Market Price of Insurance Risk Implied by Catastrophe Derivatives

Alexander Muermann*

Abstract

Insurance derivatives facilitate the trading of insurance risks on capital markets, such as catastrophe derivatives that were traded on the Chicago Board of Trade. Simultaneously, insurance risks are traded through reinsurance portfolios. In this paper we make inferences about the market price of risk implied by the information embedded in the prices of these two assets.

1. Introduction

The insurance and reinsurance industry has become increasingly concerned about the concentration of exposures linked to a single event, such as a natural catastrophe or a terrorist attack. Those industries traditionally provided the vehicle in the private sector for spreading risks of such magnitudes across society. In 1992 Hurricane Andrew caused \$16 billion in insured losses, and more than 60 insurance companies became insolvent. The estimates of insured losses due to Hurricane Katrina in 2005 range from \$40 to \$60 billion. In need for alternative means of risk spreading that would add capacity to the market dealing with such extreme events, the private sector has been exploring the securitization of those risks. In addition to adding capital, a liquid catastrophe derivatives market would allow insurance and reinsurance companies to adjust their exposure to natural catastrophic risk dynamically through hedging with those standardized financial contracts at low transaction costs. In 1993 catastrophe derivatives were introduced at the Chicago Board of Trade (CBoT). These exchange-traded financial derivatives were based on underlying indexes reflecting insured property losses due to natural catastrophes that were self-reported by insurance and reinsurance companies. Insurance futures and options on insurance futures were the first type of contracts traded at the CBoT. Because of the very low trading activity, they were replaced in 1995 by catastrophe spread options based on underlying loss indexes, which are provided by Property Claim Services (PCS), an independent statistical agency. See D'Arcy and France (1992) for a detailed description of insurance futures, O'Brien (1997) for catastrophe spread options, and Cox, Fairchild, and Pedersen (2000) for a general discussion on the securitization of risks.

The pricing of catastrophe derivatives is challenging as the market is incomplete. Natural catastrophes cause jumps in the underlying indexes of random size at random points in time. It is thus not possible to determine a unique price process for catastrophe derivatives purely based on the exclusion of arbitrage opportunities. The literature on pricing of catastrophe derivatives either assumes that jump sizes of the underlying loss index are constant or specifies investors' preferences. Cummins and Geman (1995) and Geman and Yor (1997) assume that the underlying loss index follows a geometric Brownian plus a Poisson process with constant jump sizes. Although unique pricing under no arbitrage is appealing, the assumption of constant loss sizes is questionable in the context of natural catastrophic risk. Aase (1999) takes a different, more realistic modeling approach. He models the dynamics of the underlying loss index as a compound Poisson process with random jump sizes and specifies the pref-

* Alexander Muermann, PhD, is Professor of Risk Management and Insurance in the Department of Finance and Accounting at the Vienna University of Economics and Business Administration. Heiligenstädter Str. 46–48, A-1190 Wien, Austria. alexander.muemann@wu-wien.ac.at.

erences of market participants by a utility function. Price processes are then determined within a partial equilibrium framework. Embrechts and Meister (1997) generalize Aase's stochastic model by allowing for mixed compound Poisson processes with stochastic frequency rate. Cox and Pedersen (2000) determine prices of catastrophe risk bonds within a equilibrium setting.

In this paper we exploit the fact that the underlying insurance risk is simultaneously traded in reinsurance portfolios. When those portfolios are transferred to financial markets, for example, through stock trading of stock reinsurers, their prices are formed by the same mechanism that underlies the price formation of catastrophe derivatives. We utilize this link between both prices and make inferences about the implied market price of the underlying insurance risk while keeping the more realistic model of a compound Poisson process for the underlying loss index, as in Aase (1999).

In Section 2 we introduce the model for the underlying risk process and the contracts that are traded on the market. Section 3 derives the market price for catastrophe risk implied by a catastrophe call option and a reinsurance portfolio. We conclude in Section 4.

2. THE MARKET

In this section we define the stochastic process for the underlying loss index, the specifications of the catastrophe derivative and reinsurance portfolio, and their price processes.

2.1 Risk Process

Uncertainty in the market is described by a probability space $(\Omega, \mathcal{F}, P)$ on which random variables will be defined. We assume that the economy is of finite horizon $T < \infty$, and the flow of information is modeled by a nondecreasing family of σ-algebras $(\mathcal{F}_t)_{0 \le t \le T}$, a filtration. We assume that $\mathcal{F}_T = \mathcal{F}$, each $\mathcal{F}_t$ contains the events in $\mathcal{F}$ that are of P-measure zero, and the filtration is right-continuous, $\mathcal{F}_t = \mathcal{F}_{t+}$, where $\mathcal{F}_{t+} = \bigcap_{s>t} \mathcal{F}_s$.

