A Simple Model of Insurance Market Dynamics

Greg Taylor*

Abstract

This paper constructs and studies a simple but realistic model of an insurance market. The model has a minimalist construction in the sense that the number of parameters defining it is strictly limited and the elimination of any one of them would destroy its realism. There are 11 essential parameters. Each of the parameters has a physical interpretation. Some determine competitive effects within the market, some barriers to entry, and so on. The effect of each on various aspects of the market is examined in the presence of simulated loss experience. The aspects of the market considered include stability of premium rates, profitability, and market concentration. Some of the parameters are capable of use as regulatory controls. Two parameters, in addition to the original 11, are explicit price controls. Despite its simplicity, the model displays considerably complex behavior. Some results are intuitive, but some are not. For this reason, regulatory controls need to be applied with great caution lest they induce perverse effects, possibly even the reverse of those intended. The effect of the parameters on market behavior is first studied in the absence of catastrophic events from the loss experience. Subsequently, the effect of a single such event is studied.

1. Introduction

This paper constructs a model of a competitive insurance market. The intention is that the model be as simple as possible, that is, contain as few parameters as possible, while capturing all the essential dynamics of a real market. This allows the effects of individual components of the model, such as barriers to industry entry, to be studied.

Some of these effects may be relevant to public policy. For example, high insurance profit margins, or highly volatile insurance premium rates, are viewed unfavorably by consumer advocates. Higher levels of insurer failure may be viewed more unfavorably than lower levels by insurer regulators or legislatures.

The paper simulates the behavior of a market according to the defined model and examines the effects of variation of individual parameters, or small groups of parameters, that define specific economic effects on the market. It may be possible to influence these parameters, and hence the behavior of the market, if this is seen as desirable public policy.

This approach to insurance market modeling is somewhat different from previous examples in the literature. The literature contains numerous studies of particular isolated aspects of the market. The emphasis here, however, is less on the detail of any single aspect and more on the integration of all market dynamics into a single model.

In a relatively little known paper that might reasonably be considered a seminal contribution to the subject now known as dynamic financial analysis (DFA), Coutts and Devitt (1989) introduced stochastic modeling of a single insurance operation but did not link this to competitor behavior through competitive dynamics. A related paper by Daykin et al. (1987) covered similar ground.

* Greg Taylor, PhD, FIA, FIAA, FIMA, CMath, CSci, is a Director of Taylor Fry Consulting Actuaries, Level 8, 30 Clarence St., Sydney NSW 2000, Australia. He is also a Professorial Fellow in the Centre for Actuarial Studies, Faculty of Economics and Commerce, University of Melbourne, Parkville VIC 3052, Australia, and a Professorial Visiting Fellow in the School of Actuarial Studies, Australian School of Business, University of New South Wales, Kensington NSW 2033, Australia, greg.taylor@taylorfry.com.au.

Daykin and Hey (1990) and Daykin, Pentikäinen, and Pesonen (1994) added simple competitive dynamics but retained the main focus on the behavior of a single insurer in a competitive market. Market cycles were regarded as exogenous. Much DFA development has been conducted since these authors, but, by its nature, this work too is usually concerned with the future financials of a single insurer, with or without recognition of external competitive influences.

2. MODEL SPECIFICATION

2.1 Outline

As mentioned in Section 1, the intention here is to construct a simple model that recognizes the major dynamics of an insurance market, and listing these may be useful. It is supposed that the insurance market, consisting of a number of insurers, operates in discrete time. The unit of time is arbitrary but may be thought of as the minimum period during which an insurer is able to reassess and respond to its market circumstances.

It is assumed that the market underwrites a single line of business. An equivalent assumption is that each insurer underwrites the same composition of business by line and that, over time, its premium rates in all lines vary by the same percentage factors. There is clear scope for generalization here.

In each period each insurer behaves as follows:

- It has certain target premium rates, which, if achieved in its underwriting, would achieve certain corporate goals, such as return on equity (Section 2.2.1).
- These target premium rates are tempered by competitive forces that are manifest in the form of the rates charged by competitors in the immediate past (Section 2.2.2).
- The resulting premium rates brought to market by the insurer will generate underwriting results for the quarter. These will be influenced by the loss experience, which is a stochastic quantity and not fully under the insurer's control. It may include catastrophe (CAT) events (Section 2.2.3).
- These results update the insurer's balance sheet, including solvency position (Section 2.2.4).
- At the end of the period, the competitive position of the market is rebalanced. The insurer will exit the market if its solvency position has fallen below a threshold of viability (Section 2.2.5).

Further, at the end of the period:

- New insurers may enter the market. They will be lured to do so if the emerging profit margins are sufficiently lucrative (Section 2.2.6).
- Market shares of all insurers now participating in the market are rebalanced. Business will be attracted from insurers offering higher premium rates to those offering lower (Section 2.2.7).

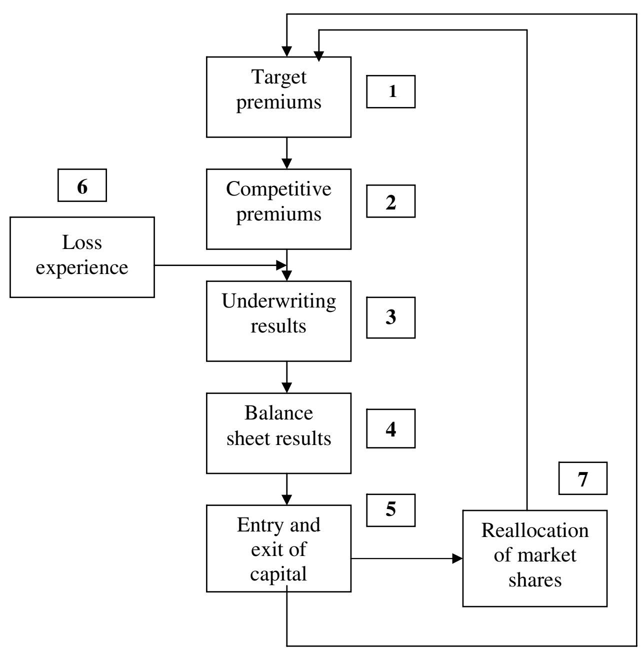
The process then recommences at the beginning.

This process is represented in flow chart form in Figure 1. The mathematical details of the dynamics appear in Section 2.2. A boxed number "n" in the figure means that the detailed description appears in Section 2.2.n.

2.2 Detail

This subsection gives the mathematical details of the market dynamics. It is reiterated that the emphasis is on simplicity—on capturing the essen-

Figure 1
Market Process over a Single Period



tial market detail with as few parameters as possible.

First, some notation: Boldface text indicates the definition of an input to the process or a system parameter.

Let

t = iteration number (suppressed within an iteration, but explicit when the relation between iterations is at issue)
I_t = number of insurers in tth iteration
i = label for ith insurer (in order of volume of exposure units)

E_{it} = number of exposure units underwritten
$\boldsymbol{E_t = \Sigma_i E_{it}}$ **= total exposure for whole market** (supposed constant over time, and so occasionally represented as just E)
K_{it} = net assets
$S_{it} = K_{it}/(P_{it}E_{it})$ = solvency ratio

$\boldsymbol{r_F}$ **= risk-free rate of return**
$\boldsymbol{r_M}$ **= stock market expected rate of return**

$\boldsymbol{\lambda_C}$ **= expected CAT claim frequency (for whole market)**
$\boldsymbol{\mu_C}$ **= expected CAT claim size (for whole market)**
$\boldsymbol{\lambda_N}$ **= expected non-CAT claim frequency per exposure unit (common to all insurers)**
$\boldsymbol{\mu_N}$ **= expected non-CAT claim size (common to all insurers)**
T_{it} = target premium per unit exposure
P_{it} = competitive premium per unit exposure

L_{Cit} = CAT incurred losses
L_{Nit} = non-CAT incurred losses

π_{it} = insurance profit
D_{it} = dividend paid.

For simplicity, it is assumed that all premiums are received at the start of a period and all associated claims paid at the end of that period. It is further assumed that each insurer invests its premiums received P in risk-free assets, and its net assets K in equities. Then let

r_{Kit} = insurer's expected return on total assets

$$= r_F + (r_M - r_F)[K_{it}/(K_{it} + E_{it}P_{it})].$$

The system will have a steady state in which, if losses and investment return occurring in each period are equal to their expected values, premiums will be sufficient to maintain a constant solvency ratio after returning an economic profit. In the competitive market this steady state will be viewed by participants as some sort of normative, or medium term average, state.

Define

K_0 = steady state capital per exposure unit
P_0 = steady state premium per exposure unit
$S_0 = K_0/P_0$ = steady state solvency ratio
r_{K0} = steady state expected return on total assets.

The undiscounted risk premium per unit exposure is $\lambda_N\mu_N + \lambda_C\mu_C/E$, and so, on the assumption of no correlation between claim costs (payable at period end) and the stock market,

$$P_0 = (\lambda_N\mu_N + \lambda_C\mu_C/E)/(1 + r_F). \quad (2.1)$$

This premium will be sufficient to pay claims. Steady state capital K_0 generates an economic return r_M, which equates to period profit. Note that tax, and particularly the issue of tax deadweight, is not considered here.

It follows that K_0 is a free variable, and there is a different steady state for each choice of K_0. The value of K_0 is determined by forces external to the market, such as regulatory constraints and consumers' concerns about security. By the definition of r_{Kit} given above,

$$r_{K0} = r_F + (r_M - r_F)[K_0/(K_0 + P_0)]. \quad (2.2)$$

System parameters are of two types:

- Those that describe the environment within which the market exists (call these *environmental parameters*) (listed in Table 1) and
- Those that describe the market dynamics within that environment (call these *dynamical parameters*).

Table 1
Environmental Parameters in Market Model

K_0	Steady state capital per unit exposure
E	Total exposure for whole market
r_F	Risk-free rate of return
r_M	Stock market expected rate of return
λ_C	Expected CAT claim frequency (for whole market)
μ_C	Expected CAT claim size (for whole market)
λ_N	Expected non-CAT claim frequency per exposure unit (common to all insurers)
μ_N	Expected non-CAT claim size (common to all insurers)

The dynamics of the model are as set out below, where the section numbering follows that in Figure 1.

2.2.1 Target Premiums

$$T_{it} = P_0 \exp\,[-k_1(S_{it} - S_0)]. \qquad (2.3)$$

Target premiums increase (decrease) as solvency falls below (rises above) steady state solvency. Thus, k_1 may be viewed as a **premium-to-solvency sensitivity** parameter.

