

PRICING WEATHER DERIVATIVES USING THE INDIFFERENCE PRICING APPROACH

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ABSTRACT

This paper adopts an incomplete market pricing model—the indifference pricing approach—to analyze valuation of weather derivatives and the viability of the weather derivatives market in a hedging context. It incorporates price risk, weather/quantity risk, and other risks in the financial market. In a mean-variance framework, the relationship between the actuarial price and the indifference price of weather derivatives is analyzed, and conditions are obtained concerning when the actuarial price does not provide an appropriate valuation for weather derivatives. Conditions for the viability of the weather derivatives market are examined. This paper also analyzes the effects of partial hedging, natural hedges, basis risk, quantity risk, and price risk on investors' indifference prices by examining the distributional impacts of the stochastic variables involved.

1. INTRODUCTION

Weather derivatives represent a recent development in hedging the risk arising from uncertain weather conditions. According to the results of a 2006 survey for the Weather Risk Management Association conducted by PriceWaterhouseCoopers, the notional value of weather derivatives traded on the Chicago Mercantile Exchange (CME) and the over-the-counter market for the summer and winter seasons of 2004–2005 was \$9.697 billion, and this grew to \$45.244 billion for the summer and winter seasons of 2005–2006. The CME reported 1,041,439 trades in summer 2005 and winter 2005–2006 combined, with 2005–2006 CME trades up over 300% compared to 2004–2005.

Weather derivatives are important in weather risk management for several reasons.¹ First, global businesses have large exposure to increasingly uncertain global weather conditions. For example, beyond the energy and power industries where weather derivatives started, other industries such as agriculture, insurance, tourism, and retail businesses are also directly or indirectly affected by weather (e.g., Dischel 1998; Zeng 2000). The U.S. Department of Commerce has estimated that the weather affects about one-third of the U.S. economy. Second, compared to weather-related insurance products, weather derivatives are more economical and flexible and require no proof of loss (Dischel 1998). Furthermore, as noted by many authors (e.g., Zeng 2000; Platen and West 2004), the correlation between weather derivatives and other financial instruments appears to be low enough to make them attractive assets for diversification and portfolio formation (Brockett et al. 2006). Despite its phenomenal growth in recent years, the development of the weather derivatives market is still impeded by the scarcity of widely accepted standard pricing techniques (e.g., Dischel 1998; Platen and West 2004).

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¹ See Brockett, Wang, and Yang (2005) for a detailed description and overview of the weather derivatives market.

Not surprisingly, the quest for new pricing techniques is imperative for the weather derivatives market (e.g., Dornier and Queruel 2000; Cao and Wei 2003).

The most distinctive feature of weather derivatives is that, unlike conventional derivatives, their payoffs are linked to weather events rather than the price of an underlying security or commodity. The commonly used underlying weather indexes—for example, heating degree days (HDDs) and cooling degree days (CDDs)²—are nontradable, and there is typically little liquidity in the weather derivatives market (Davis 2001). Accordingly the traditional no-arbitrage pricing models of financial derivatives, such as the Black-Scholes (1973) model, cannot be directly applied to the pricing of weather derivatives (see, e.g., Dischel 1998; Zeng 2000; Cao and Wei 2003). Dornier and Queruel (2000) also note that applying the idea of delta hedging to create an equivalent risk-neutral portfolio for pricing weather derivatives in an incomplete market without an underlying traded asset is not possible.

On the other hand, although actuarial valuation approaches have been very successful in the insurance market, they are not appropriate for pricing weather derivatives (Sloan, Palmer, and Burrow 2002). One striking feature of the actuarial valuation principles is that they are formulated within a framework that generally ignores the financial market. However, weather events evidently affect the prices of some liquid assets in the financial market, and weather derivatives can therefore be partially hedged by these liquid assets that are stochastically related to the payoff of weather derivatives (Craft 1998; Hirshleifer and Shumway 2003; Kamstr, Kramer, and Levi 2000; Roll 1984; Saunders 1993).

The weather derivatives market is a classic incomplete market (Davis 2001). In principle, incomplete market pricing models are more appropriate for the valuation of weather derivatives because they acknowledge both the hedgeable and unhedgeable parts of a risk. Various alternative pricing mechanisms have been developed for the incomplete market, such as super-replication (El Karoui and Quenez 1995), quadratic approaches (Follmer and Sondermann 1986; Schweizer 1988, 1991; Bouleau and Lamberton 1989; Duffie and Richardson 1991), quantile hedging and shortfall minimization (Follmer and Leukert 1999, 2000; Cvitanic 1998), marginal utility approach (Davis 1998), and indifference pricing (Hodges and Neuberger 1989; Davis, Panas, and Zariphopoulou 1993).

The particular problem of valuation of weather derivatives has also been explored in the literature. For example, Cao and Wei (2003) propose and implement an equilibrium representative agent framework for pricing weather derivatives. They generalize the Lucas (1978) model to include weather as a fundamental variable in the economy. Davis (2001) gives an exploration of weather derivative pricing using the marginal utility approach from mathematical economics, based on the idea that agents in the weather derivatives market are not representative but face very specific risks related to the effect of weather on their business. Based on a utility maximization point of view, Barriou and El Karoui (2002) determine the optimal profile and value of a derivative written on an illiquid asset, such as a catastrophic or a weather event.

The purpose of this paper is to apply the indifference pricing approach to value weather derivatives and use weather futures/forwards and weather options as examples to illustrate the model. This paper values weather derivatives in a hedging context, in which hedgers use weather derivatives to hedge their weather risks and maximize their expected utility. In practice, weather derivatives are commonly valued by the expected payoff under the physical measure, discounted at the riskless rate (Davis 2001): that is, they are valued by the actuarial approach.

