Natural Hedging of Life and Annuity Mortality Risks

Samuel H. Cox* and Yijia Lin†

Abstract

The values of life insurance and annuity liabilities move in opposite directions in response to a change in the underlying mortality. Natural hedging utilizes this to stabilize aggregate liability cash flows. We find empirical evidence that suggests that annuity writing insurers who have more balanced business in life and annuity risks also tend to charge lower premiums than otherwise similar insurers. This indicates that insurers who have a natural hedge have a competitive advantage. In addition, we show how a mortality swap might be used to provide the benefits of natural hedging.

1. Introduction

If future mortality improves relative to current expectations, life insurer liabilities decrease because death benefit payments will be later than expected. However, annuity writers have a loss relative to current expectations because they have to pay annuity benefits longer than expected. If the mortality deteriorates, the situation is reversed: life insurers have losses and annuity writers have gains. Natural hedging utilizes this interaction of life insurance and annuities to a change in mortality to hedge against unexpected changes in future benefit payments.

The purpose of this paper is to study natural hedging of mortality risks and to propose mortality swaps as a risk management tool. Few researchers investigate the issue of natural hedging. Most of the prior research explores the impact of mortality changes on life insurance and annuities separately, or investigates a simple combination of life and pure endowment life contracts (Frees, Carriére, and Valdez 1996; Marceau and Gaillardetz 1999; Milevsky and Promislow 2001; Cairns, Blake, and Dowd 2004). Studies on the impact of mortality changes on life insurance focus on ''bad'' shocks, while those on annuities focus on ''good'' shocks.

Wang, Yang, and Pan (2003) analyze the impact of the changes of mortality factors and propose an immunization model to hedge risks based on the mortality experience in Taiwan. However, life insurance and annuity mortality experience can be very different, so there is ''basis risk'' involved in using annuities to hedge life insurance mortality risk. Their model cannot pick up this basis risk.

Marceau and Gaillardetz (1999) examine the calculation of the reserves in a stochastic mortality and interest rates environment for a general portfolio of life insurance policies. In their numerical examples, they use portfolios of term life insurance contracts and pure endowment polices, similar to Milevsky and Promislow (2001). They focus on convergence of simulation results. There is a hedging effect in their results, but they do not pursue the issue.

Our paper proceeds as follows: In Section 2 we use an example to illustrate the idea of natural hedging. In Section 3, using market quotes of single-premium immediate annuities (SPIAs) from A. M. Best, we find empirical support for natural hedging. That is, insurers who have better naturally hedge mortality risks tend to have a competitive advantage over otherwise similar insurers. In Sections 4 and

* Samuel H. Cox, FSA, MAAA, PhD, is Dr. L. A. H. Warren Chair Professor of Actuarial Science, University of Manitoba, sam_cox@umanitoba.ca.
† Yijia Lin, PhD, is Assistant Professor, Department of Finance, University of Nebraska–Lincoln, ylin@unlnotes.unl.edu.

5 we propose and price a mortality swap between life insurers and annuity insurers. Section 6 is the conclusion and summary.

2. INTRODUCTORY EXAMPLE

If mortality improves, what happens to the insurer's total liability? We know that, on average, the insurer will have a loss on the annuity business but not on the life insurance business. And if mortality declines, the effects are interchanged. This section illustrates the idea of a natural hedge.

2.1 The Portfolio

At time 0, consider an insurer's portfolio of life contingent liabilities consisting of whole life policies written on lives at ages 25, 30, 35, 40, 45, 50, 55, and 60 and single-premium immediate life annuities written on lives at ages 65, 70, 75, 80, 85, 90, 95, and 100. The interest rate is flat at 8%. Life insurance premiums and annuity benefits are paid annually. Death benefits are paid at the end of the year of death.

The net annual premium rate for one unit of whole life benefit is determined so that the present value of net premiums is equal to the present value of benefits. The annuity policy is purchased with a single payment. The expected loss of the portfolio is zero. However, this expectation is calculated under the assumption that the mortality follows the tables assumed in setting the premiums. If we replace the before-shock lifetimes with the after-shock lifetimes, what happens to the loss?

The overall mortality shock effect on an insurer is determined by its business composition, that is, the ratio of annuity business to life business. To study different outcomes under different business compositions and illustrate the idea of natural hedging, we introduce a variable r that is the ratio of annuity business to whole life business. For example, given $r = 1$ at time 0, if death benefits and annuity annual payments follow Table 1 for different ages, the present values of liabilities (or premiums) of whole life insurance and of annuities are equal. Suppose there is a mortality shock striking all ages evenly. We expect that an insurer with such a balanced business (where $r = 1$) will have the lowest volatility in cash flows. If $r = 0.5$ (or $r = 2$), it means the annuity business is half of (or twice) life business. A mortality shock in this case may impose a bigger problem in cash flow stability than when $r = 1$. However, in reality mortality operates within a complex framework and is influenced by socio-economic factors, biological variables, government policies, environmental influences, health conditions, and health behaviors (Rogers 2002). Therefore, we use past mortality shocks to illustrate our idea of natural hedging.

2.2 Good Shocks

If mortality improves unexpectedly, we call it a "good" shock. The following example shows the impact of a good shock on an insurer's life insurance and annuity business, respectively, and on its overall portfolio following Table 1.

Assume that at time 0 the insurer sets an annual fixed premium of whole life insurance according to the 1990–95 U.S. SOA Basic Male Life Insurance Table and charges a lump-sum annuity premium based on the 1996 U.S. Male Annuity Basic Table. Right after the policy is issued, the mortality of life insureds and annuitants improves at different rates at each attained age. We further assume that the mortality improvement rate at a given age equals the average annual improvement rate calculated from four historical mortality tables (two for life insurance and two for annuities) described below.

Different ages and insurance cohorts (i.e., life insured or annuitant) have different mortality improvement rates. For example, one-year death rate for male age (25) is 0.00193 based on the 1958 U.S. CSO Male Life Insurance Table, but it decreases to 0.00154 in the 1990–95 U.S. SOA Basic Male Life Insurance Table. Therefore, the average annual improvement rate of male age (25) is 0.58% ((0.00154/0.00193 − 1)/35). According to these two life insurance tables, the average annual improvement rate for male age (40) is 0.40% during the same period. On the annuity side, based on

Table 1
Portfolio of Whole Life Insurance and Annuities (Dollar Amounts in Thousands)