2.2.2 Competitive Premiums

$$P_{it} = \max(k_{11}P_0, \min(T_{it}[T_{it}/\text{avg}(P'_{i,t-1})]^{-k_2}, k_{12}P_0)),$$
$$0 < k_2 < 1, \text{ for new companies in their first period of existence} \qquad (2.4)$$
$$= \max(k_{11}P_0, \min(k_{13}(T_{it}[T_{it}/\text{avg}(P'_{i,t-1})]^{-k_2}) + (1 - k_{13})P_{i,t-1}, k_{12}P_0)) \text{ afterwards}, \qquad (2.5)$$

where $\text{avg}(P'_{i,t-1})$ denotes the average of $P_{i-2,t-1}$, $P_{i-1,t-1}$, $P_{i,t-1}$, $P_{i+1,t-1}$, $P_{i+2,t-1}$, with nonexistent values (e.g., $P_{-2,t-1}$) deleted from the average.

The main parameter in conceptual terms here is k_2. The member involving it shows that insurer i is competitively influenced by the two next larger and two next smaller insurers (recall that i indexes insurers by volume of exposure).

As the average premium of the market segment consisting of insurer i and these four competitors, as observed in the previous period, falls below the target premium of insurer i, this insurer is influenced to bring to market premiums below its target, and conversely when competitors' premiums rise above insurer i's target.

The parameter k_2 is restricted to the interval (0, 1). The extreme cases $k_2 = 0$ or 1 would be those in which, respectively:

- Market premium rates were totally insensitive to competition and
- Market premium rates were so sensitive that in any period each insurer in the market simply set its premium rate to equal that observed in its own five-insurer market segment in the preceding period.

Thus, k_2 may be viewed as **competition intensity** parameter.

Parameter k_{13} distinguishes separate cases for extant and start-up insurers. It exists only in the former case, where $1 - k_{13}$ represents competitive inertia in the sense that the larger is $1 - k_{13}$, the less the competitive effect just described. In fact, the insurer prices by giving weight $1 - k_{13}$ to its premium rates of the preceding period and weight k_{13} to the competitive premium rates described above. In this sense k_{13} denotes **lack of competitive inertia.**

The start-up insurer, on the other hand, has no prior period's premium rates to create this inertia and in effect is taken to adopt $k_{13} = 1$.

The parameters k_{11} and k_{12} simply set **competitive premiums lower and upper bounds,** where the bounds are expressed as multiples of steady state premiums.

It is noteworthy that the structure of competitive premiums in equations (2.4) and (2.5) creates positive feedback in the system. As one insurer lowers premium rates, its peers will be induced to do likewise, creating increased market pressure for the first insurer to lower rates further.

2.2.3 Underwriting Results

$$\pi_{it} = (K_{it} + E_{it}P_{it})(1 + r_{Kit}) - L_{it} - K_{it}. \qquad (2.6)$$

$$D_{it} = \max\,(0, \min[K_{it} + \pi_{it} - k_3P_{it}E_{it}, k_{10}(K_{it} + \pi_{it} - S_0P_{it}E_{it})]. \qquad (2.7)$$

The definition of r_{Kit} assumes that each insurer invests net assets K in a cross section of the equity market, and the technical reserves $E_{it}P_{it}$ in risk-free assets. Equation (2.6) assumes that the expected rate of investment return is realized in each period.

This has the effect of factoring investment risk out of the market dynamics. However, as investment risk is supposed to be random, this should not change the fundamental properties of those dynamics, but simply remove an element of volatility.

At the end of each period, the net assets of insurer i amount to $K_{it} + \pi_{it}$, opening net assets for the period having been augmented by period profit. This is the amount available to pay a dividend.

It is assumed that the dividend will be limited so that the remaining net assets do not fall short of a multiple k_3 of period premium. Thus, k_3 is a **floor solvency ratio** (for dividend purposes).

The second argument of the min function in equation (2.7) conditions each dividend payment on the steady state solvency ratio. In effect, it sets this as a target solvency ratio, and no dividend will be paid if the closing solvency ratio is less than the target. If the closing solvency ratio exceeds the target, then k_{10} functions as a **dividend payout ratio** (relative to the excess over target solvency).

2.2.4 Balance Sheet Results

$$K_{i,t+1} = K_{it} + \pi_{it} - D_{it}. \quad (2.8)$$

This relation simply states that closing net assets are equal to opening net assets increased by period profit and decreased by any dividend payment.

2.2.5 Entry and Exit of Capital

Insurer i exits if

$$K_{it} < k_3 E_{it} P_{it}. \quad (2.9)$$

This relation states that any insurer will exit the market if its solvency ratio falls below the floor k_3. Thus, k_3 is also a **minimum viable solvency ratio.**

Let

$$\pi_t = \Sigma_i \, \pi_{it} = \text{market-wide insurance profit}, \quad (2.10)$$

$$P_t = \Sigma_i \, E_{it} P_{it} / E_t$$
$$= \text{market average premium per exposure}. \quad (2.11)$$

If

$$\pi_t / E_t P_t > k_4 \text{ and } \pi_{t-1} / E_{t-1} P_{t-1} > k_4, \quad (2.12)$$

then m_{t+1} new insurers will be introduced into the market, where

$$m_{t+1} = \text{int}[k_5 \, \Sigma_{j=0}^{1} \, (\pi_{t-j} / E_{t-j} P_{t\text{-}j} - k_4)] \quad (2.13)$$

with int[x] defined as the integral part of x.

Each of these new insurers i is assumed to have initial capital

$$K_{i,t+1} = k_6 \, \Sigma_j \, K_{jt}, \quad (2.14)$$

where j runs over all insurers in the market in period t. Each of these insurers commences with premium per exposure P_t.

By (2.12), capital is attracted into the market if the market-wide profit margin exceeds a threshold k_4 in each of two consecutive periods. In this sense, k_4 denotes the **threshold capital attraction profit margin.**

By (2.13), the number of new insurers attracted into the market is (subject to its integral nature) proportional to the sum, over the prior two periods and over all insurers then in the market, of the excess profit margins over the threshold. The constant of proportionality k_5 may be regarded as a **new capital attraction per unit market profitability** parameter.

The capitalization of new entrants to the insurance market must bear some relation to the strength capitalisation of the market generally if they are to be able to compete. Relation (2.14) postulates that each new insurer is capitalized at $100k_6$% of the preceding period's total market capitalization. Thus, k_6 is a **new entrant capitalization** parameter.

2.2.6 Loss Experience

Let

n_C = Realized CAT claim count (for whole market) $\sim$ Poisson (λ_C)

m_C = Realized CAT claim size (for whole market) $\sim$ Pareto (mean = μ_C, minimum = \$100 million)

n_{Nit} = Realized non-CAT claim count (*not* per exposure unit) (independent drawings for different insurers) $\sim$ Poisson ($E_i \lambda_N$)

m_{Nit} = Realized non-CAT claim size (independent drawings for different insurers) $\sim$ Gamma (mean = μ_N, coefficient of variation = 1,000%).

Then

$$L_{it} = n_C m_C (E_{it} / E_t) + n_{Nit} m_{Nit}. \quad (2.15)$$

2.2.7 Reallocation of Market Shares

Let

$$\rho_{it} = E_{it} / E_t = \text{market share}.$$

Define the (unscaled) transfer function, from insurer r to s as

Table 2
Dynamical Parameters in Market Model

k_1	Premium-to-solvency sensitivity
k_2	Competition intensity
k_3	Floor solvency ratio (for dividend purposes)
k_4	Threshold capital attraction profit margin
k_5	New capital attraction per unit market profitability
k_6	New entrant capitalization
k_7	Market price sensitivity
k_8	Market presence limit
k_9	Minimum viable market share
k_{10}	Dividend payout ratio (relative to the excess over target solvency)
k_{11}	Competitive premiums lower bound
k_{12}	Competitive premiums upper bound
k_{13}	Lack of competitive inertia

Table 3
Policy Variables

k_3	Floor solvency ratio (for dividend purposes)
k_4	Threshold capital attraction profit margin
k_6	New entrant capitalization
k_9	Minimum viable market share
k_{10}	Dividend payout ratio (relative to the excess over target solvency)
k_{11}	Competitive premiums lower bound
k_{12}	Competitive premiums upper bound

$$\alpha_{rst} = \max(0, P_{rt} - P_{st}) \max(k_8, \rho_{st}). \quad (2.16)$$

Then define

$$\alpha_{rt} = \Sigma_s \alpha_{rst}, \quad (2.17)$$

$$\tau_{rst} = [1 - \exp(-k_7\alpha_{rt})]\alpha_{rst}/\alpha_{rt}. \quad (2.18)$$

Define the (unscaled) reallocated exposures as

$$E^*_{i,t+1} = E_{it}[1 - \Sigma_s \tau_{ist}] + \Sigma_r E_{rt}\tau_{rit}. \quad (2.19)$$

Then suppose that the resulting market shares are rescaled as follows:

$$\rho_{i,t+1} = E^*_{i,t+1}/\Sigma_r E^*_{r,t+1}. \quad (2.20)$$

The sequence (2.16)–(2.20) can be explained as follows. The transfer function α_{rst} is nonzero only if $P_{rt} > P_{st}$. In this case some business is expected to flow from insurer r to insurer s, and the transfer function is intended to control the strength of the flow.

Note the factor of $\max(k_8, \rho_{st})$ in the transfer function. This recognizes an assumption that the volume of business transferring to insurer s will depend not only on the differential in premium rates but also on the market presence of insurer s, represented by its market share ρ_{st}. For example, if s and s^* are large and small insurers, respectively, with $P_{rt} > P_{st} = P_{s^*t}$, then the volume of business attracted from r to s will exceed that attracted from r to s^*.

It is assumed, however, that there is a lower limit on this effect of market presence. Otherwise insurers with small market shares would never attract any significant amounts of business. Below a certain market share, k_8, the transfer function is just as if market share were k_8. Thus, k_8 is **market presence limit** parameter.

The strength of the flow of business from insurer r to insurer s is described by (2.18). Here the square bracketed member represents the total transfer of business *away* from insurer r. It is zero when $P_{rt} = P_{st}$ and increases steadily as P_{st} decreases relative to P_{rt}. The ratio following the square bracket apportions this loss of business to other insurers in proportion with the transfer functions α_{rst}.

Exposure volumes for the next period are determined accordingly in (2.19). Volumes calculated this way will not necessarily sum to the total market volume, and so are rescaled to do so by (2.20).

In this system of rebalancing of market shares, the strength of flows between insurers is governed by the parameter k_7, which may be viewed as a **market price sensitivity** parameter.

Note that, if both max(·) terms of (2.16) are ignored, the square bracketed member of (2.18) becomes $[1 - \exp[-k_7 \Sigma_s (P_{rt} - P_{st})]]$. A single term of the summation gives $[1 - \exp[-k_7(P_{rt} - P_{st})]]$, which is just the rate of transfer of business from insurer r to insurer s (for $P_{rt} > P_{st}$) when price elasticity is k_7. The summation within the square bracket makes allowance for all possible destinations of lapsed business of insurer r.

