or credit to surplus is treated as a component of income.

ILLUSTRATION 2

Balance Sheet Perspective: The assets required depend on the surplus and reserve requirements. The reserve requirements stem from unearned premium and incurred losses that have not yet been paid. At policy inception, the unearned premium reserve is \$1,000, and the held loss reserves are zero. The surplus requirements are 25% of the unearned premium reserve plus 15% of held loss reserves; hence, the surplus is \$1,000 × 25% + \$0 × 15% = \$250. Therefore, the assets required at time zero are

$$\underbrace{1{,}000}_{\substack{\text{Unearned}\\\text{premium}\\\text{reserve}}} + \underbrace{0}_{\substack{\text{Held loss}\\\text{reserve}}} + \underbrace{250}_{\substack{\text{Surplus}\\\text{requirement}}} = 1{,}250.$$

The net cash flows are \$1,000 of collected premium less \$275 of acquisition expenses equals \$725. At policy inception we have no investment income and the taxable underwriting income is −\$75 (see Illustration 1), taxable investment income is zero, and the assumed tax rate is 35%; therefore, taxes are −(−\$75 × 35%) = \$26.25.[6]

Deferred tax assets arise from two sources: the revenue offset provision and loss reserve discounting. At time zero, no losses have yet been incurred, and the deferred tax asset due to loss reserve discounting is zero. The deferred tax asset due to the revenue offset provision is equal to 20% of the change in the unearned premium reserve times the tax rate, or 20% × \$1,000 × 35% = \$70. The total deferred tax asset is \$70 + \$0 = \$70.

Therefore, the available assets are equal to

$$\underbrace{0}_{\substack{\text{Previous}\\\text{required}\\\text{assets}}} + \underbrace{1{,}000 - 275 + 26.25}_{\text{Net cash flow}} + \underbrace{70}_{\substack{\text{Change}\\\text{in DTA}}} = 821.25.$$

The implied equity flow at time zero is

$$\underbrace{821.25}_{\substack{\text{Available}\\\text{Assets}}} - \underbrace{1{,}250.00}_{\substack{\text{Required}\\\text{Assets}}} = -428.75.$$

[6] The sign convention is as follows: payments from the company to the IRS are negative amounts, and refunds from the IRS to the company are positive amounts.

Table 1
Income at Policy Inception

	Item	Amount
	Written Premium	$1,000.00
less	ΔUnearned Premium Reserve	1,000.00
less	Expenses	275.00
less	Loss + ALAE	0.00
less	ΔHeld Loss Reserve	0.00
plus	Investment Income	0.00
plus	Taxes	−26.25
plus	ΔDeferred Tax Asset	70.00
	Income	−$178.75

Income Statement Perspective: In this case the implied equity flows are equal to the income during the period less the change in the surplus requirement. At time zero the change in surplus requirement is equal to $250. Income earned is equal to −$178.75 (see Table 1). Hence, the implied equity flow at time zero is

$$-178.75 - 250.00 = -428.75. \qquad \square$$

The implied equity flows along with the major components (from a balance sheet perspective) are shown in Table 2. The first entry (−$428.75 in col. [7]) represents a capital contribution from the equity holders to the company. The subsequent (positive) entries represent capital distributions from the company to the equity holders.

The internal rate of return of the implied equity flows is 12.75% per annum, or 6.18% each half year. The net present value of the implied equity flows at the 10.25% (per annum) cost of equity capital is $14.02 (see Table 2 col. [7]). Since the internal rate of return exceeds the cost of equity capital or equivalently the net present value is greater than zero, the insurance policy is expected to be profitable.

The equity holders expect a return exceeding their cost of capital. The net present value of $14.02 represents the increase in net worth. The implied equity flows shown in Table 2 are the starting point

Table 2
Underlying Policy's Implied Equity Flows

Time (1)	UW Cash Flow (2)	Investment Income (3)	Asset Flow (4)	Taxes (5)	Deferred Tax Asset Flow (6)	Implied Equity Flow (7)
0.0	725.00	0.00	1,250.00	26.25	70.00	−428.75
0.5	−150.00	47.20	−251.25	−32.45	−32.73	83.28
1.0	0.00	38.46	−251.25	−29.39	−32.73	227.60
1.5	0.00	29.72	0.00	−8.13	11.38	32.97
2.0	0.00	29.26	0.00	−7.97	11.38	32.67
2.5	0.00	28.81	0.00	3.57	−13.65	18.73
3.0	−650.00	29.35	−747.50	3.38	−13.65	116.58
3.5	0.00	0.00	0.00	0.00	0.00	0.00
Sum	−75.00	202.80	0.00	−44.74	0.00	**83.08**
NPV (at cost of capital)						**14.02**

For this exhibit assume that columns (2)–(6) are given.
The defining equations are in Section 3.1:
(2) = eq. (3.3)
(3) = eq. (3.4)
(4) = eq. (3.2)
(5) = eq. (3.5)
(6) = eq. (3.6)
(7) = −[(4) − (2) − (3) − (5) − (6)] or eq. (3.1).

of the retrospective analysis. At any point in time, the entries *before* this point are known, and the entries *after* this point are *expected values*. As time goes by, the uncertainty surrounding the implied equity flows is resolved. Therefore, until all cash flows are known, we speak about *expected profitability*.

At the end of each period we incorporate all available information and recompute the expected future implied equity flows. Deviations from previous estimates are recognized in the current valuation period. Retrospective analysis is the means to quantify these changes.

3. THE UNDERLYING MODEL

In this section we introduce the model and the inputs used to generate the main examples presented later. This model is a simplified version of a comprehensive pricing model documented in Feldblum and Thandi (2003). The reader can re-create the model on a spreadsheet based on the formulas presented below. We should point out that many formulas are different depending on whether the valuation date is at year-end or midyear.

Also, some calculations, like the deferred tax asset due to loss reserve discounting or taxable income at a midyear valuation, depend on the expected value at the next valuation (e.g., see eq. [3.19]). When analyzing the effect of various events on the implied equity flows and the EVA stream one must be careful of the model implementation. In particular, as we cycle through (1) replacing expectations at time t with actual experience, (2) revising (if necessary) our expectations at times greater than t, and (3) adjusting the EVA stream at time t, we must make sure that any calculations at time t or prior are frozen; that is, they are not allowed to change even if we change our expectations on future time periods.

To keep the model simple, we will assume that cash flows and income impacts occur simultaneously. For example, when a company writes a policy with a written premium of $1,000 we assume, for modeling purposes, that the entire premium amount is collected immediately. In practice this is usually not the case. The policy premium might be collected over the term of the policy in equal installments. Although this might seem an oversimplification, it is not. In the case where the entire premium is not collected up front, the company shows a premium receivable for the uncollected amount. If this receivable is an admitted asset in the statutory balance sheet, then there is no effect on the implied equity flows. If the receivable is not an admitted asset, then the implied equity flow increases.

3.1 Defining Equations

The main purpose of the model is to compute the implied equity flows. Therefore, our description will start there and move backwards through the formulas. In this model we shall assume that cash flows and the income impacts occur at the same time.

The implied equity flow (EF) depends on the following five flows:

- Asset flow: AF
- Underwriting Cash flow: CF
- Investment Income flow: IIF
- Tax flow: TAXF
- Deferred Tax Asset flow: DTAF

as follows:

$$\mathrm{EF}_t = -(\mathrm{AF}_t - \mathrm{CF}_t - \mathrm{IIF}_t - \mathrm{DTAF}_t - \mathrm{TAXF}_t). \tag{3.1}$$

A negative sign implies that the equity holders provide money to the insurance company, and a positive sign means that the equity holders receive money from the company.

