

THE DB UNDERPIN HYBRID PENSION PLAN: FAIR VALUATION AND FUNDING

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ABSTRACT

In this paper the defined benefit underpin guarantee is valued as a financial option, within the traditional funding paradigms of actuarial science. Assuming fixed interest rates, and assuming that salaries can be treated as a tradable asset, we value the guarantee using fair value principles. Contribution rates are developed for the Entry Age Normal, Projected Unit Credit, and Traditional Unit Credit funding methods. In addition, for the accruals methods, we demonstrate the implied hedging strategy. The traditional unit credit offers the best method of these three, as it is consistent with the principles of financial economics, and the resulting contributions more naturally follow the cost of the emerging benefit, without creating expensive barriers to new hires. The method generates significant contribution volatility, and we demonstrate how this can be reduced with suitable benefit design and ongoing risk management.

1. INTRODUCTION

It is widely acknowledged that pension plan design is experiencing a period of significant change in much of the developed world. Many final salary defined benefit (DB) plans have been closed or wound up in the past few years. New plans are overwhelmingly designed as defined contribution (DC), and yet there is a real concern by employees and employers about the problems of placing all the investment and interest rate risk on the shoulders of the individual employee, and the associated inequities in DC pensions arising from accidents of timing with respect to retirement. The Society of Actuaries has set up a major initiative, Retirement 20/20 (Society of Actuaries 2007), to address the urgent emerging issues. We have seen in the early 2000s a period of negative investment returns coupled with very low interest rates, the double whammy for DC plan members who retired at that time. The more recent economic conditions have proved challenging for both traditional DB and DC plans.

Hybrid plans offer benefits for which the investment risk is shared between the plan sponsor and the employees, by combining elements of DB and DC plan design. One hybrid that has proved attractive in some countries, including Canada, the United Kingdom, and Australia, is the DC plan with a DB underpin, known in the United States as a floor-offset plan. A description of the worldwide development of hybrid plans, including the DB underpin, is given in Wesbroom and Reay (2005). Several Canadian universities' employee pension plans follow this design; a good example is the McGill University plan (McGill 2006). In the McGill design, the employee and employer contribute to the employee's DC account. The employee has some investment discretion, but for the purpose of calculating the DB underpin benefit, the employee's actual DC fund is not used. Instead, a notional DC account is established for each employee, under which the contributions are all assumed to be invested in the pension plan's balanced fund. At retirement the (hypothetical) amount in the balanced fund is annuitized at the then current rate, and the resulting pension is compared with the pension that the employee would have been entitled to under a final salary DB plan, with an accrual rate of around 1.5% (actually the

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accrual rate is salary dependent through a two-tier design, allowing for integration with federal benefits). If the DB formula gives a higher pension, this is the pension that the employee receives. It is relatively rare that this occurs. We have used a simplified version of the McGill design in the examples in this paper.

The DB minimum offers the employee some downside protection in fairly extreme investment circumstances, and the DC benefit allows members to participate in the upside of the DC plan where the investment experience is favorable. The hypothetical account avoids the problem that members might take excessive risks with their fund allocations, in the knowledge that the downside risk is covered by the DB underpin. In other hybrid plans (such as that of York University) the members are not given any investment flexibility, all contributions are deposited in a central, balanced fund, and there is no need to track a second, hypothetical account for the purpose of the underpin.

The cost of providing the guarantee is paid by the employer in additional contributions. Currently, it appears that these contributions are assessed using traditional actuarial methods, supplemented with deterministic or stochastic scenario testing. In this paper we propose a method for recognizing the minimum benefit as an option. We use financial theory to value and hedge the option under some fairly idealized assumptions.

This paper is designed to be read by both practitioners and researchers. For the benefit of pension practitioners, we have included information on option pricing of exchange options that is relevant to our work but is not new. For the benefit of finance researchers and practitioners, we have described, relatively briefly, some of the approaches that are used in pension plan funding. The contribution of this paper is the combination of financial engineering of options and recognized pension-funding principles.

Little previous published research exists on this topic. Sherris (1995) considered pricing the hybrid benefit under a number of more or less risk-neutral measures but did not consider funding or risk management issues. Blake (1998) calls the benefit a “target minimum pension” (TMP) benefit; he values the benefits using option pricing but does not consider funding. His approach would be similar to the EAN method described below.

In Section 2 we list the major assumptions and simplifications used. In Section 3 we demonstrate the application of option pricing to the valuation of the underpin benefit, where the option valued takes all past and future service and contributions into consideration. This is analogous to the EAN funding framework for a traditional DB plan. In Section 4 we move to an accruals basis and show how the option pricing and hedging methodology can be incorporated in the Projected Unit Credit funding approach. In Section 5 we show that there are advantages in using the option pricing and hedging within a Traditional (or Current) Unit Credit funding paradigm. In Section 6 we discuss methods for managing the level and volatility of the contributions under the traditional unit credit approach. Finally, in Section 7 we summarize the results and discuss how the work might be developed further.

2. MODEL ASSUMPTIONS AND NOTATION

We will value the excess of the guaranteed DB benefit over the DC benefit for an employee who enters the plan at age x_e and retires at age x_r . We assume (for simplicity) that there are no exits between entry and retirement. It is quite straightforward to relax this assumption if exits are independent of the option value. It is also a commonly used simplifying assumption in pension plan valuation; see, for example, Anderson (2006).

Let $T = 12(x_r - x_e)$ denote the total number of months of plan membership. We will work in months as we will assume in later sections that the hedge for the underpin is rebalanced monthly, and using monthly time steps rather than annual makes the notation somewhat neater.

We use the following notation for the DB benefit: The random process S_t denotes the employee's salary t months after entry. For simplicity, we assume that this also increases monthly. The monthly salary increase is modeled as a lognormal process, and we do not adjust for promotional or an age-related salary increase. It would be quite straightforward to construct a salary model with a determin-

istic promotional or age-related element and a stochastic exogenous component, but we remain with the simpler model here.

Let α denote the DB accrual rate per month of service. We let α_{xr} denote the value at exit of an annuity of 1 per year paid according to the pension plan rules. For the development in the following sections, we assume that the cost of the annuity is fixed. In future work we intend to explore the impact of random annuity rates.

The stochastic process A_t represents an index corresponding to the return on the (possibly notional) DC balanced fund. A_t is the accumulation at t months of \$1 invested in the underlying DC fund at time 0.

The value at exit of the DB plan benefit is then DB_T , for example, where

$$DB_T = \alpha TS_T \alpha_{xr}.$$

We have used here a final pay pension plan design, which makes both the explanation and the mathematics simpler. In practice, DB benefits would more commonly be based on final average salary over some period (perhaps three or five years) immediately preceding exit. In Section 6 we explore the effectiveness of averaging in reducing the variability of the contribution rate.

For the DC benefit value, assuming monthly contributions, we have, for $1 \leq t \leq T$:

$$DC_t = \sum_{j=0}^{t-1} c S_t \frac{A_t}{A_j},$$

where c is the monthly contribution rate for the DC benefit, assumed fixed.

The guarantee at retirement then has payoff

$$\max(DB_T - DC_T, 0),$$

that is, the employee receives the DC account value, plus the amount sufficient to pay the additional pension to make the total pension up to the DB level when necessary. The random processes that are of interest here are the fund accumulation process A_t and the salary process S_t .

The DB underpin option $\max(DB_T - DC_T, 0)$ resembles an exchange option (also known as a Margrabe option), which offers the higher of two risky assets. However, a major complication here is that both assets DB and DC depend on the underlying salary process. Even with very simple assumptions for the underlying processes, the option is rather more complex than a standard exchange option.