This paper extends the literature by incorporating price risk, weather/quantity risk, and other risks in the financial market by analyzing the relationship between the actuarial price and the indifference price of weather derivatives and by examining the conditions under which the actuarial price does not yield an appropriate valuation for weather derivatives. The viability of the weather derivatives market

² Because one need not heat if the temperature is above 65°F, daily HDD = $\text{Max}(0, 65^\circ\text{F} - \text{daily average temperature})$. Similarly, because one does not need to cool if the temperature is below 65°F, daily CDD = $\text{Max}(0, \text{daily average temperature} - 65^\circ\text{F})$. Weather derivatives are based on the accumulation of HDDs or CDDs during a certain period (contract period), e.g., one calendar month, or a winter/summer season. To calculate the degree days over a period, simply aggregate the daily degree days of each day in that period.

is also analyzed by examining the indifference prices of investors in this market. The weather derivative market is a new derivative market and still immature, and the examination of market viability provides important insights for the investors in the weather derivative market. In addition, the indifference prices derived in this research can serve as important benchmark prices for the market players.

Partial hedging, natural hedges, and basis risk are all important issues in corporate hedging. However, to the best of our knowledge none of the previous research has examined their impacts on the value of weather derivatives.³ Weather derivatives can be hedged partially in the financial markets by those liquid assets that are stochastically related to weather indexes. A natural hedge occurs when one risk is offset by another one in the hedger's business. For instance, a company whose commodity price and quantity demand move in the opposite directions holds a natural hedge. Basis risk occurs when a weather contract is written in a different location than the area that a hedger wishes to cover. By examining the distributional impacts of the stochastic variables involved, this paper also extends the literature by analyzing the effects of partial hedging, natural hedges, and basis risk, as well as quantity risk and price risk on an investors' indifference prices.

The next section introduces the general indifference pricing approach and presents the specific indifference pricing model used in this research. The results of the derivations of indifference prices and market viability are also provided in this section. Section 3 analyzes the distributional impacts of the stochastic variables on both linear and nonlinear weather contracts. We conclude with a summary and some ideas for future research.

2. MODEL SPECIFICATION AND INDIFFERENCE PRICES OF WEATHER DERIVATIVES

The indifference pricing approach stems from the basic principle of equivalent utility.⁴ This method is based on expected utility arguments and produces the reservation prices. It is built around the investors' preferences toward risks that cannot be eliminated because of market incompleteness. The indifference pricing approach has been applied to pricing traditional financial derivatives in incomplete market (e.g., Hodges and Neuberger 1989; Davis, Panas, and Zariphopoulou 1993; Eydeland and German 1999; Henderson 2002; Owen 2001; Zariphopoulou 2001), to pricing insurance products in the financial market (e.g., Moore and Young 2003; Young and Zariphopoulou 2002; Moller 2003; Young 2003), and to pricing some other derivatives with exotic underlyings (e.g., Barrieu and El Karoui 2002; Musiela and Zariphopoulou 2002).

To illustrate the general indifference pricing approach,⁵ this paper assumes a dynamic market setting that consists of two assets, a stock with a price process $S = (S_t)_{0 \leq t \leq T}$, where T is a fixed finite time horizon, and a savings account with a price process that is constant and equal to one, as well as a nontraded asset Y_t on which a European-type claim is written. The payoff of this European derivative is denoted by $g(Y_T)$, payable at time T . Moreover, no trading of the derivative is allowed after its inscription/purchase. The individual risk preferences are modeled via a utility function u . In this model the investor seeks to maximize the expected utility of terminal wealth with and without taking into account the European claim.

The optimization problem without considering this financial claim is the classical Merton model of optimal investment (Merton 1969, 1971), namely,

$$V(\varpi) = \sup_{\vartheta} Eu(\varpi + \int_0^T \vartheta_t dS), \quad (2.1)$$

where ϖ denotes the initial wealth of the investor and ϑ_t represents his or her investment strategy (or the composition of his or her investment portfolio of the risky asset and the risk-free asset at time t).

³ Golden, Wang, and Yang (2007) examine credit and basis risk for weather derivatives, and Golden, Yang, and Zou (2009) discuss the effectiveness of using basis derivatives for managing weather-related risks.

⁴ See Gerber and Pafumi (1998) for a detailed overview of the principle of equivalent utility.

⁵ In the introduction of the general indifference pricing approach, this paper presents the continuous-time model. The discrete-time model adopted in this paper will be presented later as a special case.

Considering the possibility of buying/selling δ units of this financial claim, the buyer's and seller's optimization problems are defined, respectively, by

$$V^b(\bar{w}) = \sup_{\vartheta} Eu \left[\bar{w} + \int_0^T \vartheta_t dS_t - \delta\pi^b + \delta g(Y_T) \right] \quad (2.2)$$

and

$$V^s(\bar{w}) = \sup_{\vartheta} Eu \left[\bar{w} + \int_0^T \vartheta_t dS_t + \delta\pi^s - \delta g(Y_T) \right], \quad (2.3)$$

where π^b and π^s denote the prices of buying and selling one unit of this financial claim, respectively.

The seller's (buyer's) indifference price of the European claim $g(Y_T)$ is defined as $\pi^s(\pi^b)$, such that the investor is indifferent to the following two scenarios: (1) optimize his or her expected utility without employing the derivative and (2) optimize his or her expected utility taking into account, on one hand, the liability (payoff) $g(Y_T)$ at expiration T and, on the other hand, the compensation π^s (cost π^b). Therefore, the indifference prices $\pi^s(\pi^b)$ must satisfy

$$\sup_{\vartheta} Eu \left(\bar{w} + \int_0^T \vartheta_t dS_t \right) = \sup_{\vartheta} Eu \left[\bar{w} + \int_0^T \vartheta_t dS_t + \delta\pi^s - \delta g(Y_T) \right] \quad (2.4)$$

and

$$\sup_{\vartheta} Eu \left(\bar{w} + \int_0^T \vartheta_t dS_t \right) = \sup_{\vartheta} Eu \left[\bar{w} + \int_0^T \vartheta_t dS_t - \delta\pi^b + \delta g(Y_T) \right]. \quad (2.5)$$

The most popular weather derivatives are based on temperature indexes such as HDDs, CDDs, and cumulative average temperature (CAT). Weather derivatives are designed to hedge weather risks for a specific time period (a contract term). The most popular contract terms are a week, a month, or a season. The HDD or CDD index values are the sum of each daily HDD or CDD value, whereas the CAT index is the accumulation of each daily average temperature, during a specific contract term. Because of this structure of weather derivatives, hedgers generally hold weather derivatives for the whole contract term (or the hedging period) till maturity.⁶ Therefore, this paper adopts a two-date model that is more appropriate than the multiperiod or the continuous-time setting for this research.

