The Actuary

The Newsletter of the Society of Actuaries

VOL. 20, No. 6 June, 1986

FINANCIAL REPORTING SECTION

By William Schreiner

The Life Insurance Company Financial Reporting Section will celebrate its fourth birthday in October with over 1,500 members. The interests of the Section members are wide-ranging and include statutory accounting, the valuation actuary, U.S. GAAP accounting, income tax accounting, Canadian accounting, management reporting and so forth. The majority of the membership is employed by insurance companies (77%), with a significant minority (19%) in the consulting field; Canadian and non-North American members make up 16% of the Section.

Because of the diversity of Section membership and interests, nominations to the Section Council — the Section's nine-member governing board — have followed an objective of providing representation of various membership segments. Thus, the current Council is composed of three individuals who are employed by stock companies, three by mutual companies, one by an accounting firm, one by a consulting actuarial firm, and one by an industry association; two of the members are Canadians.

During the first three years, the Section's major efforts have been directed towards providing educational and informational programs for the benefit of the members. An active and effective program committee has contributed significant tracks relating to financial reporting to the spring and annual Society meetings programs. For example, this fall's Chicago meeting will contain 18 programs developed by the Life Insurance Company Financial Reporting Section. Included in the lineup are: capital management, mergers and acquisitions, New York's new annuity

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CONTINUING EDUCATION CONTINUES TO GROW WITH SOCIETY

By John E. O'Connor, Jr.
Executive Director

The growth and evolution of the actuarial profession in recent years have created an increasing interest in continuing education and professional development.

In response to this interest, the Society of Actuaries added a Continuing Education Department to its office structure in January 1985. Barbara Choyke, a continuing education professional, was hired to serve as the Society's Manager of Continuing Education. In her capacity, Barbara serves as a liaison to the Committee on Continuing Education, the Services to Members Policy Committee, the Program Committee and various Sections by developing and recommending continuing education programs and policies. Her additional responsibilities include: developing seminar topics and presentation styles, recruiting faculty and conducting research in continuing education.

Under Barbara is Ann Polodna, the Seminar Coordinator, and Joyce Paschall, the Seminar Assistant. Ann oversees the administrative work of seminars by coordinating hotel arrangements, brochure preparation, handout materials and marketing strategies. Joyce assists both Barbara and Ann in the administration of the continuing education program and serves as the registrar for all seminars.

Since 1980, the Seminar Program has grown from an offering of 10 topics in 21 locations to last year's total of 17 topics in 41 locations. Attendance also has increased by over 40% in the past five years.

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EXPOSURE DRAFT OF LIFE INSURANCE COMPANY VALUATION PRINCIPLES

by Gary Corbett

The Committee on Life Insurance Company Valuation Principles (COLICVP) has recently mailed to Society members an Exposure Draft of Valuation Principles for Life Insurance Companies. Comments on the draft principles are to be submitted by July 1, 1986 to Robert D. Shapiro, Merrill Lynch Capital Markets, One Plaza East, 330 Kilbourn Avenue, Milwaukee, WI 53202.

In this article I want to:

1) Encourage members to comment on the draft principles.

2) Encourage discussion of the Society's role in identifying, developing and publishing actuarial principles.

COLICVP was charged by the Board of Governors to develop principles applicable to the valuation of life insurance companies for the measurement of solvency and solidity. Developing the principles themselves was a difficult enough task but, as the first Society committee to be charged with the development of actuarial principles, the Committee was forced to wrestle with such basic questions as: What are principles? Can principles be distinguished from standards and practices?

In early 1985 a Task Force on Actuarial Principles was charged by the Board with recommending the Society's role in establishing actuarial principles and how this role was to be performed. This Task Force submitted a report to the Board in August 1985 recommending that the Society "pursue explicit expression of actuarial principles through a promulgation process". The Society would thus add promulgation of actuarial principles as a third activity to its

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The Actuary

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ACTUARIAL RESEARCH

The Society's traditional emphasis on Education and Examination, and its more recent emphasis on Continuing Education, may obscure the fact that the development of new knowledge, not only the passing on of what has already been learned, is the life blood of any profession. Actuaries have not been as interested, or as successful, when it comes to research, though there has been more attention of late.

The Society (and its predecessors) have long published the research efforts of individual actuaries in a refereed journal; and have collected, compiled, and published mortality and morbidity experience. Until 20 years ago, the *Transactions*, the *Reports Numbers*, and the journals of the Casualty Actuarial Society and the Conference of Actuaries in Public Practice, represented most of the profession's research output.