In the following sections we discuss the valuation and risk management of this option using a financial engineering framework. It is convenient and attractive to place the different approaches in the context of the valuation and funding of a conventional DB pension benefit.

We make some simplifying assumptions, all of which may be relaxed in subsequent work. We have already mentioned that the annuity factor at retirement is assumed to be fixed. We assume that the DC fund is invested in traded assets, and more unusually, for the Entry Age Normal and Projected Unit Credit approaches, we also assume that salaries are traded assets. In fact, of course, salaries are not traded. However, salaries show a very high correlation to price inflation, and inflation can be considered a tradable asset where inflation-linked bonds are issued. Such bonds are available in Canada, the United States, and the United Kingdom. Using inflation-linked bonds, salary can be partially hedged, but this is not an issue we explore further in this paper. Further discussion of the issue of the partial hedging of salary can be found in Chen (2007). This is an important topic of future research; we are conscious that this paper represents a first step and not the last word in this area.

We assume in the numerical examples that the monthly DC fund return follows a lognormal process with volatility of 15% per year, corresponding to the volatility of a balanced fund with a mix of equities and bonds. We assume also, as discussed above, that monthly salaries follow a lognormal process with volatility of 4% per year. The processes are dependent, through the correlation coefficient $\rho = 0.22$, of the underlying normal processes. The correlation figure is supported by Lucas and Zeldes (2006). The other salary assumptions are broadly taken from Canadian national statistics on salaries from 1954

to 2002, as published in the 2003 *Report on Canadian Economic Statistics* (Canadian Institute of Actuaries 2003). We assume a constant risk-free rate of 5% per year continuously compounded.

The examples are based on a DC plan with a total, fixed DC annual contribution rate of 12.5% of salary, paid monthly. It is not important in the calculations whether this is an employer or employee rate, but in fact is taken as an approximation to the (integrated) McGill DC contribution rate. The DB underpin offers an accrual rate of 1.5% per year of service. We assume that the annuity factor at retirement is 10.0.

Note that, although all of the parameters listed here are given as annual rates, the parameters used in the formulas below are all equivalent monthly values.

2.1 The Exchange Option

In this section we review, very briefly, the valuation of the original European exchange option. Details may be found in Margrabe (1978).

The option is on the excess at time T of one risky asset, $S_1(t)$ over, for example, another, $S_2(t)$. Under the original framework, both assets follow correlated geometric Brownian motion, so that at any future time T , the distributions of $S_1(T)$ and $S_2(T)$ are both lognormal. The value of the option at $t = 0$, for example, can be written as¹

$$\begin{aligned} & E_Q[e^{-rT}(S_1(T) - S_2(T))^+] \\ &= E_Q \left[e^{-rT} S_2(T) \left(\frac{S_1(T)}{S_2(T)} - 1 \right)^+ \right] \\ &= (S_1(0)N(d_1) - S_2(0)N(d_2)), \end{aligned} \tag{2.1}$$

where

$$\begin{aligned} d_1 &= \frac{\log(S_1(0)/S_2(0)) + \frac{1}{2} \sigma^2 T}{\sigma \sqrt{T}}, \\ d_2 &= d_1 - \sigma \sqrt{T}, \end{aligned}$$

and

$$\sigma^2 = (\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2).$$

For this analytic result we use the facts that under the risk-neutral measure, the present value of a risky asset due at T is the current value of the asset, and that the ratio $S_1(T)/S_2(T)$ has a lognormal distribution with parameters $\mu = 0$ and $\sigma = \sqrt{(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)}$.

3. ENTRY AGE NORMAL

Under the Entry Age Normal (EAN) funding method for conventional final salary DB benefits, the value of the total accrued benefit is amortized in the form of a level percentage of salary contribution rate. The method is described in more detail in, for example, Aitken (1995) and Anderson (2006). EAN funding is not widely used for private plans in North America, though it is still used elsewhere and is widely applied to public plans in the United States. Collinson (2001) cites several European countries using EAN. However, Enderle et al. (2006) demonstrate that the EAN method is incompatible with corporate financial economics, and the method is considered obsolescent or even obsolete in many jurisdictions.

¹ Note that $(x)^+ = \max(x, 0)$.

Our purpose in using EAN is that it is a method that starts from the valuation of the whole benefit accrued by retirement. This is the option valued by both Blake (1998) and Sherris (1995). It offers a relatively straightforward starting point, for which we do not have to take into consideration the accruing nature of the benefit, treating the full service benefit as a liability to be valued, and funded with a level contribution rate.

Applying the EAN method to the underpin option valuation then, we find the value at entry of the full option and express that value as a constant contribution rate payable throughout the employee’s term of membership of the plan.

The value of the DB underpin option is

$$\begin{aligned}
 & E \left[e^{-rT} \left(\alpha TS_T a_{xr} - \sum_{j=0}^T c S_j \frac{A_T}{A_j} \right)^+ \right] \\
 & = E_Q \left[e^{-rT} \left(\alpha TS_T a_{xr} - c A_T \left(\sum_{j=0}^T \frac{S_j}{A_j} \right) \right)^+ \right].
 \end{aligned}$$

The exchange is slightly more complicated than the regular exchange option through the factor $(\sum_{j=0}^T S_j/A_j)$. Even though each term in the sum is lognormal, the sum of dependent lognormal random variables is not lognormal and is not a very tractable function. This term indicates that the option is path dependent, because different paths for the ratio S_t/A_t will give different payoffs for the guarantee. Although the guarantee is not analytically tractable, we can use a Monte Carlo simulation to estimate the value.

In Figure 1 we show the value at entry of the guarantee accrued over the entire working lifetime, for ages at entry from 25 to 64. We assume no exits, and all the other assumptions are listed in Section 2. The estimates are determined using 10,000 simulations.

The guarantee cost is expressed as a multiple of the initial salary. The figure shows that the option is valuable for younger lives, with a value of 1.7 times the starting salary at age 25, reducing to 0.025 times the starting salary for a life entering the plan at age 64. The standard errors for the estimates of the guarantee cost at entry range from 0.015 for age 25 to 0.0001 at age 64. These numbers could be interpreted as fair or market valuations at entry of the option that matures at retirement.

Following the EAN approach, the next step is to express the cost as a level contribution over the period of plan membership. For this we must divide the lump-sum value by the value of the salary-

Figure 1
Value at Entry of DB Underpin Benefit, per Unit of Salary, Based on 10,000 Simulations

