

Contributions
to the
Theory
of
Life Insurance Premium Reserves

by

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Translators' Note

All of the financial figures of the original paper have been reproduced in the translation. This has been achieved by substituting dollars for the Prussian taler. The units of Prussian currency at the time were as follows:

1 Taler

30 Silbergrosschen per Taler

12 Pfennig per Silbergrosschen

A single taler in 1861 was worth \$.75 in U.S. currency. Reconstructions of the Consumer Price Index for that era show a CPI of 27 for 1861 where the CPI of 1967 is 100.

All of the tables in the original paper have been recalculated using the spreadsheet software Quattro. Except where indicated, the only differences between the recalculated values and the original values are due to rounding.

Introduction

More than any other corporations life insurance companies should be held to exact and strict accounting rules. The entry into a life insurance contract is an act of self-denial and an example of the noblest love of spouse or child. Often the policy is the only inheritance and the sole resource of widows and orphans. Since life insurance institutions draw in a considerable portion of the savings of thousands of policyholders on the basis of contracts several of which will not expire before a half or sometimes almost a whole century. Therefore, their solidity and solvency must be mathematically provable and indisputable.

These undeniably true words which were spoken by the New York State Insurance Commissioner when he withdrew the license of the American Mutual Life Insurance Company of New Haven, Connecticut, to do business in the State of New York. In the spirit of these words, we will present several considerations

about two different methods of calculating the premium reserves of life insurance companies in the following pages.

In the year 1857, Mr. Wilhelm Lazarus, in a thorough article (See Rundschau der Versicherungen, 1857, p317.) had already called attention to the misleading principle of estimating the present value of obligations at a lower interest rate than the present value of the yet to be collected premiums and demonstrated the consequences of such misleading calculations on the balance sheet of a company which thus calculates. Yet since that time, practically nothing has happened to prevent the use of this misleading principle. Here and there, one finds in professional journals the remark that the premium reserve figured on gross premiums is too small, but that is all one finds. In more recent times, prompted by the publication of the balance sheets of some English companies, some voices have been raised against the mischief of these calculations. With this, other aspects of the life insurance business have been drawn into the conversation and, here and there, mistaken concepts have become widely shared. For example, it was asserted that the recent custom of paying agents a high commission for the acquisition of a life insurance contract, about 1 % of the insured amount, seems to make it impossible to set up an

actuarially sound premium reserve. And thus companies that pay such commissions are forced to calculate premium reserves too low, if not even according to the principle of gross premiums. The presentation of such mistaken, when not intentionally misleading assertions, are well suited to deeply affect the trust of the public in life insurance companies and will harm exactly those companies that have published their business details with the greatest openness and make no secret of their commissions. We attempt to contribute, to the best of our ability, the correct view about the premium reserves of life insurance companies and in the first segment of the following monograph provide the proof that even for the custom of a closing commission, a premium reserve can be established that has a completely calculable magnitude. In the second segment, we will briefly discuss the method of calculation with gross premiums, and we believe that we thereby show that many phenomena existing in the real world are natural consequences of this misleading system.

I. Premium Reserves and Closing Commissions

In a life insurance company, the premium reserve plays a large role. The annual premium for an insurance policy [in this we have in mind the usual whole life insurance policy with annual premium payment] remains constant for the whole term of the insurance policy while the risk that the insured amount becomes due by the death of the insured grows with each year. And if an insurance policy were always issued for one year only, it would require an increase in premium each year. Therefore a premium that remains constant for the whole term of the insurance policy is greater for the first year after the issue of the insurance policy than the yearly renewable term (YRT) premium; later the premium is smaller than the YRT premium. If a certain number of persons enter an insurance company, then the company will create a considerable surplus after covering the occurring cases of death among those persons. And in the later years this surplus has to cover the added expenditures when the company must pay more for cases of death than it receives in premiums. For example, the attached Table II shows that a company which insures a number of 30 year old persons with a sum of \$500,000, collects more in premium during the first 25 years than it has to pay out for cases of death. From the 26th year on, the expenses for deaths overtake the premium income. For

example, in the 40th year, the excess of the death claims over the premiums is \$8,679.

Therefore as the compelling consequence of the level premium method, a life insurance company must set up a premium reserve company. In each case, the amount of the premium reserve fund cannot be arbitrarily set as it suits the company, but it has to be mathematically and precisely determined, just as the annual premium is calculated mathematically and precisely on the basis of a mortality table and a certain rate of interest. For example, the insurance company may not lower the premium reserve fund, speculating that the mortality among the insured will be more advantageous than could be expected from the mortality table used in the calculation of the premium or that it will, in consequence of fortunate speculations, obtain a higher interest income with their monies and thus be easily able to cover the shortage in the premium reserve funds. The premium reserve fund must be set up much more from the paid in premiums of the existing insurance policies and the calculable interest after deduction of the same calculable future expenses for death, and to be sure the calculable expenses, not the actually caused ones for otherwise a sustained excess mortality among the insureds could, if not exhaust the premium reserve fund, at least

significantly lower it below the level required for the future performance of the company. The premium reserve fund is not to be set up from the premiums paid by the insured but from the net premiums. The premiums arrived at by exact calculation (net premiums) are to be increased by a loading which should reimburse the company for business expenses and the risk that mortality among the insureds exceeds the expected limits. The loading, the size of which will be established in the individual companies according to more or less arbitrary rules of thumb, goes to the company, and the premium reserve fund is calculated from the remaining net premiums after the deduction of this loading. As soon as the net premium described above is determined, the premium reserve will result as a mathematical consequence at every point of time during the term of the insurance policy.

As a rule the net premium is so established that it remains constant for the whole term of the insurance policy, just as the loaded premium paid by the insured remains constant. But clearly there is no reason not to include other considerations relating to the amount of net premium if only these are rationally calculated, completely covering the obligations of the company. There are insurance policies with rising or falling premium payments whereby naturally not only the loaded

premium changes in the different periods during the term of the insurance policy, but also the net premium.

To arrive at that firmly established net premium that we believe we must prefer in general and prefer especially in the current situation in the business, we put forward several considerations:

The participation of the German public in life insurance has increased in a gratifying fashion in recent years. Not only do some of the few older German companies do more significant business than in earlier decades, but also the newer German companies and besides them a considerable number of foreign companies work with generally growing success. The Germania has even experienced such a spectacular participation by the public that last year (1862) it issued insurance contracts in a sum of significantly more than \$6 million and in the month April of this year alone, in a sum of more than \$1 million. This glorious upswing in the life insurance business is not only the consequence of the correct appreciation and high significance which life insurance has from the individual, the family, and the state and which is permeating all levels of society, but this upswing is also brought about by the efforts of the executives of the life insurance companies and their agents.

These executives introduced the practice of paying the agent a higher commission at the issue of the insurance policy [previously the agent had only received a commission which remained constant for the term of the insurance policy. The agent kept a portion of the premiums he collected and thus achieved for himself a worthwhile income only after years and years]. This succeeded in attracting active businessmen as agents who then, being placed in a position to quit their other business, were able to use their whole strength for the attraction of new insurance policies. The success,¹ that the introduction of this commission practice has had until now is its own best justification. But complaints against the commission method force another question: how does a company defray the agency costs which must grow proportionately for a large scale addition of new insurance policies. We admit that if a company is required not only to post an actuarially sound premium reserve [naturally this must be required] but if it is also required to set the amount of the premium reserve on the basis of the determined, prescribed, or traditional calculation model (net level premium), then company can thereby run into

¹. The introduction of a closing commission recommends itself on other grounds also. For example, how is an agent to be repaid who in a short time has made significant sales when he is forced (by death or other conditions) to give up the business?

difficulties. The expenses of an insurance policy, for the coverable mortality for the first year and for the premium reserve set aside at the end of the first year according to any one calculation model can and must under all circumstances be greater than the premium income for this insurance policy. The thus growing deficit will naturally be the greater, the greater the increase in new insurance policies is, and when the increase is sufficiently large, the deficit too will become so large that even a company with significant means could run into difficulties. It should be clear even to the lay person that a contradiction emerges here. On the one hand, one wishes ample increases in new insurance policies; on the other hand, one would be financially embarrassed by this and this could happen even [as will be demonstrated in the following] with commission rates that have not been set too high.