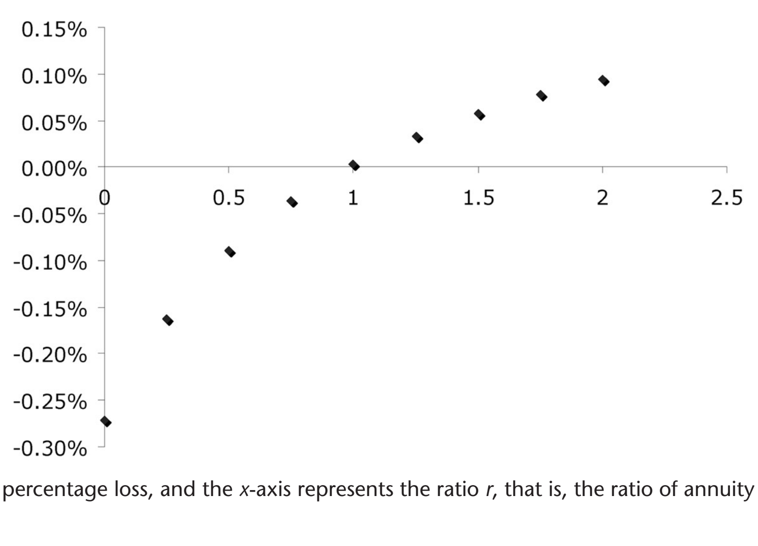
Whole Life		Annuities	
Age	Death Benefit	Age	Annual Payment
25	$992,832	65	$78,135*r*
30	1,100,000	70	21,000*r*
35	1,200,000	75	8,000*r*
40	1,375,000	80	5,000*r*
45	1,200,000	85	2,500*r*
50	1,100,000	90	5,000*r*
55	1,050,000	95	2,500*r*
60	80,000	100	1,500*r*

the 1963 U.S. Male Annuity Basic Table and the 1996 U.S. Male Annuity Basic Table, the average annual improvement rate of male age (65) is 1.52% and 1.12% for male age (80) from 1963 to 1996. We use these historical annual improvement rates for different ages of life insureds and annuitants as time passes to show the impact of a good mortality shock.

Figure 1 shows the percentage deviation of the present value of benefits from whole life insurance premiums and that of annuity payments from the total annuity premium collected at time 0. We also show the percentage of deviation from the present value of total premiums collected. Each result includes a given ratio of annuity business to life insurance business, modeled by the ratio of annuity liabilities to whole life liabilities following the tables (i.e., the 1996 U.S. Male Annuity Basic Table and the 1990–95 U.S. SOA Basic Male Life Insurance Table) at time 0.

With the business ratio $r = 0.25$, which means that whole life insurance accounts for the majority of the business, annuity business will lose 0.28% of the total premium collected at time 0. Life business will gain 0.27% because whole life death benefits will be paid later than expected. However, the shock causes a much smaller effect on the portfolio including both whole life and annuities. In this scenario, a 0.16% gain is shown in Figure 1. Similarly, given a more balanced business with $r = 1$, the good shock imposes almost no effect on the insurer's total cash flows, which is shown as the cross point in the x-axis in Figure 1. We conclude that writing both life and annuity business reduces cash flow

Figure 1
Percentage Deviation of Present Value (at Time 0) of Sum of Whole Life and Annuity Payments from That of Total Premiums Collected Caused by a Good Mortality Shock



Note: The y-axis represents the percentage loss, and the x-axis represents the ratio r, that is, the ratio of annuity business to whole life business.

volatility caused by a good shock, which, in most cases, lasts for a long time (e.g., until life insureds or annuitants die, as shown in our example). What happens if there is a one-time (e.g., lasting only one year) bad shock?

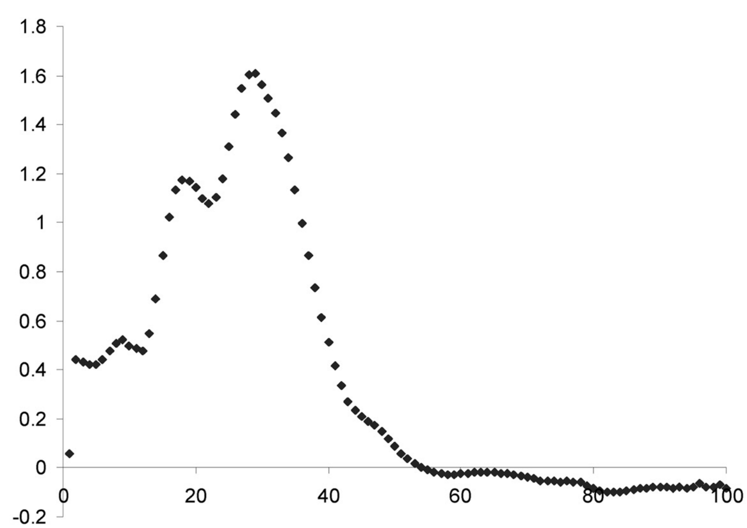
2.3 Bad Shocks

Although mortality among the general and insured populations has witnessed remarkable improvements over the last several decades, mortality could continue to improve or it could worsen. Some events (e.g., an epidemic) act like a common shock to the entire mortality curve and decrease the expected lifetimes of a large number of the population. We call these events "bad" shocks. We show the impact of a bad mortality shock using the 1918 worldwide flu epidemic as an example and the same business composition and other assumptions we used in Section 2.1.

Figure 2 shows how the 1918 worldwide flu epidemic increased one-year death rates for different ages—very unevenly. The flu struck ages 0–50 more seriously than older ages. According to Taubenberger and Morens (2006), this may have been due to immunity that the older people had acquired as survivors of earlier flu epidemics. Alternatively, Noymer and Garenne (2000) provide evidence that this is a result of increased lethality due to interaction of the flu and tuberculosis. The second analysis also provides an explanation as to why the 1918 flu struck men more severely than women, an effect we have not addressed: the incidence of tuberculosis in men was higher than in women.

We assume that right after the insurer issues whole life and annuities policies shown in Table 1, a flu epidemic breaks out. It lasts one year and increases the death rates for different ages following the pattern of the 1918 worldwide flu. Figure 3 shows that no matter what its business composition, the insurer always has a loss. For example, when $r = 1$, the overall loss is 0.48%. Why does it happen? Why is the overall impact not zero? It lies in the fact that this insurer mainly writes life business to younger ages and sells annuity policies to older annuitants. A flu-type bad shock causes an unexpected loss in its life business, which its annuity business does not hedge because the death rates did not increase in older ages. In this example the insurer does not obtain the desired natural hedging effect. However, writing both life insurance and annuities does help reduce cash flow volatility. For example, if $r = 1$, although the loss in life insurance is 0.91%, the loss of the portfolio is only 0.48%.