Early movement in the research direction came in the '60s with the organization of annual Actuarial Research Conferences at campus locations, followed by the publication of *ARCH*, both under the sponsorship of what was then known as the Research Committee, and is now called the Committee on Research on Theory and Applications. The 21st such Conference will be held this fall. Worthy as this effort has been, it has remained the domain of a small group of academically oriented actuaries. *ARCH* is probably too esoteric for the main body of practicing actuaries, with whom *ARCH* has never really caught on.

The notable research development of the '70s was the formation of the Actuarial Education and Research Fund, a profession-wide attempt to encourage the development of new actuarial knowledge. Although AERF clearly has potential, its progress has been slow. Some may be discouraged as to its actual accomplishments during the decade since it began. We are told that the Directors of AERF are searching for a better definition of its role.

In the early '80s the Society began to organize for a serious research effort, directed particularly toward the improvement of its experience studies. In the formal structure we now find (see pages 20-23 of the *Yearbook*) a Vice-President for Research and Studies, a Research Policy Committee, and no less than 15 other Committees or Sub-Committees under the Research and Studies label. The Society also has a full-time Research Director in its Itasca office. If sheer organization can get the research accomplished, the prognosis would seem to be good.

There are other encouraging signs. Some of the Society Sections are taking an interest, and there is a suggestion for a journal less formal than the *Transactions*.

For all that, the recent product of actuarial research is nothing to brag about. *Transactions* papers today scarcely exceed their number when the profession was much smaller. Few actuarial books are being published, and most of these are more education than research. Meanwhile expansion of non-actuarial fields seems to narrow the actuarial role. There is some suspicion that in developing new knowledge the profession has not kept pace.

Perhaps the greatest need is the identification of areas in which actuarial research is most urgently needed. Once an area is well identified, it should not prove too difficult to harness the problem-solving capabilities of actuaries, and to get important research done.

C.L.T.

Mail Alert

The Second Ballots for the Society's 1986 elections will be mailed to Fellows on July 29. To be valid, ballots must be returned to the Society office by Aug. 29.

ACTEX STUDY MANUALS

ACTEX Study Manuals will be available for all November exams except parts 9 and EA-2. New editions for parts 1, 7, and 8. Particulars in your study note package, or from The Actuarial Book Store, P.O. Box 318, Abington, CT 06230, (203) 974-3540. □

SOLUTION MANUALS— PROBLEM WORKSHOP

Solution Manuals by Dr. Ralph Garfield are now available: Part 2 (Nov. '81 and May '82 exams) $18. Part 3 (Nov. '84 exam) $14; (Nov. '85 exam) $14 (May '86 exam - available August) $16 Part 4 (Nov. '84 exam and SOA practice exam) $32; (Nov. '85 exam) $16; (May '86 exam - available August) $16. EA-2 (Nov. '84 and '85 exams) $30.

Problem Workshop — An intensive 2½ day workshop for the EA-2 exam will be offered in New York City on Oct. 24, 25, 26.

For details, write to A.S.M. P.O. Box 522, Merrick, NY 11566. □

The Actuarial Profession: 100 Years Old in 1989

In celebration of the actuarial profession's 100th birthday, there will be a gala centennial celebration in Washington, D.C., on June 12-14, 1989. A Steering Committee, representing the North American actuarial organizations, is already hard at work to make this a truly outstanding event. Mark your calendar now, even though this celebration is still three years into the future.

Late in 1986 actuaries will be invited to prepare papers on topics selected by the Program Committee as relevant to the theme and content of the centennial program.

Financial Reporting Section

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valuation regulation, sources of profit, and GAAP for universal life.

One of the innovations that will be introduced at the Chicago meeting is a Section breakfast that will be set up in discussion groups to facilitate the exchange of ideas in an informal setting. The breakfast will be immediately followed by the Section's annual meeting and a panel on "Current Topics in Financial Reporting".

The Section's largest undertaking to date was the 1985 St. Louis meeting devoted to financial reporting issues. Over 600 Society members attended the two-day meeting at which 48 panels and workshops were available. The meeting was very well-received and the Section Council is currently working to gain approval to organize a similar meeting in 1987, or 1988 at the latest.