It would be wonderful if a solution for this could not only be found but if it offered itself, and this is really the case.

The obvious thought is that under the old, usual method of calculation which at the close of an insurance policy posts the costs growing out of that closing, possibly in a prepaid commission account, and this account is entered as an asset of the company. The sums brought into this account in an individual year would then have to be amortized in a certain

number of years, or what would be preferable is that from the total amount of the prepaid commission each year a certain and adequate portion of the premium income would be written off. This model was publicly discussed on April 26 of this year in an article in the German insurance paper of Breslau, but for various reasons we cannot sanction this. For one, there is no measure and no limit for the amount of those costs that could be placed in this prepaid commission account. In essence this account would have the effect of lessening the base capital or paid in the premium reserve; and then it would certainly be possible to endanger the stability of the company through excessive expenses for the acquisition of new insurance policies. Second, the artificial asset of prepaid commissions should really be gradually amortized. Since this requires a relatively high fraction of the annual premium, this means that the coverage of the other costs and the gain for a long series of years, respectively, are unnecessarily lowered. Finally, thirdly, there could and would be found, in the account of the prepaid commissions, sums prepaid for such insurance policies

that have lapsed. Later insureds then have to pay subsequently the costs which have been incurred by earlier insureds.²

². If we seek to rationally establish the amortization of the account of the prepaid commission, especially in a way that is free from the disadvantages of the second and third objection then automatically one arrives at the procedure proposed by us.

In the following simple way one develops a model which is completely untouched by these disadvantages. One calculates the net premium for an insurance policy in such a way that in the first year it is smaller by a certain amount³ than for the following years. We omit the development of the necessary formula here. It is, by the way, extremely simple. We get our net premium as the usual premium which remains constant for the whole term of the insurance policy. Take the amount by which the first annual premium should be smaller and divide it by an annuity due for the issue age and add it to the usual net premium.

I = the amount of closing costs to be amortized
per \$100 of insurance

P_x = the net annual premium for \$100 of insurance

a_x = the first net annual premium

³. We intend here for a partial premium which keeps the same value for each issue age, which is a different fraction of the premium for each issue age, and which is a constant percent of the insured sum. If the amount by which the first annual premium shall be smaller than the following annual premium is a percentage of the premium, the formula for the calculation changes somewhat. In such a case, the following considerations do not fit exactly.

$$= P_x + I / \ddot{a}_x - I$$

β_x = the second and subsequent net annual premium

$$= P_x + I / \ddot{a}_x$$

For example, calculating with 3 1/2 percent interest and using the Combined Experience Table, the net annual premium at issue age 40 for an endowment for \$100 at age 90 [and in the following, this is what we always have in mind] is 2.484 percent. Should the first annual premium be smaller by 1 percent of the insured sum, then we get as the premium for the following years 2.484 + 1 divided by the annuity due at age 40 which ends at 90, that is, 2.484 + 0.059 and this as premium for the first year 2.543 - 1. = 1.543. In contrast, if the first premium should be smaller by 1 1/4 percent of the insured sum, then the first premium is 2.484 + 5/4 * 0.059 - 1.25 = 1.308 and the premium for all the following years is 2.484 + 5/4 * 0.059 = 2.558.

Naturally also the insured pays the same premium in the first year which he will have to remit in the following years, and the company covers the cost arising from acquisition of the insurance policy with the portion of the premium available to it in the first year. The premium reserve will be calculated just as with the usual premium, that is, one finds the premium

reserve for an insurance policy which has been in existence for a number of

$$\begin{aligned} {}_tV_x &= \text{the net level premium reserve at age } x \\ &= A_{x+t} - P_x \ddot{a}_{x+t} \end{aligned}$$

A_{x+t} = the net single premium at age $x+t$

$$\begin{aligned} {}_tV_x' &= \text{the Zillmer premium reserve at age } x+t \\ &= A_{x+t} - \beta \ddot{a}_{x+t} \end{aligned}$$

While the formulas are developed prospectively here, Zillmer's premium reserve calculations are actually done retrospectively as was common in the 19th century.

years by deducting from the net single premium at the attained age the product of the net annual premium and an annuity due at the higher age. The premium reserve will be somewhat smaller here than according to the previous method, but exactly fulfills the demands:

- 1) that it is formed from the net premiums, after one takes deductions of the calculable expenses for death, and with the addition of the calculable interest, or
- 2) in other words, that the premium reserves together with the premium to be paid by the insured in the future and, that is of course, with the net premiums, completely

covered the obligations of the company. Since the premium reserves are somewhat smaller than according to the old (net level premium) method, the net premiums are by a small, but corresponding, part higher.

Before we enter into further considerations of the premium reserve, we want to say something more about the first year premium. For this premium, for each issue age, there exists a minimum, and this is given by the premium for one year of term insurance. If the premium for the first year were much smaller than the corresponding premium for one year of term insurance, then the mortality of the first year would not be completely covered by the premium, and the insurance company would have to cover the excess mortality out of other means which under circumstances could become not only uncomfortable, but even dangerous. Insofar as the minimum first year premium is given by the premium for one year of term insurance, the maximum of the difference between the premium for subsequent years and the premium for the first year is given at the same time for every age. If we call this difference X , then obviously the usual premium increased by X divided by the annuity due at the issue age must be just as large as the premium for one year of term insurance increased by X .

$$P_x + X / \ddot{a}_x \geq A_{x:1} + X$$

Solving for X gives

$$X \leq (P_x - A_{x:1}) \ddot{a}_x / (\ddot{a}_x - 1)$$

If we solve the resulting equation, we find X, that is, the maximum of the costs that may be incurred at the closing of an insurance policy equal to the difference between the usual net premium and the premium for one year of term insurance, this difference multiplied by the annuity due at the issue age and divided by the annuity due lessened by one or immediate annuity at the same issue age.

The formula for the maximum of the closing costs can be expressed in many ways. So this maximum, for example, is equal to the quotient of the annuity due at the issue age lessened by one and the annuity of the next higher age lessened by one, that is, for the insurance sum one, or it is equal to the difference of the usual net premium at a one year higher age and the usual net premium of the issue age multiplied by the annuity due at the issue age or, and this is the simplest expression, the maximum of the closing costs is equal to the net premium for the one year higher age lessened by the premium of the issue age for a one year term insurance policy. This simple form for the maximum of the closing costs can also be directly deduced. The

maximum that the company can spend for the acquisition of an insurance policy is evidently reached when at the end of the first year, the premium reserve is zero. The latter is the case if we take as net premium the usual net premium of the next higher age. Since the premium for the one year term insurance policy is used to cover mortality, there remains for the coverage of costs caused by closing at most the difference between the usual net premium of the next higher age and the premium for the one year term insurance policy.

The following table shows the maximal rates for the closing costs if one calculates according to the Combined Experience Table and 3 1/2 percent interest.