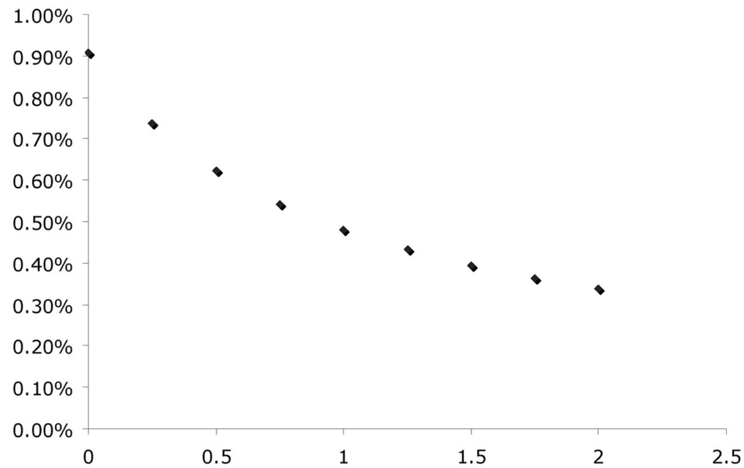
Figure 2
Impact of 1918 Worldwide Flu Epidemic



Note: The *y*-axis represents percentage increase in one-year death rate at a given age in 1918 relative to that in 1917. The *x*-axis represents the age.

Figure 3

Percentage Deviation of Present Value (at Time 0) of Sum of Whole Life and Annuity Payments from That of Total Premiums Collected Caused by a 1918 Flu-Type Bad Shock

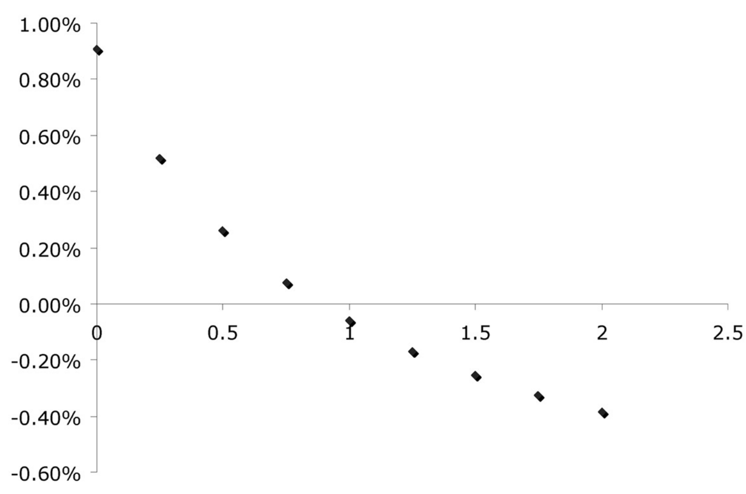


Note: The *y*-axis represents the percentage loss, and the *x*-axis represents the ratio *r*, that is, the ratio of annuity business to whole life business.

Since we have not seen an epidemic breaking out for about a century, no one has immunity to epidemics. If an epidemic occurs, it may kill different ages at the same time, including older ages. To mimic a more realistic 1918-type flu in the future, we assume that a one-year death rate increase for each age from 25 to 60 follows the population mortality deterioration for these ages from 1917 to 1918. Mortality rate changes in ages above 60 equal the average mortality deterioration in ages 25–60. The overall effects of this hypothesized shock on the portfolio for different ratios is shown in Figure 4. In this case more balanced business (i.e., r is closer to 1) helps stabilize cash flows better, which is similar to what we see in Section 2.2.

Figure 4

Percentage Deviation of Present Value (at Time 0) of Sum of Whole Life and Annuity Payments from That of Total Premiums Collected



Note: Mortality rate increase for each age from 25 to 60 follows the population mortality deterioration for those ages from 1917 to 1918. Mortality rate changes in ages above 60 equal the average mortality deterioration in ages 25–60. The *y*-axis represents the percentage loss, and the *x*-axis represents the ratio *r*, that is, the ratio of annuity business to whole life business.

3. EMPIRICAL SUPPORT FOR NATURAL HEDGING

Life insurance and annuities have become commodity-like goods, meaning that the price variable is a primary source of competition among insurance industry participants. Through various marketing campaigns, consumers are well aware whether a price offered by an insurer is attractive or not. What does theory say about natural hedging in such a market?

Financial theory tells us that systematic risk cannot be diversified away because systematic risk influences all businesses. This argument, however, was originally intended for security managers, not corporate managers (Chatterjee and Lubatkin 1992). Action by corporate managers may alter the underlying systematic risk profiles of their portfolio of business (Chatterjee and Lubatkin 1990; Helfat and Teece 1987; Peavy 1984; Salter and Weinhold 1979). Thus, while mortality risk may not be hedgeable in financial markets, it may be reduced or eliminated by some insurers through reinsurance, asset-liability management, and, we propose, mortality swaps. Thus financial theory suggests we should find evidence for natural hedging in annuity and life insurance markets.

In addition, the industrial organization literature offers numerous theoretical and empirical models that link vertical integration to a reduction of environmental uncertainty (Arrow 1975; Carlton 1979; Mitchell 1978). Applied to the insurance market, these results suggest that an insurer writing both life insurance and annuities will have reduced risk relative to similar firms writing one of these products. Now let us look at the data.

3.1 Data, Measures, and Methodologies

3.1.1 Data and Measures

The annuity prices are actual quotes for single-premium immediate annuities (SPIAs) for a 65-year-old male in each year from 1995 to 1998 (Kiczek 1995, 1996, 1997; A. M. Best 1998). Each year A. M. Best Company surveyed about 100 companies to obtain quotes for the lifetime-only monthly benefits paid to a 65-year-old male with $100,000 to invest.

Table 2 describes the annuity payment data and company annual statement data collected by the National Association of Insurance Commissioners for our two subsamples. We transformed each quote from a rate of payment per month to an equivalent price, as follows. If m is the monthly payment rate per $100,000 and Price is what the company would charge for an annuity of 1 per year for the annuitant's lifetime, then

$$100{,}000 = 12m \text{ Price},$$

$$\text{Price} = \frac{100{,}000}{12m}.$$

The ratio of life insurance reserves to total annuity and life insurance reserves, denoted Ratio, reflects the level of natural hedging provided by life insurance business to annuity business. The idea is that, of two otherwise similar annuity writers, the one with the higher Ratio value has a better hedge against longevity risk. The better hedge may allow for lower provision for risk in its premiums, and thus lower prices. In addition, this ratio determines the degree to which the insurer writes annuity business. For example, if the ratio is less than 0.90, the company annuity reserve is more than 10% of the total.