In this two-date model, the financial market consists of two assets, a risky asset (such as a stock, the market portfolio, or a subset of the market portfolio) with a random rate of return r at date 1 and a savings account with a gross risk-free rate of return r_f , as well as a random weather index y on which a weather derivative can be written. The payoff of this weather derivative is denoted by R_y . In this research the investors' risk preferences are modeled by the mean-variance utility function

$$u(x) = E(x) - \lambda\sigma^2(x), \quad (2.6)$$

with the risk aversion parameter $\lambda > 0$. Higher expected wealth improves firm value, whereas higher volatility imposes costs due to increases in the likelihood of financial distress and/or effects on future investment incentives. The mean-variance utility function has been widely used in the literature (e.g., Duffie and Richardson 1991; Vukina, Li, and Holthausen 1996; Doherty and Richter 2002; Bessembinder and Lemmon 2002); however, other objective functions such as the exponential utility and the power utility could be applied similarly.

In our models the investor seeks to maximize the mean variance of his or her terminal wealth at date 1 under the scenarios with and without taking into account the weather derivative. The buyer (or

⁶ Even assuming that the weather derivative is bought/sold initially and the position in that single instrument is unchanged, it is certainly possible that the investor may dynamically rebalance the weights in other securities in the portfolio. We leave the impact of the dynamic rebalancing to future work. We are using a static model because it is also mathematically simpler, and possibly easier to implement.

hedger)⁷ considered in this research is typified by an electric power provider with random power demand q and random unit power price p at date 1,⁸ and the seller (or issuer) of weather derivatives can be an investment bank, an insurance company, or an energy trading company. Our model incorporates price risk (p), weather/quantity risk (q), and other risks (r) in the financial market and values weather derivatives in a hedging context in which hedgers use weather derivatives to hedge their weather risks and maximize their utility of terminal wealth.

First, we analyze the buyer's indifference prices of weather derivatives. Without weather hedging, the optimal portfolio choice problem of the buyer is

$$V_1^b = \underset{a}{\text{Max}} u[(\varpi - a)r_f + ar + pq], \tag{2.7}$$

where $(\varpi - a)r_f + ar + pq$ is the buyer's terminal wealth at date 1, in which a is the amount invested in the risky asset, $\varpi - a$ is the amount invested in the risk-free asset, ϖ denotes the initial wealth of the investor, and pq is the buyer's revenue from power supply. Taking into consideration using a weather derivative to hedge the weather risk, the optimal portfolio choice problem of the buyer becomes

$$V_2^b = \underset{a}{\text{Max}} u[(\varpi - \delta\pi^b)r_f + a(r - r_f) + pq + \delta R_y], \tag{2.8}$$

where π^b is the buying price of the weather derivative. Under the mean-variance utility function, the buyer's indifference price of the weather derivative is

$$\pi^b = \frac{1}{r_f} \left[\mu_{R_y} - \delta\lambda^b\sigma_{R_y}^2 - 2\lambda^b\sigma_{pq,R_y} - \frac{\sigma_{r,R_y}(\mu_r - r_f - 2\lambda^b\sigma_{r,pq} - \delta\lambda^b\sigma_{r,R_y})}{\sigma_r^2} \right]. \tag{2.9}$$

Because of the prominent role of the risk aversion parameter λ , the investors' indifference prices can be highly subjective. Generally, one participant's indifference price should not be used to determine the general viability of a market. However, the weather derivatives transactions are mostly over-the-counter customized deals between a buyer and a seller even though some weather derivatives are listed for trading at some exchanges such as the CME. Investors' indifference prices provide important benchmarks to the market players in the customized weather derivatives markets.⁹

For the market to be viable, the market price of the weather derivative must be between the seller's indifference price and the buyer's indifference price: that is, the market price is no higher than the buyer's indifference price. Therefore, if the buyer's indifference risk premium—the difference between π^b and the risk-free-rate discounted price μ_{R_y}/r_f —is negative:

$$\frac{1}{r_f} \left[\delta\lambda^b\sigma_{R_y}^2 + 2\lambda^b\sigma_{pq,R_y} + \frac{\sigma_{r,R_y}(\mu_r - r_f - 2\lambda^b\sigma_{r,pq} - \delta\lambda^b\sigma_{r,R_y})}{\sigma_r^2} \right] > 0, \tag{2.10}$$

⁷ Generally, hedgers take a short position in forwards/futures, and the payoff of a long position in a forward/future is $\gamma - K$. In this paper the payoff of a long position in a forward/future is specified as $K - \gamma$ so that hedgers buy long (issuers sell short) both forwards/futures and put options, which then can be analyzed in a single buyer's model. Otherwise we have to present two models for hedgers alone: one for buying put options and the other one for selling forwards/futures (and for issuers: one model for selling put options and the other one for buying forwards/futures). This avoids much clumsiness by presenting fewer models/equations. However, this specification does not affect our analyses of the indifference prices of weather contracts.

⁸ p and q are introduced separately because weather derivatives are designed to hedge quantity risk instead of price risk.