Issue Age x	Maximum Rate of Closing Cost	
	As % Ins Sum	As % Premium
10	0.48%	42.4%
15	0.59%	46.7%
20	0.71%	50.1%
25	0.86%	53.3%
30	1.04%	56.0%
35	1.27%	58.5%
40	1.58%	61.1%
45	1.95%	62.3%

50	2.33%	60.2%
55	2.77%	57.0%
60	3.28%	52.8%

$$\begin{aligned} \text{Maximum Closing Costs} &= P_{x+1} - A_{x:1} \\ \text{Maximum Closing Costs as a Percent of Premium} \\ &= (P_{x+1} - A_{x:1}) / P_{x+1} \end{aligned}$$

It follows from this table that if the average issue age is 37 years or a little more, closing costs of a little more than 1 3/8 percent of the insured sum can be applied; if we take into consideration that the maximum rate for higher ages grows in an increasing relationship and that furthermore the younger issue ages purchase smaller sums of insurance, then we may assert that an insurance company which figures according to the Combined Experience Table and an interest rate of 3 1/2 percent can establish 1 1/2 percent of the insured sum as the average maximum that it may spend for the acquisition of an insurance policy and that it may cover by a corresponding calculable portion of the net premium.

It does not follow from this that 1 1/2 percent of the insured sum has to be figured as the closing cost; on the contrary, I hold it advisable to stay with 1 percent or 1 1/4 percent of the insured sum. It is possible for the average issue age to decrease if younger persons participate in insurance in greater numbers or if younger persons insure themselves with larger amounts than the present experience has shown. Then it is possible for the results of the calculation to develop so that the average issue age decreases and the calculable rate of closing costs has become too high.

Since even in the case where we calculate the closing costs with only 1 - 1 1/4 percent of the insured sum, with high probability but not with absolute certainty we are secured against the danger that could emerge from a decline in the average issue age; therefore I consider a precautionary measure necessary which will be discussed in the following:

If we figure as closing costs more than the maximum rate for the youngest issue age, then at the younger ages the net premium to be used for the first year for the risk is smaller than the premium for a one year term insurance policy at the issue age or, in other words, the premium reserve at the end of the first insurance year becomes negative and because of this the net premium to be used in following years would be greater than the usual net premium for an issue age one year higher. If the company includes the negative premium reserve for a younger age in the total sum of the premium reserves this lowers the total premium reserves exactly by as much as the company has too little for the calculable mortality in the course of the year [of course, drawing interest until the end of the year]. The higher the closing costs are calculated, the greater will be the amount of the negative premium reserves and by so much more would the total sum of the premium reserves be forced down thereby. Add to this the fact that the cost must be written off against the company's capital for those insurance policies which lapse after a 1 year existence. In consideration of these circumstances, I hold it essential that a company does not include the negative premium reserves in the total premium reserves, but calculates the premium reserve for the end of the year as zero.⁴ From this grows for the company an expense which exceeds the calculable expense allowance. But this is easy to handle, as will be demonstrated, so long as we can assume that the calculable closing costs have not been set too high. If the conditions change so that it appears that the closing costs calculated up to now have been set too high, this will become evident when the portion of income that would be used for the formation of the profit is reduced while the premium reserve remains untouched and the company has, at the same time, a built in regulator for the magnitude of the closing costs.

The insurance policies that had a negative premium reserve at the end of the first year are increasing more in the second year because of the consideration of the calculable closing costs. If at the end of the second year, the premium reserve should

⁴. If at the end of the business year an insurance policy has not been in existence for a full insurance year, the premium reserve for the end of the business year naturally has to be calculated according to the calculations of fractional duration premium reserve. If, for example, we calculate the premium reserves as if the full annual premiums had been paid at the beginning of each insurance year for such an insurance policy for which the premium reserve would be negative or rather zero at the end of the insurance year we must premium reserve enough so that the calculable mortality for the not yet elapsed fraction of the first insurance year is covered.

still be negative then instead of the negative value again we set the premium reserve to zero and pay the calculable premium reserve only at the end of the third year. This last happened only with the very lowest issue ages as the following table shows:

Issue Age	Net Level Premium Reserves for a Policy for \$100		
	Durations		
	1	2	3
15	0.584	1.187	1.809
16	0.606	1.232	1.878
17	0.630	1.279	1.949

Issue Age	Zillmer Premium reserves @ 1 1/4 % Closing Costs		
	Durations		
	1	2	3
15	-0.659	-0.049	0.581
16	-0.636	-0.002	0.651
17	-0.612	0.045	0.723

$${}_tV_x' = {}_tV_x - I (1 - {}_tV_x)$$

In general, the expense that the company incurs by not setting up the actual negative premium reserves but setting them to zero, that is, for some of the insurance policies the premium reserves are calculated as greater than they actually are, is not significant. If we calculate on the basis of 1 1/4 percent closing cost, the premium reserves will already be positive at the end of the first year for an issue age of 35 years; if we calculate on the basis of 1 percent, the premium reserves will already be positive for the issue age of 30. In general, the younger ages are covered with less insurance the lower the age so that where the closing expense for the individual insurance policy becomes somewhat larger, then there are only individual insurance policies that cause this expense. Add to this that in a steadily growing business, this expense is to be paid only in the first year. The net premium for the second year has to cover the mortality of that year and the negative premium reserve for the end of the first year and beyond that provide the positive premium reserve for the end of the second year. Since the negative premium reserve has already been defrayed by the company, this portion of the net premium flows towards its profit or rather this portion defrays the negative premium reserves for

the new insurance policies. In a business that grows at an increasing rate, the expense would repeat itself every year, and the increase would be the difference by which the insurance policies for the younger ages have grown more than in the previous year. To have grasp of the size of these costs we will make the specific and certainly not too low assumption that in a year company insures

\$10,000	for 20 year old persons
\$20,000	for 21 year old persons
\$30,000	for 22 year old persons
...	...
\$100,000	for 29 year old persons
\$120,000	for 30 year old persons
\$140,000	for 31 year old persons
...	...
\$200,000	for 34 year old persons

Then if we calculate closing costs as 1 percent of the insured amounts, we would have

		Negative Premium Reserve	
Issue Age	Insured Sum	As Percent of Insured Sum	In Total (\$)
20	\$10,000	-0.289%	-28.9
21	\$20,000	-0.263%	-52.5
22	\$30,000	-0.234%	-70.3
23	\$40,000	-0.205%	-82.2
24	\$50,000	-0.175%	-87.5
25	\$60,000	-0.143%	-85.8
26	\$70,000	-0.110%	-77.1
27	\$80,000	-0.075%	-60.4
28	\$90,000	-0.040%	-35.9
29	\$100,000	-0.002%	-2.2

Sum	\$550,000		-582.8
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Col. 3 = Premium reserve Factor without sign

$${}_tV_x' = {}_tV_x - I (1 - {}_tV_x)$$

Col. 4 = Col. 3 * Col. 2

If we calculate closing costs as 1 1/4 percent of the insured amount

		1 1/4 % Closing Cost Negative Premium reserve		
Issue Age	Insured Sum	As % of Insured Sum	In Total	
20	\$10,000	-0.538	-53.8	
21	\$20,000	-0.511	-102.1	
22	\$30,000	-0.482	-144.7	
23	\$40,000	-0.453	-181.4	
24	\$50,000	-0.423	-211.5	
25	\$60,000	-0.391	-234.5	
26	\$70,000	-0.358	-250.6	
27	\$80,000	-0.323	-258.6	

28	\$90,000	-0.288	-258.8
29	\$100,000	-0.250	-249.7
30	\$120,000	-0.211	-253.0
31	\$140,000	-0.170	-237.4
32	\$160,000	-0.127	-203.1
33	\$180,000	-0.082	-147.0
34	\$200,000	-0.033	-66.8
Total	\$1,350,000		-2,853.0

If a company in every year issues \$550,000 more insurance inforce than in the preceding year for the issue ages under 30, then it would have, at 1 percent closing costs, an expenditure that exceeds the calculable only by \$586, that is, this expenditure does not even amount to 1/4 percent of the premium income as soon as the premium income has climbed over \$250,000, or if the company figures on 1 1/4 percent closing costs, it would have an annual expenditure of \$2,870 when in every year it issues policies for \$1,350,000 more than in the previous year for the issue ages under 35, thus less than 1 percent of the premium income when this has exceeded \$300,000.