The Section has also actively assisted in the development of the popular valuation actuary seminars and is planning additional efforts in the area of continuing education.

The Financial Reporter, the newsletter of the Section, was started in 1984 and has provided members with news of developing issues in the U.S. and Canada, including the valuation actuary and GAAP accounting for universal life and annuities, as well as news of Section activities and coming events. The progress of the issue of accounting for universal life insurance policies through the American Institute of Certified Public Accountants and the Financial Accounting Standards Board has been followed in detail and the newsletter will remain a key source of current information on the subject until it is resolved by the Board, presumably some time in 1987.

Also, the last issue featured a debate on the merits of the Joint Committee's proposal on the role of the valuation actuary in which Walt Rugland urged acceptance of the concept, while this writer suggested that significant modifications should be made in the approach. A number of thoughtful responses, which will be published in the next issue of the newsletter, were received from readers. In addition, efforts are currently underway to expand the newsletter from its current January/August schedule.

The next major effort of the Section will be in the area of the examination syllabus. An E&E Advisory Committee has been formed and is charged with providing a thorough analysis of existing examination material to produce proposals for improvements. In addition, liaison has been established with the SOA Research Policy Committee with the objective of identifying financial reporting issues needing research support.

One of the major continuing efforts of the Section Council will be to increasingly draw on the talent and expertise of Section members to improve the programs of the Section. While much of the work of the Section is unheralded and goes on behind the scenes of existing Society programs, the response of our members to requests for their time and expertise has been superb. Many Section members have already willingly provided their time and effort on behalf of the membership through staffing committees, speaking at meetings and seminars, writing articles, and service on the Council.

It is the Council's desire to expand the utilization of this resource and it plans to distribute a questionnaire this summer to all members to seek their ideas and to identify areas where each individual would be willing to make a contribution. We hope that when the questionnaire arrives Section members will take a few minutes to complete it. In the meantime, if you have any questions, ideas or suggestions that you would like to share with the Council, you are invited to get in touch with a Council member (see page 30 of the *Yearbook*). We look forward to hearing from you. □

LIFE INSURANCE FINANCIAL STATEMENTS: KEYS TO SUCCESSFUL REPORTING

By R. Arthur Saunders, FSA, FCIA
(Teach'em, Inc., Chicago, IL)
Reviewed by Anthony B. Richter

Did you know that —

- double-entry bookkeeping was invented in Renaissance Italy?
- in the entire history of Canada, there have only been *seven* Superintendents of Insurance, *six* of whom were actuaries?
- in the NAIC Early Warning System for U.S. insurance companies, one of the tests for financial stability is the ratio of real estate investments to capital and surplus? (It should be less than 100%.)

No, these facts are not from a book on insurance trivia. Far from it, in fact. They come from *Life Insurance Financial Statements: Keys to Successful Reporting,* by R. Arthur Saunders, FSA, FCIA. The author's new book on life insurance company annual statements, both U.S. and Canadian, fills a need for up-to-date information that has too long been unfilled. This book will be included in the Society's exam syllabus, and the Society's seal appears on the title page and the back cover.

The author gives equal time to U.S. and Canadian financial statements, and this will prove useful to those interested in differences of philosphy and structure between the two countries. (The reviewer was somewhat envious to learn that Canadian actuaries have relatively more freedom to set Annual Statement actuarial assumptions than their U.S. counterparts.)

The book also includes an especially welcome chapter, a lucid explanation of GAAP accounting, with examples, and a thorough explanation of the difference between GAAP and statutory accounting. The chapter will provide an understanding of the difficulties of matching revenues and costs in life insurance accounting.

A substantial portion of the book's 305 pages (which are followed by a glossary and an index) are taken up with numerical examples of transactions and hypothetical financial exhibits. (Some of the examples are quite detailed, and the reader might get bogged down.) The treatment of the material is exhaustive, and the book provides a "one-stop" opportunity for familiarity and understanding of the Annual Statement and the various supporting exhibits.

Mr. Saunders is to be congratulated for a genuine work of scholarship. Notwithstanding revisions that may be needed due to changes in the structure of the Annual Statement itself (changes are constantly being discussed by the NAIC), this book should remain the definitive work on life insurance accounting for many years. □

Continuing Education

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This Spring's seminar program offers 10 topics in 21 locations as well as two Section-sponsored meetings. The Individual Life and Annuity Product Development Section presented "Interest Sensitive Product Development: Practical Considerations", and the Reinsurance Section sponsored "Underwriting and Actuarial Implications of AIDS and AIDS-Related Illnesses", both in conjunction with the Boston Spring meeting.

