Modeling Disability in Long-Term Care Insurance

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Abstract

Long-term care (LTC) costs and, in particular, those arising under an LTC insurance contract, are difficult to estimate. This is because of the complex effects of the processes of aging—disability and cognitive impairment. As disability is a gradual, as opposed to a discrete, process, and as the effects are sometimes reversible, a fairly complex model is necessary to capture its nature. This paper concentrates on modeling the disability process of aging only and, in particular, fully incorporates the recovery process as dictated by the data. With the recovery process modeled, the effect on the estimated model costs of disability of the common simplifying assumption that recoveries can be ignored is easily assessed.

This paper has twin objectives: (1) to present novel methodology, the penalized likelihood, for using interval-censored longitudinal data, such as the National Long-Term Care Study, to parameterize Markov models; and (2) to estimate the costs arising under an LTC insurance contract in respect of disability. The model is also used to show that ignoring recovery from disability can lead to significant overestimation of LTC insurance costs—suggesting that claims underwriting in LTC insurance may be an important factor in managing claims costs.

1. Introduction

1.1 Modeling Long-Term Care Costs

Modeling the processes that lead to claims under a long-term care (LTC) insurance policy is complex. There are different underlying causes of claiming (e.g., physical versus mental deterioration), and the processes being modeled may involve progression through a number of states of health (e.g., progression through states of varying disability), rather than being binary as in life insurance (alive–dead), which, in turn, make it difficult to specify objective claims criteria. In addition, events leading to claiming are often reversible (e.g., people can recover from some types of disability)—a factor that can be difficult to allow for, unless a suitable modeling framework is used. Indeed, previous researchers have often assumed that recovery from disability is not possible, or their models incorporate it in a very approximate manner (Alegre et al. 2002; Dullaway and Elliot 1998; Haberman and Pitacco 1999; Nuttall et al. 1994; Rickayzen and Walsh, 2002). The continuous time model proposed in this paper has no such restriction, and recovery from disability is fully incorporated in the model. This allows the effect of ignoring recoveries to be quantified, as well as provides some insight into the importance of claims underwriting in LTC insurance.

In the United States, interval-censored longitudinal data on disability have been available since the mid-1980s, from the National Long-Term Care Study (NLTCS) in 1982, 1984, 1989, and 1994. Interval censored means individuals in the study are interviewed at fixed time points, resulting in their disability status being known at these points in time, with the number and timing of changes in some status of interest (e.g., disability status) unknown. These extensive studies, undertaken at great cost, include data on more

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than 35,000 lives—making them unique in scope. There were linked series of questionnaires establishing loss of activities of daily living (ADLs) and instrumental ADLs, and institutionalization. In a Markov framework, as used in this paper, interval-censored longitudinal data give rise to estimates of a transition probability matrix over an extended time period. There is then the problem of how to ''convert'' these probabilities, within the Markov framework, to realistic (positive and real) parameter estimates (transition intensities), which are more useful for actuarial applications. The method we propose is intuitive, produces reasonable estimates, and is flexible enough to work where other methods cannot be applied.

1.2 Outline of Paper

The goal of this paper is to describe and parameterize a continuous-time Markov model of the disability process and to look at the cost of disability under an LTC insurance contract. The data used to estimate the model parameters are from the 1982, 1984, 1989, and 1994 NLTCS in the United States (1997 NLTCS Public Use CD)—but this paper focuses on the work done with the 1982 and 1984 NTLCS in particular. Though not discussed in this paper, NLTCSs have also been undertaken in 1999 and 2004.

We start by introducing LTC insurance contracts in Section 2. Then in Section 3, after describing the data and some previous research that has used the data, the model and statistical framework are introduced. In Section 4, after discussing the reasons standard maximum likelihood estimates cannot be calculated from the data, we look at how previous researchers have dealt with this problem. A novel method is then proposed and implemented to obtain estimates of the model's parameters. In Section 5, confidence intervals for the parameters are estimated and then used in the graduation process. We use the parameterized model in Section 6 to calculate the expected present value (EPV) of model LTC benefits in respect of disability. In particular, we look at single premiums for a range of sample policies and investigate the effect on model premiums of ignoring recovery from disability. Conclusions and discussion are provided in Section 7.

2. LONG-TERM CARE INSURANCE

Most long-term care providers in the United Kingdom offer a range of policy options for customers to tailor LTC policies to their personal needs. For interested readers, Dullaway and Elliot (1998) provide a comprehensive discussion of the U.K. LTC market and the data sources used for pricing and reserving. For modeling purposes we look only at the basic features of LTC policies (in Section 2.1). The model we present later does not aim to be product specific, rather it is based on the underlying process so it can be modified to reflect many LTC product designs. For example, it could as easily be used for pricing immediate care annuities (an annuity payable to someone already in need of care) as for pricing LTC policies.

2.1 Long-Term Care Products

The basis for an LTC product is that a healthy person pays a single premium, or regular premiums up to a certain age or the start of benefit payment, in return for the entitlement to benefit when the need arises. The benefits, a sum assured per annum, will usually be paid for the duration of time that a person meets the claims criteria and will cease on recovery or death. The sum assured is usually indexed in some way to allow for health care cost inflation and is limited to the reasonable cost of care. A policyholder may become eligible for benefits in two ways: through disability or through loss of mental function (cognitive impairment), both of which we discuss in more detail next. Disability is measured in terms of an inability to perform certain activities of daily living (ADLs).

2.2 Activities of Daily Living

A typical set of ADLs is that used as a benchmark by the Association of British Insurers: washing, dressing, mobility, toiletting, feeding, and transferring. A typical LTC policy would pay benefits upon failure of a given number of ADLs, often three or four. Sometimes, a reduced benefit might be paid on failure of a smaller number of ADLs.

Failure of ADLs is not irreversible; Manton (1988) and Manton, Corder, and Stallard (1993) show quite high rates of recovery. Evidently, LTC insurance shares many of the features of permanent health insurance (PHI)—both in the difficulty of claims underwriting and in statistical analysis.

An insurance policy that pays claims depending on a complex series of events is conveniently represented by a multiple-state model, which is the approach we adopt in Section 3. The order in which ADLs fail is then of potential importance, leading to the unpleasant possibility that 6! = 720 states might be needed to represent all possibilities, even ignoring recoveries. Some studies have suggested that ADLs typically fail in a given order. Dullaway & Elliott (1998) state, without mentioning a source, that ADLs usually fail in the order given above and that recoveries are typically in reverse order. Dunlop, Hughes, and Manheim (1997) suggest the following orders, based on a 1984–1990 study with 5,151 subjects:

- mobility, washing, transferring, dressing, toileting, feeding.
- mobility, washing, transferring, toileting, dressing, feeding.

These two lists agree on the first three ADLs to fail and, therefore, about the events that would trigger a claim under a typical LTC policy. It is therefore a reasonable simplification to consider only the number of failing ADLs and to ignore the order of failure.

2.3 Cognitive Impairment

The term *cognitive impairment* is usually defined explicitly in an LTC policy, to the extent that the person insured is eligible for benefits when they are in need of continual care or supervision due to deterioration in, or loss of mental capacity from, an organic cause.

Cognitive impairment is usually measured in terms of the score attained on a mental state examination, such as the mini-mental state examination. There will usually be some cutoff score, beyond which the person becomes eligible for benefits, although evidence from relevant health care professionals is also used. Clearly, assessment is liable to be imprecise, making it difficult to decide on an exact date of inception of cognitive impairment, if such a thing exists. Often benefits will be triggered by a loss of ADLs caused by cognitive impairment, rather than cognitive impairment itself.

3. MODEL FOR LONG-TERM CARE INSURANCE

3.1 U.S. National Long-Term Care Study

The primary source of useful data on ADLs is the series of 1982, 1984, 1989, and 1994 U.S. National Long-Term Care Study (NLTCS), based at Duke University. Because these data sets have been discussed in detail elsewhere (Manton 1988; Manton et al. 1993), we only provide an overview of the data here.

Data from each pair of consecutive study have been used to parameterize the model (see Pritchard 2002 for more details and parameter estimates). However, for brevity, we illustrate the methods in detail using only the 1982 and 1984 study, which have already been discussed at length (see Section 3.2).

In 1982, to identify approximately 6,000 chronically disabled persons, more than 35,000 Medicare records were screened, also identifying more than 1,900 institutionalized lives. The cutoff date used for this survey was April 1, 1982, so people were categorized as of this date. The 1984 NLTCS was designed to track changes in functional level and returns to the community, as were the 1989 and 1994 study. They are longitudinal in the sense that they follow the same cohort of lives, but they only report ''snapshot'' views of the cohort at fixed times—they do not continuously track the cohort, but individual lives are traceable at each survey. Therefore, complete life histories are not available. The 1984, 1989, and 1994 study incorporated aged-in samples to maintain adequate coverage of the U.S. population aged 65 years and over. For example, the sample used in 1982 who were over 65 years old would be over 67 years old in 1984, so to maintain a sample of over-65-year-old lives in 1984 a new sample, the aged-in population, of people aged 65 to 66 years old in 1984 was combined with the original sample.

The same measures of disability were used in all years and were defined by the inability to perform one or more of eight instrumental activities of daily living (IADLs, including light housework, laundry, meal preparation, grocery shopping, getting around outside, getting to places outside within walking distance, money management, using the telephone) or one or more of six ADLs (eating, getting in and out of bed, getting around inside, dressing, bathing, getting to the bathroom or using the toilet) without using personal assistance or special equipment.

3.2 Previous Research on Disability Using the NLTCS

These data have already been used for research into trends of disability. Papers by Manton (1988) and Manton et al. (1993) are of particular interest as they look at transition probabilities between disability levels. In both of these studies, disability was grouped into six categories: healthy, loss of IADL only, loss of 1–2 ADLs, loss of 3–4 ADLs, loss of 5–6 ADLs, and institutionalized (Inst'd).

Manton (1988) studied changes in functional level between 1982 and 1984, producing adjusted and unadjusted two-year transition probabilities between disability levels, stratified by gender and age (grouped into 65–74, 75–84, and 85+). His aim was to look at trends of disability in the general U.S. population using longitudinal data.

Manton et al. (1993) studied the change in incidence of chronic disability between 1982, 1984, and 1989, providing two- and five-year transition probabilities for changes in functional level, stratified by age (as above) but not by gender. This paper looked at changes in trends of disability over time by comparing transition probabilities between 1982–1984 and 1984–1989. We look at this paper in more detail as it raises some interesting methodological questions relevant to our research.

After adjusting for censored data, Manton (1988) and Manton et al. (1993) calculated maximum likelihood estimates of the t-year probability of a person age x moving from state i to state j ($P^{ij}_{x\,x+t}$)

$$P^{ij}_{x\,x+t} = \frac{n^{ij}_{x\,x+t}}{\sum_j n^{ij}_{x\,x+t}} \tag{3.1}$$

where, for example, in the case of the 1982 and 1984 NLTCS, $n^{ij}_{x\,x+t}$ is the number of people in state i, age x in 1982, and in state j, age $x + 2$ in 1984.

To compare transition probabilities between the two time periods 1982–1984 and 1984–1989, it is necessary to transform the five-year transition probabilities (from 1984–1989) into two-year transition probabilities, or vice versa. Assuming constant intensities, two-year transition probabilities can be calculated by raising the matrix of five-year transition probabilities to the power 2/5 (or five-year transition probabilities, by raising the matrix of two-year transition probabilities to the power 5/2). However, some of the two-year transition probabilities calculated in this way are negative. A simple solution to this is to set any negative probabilities to 0, as in Manton et al. (1993). However, this presents more difficulty if we are interested in the intensities because

1. In a multiple state model in which all nondead states communicate, if one transition probability is zero, then all transition probabilities are necessarily zero.
2. It implies that at least some of the intensities calculated from this data will be negative (or complex) (see Section 4 for more detail), as a set of positive intensities cannot produce negative probabilities.

This problem arises because not all discrete-time homogeneous Markov chains have transition probabilities consistent with continuous-time Markov processes. In fact, it would be slightly surprising if real data led to a set of transition probabilities that were consistent in this way.

The difference between this paper and the research described earlier is that we aim to

1. Model the underlying disability process by estimating the intensities of disability.
2. Set out consistent methodology for dealing with such data that would be of use for modeling purposes and for the calculation of transition probabilities.

Another approach, closer in nature to the methodology taken in this paper, is given in Robinson (1996). The focus of this work is to estimate the transition intensities underlying matrices of transition probabilities between various health statuses based on the 1982 and 1984 NLTCS. By assuming parametric forms for the transition intensities (arrived at, in part, by trial and error), the author avoids the possible complications associated with transforming a matrix of transition probabilities to one of transition intensities (see Section 4.2 for details). Once the parametric forms are specified, it is then a straightforward task to extract the maximum likelihood estimates of the parameters.