One could say that the company can figure the maximum rates for closing costs for every age, the total premium reserve of the new insurance policy would then amount to exactly zero, and the net premium of each individual issue age

would then become equal to the usual net premium of the next higher age. If a company calculates its premiums from the start so that it every time adds a certain percentile loading, then the premiums for the higher issue ages would become very expensive [besides this for safety it would have to make the size of the commissions for the agents likewise dependable on the issue age which as already mentioned could lead to all sorts of difficulties]. The following table shows the various percentage rates which would remain for the company from the loaded premium when the loaded premium is calculated in the usual method according to the Combined Experience Mortality Table and 3 1/2 percent interest and a loading of 12 1/2 percent. According to the usual method, the company would lose a ninth part or 11 1/9 percent of the premiums.

	For Administrative Cost Stated as a Percent of the Loaded Premium, if the First Year Premium Reduced by:		
Issue Age	Max Rate	1 % Insured Sum	1 1/4 % Insured Sum
20	8.94%	8.04%	7.28%

30	8.46%	8.55%	7.91%
40	7.80%	9.01%	8.49%
50	7.14%	9.41%	8.99%
60	6.53%	9.71%	9.36%

$$\text{Remainder} = (G_x - I - A_{x:1}) / G_x$$

This formula does not work for the second and third columns.

Additionally this table shows that the agency commissions which according to the earlier method were measured according to a determined percentage rate of the premium, according to the new method could be easily so determined that it becomes more advantageous for the company. After a significant commission in the first year after the closing of the insurance policy the agent will be readily satisfied with a smaller commission for the later years. If, for example, earlier a company paid as agents commission 6 percent of the premium, it would have only 5 1/9 percent for the other administrative costs. Now, if besides the closing commission it pays only 2 to

3 percent premium commission, there remains for it for most issue ages more than 5 1/9 percent.

As it concerns the premium reserve for an insurance policy that has existed for a number of years, the premium reserve would be somewhat less than according to the previous method. The difference between the two premium reserves will become smaller each year. As can easily be proved with the help of mathematical symbols, when the calculable rate of the closing commissions is equal to "a" percent of the insured amount, then the difference in the premium reserves is "a" percent of the insured sum lessened by "a" percent of the premium reserve calculated according to the old method.

$${}_tV_x - {}_tV_x' = I (1 - {}_tV_x)$$

Since the premium reserve gets bigger every year, the difference will become less with every year. For example, if the old premium reserve for an insurance policy of \$100 has the value of:

\$25.00 then the new premium reserve with 1 % closing costs is \$24.25

50.00	,,	,,	,,	,,	,,	,,	,,	,,	49.50
75.00	,,	,,	,,	,,	,,	,,	,,	,,	74.75
100.00	,,	,,	,,	,,	,,	,,	,,	,,	100.00

The total premium reserve of an insurance company for all insurance policies whose premium reserves are calculated according to the new principle has likewise a smaller value than if it were calculated according to the old method because each individual premium reserve is smaller. The difference will be relatively greater where the greatest number of insurance policies have not existed for very long. The difference here is also "a" percent of the insured sum lessened by "a" percent of the premium reserve calculated according to the old method when "a" percent of the insured sum was calculated as closing costs. For example, at the end of the year 1861, the Life Insurance Company of Gotha had insurance inforce of \$38,800,000 and a premium premium reserve of about \$8,700,000. If the Life Insurance Company of Gotha had figured from the beginning 1 percent closing costs, its premium reserves would be about:

(\$388,000 - \$87,000) that is, \$301,000 smaller.

Table I (attached) follows the premium reserve for an insurance policy of \$100 for the issue ages of 30,40,and 50 through the whole term of the insurance policy. We see that the premium reserves with calculation of closing costs grow faster than the old premium reserves as it must be since from the second policy year on there is a higher, calculated net

premium. The Tables II, III, and IV each follow a company of persons of the same age that insure themselves at the same time, and Table V gives a picture of the results achieved by an insurance company when it reinsures every year such cohorts as are followed individually in Tables II, III, and IV. It shows that after a sequence of 60 years, with an insurance inforce of about \$41,900,000, the old premium reserve would have grown to \$12,644,900. The premium reserve with 1 percent closing costs would have grown to \$12,352,000, and the premium reserve with 1 1/4 percent closing costs would have grown to \$12,279,100. The difference between the old and the new premium reserve is, according to the above, with 1 percent closing costs $\$419,102 - \$126,449 = \$292,653$ with 1 1/4 percent closing costs $\$419,102 - \$126,449) * 5/4 = \$365,817$.

With the assumptions of Table V, a stationary state comes after 60 years, that is, as the consequence of annual cases of death, respectively, expirations of insurance policies, the same sum comes due which is annually newly insured. Therefore, in the future the insurance inforce, premium income, and premium reserve remain constant as long as the assumptions are met. In such a state, obviously, the net premiums and the calculable interest on those premiums and on the premium reserve capital must be as great as the annually payable

insurance sum. From the existing insurance inforce after 60 years come:

\$16,947,500 to members that join at age 30

\$20,055,100 to members that join at age 40

\$ 4,907,600 to members that join at age 50

If we now add to these values the expected premium and interest income sums and calculate premium incomes as follows:

	Net Premium if the First Year's Premium is Reduced by		
Issue Age	0 %	1 %	1 1/4 %
30	1.7973	1.8490	1.8620
40	2.4841	2.5428	2.5575
50	3.7083	3.7792	3.7969

-

$$\beta_x = P_x + I / \ddot{a}_x$$

-

We get the following income schedule which should cover the operations of the company:

1) when the premium reserve is calculated without closing costs

Net Premium	\$1,021,661
3 1/2 percent interest thereon	35,758
3 1/2 percent interest on premium reserve	442,547

Total

\$1,499,966

2) if the premium reserve is calculated with 1 percent closing costs

Net Premium (after deduction of \$15,000)	\$1,031,547
3 1/2 percent interest thereon	36,104
3 1/2 percent interest on the premium reserve	
432,327	

Total	\$1,499,978

3) if the premium reserve is calculated with 1 1/4 percent closing costs

Net Premium (after deduction of \$18,750)	\$1,034,019
3 1/2 percent interest thereon	36,190
3 1/2 percent interest on the premium reserve	
429,768	

Total	\$1,499,977

The small differences of \$34, \$22, and \$23 respectively against the required sum of \$1,500,000 have their origin only in the round off that the numbers in Table V were subjected to (this is an alternative proof of the adequateness of the various premium reserves).

The above demonstrates clearly that a premium reserve calculated with a rate of closing costs that is not too high is completely sufficient for the solvency of the company if the insurance policies exist until their normal end. If an insured allows his insurance to lapse while he is still alive, then this would not be awkward for the company especially when it has used the precautionary rule that the negative premium reserves at the younger issue ages are not put up with the calculable amount but with the amount of zero, and when the agent does not receive a corresponding portion of the closing commissions or must pay out respectively in the case where the insurance lapses during the first year of its existence. On the contrary the collective premium reserves on such insurance policies go to the company and flow to its profits. If at lapsing the insurance has been in existence long enough so that the insured can demand a buy back price, only the lower premium reserve can come into the calculations. As far as we know a company is (not)obliged in such a case to pay the whole premium reserve, only a certain portion of the premium reserve. If a company, for example, customarily pays 3/4 of the premium reserve, then it can pay back just as much as when the premium reserve was calculated without closing costs as soon as the premium reserve has grown, for a \$100 insured sum with 1

percent closing costs, to over \$3 and, with 1 1/4 percent closing costs, to over \$3 3/4 or, when the premium reserve without closing costs, had grown to \$4 or \$5, respectively, and it can pay back better since it has already paid itself for all closing costs.