A recent trend in the Society's continuing education effort has been toward co-sponsorship of symposiums with sections, committees and other associations. These symposiums offer the opportunity for a larger audience to benefit from a panel of faculty with their own expertise in a particular topic. Last year's Symposium for the Valuation Actuary attracted 288 attendees and the Healthcare in the Future symposium was attended by 320 actuaries and hospital administrators. A bonus to these meetings are the transcripts which are produced afterward which further add to their educational benefit. Both programs will be repeated in the Fall of 1986 and Spring of 1987, respectively.

Seminar development begins with the assignment of topics and then recruitment of faculty. A question most often asked by the membership of the seminar planning process is why a particular date or location was selected. Locations are often selected based on membership and audience distribution. The highest concentration of Society members exists in Eastern and Central U.S. cities (30% and 20%, respectively). A larger percentage of seminars is held in these cities; however, every effort is made to schedule seminars in Western, Southern and Canadian cities. A major concern of date selection is to avoid other actuarial meetings and holidays; however, sometimes hotel and faculty availability are determining factors.

A Faculty Training Program designed to "fine tune" seminar presentations is in progress. Barbara plans to improve the quality of seminars by working more closely with faculty and recommending effective presentation styles, proper use of audio/visuals and other techiques. In addition to the Faculty Training Program, members will see more workshop-type seminars offering "hands-on" experience and training held this Fall and next Spring.

Because many seminar topics deal with issues relevant to accountants as well as actuaries, the Illinois Department of Registration and Education for Public Accountant Continuing Education this year approved the Society as a sponsor for granting CPE credit. Actuaries or CPA's employed by accounting firms who must satisfy continuing education requirements can now receive CPE credits at Society seminars.

Within the year, members can look forward to the PAT Series (Professional Actuarial Training), which will be a series of modules or courses providing an update on basic education issues melding theory with practice. The PAT Series will be repeated each year and have a set curriculum.

If you would like more information on present or upcoming continuing education programs, please contact Barbara, Ann or Joyce at the Society office. □

DEATHS

Herbert C. Dunkley	ASA 1932
H. Linden Sharpe	FSA 1939

Exposure Draft

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already accepted responsibilities for education and research. The Task Force believed that the expression of actuarial opinions "would allow for better focus of educational developments with regard to appropriate developments in the underlying fabric of actuarial science. It would add an active dimension to the basis of education, practice definition, and professional stature".

This recommendation was discussed by the Executive Committee in August and by the Board in October. Some concerns raised were:

1) The promulgation of principles would inhibit progressive actuarial thinking and practices.

2) Principles cannot be separated from standards and therefore should be left to the standard-setting national bodies.

3) A statement purporting to be a principle is either obvious and thus unnecessary, or not obvious and thus not an accepted principle.

4) Enforcement of adherence to principles is inappropriate to a scientific organization.

In addition to the supporting reasons advanced in the Task Force report, others were:

1) The Society cannot permit itself to be placed in the position of educating actuaries in practices which may not be based on sound actuarial principles.

2) Enforcing adherence to Society promulgated principles could be the responsibility of the standard-setting national bodies. Members of these bodies, in setting standards, would be required to follow actuarial principles, as promulgated by the learned actuarial bodies (CAS and SOA).

The Board did not accept the recommendation of the Task Force. However, the Board subsequently approved the exposure by COLICVP of the valuation principles, recognizing that any promulgation of these principles would be dependent on the Board's ultimate acceptance of the major recommendations of the Task Force.

The Executive Committee and the Board will be discussing the Society's role in promulgating actuarial principles later this year. We shall also consider the possibility of scheduling a panel discussion on this topic at the Annual Meeting. To aid us in these discussions, we would appreciate your comments. Please send them to me at my *Yearbook* address by Aug. 1. I will ensure they are brought to the attention of the Executive Committee and of the Board. □

LETTERS

Voting

Sir:

Your article on the exercising of voting rights by actuaries in the Society's election in recent years did a disservice to many actuaries. For a number of us, the changing times have brought our own professional careers into byways that are well outside the main stream of actuarial thought. This does not imply any lack of interest in or respect for the profession which underlies our careers; it only signifies that actuaries, like other professional groups, are made up of a diverse assembly of multi-faceted individuals

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Letters

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who elect, at one time or another, to emphasize one of their skills vis-a-vis others.