In an efficient, competitive insurance market, the price of insurance will be inversely related to firm default risk (Phillips, Cummins, and Allen 1998; Cummins 1988; Merton 1973): that is, a company with more default risk would have lower annuity prices. We use the A. M. Best rating, for the year prior to the quote, as a measure of default risk of annuity insurers. We use the rating to define a numerical variable, Lrate, as follows:

Table 2
Descriptive Insurer Statistics for 1995 to 1998 (Dollar Amounts in Millions)

Panel A				
		Ratio $<$ 0.90 (N = 299)		
Variable	Description	Mean	Minimum	Maximum
m	Annuity payment rate	765	653	992
Price	Equivalent price	10.93	8.40	12.76
Resann	Annuity gross reserve	3,490,298	5,192	43,011,379
Reslife	Life insurance gross reserve	2,741,170	0	45,174,284
Ratio	Reslife/(Reslife + Resann)	0.44	0.00	0.89
Tasset	Total assets	9,069,208	33,351	127,097,380
Lrate	A. M. Best rating, 1 year prior	1.59	1	4
Comexp	Commission + Expenses/Net premiums	23.21%	2.50%	102.20%

Panel B				
		Ratio $<$ 0.75 (N = 243)		
Variable	Description	Mean	Minimum	Maximum
m	Annuity payment rate	765	653	992
Price	Equivalent price	10.93	8.40	12.76
Resann	Annuity gross reserve	4,082,579	10,535	43,011,379
Reslife	Life insurance gross reserve	2,412,583	0	45,174,284
Ratio	Reslife/(Reslife + Resann)	0.35	0.00	0.75
Tasset	Total assets	9,079,913	33,351	127,097,380
Lrate	A. M. Best rating, 1 year prior	1.58	1	4
Comexp	(Commission + Expenses)/Net premiums	21.95%	2.50%	93.50%

$$\begin{array}{rr} \text{A++, A+} \rightarrow \text{Lrate} = 1 & \text{A, A}- \rightarrow \text{Lrate} = 2 \\ \text{B++, B+} \rightarrow \text{Lrate} = 3 & \text{B, B}- \rightarrow \text{Lrate} = 4 \\ \text{C++, C+} \rightarrow \text{Lrate} = 5 & \text{C, C}- \rightarrow \text{Lrate} = 6 \\ \text{D} \rightarrow \text{Lrate} = 7 & \text{E, F, or S} \rightarrow \text{Lrate} = 8. \end{array}$$

This leads to the hypothesis that higher values of Lrate occur with lower annuity prices.

Other factors that may affect the annuity prices are also included in our regression model. We use the logarithm of the company's total gross annuity reserve, log(Resann), to represent the degree to which the company writes annuity business. The logarithm of the company's total assets, log(Tasset), controls for the size of the company. The sum of commissions and expenses divided by total written premium, Comexp, measures the company's expenses. Higher expenses should be related to higher annuity prices. The sample includes 322 complete observations from which we extracted two subsamples of companies that write more than a small amount of annuity business (defined below).

3.1.2 Methodologies

We use the pooled ordinary least squares (OLS) technique to investigate the relation between annuity prices and natural hedging, controlling for size, default risk, expenses, and year effects. This is the model relationship:

$$\begin{aligned} \text{Price} = \alpha + \beta\text{Ratio} + \gamma_1 \log(\text{Resann}) + \gamma_2 \log(\text{Tasset}) + \gamma_3\text{Lrate} \\ + \gamma_4\text{Comexp} + \delta_1 D_{1998} + \delta_2 D_{1997} + \delta_3 D_{1996} + \varepsilon, \end{aligned} \tag{3.1}$$

where

$$D_i = \begin{cases} 1 & \text{if the quote was observed in year } i \\ 0 & \text{if the quote was observed in another year} \end{cases}$$

and ε is the error term.

Under our hypothesis, the coefficient β should be negative, indicating that natural hedging allows for lower annuity prices. We run the regression for two subsamples determined by the proportion of annuity business the company has written, as represented by Ratio.[1] The one sample regression includes sample observations for which Ratio < 0.90, that is, at least 10% of the company's reserve is for its annuity business. This sample rules out companies with very little annuity business. The sample size is $N = 299$.

The other subsample consists of those observations for which the company has Ratio < 0.75 so that its life insurance reserves are less than 75% of its total reserves. The size of this sample is $N = 243$. If we eliminate more companies in the range 0.5 < Ratio < 0.75, where we might see a stronger effect of natural hedging, the sample sizes are too small to get adequate significance.

3.2 Findings and Implications

The coefficient estimates are presented in Table 3. Each estimated coefficient is large relative to its standard error. All of the results are significant at the 5% level or better. White statistics (Greene 2000) indicate that the distribution of errors is not heteroscedastic; OLS is appropriate. The signs of all the coefficients are consistent with our hypothesis. Since β is negative, annuity writers with more life insurance business tend to have lower annuity prices than similar companies with the same size and rating but less life insurance business. Our story about natural hedging is consistent with this empirical conclusion. The regression results suggest that annuity-writing companies benefit from natural hedging, although these companies may not be making explicit natural hedging decisions.

When annuity business increases relative to life insurance business, the need for longevity risk hedging increases. When we focus on those observations with the proportion of annuity reserve to the sum of annuity reserve and life insurance reserve higher than 25%, that is, Ratio < 0.75, its coefficient (−0.5487) is larger than the coefficient (−0.4777) obtained for the case Ratio < 0.90. This suggests that when an annuity writer sells relatively more annuities, the increase in the life insurance has a higher marginal effect in lowering the annuity price.

How do we interpret the coefficient of Ratio (−0.5487) when Ratio < 0.75? Suppose a life insurer has 5% of its business in life insurance and 95% of its business in annuities. It sells life-only SPIAs to males aged 65 at the market average monthly payout of $765. If it can realize full natural hedging, that is, 50% of business in life insurance and 50% business in annuities, its SPIA monthly payouts can be increased by $18, that is, from $765 to $783 because it can reduce the risk premium in its price. Its SPIA prices can be more attractive than those of similar competitors.

Table 3

Pooled OLS Regression: Relationship between Annuity Price and Natural Hedging

Variable	Price	
	Ratio < 0.75	**Ratio < 0.90**
Intercept	10.2900	9.6432
Ratio	−0.5487	−0.4777
log(Resann)	−0.3171	−0.1648
log(Tasset)	0.3223	0.1956
Lrate	−0.1209	−0.1073
Comexp	0.9803	0.8045
N	243	299
R^2	0.3350	0.2913

[1] The proportion of annuity reserves to the sum of annuity reserves and life insurance reserves measures the longevity risk exposure of an annuity insurer; this ratio is the complement of the variable Ratio: Resann/(Reslife + Resann) = 1 − Ratio.

The regression results suggest that when a life insurer writes more annuities, its annuity price goes down because the sign of the annuity business variable, log(Resann), is negative. The size of an insurer is positively related to the price of its SPIA, which may reflect the market power of bigger firms. This is consistent with prior research (Sommer 1996; Froot and O'Connel 1997).

Some previous evidence to support the hypothesis that insurance prices reflect firm default risk can be found in Berger, Cummins, and Tennyson (1992) and Sommer (1996). Our default risk measure Lrate has a negative and significant coefficient, which is consistent with the earlier results. The sign of the coefficient of the expense variable log(Comexp) is positive and significant. Higher expenses are consistent with higher prices.