The asset flow (AF) is defined as the change in held assets (HA) between periods:

$$\mathrm{AF}_t = \mathrm{HA}_t - \mathrm{HA}_{t-1/2}. \tag{3.2}$$

The underwriting cash flow (CF) is composed of the written premium (WP), acquisition expenses (AE), general expenses (GE), and paid losses (PL).[7]

$$\mathrm{CF}_t = \mathrm{WP}_t - \mathrm{AE}_t - \mathrm{GE}_t - \mathrm{PL}_t. \tag{3.3}$$

The investment income flow (IIF) is equal to the income-producing assets at the *beginning* of the period times the benchmark investment rate of return (IR):

$$\mathrm{IIF}_t = \mathrm{IP}_{t-1/2} \times \mathrm{IR}_t. \tag{3.4}$$

The tax flow (TAXF) is the sum of the taxes paid on underwriting income (UWT) and investment income (IIT):

$$\mathrm{TAXF}_t = -(\mathrm{UWT}_t + \mathrm{IIT}_t). \tag{3.5}$$

The last flow item is the deferred tax asset flow (DTAF). Similar to the held asset flow, it is equal to the change in deferred tax asset (DTA) between adjacent periods:

$$\mathrm{DTAF}_t = \mathrm{DTA}_t - \mathrm{DTA}_{t-1/2}. \tag{3.6}$$

The assets the insurance company is required to hold, that is, the held assets (HA) are equal to the sum of the unearned premium reserve (UEPR), the held loss reserve (HR), and surplus (SP):

$$\mathrm{HA}_t = \mathrm{UEPR}_t + \mathrm{HR}_t + \mathrm{SP}_t. \tag{3.7}$$

The income-producing assets (IP) are equal to the held assets (HA) less the non-income-producing assets (NIP):

$$\mathrm{IP}_t = \mathrm{HA}_t - \mathrm{NIP}_t. \tag{3.8}$$

Non-income-producing assets (NIP) are equal to the total deferred tax assets, that is,[8]

$$\mathrm{NIP}_t = \mathrm{DTA}_t. \tag{3.9}$$

For the underwriting tax paid (UWT), we need to consider two cases: midyear valuations and year-end valuations. Taxable income is always calculated as of year-end, but in our model the company pays taxes semiannually. So at a midyear valuation we estimate the taxes for the whole year and pay only half of it. At year-end we recalculate the taxes for the whole year and subtract the taxes that we already paid at midyear:

$$\mathrm{UWT}_t = \begin{cases} \mathrm{UWTI}_{t+1/2} \times \mathrm{TR}_t/2 & \text{if } t \text{ is a midyear valuation} \\ \mathrm{UWTI}_t \times \mathrm{TR}_t - \mathrm{UWT}_{t-1/2} & \text{if } t \text{ is a year-end valuation.} \end{cases} \tag{3.10}$$

Taxable underwriting income (UWTI) is calculated only at year-end. Its components are written premium (WP), acquisition and general expenses (AE and GE respectively), paid losses (PL), the change in tax reserves (TaxRes), and 80% of the change in the unearned premium reserve (UEPR) (see the second and last summands in the next formula):

$$\begin{aligned} \mathrm{UWTI}_t = {} & \mathrm{WP}_t - (\mathrm{UEPR}_t - \mathrm{UEPR}_{t-1}) - (\mathrm{AE}_t + \mathrm{AE}_{t-1/2}) - (\mathrm{GE}_t + \mathrm{GE}_{t-1/2}) \\ & - (\mathrm{PL}_t + \mathrm{PL}_{t-1/2}) - (\mathrm{TaxRes}_t - \mathrm{TaxRes}_{t-1}) + 20\% \times (\mathrm{UEPR}_t - \mathrm{UEPR}_{t-1}). \end{aligned} \tag{3.11}$$

Tax reserves are proportional to the held loss reserves of the company. The proportionality factor is given by IRS regulations, and we denote it by IRSf. Also, tax reserves are computed only at year-end:

[7] Within paid losses we include allocated loss adjustment expenses (ALAE).

[8] For ease of presentation we have not included other non-income-producing assets (e.g., premium receivables) in our model.

$$\text{TaxRes}_t = \text{HR}_t \times \text{IRSf}_t \text{ only if } t \text{ is a year-end valuation.} \quad (3.12)$$

The nominal loss reserves (NR) are equal to the sum of all *future paid losses* corresponding to premium earned to date. At time $t = 0$ the nominal loss reserves are zero because our policy was issued on the last day of the year; therefore, as of this valuation date, we have not yet earned any premium. Also, at the first midyear valuation we have earned half of our premium, and so our nominal loss reserves are equal to half of the sum of all future losses:

$$\text{NR}_t = \begin{cases} 0 & \text{if } t = 0, \\ (\Sigma_{k>t} \text{PL}_k)/2 & \text{if } t = 1/2 \\ \Sigma_{k>t} \text{PL}_k & \text{otherwise.} \end{cases} \quad (3.13)$$

Held loss reserves (HR) are equal to the reserve adequacy ratio (rar) times the nominal loss reserves:

$$\text{HR}_t = \text{rar}_t \times \text{NR}_t. \quad (3.14)$$

At policy inception the unearned premium reserve (UEPR) is equal to the written premium (WP) and decreases linearly over the term of the policy. Hence, at midyear we will have only half of the written premium in the unearned premium reserve, and at year-end it will diminish to zero:[9]

$$\text{UEPR}_t = \begin{cases} \text{WP}_t & \text{if } t = 0, \\ \text{WP}_{t-1/2}/2 & \text{if } t = 1/2, \\ 0 & \text{otherwise.} \end{cases} \quad (3.15)$$

We have chosen to model the surplus requirements (SP) for our policy as a percentage of the unearned premium reserve (UEPR) plus a factor times the held loss reserves (HR). Thus at valuation time t we have

$$\text{SP}_t = \text{sur}_t \times \text{UEPR}_t + \text{srr}_t \times \text{HR}_t, \quad (3.16)$$

where sur is the surplus to unearned premium reserve factor and srr is the surplus to held loss reserves factor.

The investment taxes are equal to the tax rate (TR),[10] times the investment income generated over the period (IIF):

$$\text{IIT}_t = \text{TR}_t \times \text{IIF}_t. \quad (3.17)$$

The deferred tax asset due to loss reserve discounting (DTA^{RD}) is equal to the reserve discount that reverses *within the next twelve months* times the tax rate:

$$\text{DTA}_t^{\text{RD}} = \{(\text{HR}_t - \text{HR}_{t+1}) - (\text{TaxRes}_t - \text{TaxRes}_{t+1})\} \times \text{TR}_t. \quad (3.18)$$

This definition is valid only at year-end valuations where $t > 0$. For midyear valuations we take the average of the deferred tax assets due to loss reserve discounting at the previous and next year-end valuations, that is,

$$\text{DTA}_t^{\text{RD}} = (\text{DTA}_{t-1/2}^{\text{RD}} + \text{DTA}_{t+1/2}^{\text{RD}})/2. \quad (3.19)$$

The initial value—at time $t = 0$—is set equal to zero.

The deferred tax asset due to the revenue offset (DTA^{RO}) is equal to 20% times the unearned premium reserve (UEPR) times the tax rate. IRS regulations mandate the factor of 20%:

$$\text{DTA}_t^{\text{RO}} = 20\% \times \text{UEPR}_t \times \text{TR}_t. \quad (3.20)$$

Finally, the total deferred tax asset (DTA) is equal to the sum of the revenue offset (DTA^{RO}) and the reserve discounting (DTA^{RD}):

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[9] We are implicitly assuming that our policies are one-year contracts.

[10] Although the income tax rate on investment income often is different from that on underwriting income, we are using the same tax rate for both. The model can easily handle a different tax rate on investment income.

Table 3
Initial Model Parameters

Time	IR	TR	rar	sur	srr	IRSf
0.0	4.00%	35.00%	100.00%	25.00%	15.00%	0.00
0.5	4.00	35.00	100.00	25.00	15.00	
1.0	4.00	35.00	100.00	25.00	15.00	0.86
1.5	4.00	35.00	100.00	25.00	15.00	
2.0	4.00	35.00	100.00	25.00	15.00	0.88
2.5	4.00	35.00	100.00	25.00	15.00	
3.0	4.00	35.00	100.00	25.00	15.00	0.90
3.5	4.00	35.00	100.00	25.00	15.00	

$$\mathrm{DTA}_t = \mathrm{DTA}_t^{\mathrm{RO}} + \mathrm{DTA}_t^{\mathrm{RD}}. \tag{3.21}$$

3.2 Initial Parameters and Variable Names

The initial parameters for the illustrations used in the paper are summarized below:

Time	WP	AE	GE	PL
0.0	1,000.00	275.00	0.00	0.00
0.5	0.00	0.00	150.00	0.00
1.0	0.00	0.00	0.00	0.00
1.5	0.00	0.00	0.00	0.00
2.0	0.00	0.00	0.00	0.00
2.5	0.00	0.00	0.00	0.00
3.0	0.00	0.00	0.00	650.00
3.5	0.00	0.00	0.00	0.00

Table 3 summarizes the investment rate of return per six-month period, the marginal tax rate, various ratios, and the IRS discount factors.