The risk faced by the insurance and reinsurance industry is inherent in their exposure to accumulated insured property losses. Because natural catastrophes cause claims of extreme magnitude, we follow the approach of Aase (1999) and assume that the process of accumulated insured property losses is a compound Poisson process $X = (X_t)_{0 \le t \le T}$. The random variable X_t thus represents the sum of insured property losses to the industry incurred in $(0, t]$:

$$X_t = \sum_{\{k \mid T_k \le t\}} Y_k = \sum_{k=1}^{N_t} Y_k, \tag{2.1}$$

where T_k is the random time point of occurrence of the kth catastrophe that causes a corresponding insured property loss of size Y_k, and N_t is a random variable counting catastrophes up to and including time t. We assume that the counting process $N = (N_t)_{0 \le t \le T}$ is a Poisson process with intensity λ, and $Y_1, Y_2, \ldots$ are nonnegative, independent, and identically distributed random variables, all independent of the counting process N. Let G be the distribution function of Y_k with support $[0, \infty)$. The process X is fully characterized by the pair $(\lambda, dG(y))$, which are called the characteristics of X.

Under these assumptions X is thus a time-homogeneous process with independent increments and belongs to the class of Lévy processes. Actuarial studies (see Levi and Partrat 1991) have shown that these assumptions are reasonable in the context of losses arising from windstorm, hail, and flood. Earthquakes are described as events arising from a superposition of events caused by several independent sources. Accumulated insured losses due to earthquakes can therefore be approximated by a compound Poisson process.[1] The assumption of time homogeneity is questionable for the case of hur-

[1] Cossette, Duchesne, and Marceau (2003) propose an interesting catastrophe risk model that allows for dependence of claims caused by a single catastrophe. In their model damage ratios of insured properties are driven by the catastrophe's intensity.

ricanes, which occur seasonally. In the context of catastrophe derivatives, however, contracts linked to regions that are exposed to hurricane risk all track quarterly loss periods to account for seasonal effects.

We assume that the past evolution and current state of the risk process X is observable by every agent in the economy: that is, X is assumed to be adapted to the filtration $(\mathcal{F}_t)_{0 \le t \le T}$. We thus exclude any effects that asymmetric information may have on the market through moral hazard or adverse selection problems. For simplicity, it is assumed that X generates the flow of information, $\mathcal{F}_t = \sigma(\sigma(X_s, s \le t) \cap \mathcal{N})$, where $\mathcal{N}$ denotes the events of P-measure zero.

2.2 Equivalent Probability Measures

Changing the probability measure plays a central role in the context of no-arbitrage valuation of contracts because their discounted price processes are martingales under the appropriate probability measure. For compound Poisson processes, Delbaen and Haezendonck (1989) characterized the set of probability measures Q on $(\Omega, \mathcal{F})$ that are equivalent to the "reference" measure P and that preserve the structure of the underlying risk process X, such that X is a compound Poisson process under the new probability measure Q. Aase (1992) showed that this set can be parameterized by a pair $(\kappa, v(\cdot))$, where κ is a strictly positive constant κ and $v : \mathbb{R}_+ \to \mathbb{R}$ is a measurable function that is strictly positive on the support of G with $\mathbf{E}^P\,[v(Y_1)] = 1$. The density process $\xi_t = \mathbf{E}^P[\xi_T | \mathcal{F}_t]$ of the Radon-Nikodym derivative $\xi_T = dQ/dP$ is then given by

$$\xi_t = \exp\left(\sum_{k=1}^{N_t} \ln(\kappa v(Y_k)) + \lambda(1 - \kappa)t \right), \tag{2.2}$$

for any $0 \le t \le T$ under the assumption that $\mathbf{E}^P[\exp(\Sigma_{k=1}^{N_t} \ln(\kappa v(Y_k)))] < \infty$. Let $P^{\kappa,v}$ denote the equivalent probability measure that corresponds to the constant κ and the function $v(\cdot)$. Under the new measure $P^{\kappa,v}$, the process X is then a compound Poisson process with characteristics $(\lambda^*, dG^*(y))$, where $\lambda^* = \lambda\kappa$ and $dG^*(y) = v(y)dG(y)$.[2]

Aase (1992, 1993) interprets the Radon-Nikodym derivative (2.2) as the marginal disutility of the market that endogenously arises in equilibrium of a dynamic pure risk exchange economy. He calls κ the market price of frequency risk and $v(\cdot)$ the market price of claim size risk.

