



SOCIETY OF ACTUARIES

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## Unruh

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is on a particular track, its future values should be based on that track. When it changes tracks, future values should change with the new track. The expense allowance calculated at issue should be based on the automatic track. Future changes producing an increase in net premium should allow a related increase in expense allowance. However, no decrease in expense allowance should be required if a change results in a decrease in net premium.

(2) For open policies that provide contractually for the possibility of changes in benefits, premiums, or premium-paying period, but do not specify when the change is to occur or what it is going to be, a traditional prospective adjusted premium approach is suggested. This approach should assume changes not stated numerically in the policy would not occur. When a change in premiums or benefits does occur, new adjusted premiums should be calculated such that their present value equals the present value of future benefits less the current minimum cash value plus any new expense allowance resulting from the change, again assuming no further changes beyond the point of recalculation. This procedure would be used for minimum values for most life-cycle and cost-of-living policies.

The Committee looked beyond the special plans that have been discussed and recommended that for unique policies not yet developed, a central technical entity (such as the NAIC Central Office) could be empowered to review policies to determine if they are actuarially sound and meet certain standards of equity and to approve other technical aspects. This approach would facilitate approval of complex policies and would promote feasibility of product design while decreasing risk of abuse.

This reviewer is somewhat disappointed that the Committee did not recommend also allowing the use of an assumption of a percentage increase in the index in the case of cost-of-living increases so that minimum values could be determined at issue. The proposed minimums are complex and could be very difficult for the consumer to understand. However, the proposed minimums would accommodate a modification of the in-

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## THE INTERNATIONAL ACTUARIAL NOTATION

by Frank P. Di Paolo

In the December 1972 issue of *The Actuary*, John Boermeester reviewed succinctly the origin and history of the International Actuarial Notation, also known as the Halo Notation. According to John Boermeester, the Halo Notation was invented by David Jones who first used it in *The Value of Annuities and Reversionary Payments* published in Great Britain in 1844. With minor modifications, this notation was officially adopted by the Institute of Actuaries in 1872 when it appeared in an Appendix to the *Institute of Actuaries Life Tables*. In 1898 the Halo Notation was adopted by the International Congress of Actuaries and, except for a number of minor changes which were made in 1954 during the XIV International Congress of Madrid, it is still in use today.

The Halo Notation is basically an ideograph. The symbol representing the function is in the centre of the ideograph, while the symbols representing the parameters are shown, in accordance with well defined rules, as suffixed or prefixed subscripts and superscripts. For example, in the case of  $n|a_x^{(m)}$ ,  $a$  is the function symbol and  $x$ ,  $m$  and  $n$  are the parameters.

The Halo Notation is very concise and unambiguous and therefore it can be easily read and quickly understood. A more diffused notation would be somewhat time-consuming to read and may not be as quickly understood. Indeed, this is perhaps the main reason why the Halo Notation has gallantly survived over 130 years. However, the Halo Notation cannot be easily typed with common typing devices nor can it be easily incorporated into computer programmes.

In an attempt to remedy the shortcomings of the Halo Notation with respect to programmability, a number of actuaries have suggested linearizing the actuarial notation. In fact, since 1968, at least nine papers have been published on the subject. These papers are:

1. *Proposals for an International Actuarial Publication Language and its Representation in Computer Compatible Form* by Carl Boehm (Germany), Georg Reichel (Germany) et al. published in the *Transactions of the XVIII International Congress of Actuaries*, Munich 1968.

2. *A New Actuarial Notation* by the General Functions Sub-Committee of the Institute of Actuaries of Australia and New Zealand, published in October 1971.

3. *ACT, An Actuarial Programming Language* by David R. W. Jamieson (Canada), published in the January 1972 issue of *The Actuary*. It must be noted that *ACT* is not meant to be a publication language but only a programming language.

4. *Some Proposals for a Revision of the International Actuarial Notation* by P. J. Turvey (Great Britain) published in the *Journal of the Institute of Actuaries Students' Society* March 1972.

5. *Proposals for an International Actuarial Publication Language* by Carl Boehm (Germany), J. Adams (Belgium) et al. published in the *Transactions of the XIX International Congress of Actuaries*, Oslo 1972.

6. *A Contribution to the Discussion on a New International Actuarial Notation* by J. Engelfriet (Netherlands) and A.I.M. Kool (Netherlands) published in the October 1973 issue of the *BLATTER*.

7. *An Actuarial Notation Based on Symbolic Logic* by G. C. Taylor (Australia) Research Paper No. 50 published by the Macquarie University in April 1974.

8. *Thoughts on the Harmonization of Some Proposals for a New International Actuarial Notation* by C. Boehm (Germany), J. Engelfriet (Netherlands) et al. published by the authors in 1975.

9. *A Revised Actuarial Notation*. A suggestion by the Notation Sub-Committee of the Institute of Actuaries of Australia and New Zealand (Edition 10, October 1975).