Finally, all variable names are summarized along with their meaning:

AE	Acquisition expenses
AF	Asset flow
CF	Underwriting cash flow
DTA	Total deferred tax asset
$\mathrm{DTA}^{\mathrm{RD}}$	Deferred tax asset due to loss reserve discounting
$\mathrm{DTA}^{\mathrm{RO}}$	Deferred tax asset due to revenue offset
DTAF	Deferred tax asset flow
EF	Implied equity flow
GE	General expenses
HA	Held assets
HR	Held loss reserve
IIF	Investment income flow
IRSf	IRS discount factor
IIT	Investment income tax
IP	Income-producing assets
IR	Investment rate of return
NIP	Non-income producing assets
NR	Nominal loss reserve

PL	Paid losses
rar	Reserve adequacy ratio
SP	Surplus
srr	Surplus to held loss reserve ratio
sur	Surplus to unearned premium reserve ratio
TAXF	Tax flow
TaxRes	Tax reserve
TR	Marginal tax rate
UEPR	Unearned premium reserve
UWT	Underwriting tax paid
UWTI	Underwriting taxable income
WP	Written premium

4. ACCOUNTING SYSTEMS AND EVA

The stream of implied equity flows reflects profit or loss embedded in the underlying insurance policy. For our illustration, this stream is

$$(-428.75) + 83.28 + 227.60 + 32.97 + 32.67 + 18.73 + 116.58 = 83.08.$$

The expected profit is realized over three years. The incidence of profits, as well as the EVA in each valuation period, depends on the accounting system.

For example, GAAP and SAP defer the recognition of income. Tax accounting, fair value accounting, and net present value accounting (introduced below) recognize income up-front. The internal rate of return accounting (also introduced below) recognizes income throughout the life of the policy at a constant percentage of the capital used.

In all of these accounting systems the stream of EVA will be different, but the NPV, at the cost of capital, of this stream will be invariant across these systems and equal to the NPV of the implied equity flows. In essence, the rules of a particular accounting system move income from period to period, but they cannot change the value added or destroyed over the lifetime of the policy.

The EVA during an accounting period measures the net worth created or destroyed. EVA is defined as the after-tax net income minus the dollar cost of capital, where the dollar cost of capital is the required return on capital times the amount of capital held:

$$\text{EVA}_t = \text{after-tax net income}_t - \text{cost of capital} \times \text{capital held}_t.$$

ILLUSTRATION 3

Suppose that the after-tax net income for the current period is \$2 million and that the amount of capital at the start of the period is \$15 million. If the cost of capital for the company is 10%, then the EVA for the period is

$$2 - 10\% \times 15 = 0.5 \text{ million}.$$

If the cost of capital is 20%, then the EVA is −\$1 million. □

One may think of EVA as the residual income left after subtracting the cost of the capital used to support the operations.

4.1 The NPV Accounting System

The NPV accounting system shows the total EVA of the insurance contract when the business decision is made, that is, at policy inception. At subsequent valuation dates, the expected EVA is zero.

Under this accounting system the after-tax net income at inception of the contract is equal to the NPV, calculated at the cost of capital, of the implied equity flows (see Table 4, col. [3]). At other

Table 4
NPV Accounting System

Time (1)	Implied Equity Flow (2)	After-Tax Net Income (3)	Capital Contribution (4)	Equity Fund (5)	Return (6)	EVA (7)
*			428.75	428.75		
0.0	−428.75	14.02	14.02	442.77	3.27%	14.02
0.5	83.28	22.14	−61.14	381.63	5.00	0.00
1.0	227.60	19.08	−208.52	173.12	5.00	0.00
1.5	32.97	8.66	−24.31	148.81	5.00	0.00
2.0	32.67	7.44	−25.23	123.58	5.00	0.00
2.5	18.73	6.18	−12.55	111.03	5.00	0.00
3.0	116.58	5.55	−111.03	0.00	5.00	0.00
3.5	0.00	0.00	0.00	0.00		0.00
Sum	83.08	83.08				
NPV	14.02					14.02

* Time just before inception of the contract.
(2) Implied equity flows.
$(3)_{0.0}$ = net present value of col. (2) at the Cost of Capital. For $t > 0$ we have $(3)_t = (5)_{t-1/2} \times$ Cost of Capital.
$(4)_* = -(2)_{0.0}$, $(4)_{0.0} = (3)_{0.0}$, and for $t > 0.0$ we have $(4)_t = (3)_t - (2)_t$.
$(5)_* = -(2)_{0.0}$ and for $t \geq 0.0$ we have $(5)_t = (5)_{t-1/2} + (4)_t$.
$(6)_t = (3)_t/(5)_{t-1/2}$.
$(7)_{0.0} = (3)_{0.0}$, and for $t > 0.0$ we have $(7)_t = (3)_t - (5)_{t-1/2} \times$ Cost of Capital.

valuation dates the after-tax net income is equal to the equity fund at the start of the period times the cost of capital. Hence, at future valuation dates the return in column (6) is equal to the cost of capital.

Column (5) shows the total amount of capital under this accounting system. The period labeled with an asterisk represents the time right before policy inception, when equity holders must contribute funds to meet the asset requirements. The EVA (col. [7]) at policy inception is equal to the net present value, at the cost of capital, of the implied equity flows. During subsequent accounting periods, the expected EVA is zero.

As time goes by, actual results replace expected results, and the actual EVA in each period is likely to change from its expected value. In fact, only if actual experience matches expectations, then the EVA will be zero.

4.2 The IRR Accounting System

The IRR accounting system shows a constant rate of return throughout the life of the policy. The return is measured as the after-tax net income divided by the amount of capital at the start of the period (see Table 5). A similar approach is discussed in Bingham (1993a,b) and expanded in Bender (1997):

- The after-tax net income at policy inception is zero. At subsequent valuation dates, the after-tax net income is equal to the IRR of the implied equity flows times the equity fund at the beginning of the period. The return shown in column (6) is constant and equal to the IRR of the implied equity flows.
- The EVA at inception of the contract is zero. For any other period, the EVA is the after-tax net income (col. [3] of Table 5) less the cost of capital times the amount of capital employed (col. [5] of Table 5) under this accounting system.[11]

ILLUSTRATION 4

Suppose we want to compute the after-tax net income and the EVA at time 1.0. From Table 5 we know that the amount of equity at time 0.5 is equal to $371.98. The internal rate of return of the implied

[11] There is an equivalent definition of EVA highlighting the difference between the cost of capital and the return generated under the accounting system. EVA is equal to the difference between the return (col. [3] of Table 5) and the cost of capital times the amount of capital used by the accounting system. One can see the equivalence of these definitions by unwinding the recursive definitions given in Table 5. See also Schirmacher (2002).