2.3 Assets and Price Processes

We assume that three assets are traded in the capital market: a risk-free bond, a reinsurance portfolio, and a catastrophe derivative. The risk-free bond accumulates interest at a deterministic rate r, continuously compounded. The reinsurance portfolio specifies a premium process $p = (p_t)_{0 \le t \le T}$ for the industry's overall insured losses $X = (X_t)_{0 \le t \le T}$. The premium p_t defines the price at time t for the remaining risk, $X_T - X_t$.[3] The catastrophe derivative is a European-style derivative with maturity T that is written on the same underlying risk process X.[4] The payoff of the catastrophe derivative thus depends on the realization of X_T only and specifies a price process $\pi = (\pi_t)_{0 \le t \le T}$. Let $\phi : \mathbb{R}_+ \to \mathbb{R}$ be a measurable function that specifies the payoff at maturity to the buyer of the catastrophe derivative, that is, at T the buyer receives $\phi(X_T)$. The price π_t defines the price at time t that the buyer has to pay to enter into the financial contract.

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[2] The Esscher transform specifies a particular change of measure in this context for the parameters $\kappa = 1$ and $v(y) = e^{\alpha y}/\mathrm{E}[e^{\alpha Y_1}]$. Esscher transforms have been applied to premium calculation principles and to option pricing (see Delbaen and Haezendonck 1989; Gerber and Shiu 1994; Bühlmann et al. 1998).

[3] We do not consider reinstatement provisions, which are a common feature in reinsurance contracts. A reinstatement provision puts a limit on the number of events and/or aggregate losses that will be paid under the reinsurance contract. See Anderson and Dong (1998) for the pricing of reinsurance with reinstatement provisions using the output of a catastrophe model.

[4] An important assumption of our analysis is that the catastrophe derivative does not replace the reinsurance contract. Both contracts coexist with the same underlying risk, and both prices are therefore observable. We envision a situation in which the risk is transferred to a reinsurance company for a premium p_t. Simultaneously part of the risk is then further transferred to the capital market, where the price of the insurance derivative π_t is observed.

In the absence of arbitrage strategies the fundamental theorem of asset pricing states that both discounted price processes are martingales equivalent probability measure $P^{\kappa,v}$.[5] The two price processes can therefore be represented as

$$p_t^{\kappa,v} = e^{-r(T-t)}\mathbf{E}^{P^{\kappa,v}}[X_T - X_t|\mathcal{F}_t] \tag{2.3}$$

and

$$\pi_t^{\kappa,v} = e^{-r(T-t)}\mathbf{E}^{P^{\kappa,v}}[\phi(X_T)|\mathcal{F}_t] \tag{2.4}$$

for all $0 \le t \le T$.

3. IMPLIED MARKET PRICE OF RISK

We now examine the market price of catastrophe risk implied by observed prices of the reinsurance portfolio and a catastrophe call option. Prices $p_t^{\kappa,v}$ and $\pi_t^{\kappa,v}$ are given, and pricing formulas (2.3) and (2.4) will be used to make inferences about the characteristics $(\lambda^*, dG^*(y))$ and thereby the market price of frequency and claim size risk, κ and $v(\cdot)$.

Proposition 1

Suppose a reinsurance portfolio and an out-of-the-money catastrophe call option with strike price K is traded in the capital market with observed prices $p_t^{\kappa,v}$ and $\pi_t^{\kappa,v}$ at time t. Assume that one catastrophe is sufficient to bring an out-of-the-money call option in the money, $G(K - X_t) = 0$. Then the market price of frequency risk and the risk-adjusted expected loss size are given by

$$\kappa = \frac{1}{\lambda(T-t)} \ln\left(\frac{K - X_t}{K - X_t - e^{r(T-t)}(p_t^{\kappa,v} - \pi_t^{\kappa,v})}\right) \tag{3.1}$$

and

$$\mathbf{E}^{P^{\kappa,v}}[Y_1] = e^{r(T-t)}p_t^{\kappa,v}\ln\left(1 - \frac{e^{r(T-t)}(p_t^{\kappa,v} - \pi_t^{\kappa,v})}{K - X_t}\right). \tag{3.2}$$