II

Mr. Wilhelm Lazarus in the above mentioned paper "The Security of Life Insurance Institutions" (Rundschau der Versicherungen, 1857, p.317) called attention to it, noting that in the closing of the books of the Gresham Life Insurance Society on 12 November 1855, the liabilities of the society toward its insureds, even though it had been in existence for 7 years already, were valued lower than the value of premiums yet to be collected and yet a significant sum of the premiums to be used for the coverage of obligations had already been collected. A similar relationship shows itself in the balance sheet of the Great Britain published some time ago. After an almost 20 year existence, instead of a premium reserve, the Great Britain had as an asset the value of future premium income 529,469 pounds and as a liability the present value of the insured sum 494,707 pounds. Not only is there no premium reserve on hand, but the company figures for itself a net worth or profit of 34,762 pounds or \$231,700 which sum is the present value of what

Note: 34,762 pounds is 231,700 talers. This is 6 2/3 talers per pound. The exchange rate to U.S. dollars at this time was

5 dollars per pound. Thus the exchange rate was 1 1/3 taler per dollar or \$.75 for a taler.

-
the company will collect in excess of what it expects to pay at the close of the existing insurance policies.

It is known that in the calculation of the net premium, the obligations of the insured are equalized with the obligations of the company. If we continue on to the premium reserve calculation with net premium, then under special circumstances, as was shown in the first segment, the premium reserve of a portion of the insurance policies flowing out of the current year can become negative in calculation but this negative amount is of little significance as opposed to the premium reserves of all the insurance policies of that year and disappearing as opposed to the total premium reserves of all insurance policies from various years. It is then impossible, using calculations with net premiums, for the premium reserve to work out in the way it did with the Great Britain; and, in practice, a significant number of English companies calculate their premium reserves so that the premiums to be paid in the future by the insureds are calculated not according to the net

premiums but according to a much higher premium and as a rule according to the loaded premium while the obligations of the company are valued according to the net premiums. As a consequence of this calculation, the premium reserve does not meet the demand that it be formed out of portion of the premium demanded for the risk after a one-time deduction of expenses covering calculable mortality and the addition of calculable interest. This method of calculating the premium reserves is clearly misleading. Apparently it is calculated according to the same principles as in the calculation with net premiums. One deducts from the value of the insured sum [the net single premium for insurance] the value of the premiums to be collected in the future, only instead of the net premium one takes the gross premiums. Herein lies the mistake. This principle for the calculation of premium reserves is generally only correct [even if it can be directly deduced] because and so far as it is a reformulation of the other principle according to which the premium reserve consists of the portion of the net premium not used up for mortality and the duly collected interest. If a company would enjoy a growing advantage out of closed insurance policy as soon as possible, then there is only the following way to sanction this procedure. So long as the loaded premium paid by an insured is

higher than the net premium of that age into which the insured is gradually moving, the company figures as net premium the premium for one year term insurance, the excess of the loaded premium over the one year term premium it figures as profit. Once the insured has reached that age for which as issue age the net premium of the whole life insurance equals that premium which the insured pays, the whole life premium is charged. During the first (5 to 7) years when this forms absolutely no premium reserve, and only later would the company have a portion of the premium for premium reserve. Even with this model which borders on the imaginable, there is no talk about negative premium reserves. This is a new proof that this method of the English companies for calculating premium reserves with gross premiums is misleading.

The just described way of enjoying the advantage from an insurance policy as quickly as possible is surpassed by the method using gross premiums. Here in the moment of the closing of an insurance policy, a profit appears for as many percent of the total value of the future premium payments as the percentage loading included in the premiums, or since the value of the future premium payments measured according to net premiums is equal to the net single premium for the insurance there is a profit in the amount of just as much a percent of

the net premium, as the loaded premium is higher than the net premium; for example, if a company figures according to the Combined Experience Table with an interest rate of 4 1/2 percent [in many cases, a higher interest rate is figured and the thus reduced net premiums are loaded on this much more] and 30 percent load of the net premiums, at the moment of the closing of the insurance policy the profit amounts to:

Issue Age	Profit as Percent of Insured Sum
20	6.63%
30	8.16%
37	9.60%
40	10.34%
50	13.38%
60	17.02%

 -

$$(.30) A_x / 100$$

 -

If such a company in a year closes insurance policies in the amount of \$5,000,000 [on the average issue age 37], in so doing it figures for itself a profit of not less than \$480,000, that is,
 $\$5,000,000 * 9.60\%.$ ⁵

Therefore the first method cannot be sanctioned because nothing remains for the company to take care of business expenses if the net premium to be calculated for obligations of the company [premium reserve and mortality] during the term of the contract reaches the magnitude of the loaded premium. If the company has significant stock capital, it can cover those

⁵. Aside from the other consequences that such figures must have on the financial development, this procedure seems to be unjustified, not to use a stronger expression, because this profit can only be realized when the insureds maintain their insurance in force until the regular expiration or even could be forced to maintain them in force. But the company knows that a number of the insurance policies will lapse and in spite of this ascribes the profit of these to itself. It is really very mildly judged when one asks as already Mr. Lazarus did in his essay "What would one say when a merchant puts as an increase of his assets profit which is hoped for out of an initiated, but unconsummated deal?" However the merchant could achieve this goal while the insurance company knows in advance that it will not achieve its goal in so and so many cases.

costs with the interest on this capital when the business expansion is small. If the expansion of the business exceeds a certain limit then the interest on even the most significant capital will not be sufficient. One cannot sanction a model which, with the expansion of the business, does not offer growing means for the coverage of business costs -- with traditionally figuring companies, the means for the coverage of business costs and beyond that for the formation of profits, increase first in a determined relationship to premium income and then in a certain relationship to premium premium reserve in that the interest realized above the regular percentage rate [and this will probably not be higher than 3 1/2 percent in solid companies] flow not to the premium reserve but to the administrative accounts, respectively, the profit. This last income as a rule is completely missing in the irrationally figuring companies because usually they have a calculated interest rate that is so high that the actual interest on the premium reserve capital is actually much lower than that required by calculation.

The method of premium reserve calculation from the gross premiums suffers from the same problem, that it leaves nothing left for coverage of administrative costs from the premiums that will come in the future. It brings with it several

disadvantages, that bring with them a soon and sure ruin of the company.

First we will compare the magnitude of the premium reserves that have been calculated according to the method of gross premiums with the magnitude of rationally calculated premium reserves. Table I shows in its fourth column the premium reserve for issue ages 30, 40, and 50 on the basis of a premium provided with an interest rate of 4 1/2 percent and a loading of 30 percent. The table shows that in the first 5 to 7 years, the premium reserve is negative. Later the premium reserve achieves a positive value and when the insurance policy continues until its end, the premium reserve finally grows to the full amount of the insurance sum. Table II shows in its seventh column the premium reserves which the company puts up for the number of persons who at issue time had been 30 years old and insured altogether for \$500,000 when the appropriate insurance policies are followed up to the natural end. Likewise Table III and IV show similar results for people who at issue age are 40 respectively 50 years old. Columns 8 and 9 in Tables II through IV also demonstrate the asset and liability postings that are so beloved in the English accounting. The difference of the numbers from columns 8 and 9 give the premium reserves under column 7. Table V contains the

summary of the three preceding tables. When we add numbers of the individual columns from the top down, we get those results that an insurance company achieves when it insures such groups of insurance policies each year. The row sums combined out of the 3 tables gives then the numbers for Table V. The premium rates [Column 3] that were the basis for the premium income are the premiums of 4 1/2 percent raised by 30 percent. The columns 8 and 9 were omitted but they could be developed by the addition of the 3 Tables II to IV.

Table V shows that the total premium reserve becomes positive only in the 13th year, and that finally after 60 years, where under the assumed circumstances, likewise takes place a stationary condition, the premium reserve is about \$4,000,000 smaller than the premium reserve found by rational calculation in columns 4, 5, and 6. This difference amounts to about 10 percent of the insurance sum, more than 30 percent of the rational premium reserve, and about 350 percent of the premium income. These relationships become even more colossal if we presume a steadily growing business as it happens in Table IV. Here after 30 years the difference between the premium reserves from the gross premium and the rationally calculated premium reserve already amounts to about \$9,000,000, that is, more than 50 percent of the rational premium reserve.