⁹ Indifference prices are also very important even for the exchange market. Hedgers have to know their indifference prices to determine if participating in the exchanges' market will achieve their hedging objectives and maximize their utility. In this sense the market makers have to set the market clearing price in the range of investors' subjective prices (i.e., indifference prices) to attract enough participants for the market to be viable. However, the market clearing price will be determined by indifference prices of a majority of investors instead of only a few potential participants.

that is,

$$\lambda^b [\delta(\sigma_{r,R_y}^2 - \sigma_r^2 \sigma_{R_y}^2) + 2(\sigma_{r,R_y} \sigma_{r,pq} - \sigma_r^2 \sigma_{pq,R_y})] < \sigma_{r,R_y} (\mu_r - r_f), \quad (2.11)$$

then $\pi_M < \mu_{R_y}/r_f$: the market price of the weather derivative, π_M , must be lower than its expected discounted value discounting at the risk-free rate and under the physical measure (actuarial price), and thus the actuarial price is not an appropriate valuation of the weather derivative.

Consider a weather forward contract with tick size of \$1 and delivery price K . The buyer's (or hedger's) payoff for this contract is specified as $R_y = K - y$, where y is the weather index. The indifference price of the weather forward can be expressed as

$$\pi_f^b = \frac{1}{r_f} \left[(K - \mu_y) - \delta \lambda^b \sigma_y^2 + 2\lambda^b \sigma_{pq,y} + \frac{\sigma_{r,y} (\mu_r - r_f - 2\lambda^b \sigma_{r,pq} + \delta \lambda^b \sigma_{r,y})}{\sigma_r^2} \right]. \quad (2.12)$$

The indifference forward price F^b is the value of K that makes π_f^b equal to 0, that is,

$$F^b = \mu_y + \delta \lambda^b \sigma_y^2 - 2\lambda^b \sigma_{pq,y} - \frac{\sigma_{r,y} (\mu_r - r_f - 2\lambda^b \sigma_{r,pq} + \delta \lambda^b \sigma_{r,y})}{\sigma_r^2}. \quad (2.13)$$

Similarly, for the market to be viable, the market forward price must be no lower than the buyer's indifference forward price. Therefore, if the buyer's indifference forward risk premium is positive:

$$\delta \lambda^b \sigma_y^2 - 2\lambda^b \sigma_{pq,y} - \frac{\sigma_{r,y} (\mu_r - r_f - 2\lambda^b \sigma_{r,pq} + \delta \lambda^b \sigma_{r,y})}{\sigma_r^2} > 0, \quad (2.14)$$

that is,

$$\lambda^b [\delta(\sigma_y^2 \sigma_r^2 - \sigma_{r,y}^2) + 2(\sigma_{r,y} \sigma_{r,pq} - \sigma_r^2 \sigma_{pq,y})] > \sigma_{r,y} (\mu_r - r_f), \quad (2.15)$$

then $F_M > \mu_y$. Hence the market forward price of the weather index, F_M , must be higher than its expected value under the physical measure.

Next, we analyze the seller's indifference price of weather derivatives. Without taking into account a weather derivative, the optimal portfolio choice problem for the seller is

$$V_1^s = \underset{a}{\text{Max}} u[\bar{w}r_f + a(r - r_f)]. \quad (2.16)$$

Taking into account a weather derivative, the optimal portfolio choice problem for the seller becomes

$$V_2^s = \underset{a}{\text{Max}} u[(\bar{w} + \delta \pi^s)r_f + a(r - r_f) - \delta R_y], \quad (2.17)$$

where π^s is the selling price of the weather derivative. Accordingly, under the mean-variance utility function, the seller's indifference price of the weather derivative is

$$\pi^s = \frac{1}{r_f} \left[\mu_{R_y} + \lambda^s \delta \sigma_{R_y}^2 - \frac{\sigma_{r,R_y} (\mu_r - r_f + \lambda^s \delta \sigma_{r,R_y})}{\sigma_r^2} \right]. \quad (2.18)$$

For the market to be viable, the market price of the weather derivative must be no lower than the seller's indifference price. Therefore, if the seller's indifference risk premium is positive:

$$\lambda^s \delta \sigma_{R_y}^2 - \frac{\sigma_{r,R_y} (\mu_r - r_f + \lambda^s \delta \sigma_{r,R_y})}{\sigma_r^2} > 0, \quad (2.19)$$

that is,

$$\lambda^s > \frac{\sigma_{r,R_y} (\mu_r - r_f)}{\delta (\sigma_{R_y}^2 \sigma_r^2 - \sigma_{r,R_y}^2)}, \quad (2.20)$$

then $\pi_M > \mu_{R_y}/r_f$. Thus, the market price of the weather derivative, π_M , must be higher than its expected discounted value discounting at the riskless rate and under the physical measure. This inequality holds when $\rho_{r,R_y} < 0$. Hence, when the return of the risky asset and the payoff of the weather derivative are negatively correlated, the actuarial price is not an appropriate market price for this weather derivative.

Similar results apply to the seller's (or issuer's) indifference forward price. The seller's indifference price for a weather forward with payoff $R_y = K - y$ can be expressed as

$$\pi^s = \frac{1}{r_f} \left[K - \mu_y + \lambda^s \delta \sigma_y^2 + \frac{\sigma_{r,y}(\mu_r - r_f - \lambda^s \delta \sigma_{r,y})}{\sigma_r^2} \right]. \quad (2.21)$$

The indifference forward price is

$$F^s = \mu_y - \lambda^s \delta \sigma_y^2 - \frac{\sigma_{r,y}(\mu_r - r_f - \lambda^s \delta \sigma_{r,y})}{\sigma_r^2}. \quad (2.22)$$

In a viable market the market forward price must be no higher than the seller's indifference forward price. Therefore, if the seller's indifference forward risk premium is negative:

$$\lambda^s \delta \sigma_y^2 + \frac{\sigma_{r,y}(\mu_r - r_f - \lambda^s \delta \sigma_{r,y})}{\sigma_r^2} > 0, \quad (2.23)$$

that is,

$$\lambda^s > \frac{\sigma_{r,y}(\mu_r - r_f)}{\delta(\sigma_{r,y}^2 - \sigma_y^2 \sigma_r^2)}, \quad (2.24)$$

then $F_M < \mu_y$, the market forward price of the weather index, F_M , must be lower than its expected value under the physical measure, if the market does exist. This inequality holds when $\rho(r; y) > 0$: when weather events have a positive impact on the return of the risky asset in the financial market, actuarial forward price is not an appropriate market forward price in the weather derivatives market.