If $\rho_i < k_9$, then insurer i exits, and the market shares are rescaled:

$$E_{i,t+1} = E\rho_{i,t+1}. \quad (2.21)$$

It is assumed that unsuccessful insurers do not continue to dwindle to infinitesimal sizes, but exit the market after falling below some critical market share. The parameter k_9 represents the **minimum viable market share.**

The dynamical parameters k_1 to k_{13} are summarized in Table 2. Some of the variables in Table 2 are subject to regulation and so may be regarded as policy variables, listed in Table 3.

A variable has been included in Table 3 only if capable of *direct regulation.* The variables omitted are essentially those controlling the competitive dynamics of the market. Some of the variables omitted might be influenced indirectly by policy. For example, premium-to-solvency sensitivity (k_1) could be influenced by the imposition of solvency-dependent risk capital charges.

3. SIMULATION SETUP

Market behavior has been simulated over the periods $t = 1, 2, \ldots, 60$ by means of the model described in Section 2. Loss experience has been generated as random drawings of the quantities n_C, m_C, n_{Nit}, and m_{Nit}. A *base case,* involving a base set of parameters, has been simulated, and then other cases obtained by varying the parameters singly or in small related groups. The same drawing of loss experience is used for all simulations.

Initially, this is done for the case $n_C = 0$, that is, when the effect of catastrophes is excluded. The whole procedure is then repeated with catastrophes included. The base case is chosen as representing a reasonably stable and uneventful market. Table 4 sets out the values adopted for the environmental parameters, and Table 5 the dynamical parameter values for the base case.

Table 6, which lists the initial values of the free variables in the market, indicates that each of the 100 initial market participants underwrites 75,000 exposure units. The premium of \$160 never appears in the market but functions as $P_{i,0}$ in the application of equation (2.3) for $t = 1$. It is chosen to differ significantly from the steady state premiums in the examples below, to illustrate the reversion to steady state over time in the case of a stable system.

Table 4
Environmental Parameter Values

	Parameter	Value
K_0	Steady state capital per unit exposure	\$145
E	Total exposure for whole market	1.5 million
r_F	Risk-free rate of return	4%
r_M	Stock market expected rate of return	12%
λ_C	Expected CAT claim frequency (for whole market)	2%
μ_C	Expected CAT claim size (for whole market)	\$120 million
λ_N	Expected non-CAT claim frequency per exposure unit (common to all insurers)	12%
μ_N	Expected non-CAT claim size (common to all insurers)	\$1,000

Table 5
Base Case Dynamical Parameter Values

k_1	0.667
k_2	0.25
k_3	0.1
k_4	0.2
k_5	30
k_6	0.000333
k_7	0.1
k_8	0.01
k_9	0.0006
k_{10}	0.7
k_{11}	0
k_{12}	1000
k_{13}	0.75

Table 6
Initial Values of Market

Number of market participants	20
Market share of each participant	5%
Premium per exposure unit	\$160
Opening capital (total market)	\$270 million

Note that the price controls k_{11} and k_{12} are, in effect, switched off in the base case. (Compare the steady state premium of \$115.4 calculated in accordance with equation 2.1.)

Different variations in the dynamical parameter values appearing in Table 5 affect the following aspects of the market:

- Length of market cycles
- Depth of market cycles
- Market concentration (as represented by the Herfindahl-Hirschman index)
- Longevity of initial participants in the market
- Longevity of new entrants to the market
- Diversity of premium rates on offer
- Market profitability
- Number of market participants
- Market solvency
- Insolvency rate
- Expected policy holder deficit.

4. SIMULATION RESULTS

4.1 Base Case

Base case parameters are set out in Table 5, and it is assumed for the present that $n_C = 0$. Figures

2–6 illustrate the major dynamics of this market. The general picture is one of a typical market, with:

- Generally stable premium rates and solvency (Fig. 2)
- Largely stable number of market participants but with the occasional entrant or exit (Fig. 3)
- A marked diversity of premium rates available in the market (Fig. 4)
- An average profit margin that is variable but generally positive (Fig. 5)
- Some market concentration over time but only of a moderate sort (Fig. 6).

Figure 2
Market Solvency

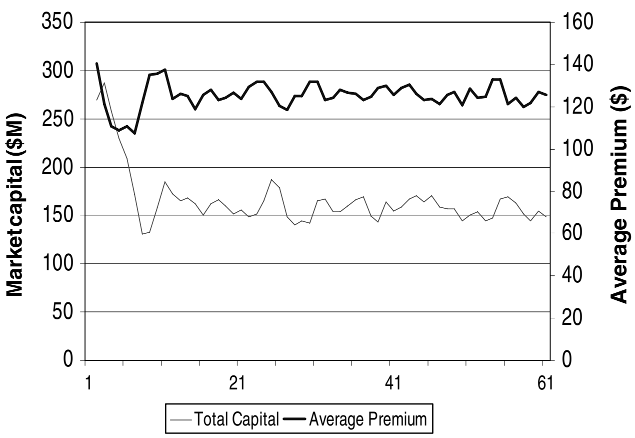


Figure 3
Number of Market Participants

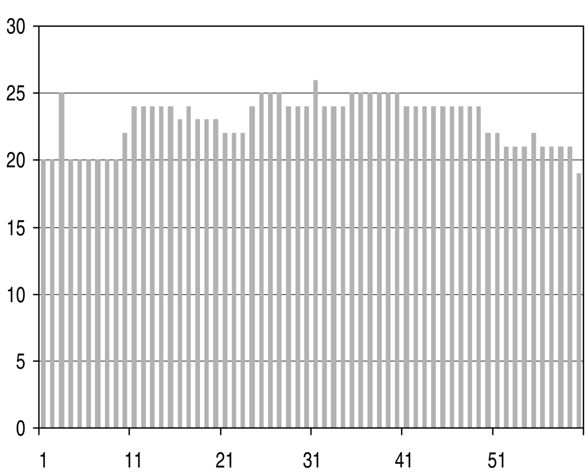


Figure 4
Diversity of Market Premium Rates

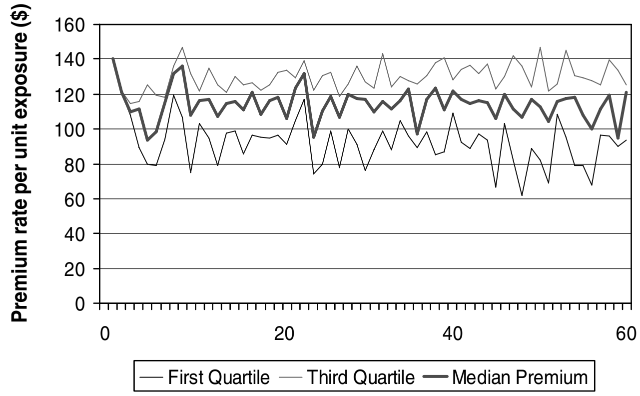


Figure 5
Average Profit Margin

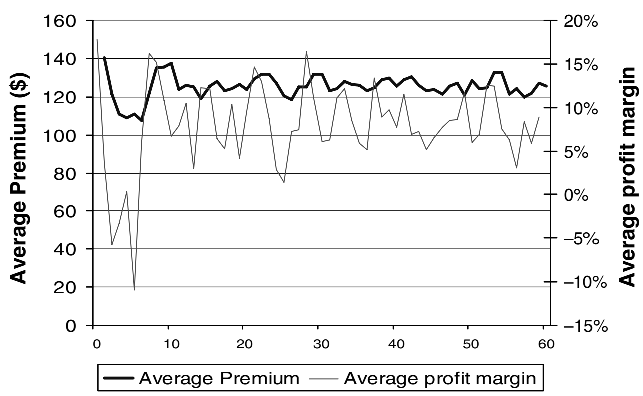
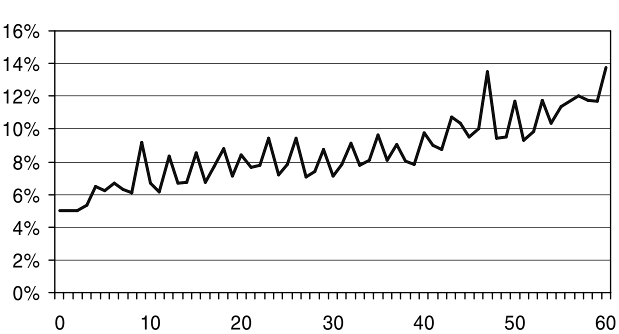


Figure 6
Herfindahl-Hirschmann Index



4.2 Cyclic Market Behavior

Certain parameter values in the model cause market premium rates to exhibit cyclic behavior. This is illustrated in Figure 7, where decreasing values of k_2 are associated with premium cycles of increasing amplitude. The plot for the base value of k_2 is shown as a heavy line.

The empirical autocorrelation function (ACF) for average premium is shown for the same values of k_2 in Figure 8, where the horizontal axis displays the autocorrelation lag. Again, increasing amplitude is associated with decreasing k_2.

The discussion immediately following equation (2.4) indicates that k_2 is a competition intensity parameter and that reduction in its value represents a weakening of competition in the market. The numerical results here therefore imply that weakened competition (from the base case) will induce cyclic behavior in market premium rates.

It is interesting to note that Figure 8 implies a periodicity of six or seven time intervals. This value is not unique to variation of k_2. As will be seen, when the market exhibits cyclic behavior, induced by whichever parameter, the cycles are usually of about this periodicity.

A "time interval" here is the length of time required for each rebalancing of the market under the competitive dynamics. The value of this interval is unknown. It is interesting to note, however, that, if it were a year, the above occurrence of a six- or seven-year cycle would agree with the empirical results of Cummins and Outreville (1987), who found market cycles of about this period in a number of countries.

4.3 Diversity of Premium Rates

Figures 9–11 illustrate the fact that diversity of premiums available in the market increases as the market price sensitivity parameter k_7 is increased. Note that Figure 10 reproduces Figure 4.