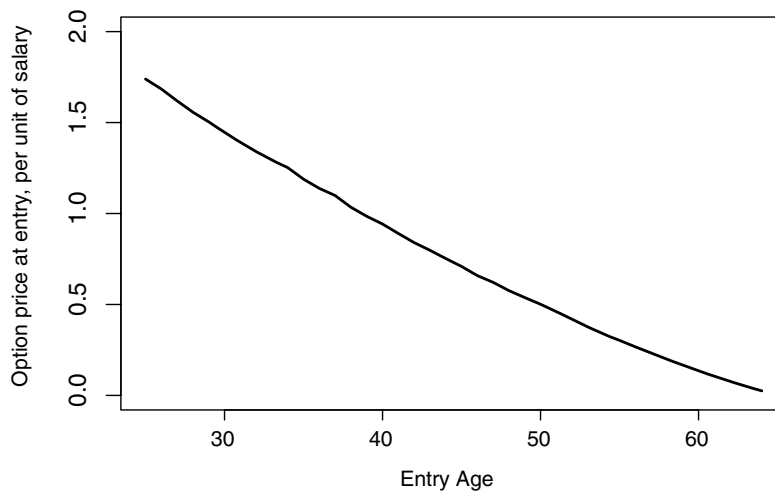
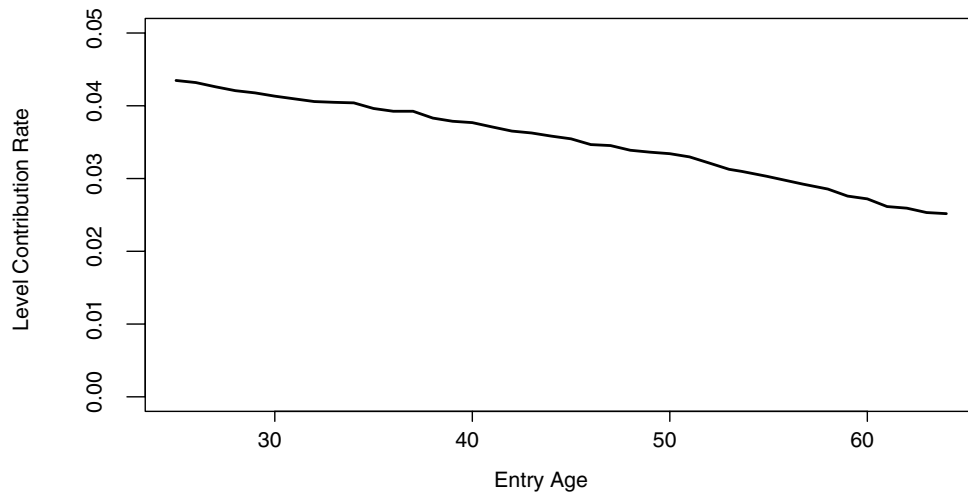


Figure 2
Level Annual Contribution Rate to Finance the DB Underpin, by Age at Entry



indexed annuity. This annuity is also valued under the risk-neutral measure, and also assumes no exits, giving a value

$$E_Q \left[\sum_{j=1}^T e^{-rj} S_j \right] = TS_0,$$

because under the risk-neutral measure the discounted value at time 0 of a risky asset due at $t > 0$ is always simply the current value at 0 of the asset, that is, $E_Q[e^{-rt}S_t] = S_0$ for all $t > 0$.

The level contribution rate to fund the benefit at different entry ages is given in Figure 2. This shows that the level contribution rate also decreases with the entry age. For the more common entry ages, such as from 30 to 45, the additional contribution rate required to fund the DB underpin is between 4.3% and 3.5% of salary.

These rates are rather higher than the current valuations indicate. However, the “no exit” assumption is conservative. Withdrawals are substantially cheaper than retirement benefits, if the option is permitted for withdrawals. Assuming that, for instance, 50% of new entrants do not stay to retirement would reduce the cost to rates that are consistent with current funding.

Although the option valuation and the contribution rate can be found quite simply using Monte Carlo methods, there are problems, both technical and philosophical, with this approach. The problem with the EAN approach is that it treats the benefit based on past service and the benefit from future service on an equal footing in the valuation. Recall that the employer maintains an option to close the plan to new accruals or to close it altogether. It is therefore inconsistent with the nature of the developing liability to treat accrued benefits, which are a debt on the plan assets, on an equal footing with future service benefits, which are not. Thus, the EAN valuation cannot be interpreted as a fair value of the benefit. That can be found only by considering the option value on an accruals basis, under which the liability is not recognized until the relevant service has been completed, when the pension benefit changes from an *intention* (future service) to a *guarantee* (past service). This issue is covered in more detail in Enderle et al. (2006).

4. PROJECTED UNIT APPROACH

For a final salary DB plan, there are two approaches to accruals funding. The first is the projected unit credit (PUC) approach, for which the benefit valued is based on the final projected salary. The second

is the traditional or current unit credit approach (TUC), where the salary is not projected. Instead, the benefit is valued without future salary increase. In this section we propose a method for the valuation and funding of the guaranteed minimum benefit analogous to the PUC method. In Section 5 we adapt the methodology from this section for the TUC approach.

Under the PUC approach we value the option based on accrued contributions for the DC benefit and based on past service and projected salary for the DB benefit: that is, unlike the EAN method, we make no allowance in the valuation at t for any contributions to the DC account due after time t , and we make no allowance for any DB benefit that might be earned in respect of service after t . In accordance with a traditional PUC valuation approach, we do allow for future salary growth for the DB benefit. For the purpose of modeling the funding requirements, we assume monthly valuations and contributions, both to the DC and DB accounts.

Let \mathcal{F}_t denote the relevant information available at t ; here that means that the valuation of the benefit given \mathcal{F}_t assumes knowledge of S_{t_w} for all $t_w \leq t$ and A_t . At the end of the t th month, then, we have a total accrued guarantee liability with value

$$H(t) = E_Q \left[e^{-r(T-t)} \max \left(\text{DB}_t \frac{S_T}{S_t} - \text{DC}_t \frac{A_T}{A_t}, 0 \right) \middle| \mathcal{F}_t \right],$$

where

$$\begin{aligned} \text{DB}_t &= \alpha t S_t a_{x:r}, \\ \text{DC}_t &= \sum_{j=0}^{t-1} c S_j \frac{A_t}{A_j}. \end{aligned}$$

Thus, $\text{DB}_t(S_T/S_t)$ represents the projected amount at T of the DB benefit accrued up to t , DC_t represents the accumulation at t of all contributions up to t , and $\text{DC}_t(A_T/A_t)$ represents the projected amount at T of DC contributions up to time t . Note that, given \mathcal{F}_t , DB_t and DC_t are known, not random. Under the assumption that salaries (S_t) and DC fund growth (A_t) are jointly lognormally distributed, then $\text{DB}_t(S_T/S_t)$ and $\text{DC}_t(A_T/A_t)$ are also jointly lognormally distributed (given \mathcal{F}_t), and we recognize $H(t)$ as the value of a standard exchange option, to which we can directly apply Margrabe's formula from equation (2.1), so that

$$\begin{aligned} H(t) &= \alpha t a_{x:r} S_t N(d_1(t)) - \sum_{j=0}^{t-1} c S_j \frac{A_t}{A_j} N(d_2(t)) \\ &= \text{DB}_t N(d_1(t)) - \text{DC}_t N(d_2(t)), \end{aligned}$$

where

$$d_1(t) = \frac{\ln \left(\frac{\text{DB}_t}{\text{DC}_t} \right) + \frac{1}{2} \sigma^2 (T - t)}{\sigma \sqrt{T - t}}$$

and

$$d_2(t) = d_1(t) - \sigma \sqrt{T - t} \quad \text{and} \quad \sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2.$$

The accrued pension based on projected salaries is also known as the Projected Benefit Obligation (PBO). The fair value of this guarantee is then measured by $H(t)$. To hedge the option, we assume that value is invested in the salary asset and the underlying fund asset. The replicating portfolio at t is long $\mathcal{S}\{\text{DB}_t N(d_1(t))\}$ in the salary asset and is short $\mathcal{S}\{\text{DC}_t N(d_2(t))\}$ in the DC fund underlying asset.

Given the valuation $H(t)$ at t , the next task for the plan actuary is to design an appropriate funding plan for the guarantee cost. On the accruals principle for a traditional DB valuation, benefits are funded as the relevant service is earned, so, apart from post hoc smoothing mechanisms, the contribution

each year is sufficient to fund the increase in liability in that year. We use the same broad approach to the funding of the guarantee under the accruals approach, although we assume a monthly time step rather than annual.

Each month the accrued benefits increase through additional service (on the DB side) and additional contributions (on the DC side). So each month the option changes. We assume that each month the hedge brought forward from the previous month is combined with whatever additional cash flow is required to set up the new hedge. The monthly cost of the benefit is the resulting cash flow, and we divide this by the projected salary at the month end to express the cost as a contribution rate.