4. MORTALITY SWAPS

In Section 3 we conclude that natural hedging may allow an annuity writer to lower its prices. However, it may be too expensive and unrealistic for an annuity writer to utilize natural hedging by changing its business composition. If we consider a corporate pension plan as an annuity writer, it may not even be legal for it to issue life insurance. Even for an insurer specializing in annuities, entering the life insurance business may not be practical. Moreover, natural hedging is not a static process. Dynamic natural hedging is required for new business. If an insurer is able to take advantage of natural hedging at a low cost by financial innovation, it can gain competitive advantage in the market by selling annuity products at lower prices. We propose mortality swaps to accomplish this goal.

4.1 Basic Ideas

We are suggesting that a market for mortality swaps may develop in which brokers and dealers offer swaps to annuity writers and separately to life insurers. The broker may match each annuity deal with a life deal or manage its portfolio of mortality swaps on an aggregate basis. As a start toward development of such a market we propose a market-based approach to value each side of a mortality swap in Section 5. The annuity (or survivor risk) side is priced in a way that is consistent with observed prices in the annuity market. Similarly the mortality (or death risk) side is consistent with the life insurance market.

Dowd et al. (2004) propose the possible uses of survivor swaps as instruments for managing, hedging, and trading mortality-dependent risks. Their proposed survivor swap involves transferring a mortality risk related to a specific population with another population, that is, one specific longevity risk for another specific longevity risk. Their mortality swap can be used to diversify the longevity risks. However, the survivor swap does not hedge a good shock or a bad shock that strikes across populations (a systematic risk). Our approach is different, and it can hedge these shocks. Without any collateral, the swap payments are subject to counterparty risk: that is, one party, the broker or the insurer, may default. We ignore this issue and assume all parties fulfill their contractual obligations. Mortality swaps are described as follows.

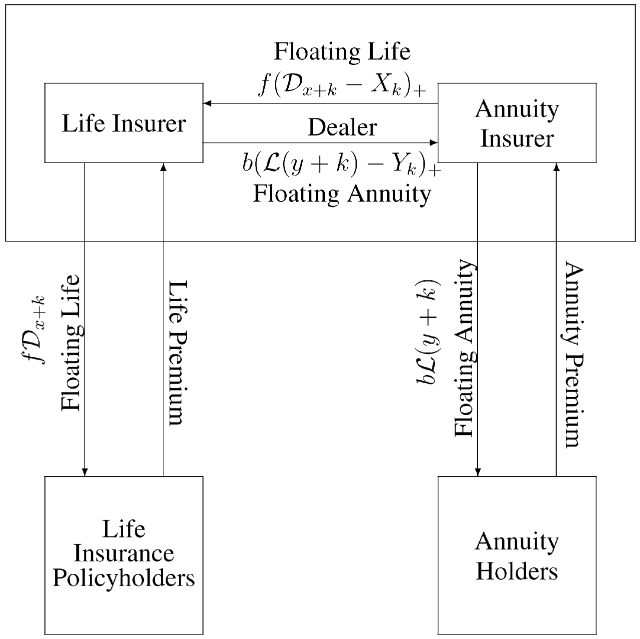
4.2 Mortality Swap Design

Each year the annuity writer pays floating cash flows to the life insurer based on the actual number of deaths in the life insurer's specified portfolio of policies. This provides a benefit to the life insurer if mortality deteriorates, which the annuity writer may pay from its gain due to reduced annuity benefits.

At the same time the life insurer pays an annuity benefit based on the actual number of survivors in the annuity writer's specified portfolio of annuities. If, for example, mortality improves, the life insurer pays the annuity writer, but it has a gain on its own life insurance policies. There are no other swap payments. Figure 5 illustrates the mortality swap cash flows.

To keep the notation fairly simple, we assume that all of the lives in the life insurance portfolio are subject to the same mortality table, denoted by (x), for pricing purposes. At time 0 we have a portfolio of ℓ_x lives each insured for an amount f. The random number of survivors to age $x + k$ is denoted $\mathscr{L}(x + k)$. The number of deaths in the year $(k, k + 1)$ is $\mathscr{D}_{x+k} = {}_1\mathscr{D}_{x+k} = \mathscr{L}(x + k) - \mathscr{L}(x + k + 1)$.

Figure 5
Mortality Swap Diagram



The conditional distribution of $\mathscr{L}(x + k)$, given that the distribution follows the table Θ specified at time 0, is binomial with parameters $m = \ell_x$ and $q = {}_kp_x - {}_{k+1}p_x$, with q calculated using the mortality table Θ. Of course, uncertainty in the table Θ at time 0 means that $\mathrm{E}[\mathscr{D}|\Theta]$ is as random as the table is. However, for the purposes of designing a swap, assume that the parties agree on the payment strike prices. One way to do this would be to agree on a table $\Theta = \theta$ and set strike level X_k for life insurance at the median or some higher percentile.

The life insurer's aggregate death benefit payment in year $k + 1$ amounts to $f\mathscr{D}_{x+k}$. The insurer's risk is that actual life insurance benefit payments exceed the expected value, and so it swaps, agreeing to pay annuity payments (defined below) in exchange for a payment in year $k + 1$. The payment from the swap is

$$f(\mathscr{D}_{x+k} - X_k)_+ = \begin{cases} f(\mathscr{D}_{x+k} - X_k) & \text{if } \mathscr{D}_{x+k} > X_k \\ 0 & \text{otherwise} \end{cases} \quad \text{for } k = 0, 1, \ldots . \tag{4.1}$$

The swap payment limits the net aggregate life insurance benefit from the life insurer to the expected value in each year. Of course, there is a price: The life insurer must provide annuity swap payments when the annuity writer needs them.

The annuity writer pays a benefit b per year to each survivor of an initial cohort of ℓ_y lives, all subject to the same table at time 0 (but usually different from the lives on which the life insurance is written). The number of survivors at time k is denoted $\mathscr{L}(y + k)$. Therefore, the total benefit paid at time k is $b\mathscr{L}(y + k)$. The annuity writer gets relief when it exceeds the set strike level Y_k. This defines the life insurer's swap payment to the annuity writer:

$$b(\mathscr{L}(y + k) - Y_k)_+ = \begin{cases} b(\mathscr{L}(y + k) - Y_k) & \text{if } \mathscr{L}(y + k) > Y_k \\ 0 & \text{otherwise} \end{cases} \quad \text{for } k = 0, 1, \ldots . \tag{4.2}$$

The net cash flow that the annuity writer pays in the year $(k, k + 1)$ is

Annuity benefits	$b\mathcal{L}(y + k)$
To the life insurer	$f(\mathcal{D}_{x+k} - X_k)_+$
From the life insurer	$b(\mathcal{L}(y + k) - Y_k)_+$
Total	$\begin{cases} b\mathcal{L}(y + k) & \text{if } \mathcal{D}_{x+k} \le X_k,\ \mathcal{L}(y + k) \le Y_k \\ b\mathcal{L}(y + k) + f(\mathcal{D}_{x+k} - X_k) & \text{if } \mathcal{D}_{x+k} > X_k,\ \mathcal{L}(y + k) \le Y_k \\ bY_k & \text{if } \mathcal{D}_{x+k} \le X_k,\ \mathcal{L}(y + k) > Y_k \\ bY_k + f(\mathcal{D}_{x+k} - X_k) & \text{if } \mathcal{D}_{x+k} > X_k,\ \mathcal{L}(y + k) > Y_k. \end{cases}$

The annuity writer pays no more than bY_k in net annuity benefits, although it may make a swap payment because of excessive deaths in the life insurance portfolio.