The methodology described in this paper has the following advantages over this approach:

1. By directly estimating the intensities of disability, no a priori shape is imposed on the transition intensities. Once the point estimates and their confidence intervals are calculated, the investigator can choose how to proceed. In this investigation, we chose simple graduation formulas, the form of which was dictated by the shape of the maximum likelihood point estimates.
2. It is not necessary to assume that types of transition intensities behave in a similar manner. For example, in Robinson (1996), the value of one of the parameters is determined by whether the transition intensity represents mortality, recovery, or any other transition.
3. It produces point estimates of the intensities of disability and confidence intervals—which are, themselves, of interest. For example, they could be used to investigate whether there are any similarities between types of transition intensities.

3.3 Classification of Disability

We grouped disability into the same 6 categories, as did Manton (1998) and Manton et al. (1993). We chose the same set of groupings because

1. Grouping disability in this way had desirable features, this grouping was predictive of mortality and disability.
2. The groups were not so small as to reduce their credibility and gave a reasonable number of groups to model.
3. The raw data were already classified into these groupings, making data extraction easier.
4. This previous research could then be used as a check.

There are many other ways and instruments to classify disability; however, the use of ADLs is consistent with methods used by insurance companies in some countries, including the United Kingdom (see Section 2.2). For more information on the classification of disability see Manton (1988).

For more specific modeling, it may be necessary to split some of these states further, providing the data are available. For example, to model a product that pays out half the sum assured on the loss of two ADLs and the full sum assured after the loss of three or more ADLs, the 1–2 ADL group would need to be split into two separate groups. However, this grouping is sufficient for our purposes—we shall model contracts that pay out the full sum assured on the loss of three or more ADLs or while institutionalized. It should be noted that not all lives that are institutionalized would be eligible for benefits under a LTC insurance contract—data permitting, we could reduce the benefits payable to reflect the proportion of institutionalized lives not eligible for benefits. However, since our aim is to model the general features of an LTC insurance contract and not any specific contract, we will not make any adjustments for this. An alternative approach given in Robinson (1996), avoids the use of health care setting by defining all states in terms of health status.

Cognitive impairment is also assessed in the NLTCS and could be used to incorporate cognitive impairment into a model of LTC; however, this paper focuses on modeling disability as defined by the states discussed earlier.

Before the data could be used, adjustments had to be made for nonresponders in each of the survey years. Readers interested in these adjustments can find the details in Pritchard (2002). Table 1 summarizes the raw data after adjusting for censored cases. For the same reasons as in Manton (1988), two groups of people were classified separately, as their experiences were unlike those of any other groups:

1. Nonresponders in 1982 had a worse disability experience and higher mortality, which can be explained by disability or poor health being a reason for nonresponse.
2. People institutionalized after April 1, 1982, but before the actual survey took place, had very high mortality rates and high recovery rates, which may be expected from admissions to institutions for acute medical reasons.

Both of these groups are left out of the analysis due to their unique experiences.

Table 1
Summary of Data from the 1982 and 1984 NLTCS, Adjusted for Censored Data

1982 Status	1984 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead	Total
Healthy	10,269.1	492.5	409.2	123.3	121.3	196.4	876.6	12,488.6
IADL Only	285.2	563.2	348.7	83.2	83.2	104.1	280.0	1,748.0
1–2 ADLs	120.7	231.0	693.1	243.6	141.7	158.5	417.3	2,006.3
3–4 ADLs	16.6	33.2	150.7	201.7	167.0	84.2	211.3	865.3
5–6 ADLs	16.8	36.8	68.4	93.7	313.9	101.1	389.4	1,020.4
Inst'd as of April 1, 1982	12.0	10.0	9.0	15.1	14.1	959.5	738.2	1,758.2
Inst'd after April 1, 1982	8.2	9.2	10.2	6.1	7.1	120.9	130.9	292.9
82 nonresponders	26.7	16.0	30.7	21.4	28.1	58.8	123.0	305.0
Total	10,755.7	1,392.3	1,720.5	788.4	877.1	1,783.9	3,167.0	20,485.0

3.4 Model of Disability

A suitable type of model, which easily represents the complex process of disability and naturally allows for the inclusion of insurance costs, is the continuous-time, multiple-state model (see Macdonald 1996 or Waters 1984 for general discussions of these models).

The main disadvantage of using such models is that they require the estimation of a large number of transition intensities, which, in turn, requires detailed data. However, in the short run, models can be tailored, to a certain extent, to the available data. In the long run it may be more appropriate to tailor data collection to the demands of suitable models, which can only be done once this demand is known—a result of specifying an appropriate model in the first place. In particular, simpler, more product-specific models may not have the flexibility to model new insurance products, creating the need, with each new product, to design a new model. This may be especially important in the potentially diverse LTC insurance market.

Figure 1 shows a model of disability, defined in terms of loss of activities of daily living. There are seven states in this model, which we denote as follows: Healthy—state 1; IADL only—state 2; 1–2 ADLs—state 3; 3–4 ADLs—state 4; 5–6 ADLs—state 5; institutionalized—state 6; and dead—state 7. The reasons for the grouping of disability states in this way were discussed in Section 3.3. The states can easily be expanded, if necessary, using the same methodology if the data are available. Disability is a reversible process (see Section 4.1) and is modeled as such, although this does require the estimation of a large number of transition intensities. However, given sufficient data, this is just a matter of "number crunching." Cognitive impairment is also covered by LTC insurance (discussed in Section 2.3), but only claims arising from disability will be considered in this paper. This will include a proportion of claims whose underlying cause is cognitive impairment—those persons who lose sufficient ADLs because of cognitive impairment, which makes them eligible for benefits, before being eligible on account of cognitive impairment itself.

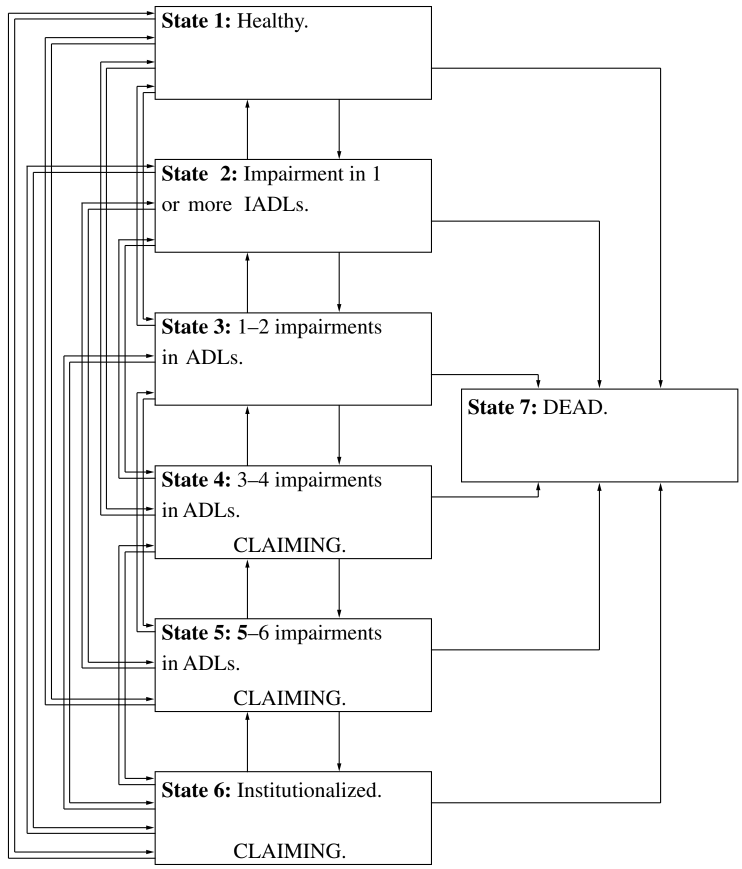
This model is set in continuous time, which ensures that there are no complications about which of the many possible transitions occur in which order. In such a framework, movements between states are defined by transition intensities, or just "intensities" for short. These can be thought of as the instantaneous probabilities of moving between states. More formally, the intensity between state i and state j for a life age x, μ_x^{ij}, is defined as

$$\mu_x^{ij} = \lim_{dt \to 0} \frac{P_{x\,x+dt}^{ij}}{dt}$$

where $P_{x\,x+dt}^{ij}$ is the same as in equation (3.1).

A final comment on the model presented here: A comprehensive model of LTC costs would need to include all forms of cognitive impairment as well as disability. However, this would require data, at the individual level, of progression through ADLs and all forms of cognitive impairment. Until such data become available (from demographic studies or as the experience of in-

Figure 1
Model of Disability for the Lifetime of an Individual and LTC Insurance



sured lives increases), such detailed models cannot be utilized. A major cause of claiming in LTC insurance is onset of Alzheimer's disease; for a discussion of Alzheimer's disease and LTC insurance see Macdonald & Pritchard (2001).

3.5 Outline of Method

The data do not provide sufficient information to estimate the transition intensities directly. This is because the usual maximum likelihood estimators (occurrence–exposure rates) require information at the individual level of the numbers and times of transitions between states (Waters 1984). The NLTCS do not provide this much detail (see Section 3.1), since they do not keep track of when individual lives move between states.

As in Section 3.2, we calculate the two-year transition probabilities of disability and recovery (in Section 4.1) and show in Section 4.2 that these are not consistent with a set of positive, continuous-time, Markov-chain intensities (as some of the implied intensities are negative). To avoid negative intensities, we use a penalized maximum likelihood method to constrain the estimates to lie in the feasible region of the parameter space; the unconstrained estimates are still a useful starting point for the fitting process. We do this in Section 4.3, where we describe a method for obtaining the maximum likelihood estimates. Then in Section 5.1, we calculate approximate confidence intervals for these estimates and graduate them in Section 5.2.

4. ESTIMATION OF TRANSITION INTENSITIES

As mentioned earlier, the methods used will be illustrated with data from the 1982 and 1984

NLTCS only, for male and females combined and for the five age groupings: 65–69 years, 70–74 years, 75–79 years, 80–84 years, and 85+ years. Results for males and females separately, for the other survey years, and for the data grouped into three age bands (65–74 years, 75–84 years, and 85+ years) can be found in Pritchard (2002).

4.1 Calculation of Two-Year Transition Probabilities

The two-year transition probabilities for men and women [equation (3.1)], are given in Table 2. It is noticeable that the probability of recovery from disability can be fairly high, especially for younger age groups, justifying the use of a disability-recovery model.

It is worth noting a few features of mortality:

1. As expected, it generally increases with age for all levels of disability.
2. As might be expected, it also increases with disability level for almost all age groups (i.e., people with greater disability have a higher two-year probability of death), the notable exceptions being the similar two-year probability of death for people with 1–2 ADLs and 3–4 ADLs in some age groups and the low mortality for 65–69-year-old institutionlized lives.

Point 2, supports the inclusion of IADL only as a separate state in the process of disability, as noted by Manton (1988).

4.2 Transformation of Transition Probabilities to Transition Intensities

If we assume the intensities are constant for each age group in the NLTCS data, we can easily compute the matrix of intensities from the matrix of two-year transition probabilities, calculated in Section 4.1. If $P(t)$ is the matrix of transition probabilities, and Q is the matrix of constant intensities (expressed as rates per annum), we have

$$P(t) = \exp(Qt) \tag{4.1}$$

The usual problem is to calculate $P(t)$ from Q; here, it is the other way round. This is not an original problem—the theoretical issues of embeddability (Are the observations consistent with a continuous time Markov model?) and identification (Is the matrix of intensities unique?) are discussed in detail in Singer and Spilerman (1976). They also discuss methods for calculating the matrix of intensities from a matrix of transition probabilities and touch on the issue of data errors causing the observations from an embeddable process to be nonembeddable. However, Singer and Spilerman do not discuss the practical problem of what to do if a matrix of intensities has negative or complex entries, when it is required that all intensities be real and positive. Israel, Rosenthal, and Wei (2001) investigate this issue and also provide a good summary of previous work done in this area. They describe four methods for dealing with negative (but not complex) intensities:

1. The method originally given by Jarrow, Lando, and Turnbull (1997), who, in parameterizing their credit risk model, assume that the probability of a company moving between more that one level of credit rating in a year is "small" (i.e., can be ignored)—this assumption reduces the estimation problem to a set of equations that can easily be solved.
2. Set all of the off-diagonal negative intensities equal to zero, and adjust the diagonal entries accordingly.
3. Set all of the off-diagonal negative intensities equal to zero, and "redistribute" these negative intensities over the positive off-diagonal intensities in proportion to their size (leaving the diagonal intensities unaffected).
4. In a remark, it is mentioned that method 3 could be improved by "optimizing this choice as a multivariate function," but they note that the improvements were rarely substantial and do not provide details on how this might be done.