Voting, whether in a Society election or in the political arena, requires an informed electorate. Many of us are not informed on the professional or personal qualifications of most of the people on the ballot. We are then, unfortunately, reduced to the position of having to hope that those who do vote are making intelligent decisions not emotional ones and opt not to add our personal "static" to the voting process.

Thus far, the Society has been fortunate, despite your lament about the percentage of Fellows casting votes. We have been well served by those who have been elected. This may be a tribute to the wisdom of those who vote or we may be the benefactors of some of the actuarial nature of a Delphic projection where the limitations of each individual voter including his prejudices, misunderstandings and misinformation balance each other out.

To paraphrase that great mind of the twentieth century, Burt Lance, "it ain't broke, so don't fix it".

Joseph H. Dowling

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Sir:

Concerning the recent article on the falling percentage of FSA who vote in annual elections:

First, this result is not terribly surprising. Several factors—the increased number of FSAs, the declining percentage of FSAs who attend Society meetings regularly, and others—make this a likely result. FSAs in general, especially those who have not been members for long, are simply less likely to know the candidates than was the case when the size of the Society was smaller.

The pictures that come with the second ballot are probably a help (in that they jog memories as to who a particular candidate is), but I believe the Board made a mistake a couple of years ago when they rejected the idea of including, with the second ballot material, a brief statement from each candidate. I have no idea what candidates would have said, but personal statements might have helped break through the sense of anonymity that is a probable cause of the lower voting percentage.

Daniel J. McCarthy

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Comparing The Actuarial Exams

Sir:

After reading Rick Roeder's comparison of the actuarial and CPA exams in the March issue, I couldn't resist the opportunity to comment on the Canadian environment. By way of background, I should point out that I

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MATH ODDITIES

Readers having responded more favorably than otherwise to the April suggestion for a column on mathematical facts, oddities, problems, puzzles, and the like, *The Actuary* begins what we hope may be a continuing feature, to alternate with the WORKDAY PROBLEMS column first appearing in May.

The Fifth Power of an Integer

Jerrold M. Levy noted the following in a Detroit newspaper:

Q. Very quickly, what single digit number raised to the 5th power is 32,768?
A. Since any single digit number raised to the 5th power ends in that number, the answer must be 8.

Mr. Levy, not being aware of this characteristic of the fifth power of small integers, undertook to prove the underlying proposition along these lines:

> Let x be an integer less than 10. Assume that x^5 can be put into the form $x + 10 \cdot p(x)$, where p(x) is a 5th degree polynomial. Then derive p(x), and show that its value, for x = 1, 2, 3, ...9, is integral.

Mr. Levy derives p(x) in terms of the factorial products $(x-2)^{(k)}$ as

$$p(x) = 1/10\,(x-2)^{(5)} + 2(x-2)^{(4)} + 25/2\,(x-2)^{(3)} + 57/2\,(x-2)^{(2)} + 21(x-2) + 3$$

and p(x) is indeed integral, term by term, for all single digit x.

A reader suggests that the Levy proof can be made easier by writing p(x) as

$$p(x) = \frac{x^5 - x}{10} = \frac{x(x-1)(x+1)(x^2+1)}{10},$$ the numerator of which is easily shown to be a multiple of 10 for all single digit x.

Then the same reader makes three other points:

(1) An ample numerical proof is simply to punch out x^5 for single digit integers on a calculator.

(2) The binomial theorem shows us that the 5th power of *any* integer that ends in x will also end in x. Hence $(428)^5 = (420 + 8)^5$ will also end in 8.

(3) The number theory of congruences, and a theorem ascribed to Euler, lurk in the background.

Still further analysis of this 5th power proposition leads to some interesting extensions.

(a) Since an integer ending in x repeats the x in the final digit of the 5th power, it will also repeat in the 9th, 13th, 17th, etc. powers.

(b) For the special cases where x = 0, 1, 5, or 6, *all* powers of an integer ending in x will likewise end in x.

(c) For the special cases of x = 4 or 9, every second power repeats, so the proposition holds for all odd powers - 3, 5, 7, 9, etc.