PROOF

The underlying risk process X is a Markov process with independent and stationary increments that generates the filtration $(\mathcal{F}_t)_{0\le t\le T}$. Prices as given by (2.3) and (2.4) can thus be expressed as

$$p_t^{\kappa,v} = e^{-r(T-t)}\mathbf{E}^{P^{\kappa,v}}[X_{T-t}] = e^{-(\lambda^*+r)(T-t)} \sum_{k=1}^{\infty} \frac{(\lambda^*(T-t))^k}{k!} \int_0^\infty y\, dG^{*k}(y) \tag{3.3}$$

and

$$\begin{aligned}\pi_t^{\kappa,v} &= e^{-r(T-t)}\mathbf{E}^{P^{\kappa,v}}[\phi(X_{T-t} + X_t)] \\ &= e^{-(\lambda^*+r)(T-t)}\left(\phi(X_t) + \sum_{k=1}^{\infty} \frac{(\lambda^*(T-t))^k}{k!} \int_0^\infty \phi(y + X_t)\, dG^{*k}(y)\right),\end{aligned} \tag{3.4}$$

where $G^{*k}(\cdot)$ is the distribution function of the k-fold convolution. The price of a catastrophe out-of-the-money call option with strike price K and maturity T is given by

$$\pi_t^{\kappa,v} = e^{-(\lambda^*+r)(T-t)} \sum_{k=1}^{\infty} \frac{(\lambda^*(T-t))^k}{k!} \int_0^\infty (y + X_t - K)^+\, dG^{*k}(y). \tag{3.5}$$

The assumption $G(K - X_t) = 0$ implies $G^*(K - X_t) = 0$ as the probability measures P and $P^{\kappa,v}$ are equivalent. Pricing formulas (3.4) and (3.3) then yield

[5] We note that sometimes the stronger requirement of "no free lunch with vanishing risk" is needed for the existence of an equivalent martingale measure (see Delbaen and Schachermayer 2006).

$$\pi_t^{\kappa,v} = e^{-(\lambda^*+r)(T-t)} \sum_{k=1}^{\infty} \frac{(\lambda^*(T-t))^k}{k!} \int_0^{\infty} (y + X_t - K)\, dG^{*k}(x)$$
$$= p_t^{\kappa,v} - e^{-r(T-t)}(K - X_t)(1 - e^{-\lambda^*(T-t)}). \tag{3.6}$$

Solving for the implied risk-adjusted intensity λ^* yields

$$\lambda^* = \frac{1}{(T-t)} \ln\left(\frac{K - X_t}{K - X_t - e^{r(T-t)}(p_t^{\kappa,v} - \pi_t^{\kappa,v})}\right). \tag{3.7}$$

The market price of frequency risk κ implied by $p_t^{\kappa,v}$ and $\pi_t^{\kappa,v}$ proves (3.1).

Because the risk process X has stationary and independent increments, the martingale property implies that the premium process is of the form

$$p_t^{\kappa,v} = e^{-r(T-t)}\mathbf{E}^{P^{\kappa,v}}[X_1](T-t) = e^{-r(T-t)}\lambda^*\mathbf{E}^{P^{\kappa,v}}[Y_1](T-t). \tag{3.8}$$

We derive (3.2) by substituting λ^* from equation (3.7). ■

Under the assumption $G(K - X_t) = 0$, the information contained in the prices of the catastrophe call option and the reinsurance portfolio is sufficient to derive the market price for frequency risk and the risk-adjusted expected loss size.

From Proposition 1 we obtain the following comparative statics. The market price of frequency risk is increasing in the difference in prices, $p_t^{\kappa,v} - \pi_t^{\kappa,v}$, and in the risk-free interest rate, r; and it is decreasing in the "physical" frequency, λ, and in the difference $K - X_t$, keeping everything else fixed respectively. The risk-adjusted expected loss size is increasing in the difference $K - X_t$, and in the catastrophe call option price, $\pi_t^{\kappa,v}$, again keeping everything else fixed respectively. Furthermore, the risk-adjusted frequency λ^* is larger than the "physical" frequency λ if and only if

$$p_t^{\kappa,v} - \pi_t^{\kappa,v} > e^{-r(T-t)}(K - X_t)(1 - e^{-\lambda(T-t)}), \tag{3.9}$$

and the risk-adjusted distribution has a higher expected loss than the "physical", $\mathbf{E}^P[Y_1]$, if and only if

$$p_t^{\kappa,v} - \pi_t^{\kappa,v} < e^{-r(T-t)}(K - X_t)\left(1 - \exp\left(-\frac{e^{r(T-t)}p_t^{\kappa,v}}{\mathbf{E}^P[Y_1]}\right)\right). \tag{3.10}$$

To infer more information about the market price of severity risk, we have to make assumptions about the distribution of loss sizes and its change under the change of probability measure.