Figure 7
Average Premium for Varying k_2

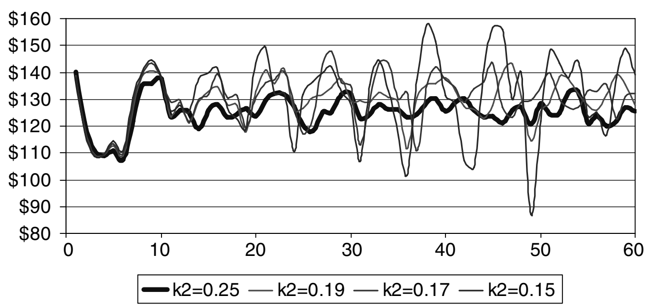


Figure 8
Premium Autocorrelation Function for Varying k_2

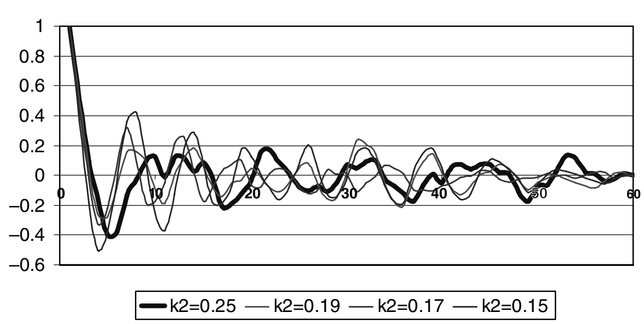


Figure 9
Diversity of Market Premium Rates
k_7 = 0.04

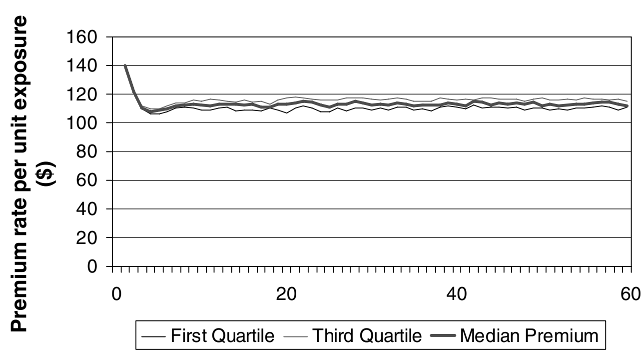


Figure 10
Diversity of Market Premium Rates
k_7 = 0.10

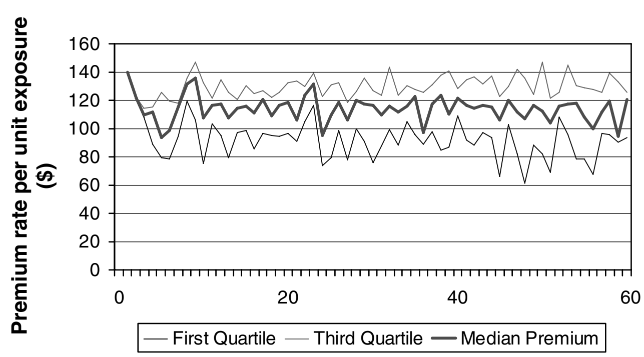


Figure 11
Diversity of Market Premium Rates
k_7 = 0.17

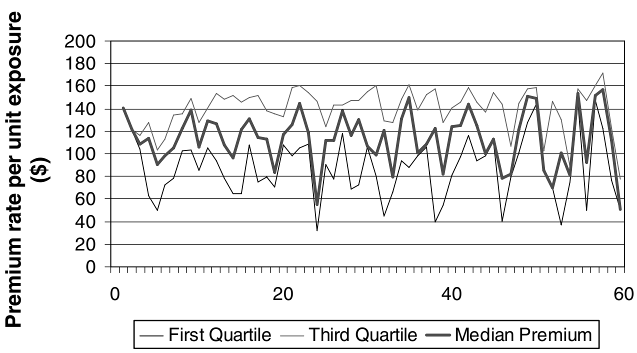


Figure 11 appears to have become cyclic. This is indeed the case, as is illustrated by Figure 12, which plots the ACF of average premium in the market. It may be checked that cyclic behavior steadily intensifies as k_7 increases above the base value of 0.10.

4.4 Number of Market Participants and Market Concentration

4.4.1 Dividend Payout Ratio

The number of market participants and market concentration are strongly affected by the dividend payout ratio k_{10}. Figures 13–18 illustrate the effect of increasing k_{10} from its base value of 70% to 90%. Note that Figures 13 and 14 reproduce Figures 3 and 6.

It can be seen that the increase in k_{10} from 0.7 to 0.8 causes an appreciable increase in the number of market participants, although without a re-

Figure 12
Average Premium Autocorrelation Plot
k_7 = 0.17

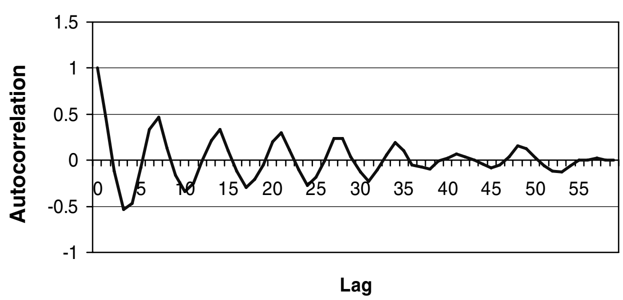


Figure 13
Number of Market Participants
k_{10} = 0.7

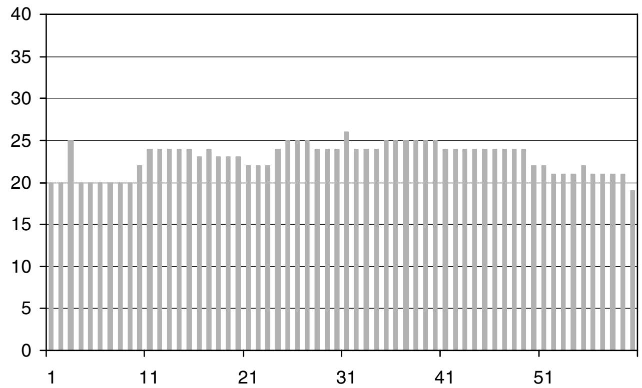


Figure 14
Herfindahl-Hirschmann Index
k_{10} = 0.7

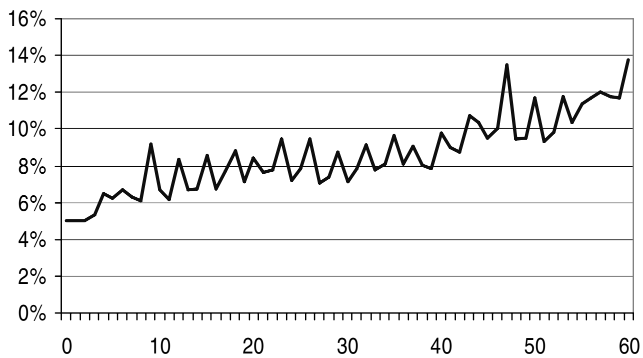


Figure 15
Number of Market Participants
k_{10} = 0.8

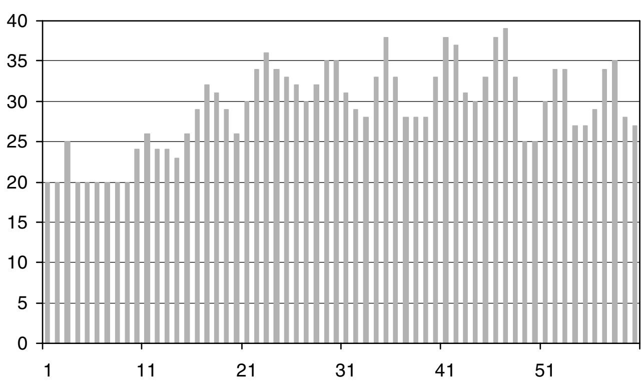


Figure 16
Herfindahl-Hirschmann Index
k_{10} = 0.8

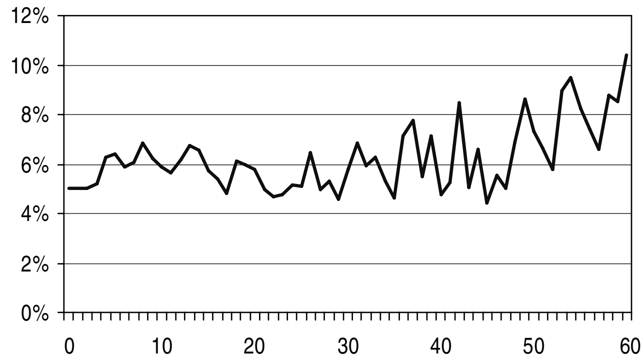


Figure 17
Number of Market Participants
k_{10} = 0.9

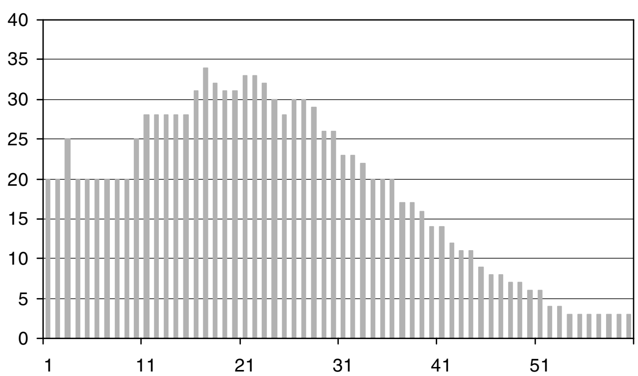
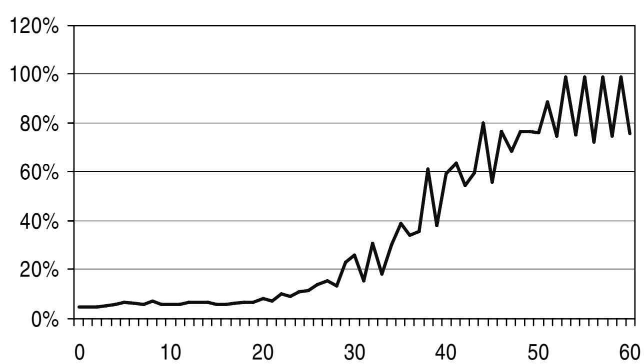


Figure 18
Herfindahl-Hirschmann Index
k_{10} = 0.9



duction in the concentration index and at the expense of the introduction of cyclic behavior. The latter is visible in Figure 15 and is confirmed by Figure 19.

A further increase of k_{10} from 0.8 to 0.9 causes an initial increase followed by a crash in the number of market participants (Fig. 17). The cyclic market behavior is eliminated, and average premium in the market follows a process more resembling AR(1) (Fig. 20).

4.4.2 New Capital Attraction per Unit Market Profitability

It is interesting that the number of participants in the market is not permanently increased by increasing the parameter k_5 governing new capital attraction per unit market profitability. New participants are certainly attracted when this parameter is increased from its base value of 30 to 45, but intense competition sets in, creating violent market cycles (Fig. 21), leading to the demise of many.