Table 5
IRR Accounting System

Time (1)	Implied Equity Flow (2)	After-Tax Net Income (3)	Capital Contribution (4)	Equity Fund (5)	Return (6)	EVA (7)
*			428.75	428.75		
0.0	−428.75	0.00	0.00	428.75	0.00%	0.00
0.5	83.28	26.51	−56.77	371.98	6.18	5.07
1.0	227.60	23.00	−204.60	167.38	6.18	4.40
1.5	32.97	10.35	−22.62	144.76	6.18	1.98
2.0	32.67	8.95	−23.72	121.03	6.18	1.71
2.5	18.73	7.48	−11.24	109.79	6.18	1.43
3.0	116.58	6.79	−109.79	0.00	6.18	1.30
3.5	0.00	0.00	0.00	0.00		0.00
Sum	83.03	83.03				
NPV	14.02					14.02

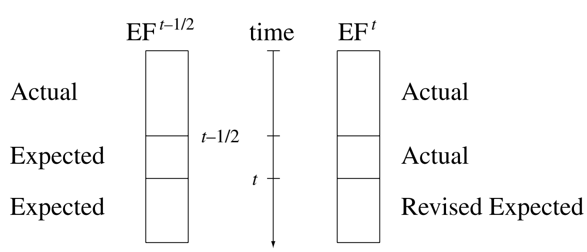
* Time just before inception of the contract.
(2) Implied equity flows.
$(3)_{0.0} = 0$. For $t > 0$ we have $(3)_t = (5)_{t-1/2} \times$ the internal rate of return of col. (2).
$(4)_* = -(2)_{0.0}$, $(4)_{0.0} = (3)_{0.0}$, and for $t > 0.0$ we have $(4)_t = (3)_t - (2)_t$.
$(5)_* = -(2)_{0.0}$ and for $t \geq 0.0$ we have $(5)_t = (5)_{t-1/2} + (4)_t$.
$(6)_t = (3)_t/(5)_{t-1/2}$.
$(7)_{0.0} = (3)_{0.0}$, and for $t > 0.0$ we have $(7)_t = (3)_t - (5)_{t-1/2} \times$ Cost of Capital.

equity flows is 6.18%. The after-tax net income at time 1.0 is equal to \$371.98 × 6.18% = \$23.00.[12] The EVA is then equal to \$23.00 − \$371.98 × 5% = \$4.40. □

5. QUANTIFYING CHANGES IN EVA

Changes in the implied equity flows affect the stream of EVA.[13] As time goes by, actual cash flows replace projected cash flows, and the estimates of future cash flows are updated. In the illustration here, we incorporate all available information at each valuation date. Changes in the implied equity flows are reflected in changes in the EVA stream.

To calculate the effect on EVA, we compare the stream of implied equity flows from one valuation date to the next. As a convenient way of referring to the entire stream of implied equity flows at different valuation dates, we use EF^t to represent the entire stream viewed as of the valuation date t. All entries *before* the valuation date (including the entry at the valuation date) are actual numbers, and the remaining entries are expected quantities. The following diagram shows schematically the implied equity flows $EF^{t-1/2}$ and EF^t:



[12] We used a higher precision in the calculation than that displayed here.
[13] This section is similar to source of earnings analysis used by life insurance companies; see Tan (1989), Eckman (1990), and Feldblum (2006, appendix).

Table 6
Implied Equity Flows at Valuation Dates 0.5 and 1.0

Time	Equity Flows	
	$EF^{0.5}$	$EF^{1.0}$
0.0	−428.75	−428.75
0.5	83.28	83.28
1.0	227.60	*224.47*
1.5	32.97	*30.55*
2.0	32.67	*30.29*
2.5	18.73	*16.38*
3.0	116.58	*114.20*
3.5	0.00	0.00
Sum	83.08	70.43
NPV	14.02	3.53
IRR	6.18%	5.30%

We should point out that the initial segment of the streams $EF^{t-1/2}$ and EF^t that are *both* labeled *actual* have identical entries. Therefore, when we compare both streams these segments cancel each other out, and we need only concentrate on the remaining entries.

Our discussion will be centered around the two accounting systems introduced in the previous section. Table 6 shows the implied equity flows $EF^{0.5}$ and $EF^{1.0}$. The first two entries for both streams are identical, and the remaining entries differ. The difference in the implied equity flows stems from a drop in investment income. At inception of the contract we assumed an investment yield of 4% per period. At time 1.0 we changed it to 3.5% for the current and future periods. The table also shows the nominal sum of all entries, the NPV (at the cost of capital of 5% per period), and the IRR for both streams. With this information we can begin our analysis.

5.1 The NPV Perspective

The NPV perspective is the simplest setting for quantifying changes in the EVA stream. Any change to the implied equity flows between two valuation dates will be fully reflected during the current accounting period. This applies regardless of when the difference actually occurs. For example, suppose that during the current valuation at time t we realize that the very last entry between $EF^{t-1/2}$ and EF^t differs. Then this difference must be reflected in the economic value added during *this period*.

Table 6 shows that at time 1.0 and onwards there are significant changes in the implied equity flows from one valuation to the next. All these changes will be recognized in the EVA stream during the current accounting period.

The entire EVA stream—as of the valuation date t—is denoted by EVA^t. Let us refer to specific entries within this stream using a subscript. For example, $EVA^t_{1.0}$ refers to the time 1.0 entry of the EVA stream with information up to the valuation date t.

As of the previous valuation date, that is, $t = 0.5$, the expected EVA at time 1.0 was zero, that is, $EVA^{0.5}_{1.0} = 0$ (see Table 4, col. [7]). Now that the implied equity flows have changed, what should the actual value be?

For the NPV accounting system it is equal to the difference between the NPV of the implied equity flows streams $EF^{0.5}$ and $EF^{1.0}$ valued at time 1.0. The NPV calculation uses the cost of capital as the discount rate.

ILLUSTRATION 5

The *current and future* implied equity flows (from Table 6) are:

Time	1.0	1.5	2.0	2.5	3.0	3.5
$EF^{0.5}$	227.60	32.97	32.67	18.73	116.58	0.00
$EF^{1.0}$	224.47	30.55	30.29	16.38	114.20	0.00

The NPV of these streams (discounted to time 1.0), calculated at the cost of capital, is equal to

$$227.60 + \frac{32.97}{1 + 5\%} + \frac{32.67}{(1 + 5\%)^2} + \frac{18.73}{(1 + 5\%)^3} + \frac{116.58}{(1 + 5\%)^4} = 400.72$$

and

$$224.47 + \frac{30.55}{1 + 5\%} + \frac{30.29}{(1 + 5\%)^2} + \frac{16.38}{(1 + 5\%)^3} + \frac{114.20}{(1 + 5\%)^4} = 389.14,$$

respectively. Therefore, the *change in value* at time 1.0 for the EVA stream is equal to the difference between \$389.14 and \$400.72, that is, −\$11.58. Hence, the EVA at time 1.0 is

$$\underbrace{0.00}_{EVA^{0.5}_{1.0}} + \underbrace{(-11.58)}_{\Delta EVA} = \underbrace{-11.58}_{EVA^{1.0}_{1.0}}. \qquad \square$$

At any subsequent valuation, if there is another change in the implied equity flows, then we would calculate the change in EVA in the same manner as in the previous illustration. Thus the EVA stream under the NPV accounting system will consist of zero and nonzero entries. The nonzero entries signal the accounting periods where changes to the implied equity flows have been recognized.

5.2 The IRR Perspective

The IRR accounting system shows a constant rate of return equal to the IRR of the implied equity flows throughout the life of the insurance contract. The after-tax net income in this accounting system is a constant percentage of the equity fund.

ILLUSTRATION 6

The IRR of the implied equity flows in column (2) of Table 5 is 6.18%. The rate of return in column (6) is equal to this IRR. □

A change in the implied equity flows is reflected in the returns and in the EVA stream. Three ways (among others) of incorporating these changes are the following:

1. Change all entries (past, current, and future) so that the returns are equal to the IRR of the new implied equity flows.
2. Change the current and future entries so that the returns over the remaining (future) periods are constant.
3. Change the current period's return so that the remaining entries reflect the IRR on the new implied equity flows from inception of the policy. In GAAP parlance, the current entry is a *true-up* item, or a balancing entry between the old and the new internal rate of return.