Finally we can come to this probability problem:

Given that $z = u^v$, where u and v are random positive integers, what is the probability that the final digit of z will be x, where x = 0, 1, 2, ...9? Can you prove that these probabilities are:

20% for x = 1 or 6
10% for x = 0, 4, 5, or 9
5% for x = 2, 3, 7, or 8? □

Letters

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obtained my C.A. (Chartered Accountant) designation prior to qualification as an FSA. Also, unlike Mr. Roeder, my undergraduate degree was in commerce, but I did include several actuarial science courses in my program, and as a result, I had passed the first four parts by graduation.

I have to agree with Mr. Roeder that the major difference in the exams is in the breadth of the material. This should be expected since the entire scope of the accounting syllabus (including auditing, taxation, professional ethics, financial reporting, etc.) is tested in four consecutive days, whereas the actuarial material is spread out in neater (and often more difficult) little chunks.

In Canada, the accounting exams are normally all essay type questions, as opposed to the blend of essay and multiple choice to which actuaries are accustomed. Furthermore, the accounting questions are often not as well directed as the actuarial counterparts. Indeed, marks are often allocated to the correct identification of the problem!

In terms of preparation time required, CA students usually begin their study schedule approximately one year in advance of the exams. For the actuarial exams at the Fellowship level, I generally found 3.5 to 4 months prior to the exam to be a suitable starting date.

Although I was probably too busy to notice, during the CA exams I failed to observe people "aimlessly walking around with a cup of coffee". Quite to the contrary of Mr. Roeder's experience, I usually felt much time pressure, especially in comparison to the multiple choice papers of the Fellowship exams where it is not uncommon to find people handing in their papers early (and still passing). I can only conjecture that in the U.S., where it is my understanding that partial credits may be obtained by passing, say, two out of the four papers, some candidates put all their emphasis in passing those two days, and wait until the next sitting to pass the remainder. In Canada, partial credits are not available, and so a somewhat more consistent performance is required on all four days.

As Mr. Roeder noted, the number of people attempting the accounting exams

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Exam Memories

Sir:

For my solution to Henry Unruh's problem, I place the black balls on the line and make the following observations:

(1) Each of the $m-1$ spaces between the black balls will produce 2 contacts if occupied by one or more white balls; and the space at either end will similarly produce 1 contact, and

(2) The number of different ways of distributing n white balls in j of those spaces, none of the j spaces to be empty, is $C(n-1,j-1)$.

Let NC be a random variable equal to the number of contacts. Then it is not difficult to show, using (1) and (2), that

$$\Pr(NC=2j) = K[C(m-1,j)\cdot C(n-1,j-1) + C(m-1,j-1)\cdot C(n-1,j)]$$
$$\text{and } \Pr(NC=2j+1) = K[2C(m-1,j)\cdot C(n-1,j)]$$
$$\text{where } K \text{ is the reciprocal of } C(m+n,m)$$

So much for the easy part of the problem. My solution to the rest of Henry's problem is very long, so I will give only the results.

Start by defining the quantities k and r as follows:

$k = [mm/(m+n)]$, where the brackets indicate "largest integer in"

$r = m + n - 2k$

It can be shown that at least one of $2k$, $2k+1$, and $2k+2$ must be a modal value, and that $2k-1$ is the only other possibility.

Algebraic manipulation of the two probability expressions above will yield the following:

$$\Pr(NC = 2k) = A\cdot rk/((m-k)(n-k))$$
$$\Pr(NC = 2k+1) = A\cdot 2$$
$$\Pr(NC = 2k+2) = A\cdot(r-2)/(k+1)$$
$$\text{where } A = C(m-1,k-1)\cdot C(n-1,k-1)\,(m-k)\,(n-k) \,/\, k^2\cdot C(m+n,\,m)$$

Hence the modal value(s) among $2k$, $2k+1$, $2k+2$, can be found by finding the largest values among the three quantities above. (In finding this largest value it is not necessary to determine A). $2k-1$ will be a modal value only if $m=n$ and is even, in which case all of $m-1$, m, and $m+1$ are modes.

A more interesting question is what combinations of modal values are possible, and for what values of m and no do they occur? Here is a report on my progress so far (and the probability of further progress appears low).

(1) The distribution must be unimodal, bimodal, or trimodal (never more).

(2) It will be trimodal in only two circumstances, (a) if $m=n$ and is even, as before noted, or (b) if m and n are integers of the form $2(2q-1)(q+1)$ and $2(2q+1)(q-1)$, where q is an integer greater than 1. Here the modal values will be $2k$, $2k+1$, and $2k+2$. The first few values of such (m,n) are (18,10), (40,28), (70,54), etc.