3.1 Pareto Distribution

Suppose that the size of insured property losses is distributed according to a Pareto distribution under the physical probability measure P,

$$\frac{dG(y)}{dy} = \alpha \cdot \frac{c^\alpha}{y^{\alpha+1}}, \tag{3.11}$$

with shape parameter $\alpha > 0$, scale parameter $c > 0$, and support $y \in [c, +\infty)$. The condition $G(K - X_t) = 0$ implies $c \geq K - X_t$. Furthermore, assume that the change in probability measure preserves the property that insured losses are distributed according to a Pareto distribution and results in a shape parameter α^* under $P^{\kappa,v}$. Because P and $P^{\kappa,v}$ are equivalent probability measures, their support and thus their scale parameter c is identical. The expected value of the loss size under $P^{\kappa,v}$ is $E^{P^{\kappa,v}}[Y_1] = \alpha^*c/\alpha^*-1$ for $\alpha^* > 1$.[6] Equation (3.2) yields

[6] For $\alpha^* \leq 1$ the expected value is infinite.

$$\alpha^* = \frac{e^{r(T-t)}p_t^{\kappa,v}\ln\left(1 - \dfrac{e^{r(T-t)}(p_t^{\kappa,v} - \pi_t^{\kappa,v})}{K - X_t}\right)}{e^{r(T-t)}p_t^{\kappa,v}\ln\left(1 - \dfrac{e^{r(T-t)}(p_t^{\kappa,v} - \pi_t^{\kappa,v})}{K - X_t}\right) - c}. \tag{3.12}$$

The implied market price of severity risk $v(\cdot)$ is then given by

$$v(y) = \left(\frac{\alpha^*}{\alpha}\right) \cdot \left(\frac{c}{y}\right)^{\alpha^*-\alpha}, \tag{3.13}$$

where α^* is defined by (3.12). The risk-adjusted loss size distribution is riskier than the "physical" in the sense of having a higher mean and a higher variance if $\alpha^* < \alpha$. This is the case if and only if

$$p_t^{\kappa,v} - \pi_t^{\kappa,v} < e^{-r(T-t)}(K - X_t)\left(1 - \exp\left(-\frac{e^{r(T-t)}p_t^{\kappa,v}}{\dfrac{\alpha c}{\alpha - 1}}\right)\right).$$

3.2 Erlang(n) Distribution

If the size of insured property losses is distributed according to an Erlang(n) distribution under the physical probability measure P, then

$$\frac{dG(y)}{dy} = \frac{c^n}{\Gamma(n)}\, e^{-cy}y^{n-1} \tag{3.14}$$

with shape parameter $n \in \mathbb{N}$, scale parameter $c \in \mathbb{R}_+$, and support $(0, +\infty)$. Note that the condition $G(K - X_t) = 0$ is not satisfied. Aase (1999) derives a bound for the approximation error of this assumption for loss sizes that are distributed according to an Erlang(n) distribution. He concludes that the approximation works generally well unless X_t is small relative to K. We therefore continue under the assumption that the approximation error is small. Again we assume that the change in probability measure preserves the property that insured losses are distributed according to an Erlang(n) distribution and results in a scale parameter c^* under $P^{\kappa,v}$. This invariance property is valid, for example, if investors' preferences exhibit constant absolute risk aversion (see Aase 1999). The expected value of the loss size under $P^{\kappa,v}$ is $E^{P^{\kappa,v}}[Y_1] = n/c^*$, and equation (3.2) yields

$$c^* = \frac{n \ln\left(\dfrac{K - X_t}{K - X_t - e^{r(T-t)}(p_t^{\kappa,v} - \pi_t^{\kappa,v})}\right)}{e^{r(T-t)}p_t^{\kappa,v}}. \tag{3.15}$$

The implied market price of severity risk $v(\cdot)$ is then given by

$$v(y) = \left(\frac{c^*}{c}\right)^n e^{-(c^*-c)y}, \tag{3.16}$$

where c^* is defined by (3.15). The risk-adjusted loss size distribution is riskier than the "physical" in the sense of having a higher mean and a higher variance if $c^* < c$. This is the case if and only if

$$p_t^{\kappa,v} - \pi_t^{\kappa,v} < e^{-r(T-t)}(K - X_t)\left(1 - \exp\left(\frac{e^{r(T-t)}p_t^{\kappa,v}}{n/c}\right)\right). \tag{3.17}$$

4. CONCLUDING REMARKS

In this paper we made inferences about the market price of catastrophe risk from observed prices of catastrophe derivatives and traded reinsurance portfolios. In future research it would be interesting to include basis risk in catastrophe derivatives and/or investment risk in the reinsurance portfolio to account for a potential mismatch between the exposures of reinsurance portfolios and the underlying loss indexes.

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