That is, at t , the hedge established at $t - 1$ months and brought forward to t has value

$$Hbf(t) = DB_{t-1} \frac{S_t}{S_{t-1}} N(d_1(t-1)) - DC_{t-1} \frac{A_t}{A_{t-1}} N(d_2(t-1))$$

as the salary part of the hedge rises in line with salaries, and the (short) DC asset part of the hedge rises in line with the DC assets. The new hedge costs $H(t)$, so the cash flow for the guarantee at t is

$$CF(t) = H(t) - Hbf(t),$$

and the resulting additional contribution as a proportion of salary is

$$cr(t) = \frac{CF(t)}{S_t}.$$

That is, each month the simulated contribution rate is sufficient to fund the difference between the value of the option hedge brought forward from the previous month and the option hedge required based on the accrued DB and DC benefits at that time. This assumes explicitly that the contribution is invested in the hedge portfolio each month.

In Figure 3 we show the resulting additional guarantee contributions, averaged over 100,000 simulations. The simulations follow the real-world probabilities (P -measure), and the option values are calculated under a Q -measure. Under the P -measure, DC assets and salaries are assumed to be jointly lognormally distributed, as under the risk-neutral measure, with the same covariance matrix for the underlying normal distribution. For the P -measure we assume the μ parameters for the underlying bivariate normal distribution are 8% per year for the DC assets and 5% per year for the salaries.²

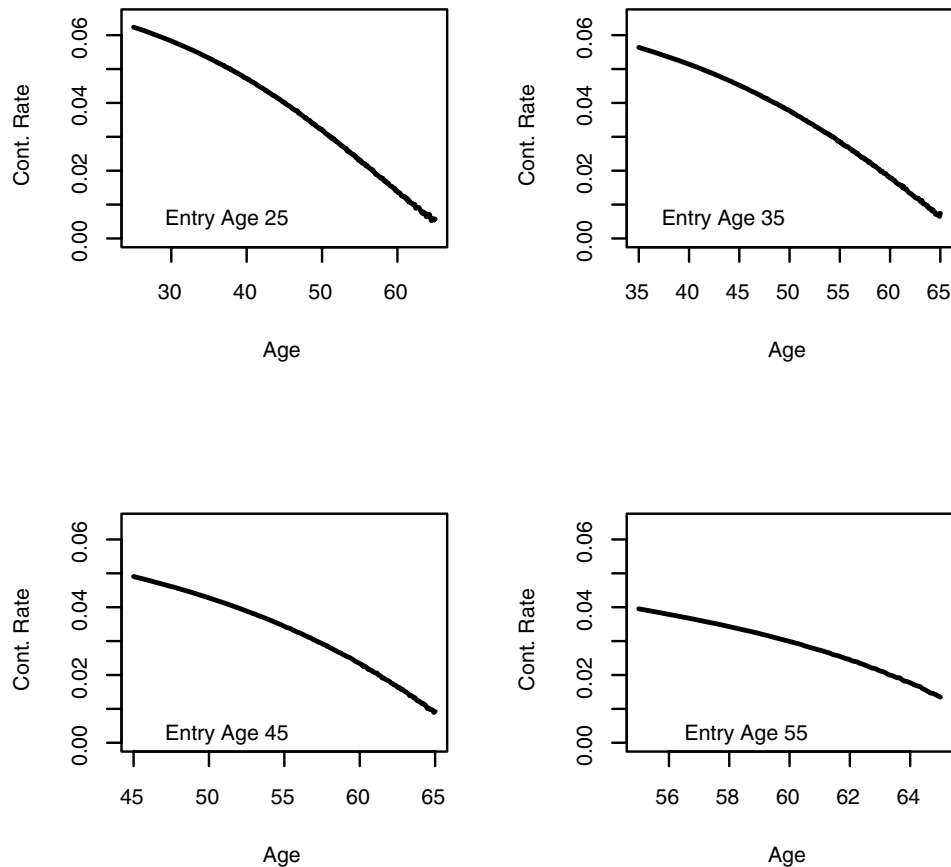
The additional contribution starts high, and then, on average, decreases as the employee approaches the normal retirement age. For example, for lives entering at age 35, the average contribution required in the first month of employment is around 5.6% of salary; just before retirement it averages around 0.7%. Overall, the average contribution rate is around 3.5%, which compares with the 4.0% indicated by the EAN method above.

The contribution rates appear high compared with current contribution rates for underpin benefits. Furthermore, the timing of costs is not ideal, with very high costs at the start of the employee's career, making hiring more expensive. Any such hedge established for an employee who subsequently withdraws would result in a substantial waste.

The reason for the very high cost at entry is the fact that under this approach we are hedging the double uncertainty of salaries and the DC returns, over a very long term. Although the DC returns are (to some extent) outside the control of the company, salaries may not be. Salaries may be treated as endogenously determined if the company has complete control over salary growth; they may be treated as exogenously determined if the effect of industry and market forces overwhelms the autonomy of the employer in setting salaries. The PUC approach assumes the exogenous model, that is, that salary growth is a force that needs to be hedged, rather than a variable that can be controlled by the man-

² This assumption makes the salaries unattractive as a traded asset, earning the same mean return as the risk-free rate, but with risk. Under the CAPM this would require the salary asset to have $\beta=0$, which is unlikely. This becomes less problematic if we relax the assumption that salary is traded. See Chen (2007) for details.

Figure 3
**Contribution Rates to Finance the DB Underpin, Using PUC Approach, 100,000 Simulations,
 for Entry Ages 25, 35, 45, and 55**



agement. It might be more appropriate, from an economic standpoint, to consider the total remuneration package as exogenously determined. This total remuneration cost would then be used to support both pay increases and the consequential cost of increasing the pension benefits. In this case spikes in the total remuneration cost do not necessarily result in spikes in the pension cost. Although we do not pursue the total remuneration concept further in this paper, we note that both the endogenous salary and the exogenous total remuneration theory, endogenously apportioned to salaries and benefits, are more consistent with the traditional or current unit credit approach than with the PUC.

5. TRADITIONAL UNIT CREDIT

Under the traditional unit credit (TUC) approach, the valuation does not take into consideration any future salary increases. Philosophically this is consistent with the accruals approach, in that we do not fund or hedge any benefit until it has accrued, and the TUC approach is most consistent with the fair valuation of the pension liability from a financial economics perspective. The TUC method recognizes that the balance between salaries and pensions costs is under the control of the employer and recognizes the employer option to freeze the plan at current salary levels. See Enderle et al. (2006) for more details.

At the end of the t th month, we have a total accrued guarantee liability with value

$$H(t) = E_Q \left[e^{-r(T-t)} \max \left(DB_t - DC_t \frac{A_T}{A_t}, 0 \right) \middle| \mathcal{F}_t \right].$$

At $t < T$ only the value of A_T is uncertain; DB_t , DC_t , and A_t are all known. We recognize this payoff as a European put option with strike DB_t . Under the assumption that DC fund accruals are lognormally distributed, we can directly apply the Black-Scholes formula, so that the fair value of the guarantee at t is

$$H(t) = e^{-r(T-t)} DB_t N(-d_2(t)) - DC_t N(-d_1(t)),$$

where

$$d_1(t) = \frac{\ln \left(\frac{DC_t}{DB_t} \right) + (r + \frac{1}{2} \sigma_a^2)(T - t)}{\sigma_a \sqrt{T}}$$

and

$$d_2(t) = d_1(t) - \sigma_a \sqrt{T - t}.$$

As in the PUC case, the actuary will seek to construct a funding schedule that funds the liabilities as they are accrued. We assume again that the hedge value $H(t)$ is used to purchase the replicating portfolio, which is long $\$ \{e^{-r(T-t)} DB_t N(-d_2(t))\}$ in zero-coupon bonds maturing at T , and is short $\$ \{DC_t N(-d_1(t))\}$ in the DC fund underlying asset.