(3) The distribution can be bimodal at $2k$ and $2k+1$, at $2k$ and $2k+2$, or at $2k+1$ and $2k+2$. These are the only possibilities.

(4) The distribution will be bimodal at $2k+1$ and $2k+2$ if and only if $m+n = 4q$, q is a positive integer, $m \neq n$, and $2q - \sqrt{2(q+1)} < n < 2q + \sqrt{2(q+1)}$. The first few examples here are (3,1), (5,3), (6,2), (7,5), (8,4), (9,7), and (10,6).

(5) It is surprising that the distribution can be bimodal at $2k$ and $2k+2$. A computer run indicated that, for m and n each less than 1000, there were only 6 such cases, namely, (144,45), (385,145), (399,252), (455,203), (741,416), (868,412). At (144,45), $k=34$, the modes are at 68 and 70, and the probability of 68 or 70 contacts is 70% greater than the probability of 69!

(6) A few examples where the distribution is bimodal at $2k$ and $2k+1$ are (9,5), (10,7), (20,14), (27,22), and (30,21).

(7) The distribution will be unimodal at $2k+1$ if $m=n$ and is odd. In that one case, $k = (m-1)/2$, and the unique mode is at m.

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Comparing Actuarial Exams

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is significantly greater than those sitting for actuarial exams. I made it a point of visiting the CA exam site a couple of weeks prior to the first exam day in an attempt to reduce the sensation of being overwhelmed by the size of the arena.

Both the accounting and actuarial professions are rightfully demanding in their exams for admission, and in the final analysis, the feeling of isolation is about the same when you're locked up in a room studying and missing out on the World Series or Stanley Cup playoffs. Neither profession has an easy path for admission, and of course, the responsibility for professional development continues long after the diplomas are framed and hung proudly on office walls.

Ben Mackler

Exam Memories

(Continued from page 6)

As you can see, I left a few holes for others to fill. As a final comment, let me express the hope that Henry Unruh has forgotten all the other problems from the old exams.

Walter Shur

Editor's Note: Mr. Shur tells us that he agrees with most, but not all, of the Arvanitis analysis (April). Although the even-number-of-contacts equations are correct, it is not possible to determine the even-distribution mode by comparing P(2k) with P(2k − 1). For example, if m = 4, n = 3, M(k) = (7 − 2k)/2k, making the critical largest k = 1, and suggesting that 2 is the mode of the even distribution. While 2 is the last even contact more probable than the next preceding odd, 4 is the last even contact more probable than the next preceding even — and hence the mode of the even distribution.

Exam Memories

Sir:

I wonder how readers' solutions to the "m-black and n-white ball problems" would compare with the "official" solution in the 1938 "Problems and Solutions" published by the Society's predecessor organizations. As I remember the solution was lengthy, with characteristics of the Arvanitis and Shur analyses. Surely solving this problem in roughly 18 minutes under examination conditions was a pretty tall order.

Henry Unruh

Editor's Note: With the publication of Mr. Shur's lengthy analysis, and Mr. Unruh's short comment, *The Actuary* now takes a breather as to this exasperating probability problem. Before we do so, however, we wish to acknowledge recently received letters from Michael Gastineau and Jack Elkin; and a further analysis from Ernest Arvanitis.

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Sir:

I really enjoyed the March article. I, a recent ASA and a new employee of a "Big 8" accounting firm, have often been asked to compare the CPA exam to the actuarial exams and have found it difficult to respond. I feel I will be more able to compare the Fellowship exams with the CPA exam, as they both consist of multiple choice and essay questions. However, after completing the first five exams, I am a little disappointed with Mr. Roeder's analogy.

Are we actuarial students, or former students, actually "Actuarial Gods" — able to withstand the toughest questions on the "Ten Toughest Tests" in the universe? I hardly believe this to be true and in defense of my "CPA brethren", I'd like to make a few points for the CPA exam.

(1) Are calculators allowed to be used on the CPA exam? No. Lord knows how we actuaries could pass an exam without our "Official Society Calculator".

(2) The actuarial exams are for one day and the longest are in total 5 hours. This is a far cry from 2½ days of testing with the "Accounting Practice" sections alone consisting of 5 straight hours.