ILLUSTRATION 7

The implied equity flows have an IRR of 6.18% (per period) at inception of the contract. At time 1.0 the implied equity flows change, and their new IRR is equal to 5.30%. The following table shows the rates of return under the three options discussed above:

	Rates of Return per Period			
Time	Original	Option 1	Option 2	Option 3
0.0	0.00%	0.00%	0.00%	0.00%
0.5	6.18	5.30	6.18	6.18
1.0	6.18	5.30	4.38	4.23
1.5	6.18	5.30	4.38	5.30
2.0	6.18	5.30	4.38	5.30
2.5	6.18	5.30	4.38	5.30
3.0	6.18	5.30	4.38	5.30
3.5	0.00	0.00	0.00	0.00

The column labeled *original* shows the expected returns before any changes. Since the implied equity flows change at time 1.0, the entries at time 0.0 and 0.5 are actual numbers. Under *option 1* we would ignore these actual numbers and overwrite them with the current internal rate of return of the implied equity flows. For *option 2* we overstated the returns in previous periods (6.18% vs. 5.30%), and so to compensate we need a lower IRR (4.38% vs. 5.30%) for the remaining periods. *Option 3* shows the current IRR of the implied equity flows for all *future* periods, namely, 5.30% at times $t \geq 1.5$. During the current period ($t = 1.0$) we fully correct the previous period's overstated returns. □

Option 1 is suitable for *post mortem* actuarial analyses of profitability. It is not a reasonable accounting option, since we cannot restate accounting results from previous periods. Option 2 is misleading because the results suggest that the implied equity flows have an IRR of 4.38%, when in reality their IRR is 5.30%. Option 3 presents perhaps the most accurate picture of the implied equity flows. Whenever a change to the implied equity flows occurs, we reflect all previous differences during the current accounting period. Future periods show the true IRR for the entire time horizon. The next illustration shows the calculations necessary to quantify the changes in the EVA stream under option 3.

ILLUSTRATION 8

Suppose that at time 1.0 the implied equity flows change as shown in Table 6. From Table 5 we know that the equity fund at time 0.5 is equal to \$371.98. We may also derive the equity fund at time 0.5 by calculating the present value of the *future* implied equity flows. Most of the time the appropriate discount rate is equal to the internal rate of return of the implied equity flows.[14] For example, the IRR of the implied equity flows ($EF^{0.5}$) is equal to 6.18%, and so the equity fund, at time 0.5, is given by[15]

$$\frac{227.60}{1 + 6.18\%} + \frac{32.97}{(1 + 6.18\%)^2} + \frac{32.67}{(1 + 6.18\%)^3} + \frac{18.73}{(1 + 6.18\%)^4} + \frac{116.58}{(1 + 6.18\%)^5} = 371.98.$$

At time 1.0 the implied equity flows change (from $EF^{0.5}$ to $EF^{1.0}$), and so the return at this time will not equal the IRR of the implied equity flows. The IRR of the new implied equity flows (that is, $EF^{1.0}$) is equal to 5.30%. We have to calculate the appropriate return at time 1.0. We may do this by solving the following equation:

[14] We can use only the IRR of the implied equity flows as long as there have been no changes. Once the implied equity flows change, we can still discount them to obtain the equity fund, but we have to use the stated returns as the appropriate discount factors.

[15] This fact follows by unraveling the recursive definitions for the entries of the IRR accounting system given in Table 5.

$$371.98 = \frac{224.47}{1+r} + \frac{30.55}{(1+r)(1+5.30\%)} + \frac{30.29}{(1+r)(1+5.30\%)^2} + \frac{16.38}{(1+r)(1+5.30\%)^3} + \frac{114.20}{(1+r)(1+5.30\%)^4},$$

where r represents the return at time 1.0. The solution to this equation is $r = 4.23\%$.

We now proceed to calculate the remaining entries of the EVA stream. At time 1.0 the after-tax net income is equal to the return times the equity fund at the start of the period, that is,

$$371.98 \times 4.23\% = 15.73.$$

The dollar cost of capital for this period is equal to the cost of capital times the equity fund at the start of the period, namely,

$$5.00\% \times 371.98 = 18.60.$$

Hence, the EVA at time 1.0 is given by the difference between the after-tax net income and the dollar cost of capital for this period:

$$15.73 - 18.60 = -2.86.$$

At time 1.5 the return is equal to the IRR of the implied equity flows, that is, 5.30%. The equity fund at time 1.5 is equal to the discounted implied equity flows, that is,

$$\frac{30.55}{1+5.30\%} + \frac{30.29}{(1+5.30\%)^2} + \frac{16.38}{(1+5.30\%)^3} + \frac{114.20}{(1+5.30\%)^4} = 163.24.$$

Thus the EVA at time 1.5 is equal to

$$\underbrace{163.24 \times 5.30\%}_{\text{after-tax net income}} - \underbrace{163.24 \times 5.00\%}_{\text{dollar cost of capital}} = 0.49.$$

At time 2.0 the return is also 5.30%, and the equity fund is given by

$$\frac{30.29}{(1+5.30\%)} + \frac{16.38}{(1+5.30\%)^2} + \frac{114.20}{(1+5.30\%)^3} = 141.35.$$

The EVA is then equal to

$$141.35 \times (5.30\% - 5.00\%) = 0.43.$$

The following table summarizes the EVA and the changes that occur due to the difference in the implied equity flows. The EVA stream before any changes is taken from Table 5.

	EVA Stream		
Time	Before	After	Change
0.0	0.00	0.00	0.00
0.5	5.07	5.07	0.00
1.0	4.40	−2.86	−7.26
1.5	1.98	0.49	−1.49
2.0	1.71	0.43	−1.29
2.5	1.43	0.36	−1.07
3.0	1.30	0.33	−0.97
3.5	0.00	0.00	0.00

□

The above illustration shows how to determine the EVA stream from the new implied equity flows. We then compute the change as the difference between the previous valuation results and the current valuation. In Section 8 we present the procedures used to quantify this change directly from the implied equity flows.

6. DEVIATIONS IN INVESTMENT RETURNS

Interest rate changes affect the implied equity flows, since an increase in the investment yield produces greater investment income. The *after-tax* investment income flows through to the equity holders.

We differentiate among three types of scenarios:

- Short-duration assets
- Longer-duration assets held at amortized values
- Longer-duration assets held at market values.

For short-duration assets like Treasury bills and cash equivalents, there are no material capital gains and losses stemming from interest rate changes. Investment gains or losses compared to the expected values stem from changes in the investment yield.

For longer-duration assets held at amortized value on the statutory balance sheet, the unrealized capital gains and losses stemming from interest rate changes do not affect the implied equity flows, and they do not affect the federal income tax cash flows. We use the book yields and book values for these assets; the retrospective analysis shows no effect on the IRR.

Statutory valuation rules determine the flow of money from the insurance company to its equity holders. The implied equity flows follow the statutory valuation, not the GAAP valuation. Therefore in this paper we use statutory valuation rules, not GAAP valuation rules and not fair value valuation principles. Most investment-grade bonds are held at amortized value on the statutory balance sheet and at market value on the GAAP balance sheet; see SFAS 115 for the exceptions.

For longer-duration assets held at market value, the unrealized capital gains or losses stemming from the interest rate changes directly affect the held assets, deferred tax assets and liabilities, and implied equity flows. Examples of such assets are noninvestment grade bonds. The discussion below of common stocks applies to these assets as well.

ILLUSTRATION 9: COMMON STOCKS

The company owns \$100 million of common stocks on December 31, 20XX + 1. The expected capital accumulation rate is 10% per annum. The expected dividend yield is 2% per annum. At the beginning of 20XX + 2 the stocks suddenly rise in value to \$120 million, for a 20% capital accumulation increase. The dividends are \$2 million.

The pretax difference (actual from expected) in common stock values is \$10 million. There is no tax paid, but there is a deferred tax liability of \$3.5 million.[16] The net implied equity flow to the equity holders is \$6.5 million.

The extra assets are returned to the equity holders in the current period. The assets required are not affected by the investment gain or loss in the current period. One might argue that the investable assets decrease, since the deferred tax asset is noninvestable. However, it is equally possible that the deferred tax asset offsets other deferred tax liabilities of the company. It is easiest to model the implied equity flows as if the capital gain or loss is realized in the year in which it occurs. □

The investment income is equal to the investment yield times the income-producing assets. The income-producing assets are determined from statutory constraints; they do not depend on past investment performance. Changes in the amount of investment income result strictly from changes in the investment yield.

[16] Assuming a tax rate of 35% is applicable to investment activities.