The life insurer's net cash flow in the year $(k, k + 1)$ is

Death benefits	$f\mathcal{D}_{x+k}$
From annuity writer	$f(\mathcal{D}_{x+k} - X_k)_+$
To annuity writer	$b(\mathcal{L}(y + k) - Y_k)_+$
Total	$\begin{cases} f\mathcal{D}_{x+k} & \text{if } \mathcal{D}_{x+k} \le X_k,\ \mathcal{L}(y + k) \le Y_k \\ fX_k & \text{if } \mathcal{D}_{x+k} > X_k,\ \mathcal{L}(y + k) \le Y_k \\ f\mathcal{D}_{x+k} + b(\mathcal{L}(y + k) - Y_k) & \text{if } \mathcal{D}_{x+k} \le X_k,\ \mathcal{L}(y + k) > Y_k \\ fX_k + b(\mathcal{L}(y + k) - Y_k) & \text{if } \mathcal{D}_{x+k} > X_k,\ \mathcal{L}(y + k) > Y_k. \end{cases}$

In each year the swap rearranges the sum of annuity and life insurance benefit payments. The sum is always $b\mathcal{L}(y + k) + f\mathcal{D}_{x+k}$, but the parties swap adverse outcomes. They need not swap all of their business, and the contract could specify upper bounds on the annual swap payments.

The more closely related are the underlying life and annuity portfolios, the better will be the hedge. Factors can move mortality either way. The 1918 flu epidemic is an example that had a negative impact across populations and would probably hit both life and annuity portfolios. However, the flu effects varied a great deal by age with people at ages 0–50 hit much harder than older lives, as shown in Figure 2. As we show in Section 2, the proposed swap would not fully protect the life insurer in this case.

5. PRICING

Cairns, Blake, and Dowd (2004) discuss a theoretical framework for pricing mortality derivatives and valuing liabilities that incorporates mortality guarantees. Their stochastic mortality models require certain "reasonable" criteria in terms of their potential future dynamics and mortality curve shapes. Mortality changes in a complex manner, influenced by socioeconomic factors, biological variables, government policies, environmental influences, health conditions, and health behaviors. Not all of these factors improve with time, and, moreover, opinions on future mortality trends vary widely (Buettner 2002; Hayflick 2002; Goss, Wade, and Bell 1998; Rogers 2002). Even if we could settle on a dynamic framework, estimation of the parameters may be very difficult. We take a different, static approach.

5.1 Wang Transform

Wang (1996, 2000, 2001) has developed a method of pricing risks that unifies financial and insurance pricing theories. This method can be used to price mortality bonds (Lin and Cox 2005). Now we apply this method to mortality swaps. Consider a random payment X. If the cumulative density function is $F(x)$, then a "distorted" or transformed distribution $F^*(x)$ is determined by parameter λ according to the equation

$$F^*(x) = \Phi[\Phi^{-1}(F(s)) - \lambda], \tag{5.1}$$

where $\Phi(x)$ is the standard normal cdf. If the payment X is made at time T, the idea is to determine λ so that at time 0 the price of X is its discounted expected value using the transformed distribution. Then the formula for the price is

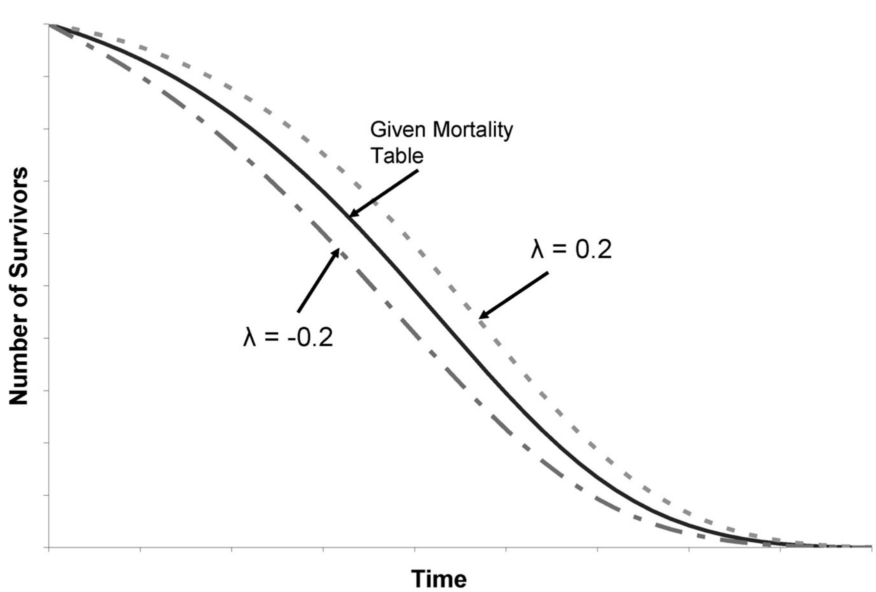
$$v_T \mathrm{E}^*(X) = v_T \int x \, dF^*(x), \tag{5.2}$$

where v_T is the discount factor determined by the market for risk-free bonds at time 0. Thus, for an insurer's given liability X with cumulative distribution function $F(x)$, the Wang transform will produce a ''risk-adjusted'' distribution function $F^*(x)$. The mean value under $F^*(x)$, denoted by $\mathrm{E}^*[X]$, is the risk-adjusted ''fair value'' of X at time T. Wang's papers describe the utility of this approach. Our idea is to use observed annuity prices to estimate the market price of risk for annuities, then use the same distribution to price the annuity side of a mortality swap. For the life insurance side we use the same idea. Term life insurance prices determine the market price of risk in the life insurance market, giving us the appropriate distribution for pricing the life insurance side of the swap.