Clearly a weather derivative market transaction is possible only if the buyer's indifference price is no lower than the seller's indifference price. Therefore, another necessary condition for market viability should be $\pi^b \geq \pi^s$. This necessary condition can be stated as

$$\frac{\lambda^b + \lambda^s}{\lambda^b} \leq \frac{2\sigma_{r,R_y}\sigma_{r,pq} - 2\sigma_{pq,R_y}\sigma_r^2}{\delta\sigma_{R_y}^2\sigma_r^2 - \delta\sigma_{r,R_y}^2}. \quad (2.25)$$

3. DISTRIBUTIONAL IMPACTS OF THE STOCHASTIC VARIABLES

This section analyzes the effects of partial hedging, natural hedges, basis risk, quantity risk, and price risk on the buyer's indifference forward premium and indifference prices of put options. Specifically we analyze the distributional impacts of the stochastic variables involved. Distributional impacts on the seller's indifference forward premium and indifference prices of put options can be analyzed similarly.

For a weather forward contract with tick size of \$1, delivery price of K , and payoff $R_y = K - y$, where y is the weather index, the buyer's indifference forward premium, FP^b , is

$$FP^b = F^b - \mu_y = \delta\lambda^b\sigma_y^2 - 2\lambda^b\sigma_{pq,y} - \frac{\sigma_{r,y}(\mu_r - r_f - 2\lambda^b\sigma_{r,pq} + \delta\lambda^b\sigma_{r,y})}{\sigma_r^2}. \quad (3.1)$$

For simplicity and without loss of generality, let $\delta = 1$ and $\lambda^b = 1$. Restated in the volatility of the stochastic variables and their correlations, the indifference forward premium can be expressed as

$$FP^b = (1 - \rho_{r,y}^2)\sigma_y^2 - \frac{\mu_r - r_f}{\sigma_r} \sigma_y \rho_{r,y} + 2\sigma_y \rho_{r,y} \rho_{r,pq} \sigma_{pq} - 2\sigma_y \rho_{pq,y} \sigma_{pq} \quad (3.2)$$

Assume that the stochastic variables are multivariate normally distributed (other distributional models can be analyzed by conducting a series of simulations). Following Vukina, Li, and Holthausen (1996),

$$\rho_{pq,y} = \frac{\mu_p \sigma_q \rho_{q,y} + \mu_q \sigma_p \rho_{p,y}}{\sigma_{pq}} \quad (3.3)$$

and

$$\rho_{pq,r} = \frac{\mu_p \sigma_q \rho_{q,r} + \mu_q \sigma_p \rho_{p,r}}{\sigma_{pq}}. \quad (3.4)$$

Therefore,

$$FP^b = (1 - \rho_{r,y}^2)\sigma_y^2 - \frac{\mu_r - r_f}{\sigma_r} \sigma_y \rho_{r,y} - 2\mu_p \sigma_y \sigma_q \rho_{q,y} - 2\mu_q \sigma_y \sigma_p \rho_{p,y} \\ + 2\mu_p \sigma_y \sigma_q \rho_{r,y} \rho_{q,r} + 2\mu_q \sigma_y \sigma_p \rho_{r,y} \rho_{p,r} \quad (3.5)$$

Weather derivatives are designed to hedge weather risks, which are primarily considered as volume/quantity risks caused by weather events.¹⁰ Basis risk occurs when a weather derivative is written on an index that is calibrated in a different location than the area that a hedger wishes to cover. Basis risk of the weather forward contract can be represented by the correlation coefficient between the weather index and the quantity demanded, $\rho_{q,y}$ (higher $\rho_{q,y}$ indicates lower basis risk). Note that $\partial FP^b / \partial \rho_{q,y} = -2\mu_p \sigma_q \sigma_y < 0$. Therefore, ceteris paribus, the buyer's indifference forward premium increases in basis risk (decreases in $\rho_{q,y}$): the higher the basis risk, the lower the buyer's indifference forward premium.

A dirty hedge occurs when a derivative security cannot be fully hedged with liquid exchange-traded securities and/or its risk exposure changes over time or in response to changes in a secondary underlying variable. Weather derivatives can also be used as a "dirty hedge" of power price risk if the correlation between the power price and the weather index is not zero ($\rho_{p,y} \neq 0$). Note that $\partial FP^b / \partial \rho_{p,y} = -2\mu_q \sigma_p \sigma_y < 0$. Therefore, ceteris paribus, the buyer's indifference forward premium decreases in the correlation between the power price and the weather index $\rho_{p,y}$ (increases in the basis risk of this "dirty hedge").

In our model a natural hedge happens when $\rho_{q,y} \rho_{p,y} < 0$. For a power provider, it is very likely that the correlation between the weather index and the quantity demanded is positive ($\rho_{q,y} > 0$). Therefore, a natural hedge occurs when the correlation between the weather index and the power price is negative ($\rho_{p,y} < 0$), whereas there is no natural hedge if $\rho_{p,y}$ is also positive. Ceteris paribus, the buyer's indifference forward premium is higher in the case when a natural hedge exists than that when there is no natural hedge for the power provider.

