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"Asset Allocation with Hedge Funds on the Menu," Phelim Boyle and Sun Siang Liew, October 2007

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The authors are to be congratulated for this timely paper. This discussion is motivated by the statement on page 8 that "it would be useful if we could tell . . . which regime we are in at a particular time." We wish to point out that, in theory, the volatility parameter can be read off from any given sample path: it can be identified by means of the *quadratic variation* of the natural logarithm of the asset price.

Let $S(\tau)$ be the price of the asset or hedge fund at time τ, measured in years. Let

$$X(\tau) = \ln[S(\tau)/S(0)] \tag{D.1}$$

be the continuously compounded return over the time period $[0, \tau]$. A usual assumption in finance is that $X(\tau)$ satisfies a stochastic differential equation such as

$$dX(\tau) = \mu(\tau)d\tau + \sigma(\tau)dW(\tau), \tag{D.2}$$

where $\{W(\tau)\}$ is a standard Brownian motion. For the regime-switching model in the paper, it is assumed that μ and σ are constant for each month, that is,

$$\mu(\tau) = \mu(\lfloor 12\tau \rfloor/12)$$

and

$$\sigma(\tau) = \sigma(\lfloor 12\tau \rfloor/12).$$

Actuarial students have learned from Exam MFE/3F (McDonald 2006, pp. 658–59) the multiplication rules

$$d\tau \times d\tau = 0,$$
$$d\tau \times dW(\tau) = 0,$$
$$dW(\tau) \times dW(\tau) = d\tau.$$

Thus,

$$[dX(\tau)]^2 = [\sigma(\tau)]^2\, d\tau, \tag{D.3}$$

and hence,

$$\int_a^b [dX(\tau)]^2 = \int_a^b [\sigma(\tau)]^2\, d\tau. \tag{D.4}$$

If a and b are time points in the same month, then $\sigma(\tau)$ is the same for all $\tau \in [a, b]$ by assumption, and hence by (D.4) it is given by

$$\frac{1}{b-a}\int_a^b [dX(\tau)]^2. \tag{D.5}$$

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Note that (D.5) remains a valid formula for determining the volatility parameter, no matter how short the time interval $[a, b]$ is. Also, one can derive (D.5) without the multiplication rules; see Section 10.13, ''A Layman's Guide to Brownian Motion (Wiener Process),'' in Panjer (1998).

Finally, we wish to point out the following problem in the ''Sample Questions and Solutions'' of Exam MFE/3F:

$$\lim_{n\to\infty} \sum_{j=1}^{n} [X(jT/n) - X((j-1)T/n)]^2 = \sigma^2 T. \tag{D.6}$$

A derivation is as follows. From

$$X(t+h) - X(t) = \mu h + \sigma[W(t+h) - W(t)],$$

we have

$$[X(t+h) - X(t)]^2 = \mu^2 h^2 + 2\mu h\sigma[W(t+h) - W(t)] + \sigma^2[W(t+h) - W(t)]^2.$$

With $h = T/n$,

$$\sum_{j=1}^{n} [X(jT/n) - X((j-1)T/n)]^2$$

$$= \mu^2 T^2/n + 2\mu(T/n)\sigma[W(T) - W(0)] + \sigma^2 \sum_{j=1}^{n} [W(jT/n) - W((j-1)T/n)]^2. \tag{D.7}$$

As $n \to \infty$, the first two terms on the right-hand side of (D.7) become 0, and the sum on the right-hand side of (D.7) becomes T in accordance with formula (20.6) of McDonald (2006, p. 653).

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''Markov Aging Process and Phase-Type Law of Mortality,'' X. Sheldon Lin and Xiaoming Liu, October 2007

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In recent years there has been increasing interest in stochastic modeling of human mortality. This interesting paper by Dr. Lin and Ms. Liu contributed a new analytical mortality law, which has explicitly taken account of the underlying unobservable physiological processes. In this discussion we further explore the computations involved in estimating the model parameters.

In equation (4.2) in Lin and Liu's paper, we see that the survival function under the proposed phase-type law of mortality involves the matrix exponential of $x\boldsymbol{\Lambda}$. The exponential of an $n \times n$ square matrix A is defined by the usual series expansion

$$e^{A} = \boldsymbol{I} + \boldsymbol{A} + \frac{1}{2!}\boldsymbol{A}^2 + \frac{1}{3!}\boldsymbol{A}^3 + \cdots + \frac{1}{r!}\boldsymbol{A}^r + \cdots.$$

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