Figure 6 shows the basic idea of the Wang transform. Annuity writers adjust the distribution to compensate for the risk that the annuitants live longer than expected. In this case, according to equation (5.1), the market price of risk λ will be positive, for example, $\lambda = 0.2$ in our example. In other words, the expected number of survivors under the transformed distribution is higher than what a given table suggests. On the other hand, life insurers are compensated for the risk that the insured dies sooner than expected. So in the life insurers' transformed distribution, the number of survivors will be less than that derived from the mortality table: that is, the market price of risk λ is negative (e.g., -0.2 in our example), and its survivor curve lies below that from the given table.

The Wang transform is based on the idea that the life insurance or annuity market price takes into account the uncertainty in the mortality table, as well as the uncertainty in the lifetime of a life insured or an annuitant once the table is given. We are assuming that investors accept the same transformed distribution and independence assumption for pricing mortality swaps.

Figure 6
Basic Idea of the Wang Transform



5.2 Market Price of Risk

For the annuity distribution function $F_a(t) = {}_tq_{65}$, we use the 1996 IAM 2000 Basic Table for a male life age 65. Then assuming an expense factor equal to 4%, we use the 1996 market quotes of qualified immediate annuities (Kiczek 1996) and the U.S. Treasury yield curve on December 30, 1996, to get the market price of risk λ_a by solving the following equation:[2]

$$1000(1 - 0.04) = 7.48 \sum_{j=1}^{\infty} v_{j/12}[1 - F_a^*(j/12)]. \tag{5.3}$$

The sum is over months, measuring time in years, so $v_{j/12}$ is the discount factor at time 0 for a payment of 1 after $j/12$ years. And $[1 - F_a^*(j/12)]$ is the survival probability usually denoted ${}_{j/12}p_{65}$, but using the transformed distribution. This has to be solved numerically for λ_a.

We determine that the market price of risk for annuitants is $\lambda_a = 0.2134$. We think of the 1996 IAM 2000 Basic Table as the actual or physical distribution, which requires a distortion to obtain market prices.

Similarly, for the life insurance distribution function $F_l(t) = {}_tq_{35}$, we use the 1990–95 SOA Basic Table for a male life age 35. A. M. Best (1996) reports 97 companies' market quotes of 10-year level $250,000 term life insurance for both preferred nonsmokers and standard smokers in 1996. The 1990–95 SOA Basic Table is based on data that include both smokers and nonsmokers. We calculated a weighted average of term life insurance prices for each company using weights based on the incidence of smoking in the U.S. population as reported by the Centers for the Disease Control in 1995.[3] Assuming an expense factor equal to 10%, we use the average of smoking-weighted market quotes ($456.73) and the U.S. Treasury yield curve on December 30, 1996, to calculate the market price of risk λ_l by solving the following equation:

$$456.73 \times (1 - 0.10) = 250{,}000 \sum_{k=0}^{9} v_{k+1}[F_l^*(k + 1) - F_l^*(k)], \tag{5.4}$$

where v_{k+1} is the discount factor for a payment after $k + 1$ years. Our calculated λ_l is equal to 0.1933. A positive life insurance market price of risk means the market anticipates mortality improvement for insured lives, relative to the base table.

5.3 Mortality Swap Pricing

We now can price each side of the swap. This will allow us to determine factors b and f for which the two market prices are equal so no cash is paid by either party to initiate the contract; they make only contractual swap payments.

The market price of the payment $f(\mathcal{D}_{x+k} - X_k)_+$ for year k is its discounted expected value $f\mathrm{E}^*[(\mathcal{D}_{x+k} - X_k)_+]$. The "*-distribution" of $\mathcal{D}_{x+k}$ is binomial with parameters $m = \ell_x$ and $q = F_l^*(k + 1) - F_l^*(k)$. The expected value can be calculated as follows:

$$\mathrm{E}^*[(\mathcal{D}_{x+k} - X_k)_+] = \sum_{j=X_k}^{m} (j - X_k)p_j,$$

where

$$p_j = \Pr(\mathcal{D}_{x+k} = j) = \binom{\ell_x}{j} q^j(1 - q)^{\ell_x - j}.$$

[2] The analogous equations in Lin and Cox (2005) are incorrect, although their results are solutions of the correct equations as stated here.

[3] Source: www.cdc.gov. About 27% smokers, 27.5% former smokers, and 45.5% nonsmokers.

Thus we can calculate the market price of the life insurance benefit swap payments as

$$f \sum_{k=0}^{\infty} v_{k+1} \mathrm{E}^{*}[(\mathcal{D}_{x+k} - X_k)_+].$$

We apply this same technique to calculate the market value of the annuity benefit swap payments as

$$b \sum_{k=0}^{\infty} v_{k+1} \mathrm{E}^{*}[(\mathcal{L}(y + k) - Y_k)_+].$$

We need only change the parameters in the calculation of $\mathrm{E}^{*}[(\mathcal{L}(y + k) - Y_k)_+]$ since the distribution of $\mathcal{L}(y + k)$ is binomial with $m = \ell_y$ and $q = 1 - F_a^{*}(k) = {}_kp_y$ (in the transformed table).

Suppose that at time 0 there are $m = 10{,}000$ male annuitants age 65 and $m = 10{,}000$ lives in life insurance on male insured age 35 in a portfolio. For a death benefit of $100,000 per insured, the present value of expected swap payments to the life insurer on those 10,000 lives is $52,775. This is only about 1.6% of the net term life insurance premium.

For the annuity payment side of the swap, the price of mortality swap on 10,000 lives, given annual benefit $1 per annuitant, is $1690. To adjust the average amount of insurance f per term policy to make each side of the swap have the same price, we have

$$1690 = f \frac{52{,}775}{100{,}000},$$

or $f = 3203$. Thus for each dollar of annual annuity benefit we must have $3203 of death benefit to make the prices of the mortality swap to each side equal at time 0. Then each party will have the benefit of natural hedging for 10 years. Even though the prices are equal at time 0, the mortality may move one way or the other, so that the future market value favors one party or the other. The swap has to be revalued each year to properly reflect each company's position, as it may be either an asset or liability.

6. CONCLUSIONS AND DISCUSSION

Natural hedging utilizes the interaction of life insurance and annuities to a change in mortality to stabilize aggregate cash outflows. Our empirical evidence suggests that natural hedging is an important factor contributing to annuity price differences after we control for other variables. These differences become more significant for those insurers selling relatively more annuity business. We expect that future study on life insurers may reach the same conclusion.

Most insurance companies still have considerable net exposures to mortality risks even if they reduce their exposure by pooling individual mortality risk and by balancing their annuity positions against their life positions (Dowd et al. 2004). Natural hedging is feasible, and mortality swaps make it available widely.

We show how to design a mortality swap between a life insurer and an annuity writer to create a natural hedge. Compared with traditional reinsurance and other derivatives, such as mortality bonds, mortality swaps may be arranged at possibly lower costs and in a more flexible way to suit diverse circumstances. Thus, there are good reasons to anticipate increased activity in mortality swaps between life insurers and annuity insurers.