To examine further what influence the calculation with gross premiums has on the development of the business, we will first assume that no insurance policy expires before its regular end. If the company does for a longer time equal business then, and this is certainly an unnatural phenomenon, the profit lessens from year to year. If, for example, we use the numbers of Table V and assume the existence of a fund from which the company can withdraw the artificially calculated profit for as long as the actually collected premium moneys do not reach so far and if we further assume that all funds draw interest at 4 1/2 percent, and if we calculate agency commissions as 1/2 of the first annual premium and only 1 percent of all later premiums and as administrative costs annually \$20,000 multiplied by .1 percent of the insurance sum, the company will achieve profits as they are put together in the following table:

Business Year	Profit	Premium Income	Profit as a Percent of Premium Income
1	\$120,000	\$43,800	274%
5	\$111,400	\$213,900	52%
10	\$101,700	\$413,800	25%
15	\$92,200	\$597,300	15%
20	\$83,400	\$760,700	11%
30	\$67,600	\$1,012,400	7%
60	\$50,500	\$1,209,500	4%

(table from page 28)⁶

Profit therefore decreases by about 58 percent in the course of time since with an equal or even business, the profit becomes less each year. It is a necessary consequence that the

⁶. Here the profit is greater than the premium income. It generally happens often with the companies that calculate in this way that they have more profit than they own in cash or money value. Therefore one frequently finds that colossal profit postings have been held back for later distribution.

whole profit that a policy can bring with regular existence will be used up in the year of the end of the insurance policy.

If the business continues in the same way, there would result year after year the same profit if the business expense remains the same. But these business expenses grow with the expansion of the business; therefore the profit must decline.

Conversely, if a company wants to achieve equal profits in each following year, it must do more and more business so that the profit will not sink under a certain percent of the premium income [In the last case, the profit from a certain point in time on would have to grow proportionately to the premium income].

This appearance which characterizes the whole system as unhealthy emerges even without lapses. But what happens when a large part of the insurance policies lapse time and again? Then here emerges the main problem of this system. The lapsing of insurance policies occurs, to a large extent, in insurance policies that are not very old yet, where as a rule the premium reserve is still negative. With the lapsing of such an insurance policy, the negative premium reserve, respectively, the excessive asset posting disappear; the company now must cover this disappearance by new insurance policies which bring with them negative premium reserves and only with growth beyond

that comes the attainment of the annual profit. Since with growing expansion, the number of lapsing insurance policies also grows then the business has to grow faster and faster. Since the accomplishment of this demand requires superhuman efforts, the sad consequences of this unhealthy system are very soon demonstrated here too. Even with the calculation with gross premiums according to which the premium reserve, the debt of the company to its insureds, is established so small in the balance sheet even sometimes conversely as a debt of the insured, the companies very soon get into difficulties because the expansion is not increasing sufficiently [the losses in interest income because of the too high assumed interest rate are not even calculated here]. To figure out the proper profit, the to cover the no longer manageable deficit, the companies are forced to extrapolate all kinds of doubtful postings in accruals, and it is very difficult for the public to judge their value. When this maneuver finally will not work any more, one attempts to get rid of the whole business by ceding it to another company. It would hardly seem possible that a company would get rid of a business in such a sickly state; but the experience shows that every year in England so and so many amalgamations and business transfers occur. How a company can feel motivated to take over such an unhealthy

business cannot even be conceived in a German imagination. Such companies can only speculate that in the next years the premium income will exceed the expenditures for mortality and that they could in some one form calculate the excess part of the premium income as gain. When it finally will not go any farther, the business collapses; it does no harm when the many insureds remain unsatisfied. The executives of the business and their shareholders, these too have enjoyed their benefits.

Nothing keeps them from establishing a new business under some other name. A fraternal burial society which is arranged as badly as possible, which collects contributions according to the principle of the present need, is to be preferred to such a life insurance company. When the burial society cannot continue on the whole, the members have paid as little as possible, and certainly too little; however, the policy holders of such a company have paid sufficient amounts and yet come out empty in the end.

When an insured, who has already been insured long enough so that for him, individual calculations result in a positive premium reserve, wants to give up his insurance policy, the company according to calculations could concede a buy back price, but in reality there is no money, for against the positive premium reserve of the long term insurance falls the

negative premium reserve of a younger insured, and both premium reserves together cancel each other partly or fully.

Therefore, as a rule, one seeks vainly in the insurance terms of such companies for a rule which obliges the company to buy back the policy; many companies declare frequently, naturally without adding the true reason, that a buy back in mass would be disadvantageous to the company. Naturally when one has to pay more than one has, that can hurt.

We can establish different consequences of the unhealthy system of calculating with gross premiums. We want to mention only the following. When a company suddenly ceases every entry of new insurance policies, in the rational calculation the premium reserve will grow for a time if a stationary state had not already been entered. In any case the premium reserve grows less than when the new insurance policies are added. It is reversed in a system of gross premiums. If insurance policies are not being added any more during a series of years, the premium reserve will grow stronger than when new insurance policies are added.

Several companies, that evidently are calculating with gross premiums, declare that they in the calculations of the value of future premium payments set apart a sufficient portion

of the loaded premiums for the administrative costs. Suppose now that the value of the insured sums is calculated not only according to the same mortality table, but also the same interest rate as the value of the future premium payments [a higher interest rate in the calculation of the value of the insured sums, and many companies calculate this way, obviously would form this value as less and thereby cancel the setting aside of a part of the loaded premium for the administrative costs when not leading to worse results]; the partitioning of the whole loading would lead to a rational establishment of the premium reserve. As a rule, the premiums have colossal loadings, occasionally 40% or more of the net premiums and above; if now about 5 percent of it is partitioned, then the premiums that are to be used for the calculation of future premium income are loaded so sufficiently that the premium reserve for the individual insurance policy here also needs several years to go over from the negative to the positive. If the annual business profit with equal additions and without losses by lapsing would become equal [the same year after year], that is, in the sum, not in the percent of premium income then since a portion of the insurance policies will lapse to achieve the equal gain, the annual additions must grow or increase, and since with growing additions there also grows

the number of lapsing insurance policies, the additions must grow for so much more. Should the annual profit not sink below a certain percentage rate of the premium income, the annual additions must naturally grow so much faster. Here as above, the condition of the company is tied to the demand that the annual additions must grow faster each year. Also for this case, there result the same consequences as when the premium reserves are calculated according to the loaded premiums only here, they occur somewhat slower and milder.

If we summarize the above, then the mistakes of the premium reserve calculation from gross premiums which are higher than the net premiums calculated for the risk, but in general equal to the loaded premiums, consists of the following:

- 1) At the close of an insurance policy, there is calculated an artificial gain⁷ which consequently may not be realized and in so and so many cases is not actually realized;
- 2) The premium reserve of the individual insurance policy is in and for itself already too low besides that the negative premium reserves of the younger insurance

⁷. This gain is by as much higher the higher the loading --Therefore one finds in England almost only mutual or mixed companies. Pure stock companies would have to add the possibly smallest loading to their net premiums.

policies partly or wholly cancel the positive premium reserves of the older insurance policies;

- 3) The gain mentioned under 1) is the sole or main source a) for absorbing the administrative costs,
- b) for the coverage of the disappearing negative premium reserves as consequence of the premature expiration of the insurance policies,
- c) for the annual profit.

Since both the first postings [a) and b)] are necessary expenses and in addition growing ones, but since the profit mentioned under 1) cannot always grow further or what amounts to the same thing since the annual addition or growth cannot go higher forever with each following year, then the annual profit mentioned under c) sooner or later must let up and finally a deficit will come into existence and the company will become insolvent.