(3) If the CPA examination room is really the atmosphere of a Van Halen concert, wouldn't it be easier to concentrate taking an actuarial exam — whose acoustics could be compared to the Mormon Tabernacle — than taking the CPA exam?

One further comment, my hat is off to Mr. Roeder who passed the exam by occasionally studying between innings of the Tiger-Padre series. I went to a well known accounting university which has a CPA review course and a fine record of passing, on average, 60% of its students on the first try. I studied just as hard for the CPA exam as for any of the first 5 actuarial exams and found myself passing with only a few points more than the magic 300.

My point is that the actuarial exams may be tougher than the CPA exam in a lot of ways, but the CPA exam is by no means like a "10-point quiz". The accounting profession and CPA's are highly regarded in the business world and they've earned it. The actuarial profession is also regarded very highly, but is no better or worse than the former. And as the old joke goes — "An actuary is just an accountant who found accounting work *too* exciting".

John Christensen

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Sir:

I was fascinated to read Rick Roeder's comparing of actuarial and CPA exams.

I was, sadly, fairly "senior" when I took both the C.P.A. and the actuarial exams. However, I had no formal training in either (beyond my mathematics background): i.e., no actuarial or accounting courses. I would like to add these observations:

1. There is no doubt the actuarial exams are tougher (for one thing, there are ten of them; it is possible to pass the CPA all at a swoop as I, and presumably Rick, did).

2. Starting from scratch, it took me two years to prepare for the CPA exam. It took two and a half years to get my A.S.A.

(Continued on page 8)

Letters

(Continued from page 7)

3. I do *not* agree with Rick that one can pass the CPA exams from "practical" business knowledge. (He may or may not imply this; I'm not sure.) In fact, "practical" knowledge is often a hindrance in that you wind up doing CPA problems the unorthodox way — no hit with the graders.

4. In any case, I see no way one can do well on Business Law and Auditing (two of the four parts of the CPA exam; the others are Accounting Theory and Accounting Practice) from "practical" experience, unless you were a lawyer or an auditor.

5. There is no doubt that more time is afforded on CPA exams than actuarial exams. I was able to finish all four parts of my CPA exam. I *never* finished an actuarial exam.

6. CPA Exams tend to be much more *practical* than actuarial exams. For example, to work in tax you really *do* have to know the answers to CPA exam tax questions. The only relevant parts of my A.S.A. exams are the first half of the old Jordan book and possibly some theory of interest.

P.S. How many FSA's can even *define* a Gamma function, much less use it in their work?

Donald P. Minassian

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April Editorial

Sir:

Your editorial on Later Retirement is excellent, and I agree with it completely. I would like to see private pension planners take action in this direction, and also see Medicare parallel OASDI.

Under the practical politics of the matter, it was easier to leave age 62 alone and have the larger reduction than to move the earliest retirement age up as the normal retirement age rose.

Robert J. Myers

The Actuary

Will Be Back in September.

Have a Good Summer.

You Should Live So Long

Sir:

In the January issue, Murray Projector quotes Genesis 6:3 as follows:

> "And the Lord said: 'My Spirit shall not abide in man forever, for that he also is flesh; therefore shall his days be a hundred and twenty years.'"

Mr. Projector suggests the interpretation that 120 years is the maximum age or "omega" for man. This is an interesting idea when one considers the recorded life spans of Noah (of whose generation Genesis 6:3 speaks) and his descendants. The enclosed graph shows these life spans down to Moses, of whom Deuteronomy 34:7 states:

> "And Moses was a hundred and twenty years old when he died: his eye was not dim, nor his natural force abated."

The curve which has been fitted to the data is of the form $Y = A + B \cdot C^{-X}$. With "A" set equal to 120, the R-squared of the fit is approximately 92% (the R-squared can be increased slightly using a lower value of "A"). This is a remarkably good fit to biological data.

I am not sure what all this means except that, as always, there is more to the Bible than meets the eye. I welcome the comments of other readers.

Mark W. Campbell

Age at Death of Noah and his Descendants

	Generation from Noah	*Age At Death*
Noah	0	950
Shem	1	600
Arphaxad	2	438
Salah	3	433
Eber	4	464
Peleb	5	239
Reu	6	239
Serug	7	230
Nahor	8	148
Terah	9	205
Abraham	10	175
Isaac	11	180
Jacob	12	147
Levi	13	137
Kohath	14	133
Amram	15	137
Moses	16	120