They then go on and apply these methods to matrices of transition probabilities of a company's movements between credit ratings. To compare the methods they look at the "distance" (L^1-norm) between the original matrix of transition probabilities and the matrix of transition probabilities that is calculated from the estimated matrix of intensities using the preceding methods. (The L^1-norm is simply the sum of the absolute differences between the elements of two matrices.) Using the L^1-norm they show that method 2 gives substantially better results than method 1; in one example, the L^1-norm was just over 40

Table 2

Two-Year Transition Probabilities between Disability States Calculated from the 1982 and 1984 NLTCS, as a Percentage, for Males and Females

1982 Status	Age Group	1984 Status: Healthy	IADL Only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	90.01	2.41	1.92	0.59	0.56	0.47	4.04
	70–74	83.72	3.60	2.41	0.95	0.93	1.11	7.28
	75–79	76.74	5.40	4.08	1.30	1.24	1.94	9.30
	80–84	65.24	6.42	7.07	1.95	1.69	4.46	13.18
	85+	47.30	8.67	11.07	2.58	3.40	8.20	18.78
IADL Only	65–69	30.69	31.53	15.70	4.55	4.28	2.19	11.06
	70–74	25.31	32.42	17.86	3.27	2.82	4.94	13.39
	75–79	15.40	30.60	18.75	5.01	5.31	7.54	17.39
	80–84	9.40	29.64	25.34	4.47	5.85	8.02	17.28
	85+	2.21	28.72	21.09	7.32	6.50	9.88	24.28
1–2 ADLs	65–69	16.25	13.44	35.96	10.50	6.05	2.94	14.87
	70–74	10.49	13.22	34.40	12.35	5.72	6.92	16.89
	75–79	9.20	12.36	34.83	10.34	6.71	6.08	20.48
	80–84	5.68	10.39	31.89	11.66	5.72	9.68	24.99
	85+	3.69	6.91	28.19	13.33	10.21	13.77	23.91
3–4 ADLs	65–69	7.19	4.69	28.22	23.10	13.19	4.74	18.85
	70–74	8.40	3.31	19.30	28.75	18.52	4.00	17.72
	75–79	3.76	3.51	18.31	20.65	16.51	11.01	26.25
	80–84	2.34	4.83	14.66	21.69	21.72	12.03	22.73
	85+	1.03	3.79	5.94	18.43	23.17	16.08	31.56
5–6 ADLs	65–69	7.02	5.77	8.75	9.05	32.98	5.86	30.57
	70–74	6.13	5.07	8.92	10.24	28.49	8.02	33.12
	75–79	3.96	3.81	4.91	7.73	34.63	11.54	33.41
	80–84	3.73	4.50	6.60	10.38	26.68	11.16	36.93
	85+	1.33	0.58	5.22	7.61	26.42	12.04	46.80
Inst'd	65–69	7.54	0.97	1.63	2.30	0.12	72.22	15.22
	70–74	2.97	0.75	1.81	1.25	2.36	52.39	38.48
	75–79	1.74	1.50	0.87	0.74	1.48	56.86	36.81
	80–84	0.93	0.58	0.57	0.77	0.54	58.88	37.74
	85+	0.46	0.31	0.05	0.71	0.58	49.25	48.64

times less for method 2. Method 3, in the same example, gives a slight further improvement over method 2, and no figures were given for method 4.

Method 1 seems overly simplistic, especially when presented with other simple methods that could be used and that have been shown (by some criteria) in some examples to provide better estimates. Also, in the case of credit ratings, it may be realistic that only one transition occurs in a year, whereas for movement between disability states over a two- or five-year period, it does not seem at all plausible. Methods 2 and 3 offer simple practical solutions to the problem of negative intensities. However, the only justification for these methods is that they produce matrices of intensities, which lead to transition probabilities "close" to the original matrices of transition probabilities, in terms of the L^1 -norm. We make the following comments about these methods:

1. There is no statistical justification for using the L^1-norm. This measure of "distance" gives equal weight to all errors between probability matrices. A more useful measure should allow for the credibility of each probability (i.e., probabilities based on more data should have a greater weight attached to them).
2. There is no intuitive reason for the negative intensities to be set to 0, unless the physical process dictates this.
3. The adjustments are done on a row-by-row basis, which implicitly assumes that adjustments to one row do not affect other rows. This seems intuitively wrong: If the intensity from state A to state B is negative, then it seems

likely that by setting it to zero, there would be some effect on the reverse intensity—that from state B to state A. Neither of methods 2 or 3 allow this to happen.
4. None of the suggested methods work if the matrix of intensities has complex entries.

For any given distance function, the issues in the last three points above can simply be solved by minimizing the distance function with respect to the intensities, subject to keeping all the intensities positive. This also has the advantages that it will always produce estimates of the intensities that minimise the distance function by definition, and, by using different starting points for the minimization routine, the uniqueness of the estimates can be checked.

An obvious choice of distance function to use, allowing for point 1, is minus the likelihood function—rather than minimizing a distance function, we can maximise the likelihood function. The resulting estimates would then be the maximum likelihood estimates subject to being real and positive. The details of how this can be done are discussed in the next section. We first need to convert the matrices of transition probabilities into matrices of intensities.

The method given in Section 6.4.2 of Kulkarni (1995) is easily inverted if we notice that $P(t)$ and Qt have the same eigenvectors, and the eigenvalues of $P(t)$ are the exponentiated eigenvalues of Qt.

Table 3 gives the annual piecewise constant intensities for males and females using this method. We will refer to these as the initial estimate (IE) intensities and use the notation $\hat{\mu}_r^{i\,j}$. The diagonal entries are left blank here for clarity, although, strictly speaking, for any given state they should be minus the sum of the intensities out of that state. As predicted in Section 3.2, a number of the off-diagonal intensities are negative, which have no meaning in models of physical processes. We include the estimates here because

1. They are a good starting point for a search for positive intensities, as discussed earlier.
2. They are required to calculate confidence intervals for the intensities in Section 5.1.

4.3 MLE of Transition Intensities

Our aim is to adjust the matrices of IE intensities to eliminate the negative intensities, while keeping the resulting matrices of transition probabilities as close (in the sense of likelihood value) as possible to the original matrices of transition probabilities. More specifically we want to maximize the likelihood function, L_r, for each age group r where:

$$L_r \propto \prod_{all\ \ i,j} [P^{ij}_{rr+t}(\mu_r^{1\ 2}, \ldots, \mu_r^{n\ n-1})]^{n^{i\,j}_{rr+t}} \qquad (4.2)$$

which is equivalent to maximizing the log likelihood function, l_r:

$$l_r \propto \sum_{all\ \ i,j} (n_r^{i\ j}{}_{r+t} \log[P_r^{i\ j}{}_{r+t}\ (\mu_r^{1\ 2}, \ldots, \mu_r^{n\ n-1})]) \qquad (4.3)$$

where $t = 2$ when using the 1982–1984 NLTCS and $t = 5$ for the 1984–1989 and 1989–1994 NLTCS. The intensities, $\mu_r^{i\ j}$, are assumed to be constant for each age group, r, but can vary between age groups. We have written the $P_r^{i\ j}{}_{r+t}$ here as functions of the intensities, obtained explicitly by solving Kolmogorov's forward equations. The problem is now to find a set of *positive* intensities that maximise the log likelihood function. We can do this by finding

$$\max_{\mu_r^{i\ j} i\neq j} \left\{ \sum_{all\ \ i,j} \{n_r^{i\ j}{}_{r+t} \log[P_r^{i\ j}{}_{r+t}(\mu_r^{1\ 2}, \ldots, \mu_r^{n\ n-1})]\} + F(\mu_r^{1\ 2}, \ldots, \mu_r^{n\ n-1}) \right\} \qquad (4.4)$$

where:

$$F(\mu_r^{1\ 2}, \ldots, \mu_r^{n\ n-1}) = \begin{cases} 0 & \text{if } \mu_r^{i\ j} \geq 0\ i \neq j \\ -K + \sum_{i\neq j} \min(0, \mu_r^{i\ j}) & \text{otherwise} \end{cases}$$

for each age group r. The function F is a penalty function, which ensures that all intensities are kept positive during a computational maximization scheme. If all intensities are positive, F is 0; if one or more intensities are negative, then F has a large negative value that decreases as the intensities become less negative (which ensures the gradient is in the correct direction). The constant K is chosen with respect to the size of the log likelihood to ensure that the ''cost'' of a negative intensity is sufficiently large for the intensity to be kept nonnegative. It does not penalize a zero intensity, as this is a reasonable estimate (unlike a zero transition probability).

Table 3

Annual Intensities between Disability States Calculated from 1982 and 1984 NLTCS, for Males and Females

1982 Status	1984 Status							
	Age Group	Healthy	IADL Only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0198	0.0119	0.0027	0.0029	0.0022	0.0184
	70–74		0.0315	0.0132	0.0056	0.0059	0.0057	0.0338
	75–79		0.0507	0.0254	0.0072	0.0055	0.0092	0.0420
	80–84		0.0647	0.0523	0.0106	0.0077	0.0263	0.0599
	85+		0.1072	0.1157	−0.0071	0.0277	0.0582	0.0802
IADL Only	65–69	0.2607		0.2321	0.0339	0.0440	0.0138	0.0562
	70–74	0.2274		0.2931	−0.0139	0.0297	0.0376	0.0640
	75–79	0.1391		0.3007	0.0314	0.0518	0.0670	0.0795
	80–84	0.0857		0.4693	−0.0542	0.1085	0.0493	0.0461
	85+	0.0122		0.3906	0.0350	0.0503	0.0635	0.1223
1–2 ADLs	65–69	0.0943	0.2214		0.1966	0.0466	0.0179	0.0822
	70–74	0.0457	0.2217		0.2213	0.0192	0.0710	0.0880
	75–79	0.0635	0.2058		0.2034	0.0398	0.0320	0.1090
	80–84	0.0494	0.1804		0.2711	−0.0286	0.0816	0.1657
	85+	0.0471	0.1040		0.3084	0.0410	0.1180	0.0951
3–4 ADLs	65–69	0.0161	−0.0583	0.5779		0.2342	0.0373	0.0967
	70–74	0.0600	−0.0416	0.3288		0.3539	0.0013	0.0577
	75–79	0.0119	−0.0313	0.3876		0.3170	0.1132	0.1394
	80–84	−0.0035	0.0162	0.2897		0.5482	0.0962	0.0387
	85+	0.0032	0.0851	0.0521		0.5996	0.1823	0.0774
5–6 ADLs	65–69	0.0325	0.0788	0.0418	0.1637		0.0519	0.2283
	70–74	0.0312	0.0672	0.0797	0.1783		0.0936	0.2504
	75–79	0.0264	0.0510	0.0139	0.1490		0.1131	0.2227
	80–84	0.0354	0.0654	0.0328	0.2464		0.1137	0.2897
	85+	0.0138	−0.0149	0.0957	0.1670		0.1330	0.3682
Inst'd	65–69	0.0441	0.0081	0.0030	0.0277	−0.0047		0.0856
	70–74	0.0192	0.0034	0.0154	0.0088	0.0270		0.2533
	75–79	0.0106	0.0161	0.0028	0.0071	0.0138		0.2355
	80–84	0.0069	0.0055	0.0017	0.0099	0.0022		0.2407
	85+	0.0047	0.0031	−0.0023	0.0117	0.0028		0.3368

To implement the minimization routine discussed earlier, initial matricies of intensities are required. A natural choice for this starting point is, if it is not complex, the matrix of IE intensities from the previous section. If the matrix of IE intensities does contain complex entries (usually if one entry is complex, then they all are), then a simple starting martix can be used, with every annual intensity set to an arbitrary value of 0.01—from which the maximization routine will still converge but will take considerably longer to do so than from the IE intensities.

The estimates from this routine are the maximum likelihood estimates when the stochastic process is constrained to a Markov form. Hence, we will refer them as the MLEs and use the notation $\bar{\mu}^{i\,j}_{x+t}$. There are standard numerical routines available to solve the Kolmogorov equations and to maximize the penalized log-likelihood. See Pritchard (2002) for a discussion of some of these methods.

