THE MAGNIFICENT SEVEN

It is an honor to be invited to write a guest editorial for the *North American Actuarial Journal.* My topic is financial engineering, a new discipline of increasing importance to the actuarial profession. I will discuss some ideas that form the intellectual foundations of financial engineering. Just as engineers use breakthroughs from some area of basic science such as physics to develop products such as iPods, financial engineers use modern finance theories to develop practical applications based on these theories. A typical application would be the design, construction, pricing, and risk management of a new financial instrument. Because it is the advances in basic finance theory that form the basis for the applications in financial engineering, I thought it would be instructive to discuss some important ideas in the finance field. So here are what I consider to be the top seven contributions: the magnificent seven.

THE NO-ARBITRAGE PRINCIPLE

The popular expression of this concept is that there is no free lunch. This intuition crops up in different fields. Cournot stated this result in 1838 in connection with foreign exchange rate cycles. We have a similar result in physics: It is impossible to invent a perpetual motion machine that will generate more energy than is put into it. The finance formulation is that if there are two portfolios that have exactly identical payoffs in the future, then, in perfectly frictionless markets, they must have the same price. Modigliani and Miller used the no-arbitrage principle in their celebrated papers on capital structure.

MEAN VARIANCE PORTFOLIO SELECTION

More than 50 years ago, Harry Markowitz showed how, under certain simple assumptions, investors could pick optimal portfolios. He assumed that if investors care only about the expected return and risk (as measured by the variance) of their portfolios, the weights in the optimal portfolios could be explicitly characterized. The inputs to the optimization are the expected returns on the securities and the covariance matrix of returns. For a given level of expected return, the Markowitz solution gives the portfolio with the lowest standard deviation. Rational investors will hold only portfolios on the efficient frontier, which turns out to be the top half of a hyperbola in mean standard deviation space. This landmark paper (1952) represents the start of the modern era of scientific finance. It is still a joy to read and contains many useful insights.

CAPITAL STRUCTURE IRRELEVANCE

Modigliani and Miller established that, under perfect market conditions, neither capital structure nor dividends mattered. We can do no better than letting Miller (1991) himself explain the idea: ‘‘Think of the firm as a gigantic tub of whole milk. The farmer can sell the whole milk as is. Or he can separate out the cream and sell it at a considerably higher price than the whole milk would bring. (That's the analog of a firm selling low-yield and hence high-priced debt securities.) But, of course, what the farmer would have left would be skim milk with low butterfat content and that would sell for much less than whole milk. That corresponds to the levered equity. The M and M proposition says that if there were no costs of separation (and, of course, no government dairy-support programs), the cream plus the skim milk would bring the same price as the whole milk.''

EQUILIBRIUM

The equilibrium framework provides a powerful tool for studying markets and the determinants of prices. The foundations of general equilibrium were laid by Kenneth Arrow and Gerard Debreu in the 1950s. Equilibrium is brought about by the actions of individual buyers and sellers as they strive to maximize their welfare. At the equilibrium price, supply equals demand and the market clears. Arrow, in his 1953 path-breaking paper, showed how state-contingent claims could achieve an optimal allocation of risk. Arrow worked for a while in the insurance industry and had plans to become an actuary. He was dissuaded from doing so by Koopmans, who dismissed actuarial science, with the words: *"There is no music in it."*

Arrow-Debreu securities pay one unit if and only if a certain state occurs. In a one-period model, the portfolio of all the Arrow-Debreu securities corresponds to the risk-free bond, and the prices of these securities are proportional to martingale probabilities. This connection provides an intuitive way to interpret the concept of risk-neutral valuation.

The Capital Asset Pricing Model

William Sharpe (1964) is usually credited with the discovery of this model, but Lintner, Mossin, and Treynor also had the same idea. The CAPM is a beautiful, simple model that relates the expected return on a security to its risk in an equilibrium setting. The expected return on security j is a linear function of the expected return on the market portfolio of all securities. We have

$$\mathrm{E}[R_j] - r_f = \beta_j \mathrm{E}[R_m - r_f],$$

where R_m is the return on the market. The sensitivity of the security to the market is measured by β_j, which is proportional to the covariance between security j and the market. The CAPM implies that, in equilibrium, all investors should hold some combination of the market portfolio and the risk-free asset. Investors who can tolerate more risk will invest more in the market and less in the risk-free security. Conversely, investors who prefer less risk will put more in the risk-free security and less in the market.

The Black-Scholes-Merton Option Formula

The Black-Scholes-Merton expression for the option price is the most important formula in finance. Louis Bachelier, in his splendid 1900 thesis, made important contributions to option pricing and several other areas as well, but his work was neglected for the next 60 years. The 1960s heralded an upsurge of interest in the pricing of warrants and options. Paul Samuelson played a crucial role in these developments and almost solved the problem himself. In 1967, Edward Thorp and Sheen Kassouf published an influential book called *Beat the Market,* which explained how to set up a hedged portfolio of a stock and a warrant. Fischer Black considered a portfolio consisting of a short position in a call option and a long position in its stock, such that the combination had a zero beta. From this, he derived the equation satisfied by the option price. Black's derivation was based on the CAPM, and together with Myron Scholes he solved the equation to obtain their famous formula.

Robert Merton derived the same equation without the use of the CAPM. Merton's approach was based on the construction of a portfolio of the stock and the option and dynamically adjusting this portfolio over time. By using It_o calculus, Merton was able to do this continuously and hedge away all the random fluctuations. From the no-arbitrage result, this portfolio must earn the riskless interest rate. This led to the same equation as Black had discovered earlier. So the task begun by Bachelier was fully completed in 1969 by the three MIT professors. In 1997, Scholes and Merton were awarded the Nobel Prize for this achievement. Black would have shared the award had he not passed away two years earlier.

Portfolio Selection in Continuous Time

Robert Merton single-handedly created this field. He postulated and solved some of the fundamental problems in the area, and he applied sophisticated mathematics to derive some powerful results. Merton summarized his contribution as follows: *"I see the relative importance of my contribution to be more in selecting 'good' abstractions than in introducing and applying mathematical power."* Quite remarkably, he was able to derive some closed-form expressions for items of interest. Here is one of my own favorites.

An investor allocates her money between the market and the risk-free security. If the market return has a lognormal distribution and the investor has a power utility function with relative risk aversion γ, then she will follow a very simple strategy. At any time her position in the market will be a constant fraction of her current wealth. The constant proportion, often called the Merton ratio, is

$$\frac{\mathrm{E}[R_m] - r_f}{\sigma_m^2 \gamma}.$$

Let us assume that $E[R_m] = 0.08$, $r_f = 0.05$, $\sigma_m = 0.15$. Empirical studies suggest that $\gamma = 2$ is a reasonable estimate. This implies that optimal fraction in equities is 66.7%. Note that this ratio is close to the 60–40 equity bond rule of thumb advocated by some pension fund managers.

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I have selected one publication to represent each of the seven contributions.

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