Figure 19
Average Premium Autocorrelation Plot
k_{10} = 0.8

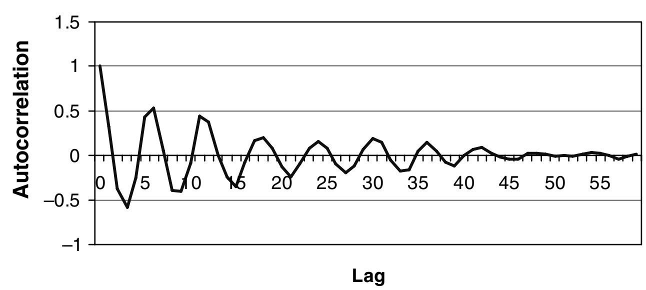


Figure 20
Average Premium Autocorrelation Plot
k_{10} = 0.9

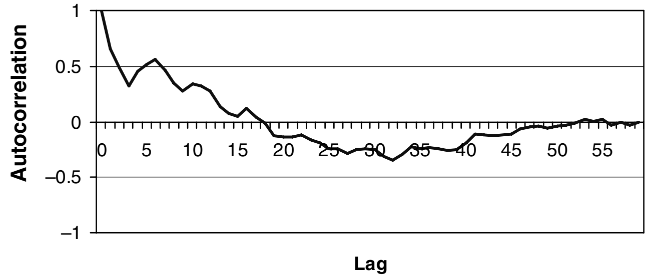
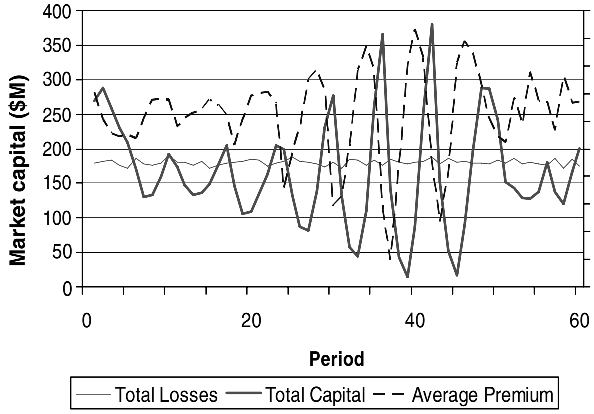


Figure 21
Market Solvency



The average insolvency rate, defined as the annual proportion of insurers (by number) going insolvent, averaged over all 60 periods, is 10%, compared with 0.15% for the base case. Ultimately, the number of participants is low (Fig. 22).

4.4.3 Minimum Viable Market Share

The effect of the minimum viable market share parameter k_9 is as expected. Increasing it reduces the number of market participants, as seen by a comparison of Figure 23 (k_9 = 0.0025) with Figure 3 (k_9 = 0.0006).

4.4.4 Market Presence Limit

The effect of the market presence limit parameter k_8 is also largely as expected. Decreasing it reduces the number of market participants, as seen by a comparison of Figure 24 (k_8 = 0.001) with Figure 3 (k_8 = 0.01).

Figure 22
Number of Market Participants

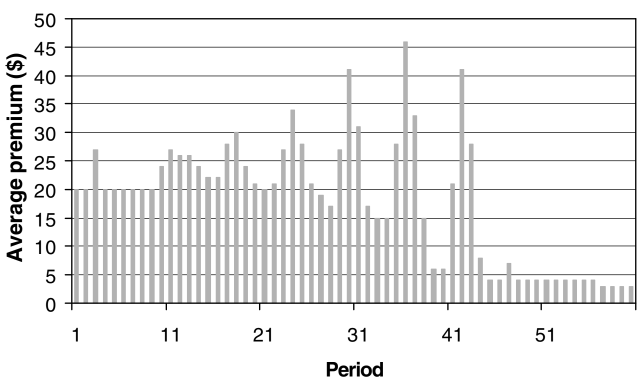
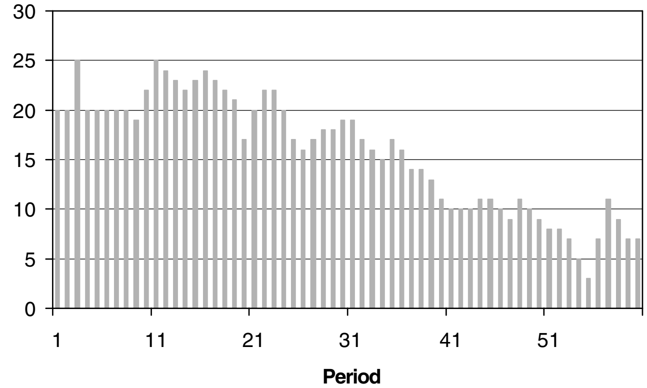


Figure 23
Number of Market Participants



Increasing k_8 above 0.01 appears to have little effect. This suggests that there may be a limit to the value of advertising simply to achieve brand prominence. Beyond that limit advertising might need to switch to some form of product differentiation to achieve effectiveness.

4.5 Effects of Competition

The effects of competition are reflected in the premium-to-solvency and competition intensity parameters k_1 and k_2. One of the major effects of these parameters is the generation of complex patterns of cyclic behavior. Figure 25 displays, for various values of k_1, the intervals of k_2 in which cyclic behavior is observed.

Figure 24
Number of Market Participants

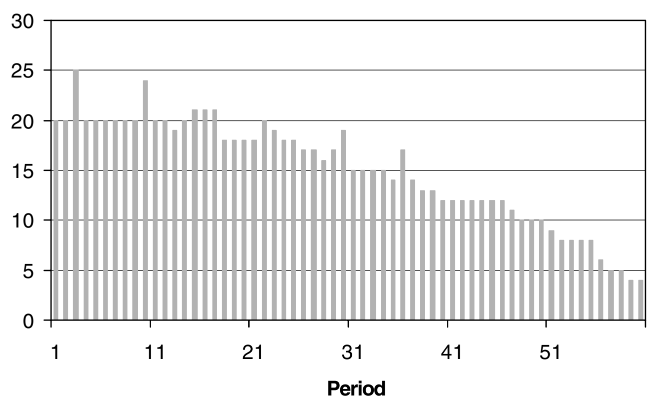
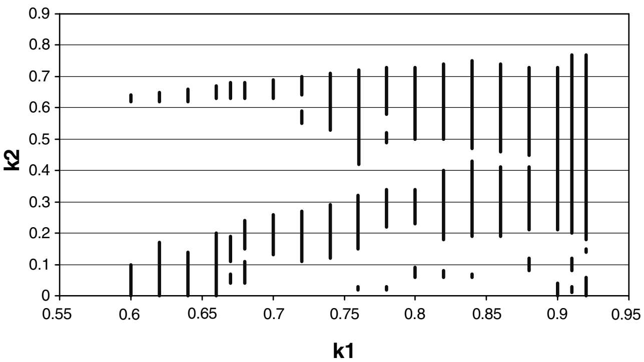


Figure 25

Regions of (k_1, k_2) Generating Cyclic Behavior



It should be said in connection with Figure 25 that there is some imprecision in the differentiation between cyclic and noncyclic cases. Generally, however, a case has been classified as cyclic if the ACF of average premium displays either

- A clear cyclic pattern (as in Figs. 12 and 19) or
- A well-defined peak well in excess of 0.2.

The regions of cyclicity exhibit catastrophe-like behavior (Poston and Stewart 1976) with bifurcation into subregions as k_1 increases, and merger of other subregions. It may be observed in the most general terms, however, that cyclic behavior becomes steadily more difficult to avoid as k_1 increases.

This means that a high preoccupation with solvency in the pricing of business by market participants is likely to lead to cyclic behavior of prices. The parameter k_1 is not one that can be regulated directly. It may, however, be subject to indirect influence by the application of some form of penalty for low solvency. Figure 25 indicates that this might have some unwelcome effects.

The same figure shows that cyclic behavior is liable to emerge if competition, as represented by k_2, is either too strong or too weak. Competition of moderate strength is more conducive to market stability.

It is not surprising that increasing the competition intensity k_2 tends to reduce the average profit of the market. Figure 26 plots the average market profit margin for the base case ($k_2 = 0.25$) and for $k_2 = 0.45$ and 0.75. It can be seen that, once the initial transients have passed, the plots clearly show this ordering with respect to k_2.

Figure 26

Profit Margin for Varying k_2

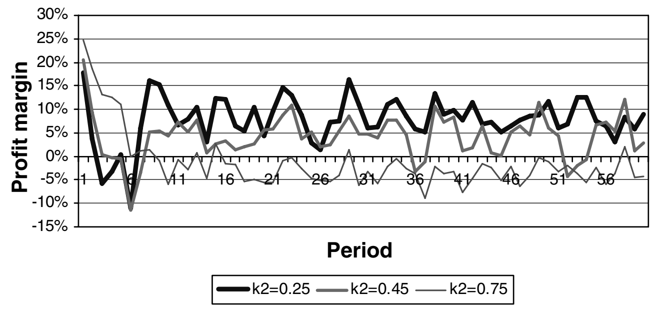


Figure 27 gives more detail on the variation of profit margin with k_2. The profit margin displayed here is averaged across the whole market, as in Figure 26, but also averaged across all 60 periods of simulated experience. This average profit margin is plotted against k_2. This is done for several values of k_1.

The steady decline of average profit margin with increasing competition intensity up to about $k_2 = 0.65$ is evident for each value of k_1 considered. The decline appears to flatten for higher values of k_2.

4.6 Regulatory Controls

4.6.1 Barriers to Entry

Setting a floor under market profitability for the introduction of new participants in the market (parameter k_4) can affect the number of participants. This is illustrated by Figures 28–31, which track market behavior as k_4 increases from 0.15 to 0.24. The case $k_4 = 0.2$ replicates Figure 3.