We then determine the contribution rate as for the PUC case. Each month we calculate the value of the hedge brought forward; the part representing the DB benefit is assumed to have increased in value according to the risk-free rate of return. The part representing the DC fund is rolled forward according to the simulated return on DC assets. The hedge required at the month end is based on the revised salary, service, and DC contributions. The difference in value is the simulated cash flow, and we divide by the simulated salary at the month end to get the simulated contribution rate.

That is, in the TUC case

$$Hbf(t) = e^{-r(T-t)} DB_{t-1} N(-d_2(t-1)) - DC_{t-1} N(-d_1(t-1)) \frac{A_t}{A_{t-1}},$$

and the cash flow and contribution rate at t are, respectively,

$$CF(t) = H(t) - Hbf(t), \quad c(t) = \frac{CF(t)}{S_t}.$$

In Figure 4 the mean contribution rate paths for the TUC method are plotted for entry ages 25, 35, 45, and 55. These should be compared with Figure 3. We see that the contribution rates fall in a very similar range as the PUC rates, but using the TUC method the cost is significantly more weighted to older ages. The cost for new entrants at younger ages is quite low: in the first year, for a new entrant age 35, the cost is less than 0.12% of salaries. For many reasons, including uncertainty about withdrawal rates, the employer's wind-up option, and the need to offer competitive nonpension benefits to new employees, which could be compromised by high pension costs, this payment path is much more attractive than the PUC approach.

The mean of the contribution rates over the whole employment period is generally lower for the TUC method than for PUC, because, under TUC, the option hedges only the market risk, not the salary risk. Hedging costs money, so hedging one risk is cheaper than hedging two, in general. However, the variability in the contribution rates near to retirement is much greater for the TUC method than for the PUC approach: for exactly the reason that we have not tried to hedge salary risk, so we retain exposure to that risk. In Figure 5 we show 100 simulated contribution paths for entry age 35. In Figure 6 we show four individual paths with both the TUC and PUC contributions plotted. It is easy to see

Figure 4
Contribution Rates to Finance the DB Underpin, Using the TUC Approach, 100,000 Simulations, for Entry Ages 25, 35, 45, and 55

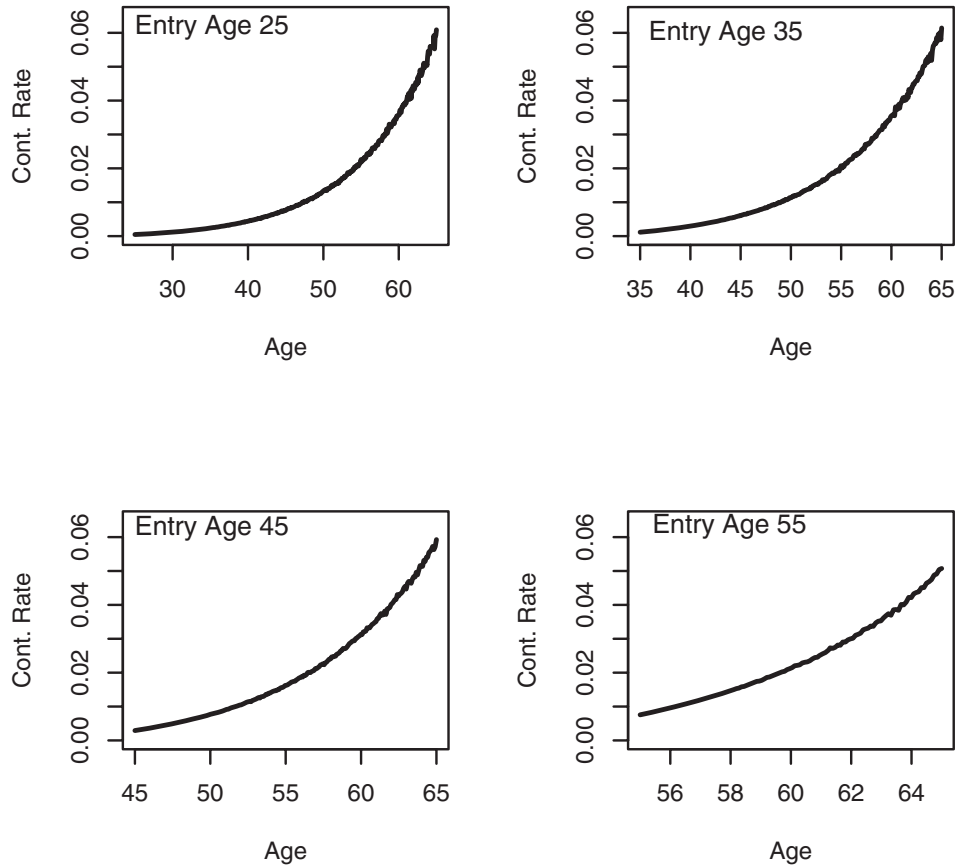


Figure 5
100 Simulated Contribution Rate Paths for Entry Age 35, Using TUC Funding

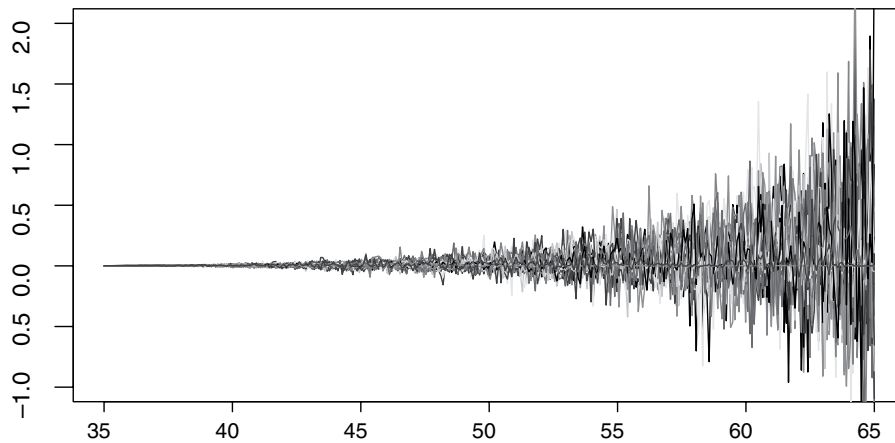
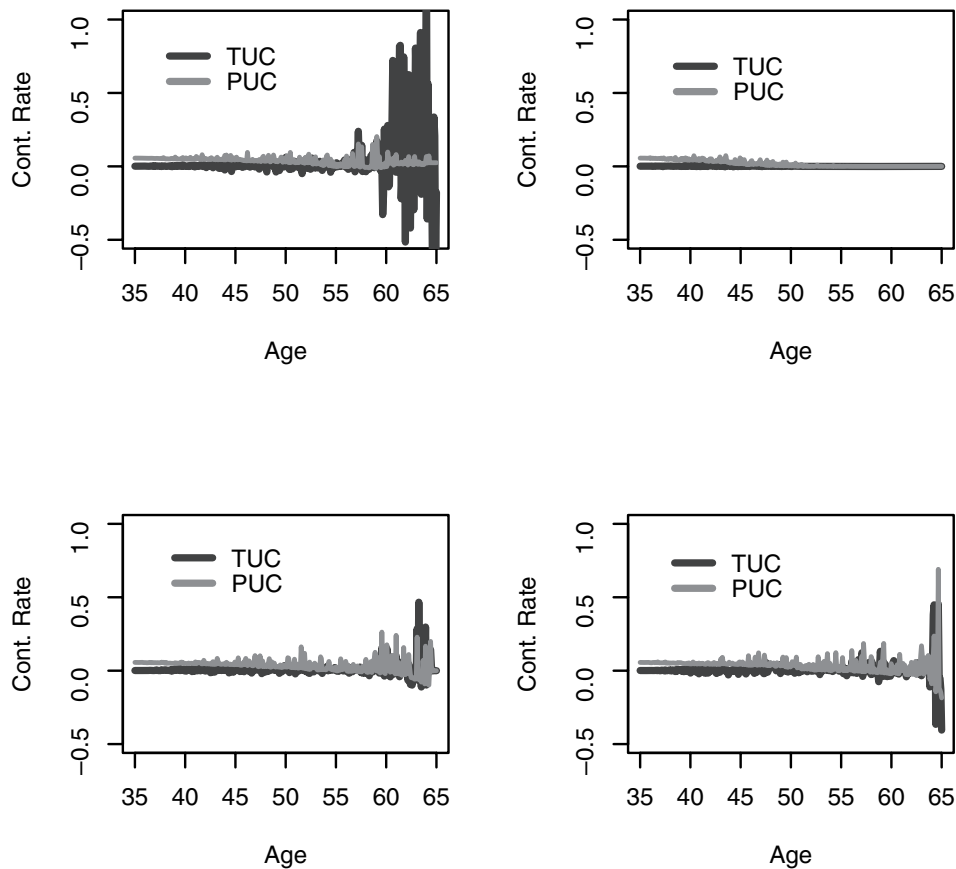