4.4 MLEs Compared with IEs

We now compare the MLEs with the intensities calculated in Section 4.2, which we refer to as the initial estimates (IEs). We also compare two-year transition probabilities calculated from these MLEs with the original two-year transition probabilities calculated directly from the data (given in Section 4.1). For males and females, the MLEs of the intensities are given in Table 4.

It is noticeable that for the majority of intensities, there is very little difference between the two methods of estimation. We know that there will be no negative intensities using the MLE method. However, in adjusting these to nonnegative values, we may expect that other intensities will change to compensate, especially intensities

Table 4

MLEs of Annual Intensities between Disability States for Males and Females Calculated from the 1982 and 1984 NLTCS

	1984 Status							
1982 Status	**Age Group**	**Healthy**	**IADL Only**	**1–2 ADLs**	**3–4 ADLs**	**5–6 ADLs**	**Inst'd**	**Dead**
Healthy	65–69		0.0198	0.0119	0.0028	0.0029	0.0023	0.0184
	70–74		0.0314	0.0134	0.0054	0.0060	0.0057	0.0338
	75–79		0.0507	0.0254	0.0072	0.0056	0.0092	0.0420
	80–84		0.0644	0.0533	0.0094	0.0084	0.0263	0.0598
	85+		0.1069	0.1115	0.0000	0.0243	0.0579	0.0807
IADL Only	65–69	0.2598		0.2285	0.0347	0.0435	0.0138	0.0562
	70–74	0.2257		0.2769	0.0000	0.0241	0.0384	0.0645
	75–79	0.1388		0.2978	0.0321	0.0515	0.0668	0.0795
	80–84	0.0871		0.4307	0.0000	0.0757	0.0493	0.0525
	85+	0.0122		0.3886	0.0323	0.0517	0.0637	0.1220
1–2 ADLs	65–69	0.0977	0.1991		0.1893	0.0500	0.0179	0.0819
	70–74	0.0490	0.2032		0.2038	0.0269	0.0699	0.0873
	75–79	0.0645	0.1940		0.2001	0.0413	0.0327	0.1089
	80–84	0.0477	0.1749		0.2275	0.0000	0.0818	0.1605
	85+	0.0469	0.1026		0.2980	0.0463	0.1186	0.0945
3–4 ADLs	65–69	0.0083	0.0000	0.5143		0.2174	0.0374	0.0989
	70–74	0.0529	0.0000	0.2829		0.3386	0.0032	0.0595
	75–79	0.0095	0.0000	0.3554		0.3102	0.1114	0.1401
	80–84	0.0000	0.0197	0.2696		0.5076	0.0963	0.0454
	85+	0.0042	0.0692	0.0563		0.5877	0.1808	0.0798
5–6 ADLs	65–69	0.0342	0.0676	0.0523	0.1586		0.0519	0.2276
	70–74	0.0337	0.0535	0.0923	0.1715		0.0930	0.2497
	75–79	0.0271	0.0442	0.0197	0.1467		0.1135	0.2225
	80–84	0.0339	0.0638	0.0408	0.2263		0.1135	0.2867
	85+	0.0130	0.0000	0.0847	0.1570		0.1337	0.3663
Inst'd	65–69	0.0442	0.0065	0.0052	0.0228	0.0000		0.0851
	70–74	0.0191	0.0036	0.0153	0.0088	0.0269		0.2533
	75–79	0.0107	0.0158	0.0030	0.0070	0.0138		0.2355
	80–84	0.0069	0.0055	0.0019	0.0094	0.0025		0.2406
	85+	0.0044	0.0022	0.0000	0.0107	0.0027		0.3368

complementary to negative intensities (one that acts between the same states but in the opposite direction, e.g., $\mu_{x+t}^{j\,i}$ is complementary to $\mu_{x+t}^{i\,j}$). We make the following observations:

1. Almost all negative intensities in the IE routine were estimated as 0 using the MLE routine. However, there are exceptions (see Pritchard 2002, for examples).
2. As expected, the largest differences are between intensities complementary to negative IE intensities (e.g., for men and women aged 80–84, the MLE of the intensity between state 4 and state 2 is substantially greater).
3. There are significant differences between estimates of some intensities that are not directly influenced by negative IE intensities (e.g., for men and women aged 80–84, the transition between 1–2 ADLs and 3–4 ADLs).

The first point appears to suggest that it may be more efficient to calculate the maximum likelihood estimates of the transition probabilities (from data), transform them into intensities, and then set any negative intensities to 0 (as in Section 4.2), rather than using a penalized maximum likelihood approach, as discussed in Section 4.2. We would argue against this method as

1. Not all of the MLEs of negative intensities were 0; all intensities should have the possibility to take a positive value.
2. There is no intuitive reason they should be set to 0, unless the physical process being modeled dictates this, which is not the case here.
3. Quite a few intensities changed significantly, compensating in some way for the intensities that were forced to be nonnegative.

4. The transformation of transition probabilities to intensities does not always produce a set of real intensities from which to start.

Table 5 provides further support for not simply setting the negative intensities to zero. The table shows the log-likelihood values (equation [4.3]) for the initial estimates (highest value by definition), the initial estimates with all negative intensities set to zero (we refer to these as adjusted estimates), and the MLEs. The MLEs always have a higher likelihood score, and the likelihood score for the other methods described in Section 4.2 will lie in between.

We can make further comparisons by using the methods described in Section 4.3 to calculate two-year transition probabilities from the MLEs—which are then comparable to the two-year transition probabilities calculated direct from the data. These transition probabilities for males and females are given in Table 6. These can be thought of as the matrices of transition probabilities closest (in the sense of likelihood function) to the original transition probabilities that are consistent with a continuous-time Markov chain with constant intensities. This means that they can also be transformed into nonnegative transition probabilities over any time span, using the methods in Section 3.2.

As expected there are no 0 transition probabilities in these estimates (even though some of the intensities are zero), and overall there is not much difference between the probabilities using these two methods of estimation. It is not estimation of the transition probabilities that is of primary interest though—the main aim is the estimation of the underlying intensities, which we can then apply to the problem of estimating the cost of disability in LTC insurance. In particular, we have calculated estimates for the piecewise constant intensities between disability states for the specified age bands—these are the crude intensities. The aim of the next section is to graduate these to get smooth, time-continuous intensities.

Table 5

Comparison of Log-Likelihood Values for the Initial Estimates,* Adjusted Estimates, and Maximum Likelihood Estimates

Gender	Age Group	Log-Likelihood Value for Initial Estimate	Adjusted Estimate	MLE
Male and Female	65–69	−4186.71	−4188.62	−4187.52
	70–74	−4780.57	−4782.02	−4781.23
	75–79	−4580.38	−4580.93	−4580.62
	80–84	−3576.82	−3580.53	−3577.39
	85+	−3283.32	−3284.97	−3284.39

*Initial estimates with negative intensities set to 0.

5. CONFIDENCE INTERVALS AND GRADUATION OF THE INTENSITIES

5.1 Confidence Intervals

If the estimated intensities were the unconstrained MLEs, then the variance estimates could easily be calculated by numerical differentiation of the log-likelihood function, but they are not quite—they are MLEs constrained to be nonnegative and would cause the first derivatives of the log-likelihood to be nonzero. An approximate method to estimate the variances of the MLEs would be to substitute the IE estimates in their place in the above procedure. However, there are two problems with this method:

1. There can be quite substantial differences between the MLEs and the IEs, which, in turn, would introduce errors into the variance estimates.
2. If the IEs are complex, then this procedure will not work.

However, this method can be adapted to calculate variance estimates for the MLEs. Omitting most of the detail (which can be found in Pritchard 2002), if we assume that the MLEs are the true underlying parameters of the process, we can use them to adjust the data to be consistent with them—this addresses both of the issues raised above. This method involves the following steps:

1. From the MLEs of the intensities, calculate the corresponding transition probabilities.
2. Use these transition probabilities to calculate the expected number of lives that move between states by multiplying the number of lives starting in a given state from the data by the corresponding transition probability.
3. As the MLEs of the intensities maximize the likelihood function using these numbers of transitions, the information matrix method

Table 6

Two-Year Transition Probabilities for Males and Females, Calculated from the MLEs of the Intensities between Disability States Calculated from the 1982 and 1984 NLTCS as a Percentage

	1984 Status							
1982 Status	Age Group	Healthy	IADL Only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	90.01	2.41	1.92	0.59	0.56	0.47	4.04
	70–74	83.73	3.60	2.41	0.95	0.93	1.11	7.28
	75–79	76.74	5.40	4.08	1.30	1.24	1.94	9.30
	80–84	65.24	6.41	7.08	1.95	1.69	4.46	13.18
	85+	47.35	8.67	10.83	2.81	3.37	8.20	18.78
IADL Only	65–69	30.67	31.61	15.67	4.55	4.27	2.19	11.05
	70–74	25.27	32.53	17.48	3.64	2.77	4.93	13.38
	75–79	15.39	30.65	18.73	5.01	5.31	7.53	17.38
	80–84	9.38	29.79	24.44	5.51	5.62	8.00	17.25
	85+	2.21	28.85	21.15	7.10	6.53	9.88	24.27
1–2 ADLs	65–69	16.29	12.84	36.49	10.47	6.08	2.95	14.89
	70–74	10.52	12.85	35.00	11.99	5.78	6.94	16.92
	75–79	9.22	12.06	35.08	10.33	6.73	6.09	20.50
	80–84	5.67	10.36	32.47	10.87	5.92	9.69	25.03
	85+	3.70	6.76	28.44	13.17	10.23	13.78	23.93
3–4 ADLs	65–69	7.15	6.27	26.77	23.56	12.68	4.73	18.83
	70–74	8.31	4.64	18.09	29.12	18.18	3.98	17.67
	75–79	3.73	4.37	17.57	20.78	16.35	10.97	26.23
	80–84	2.46	4.81	14.48	22.31	21.26	12.01	22.68
	85+	1.04	3.53	5.76	18.78	23.22	16.09	31.58
5–6 ADLs	65–69	7.03	5.54	8.85	9.01	33.15	5.86	30.57
	70–74	6.15	4.71	9.13	10.16	28.68	8.03	33.15
	75–79	3.97	3.64	4.98	7.71	34.73	11.56	33.42
	80–84	3.68	4.50	6.66	10.10	26.94	11.17	36.94
	85+	1.27	1.13	4.99	7.28	26.54	12.02	46.77
Inst'd	65–69	7.53	0.94	1.59	2.01	0.48	72.24	15.21
	70–74	2.97	0.75	1.81	1.24	2.36	52.40	38.48
	75–79	1.74	1.48	0.88	0.74	1.48	56.87	36.81
	80–84	0.93	0.58	0.57	0.77	0.53	58.88	37.74
	85+	0.44	0.25	0.18	0.69	0.55	49.26	48.63

can be used to estimate the variance (as the first derivatives of the log-likelihood will now be zero).

These variance estimates are useful for two reasons:

1. They provide a set of weights to use in the graduation process (see next section).
2. They enable us to calculate 95% confidence intervals for the IEs (for $\hat{\mu}^{ij}_{x+t}$) and to check if the MLEs ($\bar{\mu}^{ij}_{x+t}$) lie within these limits.

The 95% confidence intervals, calculated from the IEs ($\hat{\mu}^{ij}_{x+t} \pm 1.96\ \mathrm{SD}[\hat{\mu}^{ij}_{x+t}]$), are shown in Figures 2 to 7. For the numerical values of the estimated variances see Pritchard (2002).