Figure 27

Profit Margin for Varying k_1 and k_2

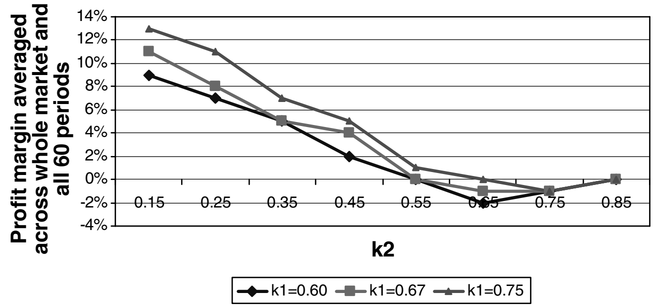


Figure 28
Number of Market Participants
k_4 = 0.150

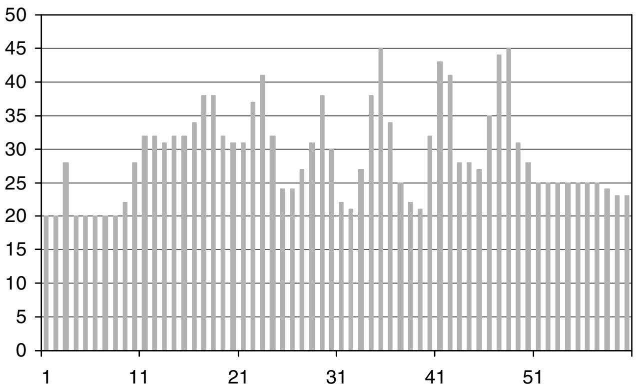


Figure 29
Number of Market Participants
k_4 = 0.175

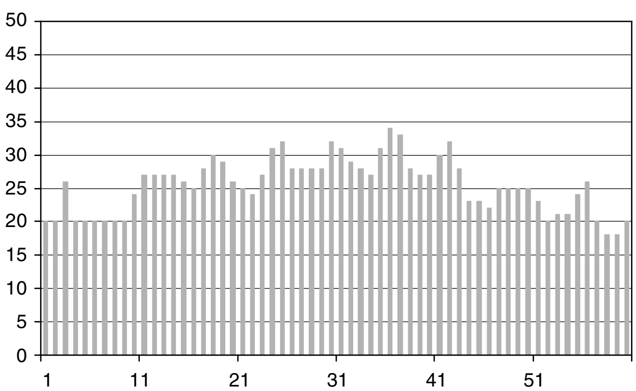


Figure 30
Number of Market Participants
k_4 = 0.200

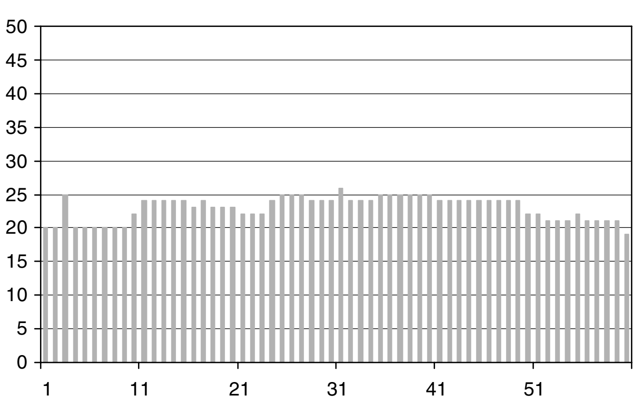
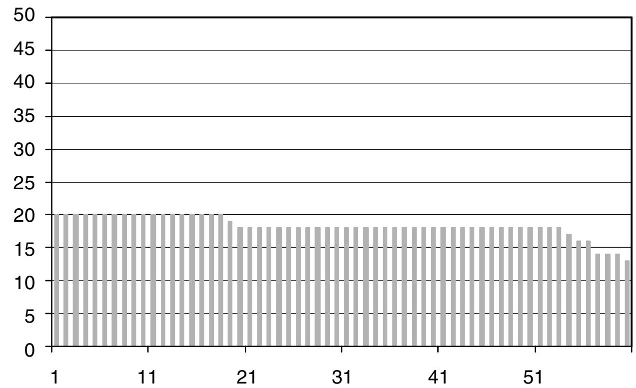


Figure 31
Number of Market Participants
k_4 = 0.240



It can be seen that, as the barrier to entry is raised, the number of market participants steadily reduces. In fact, for the case $k_4 = 0.24$ and all higher values, there are no new entrants to the market, only the occasional exit.

It is also noteworthy that, taken over the whole 60 intervals, the effect of variation in k_4 is largely transient; the final number of market participants never differs much from the initial number because a lower barrier to entry admits more new entrants, but with a lower average viability. This is illustrated by Figures 32 and 33. A different barrier to entry is discussed at the end of Section 4.6.4.

4.6.2 Price Regulation

Setting a floor under market premium rates (parameter k_{11}) can result in drastic changes to the

Figure 32
Longevity of Market Participants
k_4 = 0.15

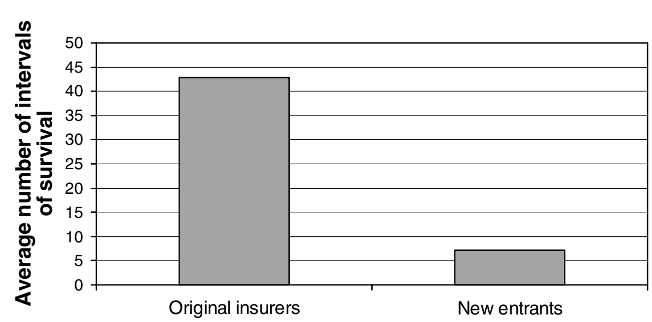
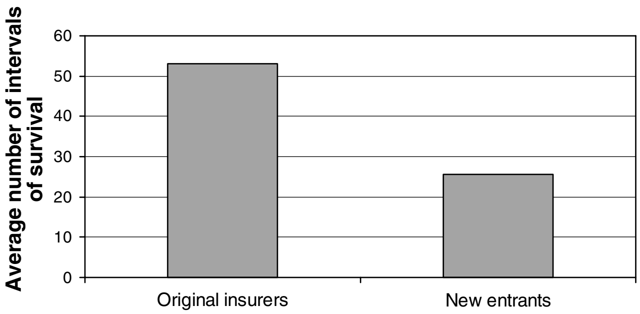


Figure 33
Longevity of Market Participants
$k_4 = 0.20$



number of participants in the market. This is illustrated by Figures 34–39, which track market behavior as k_{11} increases from 0.65 to 1. The case $k_{11} = 0$ is found in Figure 3.

Increasing minimum premium rate from a low level attracts new entrants. However, the behavior of the market becomes acutely sensitive to changes in this parameter as it approaches unity (full funding).

Moreover, increasing k_{11} beyond a certain level affects the market perversely. While new entrants are attracted, their survival is short, and ultimately the number of market participants declines. When this occurs, average market profitability declines in sympathy. This is illustrated by Figure 40, which displays the evolution of market profitability over the 60-year interval for various values of k_{11} close to unity.

Figure 34
Number of Market Participants
$k_{11} = 0.65$

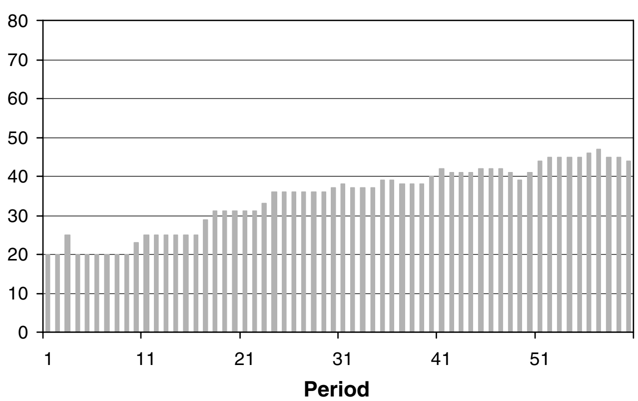


Figure 35
Number of Market Participants
$k_{11} = 0.80$

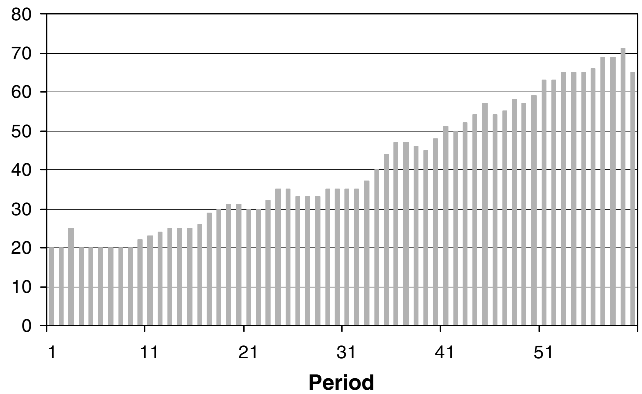


Figure 36
Number of Market Participants
$k_{11} = 0.95$

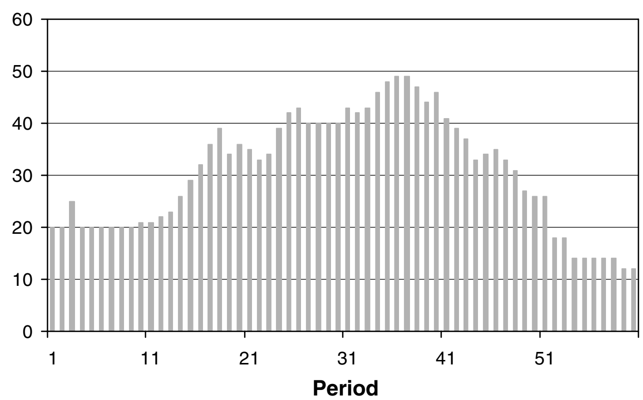


Figure 37
Number of Market Participants
$k_{11} = 0.97$

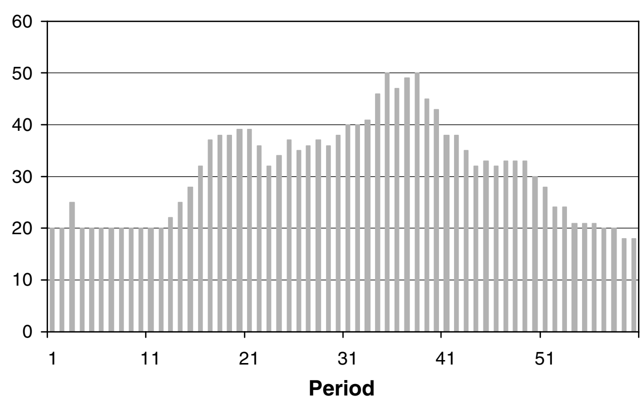


Figure 38
Number of Market Participants
k_{11} = 0.99

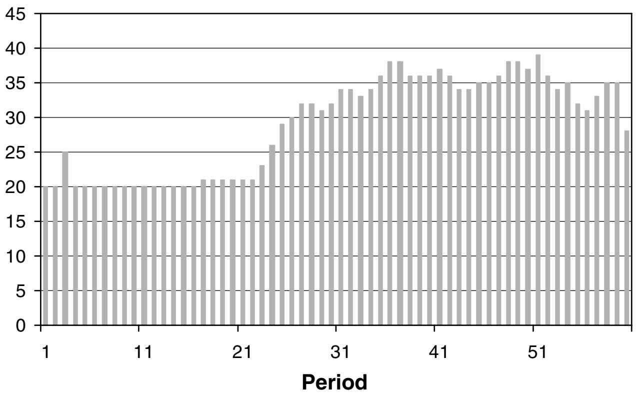


Figure 39
Number of Market Participants
k_{11} = 1.00

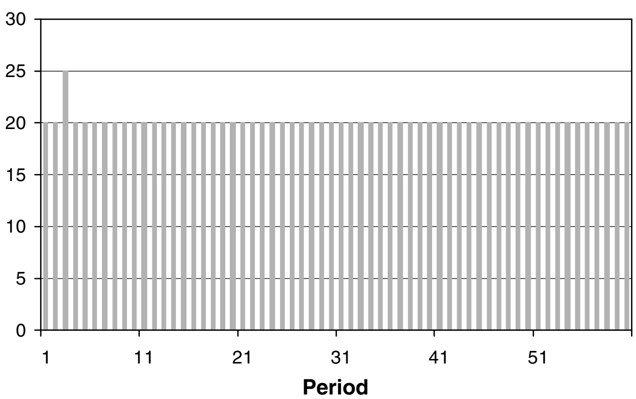
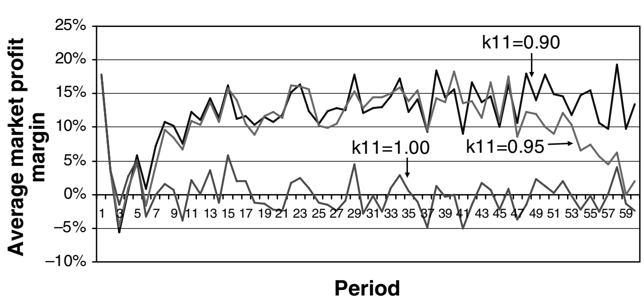


Figure 40
Market Profitability for Changing k_{11}



Setting a ceiling on premium (parameter k_{12}) does not affect the number of market participants so dramatically, but it still affects the diversity of premiums available in the market if set sufficiently low.