Table 7
Investment Income for Underlying Policy

Date	Time	Income-Producing Assets	Investment Income
Dec. 31, 20XX	0.0	1,180.00	0.00
June 31, 20XX + 1	0.5	961.48	47.20
Dec. 31, 20XX + 1	1.0	742.95	38.46
June 31, 20XX + 2	1.5	731.58	29.72
Dec. 31, 20XX + 2	2.0	720.20	29.26
June 31, 20XX + 3	2.5	733.85	28.81
Dec. 31, 20XX + 3	3.0	0.00	29.35
June 31, 20XX + 4	3.5	0.00	0.00
Sum			202.80

Income-Producing Assets = eq. (3.8).
Investment Income_t = Income-Producing $\text{Assets}_{t-1/2} \times 4\%$.

Federal income taxes are assessed on investment income when it is received. Each additional \$1 of investment income raises the implied equity flows by \$1 × (1 − tax rate). With a tax rate of 35%, each dollar of additional investment income raises the implied equity flow by \$0.65.

Similarly, a one dollar reduction of investment income reduces the implied equity flow by 65 cents.[17]

ILLUSTRATION 10

The income-producing assets and the investment income for our underlying policy are given in Table 7. Our assumption for the investment rate of return is 4% for each period. For instance, the investment income at time 1.0 is equal to \$961.48 × 4% = \$38.46.

If the investment rate of return were to change from 4% to 5% in the time interval from 1.0 to 1.5, then the investment income will increase by (5% − 4%) × \$742.95 = \$7.43. Taxable investment income also will increase by this amount, and so taxes paid increase by \$7.43 × 35% = \$2.60. Therefore, the implied equity flow for this time period will change by \$7.43 − \$2.60 = \$4.83. All other implied equity flows will not change. □

7. DEVIATIONS IN INCURRED LOSSES

Deviations in incurred losses greatly affect the implied equity flows. Their analysis is also more complex than the analysis on investment income presented in the previous section. The extra complexity arises because a change in incurred losses affects many components of the implied equity flows. For example, taxable underwriting income, held assets, and future investment income are some of the components that change. In this section we explore the way in which the implied equity flows are affected.

Changes in incurred losses arise as a change in the payment pattern (keeping ultimate losses constant), as a change in ultimate losses, or both. The change in ultimate losses can be split as

- A change in paid losses for the current period
- A change in the estimate of the loss reserve as of the valuation date.

The next illustration shows that a change in the payment pattern can be decomposed as a change in paid losses followed by a change in the loss reserve followed by another change in paid losses.

ILLUSTRATION 11

Suppose that we have the following expected payment pattern and nominal loss reserve stream:

[17] We assume that the insurance company has enough taxable income from other operations—either in the current year or in the two previous years—to be able to deduct these losses completely.

Period	1	2	3	4
Payments	1	1	1	1
Loss Reserve	3	2	1	0

Let us change the timing of our loss payments. Rather than paying one unit during period 2, we will shift it to period 3. The following table shows the result:

Period	1	2	3	4
Payments	1	0	2	1
Loss Reserves	3	3	1	0

We can decompose this timing change into a sequence of three actions. First, we reduce the payment during period 2 from one to zero. Second, we increase the loss reserve at the end of period 2 from two to three units. Finally, at the period 3 valuation we increase our payment from one to two units. □

The above illustration shows that any change in payment pattern can be understood as a sequence of changes in paid losses and changes in loss reserve estimates. Therefore, in the next sections we concentrate on analyzing these two actions.

7.1 Change in Paid Losses

Suppose that during the current period we pay one dollar more of losses and we *do not change* the estimate of our nominal loss reserve. The impact on the implied equity flows is a decrease of 65 cents.

The taxable underwriting income for the year will decrease by one dollar because of the extra dollar of losses. Therefore, if we assume a marginal tax rate of 35%, then we would reduce our tax payments by 35 cents. This reduction in paid taxes mitigates the extra dollar of underwriting losses. Hence, the implied equity flows will decrease not by the full one dollar but rather only by 65 cents.

7.2 Change in the Loss Reserve Estimate

Now suppose that we increase our nominal loss reserve estimate by one dollar. The effect of this change on the implied equity flows is complex. Not only do changes arise during this valuation period but also at every future valuation date. We will break our analysis into various pieces: taxes (on underwriting and investment income), held assets, and deferred tax assets. These are the key components that will help us determine the impact on the implied equity flows. We will first discuss underwriting tax effects. We shall defer the treatment of investment income effects until after we discuss the effects on held assets and deferred tax assets.

7.2.1 Underwriting Tax Effect

In our model we assume that the loss reserves held by the insurer are a constant percentage of the nominal loss reserves. This constant factor is called the reserve adequacy ratio, that is, the ratio of held loss reserves to nominal loss reserves. If we increase our nominal loss reserves by one dollar, then our held loss reserves will increase by one dollar times the reserve adequacy ratio.

Tax reserves are determined as the product of the insurer's held reserves times a discount factor provided by the Internal Revenue Service. Therefore, whenever our nominal loss reserves change by one dollar, the tax reserves will change by the product of one dollar, the reserve adequacy ratio, and the IRS discount factor.

A change in nominal loss reserves only affects taxable underwriting income via the tax reserves.[18] Therefore, whenever our nominal loss reserves change by one dollar, taxable underwriting income will change by the amount that tax reserves change.

ILLUSTRATION 12

Suppose that the reserve adequacy ratio is equal to 90% and the nominal loss reserves at time 1, 2, and 3 are equal to \$100, \$80, and \$60, respectively. Thus our held loss reserve at time 1 is \$90, at time 2 is \$72, and at time 3 is \$54. Also assume that the IRS discount factors are equal to 0.84, 0.86, and 0.88, respectively.

If we increase the nominal loss reserves at time 2 from \$80 to \$81, then the held loss reserve would increase from \$72 to \$72.9 and the IRS tax reserves would increase from $\$72 \times 0.86 = \$61.92$ to $\$72.9 \times 0.86 = \$62.694$. The change in tax reserves is equal to $\$62.694 - \$61.92 = \$0.774$. This is exactly the reserve adequacy ratio times the IRS discount factor, namely, $90\% \times 0.86 = 0.774$.

The change in taxable underwriting income is then $-\$0.774$ because we need to subtract the change in tax reserves. We can summarize as follows: *an increase of one dollar in nominal loss reserves decreases the implied equity flow by an amount equal to the product of the reserve adequacy ratio and the IRS discount factor.* □

7.2.2 Held Asset Effect

Our next task is to determine the impact a change in nominal loss reserves will have on held assets. Recall that the assets we need to hold at any point in time, due to statutory constraints, are equal to the sum of held loss reserves, surplus, and unearned premium reserves (UEPR). Unearned premium reserves do not play any role in this analysis because an increase in nominal loss reserves does not affect them.

In our model we determine surplus as

$$\text{Surplus} = \text{Surplus to Loss Reserve ratio} \times \text{Held Loss Reserves} + \text{Surplus to UEPR ratio} \times \text{UEPR}.$$

Therefore, an extra dollar of nominal loss reserves will increase surplus by an amount equal to the product of the reserve adequacy ratio and the surplus-to-loss reserve ratio.

ILLUSTRATION 13

Continuing with illustration 12 and assuming that the surplus-to-loss reserve ratio is equal to 15%, an increase of one dollar in the nominal loss reserve will affect surplus by an increase of $90\% \times 15\% = 0.135$. □

The change in held assets is then equal to the sum of the changes in held loss reserves and surplus. The change in our held loss reserves is equal to the product of the change in nominal loss reserves and the reserve adequacy ratio. The change in surplus is equal to the change in nominal loss reserves times the reserve adequacy ratio times the reserve to surplus ratio. Summarizing, the change in held assets is equal to

$$\text{Change in nominal loss reserves} \times \text{Reserve adequacy ratio} \times (1 + \text{Surplus to Loss Reserve ratio}).$$

7.2.3 Deferred Tax Asset Effect

The next item on our list is to determine the change in the deferred tax asset due to an increase of one dollar in the nominal loss reserve. In general, an increase in the deferred tax asset leads to a decrease in investable assets, all other things held constant. The deferred tax asset is made up of two components: one due to the revenue offset provision and the other due to the IRS loss reserve discounting. Because the unearned premium reserve does not change due to an increase in nominal loss

[18] For the definition of taxable underwriting income see Illustration 1.

reserves, it will not affect the deferred tax asset due to the revenue offset. Hence in what follows we will refer to the deferred tax asset due to IRS loss reserve discounting simply as the deferred tax asset.