The table below shows four of the most commonly used actuarial functions in Halo Notation and in linearized notation as suggested in the papers listed above.

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			(m)	
Halo Notation	$a_{\overline{x}}$	$A_x$	$a_{x:\overline{n}}$	$A_{x:\overline{n}}$
Boehm/Reichel	$a_{(x)}$	$A_{(x)}$	$a(x;n;m)$	$AE(x;n)$
Australia/New Zealand (1971)	$a_{(x)}$	$A_{(x)}$	$bf(x,n)$	$B(x,n)$
Jamieson	ANX X	AX X	ANXNM X N M	AXN X N
Turvey	$ap_{(x)}$	$a_{(x)}$	$apnm(x,n,m)$	$an(x,n)$
Boehm/Adams	$a_{(x)}$	$A_{(x)}$	$a(x,n,k)$	$AE(x,n)$
Engelfriet/Kool	$a_{(x)}$	$A_{(x)}$	$'a_{\overline{x}}(x,n,k)$	$AE(x,n)$
Taylor	$a_{(x)}$	$A_{(x)}$	*	$A(x,Ln)$
Synthesis				
Boehm/Engelfrite	$a_{(x)}$	$A_{(x)}$	$(k)a(x:n)$	$A(x,n)$
Australia/New Zealand (1975)	$a_{(x)}$	$A_{(x)}$	$a(x, \#n, f=m)$	$A(x, \#n)$

\* Taylor's paper does not give sufficient information to complete this notation.

In 1972, the President of the International Actuarial Association appointed the Sub-Committee on Notation with the responsibility of gathering opinions on the desirability of developing a new actuarial notation. The two basic questions that the Sub-Committee must answer are:

- (a) Is it desirable to linearize the International Actuarial Notation, and, if so, is there sufficient interest amongst the various national actuarial bodies to replace the Halo Notation with a linearized one?
- (b) Whether or not linearization is desirable and/or acceptable, should the scope of the actuarial notation be extended to cover fields such as pensions, disability, demography, social insurance and non-life insurance?

It would appear to be most desirable to extend the scope of the actuarial notation to other fields. The linearization issue, however, seems to be quite controversial. There are three distinct points of view:

1. The traditionalists feel that the Halo Notation has served the profession well since it first was officially adopted by the Institute of Actuaries over 100 years ago. Furthermore, the notation is concise, it can be easily read and understood and why change it simply to accommodate today's computers? Indeed, computers of five or ten years from now may be able to make use of peripheral equipment which will enable them to read and print the Halo Notation easily.

2. The reformists point out that common printing devices cannot cope with the Halo Notation. This shortcoming increases substantially the cost of printing actuarial textbooks, papers, etc. Furthermore, today's computers cannot easily digest the Halo Notation.

3. The pragmatists maintain that actuarial writers have faithfully adhered to the International Actuarial Notation since 1898 and the large amount of actuarial literature available today is full of halos. Should the actuarial notation be suddenly linearized, it would be somewhat chaotic to cross-reference new actuarial papers and textbooks with old ones. The pragmatists agree that there is a programming problem, but the problem is only of a temporary nature. Then, why not leave the Halo Notation as is and perhaps expand its scope and develop a notation vocabulary where each standard notation would be translated into various programming notations, each compatible with a specified programming language? This last view is the one favoured by most members of the Committee on Standard Notation and Nomenclature.

The I.A.A. Sub-Committee on Notation has recently circulated a questionnaire to sound out public opinion on the issues of linearization and expansion of the International Actuarial Notation. The Committee has replied to the questionnaire saying *NO* to linearization and *YES* to expansion. The reply was made as an expression of opinion of the majority of the Committee and was not made on behalf

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dex accumulation method for a cost-of-living policy where the death benefit and cash values escalate with the cost-of-living but premiums remain level. (See *Life Insurance Based on the Consumer Price Index*, by Bragg and Stonecipher, *TSA XXII*, p. 351). Presumably the minimum cash value defined by (2) above could not be greater than the comparable cash value for a level face amount policy at the same duration, and for the attained death benefit at that duration. It would be preferable and provide a greater flexibility if this type of policy could be allowed to be approved by the suggested central entity set up by the NAIC or state regulators.

The Committee is aware of the criticism of the current non-forfeiture law, that it allows certain products in today's marketplace which meet the letter but perhaps not the spirit of the law and the Report suggests ways to minimize these abuses.

The Committee re-examined the expense allowances in the present law because these were developed more than thirty years ago. More modern estimates of excess of first year expenses over renewal expenses would likely decrease as a percentage of face amount and increase as a percentage of premium. The Committee concludes that appropriate modern expense factors would be bracketed by the following for a whole life plan:

Low: \$10 per \$1,000 plus 90% of 1958 CSO 3½% net premiums

High: \$20 per \$1,000 plus 150% of 1958 CSO 3½% net premiums

(Note that the Committee recommends the use of net premiums rather than adjusted premiums for the percentage of premium allowance to simplify calculations).