A comparison of Figures 41 and 42 with Figure 4 illustrates this. In the former the premium ceiling allows a maximum margin of 5% in excess of the economic premium rate. The restricted range of premium rates is such that the upper quartile often coincides with the median. In Figure 42 a margin of up to 20% is permitted, but the loss of premium diversity is still appreciable.

4.6.3 Regulation of Price Stability

The parameter k_{13} regulates price stability. The closer it is to zero, the less the movement in premium rates from period to period. Direct insurers

Figure 41
Diversity of Market Premium Rates
k_{12} = 1.05

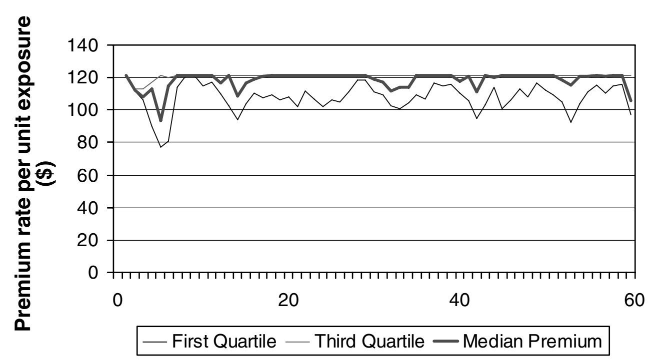
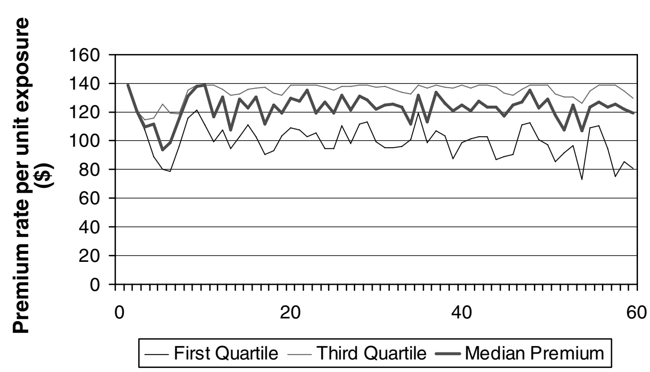


Figure 42
Diversity of Market Premium Rates
k_{12} = 1.20



often apply some pricing mechanism of this sort voluntarily to protect market share.

It would be difficult to apply the parameter at the regulatory level in exactly the form specified in (2.5). However, some variants of it, such as requiring that premium rates do not vary from one period to the next by more than p% and could be so applied.

Figure 43 tracks average market premium through the 60 periods of simulation for three values of k_{13}: 0.4, 0.75, and 0.9. All other parameters remain set at their base case values.

The plot shows increasing instability of premium rates with increasing k_{13}. Reducing this parameter to increase price stability causes a modest reduction in the number of market participants.

The same parameter can also be used to control market cycles that would otherwise exist. For example, Figure 7 shows pronounced cycles in the case $k_2 = 0.15$ and all other parameters at their base values including $k_{13} = 0.75$. Figure 44 illustrates how these cycles are steadily reduced as k_{13} is reduced to 0.6 and 0.5.

4.6.4 Solvency Maintenance

The parameter k_3 plays two roles. It sets a floor solvency ratio that must not be breached by dividend payout (see eq. 2.7). It also sets the same floor for continued underwriting. Breach of the floor leads to enforced exit from the industry.

Comparison of Figure 45 with the base case in Figure 2, and of Figure 46 with Figure 3, illustrates the effect of trebling the solvency floor, raising this parameter from its base value of 0.1 to 0.3. It is seen that a regulatory action that might have been considered responsible in fact has the effect of driving out a large proportion of

Figure 43
Average Market Premium for Varying k_{13}

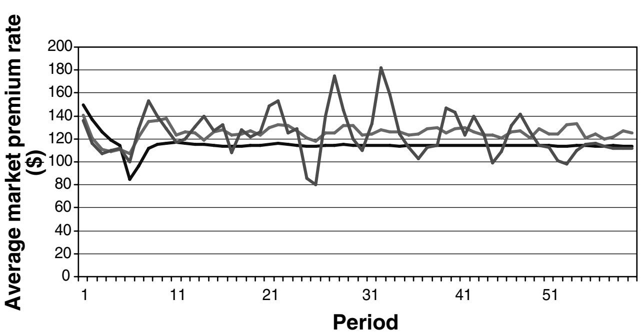


Figure 44
Average Market Premium for $k_2 = 0.15$ and Varying k_{13}

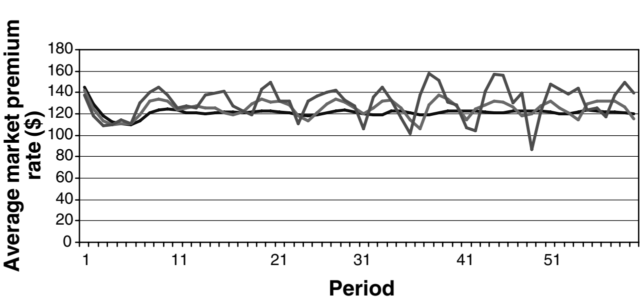


Figure 45
Market Solvency
$k_3 = 0.3$

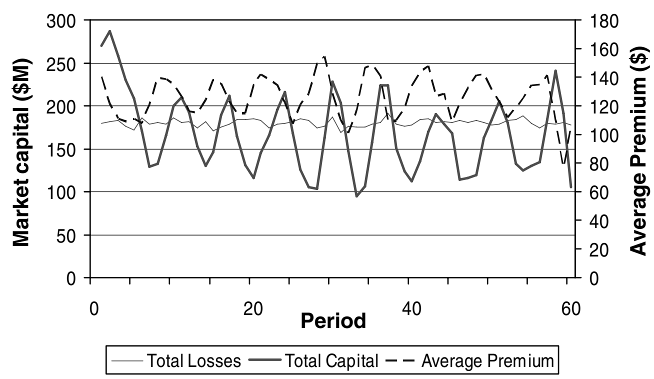
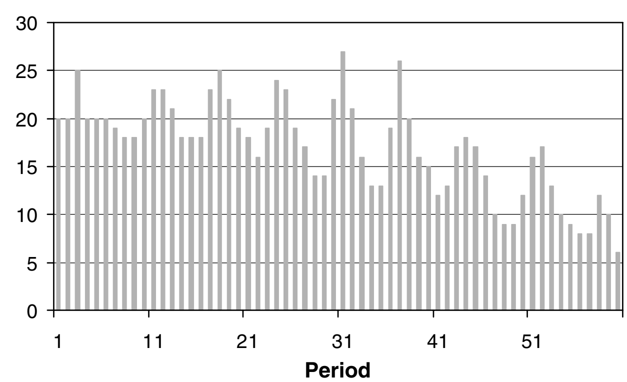


Figure 46
Number of Market Participants
$k_3 = 0.3$



market participants and creating violent market cycles in the process.

A similar effect is observed when the capital requirement for market entry is raised. Comparison of Figure 47 with Figure 2 shows the effect of trebling the market entry parameter k_6.

4.7 Catastrophe Events

As noted at the beginning of Section 4.1, all results hitherto are based on the assumption of no CAT claims ($n_C = 0$). In this discussion the value $n_C = 2\%$ is included in the simulation of loss experience. Figure 48 plots the simulated experience, consisting of the CAT experience added to that underlying the results of Sections 4.1–4.6.

The simulated CAT experience, which is conspicuous in the plot, makes up a single event in period 36 with a total cost of $107 million to the market. This increases the total cost of losses for the period by more than 50%, and the single CAT event accounts for 83% of steady state market capital.

Figures 49 and 50 illustrate the effect of the loss on the market. Initially, the loss extinguishes nearly half of the market participants and drives market solvency down to about one-third of its steady state value. However, this drives prices up to record levels, and there is a rapid influx of new participants. This accords entirely with the model of Winter (1991).

Cummins and Danzon (1991) also find empirical evidence that an insurance price crisis, such as shown in Figure 49, can arise from unexpected losses to insurers in the form of reserve adjustments. Presumably, this result can be extrapolated to unexpected losses from catastrophic events.

A series of cycles in premium rates, solvency, and number of market participants then sets in. Over the course of a cycle, premium rates vary by

Figure 47
Market Solvency
k_6 = 0.001

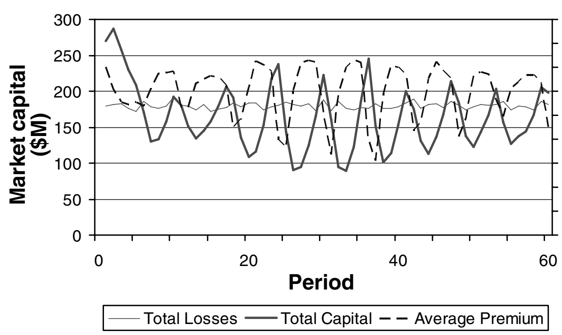


Figure 48
Loss Experience (Incl. CAT)

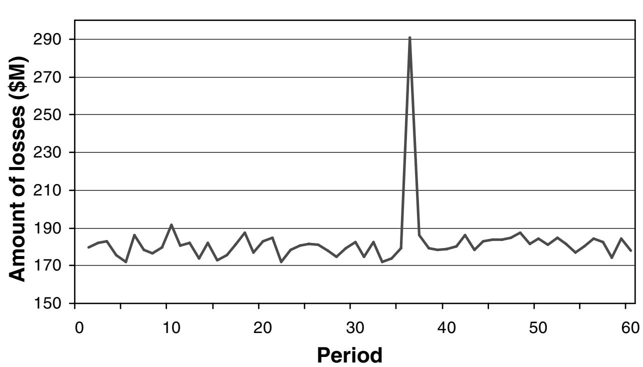


Figure 49
Market Solvency

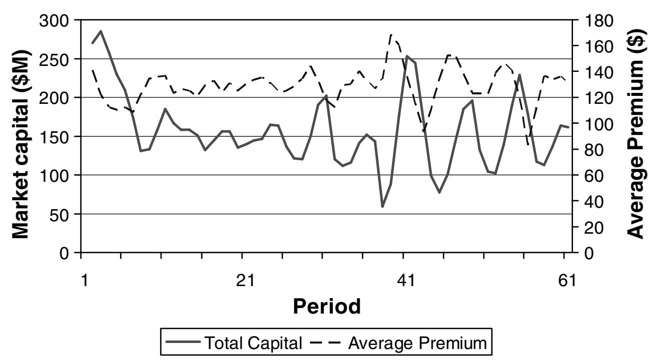
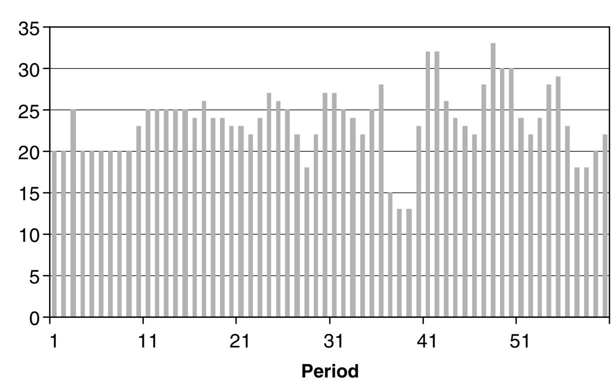


Figure 50
Number of Market Participants



25% or more of their steady state value. Market solvency varies by 70% or so of its steady state value. These results are consistent with those of Balzer (1982) and Balzer and Benjamin (1980), who show that feedback of claim costs into premiums can create cycles if the feedback lag becomes too great.