Weather derivatives can be hedged partially in the financial markets by those liquid assets that are stochastically related to the payoff of weather derivatives. This partial hedging effect can be characterized by the correlation coefficient $\rho_{y,r}$. Similarly, the hedging effects on the power provider's price risk and quantity risk using the risky asset in the financial market can be characterized by the correlation coefficients $\rho_{p,r}$ and $\rho_{q,r}$, respectively.¹¹ Note that

¹⁰ The volume/quantity risk is the risk that results from a change in the demand for goods due to a change in the weather.

¹¹ Weather risks are primarily quantity risks. Instead of treating the hedger's income as one general random endowment, we introduce price and quantity risk separately so that we are modeling quantity risk or weather risk explicitly. This makes the paper specifically related to weather derivatives or the modeling of weather risks.

$$\frac{\partial FP^b}{\partial \rho_{q,r}} = 2\mu_p \sigma_q \sigma_y \rho_{r,y}, \tag{3.6}$$

$$\frac{\partial FP^b}{\partial \rho_{p,r}} = 2\mu_q \sigma_p \sigma_y \rho_{r,y}. \tag{3.7}$$

Therefore, if $\rho_{r,y} > 0$, ceteris paribus, the buyer’s indifference forward premium increases in $\rho_{q,r}$ and $\rho_{p,r}$, vice versa. When $\partial FP^b / (\partial \rho_{r,y}) > 0$, that is,

$$\rho_{r,y} < \frac{\sigma_q}{\sigma_y} \rho_{q,r} \mu_p + \frac{\sigma_p}{\sigma_y} \rho_{p,r} \mu_q - \frac{\mu_r - r_f}{2\sigma_y \sigma_r}, \tag{3.8}$$

ceteris paribus, the buyer’s indifference forward premium increases in $\rho_{r,y}$. Otherwise the buyer’s indifference forward premium decreases in $\rho_{r,y}$: that is, the buyer’s indifference forward premium initially increases and then decreases in $\rho_{r,y}$.

The results also indicate how the price risk and quantity risk are related to the buyer’s indifference forward premium. Note that

$$\frac{\partial FP^b}{\partial \sigma_q} = 2\mu_p \sigma_y (\rho_{q,r} \rho_{r,y} - \rho_{q,y}), \tag{3.9}$$

$$\frac{\partial FP^b}{\partial \sigma_p} = 2\mu_q \sigma_y (\rho_{p,r} \rho_{r,y} - \rho_{p,y}). \tag{3.10}$$

Therefore, if $\rho_{r,y} \rho_{q,r} > \rho_{q,y}$, ceteris paribus, the indifference forward premium increases in the standard deviation of the quantity demanded σ_q , and vice versa. If $\rho_{r,y} \rho_{p,r} > \rho_{p,y}$, ceteris paribus, the indifference forward premium increases in the standard deviation of the power price σ_p , and vice versa. We can also see that, ceteris paribus, if weather events have a positive impact on the return of the risky asset in the financial market ($\rho_{r,y} > 0$), the buyer’s indifference forward premium decreases in the market price of risk of the risky asset, $\mu_r - r_f / \sigma_r$, and vice versa.

Similar distributional impacts apply to the buyer’s indifference prices of put options. For a weather put option with tick size of \$1, strike price K , and payoff $R_y^p = \max(K - y, 0)$, the buyer’s indifference price, π_p^b , is

$$\pi_p^b = \frac{1}{r_f} \left[\mu_{R_y^p} - \delta \lambda^b \sigma_{R_y^p}^2 - 2\lambda^b \sigma_{p,q,R_y^p} - \frac{\sigma_{r,R_y^p} (\mu_r - r_f - 2\lambda^b \sigma_{r,p,q} - \delta \lambda^b \sigma_{r,R_y^p})}{\sigma_r^2} \right]. \tag{3.11}$$

Because of the nonlinear structure of weather put options, the effects of the determinants (such as the correlation among the stochastic variables involved) of the indifference price cannot be clearly identified from the closed-form formula itself. Therefore, a series of simulations are conducted to clearly illustrate the effects of the determinants of a buyer’s indifference price of a put option.

We assume that the stochastic variables are multivariate normally distributed. Without loss of generality, r_f , δ , and λ^b are all assumed to be 1. The buyer’s indifference price of a put option is computed from 20,000 simulation realizations of $(r, y, p, \text{ and } q)$.¹² As in Bessembinder and Lemmon (2002), the magnitude of the simulated indifference prices can be made arbitrarily large or small by varying related parameters. However, the simulations are useful in terms of clarifying the determinants of indifference prices.

¹² 5,000, 10,000, 20,000, and 50,000 simulations are all tried, and 20,000 simulations are large enough to have a good convergence of the Monte Carlo.

Table 1
A Buyer's Indifference Price of a Put Option at Different Levels of $\rho_{q,y}$ and $\rho_{p,y}$

		$\rho_{q,y}$								
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$\rho_{p,y}$	0.0	-9.33	-7.90	-6.64	-5.59	-3.92	-2.30	-1.03	0.83	2.23
	0.1	-8.43	-5.83	-4.84	-3.50	-2.43	-0.56	1.05	2.27	3.41
	0.2	-6.15	-5.96	-3.60	-2.31	-0.46	0.67	2.91	3.95	4.85
	0.3	-5.07	-3.28	-1.95	-0.54	0.40	2.26	3.42	5.03	6.43
	0.4	-3.98	-1.62	-0.53	1.08	2.05	4.17	4.52	7.12	7.66
	0.5	-2.33	-0.58	1.38	1.69	3.28	4.75	6.45	7.45	9.22
	0.6	-0.87	0.41	2.01	3.59	5.20	6.59	7.59	8.98	10.17
	0.7	0.82	3.11	3.84	5.61	6.46	7.82	8.72	10.42	11.92
	0.8	2.58	4.16	5.32	6.55	7.79	9.12	10.58	11.36	13.25

Note: $r, y, p,$ and q are assumed to be multivariate normally distributed. $r \sim N(1.05, 0.2^2), y \sim N(100, 20^2), q \sim N(15, 3^2),$ and $p \sim N(2.5, 0.5^2).$ $\rho_{r,y} = 0.5, \rho_{p,r} = 0.5, \rho_{q,r} = 0.5,$ and $\rho_{p,q} = 0.5.$ $K = \mu_y = 100.$ All estimates are computed using 20,000 simulation runs.