The deferred tax asset at a year-end valuation is equal to the tax rate times the difference between the loss reserve discount at this year-end valuation date and the loss reserve discount at the *next* year-end valuation date. Algebraically we have

$$\text{DTA}_t^{\text{RD}} = \text{Tax Rate} \times (\text{Held Loss Reserve}_t \times (1 - \text{IRS discount factor}_t) - \text{Held Loss Reserve}_{t+1} \times (1 - \text{IRS discount factor}_{t+1})),$$

where t is the current year-end valuation date and $t + 1$ is one year later.

Suppose that at the current year-end valuation we have a change in held loss reserves equal to

$$\nabla\text{Held Loss Reserve}_t = \text{Held Loss Reserve}'_t - \text{Held Loss Reserve}_t,$$

and at the *next* year-end evaluation we have a change equal to

$$\nabla\text{Held Loss Reserve}_{t+1} = \text{Held Loss Reserve}'_{t+1} - \text{Held Loss Reserve}_{t+1}.$$

Then the change in deferred tax asset is equal to

$$\nabla\text{DTA}_t^{\text{RD}} = \text{Tax Rate} \times (\nabla\text{Held Loss Reserve}_t \times (1 - \text{IRS discount factor}_t) - \nabla\text{Held Loss Reserve}_{t+1} \times (1 - \text{IRS discount factor}_{t+1})).$$

Note that the change in the held loss reserve at time t need not be equal to the change at time $t + 1$.

ILLUSTRATION 14

As in Illustration 12, suppose that the held loss reserves at time 1, 2, and 3 are equal to \$100, \$80, and \$60, respectively. Let the IRS discount factors be 0.84, 0.86, and 0.88, and assume that the tax rate is 35%. What is the change in the deferred tax asset if the held reserves change by one dollar at both valuation dates ($t = 2$ and $t = 3$)?

Before any changes to held loss reserves occur, the deferred tax asset at time 2 is equal to

$$35\% \times (\underbrace{80 \times (1 - 0.86)}_{\substack{\text{discount at} \\ \text{time } t = 2}} - \underbrace{60 \times (1 - 0.88)}_{\substack{\text{discount at} \\ \text{time } t = 3}}) = 1.400.$$

If we increase our loss reserves by one dollar at time $t \geq 2$, then the deferred tax asset would equal

$$35\% \times (81 \times (1 - 0.86) - 61 \times (1 - 0.88)) = 1.407.$$

The difference due to the increase in held loss reserves would be \$1.407 − \$1.400 = \$0.007. This is equivalent to

$$\nabla\text{DTA}_2^{\text{RD}} = 35\% \times (1 \times (1 - 0.86) - 1 \times (1 - 0.88)) = 35\% \times 0.02 = 0.007. \quad \square$$

In the previous illustration we assumed that the increase in nominal loss reserves would occur at both valuation dates. This is not always the case. For example, consider the situation where at the current valuation date we hold some nominal loss reserves but at the next valuation we do not hold any loss reserves. This implies that we would pay off the liability completely before the next valuation date. In this case the change in deferred tax asset due to an increase of one dollar in nominal loss reserve is equal to

$$\Delta\text{DTA}_t^{\text{RD}} = \text{Tax Rate} \times (1 \times (1 - \text{IRS discount factor}_t) - 0 \times (1 - \text{IRS factor}_{t+1}))$$
$$= \text{Tax Rate} \times 1 \times (1 - \text{IRS discount factor}_t).$$

7.2.4 Investment Income Effect

Investment income at the end of the period is equal to the beginning-of-the-period investable assets times the investment rate of return. The investable assets of the company are equal to held assets less the non-income-producing assets. And the non-income-producing assets in our illustration are equal to the total deferred tax assets.[19] The deferred tax asset at time t depends on the held loss reserves at time t and at time $t + 1$. Therefore, whenever our loss reserves change, so will the investment income we earn.

For an increase of one dollar in the nominal loss reserves at times t and $t + 1$ we know that our held assets will increase by

$$1 \times \text{Reserve adequacy ratio} \times (1 + \text{Surplus to Loss Reserve ratio}),$$

and deferred tax assets will increase by

$$\text{Tax Rate} \times (1 \times (1 - \text{IRS discount factor}_t) - 1 \times (1 - \text{IRS discount factor}_{t+1})).$$

Therefore, the income-producing assets would change by the difference of the previous two expressions:

$$1 \times \text{Reserve adequacy ratio} \times (1 + \text{Surplus to Reserve ratio}) - \text{Tax Rate}$$
$$\times (1 \times (1 - \text{IRS discount factor}_t) - 1 \times (1 - \text{IRS discount factor}_{t+1})).$$

ILLUSTRATION 15

Suppose that the reserve adequacy ratio is equal to 90% and the surplus-to-loss-reserve ratio is equal to 15%. The tax rate is 35%, and the IRS discount factors are 0.86 and 0.88. Then an increase of one dollar in nominal loss reserves *now and in 12 months* would change the income-producing assets by an amount equal to

$$1 \times 90\% \times (1 + 15\%) - 35\% \times (1 \times (1 - 0.86) - 1 \times (1 - 0.88)) = 1.028. \quad \square$$

In the above illustration we assumed a one dollar increase to our loss reserves at *both* valuation dates. If we increase the loss reserve only at the first valuation date, then we can omit the last term involving the IRS discount rate. Using the same parameters as in the previous illustration, we have

$$1 \times 90\% \times (1 + 15\%) - 35\% \times 1 \times (1 - 0.86) = 0.986.$$

Because our income-producing assets have changed, our investment income will also change. If we change the nominal loss reserves at the end of the current period, then the investment income for this period *will not* change. Investment income for the current period is equal to the amount of income-producing assets at the end of the *previous* period times the investment rate of return. Therefore, investment income will change starting with the *next period*. The change will be equal to the previous period's change in income-producing assets times the investment rate of return.

Taxable investment income will change by the same amount (and at the same time) as investment income. Similarly, investment income tax will change by the product of the tax rate and the change in taxable investment income.

ILLUSTRATION 16

Suppose that the investment rate of return is equal to 4% per period and that the tax rate is 35%. Using the results from Illustration 15 we know that during the current period the income-producing assets changed by \$1.028 for every dollar of increase (at both valuation dates t and $t + 1$) in the nominal loss reserves. For the current period our investment income will not change, and taxes paid on investment income for the current period will not change. But for the next period, the change in

[19] There are other non-income-producing assets such as the premium receivable.

investment income will be equal to 4% × \$1.028 = \$0.041, and the change in investment income taxes paid will be 35% × \$0.041 = \$0.014. □

7.3 Assembling All the Pieces

We have now discussed all of the factors that affect the implied equity flows whenever we change the nominal loss reserves. We just need to put them together. The implied equity flow is made up of five flow components: asset, deferred tax asset, investment income, tax, and underwriting.

Asset flow is equal to the change in the amount of assets we are required to hold. By increasing the nominal loss reserves by one dollar the asset flow will increase by

$$\text{Reserve adequacy ratio} \times (1 + \text{Surplus to Loss Reserve ratio}).$$

Deferred Tax Asset flow equals the change in deferred tax assets arising from the loss reserve discount and the revenue offset provision. The revenue offset provision is not affected by a change in the nominal loss reserves. For each dollar that we increase the nominal loss reserves the deferred tax asset will change by an amount equal to

$$\text{Tax Rate} \times (1 \times (1 - \text{IRS discount factor}_t) - 1 \times (1 - \text{IRS discount factor}_{t+1})).$$

This formula assumes that at time t and t+1 the nominal loss reserve changes by one dollar. If at time t+1 the loss reserve does not change, then the appropriate formula would be

$$\text{Tax Rate} \times (1 \times (1 - \text{IRS discount factor}_t)).$$

Investment Income flow is equal to our investment rate of return times the income producing assets in the previous period. Therefore, during the current period there is no change in this flow item. During the next period the change will be equal to the change in the income-producing assets during the current period times the investment rate of return. The income-producing assets would change by

$$\text{Reserve adequacy ratio} \times (1 + \text{Surplus to Loss Reserve ratio}) - \text{Tax Rate}$$
$$\times (1 \times (1 - \text{IRS discount factor}_t) - 1 \times (1 - \text{IRS discount factor}_{t+1}))$$

if the nominal loss reserve changes at time t and at time $t + 1$. If the change only occurs at time t, then the income-producing assets would change by

$$\text{Reserve adequacy ratio} \times (1 + \text{Surplus to Loss Reserve ratio})$$
$$- \text{Tax Rate} \times (1 \times (1 - \text{IRS discount factor}_t)).$$

Tax flow equals the taxes paid on underwriting and investment income. An increase of one dollar in nominal loss reserves would *decrease* taxable underwriting income by

$$\text{Reserve adequacy ratio} \times (1 - \text{IRS discount factor}_t);$$

therefore, underwriting taxes would decrease by the tax rate times this factor. Investment income taxes would change by the tax rate times the change in investment income factor discussed above.