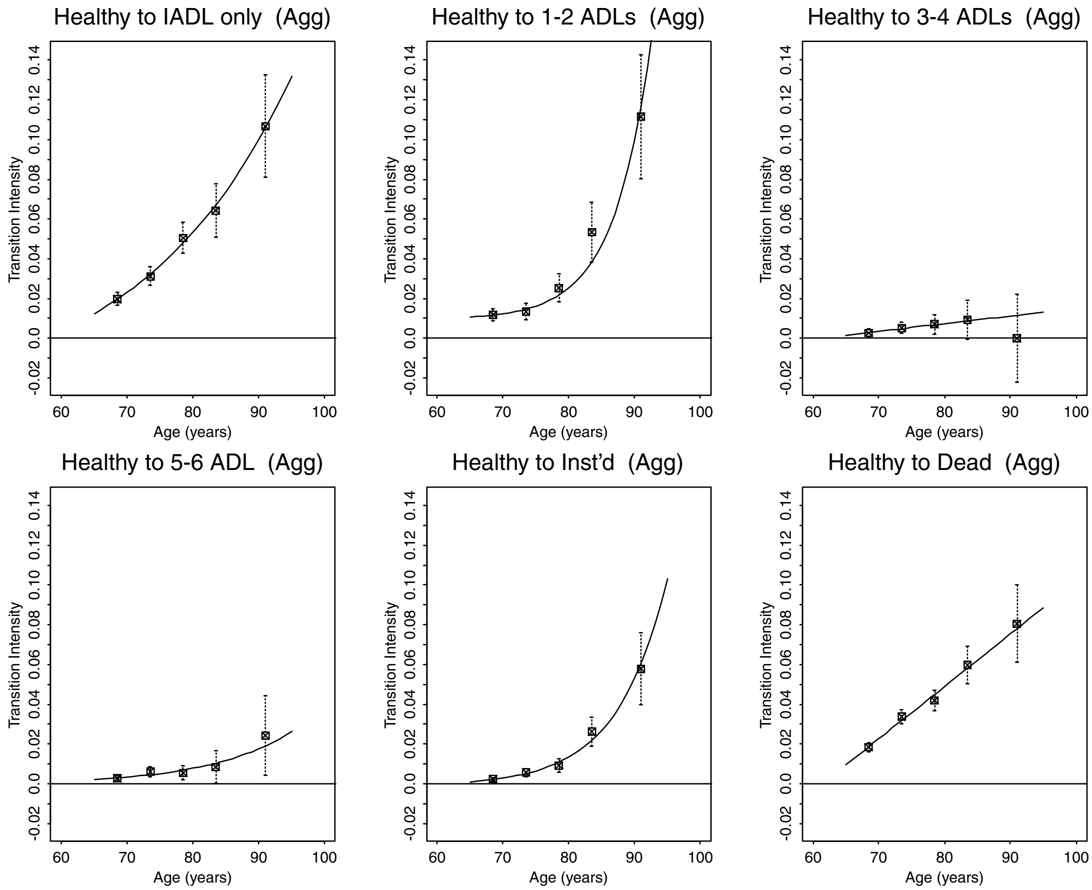
It is noticeable from the graphs that all but three of the MLEs for men and women (given in Table 4) lie within the 95% confidence limits of the IEs—when it would be expected that about nine (= 5% × 180 point estimates) would be outside the confidence intervals. This provides further support that constraining the estimation procedure is a reasonable method that produces sensible estimates.

5.2 Graduating the Intensities

We chose a very simple graduation procedure. With the data grouped in five-year age bands (65–69 years, 70–74 years, 75–79 years, 80–84 years, and 85+ years) there are five data points. We assume that the intensity for an age group is actually a point estimate of the intensity for the midpoint of that age group plus half of the duration between study, or age 90 years plus half of the duration between study for the 85+ group.

Figure 2

Point Estimates (MLE), Approximate 95% Confidence Intervals, and Parametric Fits of the Intensities Out of the "Healthy" State for Men and Women



From Table 7, which gives the mean ages for each five-year age band at the start of each survey period, this assumption seems reasonable, even though, as expected, the mean ages are slightly less than the midpoints for most age groups.

For the graduation, two parametric forms were used: either a Makeham curve or a straight line, which, from the shape of the data (illustrated in Figures 2 to 7), seem to provide adequate flexibility. It would be possible to fit more complex parametric forms (more parameters), but this would be at the cost of the smoothness of the fit, and, with only five data points, we felt this was not justified.

The method we use is, for each intensity, to fit both a straight line and a Makeham curve using weighted least squares, the weights (w^{ij}_{x+t}) being the inverse of the variance (i.e., $w^{ij}_{x+t} = 1/\mathrm{Var}[\hat{\mu}^{ij}_{x+t}]$). The parametric form which provides the best fit (in terms of the smallest sum of weighted squared residuals) was chosen. The form of the graduated intensities, $\mathring{\mu}^{ij}_{x+t}$, for $65 \le x + t \le 120$, is then, depending on which provides a better fit, either

$$\mathring{\mu}^{ij}_{x+t} = A_{ij} + B_{ij}\exp\{C_{ij}(x - 68.5 + t)\} \quad (5.1)$$

or

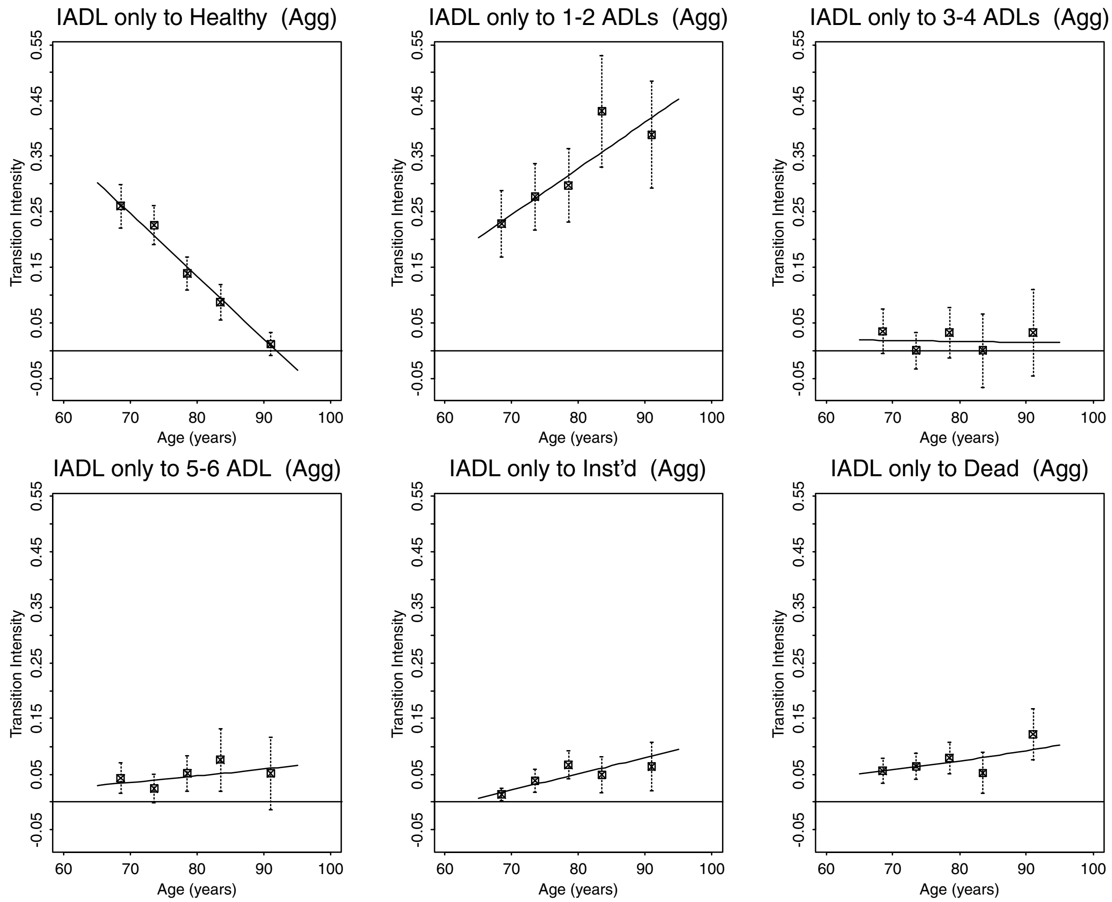
$$\mathring{\mu}^{ij}_{x+t} = A_{ij} + D_{ij}\,(x + t) \quad (5.2)$$

with a lower bound of zero on all intensities at all ages. The values for the parameters A, B, C, and D are given in Table 8 for males and females. The reasons for choosing a Makeham curve where possible and a linear form otherwise are

1. For modeling mortality, a Makeham curve is a reasonable assumption, given the number of data points.

Figure 3

Point Estimates (MLE), Approximate 95% Confidence Intervals, and Parametric Fits of the Intensities Out of the "IADL only" state for Men and Women



2. Increasing disability is a predictor of mortality and, as such, can be seen to be part of the aging process, which may then behave in a similar fashion.
3. Recovery from disability decreases with age, and a negative exponential may be appropriate.

With more data points, it may be worth investigating the trends more closely. However, in this case, we felt the data did not justify a more complex procedure.

Figures 2 to 7 give graphs of the intensities, for males and females, out of states 1–6, respectively, for the 1982–1984 NLTCS. They show the point estimates, in five-year age bands (MLEs), approximate 95% confidence intervals using the variance estimates from the previous section, and the parametric form of the intensities. While all graphs in a given figure use the same scale, it is worth noting that different figures use different scales and that the scales were chosen such that the confidence intervals could be shown on the graphs.

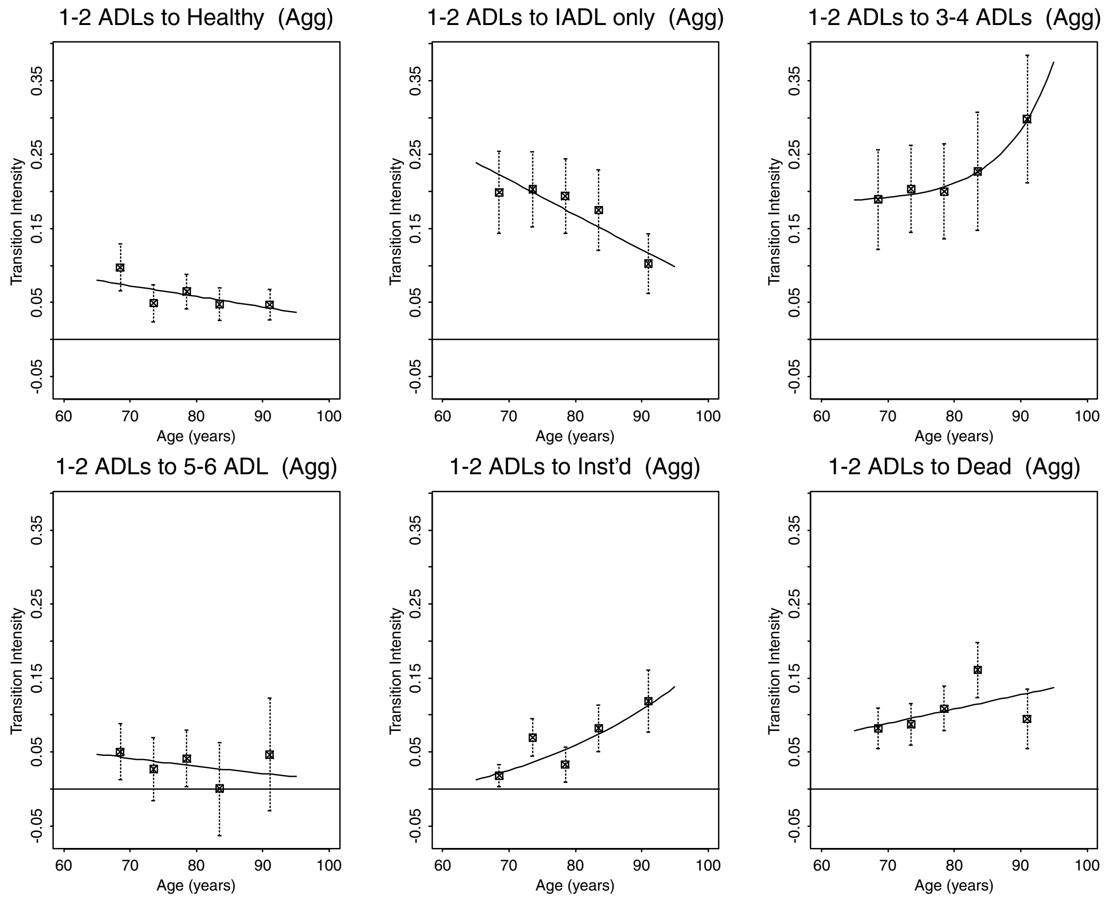
In general, the intensities are as expected: The force of disability and mortality increases with increasing age, and the force of recovery decreases with increasing age. There are only four exceptions to these rules and these all involve small intensities, calculated from relatively few data.

5.3 Overall Force of Mortality

In this section we compare the overall force of mortality from the disability model with a benchmark force of mortality. The motivation for comparing overall model mortality with a benchmark force of mortality is twofold:

Figure 4

Point Estimates (MLE), approximate 95% Confidence Intervals, and Parametric Fits of the Intensities Out of the "1–2 ADLs" State for Men and Women



1. The force of mortality has been subject to large-scale investigations, which provide benchmarks to compare the overall force of mortality from the disability model to see if it produces reasonable estimates.
2. If necessary, the intensities in the model can be adjusted to be consistent with the findings from these large-scale investigations, to make the model more reasonable for use in application (e.g., in estimating the cost of disability in a long-term care insurance contract).