Whether these are features of a healthy market is perhaps an ideological matter, but it may be useful to consider whether they can be mitigated by some form of market intervention. The most obvious form is price control, particularly the prescription of a ceiling.

This is investigated in Figures 51 and 52, where the premium ceiling parameter k_{12} is set to 1.1, and in Figures 53 and 54, where $k_{12} = 1.2$. It seen that this form of price regulation can reduce the depth of the premium cycles considerably. The price for this is a modest reduction in the number of market participants. The cycles in market solvency are not much affected.

Figure 51
Market Solvency
k_{12} = 1.1

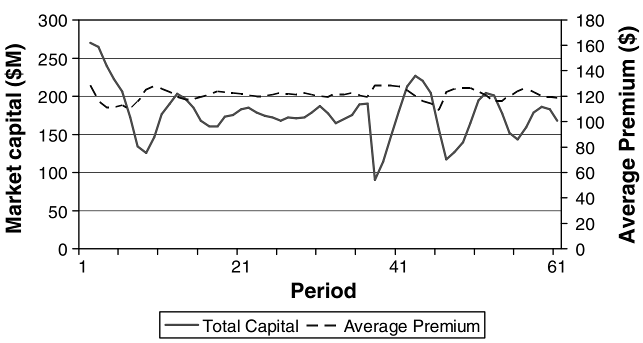


Figure 52
Nmber of Market Participants
k_{12} = 1.1

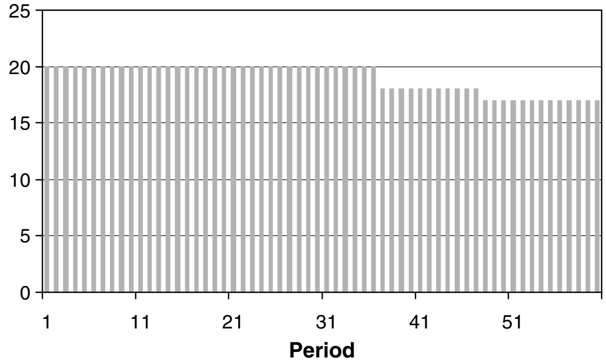


Figure 53
Market Solvency
k_{12} = 1.2

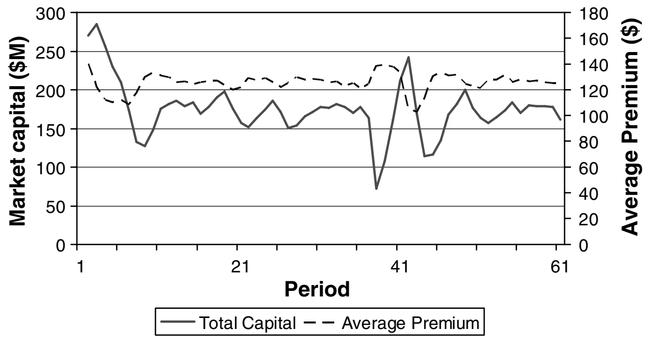
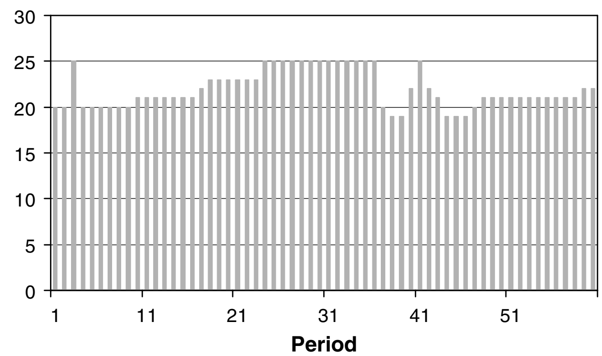


Figure 54
Number of Market Participants
k_{12} = 1.2



5. CONCLUSION

5.1 General Commentary

The model of an insurance market constructed here is minimalist in the sense that it attempts to capture all of the main features of a market using as few parameters as possible. It does so with 13 dynamical parameters. Of these, two are upper and lower limits on price, which are inessential to the functioning of the market. So it is fair to regard the market as comprising 11 parameters. It is suggested that the model is minimalist in that the elimination of any one of these 11 would be fatal to its realism.

Despite the relatively sparse parameterization, the model displays considerable complexity. Some of its results are reminiscent of catastrophe theory (Fig. 25), and a number are counterintuitive. Regulation of the market solvency provides an example. Excessive increase in the strictness of the required solvency ratio is seen in Section 4.6.4 to induce violent cyclic behavior in the average market premium rate and solvency. This is in addition to the more predictable effect of driving out a certain proportion of market participants.

5.2 Competition

Competition has occasionally been seen as a destabilizer of insurance markets. In Winter (1991), for example, competition slowly erodes market profitability until solvency becomes overly stressed. A sudden premium rate crunch then ensues, attracting new capital into the market, and the cycle recommences.

This viewpoint is valid to a limited extent. For example, Section 4.3 shows that strong competitive effects, resulting from high price sensitivity of consumers, can indeed induce cyclic behavior in average market premiums rates.

However, the effects of competition extend well beyond this. For example, Section 4.5 shows that the dependency on certain combinations of competition parameters of whether or not the market displays cyclic behavior is extremely complex.

Some of the major competitive effects are intuitive. For example, the average profit margin in the market declines with increasing competition, and it increases as insurers become more concerned with solvency (Section 4.5).

Some competitive effects are beneficial to the market, and some are not. For example, Section 4.2 shows that too low an intensity of competition leads to cyclic behavior in premium rates, and that this is eliminated in a more competitive market. On the other hand, high price sensitivity of consumers, a different aspect of competition, also leads to cycles (Section 4.3).

5.3 Use of Policy Variables

Policy variables need to be used with considerable care because their effects may be counterintuitive and even the reverse of those intended. Perhaps the most controversial use of policy variables is the application of price controls, in the form of upper or lower limits on premium rates. It is seen in Section 4.6.2 that setting a floor under premium rates does not have the expected effect if the floor is set too high.

In fact, raising the floor toward a requirement of full funding premium rates acts as a barrier to entry. The market becomes bland, with virtually no new entrants, and very stable premium rates, but no diversity between them. Moreover, average profit margin is zero.

This last effect is counter to the intention. The fact is that setting a lower floor under premium rates leads to *higher,* not lower, average rates.

Setting a ceiling on premium rates does not have unexpected consequences, but a low ceiling produces a similarly bland market.

Another quantity with major potential as a policy variable is dividend payout ratio. Section 4.4.1 shows that increasing this has counterintuitive effects. High values of the ratio induce market cycles. Even higher values eliminate these but decimate the market participants.

5.4 Catastrophe Events

Catastrophes have sometimes been viewed in the literature as responsible for market cycles, or at least for erratic behavior of premium rates. This is certainly confirmed by the model investigated here. Indeed, although the effect on the market of a single major event may be transient, it can persist for a long time (Section 4.7).

The result of this, even with a relatively low catastrophe frequency, would be that each new event would occur before the cycles induced by the previous one had died out. In this situation the market would exhibit continuous cyclic behavior. A price control, in the form of a ceiling on premium rates, can mitigate this behavior.

REFERENCES

BALZER, L. A. 1982. Control of Insurance Systems with Delayed Profit/Loss Sharing and Persisting Unpredicted Claims. *Journal of the Institute of Actuaries* 109: 285–316.

BALZER, L. A., AND S. BENJAMIN. 1980. Dynamic Response of Insurance Systems with Delayed Profit/Loss Sharing Feedback to Isolated Unpredicted Claims. *Journal of the Institute of Actuaries* 107: 513–28.

COUTTS, S. M., AND R. DEVITT. 1989. The Assessment of the Financial Strength of Insurance Companies by a Generalized Cash Flow Model. In *Financial Models of Insurance Sol-*

vency, ed. J. D. Cummins and R. A. Derrig, 1–37. Dordrecht: Kluwer.

CUMMINS, J. D., AND P. M. DANZON. 1991. Price Shocks and Capital Flows in Liability Insurance. In 18th Seminar of the European Group of Risk and Insurance Economists, Geneva Association.

CUMMINS, J. D., AND J.-F. OUTREVILLE. 1987. An International Analysis of Underwriting Cycles. *Journal of Risk and Insurance* 53: 246–60.

DAYKIN, C. D., G. D. BERNSTEIN, S. M. COUTTS, E. R. F. DEVITT, G. B. HEY, D. I. W. REYNOLDS, AND P. D. SMITH. 1987. Assessing the Solvency and Financial Strength of a General Insurance Company. *Journal of the Institute of Actuaries* 114(2): 227–310.

DAYKIN, C. D., AND G. B. HEY. 1990. Managing Uncertainty in a General Insurance Company. *Journal of the Institute of Actuaries* 117: 173–277.

DAYKIN, C. D., T. PENTIKÄINEN, AND M. PESONEN. 1994. *Practical Risk Theory for Actuaries.* London: Chapman & Hall.

POSTON, T., AND I. N. STEWART. 1976. *Taylor Expansions and Catastrophes.* London: Pitman Publishing.

WINTER, R. A. 1991. Solvency Regulation and the Property-Liability "Insurance Cycle." *Economic Inquiry* 29(3): 458–71.

Discussions on this paper can be submitted until January 1, 2009. The author reserves the right to reply to any discussion. Please see the Submission Guidelines for Authors on the inside back cover for instructions on the submission of discussions.