Premium reserves for an endowment at age 90 with a face amount of \$100 and premiums payable annually, calculated according to the Combined Experience Table.

Table I Zillmer Premium Reserve Calculations Endowment @90 Interest Rate 3.5% Issue Age 30 Net Premium 1.797252			
I=	0	I=	1
I=		I=	1.25

	Initial Reserves	Terminal Reserves	Initial Reserves	Terminal Reserves	Initial Reserves	Terminal Reserves
1	1.7973	1.0263	0.7973	0.0366	0.5473	-0.2109
2	2.8236	2.0824	1.8338	1.1033	1.5864	0.8585
3	3.8797	3.1685	2.9005	2.2002	2.6557	1.9581
4	4.9658	4.2859	3.9974	3.3287	3.7554	3.0894
5	6.0831	5.4359	5.1260	4.4903	4.8867	4.2539
6	7.2332	6.6190	6.2875	5.6852	6.0511	5.4518
7	8.4163	7.8367	7.4825	6.9151	7.2490	6.6847
8	9.6340	9.0905	8.7123	8.1814	8.4819	7.9541
9	10.8878	10.3811	9.9787	9.4849	9.7514	9.2608
10	12.1783	11.7101	11.2821	10.8272	11.0581	10.6065

<p>Table I Zillmer Premium Reserve Calculations Endowment @90 Interest Rate 3.5% Issue Age 30 Net Premium 1.797252</p>						
	I= 0		I= 1		I= 1.25	
	Initial Reserves	Terminal Reserves	Initial Reserves	Terminal Reserves	Initial Reserves	Terminal Reserves

11	3.5073	13.0794	12.6244	12.2102	12.4037	11.9929
12	14.8767	14.4899	14.0075	13.6348	13.7902	13.4211
13	16.2872	15.9415	15.4321	15.1009	15.2183	14.8907
14	17.7387	17.4306	16.8981	16.6049	16.6880	16.3985
15	19.2279	18.9528	18.4022	18.1423	18.1958	17.9397
16	20.7501	20.5055	19.9396	19.7106	19.7370	19.5118
17	22.3028	22.0830	21.5078	21.3038	21.3091	21.1090
18	23.8802	23.6846	23.1011	22.9214	22.9063	22.7307
19	25.4818	25.3086	24.7187	24.5617	24.5279	24.3750

20	27.1059	26.9545	26.3590	26.2240	26.1723	26.0414
21	28.7517	28.6203	28.0213	27.9065	27.8387	27.7281
22	30.4176	30.3045	29.7038	29.6075	29.5254	29.4333
23	32.1017	32.0050	31.4048	31.3250	31.2305	31.1550
24	33.8022	33.7198	33.1223	33.0570	32.9523	32.8913
25	35.5171	35.4489	34.8543	34.8034	34.6886	34.6420
26	37.2462	37.1891	36.6007	36.5609	36.4393	36.4039
27	38.9863	38.9387	38.3582	38.3281	38.2012	38.1754
28	40.7360	40.6982	40.1253	40.1052	39.9727	39.9569
29	42.4954	42.4647	41.9024	41.8893	41.7542	41.7455
30	44.2619	44.2360	43.6866	43.6783	43.5427	43.5389

Issue Age 30 (Continued)

	I= 0		I= 1		I= 1.25	
	Initial Reserves	Terminal Reserves	Initial Reserves	Terminal Reserves	Initial Reserves	Terminal Reserves
31	46.0332	46.0064	45.4756	45.4665	45.3362	45.3315
32	47.8037	47.7736	47.2637	47.2513	47.1287	47.1208
33	49.5709	49.5334	49.0486	49.0288	48.9180	48.9026
34	51.3307	51.2838	50.8260	50.7967	50.6998	50.6749
35	53.0811	53.0210	52.5939	52.5512	52.4721	52.4338
36	54.8182	54.7418	54.3485	54.2892	54.2310	54.1761
37	56.5391	56.4441	56.0865	56.0085	55.9733	55.8996
38	58.2413	58.1243	57.8058	57.7055	57.6969	57.6008
39	59.9215	59.7814	59.5028	59.3792	59.3981	59.2787
40	61.5787	61.4155	61.1765	61.0296	61.0759	60.9332
41	63.2127	63.0242	62.8269	62.6545	62.7304	62.5620
42	64.8215	64.6072	64.4517	64.2532	64.3593	64.1647
43	66.4044	66.1636	66.0505	65.8252	65.9620	65.7406
44	67.9608	67.6942	67.6225	67.3711	67.5379	67.2903
45	69.4914	69.1987	69.1684	68.8907	69.0876	68.8137
46	70.9959	70.6789	70.6879	70.3857	70.6109	70.3124
47	72.4761	72.1380	72.1829	71.8594	72.1096	71.7897
48	73.9353	73.5777	73.6566	73.3135	73.5870	73.2474
49	75.3750	75.0023	75.1107	74.7523	75.0447	74.6898
50	76.7995	76.4206	76.5495	76.1848	76.4870	76.1259
51	78.2179	77.8448	77.9821	77.6232	77.9232	77.5678
52	79.6420	79.2939	79.4205	79.0868	79.3651	79.0350
53	81.0911	80.7952	80.8841	80.6032	80.8323	80.5551
54	82.5925	82.3844	82.4004	82.2082	82.3524	82.1642
55	84.1816	84.1150	84.0055	83.9561	83.9615	83.9164
56	85.9122	86.0601	85.7534	85.9207	85.7136	85.8859
57	87.8574	88.3378	87.7180	88.2212	87.6831	88.1920
58	90.1350	91.1447	90.0184	91.0562	89.9893	91.0340
59	92.9420	94.8211	92.8534	94.7693	92.8313	94.7564
60	96.6184	0.0000	96.5666	0.0000	96.5536	0.0000

Table I.
 Zillmer Premium reserve Calculations
 Endowment @90, Load 30% NSP
 Interest Rate 4.5%
 Issue Age 30

	Net Premium		1.609444	
	I= 0		I= 8.1619	
	Initial Reserves	Terminal Reserves	Initial Reserves	Terminal Reserves
1	1.6094	0.8465	-6.5525	-7.2463
2	2.4560	1.7234	-5.6369	-6.2978
3	3.3329	2.6312	-4.6884	-5.3160
4	4.2406	3.5714	-3.7065	-4.2990
5	5.1808	4.5458	-2.6896	-3.2451
6	6.1552	5.5550	-1.6356	-2.1535
7	7.1645	6.6010	-0.5440	-1.0221
8	8.2104	7.6857	0.5873	0.1511
9	9.2951	8.8101	1.7605	1.3672
10	10.4195	9.9763	2.9767	2.6287
11	11.5858	11.1869	4.2381	3.9380
12	12.7963	12.4430	5.5475	5.2967
13	14.0525	13.7451	6.9061	6.7051
14	15.3546	15.0902	8.3145	8.1600
15	16.6997	16.4741	9.7694	9.6568
16	18.0836	17.8947	11.2663	11.1933
17	19.5041	19.3463	12.8027	12.7634
18	20.9557	20.8287	14.3728	14.3668
19	22.4381	22.3404	15.9762	16.0019
20	23.9499	23.8812	17.6114	17.6685
21	25.4906	25.4495	19.2779	19.3648
22	27.0589	27.0437	20.9742	21.0891
23	28.6532	28.6623	22.6986	22.8398
24	30.2717	30.3032	24.4492	24.6147
25	31.9127	31.9668	26.2241	26.4140
26	33.5762	33.6497	28.0234	28.2343
27	35.2592	35.3507	29.8437	30.0741
28	36.9602	37.0703	31.6836	31.9341
29	38.6798	38.8057	33.5435	33.8111
30	40.4152	40.5547	35.4206	35.7029
31	42.1642	42.3115	37.3123	37.6030
32	43.9210	44.0735	39.2125	39.5089
33	45.6830	45.8365	41.1183	41.4157
34	47.4459	47.5981	43.0252	43.3211
35	49.2076	49.3543	44.9306	45.2206