Table 1 displays the buyer's indifference price of a put option under different levels of $\rho_{q,y}$ and $\rho_{p,y}$.¹³ The results indicate that the buyer's indifference price of a put option increases in both the correlation coefficient of y and p and that of y and q . This reflects that hedgers are willing to pay more for weather derivatives with less basis risk. The results also indicate that the sensitivity of this indifference price to the correlation coefficient of y and $p(q)$ is almost the same at different levels of $\rho_{q,y}(\rho_{p,y}).$

Table 2 reports the buyer's indifference price of a put option under different levels of $\rho_{y,r}, \rho_{q,r},$ and $\rho_{p,r}.$ The simulation results suggest that the buyer's indifference price of a put option decreases in $\rho_{p,r}$ and $\rho_{q,r}$ when $\rho_{y,r} > 0,$ and the sensitivity is greater when $\rho_{y,r}$ is larger. The intuition is that, when $\rho_{y,r} > 0,$ a larger value in $\rho_{p,r}$ or $\rho_{q,r}$ implies a higher risk of the buyer's portfolio, and thus a lower

Table 2
A Buyer's Indifference Price of a Put Option at Different Levels of $\rho_{r,y}, \rho_{q,r}$ and $\rho_{p,r}$

		$\rho_{r,y}$								
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$\rho_{q,r}$ ($\rho_{p,r} = 0.5$)	0.0	11.67	11.17	11.06	10.79	11.04	11.55	12.28	—	—
	0.1	11.37	11.12	10.62	10.47	10.33	10.68	11.20	—	—
	0.2	11.39	10.70	10.31	10.13	10.09	10.24	10.27	—	—
	0.3	11.29	10.67	10.25	9.61	9.26	9.24	9.51	—	—
	0.4	11.49	10.59	9.75	9.36	8.84	8.57	8.55	—	—
	0.5	11.49	10.48	9.39	8.69	8.05	7.70	7.57	—	—
	0.6	11.30	10.29	8.99	8.20	7.48	6.93	6.43	—	—
$\rho_{p,r}$ ($\rho_{q,r} = 0.5$)	0.0	11.48	11.23	11.19	10.83	11.05	11.53	12.15	13.00	14.26
	0.1	11.56	11.18	10.84	10.27	10.65	10.90	11.04	11.86	12.88
	0.2	11.48	10.92	10.22	10.04	9.89	10.12	10.25	10.55	11.62
	0.3	11.81	10.54	9.89	9.53	9.32	9.39	9.34	9.88	10.31
	0.4	11.38	10.39	9.90	9.19	8.67	8.30	8.12	8.81	9.13
	0.5	11.44	10.40	9.47	8.41	7.86	7.93	7.50	7.70	8.08
	0.6	11.62	10.32	9.17	8.40	7.20	6.86	6.24	6.79	6.84
0.7	11.66	10.11	8.67	7.88	7.18	6.18	5.74	5.46	5.54	

Note: $r, y, p,$ and q are assumed to be multivariate normally distributed. $r \sim N(1.05, 0.2^2), y \sim N(100, 20^2), q \sim N(15, 3^2),$ and $p \sim N(2.5, 0.5^2).$ $\rho_{q,r} = 0.7, \rho_{p,y} = 0.5,$ and $\rho_{p,q} = 0.5.$ $K = \mu_y = 100.$ All estimates are computed using 20,000 simulation runs.

¹³ In the simulations some indifference prices are negative under some scenarios. Negative indifference prices indicate that a "buyer" would like to "get paid" to "buy" a weather contract instead of paying a price. One possible explanation is that using this weather contract to "hedge" weather risk will actually increase the hedger's total risk level. Therefore, the hedger will not use this weather contract for hedging or will require to get paid to "buy" this weather contract for taking a higher risk. It is obvious that the market will never exist under these scenarios, and no arbitrage will occur because most probably no seller is willing to "pay" to "sell" a weather contract with a nonnegative payoff to the buyer.

Table 3
A Buyer’s Indifference Price of a Put Option at Different Levels of $\rho_{q,y}$ and σ_q

		$\rho_{q,y}$									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
σ_q ($\mu_p = 2.5$)	2.00	-0.46	0.63	0.90	1.88	3.12	4.06	5.05	5.60	6.70	7.99
	2.25	-0.79	0.00	1.05	2.15	2.92	4.24	5.50	6.32	7.31	8.04
	2.50	-1.41	0.17	1.29	2.63	3.21	4.73	5.86	7.12	8.25	8.85
	2.75	-1.66	-0.16	0.98	2.15	3.67	4.68	6.14	7.46	8.39	9.82
	3.00	-1.96	-0.46	1.15	2.07	3.82	4.74	6.51	7.67	9.31	10.41
	3.25	-2.36	-0.48	1.00	2.15	3.97	5.40	6.88	8.52	9.57	10.99
	3.50	-2.73	-1.22	-0.02	2.51	4.05	5.64	7.30	8.42	10.50	11.81
	3.75	-3.21	-1.01	0.57	2.03	4.15	6.02	7.77	9.26	11.03	12.97
	4.00	-3.26	-1.38	0.59	2.41	4.27	5.83	8.13	10.01	11.68	13.53

Note: $r, y, p,$ and q are assumed to be multivariate normally distributed. $r \sim N(1.05, 0.2^2), y \sim N(100, 20^2), \mu_q = 15,$ and $\sigma_p = 0.5.$ $\rho_{r,y} = 0.5,$ $\rho_{p,r} = 0.5,$ $\rho_{p,y} = 0.5,$ $\rho_{q,r} = 0.5,$ and $\rho(p;q) = 0.5.$ $K = \mu_y = 100.$ All estimates are computed using 20,000 simulation runs.

