



SOCIETY OF ACTUARIES

Article from:

Product Development News

March 1998 – Issue No. 45

Substandard Lives: Cost of Insurance Charges for Unbundled Products—Mathematics You Can Use

by Johan L. Lotter

How good is your market conduct when you deal with substandard lives? If you are determining substandard extra cost of insurance charges for universal life or variable universal life policies on the numerical rating system, you may be using the following formula to determine your current and guaranteed charges:

$$COI_x' = (1+k)COI_x - e \tag{A}$$

where:

- COI_x' is the substandard monthly cost of insurance rate per mille
- COI_x is the standard monthly cost of insurance rate per mille
- e is an expense loading adjustment (often implemented as zero)
- $100k$ is the extra mortality percent on the numerical rating system.

Your reasoning may be that the above is consistent with the numerical rating system as interpreted by the following equation:

$$q_x' = (1+k)q_x \tag{B}$$

where:

- q_x' is the substandard mortality rate
 - q_x is the "standard" mortality rate.
- You may feel uncomfortable about the practice as described above because:
- (a) You may have questions about the application of the numerical rating to some vague "standard" mortality rate, since "standard" would have different meanings for different companies, yet the underwriting manuals used by companies are often produced by reinsurance companies and are not company "standard specific."
 - (b) You may also be aware that Equation B above breaks down for large values of k and high ages, yielding a paradoxical result when q_x' , which is a probability, exceeds unity.

If you are aware of the paradox mentioned in (b) above, you may have adopted a practical approach in which you have set an arbitrary condition such as the following:

$$COI_x' \leq \frac{1000}{12} \tag{C}$$

Implementation of Equation C sidesteps the untenable consequence that the risk charge exceeds the sum at risk at high ages. But it remains unscientific, theoretically unsatisfactory, and unfair from the policyholder's point of view.

First, at some time before the maturity date, Equation C sets, on an annual basis, the risk charges close to the annual

sums at risk, leading to visibly excessive cost of insurance charges.

Second, Equation B may imply that a life, having survived to age x , has zero probability of attaining age $x+1$, a result which many would regard as not scientifically defensible.

Third, Equation C almost forces the policyholder to surrender before the maturity date. This could lead to adverse tax and other consequences for the policyholder and to eventual dissatisfaction.

Actuaries cannot afford to regard the potential problems caused by the above "popular" approach as only becoming "real" at some point in the distant future. The mere fact that the policy was issued with treatment implied by Equation B, could lead to current market conduct questions.

The problems posed by implementation of Equations A, B, and C above, referred to hereafter as the "popular" approach, are readily eliminated by a more satisfactory theoretically "correct" approach. In what follows, we demonstrate how:

- The "correct" approach leads to a consistent, scientifically viable and useful treatment of substandard extra mortality at all ages
- The "popular" approach can be reconciled with the "correct" approach if it is acknowledged that the "popular" approach is a "first-order" approximation to the "correct" approach.

It is helpful to recognize that:

- (a) When the actuary is concerned with the equitable treatment of impaired lives, "own-company" relative mortality is at issue, absolute mortality is not.
- (b) The numerical rating system was devised to express relative mortality. It furnishes no information about absolute mortality.
- (c) For any portfolio of insured lives, relative mortality can be measured without knowing anything about the absolute mortality of the lives being studied.
- (d) A sensible way of measuring relative mortality would be simply to compare relative survival ratios of (1) those lives considered by the insurance company as substandard risks, and (2) those lives considered by the company as acceptable at standard rates, appropriately striated.
- (e) A straightforward method, involving the least number of assumptions, would be to avoid making assumptions about expected deaths and to "count" survivors among lives classified as standard risks at issue and (striations of) lives not so classified.

Such "count" would enable the actuary to directly measure relationships such as Equation D below, where the

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substandard one-year survival rate is expressed as the standard rate, raised to an exponent, $(1 + m)$; that is:

$$p_x' = p_x^{(1+m)} \tag{D}$$

where:

p_x is the one-year survival rate for unimpaired lives

p_x' is the one-year survival rate for impaired lives.

We observe that m is a useful measure of the relativity of one-year survival rates and that, if $m = 0$, the one-year survival rate for the impaired class is equal to the one-year survival rate of the standard class. If m is greater than zero, it has the effect of reducing the one-year survival rate.

Equation D immediately leads to Equation E below, which can be written in the form of Equation F:

$$(1 - q_x') = (1 - q_x)^{(1+m)} \tag{E}$$

$$q_x' = 1 - (1 - q_x)^{(1+m)} \tag{F}$$

Equation F enables ready calculation of substandard mortality rates for any age and any m .

It is instructive to consider Equation F after binomial expansion as in Equation G:

$$q_x' = (1+m) q_x - \frac{(1+m)m}{2!} q_x^2 + \frac{(1+m)m(m-1)}{3!} q_x^3 - \dots \tag{G}$$

If we ignore powers of q_x greater than unity and substitute k for m in Equation G, it reduces to Equation B (the "popular" approach). For large m and q_x , however, the second term on the right hand side of Equation G is significant and, when ignored, leads to the problems and anomalies inherent in the "popular" approach.

Once one appreciates that Equation B leads to a logical "dead end" and that assignment of a 100k percent numerical extra rating really means replacing m with k in Equations D, E, F, or G, the numerical rating is clarified with respect to its meaning and application, and one can immediately see that the "popular" approach is a first-order approximation to the "correct" approach.

The "correct" approach can be implemented as set out below:

$$\frac{COI_x^1}{1000} = 1 - \left(1 - \frac{COI_x}{1000} \right)^{(1+k)} - a(x, k)$$

where $a(x, k)$ is an adjustment "extracting" excess expense loadings (if any) in the cost of insurance rates.

While the "correct" approach is scientifically and logically defensible, the "popular" approach is not. In traditional products, the premiums calculated on the "correct" approach do not differ very much from those on the "popular" approach. In unbundled products, the deficiencies of the "popular" approach are completely and embarrassingly visible. The "popular" approach can lead to policyholder dissatisfaction when the cost of insurance deductions approach the magnitude of the sums at risk. The correct approach avoids potential market conduct difficulties.

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Pitfalls in Equity-Indexed Products

by Jay Glacy

Note: This article first appeared in the November 1997 issue of *Small Talk, the Smaller Insurance Company Section Newsletter*.



Equity-indexed products burst upon the scene in 1996 and interest in them remains high, rivaling the waves of second-to-die product development in the late 1980s and universal life product development in the early 1980s. The future of indexed products probably holds more marketplace entrants, innovative second- and third-generation designs, some unexpected regula-

tory wrinkles and, in general, more controversy. The complexities associated with equity-indexed life and annuity products already create a number of general misconceptions about them. This article identifies some key pitfalls in developing equity-indexed products and suggests some steps insurers can take to avoid unpleasant financial surprises.

Macro Product Management

A common way to think about pricing single-premium deferred-indexed annuities contemplates the purchase of a zero-coupon bond to fund nonforfeiture law minimums in conjunction with the

appropriate S&P 500 Index hedging instrument. In this simplified framework, the present value of profit is what is left over. But some important things are overlooked in this formulation. First, the question of how the insurer intends to fund the hedge purchase for those policyholders persisting beyond the first

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