The benchmark force of mortality we take from Thatcher, Kannisto, and Vaupel (1998). They compared the goodness of fit of six different models for mortality on data from 13 countries in the age ranges 80–120. For the purposes of comparison, we use the logistic model, for the reasons set out in Pritchard (2002), where the force of mortality is modeled as

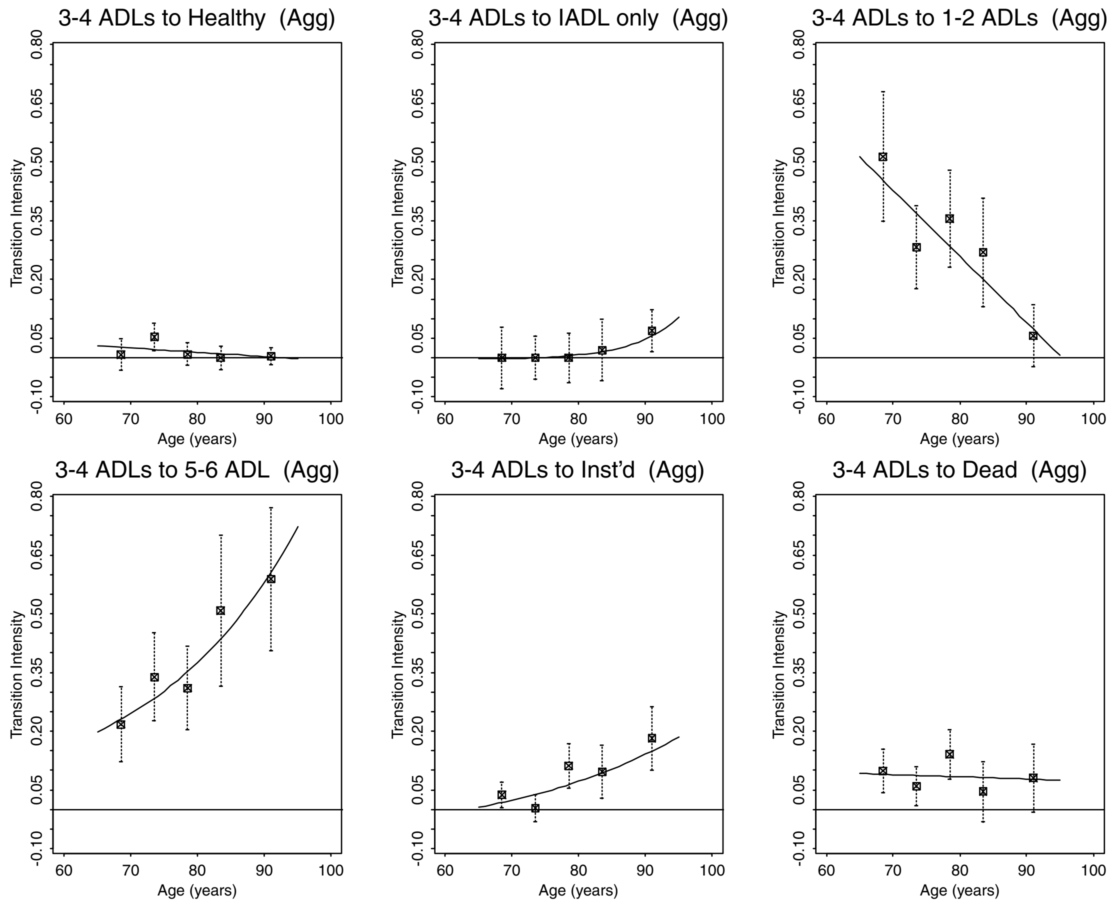
$$\mu_{x+t} = c + \frac{\alpha \exp\{b(x + t)\}}{1 + \sigma^2 \dfrac{\alpha}{b} [\exp\{b(x + t)\} - 1]} \tag{5.4}$$

and where the parameter values are given in Table 9.

Unfortunately, the models of mortality in Thatcher et al. (1998) are only fitted to males and females separately. While it would be possible to construct an aggregate benchmark force of mortality from the gender-specific ones, we felt this was not justified given that we are only using it as a benchmark. The model we use for comparison is that for females, since the majority of lives in the NLTCS are female, especially at older ages.

Overall mortality in the disability model (i.e., mortality aggregated across all disability states) can be calculated simply as the weighted sum of

Figure 5
Point Estimates (MLE), Approximate 95% Confidence Intervals, and Parametric Fits of the Intensities Out of the "3–4 ADLs" State for Men and Women



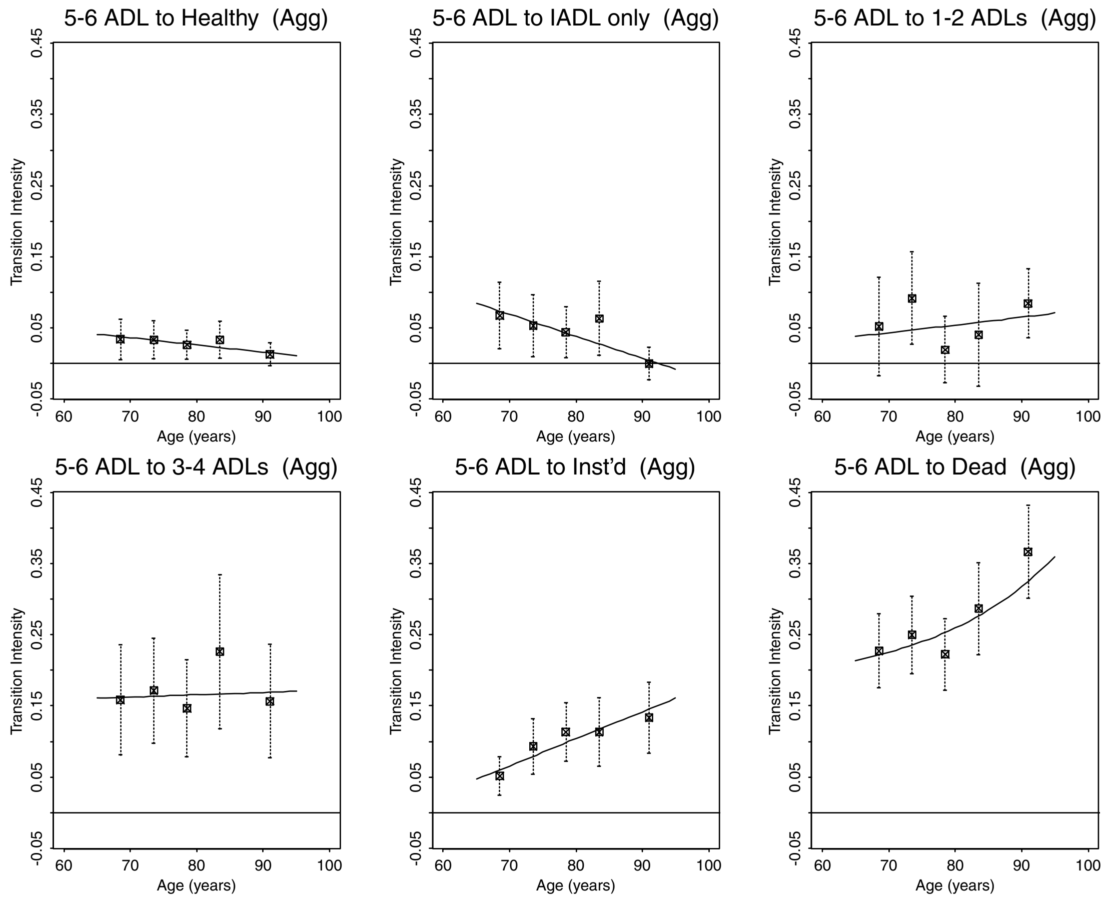
the force of mortality in each nondead state, where the weights are the probability of a life being in any given state. Assuming that a life is healthy at age 65 (a reasonable assumption for an insured life), the probability of the life being in state j at age 65+ t is then simply $P^{1\ j}_{65\ 65+t}$, the probability that the life moves from state 1 (Healthy) to state j in time t. The overall force of mortality, μ_{65+t}, at any age 65 + t can then be calculated as

$$\mu_{65+t} = \frac{\sum_{i=1}^{6} \mu^{i\,7}_{65+t} P^{1\ i}_{65\,65+t}}{\sum_{i=1}^{6} P^{1\ i}_{65\,65+t}} \qquad (5.5)$$

where $P^{1\ j}_{65\ 65+t}$ can be calculated by solving Kolmogorov's equations, for any given set of intensities and $\mu^{i\ 7}_{65+t}$ is the force of mortality in state i at age 65 + t.

Figure 8 compares overall mortality in the disability model with the benchmark force of mortality from the logistic model. It is noticeable that the aggregate mortality from the disability model is in very close agreement with the benchmark force of mortality in the age range 65–90 years. At older ages however, the benchmark force of mortality continues to increase faster than the overall force of mortality from the disability model. This may not be surprising given that the underlying intensities in the disability model are fitted to data with a maximum age of 91 years—after this, the intensities are extrapolations. This suggests that caution should be exercised when using this model to estimate insurance costs at the very highest ages. In particular, sensitivity of the costs to the mortality assumptions at these ages should be investigated

Figure 6
Point Estimates (MLE), Approximate 95% Confidence Intervals, and Parametric Fits of the Intensities Out of the "5–6 ADLs" State for Men and Women



(see Pritchard 2002 for a detailed sensitivity analysis).

5.4 Description of All Disability Models

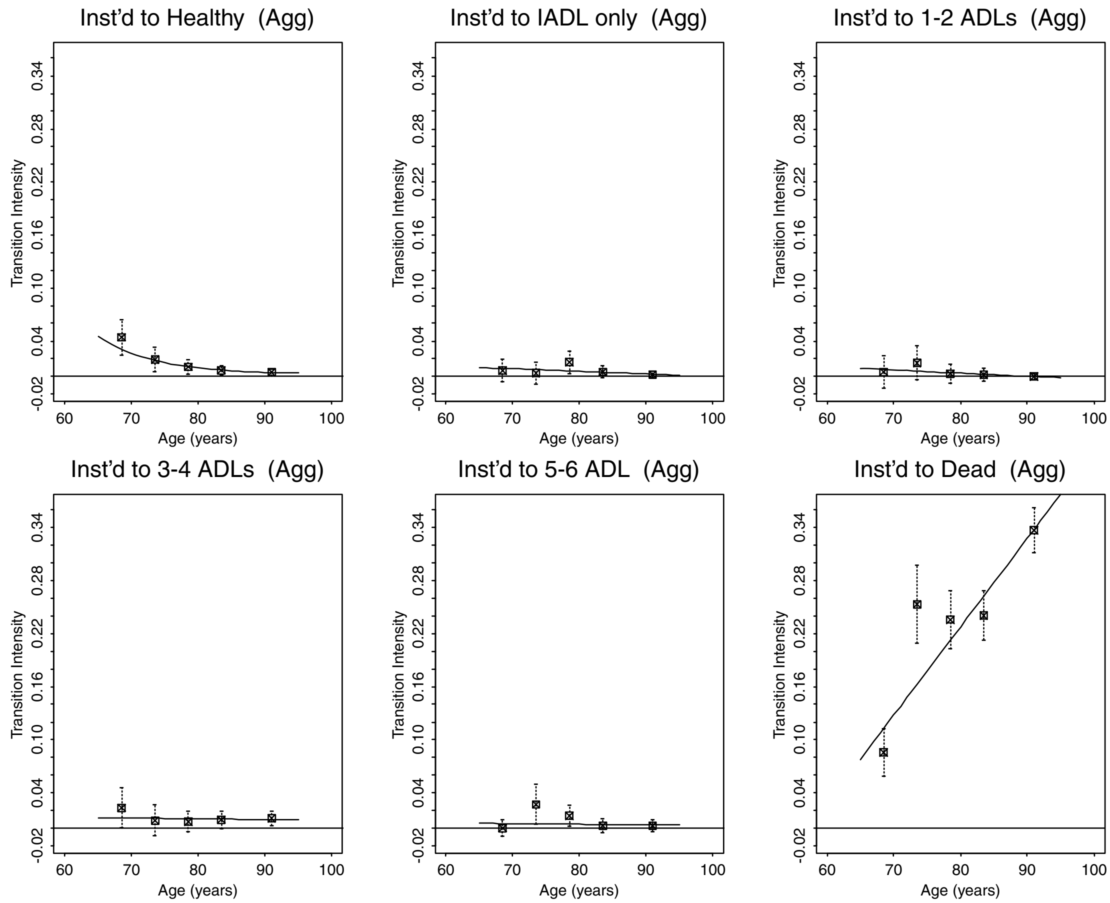
For clarity, we have so far focused on the data from the 1982 and 1984 NLTCS split into five age bands for males and females combined, demonstrating each step of the process of parameterizing the disability model. We have also parameterized the model using the same age groups for the 1984–1989 NLTCS and the 1989–1994 NLTCS; for age groups 65–74, 75–84, and 85+ years; and for males and females combined and separately. For details of all of these models and their parameters, see Pritchard (2002).