Figure 6
Sample Contribution Rate Paths Using TUC and PUC Approach, Entry Age 35



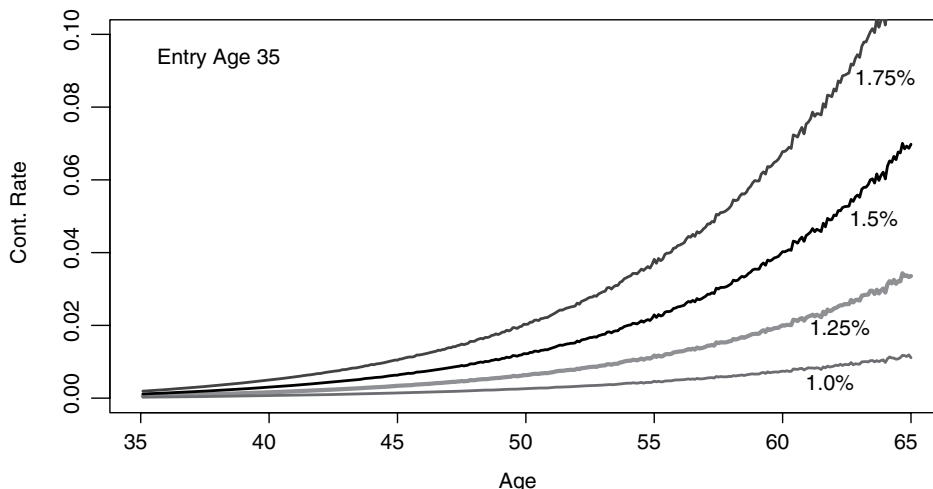
that the TUC may be more or less expensive than the PUC, and that, although it is often more variable, for some projections, such as the lower right projection in Figure 6, the PUC contribution rate path spikes higher than the TUC contribution rate path.

It is also worth noting that although contribution spikes for individual employees will be positively correlated, there will be some smoothing of the contributions for a plan with a large number of active members, as the relatively low volatility at younger ages is mixed with the higher volatility at older ages. Another relevant point is that the salary process in this model is not age related. In practice also, older employee salaries will generally be less volatile than those of younger employees. This is an implicit risk management strategy that will result in a smoother contribution path than the simplified model used here generates.

6. MANAGING THE RISK

The average costs indicated using the TUC approach seem reasonable, and the incidence, with low initial costs rising toward retirement, seems appropriate. However, the large variability may appear daunting. To address this, the funding approach demonstrated can be improved in several different ways. In addition, design elements in the hybrid plan can be used to reduce the volatility. In this section we demonstrate some simple ways to reduce the volatility.

Figure 7
Contribution Rates for Entry Age 35, Using the TUC Approach, for Different Accrual Rates, 10,000 Simulations



6.1 Accrual Rates

The results above are all based on an underpin calculated using an accrual rate of 1.5%; this is a relatively representative rate for the Canadian university plans that we have considered, but may be on the high side for private sector-sponsored plans. Changing the accrual rate has a substantial effect on the contribution rate, as demonstrated in Figure 7, where we show the effect of changing the accrual rate on the mean contribution path for a life entering at age 35, under the TUC funding approach. The 1% accrual rate cost ranges from around 0.02% in the first year of service to 1.1% in the final year.

Using a lower accrual rate impacts the variability of the costs, as well as the average contribution. In Figure 8 we show the 5% and 95% quantiles for the contribution rates for a member entering at age 35, for accrual rates of 1.5% (the base case used throughout the paper), 1.25%, and 1.0%.

Figure 8
Contribution Rate 5% and 95% Quantiles, for Entry Age 35, Using the TUC Approach, for Different Accrual Rates, 10,000 Simulations

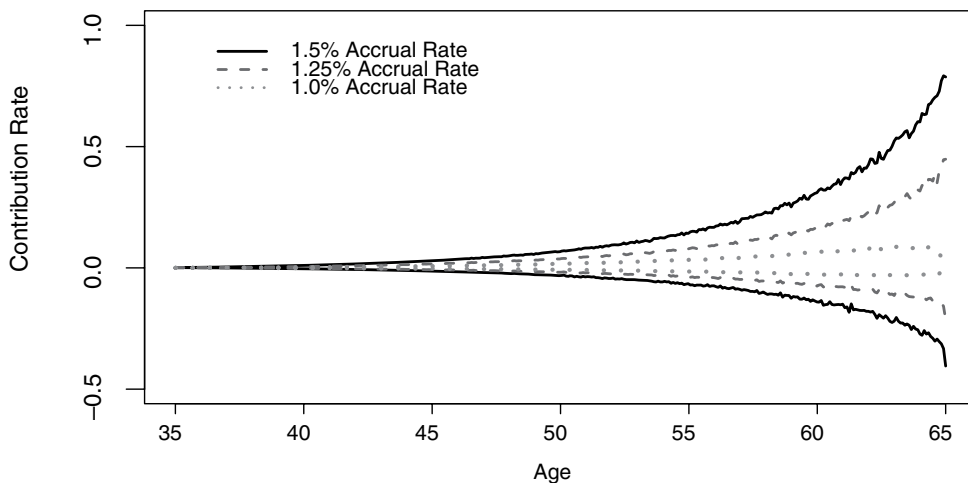
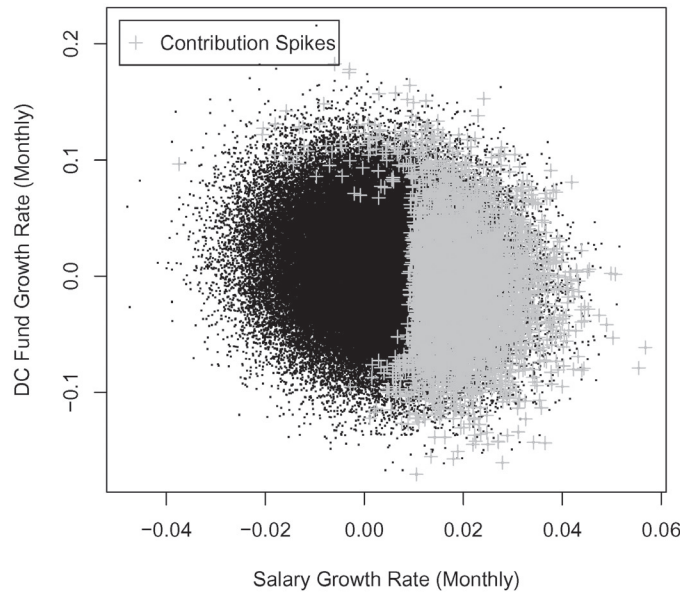