REFERENCES

ARROW, KENNETH J. 1975. Vertical Integration and Communication. *Bell Journal of Economics* 6(1): 173–83.

BERGER, LARRY A., J. DAVID CUMMINS, AND SHARON TENNYSON. 1992. Reinsurance and the Liability Crisis. *Journal of Risk and Uncertainty* 5(3): 253–72.

BEST, A. M. 1996. Term Insurance Survey Annual Renewable and 10-Year Level. *Best's Review* (*L/H*) 97(5): 61–67.

BUETTNER, THOMAS. 2002. Approaches and Experiences in Projecting Mortality Patterns for the Oldest Old. In *Living to 100 and Beyond: Survival at Advanced Ages Symposium*. Schaumburg, IL: Society of Actuaries.

CAIRNS, ANDREW J. G., DAVID BLAKE, AND KEVIN DOWD. 2004. Pricing Death: Frameworks for the Valuation and Securitization of Mortality Risk. Technical report, Heriot-Watt University, Edinburgh, U.K.

CARLTON, DENNIS W. 1979. Vertical Integration in Competitive Markets under Uncertainty. *Journal of Industrial Economics* 27(3): 189–209.

CHATTERJEE, SAYAN, AND MICHAEL LUBATKIN. 1990. Corporate Mergers, Stockholder Diversification and Changes in Systematic Risk. *Strategic Management Journal* 11(4): 255–68.

———. 1992. Vertical Strategies and Market Structure: A Systematic Risk Analysis. *Organization Science* 3(1): 138–56.

CUMMINS, J. DAVID. 1988. Risk-Based Premiums or Insurance Guaranty Funds. *Journal of Finance* 43(4): 823–39.

DOWD, KEVIN, DAVID BLAKE, ANDREW J. G. CAIRNS, AND PAUL DAWSON. 2004. Survivor Swaps. Technical report, working paper no. 7, Centre for Risk and Insurance Studies, University of Nottingham, Nottingham, U.K.

FREES, EDWARD W., JACQUES CARRIÉRE, AND EMILIANO VALDEZ. 1996. Annuity Valuation with Dependent Mortality. *Journal of Risk and Insurance* 63(2): 229–61.

FROOT, KENNETH A., AND PAUL G. J. O'CONNEL. 1997. On the Pricing of Intermediated Risks: Theory and Application to Catastrophe Reinsurance. Working paper, Harvard Business School, Cambridge, MA.

GOSS, STEPHEN C., ALICE WADE, AND FELICITIE BELL. 1998. Historical and Projected Mortality for Mexico, Canada, and the United States. *North American Actuarial Journal* 2(4): 108–26.

GREEN, WILLIAM H. 2000. *Econometric Analysis*. 4th ed. Prentice-Hall.

HAYFLICK, LEONARD. 2002. Longevity Determination and Aging. In *Living to 100 and Beyond: Survival at Advanced Ages Symposium*. Schaumburg, IL: Society of Actuaries.

HELFAT, CONSTANCE E., AND DAVID J. TEECE. 1987. Vertical Integration and Risk Reduction. *Journal of Law, Economics, and Organization* 3(1): 47–68.

KICZEK, JOHN A. 1995. Single Premium Immediate Annuity Payouts Study. *Best's Review* (*L/H*) 96(3): 51–54.

———. 1996. Single Premium Immediate Annuity Payouts. *Best's Review* (*L/H*) 97(4): 57–60.

———. 1997. Comparing the Payout. *Best's Review* (*L/H*) 97(4): 24–29.

LIN, YIJIA, AND SAMUEL H. COX. 2005. Securitization of Mortality Risks in Life Annuities. *Journal of Risk and Insurance* 72(2): 227–52.

MARCEAU, ETIENNE, AND PATRICE GAILLARDETZ. 1999. On Life Insurance Reserve in a Stochastic Mortality and Interest Rates Environment. *Insurance: Mathematics and Economics* 25(3): 261–80.

MERTON, ROBERT C. 1973. Theory of Rational Option Pricing. *Bell Journal of Economics and Management Science* 4(1): 141–83.

MILEVSKY, MOSHE A., AND S. DAVID PROMISLOW. 2001. Mortality Derivatives and the Option to Annuitize. *Insurance: Mathematics and Economics* 29(3): 299–318.

MITCHELL, EDWARD J. 1978. Capital Cost Savings of Vertical Integration. *Vertical Integration in the Oil Industry*. Washington, DC: American Enterprise Institute.

NOYMER, ANDREW, AND MICHEL GARENNE. 2000. The 1918 Influenza Epidemic's Effects on Sex Differentials in Mortality in the United States. *Population and Development Review* 26(3): 565–81.

PEAVY, JOHN W. 1984. Modern Finance Theory, Corporate Strategy, and Public Policy: Another Perspective. *Academy of Management Review* 9(1): 152–57.

PHILLIPS, RICHARD D., J. DAVID CUMMINS, AND FRANKLIN ALLEN. 1998. Financial Pricing of Insurance in the Multiple-Line Insurance Company. *Journal of Risk and Insurance* 65(4): 597–636.

ROGERS, RICK. 2002. Will Mortality Improvements Continue? *National Underwriter* 106(34): 11–13.

SALTER, MALCOLM S., AND WOLF A. WEINHOLD. 1979. *Diversification through Acquisition*. New York: Free Press.

SOMMER, DAVID W. 1996. The Impact of Firm Risk on Property-Liability Insurance Prices. *Journal of Risk and Insurance* 63(3): 501–14.

TAUBENBERGER, JEFFERY K., AND DAVID M. MORENS. 2006. 1918 Influenza: The Mother of All Pandemics. *Emerging Infectious Diseases* 12(1): 15–22. www.cdc.gov/ncidod/EID/vol12no01/05-0979.htm.

WANG, JENNIFER L., MARY L. Y. YANG, AND STEVE Y. C. PAN. 2003. Hedging Longevity Risk in Life Insurance Companies. In Asia-Pacific Risk and Insurance Association, *2003 Annual Meeting Proceedings,* Bangkok, Thailand.

WANG, SHAUN. 1996. Premium Calculation by Transforming the Layer Premium Density. *ASTIN Bulletin* 26 (1): 71–92.

———. 2000. A Class of Distortion Operations for Pricing Financial and Insurance Risks. *Journal of Risk and Insurance* 67(1): 15–36.

———. 2001. A Universal Framework for Pricing Financial and Insurance Risks. In *The 6th AFIR Proceedings,* September, pp. 679–703.

Discussions on this paper can be submitted until January 1, 2008. The authors reserve the right to reply to any discussion. Please see the Submission Guidelines for Authors on the inside back cover for instructions on the submission of discussions.