When investigating insurance costs, it will be useful to have more than one model for comparison and as a check of robustness. We will continue to look at males and females combined but use the models parameterized from all pairs of consecutive survey years and for both sets of age groupings. For brevity, we refer to the disability models parameterized from data grouped into 5-year age bands as the 19*XX*–19*YY* (5) models and those parameterised from data into 10-year age bands as the 19*XX*–19*YY* (10) models (where *XX*–*YY* = 82–84, 84–89, or 89–94).

It is worth mentioning an anomalous feature of the disability models parameterized using the 1984–1989 and 1989–1994 NLTCS. The overall force of mortality for these models was exceptionally low. This problem was traced back to the recorded numbers of deaths being very low—in fact, the recorded number of deaths in each of these five-year periods (1984–1989 and 1989–1994) is lower than for the two-year period 1982–

Figure 7

Point Estimates (MLE), Approximate 95% Confidence Intervals, and Parametric Fits of the Intensities Out of the "Institutionalized" State for Men and Women



1984. A reasonable explanation for this was that death in these later survey periods was a reason for a life being classified as "Not in survey year," a status not used in the 1984 NLTCS but heavily used in the 1989 and 1994 NLTCS.

The analysis in this paper is based on the 1997 NLTCS Public Use CD, which one of the anonymous referees helpfully identified as being incomplete, containing some temporary deletions of respondents from the NLTCS. The updated 1999 Public Use CD contains documentation that describes important supplemental data. Furthermore, they state that the number of deaths recorded in the NLTCS over the periods 1984–1989 and 1989–1994 is about 1,200–1,300 per annum.

Not being privy to this information at the time of the analysis, we adjusted the mortality for each state in the 1984–1989 and 1989–1994 models by setting them equal to the forces of mortality in the 1982–1984 models. For more details of this adjustment and checks of reasonableness, see Pritchard (2002). Given such adjustments to the 1984–1989 and 1989–1994 models, we place more reliance on the results from the 1982–1984 models, but still use the other models for comparison. Given that full data on mortality are available on the 1999 Public Use CD, it would be an interesting exercise to assess the impact of the

Table 7

Mean Ages of Lives within Each Age Group for Males and Females

Survey Year	Mean Age for Age Group				
	65–69	70–74	75–79	80–84	85+
1982	67.48	72.42	77.39	82.32	88.79
1984	67.74	72.37	77.32	82.29	88.00

Table 8
Parameter Values for the Parametric Intensities for Males and Females, Calculated from the 1982 and 1984 NLTCS, Grouped in 5-Year Age Bands

From State	To State	Parameter			
		A	*B*	*C*	*D*
Healthy	IADL only	-3.22×10^{-2}	5.19×10^{-2}	4.35×10^{-2}	—
	1–2 ADLs	9.58×10^{-3}	2.11×10^{-3}	1.74×10^{-1}	—
	3–4 ADLs	-2.34×10^{-2}	—	—	3.85×10^{-4}
	5–6 ADLs	-1.37×10^{-4}	3.16×10^{-3}	8.01×10^{-2}	—
	Inst'd	-9.05×10^{-4}	3.15×10^{-3}	1.32×10^{-1}	—
	Dead	-1.62×10^{-1}	—	—	2.64×10^{-3}
IADL Only	Healthy	1.04×10^{0}	—	—	-1.13×10^{-2}
	1–2 ADLs	-3.38×10^{-1}	—	—	8.32×10^{-3}
	3–4 ADLs	2.94×10^{-2}	—	—	-1.59×10^{-4}
	5–6 ADLs	-9.89×10^{-2}	1.33×10^{-1}	8.16×10^{-3}	—
	Inst'd	-1.81×10^{-1}	—	—	2.90×10^{-3}
	Dead	-3.19×10^{-2}	8.80×10^{-2}	1.60×10^{-2}	—
1–2 ADLs	Healthy	1.74×10^{-1}	—	—	-1.45×10^{-3}
	IADL only	5.45×10^{-1}	—	—	-4.71×10^{-3}
	3–4 ADLs	1.85×10^{-1}	5.62×10^{-3}	1.33×10^{-1}	—
	5–6 ADLs	-6.10×10^{-2}	1.04×10^{-1}	-1.11×10^{-2}	—
	Inst'd	-5.61×10^{-2}	7.72×10^{-2}	3.48×10^{-2}	—
	Dead	-4.68×10^{-2}	—	—	1.93×10^{-3}
3–4 ADLs	Healthy	1.03×10^{-1}	—	—	-1.11×10^{-3}
	IADL only	-4.26×10^{-3}	2.14×10^{-3}	1.48×10^{-1}	—
	1–2 ADLs	1.61×10^{0}	—	—	-1.69×10^{-2}
	5–6 ADLs	1.64×10^{-2}	2.13×10^{-1}	4.51×10^{-2}	—
	Inst'd	-9.20×10^{-2}	1.09×10^{-1}	3.52×10^{-2}	—
	Dead	1.27×10^{-1}	—	—	-5.50×10^{-4}
5–6 ADLs	Healthy	1.06×10^{-1}	—	—	-9.93×10^{-4}
	IADL only	2.85×10^{-1}	—	—	-3.08×10^{-3}
	1–2 ADLs	-1.81×10^{-1}	2.23×10^{-1}	4.62×10^{-3}	—
	3–4 ADLs	1.40×10^{-1}	—	—	3.16×10^{-4}
	Inst'd	-2.00×10^{-1}	—	—	3.80×10^{-3}
	Dead	1.76×10^{-1}	4.53×10^{-2}	5.28×10^{-2}	—
Inst'd	Healthy	2.39×10^{-3}	2.84×10^{-2}	-1.19×10^{-1}	—
	IADL only	2.89×10^{-2}	—	—	-2.90×10^{-4}
	1–2 ADLs	-3.10×10^{-2}	3.89×10^{-2}	-1.02×10^{-2}	—
	3–4 ADLs	-1.94×10^{-1}	2.05×10^{-1}	-3.68×10^{-4}	—
	5–6 ADLs	9.87×10^{-3}	—	—	-6.85×10^{-5}
	Dead	-5.71×10^{-1}	—	—	9.983×10^{-3}

preceding assumptions on the model parameterization and application.

Table 9
Parameter Values for Logistic Model Fitted to Data from 13 Countries at Ages 80–98 Years for Males and Females Separately over the Period 1980–1990

Parameter	Gender	
	Males	Females
a	2.99258×10^{-6}	1.46725×10^{-6}
b	1.02220×10^{-1}	1.08791×10^{-1}
c	9.93451×10^{-6}	9.99586×10^{-6}
σ	2.89834×10^{-1}	3.47119×10^{-3}

Source: Thatcher, Kannisto, and Vaupel (1998).

6. RESULTS

6.1 Modeling a Long-Term Care Contract

We assume that the LTC contract has a single premium paid at outset, so the only policy cash flows thereafter are the benefits. Benefits are payable continuously while in a claiming state, which, as discussed in Section 2.2, are the states that represent the loss of three or more ADLs (states 4, 5, and 6).

Figure 8
Comparison of Overall Mortality from Disability Model to Benchmark Force of Mortality Given by the Logistic Model by Thatcher, Kannisto, and Vaupel (1998).

The quantum of benefit is £1 per annum at inception of the policy, increasing continuously at rate δ_b per year. This is represented in the model by $b_t^j = \exp(\delta_b t)$ if $j = 4$, 5, or 6. The default we use is $\delta_b = 0.05$, representing indexation to earnings, since this is a feature of most LTC policies (Dullaway and Elliot, 1998). We use a risk-free discount rate of $\delta = 0.05$ compounded continuously throughout. The present value of the benefit is the random variable whose expected value and higher moments are obtained by solving the set of simultaneous differential equations given by Norberg (1995), the expected value being used in the traditional actuarial equation of value. We look at policies with inception ages of 60, 65, 70, and 75 years for illustration. It should be noted that the data used to parameterize the disability model start from age 65—in the calculations for an inception age of 60, the parametric transition intensities were extrapolated back to age 60. The motivation for this extrapolation is that LTC policies are generally available from age 60; however, the reader is advised to exert extra caution when interpreting the results at this age.

6.2 Costs of Disability

Table 10 gives the first three moments of the present value of benefits using the 1982–1984 models for lives entering at ages 60, 65, 70, and 75 in each state in the model for males and females combined. General trends displayed in the table are

1. The expected present value (EPV) of benefits increases with lives entering with increasing disability level, even for the nonclaiming disability states (states 2 and 3), indicating that disability may be a predictor of further disability.
2. The increase in the EPV of benefits is the greatest between lives starting in state 3 and those starting in state 4, which is as expected since lives starting in state 4 immediately start claiming benefits.

Table 10

Mean, Variance, and Skewness (q = 1, 2 and 3) of the Present Value of Disability Claims Costs for a Life Starting in Each Model State, Unit Benefit Increasing Continuously ($\delta_b = 0.05$), for Males and Females Combined Using the 1982–1984 (5) Model

Entry Age	q	State at Start of Contract: Healthy	IADL Only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd
60	1	1.9986	2.1463	2.5246	3.6596	3.8504	7.2711
	2	10.399	11.496	13.155	14.874	16.289	31.180
	3	76.511	90.090	104.908	114.024	131.132	196.274
65	1	1.9526	2.2783	2.6692	3.9193	3.9298	6.2260
	2	10.018	12.084	13.734	15.608	16.240	25.009
	3	71.879	91.554	103.526	110.932	120.269	150.968
70	1	1.9397	2.3823	2.7396	4.0996	3.8970	5.2410
	2	9.517	11.878	13.248	15.209	15.117	19.492
	3	64.646	82.764	90.761	96.534	100.925	113.449
75	1	1.9451	2.4165	2.7322	4.1575	3.7682	4.4207
	2	8.885	10.983	12.095	13.928	13.465	15.031
	3	55.846	68.967	74.400	78.203	80.904	83.364

3. The variance of the present value of benefits also tends to increase with lives entering with increasing disability level, indicating greater uncertainty for the costs of these lives.
4. The variance of the present value of benefits generally decreases for lives starting at older ages, which is also as expected since these lives have a shorter expected future lifetime and so there is less uncertainty of their costs.

Table 11 gives the EPV of benefits for lives entering at ages 60, 65, 70, and 75 in the healthy state only (which is of primary interest for insurance purposes) for all models and gender classifications. It is noticeable that there is consistency of the EPV of benefits between gender classifications. In fact, for any given starting age, across all six models,

1. The lowest EPV of benefits for females is higher than the highest for males and females combined.
2. The lowest EPV of benefits for males and females combined is higher than the highest for males alone.

Given that the three sets of models (1982–1984, 1984–1989 and 1989–1994) were parameterized using different data sets (except that the 1984–1989 and 1989–1994 models use the forces of mortality from the 1982–1984 models), the estimates of the EPV of benefits are reasonably consistent between the models. For example, for males and females combined, the greatest difference between the maximum and minimum estimate of the EPV of benefits (as a percentage of the midpoint), for all starting ages, is 11.33%—for females it is 15.59%, and for males it is 30.66%.