Figure 9
Scatterplot of Log-Growth Rates for Salaries and DC Funds, TUC Approach, Entry Age 35



6.2 Averaging the Final Salary

In Figure 9 we plot the log growth rates for salaries and the DC fund. Crosses have been superimposed where the contribution rate spikes above 50%. We see that the spikes in contributions are associated with increases of more than around 1% per month in salaries.

These salary spikes are translated into contribution spikes because, for ease of presentation and tractability, we have assumed the final benefit is based on the final salary. In practice, however, the DB benefit would be based on the average salary in some period up to retirement (perhaps one year, three years, or five years). The effect of this is to reduce the volatility of the final DB benefit and therefore

Figure 10
Comparison of 95% and 5% Contribution Rate Quantiles Using TUC Approach, No Averaging, One-Year and Five-Year Averaging of Final Salary for DB Benefit Calculation, Entry Age 35

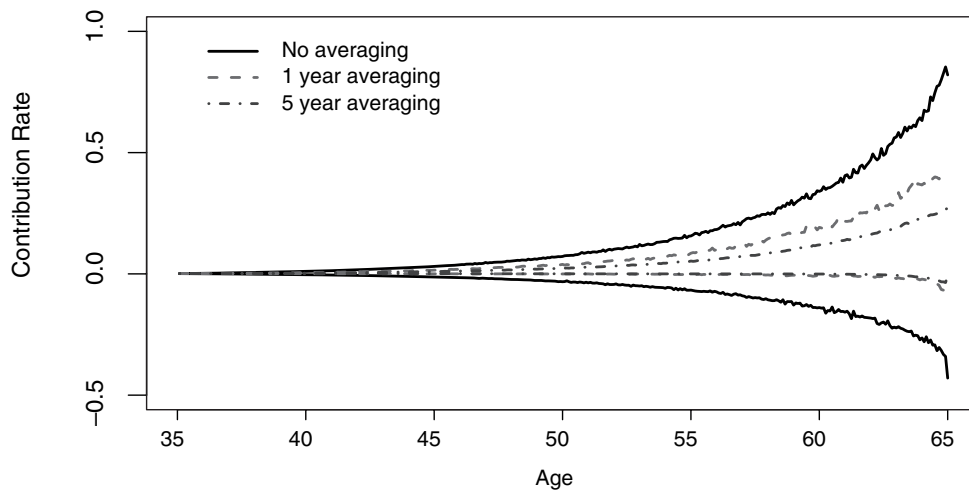
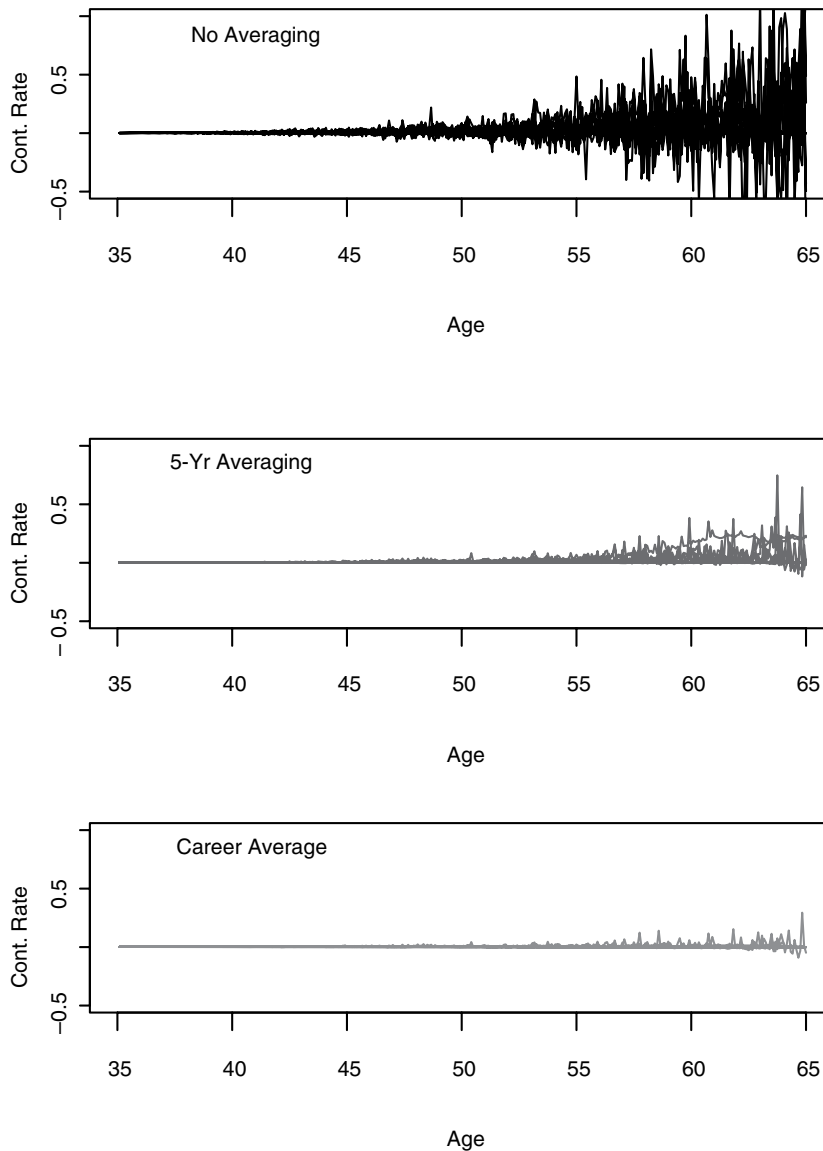


Figure 11
**Comparison of Projected Contribution Rates, 20 Sample Paths, No Averaging,
Five-Year Averaging and Career Averaging, Entry Age 35**

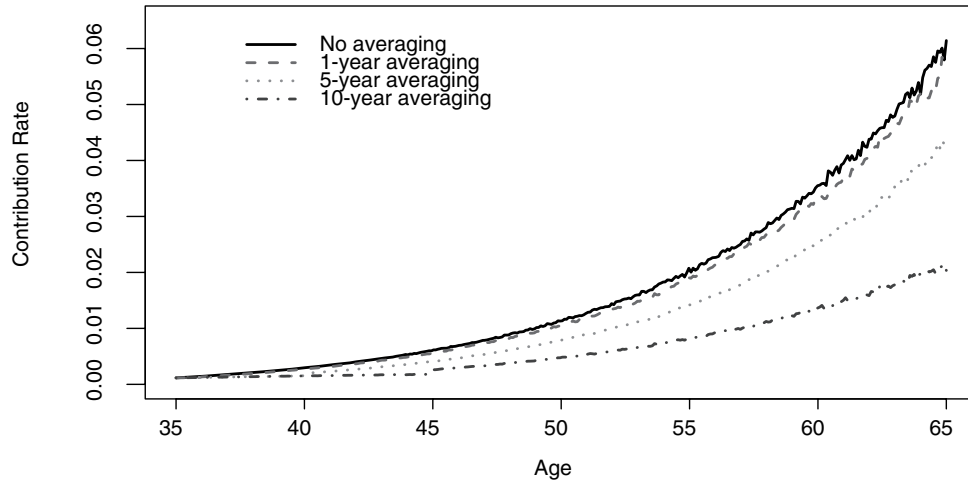


the volatility in the contribution rates. In Figure 10 we show the effect on the TUC contribution rate quantiles of defining the DB benefit as applying to the average salary over the final one year and the final five years before retirement.