Table 4
A Buyer’s Indifference Price of a Put Option at Different Levels of $\rho_{p,y}$ and σ_p

		$\rho_{p,y}$									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
σ_p ($\mu_q = 15$)	0.1	3.99	4.60	4.78	4.57	5.39	5.59	5.99	5.85	6.02	6.57
	0.2	2.99	4.11	4.65	5.03	5.50	5.90	6.59	7.17	7.48	8.31
	0.3	2.13	3.61	4.01	4.99	5.63	6.42	7.31	8.11	9.13	9.97
	0.4	1.60	3.17	4.21	5.34	5.96	7.38	8.01	9.32	10.12	11.48
	0.5	0.89	2.04	4.46	5.43	6.28	7.51	9.45	10.34	11.48	12.94
	0.6	0.62	1.71	3.14	4.60	6.55	8.48	10.07	11.70	12.53	14.92
	0.7	-0.46	1.61	2.80	5.73	7.30	9.31	11.10	12.58	14.61	16.11
	0.8	-1.03	1.36	2.85	5.22	7.39	9.94	11.38	14.09	16.14	18.25
	0.9	-1.76	0.37	2.53	5.03	8.12	10.12	12.55	15.23	17.23	19.90

Note: $r, y, p,$ and q are assumed to be multivariate normally distributed. $r \sim N(1.05, 0.2^2), y \sim N(100, 20^2), \sigma_q = 3,$ and $\mu_p = 2.5.$ $\rho_{r,y} = 0.5,$ $\rho_{p,r} = 0.5,$ $\rho_{p,y} = 0.7,$ $\rho_{q,r} = 0.5,$ and $\rho_{p,q} = 0.5.$ $K = \mu_y = 100.$ All estimates are computed using 20,000 simulation runs.

indifference price that the buyer is willing to pay for this put option. Similarly, the buyer’s indifference price for the put option increases in $\rho_{p,r}$ and $\rho_{q,r}$ when $\rho_{y,r} < 0.$ The results also indicate that the buyer’s indifference price for the put option initially decreases and then increases, in $\rho_{y,r}.$

Tables 3 and 4 display the buyer’s indifference price of a put option under different levels of σ_q and $\sigma_p.$ The results suggest that, when the correlation coefficient of y and $q(p)$ is larger than $\rho_{r,y}\rho_{q,r}(\rho_{r,y}\rho_{p,r})$ (0.25 in our simulations), the buyer’s indifference price for the put option increases in the standard deviation of $q(p).$ The sensitivity is greater at high levels of $\rho_{q,y}$ and $\rho_{p,y},$ but it is much smaller near the turning point (0.25 in our simulations). The intuition is that, when the correlation coefficient of y and $q(p)$ is larger than $\rho_{r,y}\rho_{q,r}(\rho_{r,y}\rho_{p,r}),$ a higher quantity risk (price risk) implies a lower risk of the buyer’s portfolio, and thus a higher indifference price that the buyer is willing to pay for this put option. Similarly, the buyer’s indifference price for the put option decreases in quantity risk (price risk) when the correlation coefficient of y and $q(p)$ is less than $\rho_{r,y}\rho_{q,r}(\rho_{r,y}\rho_{p,r}).$

4. CONCLUSIONS

It is well known that the weather derivatives market is a classic incomplete market. Neither complete market pricing models nor actuarial valuation approaches are appropriate to price weather derivatives.

This paper adopts an incomplete market pricing model—the indifference pricing approach—to value weather derivatives. This method incorporates price risk, weather/quantity risk, and other risks in the financial market. In a mean-variance framework the relationship between the actuarial price and the

indifference price of weather derivatives is analyzed, and some conditions are obtained concerning when the actuarial price is not an appropriate valuation notion for weather derivatives. Conditions for the viability of the weather derivatives market are also examined. These conditions relate an investor's risk aversion to the distributional characteristics of the stochastic variables involved. Important and interesting findings are documented. For example, it is shown that when weather events have a positive impact on the return of the risky asset in the financial market, the actuarial price is not an appropriate market price for weather derivatives.

This paper also analyzes the impacts of partial hedging, natural hedges, basis risk, quantity risk, and price risk on an investor's indifference price. The results indicate that a hedger's indifference forward premium increases in basis risk, and it is higher when a natural hedge exists than when there is no natural hedge for the hedger. The results also indicate that a hedger's indifference forward premium initially increases, and then decreases as a function of the correlation coefficient between the weather index and the return of the risky asset in the financial market. It increases in the correlation coefficient between the hedger's quantity demanded (or power price) and the return of the risky asset when the weather index and the return of the risky asset are positively correlated, and vice versa. Similar distributional impacts apply to a hedger's indifference prices of put options.

There are still numerous outstanding issues to be addressed before the method of the paper could be applied in practice, such as parameterization of the indifference pricing model, and the impact of dynamic rebalancing of an investor's portfolio on weather derivatives pricing. In addition, even though the analyses and results in this paper apply under the theoretical assumptions employed (such as mean-variance investors, or linearity of pricing relationships) and provide important insights about weather derivatives pricing and viability of the weather derivatives market, we expect the results would probably differ under different assumptions and suggest that investors adopt other objective functions and distributions of random variables if more appropriate when applying the indifference pricing model proposed in this paper. It would be interesting to empirically examine the pricing behavior of weather derivatives using other distributional models for the stochastic variables and other incomplete market pricing models, such as the super-replication approach and the quadratic approach. We leave exploration of the indifference pricing approach (especially for pricing weather derivatives) under other theoretical assumptions to future work.

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