Underwriting flow is composed of written premium, paid losses, and acquisition and general expenses. A change in the nominal loss reserves will not affect this flow item during the current period. If paid losses eventually reflect the reserve change, the analysis of Section 7.1 provides the answer.

8. EVA CHANGES UNDER THE IRR PERSPECTIVE

In this section we present the procedures necessary to quantify the change in the EVA stream whenever the implied equity flows change. As with the NPV perspective we will calculate various net present values. In contrast with the NPV perspective we will quantify the changes in EVA for the current period separately from the changes in future periods.

8.1 Current Period Changes

Since we are assuming that the implied equity flows change from one valuation date to the next, we will have two internal rates of return. Let us denote with IRR^t the internal rate of return of the implied equity flows as of the valuation date t.

Two factors affect the change in the current period EVA: (1) the change in the IRR, and (2) the dollar change in the implied equity flows. The adjustments necessary for these changes are given by the following formulas:

1. The adjustment necessary due to the change in the IRR is

$$\text{Adjustment(IRR)} = \mathrm{NPV}_{\mathrm{IRR}^t}(\mathrm{EF}^{t-1/2}) - \mathrm{NPV}_{\mathrm{IRR}^{t-1/2}}(\mathrm{EF}^{t-1/2}). \quad (8.1)$$

2. The adjustment due to the dollar change in the implied equity flows is

$$\text{Adjustment(EF)} = \mathrm{NPV}_{\mathrm{IRR}^t}(\mathrm{EF}^{t-1/2}) - \mathrm{NPV}_{\mathrm{IRR}^t}(\mathrm{EF}^{t}). \quad (8.2)$$

Note that the structure of the adjustments in both cases is very similar, but there are subtle differences. In the first NPV calculation we use the same equity flows ($\mathrm{EF}^{t-1/2}$) and vary the internal rate of return (IRR^t vs. $\mathrm{IRR}^{t-1/2}$). In the second adjustment we keep the IRR the same (IRR^t) but vary the implied equity flows $\mathrm{EF}^{t-1/2}$ versus EF^t.

ILLUSTRATION 17

Suppose the current valuation date is $t = 1.0$. Column (2) in Table 6 shows the implied equity flows $\mathrm{EF}^{0.5}$. These flows have an internal rate of return of $\mathrm{IRR}^{0.5} = 6.18\%$. Column (3) in the same table shows $\mathrm{EF}^{1.0}$, that is, as of the valuation date $t = 1.0$. They have an internal rate of return of $\mathrm{IRR}^{0.5} = 5.30\%$.

Now we calculate the NPV of $\mathrm{EF}^{0.5}$ twice. For the first one we use $\mathrm{IRR}^{1.0} = 5.30\%$ as the discount rate, and for the second one we use $\mathrm{IRR}^{0.5} = 6.18\%$. The first calculation yields

$$227.60 + \frac{32.97}{1 + 5.30\%} + \frac{32.67}{(1 + 5.30\%)^2} + \frac{18.73}{(1 + 5.30\%)^3} + \frac{116.58}{(1 + 5.30\%)^4} = 399.22.$$

The second calculation gives

$$227.60 + \frac{32.97}{1 + 6.18\%} + \frac{32.67}{(1 + 6.18\%)^2} + \frac{18.73}{(1 + 6.18\%)^3} + \frac{116.58}{(1 + 6.18\%)^4} = 394.97.$$

Hence, the adjustment for the current period due to the change in internal rate of return is equal to \$399.22 − \$394.97 = \$4.25.

The adjustment due to the actual dollar amount of change between $\mathrm{EF}^{t-1/2}$ and EF^t is computed as follows:

$$224.47 + \frac{30.55}{1 + 5.30\%} + \frac{30.29}{(1 + 5.30\%)^2} + \frac{16.38}{(1 + 5.30\%)^3} + \frac{114.20}{(1 + 5.30\%)^4} = 387.72$$

and

$$227.60 + \frac{32.97}{1 + 5.30\%} + \frac{32.67}{(1 + 5.30\%)^2} + \frac{18.73}{(1 + 5.30\%)^3} + \frac{116.58}{(1 + 5.30\%)^4} = 399.22.$$

Hence, the adjustment due to the change from $\mathrm{EF}^{0.5}$ to $\mathrm{EF}^{1.0}$ is equal to \$387.72 − \$399.22 = −\$11.51.

Now we have both components of the change in EVA for the *current* period. The adjustment due to change in IRR is equal to \$4.25, and the adjustment due to the dollar change in implied equity flows is equal to −\$11.51. Therefore, the change in EVA for the current period is equal to \$4.25+(−\$11.51) = −\$7.26. The EVA shown before the change in implied equity flows was \$4.40 (see col. [7] of Table 5). Hence, the revised EVA is equal to \$4.40 + (−\$7.26) = −\$2.86 as shown in Illustration 8. □

8.2 Future Period Changes

The change in EVA for subsequent periods is

$$\Delta\text{EVA} = (\text{IRR}^t - \text{Cost of Capital}) \times \text{NPV}^{\bullet}_{\text{IRR}^t}(\text{EF}^t) - (\text{IRR}^{t-1/2} - \text{Cost of Capital}) \times \text{NPV}^{\bullet}_{\text{IRR}^{t-1/2}}(\text{EF}^{t-1/2}),$$

where the ● symbol on the NPV operator means that we should discount by one more period.[20]

ILLUSTRATION 18

Again using the implied equity flows from Table 6 the change in economic value added at time 1.5 would be determined as follows:[21]

$$\begin{aligned}&(5.30\% - 5.00\%) \times \left[\frac{30.55}{1 + 5.30\%} + \frac{30.29}{(1 + 5.30\%)^2} + \frac{16.384}{(1 + 5.30\%)^3} + \frac{114.20}{(1 + 5.30\%)^4}\right]\\ &\quad - (6.18\% - 5.00\%) \times \left[\frac{32.97}{(1 + 6.18\%} + \frac{32.67}{(1 + 6.18\%)^2} + \frac{18.73}{(1 + 6.18\%)^3} + \frac{116.58}{(1 + 6.18\%)^4}\right]\\ &= 0.49 - 1.98 = -1.49.\end{aligned}$$

Thus the revised EVA at time 1.5 is

$$\underbrace{1.98}_{\text{EVA}^{0.5}_{1.5}} + \underbrace{(-1.49)}_{\Delta\text{EVA}} = \underbrace{0.49}_{\text{EVA}^{1.0}_{1.5}}.$$

The first term in the above equation comes from Table 5. All other subsequent periods are treated in the same manner.

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[20] Suppose we have cash flows $f_1, f_2, \ldots$ at equally spaced intervals. Then $\text{NPV}_r(f_1, f_2, \ldots)$ is equal to $\Sigma_{i=0}^{\infty} f_{i+1}/(1 + r)^i$, and $\text{NPV}^{\bullet}_r(f_1, f_2, \ldots)$ is equal to $\text{NPV}_r(f_1, f_2, \ldots)/(1 + r)$. That is, we discount all the cash flows one more period.

[21] In the calculation we used a higher precision than that shown here.

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