The effect is a dramatic reduction in the variability of the monthly contribution. The lower fifth percentile for the one-year and five-year average are indistinguishable in the figure; averaging over a longer period causes the upper quantile to converge to that lower quantile. In the limit, for the career average salary benefit, which applies the accrual rate to the average salary over the employee's whole career, the 5th and 95th percentiles are indistinguishable on this scale (assuming no indexation of salary). To show the effect of averaging, we have plotted 20 sample projections, using the same underlying economic scenarios, using no averaging, five-year averaging, and the career average. The results are seen in Figure 11, which shows the dramatically reduced volatility using the career average, but

Figure 12

Comparison of Projected Mean Contribution Rates, No Averaging, Five-Year Averaging and 10-Year Averaging, Entry Age 35, 1.5% Accrual Rate, TUC Funding, 10,000 Simulations



the effect does not only arise from the smoothing effect of the averaging. The effect of the positive trend in the salary process is that the career average salary is substantially lower than the final average, or the final salary, so the guaranteed benefit under the career average plan is also substantially lower than under the final average plan. Increasing the accrual rate for the longer averaging period would increase the benefit but also the volatility. In the design stage, the plan sponsor and actuary should explore the trade-offs between averaging and accrual rates to find a balance between an appropriate level of underpin benefit with an acceptable volatility in the contribution rates. In Figure 12 we show the expected contribution rates for three different averaging periods (one year, five years, and 10 years), together with the rates for the final salary benefit with no averaging. The effect on the price of using one-year averaging is not very great, but the final contribution rate is reduced by nearly 200 basis points when five-year averaging is used, and by around 400 basis points when 10-year averaging is used.

6.3 Other Risk Management Strategies

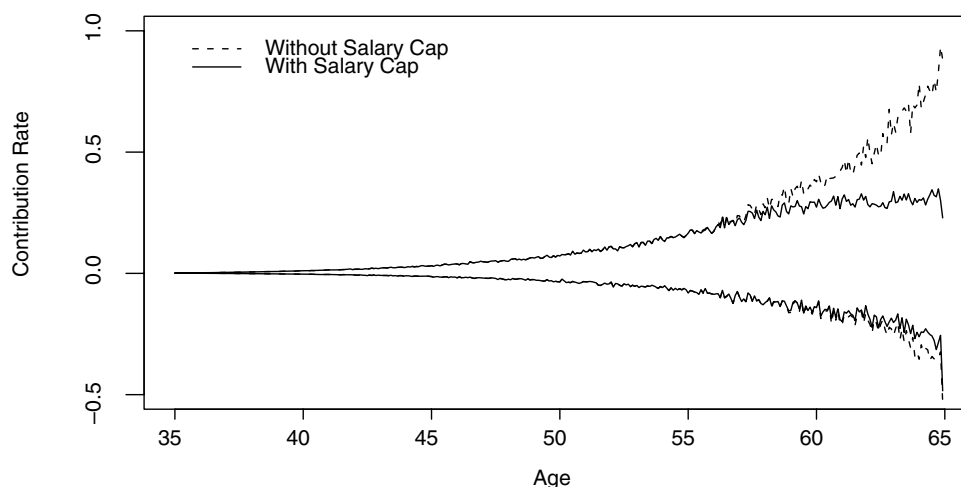
Part of the apparent volatility in the contribution rates arises from the monthly salary assumption. By smoothing out the salary increases, the volatility can be significantly reduced. For example, if we simply do not increase salaries whenever doing so would cause a contribution spike, the 95% quantile is significantly contained. This is demonstrated in Figure 13, where we compare the 95% and 5% quantile contribution rates before and after applying a 0% salary cap when a salary increase would create a large contribution rate increase.

An additional risk management strategy taken by the McGill pension plan uses age-related DC contributions. The contributions to the DC account are increased to 15% per year in the final 10 years of service. This reduces the cost a little, as the guarantee is less likely to end in-the-money, but actually does not have a very significant effect on the upper 95% quantile of contribution rates. McGill also uses five-year averaging in the DB benefits (with some indexation if inflation exceeds 3.75% per year).

It should also be noted that for any plan it is important to manage the total remuneration, including the pension accrual cost, rather than managing salaries and allowing the pension cost to move accordingly. Managing total remuneration requires an iterative calculation for this option, where the relationship between salary increases and pension accrual is not straightforward, but doing so is a significant contribution to risk management. This is a topic worthy of further research.

Figure 13

Comparison of 95% and 5% Contribution Rate Quantiles Using TUC Approach, without and with Salary Cap for Contribution Rate Smoothing, Entry Age 35



7. CONCLUSION

In this paper we have described the DB underpin hybrid pension plan benefit and indicated how financial engineering can be combined with pension funding principles to fund the guaranteed benefit.

The hybrid pension plan design is particularly attractive in a period of economic uncertainty, when neither the conventional DB nor the DC plan addresses the significant concerns of both plan sponsors and plan members. Adoption of the hybrid plan may require some adaptation of funding and valuation regulations in jurisdictions where these are framed in such a way as to allow only a very traditionalist interpretation of valuation, which may not permit a market approach to the option.

The TUC funding principle appears to offer the best approach to fair valuation and also generates a natural funding pattern that has attractive and intuitive features. The mean price of the benefit using the TUC approach is consistent with the contributions currently in force. However, the contribution variability is significant, although it can be constrained with some appropriate risk management strategies.

The approaches to smoothing the guarantee cost considered in Section 6, such as averaging the final salary for the underpin benefit, increasing the DC contribution rates near retirement, or applying a salary cap, illustrate that we have two mechanisms for reducing the variability. For ongoing risk management we should control salaries, taking into consideration the total cost of salaries and benefits. We might also adjust the contribution rate less often, so that large positive and negative movements might cancel each other out to some extent. More sophisticated hedge strategies might be adopted. But also, importantly, risk management starts at the plan design stage. By careful selection of the salary averaging period and the accrual rate in the plan design, we can significantly reduce risk.

The presentation in this paper has been considerably simplified, and further work is indicated, particularly in the modeling assumptions. It would be worthwhile to explore different models for the joint distribution of salary growth and contribution accumulations; in addition, in future work we will explore the impact of allowing interest rates and annuity prices to vary stochastically. An important topic that we have not significantly addressed is that the use of the option-pricing approach implicitly assumes that the option premium would be used to hedge the option. In the EAN and PUC cases, where we have assumed that salaries are a traded asset, this is clearly not possible. A partial salary hedge can be achieved using inflation-indexed bonds. This is described in more detail in Chen (2007). For the TUC method, however, we do not need to assume that salary is a traded asset, but even here, the simplifying

assumption about interest rates and annuity prices would make the hedge less accurate in practice than we have assumed it to be in this paper. In addition, the geometric Brownian motion assumption for the DC fund growth is likely to be too thin-tailed compared with experience, and a fatter tailed distribution, such as a regime-switching model (Hardy 2003), might be more appropriate, particularly for such long projections.

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