There is no clear pattern of the EPV of benefits with starting age. For many cases, the EPV of benefits is almost constant across starting ages. Along with increasing starting age, this may be caused by shorter expected future lifetime reducing expected costs and being compensated for by shorter expected time to disabilities, thus increasing costs. Moreover, the EPVs of benefits from any of the disability models does not depend on overall mortality (as defined in equation [5.5]), demonstrating that the patterns of disability and recovery can be more important than overall mortality in pricing an LTC contract. For example, it may be expected that a model with a high overall force of mortality would give lower estimates of the EPV of benefits (using the argument that if people are expected to die sooner, then the costs will be lower). However, the EPVs of benefits from the 1984–1989 (5) model for females, which has the highest overall mortality, are not the lowest for entry at any age and, moreover, for entry at age 75, it gives a higher estimate than does the 1982–1984 (10) model, which has the lowest overall force of mortality. Of

Table 11

EPV of Disability Claims Costs for a Life Starting in the Healthy State, Unit Benefit Increasing Continuously (δ_b = 0.05), for Males, Females, and Combined, Using All Disability Models

		EPV Using					
		1982–1984 Models		1984–1989 Models		1989–1994 Models	
Gender	Entry Age	(5)	(10)	(5)	(10)	(5)	(10)
Male and Female	60	1.9986	1.8667	1.7891	1.8678	1.9441	2.0039
	65	1.9526	1.8944	1.7549	1.8488	1.8330	1.9404
	70	1.9397	1.9136	1.7524	1.8569	1.7744	1.8975
	75	1.9451	1.9099	1.7708	1.8579	1.7439	1.8603
Female	60	2.5600	2.6288	2.7357	2.8789	2.7668	2.9927
	65	2.5982	2.5676	2.6377	2.6784	2.6478	2.7604
	70	2.6052	2.5258	2.5612	2.5365	2.5159	2.5894
	75	2.5433	2.4749	2.4661	2.4407	2.3770	2.4677
Male	60	1.2062	1.0627	0.9624	0.8855	1.0680	1.0593
	65	1.1267	1.0691	0.8824	0.9339	1.0059	1.0500
	70	1.0726	1.0624	0.8848	1.0005	0.9577	1.0381
	75	1.0364	1.0270	0.9080	1.0415	0.9285	1.0189

course, adjusting the mortality in any given model will affect the EPVs of benefits in the way described earlier.

The EPV of benefits given in Table 11 can be analyzed in more detail by looking at the EPV of benefits attributable to a given time period of the contract and specific claiming state. For example, using the 1982–1984 (5) model, the EPV of benefits for a healthy female age 60 is 2.5600, but how much of this is from claiming in state 4 in the time period 0 to 5? (i.e., between the ages 60 and 65). This can easily be calculated in the model by setting

$$b^i_{60+t} = \begin{cases} \exp(0.05t) & \text{if } i = 4 \text{ and } 0 \leq t \leq 5 \\ 0 & \text{otherwise} \end{cases} \tag{6.1}$$

This can be done for the other claiming states and for any given time period in a similar manner. For illustration, in Table 12, we look at the EPV of benefits at age 60 from the 1982–1984 (5) model for males and females combined and separately, in five-year time periods (up to age 90 and for 90+ years) and for each of the claiming states; the total (across all claiming states) for each time period and the total (across all time periods) for each claiming state are also given. It is worth noting some of the general trends from this table:

1. Over all time periods, the EPV of the benefits paid for state 6 is greater than for state 5, which, in turn, is greater than for state 4 for all gender classifications.
2. For early time periods (0–5 years and 5–10 years), the EPV of benefits is greatest while in state 4 or state 5, while after this period, the EPV of benefits is greatest for the institutionalized state, possibly as disability becomes more likely with age (and all lives are assumed to be healthy at age 60).
3. The EPV of benefits increases with time periods that cover older ages up to a maximum and then decrease; for males this maximum is the time period 15–20 years, whereas for females and both genders combined it is the time period 20–25 years. (It may be expected that the EPV of benefits for females continues to increase at older ages than males, as disability becomes more likely with age and females are expected to live longer than males.)

So, although the total EPV of benefits varies greatly by gender (for females, it is more than double that of males), the patterns by claiming state and time period show consistency between genders.

In this section, we have used the disability model to estimate the costs of disability in a long-term care contract and have looked at trends of the costs by gender, age at entry, claiming state, and time period in the contract. To check for consistency, we have looked at the EPV of benefits for models parameterized using the 1982–1984,

Table 12

EPV of Disability Claims Costs, by Time Period and Claiming State, for a Life Starting in the Healthy State at Age 60, Unit Benefit Increasing Continuously (δ_b = 0.05), for Males, Females, and Combined, Using the 1982–1984 (5) Models

Gender	Time Period	EPV of Claims from State: 3–4 ADLs	5–6 ADLs	Inst'd	All 3 States
Male and Female	0–5	0.01561	0.01720	0.00558	0.03838
	5–10	0.05313	0.04911	0.03787	0.14011
	10–15	0.08339	0.07773	0.10240	0.26352
	15–20	0.10122	0.09924	0.17764	0.37810
	20–25	0.10001	0.10529	0.22520	0.43050
	25–30	0.07807	0.08985	0.21242	0.38033
	30+	0.06041	0.08527	0.22197	0.36765
	All	0.49183	0.52368	0.98307	1.9986
Female	0–5	0.00526	0.01719	0.00088	0.02334
	5–10	0.03247	0.03901	0.01913	0.09061
	10–15	0.08203	0.07024	0.10138	0.25364
	15–20	0.12512	0.10886	0.23305	0.46703
	20–25	0.13687	0.13184	0.33364	0.60235
	25–30	0.10870	0.11920	0.32915	0.55706
	30+	0.08522	0.11375	0.36695	0.56592
	All	0.57566	0.60010	1.38418	2.5599
Male	0–5	0.02915	0.01367	0.01069	0.05351
	5–10	0.05176	0.04928	0.04917	0.15022
	10–15	0.06286	0.07358	0.09125	0.22769
	15–20	0.06618	0.08308	0.11098	0.26025
	20–25	0.05844	0.07591	0.10144	0.23579
	25–30	0.04057	0.05408	0.07132	0.16596
	30+	0.02589	0.03640	0.05047	0.11276
	All	0.33484	0.38601	0.48532	1.2062

1984–1989, and 1989–1994 NLTCS and in 5- and 10-year age groups. For brevity, further sensitivity analyses are excluded, but the interested reader can find a comprehensive sensitivity analysis of these models in Pritchard (2002).

6.3 The Recovery Process

Given the complex nature of the processes driving old age dependence, it is perhaps not surprising that, in a modeling exercise, it is common practice to make simplifying assumptions regarding these processes. Given the second-order nature of recovery (second order, since a life must become disabled before recovering), it is usually this process that is simplified or ''assumed'' away. For example, Nuttall et al. (1994), simplify their series of three-state models (healthy, disabled, and dead) by assuming that the probability of recovery (disabled to healthy) is zero; similarly, Alegre et al. (2002), parameterize a three-state model of dependency for analyzing long-term care annuities, assuming that recovery is not possible; Dullaway & Elliot (1998) convert prevalence rates to inception/termination rates by reclassifying recoveries as deaths ''to allow the methodology to work''; Rickayzen & Walsh (2002), do not ignore recovery from disability, but still adopt ''. . . a simple assumption: all people, at all ages and in all disability categories have a 10% chance of improving by one [OPCS disability] category over the course of a year''; and Haberman & Pitacco (1999) estimate the intensities in a LTC insurance model, states that ''. . . it is reasonable to disregard the possibility of recovery, at least as far as the highest levels of disability are concerned. . . .''

Is the impact of ignoring recovery from disability small enough to justify these assumptions? Or should more attention be paid to incorporating a recovery process—at the expense of additional complexity? Table 13 shows the percentage increase of the estimated single premium from ignoring recoveries (i.e., in the model, all of the recovery intensities are set to zero) compared with that allowing for recoveries in a standard LTC contract (see Section 6.1 for details).

Table 13
Percentage Increase in Single Premium from Ignoring Recoveries, Unit Benefit Increasing Continuously (δ_b = 0.05), for Males and Females Combined, Using All Disability Models

Entry Age	Percentage Increase Using					
	1982–1984 Models		1984–1989 Models		1989–1994 Models	
	(5) %	(10) %	(5) %	(10) %	(5) %	(10) %
60	29.80	31.19	17.18	18.81	23.03	22.41
65	23.91	25.17	14.92	15.29	21.34	20.16
70	17.35	19.02	12.70	12.75	18.94	17.73
75	11.37	13.42	10.75	10.34	16.64	15.20

For all models, the percentage increase of the single premium for a life age 60 from ignoring recoveries ranges from just above 17% to just over 31%. This percentage increase decreases with age to about 10% to 15% by age 75. This is as expected since recovery becomes less likely at older ages (as demonstrated by almost all of the transition intensities of recovery in Figures 3 to 7 decreasing with age); hence, ignoring it will have less effect on estimated costs.

In the context of an insured population, these figures may be seen as an estimate of the upper limit on the costs of moral hazard—if we think about the model that fully allows for recoveries as representing the situation with no moral hazard (e.g., because of strict claims underwriting) and about the model with no recoveries as representing the situation with extreme moral hazard. This suggests that poor claims underwriting may increase the EPV of claims costs by up to as much as 30% in an environment of strong moral hazard.

It is worth remembering that these results are based on population data and that different results may emerge from insured data. Indeed, moral hazard may already be a feature of an insured population to some extent. Even still, the magnitude of these differences is reasonably strong evidence that the recovery process should not be ignored and, indeed, that more effort may be justified in modeling recoveries in a more realistic manner, even if this complicates the model slightly.

We have assumed in the model of LTC presented here that all transitions, including recoveries, follow a Markov process. Stallard and Yee (1999), in an investigation by a subcommittee of the Society of Actuaries' Long-Term Care Experience Committee, suggest that the use of a Markov model for modeling recovery from institutionalization may produce misleading results. However, without the Markov assumption, estimation of the transition intensities would have been significantly more challenging, if possible, given the nature of the data, and, furthermore, this observation only applies to recovery from one of the six transient states in the model.

7. CONCLUSIONS

7.1 The Model

A novel method (penalized maximum likelihood) for using interval-censored, longitudinal data for parameterizing Markov models has been introduced and the method used on the NLTCS data to parameterize a continuous-time model of disability. Strengths of the method over previously suggested methods include

1. It can cope with the situation where the initial estimates of the intensities are complex.
2. The estimation procedure implicitly allows for the credibility of each intensity (i.e., more weight is attached to those intensities that had more observed transitions underlying them).
3. Intensities are adjusted globally (i.e., over all intensities), not on a row-by-row basis.

While it was not possible to investigate the reasonableness of the Markov assumption in the model due to data limitations, the form of the resulting graduated intensities do provide support for the method—from almost all states:

1. Force of disability increases with age.
2. Force of mortality increases with age.
3. Force of mortality increases with increasing disability level.
4. Force of recovery decreases with age.

Further, less than 5% of the graduated intensities lie outside the approximate 95% confidence intervals. All of these observations lend support to the procedure being reasonable—and one that produces reasonable results.

Common assumptions, used to simplify the complex process of disability to aid modeling, involve adjusting the recovery process in some (arbitrary) manner. Examples of this include assuming that recovery from disability is not possible or that an individual can only recover by one (model) state per annum. A simple investigation, comparing the EPV of benefits using the disability model allowing for recovery from disability against the EPV of benefits in the model with no recovery allowed, demonstrated that costs could be overstated by as much as 30% (at younger ages at entry into a lump-sum LTC plan) if recovery from disability is ignored. Even at older ages (75 years), when the rates of recovery reduce dramatically, costs could still be overstated by as much as 10%. The magnitude of these figures suggest that more time and effort may be justified in developing models that more comprehensively capture the disability process. In particular, it shows that the recovery process is not just a minor modeling inconvenience, but an integral and important part of the disability process.

7.2 Discussion

A model of long-term care based on incidence rates rather than prevalences is a major goal. From Wittenberg et al. (1998): ''In the longer term it would be valuable to develop, with the use of longitudinal data, a model that looked at trends in transition rates between health and dependency states. . . . In the absence of such analyses, various stylised assumptions have to be made about possible changes in age-specific dependency rates.'' Similarly, it is becoming more widely recognised that health cost models at the level of the individual person are needed properly to understand the components of national or regional expenditure (Smith, Rice, and Carr-Hill 2001).

Nuttall et al. (1994) pursued an intensity model of long-term care, but lacked longitudinal data, as did Rickayzen & Walsh (2002), who note that ''Future research in the area of long-term care would be greatly assisted if regular national study were undertaken which enabled longitudinal data to be collected . . . .'' They assume, however, that interval-censored, longitudinal data are sufficient to directly estimate model intensities. While such data are not sufficient, suitable methodology, such as that presented in this paper, would enable such data to be used to estimate the intensities in a suitable Markov model.

In future, almost inevitably, the type of data collected would be similar to the NLTCS (i.e., interval-censored, longitudinal data), due to cost and resource constraints. Indeed, in the United Kingdom, such a survey has been undertaken by the Medical Research Council (MRC) in the Cognitive Function and Ageing Study (CFAS). The MRC CFAS study has approximately 2,500 lives from each of six centers around the United Kingdom and includes questions on participants' ability to carry out 10 activities of daily living. Initial interviews were undertaken between 1990 and 1994, and followed up two years later—a format directly comparable to the NLTCS. A full description of the dataset can be found in McGee et al. (1998).

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