The Committee concluded that for other plans, the percentage of premium allowances should equally weight a whole life premium and the plan premium (subject to a maximum of \$55 per \$1,000). The Committee tested an expense allowance of \$10 per \$1,000 plus 100% of the weighted premium on various bases. When combined with 4½% interest and a more modern mortality table, this relatively low expense allowance provides generally lower minimum

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cash values than those provided by the current law at 3½% interest. It would hardly seem appropriate to make a change in the expense allowance without at least increasing the interest rate so as to more closely approximate the rate used in company asset shares. The Committee makes no specific recommendation for a more modern expense allowance, mortality table, or interest rate; however, these were studied and discussed in detail. It is recommended that a single set of minimum values be defined based on the highest permissible interest rate and that the present linkage between the policy valuation and non-forfeiture interest rates be eliminated. It is recommended that guaranteed paid-up insurance non-forfeiture options should be those purchased by the cash value at a rate as high as that used for cash values. It is also recommended that single premium policy minimum cash

values allow an interest rate higher than annual premium policies.

The Committee analyzed numerous other aspects. Various recommendations are made which are designed to simplify the law, to make the law more flexible and consistent, and to eliminate certain technical flaws. These include: (1) expanding the law to include deferred annuities during the buildup period on an accumulation of percentage of premium basis, (2) exemption of accident and health insurance for non-forfeiture purposes (except possibly for plans with return of premium benefits), (3) treatment of term riders as separate policies under a "severability" principle, (4) treatment of renewable and convertible term policies as a series of short term policies, (5) exemption of policies that never give rise to significant values by a direct test for triviality, (6) expansion of exemptions for term plans, (7) an alternate treatment for substandard policies so that the death benefit could be

decreased depending on the extra mortality but where minimum cash values would be identical to a standard policy, and (8) removal of anomalies that occur due to the uniform percentage of gross premium requirement.

We are indebted to Mr. Unruh and the members of the Committee for this thorough, comprehensive, and practical report and we hope that its final recommendations will induce appropriate revisions in the Standard Non-forfeiture Law. Especially needed is additional flexibility in developing and marketing new products which provide protection against erosion of insurance coverage due to inflation.

Meantime this reviewer would again urge all members of the Society to read the report and to send their comments and suggestions to:

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**Income Tax and Reserve Valuation**

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Even though the deduction for the "tabular interest" declines as the earnings rate rises above Fraser's "optimal" point of 6½%, the after-tax yield continues to rise until the earnings rate reaches 12%. The marginal tax rate exceeds 50% after the deduction reaches its maximum, but does not exceed 100% until the after-tax yield passes its "optimal" level of 12%. With the "10 for 1" approximation rule, the optimal level for the after-tax yield occurs when the current earnings rate equals 10½% plus one-half of the original assumed reserve interest rate:

Let CER be the current earnings rate  
I<sub>r</sub> be the original assumed reserve interest rate

$$\text{After-tax gain} = 1.1 \text{ CER} - [1.1 \text{ CER} - \text{CER} (1 + 10 I_r - 10 \text{ CER})] \frac{1}{2}$$

$$= (1.05 + 5 I_r) \text{ CER} - 5 (\text{CER})^2$$

Taking the partial derivative of the after-tax gain with respect to CER and set to zero,

$$\text{CER} = 10\frac{1}{2}\% + \frac{1}{2}I_r$$

It would seem that the peaking 'deduction' phenomenon described by Mr. Fraser is only incidental because the marginal tax rate increases at a constant rate. A declining after-tax income occurs for a company only at a much higher earnings rate (above 12% in this illustration).

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of the Society. Furthermore, the Committee has made it clear that if during the next few months it will become evident that the prevailing opinion amongst the Society's membership is contrary to that already expressed by the Committee, then the I.A.A. Sub-Committee will be so notified.

Thus, the Committee on Standard Notation and Nomenclature would welcome the opinion of any member of the Society on the question of linearization and expansion of the International Actuarial Notation. If you wish to express your opinion, you can do so by writing to me at Confederation Life, 321 Bloor Street East, Toronto, Canada, M4W 1H1.

**COMPETITION No. 6**

To our dismay we realized that actuaries as a group have *no plans to honor the U.S. Bicentennial*. Our Canadian members have an excuse, of course, but we are shamefaced about the lack of patriotic spirit among our U.S. members. We invite suggestions for something the Society or its members might do to make an actuarial contribution to the Bicentennial celebration.

Examples:

- A mortality table ending at age 200.
- Full retirement benefits after 13 years of service and attainment of age 50.
- Declaration of Actuarial Independence.

Perhaps the Society's motto or emblem could be given a Bicentennial theme. For each of the three submissions we like best we will award a delightful book, "American Journeys", an anthology of travel in the United States (from John Alden to Neil Armstrong).

The usual Rules will obtain and the final mailing date is May 17.

C.E.