36	50.9637	51.1015	46.8301	47.1105
37	52.7110	52.8374	48.7199	48.9880
38	54.4468	54.5578	50.5975	50.8489
39	56.1673	56.2617	52.4583	52.6918
40	57.8711	57.9486	54.3012	54.5164

Endowment @90, Load 30% NSP

Interest Rate 4.5%

Issue Age 30 (Continued)

Net Premium 1.609444

I= 0 I= 8.1619

-----|-----|
Initial Terminal Initial Terminal
Reserves Reserves Reserves Reserves
-----|-----|

41	59.5580	59.6158	56.1258	56.3197
42	61.2253	61.2627	57.9292	58.1010
43	62.8721	62.8881	59.7104	59.8591
44	64.4976	64.4925	61.4685	61.5944
45	66.1019	66.0753	63.2038	63.3064
46	67.6847	67.6380	64.9158	64.9967
47	69.2474	69.1840	66.6061	66.6688
48	70.7935	70.7148	68.2783	68.3245
49	72.3242	72.2346	69.9340	69.9684
50	73.8440	73.7533	71.5779	71.6110
51	75.3627	75.2837	73.2205	73.2664
52	76.8931	76.8472	74.8758	74.9575
53	78.4566	78.4743	76.5669	76.7174
54	80.0837	80.2054	78.3268	78.5898
55	81.8148	82.1016	80.1992	80.6407
56	83.7110	84.2472	82.2502	82.9614
57	85.8566	86.7787	84.5709	85.6996
58	88.3881	89.9251	87.3090	89.1028
59	91.5346	94.0843	90.7122	93.6015
60	95.6938	0.0000	95.2109	0.0000

Table I.
 Zillmer Premium reserve Calculations
 Endowment @90
 Interest Rate 3.5%

Issue Age 40		2.48414		1.25		
Net Premium						
I =	0	I =	1	I =	1.25	
-----	-----	-----	-----	-----	-----	
Initial	Terminal	Initial	Terminal	Initial	Terminal	
Reserves	Reserves	Reserves	Reserves	Reserves	Reserves	
-----	-----	-----	-----	-----	-----	
1	1.7973	1.5510	0.7973	0.5665	0.5473	0.3203
2	3.3482	3.1486	2.3637	2.1800	2.1176	1.9379
3	4.9458	4.7926	3.9773	3.8405	3.7352	3.6025
4	6.5899	6.4793	5.6378	5.5441	5.3998	5.3103
5	8.2765	8.2033	7.3413	7.2854	7.1075	7.0559
6	10.0006	9.9620	9.0826	9.0616	8.8531	8.8365
7	11.7593	11.7487	10.8589	10.8662	10.6338	10.6456
8	13.5460	13.5627	12.6634	12.6984	12.4428	12.4823
9	15.3600	15.4022	14.4956	14.5562	14.2795	14.3447
10	17.1994	17.2663	16.3535	16.4390	16.1420	16.2321
11	19.0635	19.1531	18.2362	18.3446	18.0294	18.1425
12	20.9504	21.0606	20.1419	20.2712	19.9398	20.0739
13	22.8579	22.9866	22.0685	22.2165	21.8711	22.0240
14	24.7839	24.9290	24.0138	24.1782	23.8212	23.9906
15	26.7262	26.8874	25.9755	26.1563	25.7878	25.9735
16	28.6846	28.8583	27.9535	28.1469	27.7707	27.9690
17	30.6556	30.8400	29.9441	30.1484	29.7663	29.9755
18	32.6373	32.8329	31.9457	32.1612	31.7728	31.9933
19	34.6301	34.8336	33.9584	34.1820	33.7905	34.0191
20	36.6309	36.8399	35.9792	36.2083	35.8163	36.0504
21	38.6371	38.8451	38.0055	38.2336	37.8476	38.0807
22	40.6424	40.8467	40.0308	40.2552	39.8780	40.1073
23	42.6440	42.8399	42.0524	42.2683	41.9045	42.1254
24	44.6372	44.8225	44.0656	44.2707	43.9227	44.1328
25	46.6198	46.7901	46.0680	46.2580	45.9301	46.1249
26	48.5873	48.7391	48.0552	48.2265	47.9222	48.0984
27	50.5364	50.6672	50.0238	50.1738	49.8956	50.0505
28	52.4644	52.5702	51.9711	52.0959	51.8478	51.9773
29	54.3674	54.4471	53.8932	53.9916	53.7746	53.8777
30	56.2444	56.2979	55.7889	55.8609	55.6750	55.7517
31	58.0952	58.1201	57.6582	57.7013	57.5489	57.5966
32	59.9173	59.9129	59.4985	59.5121	59.3938	59.4118
33	61.7102	61.6758	61.3093	61.2926	61.2091	61.1968
34	63.4731	63.4094	63.0898	63.0435	62.9940	62.9520

35	65.2066	65.1134	64.8407	64.7646	64.7492	64.6774
36	66.9107	66.7899	66.5618	66.4578	66.4746	66.3748
37	68.5872	68.4426	68.2551	68.1270	68.1721	68.0481
38	70.2399	70.0733	69.9243	69.7740	69.8454	69.6992
39	71.8705	71.6867	71.5713	71.4036	71.4964	71.3328
40	73.4840	73.2933	73.2009	73.0262	73.1301	72.9594

Issue Age 40 (Continued)

	I = 0		I = 1		I = 1.25	
	Initial Reserves	Terminal Reserves	Initial Reserves	Terminal Reserves	Initial Reserves	Terminal Reserves
41	75.0905	74.9063	74.8234	74.6554	74.7567	74.5926
42	76.7035	76.5476	76.4526	76.3130	76.3899	76.2544
43	78.3448	78.2480	78.1103	78.0305	78.0517	77.9761
44	80.0453	80.0480	79.8278	79.8485	79.7734	79.7986
45	81.8453	82.0081	81.6457	81.8282	81.5959	81.7832
46	83.8053	84.2113	83.6254	84.0534	83.5804	84.0139
47	86.0085	86.7910	85.8506	86.6589	85.8112	86.6259
48	88.5883	89.9702	88.4562	89.8699	88.4231	89.8448
49	91.7675	94.1342	91.6672	94.0756	91.6421	94.0609
50	95.9315	0.0000	95.8728	0.0000	95.8581	0.0000

Table I.
 Zillmer Premium reserve Calculations
 Endowment @90, Load 30% NSP
 Interest Rate 4.5%
 Issue Age 40

	Net Premium 2.265013		I= 10.3406	
	I=	0	I=	10.3406
	----- -----		----- -----	
	Initial	Terminal	Initial	Terminal
	Reserves	Reserves	Reserves	Reserves
	----- -----			
1	2.2650	1.3447	-8.0756	-8.8569
2	3.6097	2.7400	-6.5919	-7.3172
3	5.0050	4.1864	-5.0522	-5.7213
4	6.4514	5.6806	-3.4563	-4.0726
5	7.9456	7.2179	-1.8076	-2.3764
6	9.4829	8.7958	-0.1114	-0.6352
7	11.0608	10.4083	1.6298	1.1440
8	12.6733	12.0550	3.4090	2.9609
9	14.3200	13.7343	5.2259	4.8139
10	15.9993	15.4458	7.0789	6.7024
11	17.7108	17.1879	8.9674	8.6246
12	19.4529	18.9588	10.8896	10.5787
13	21.2238	20.7567	12.8437	12.5625
14	23.0217	22.5795	14.8275	14.5738
15	24.8445	24.4274	16.8388	16.6127
16	26.6924	26.2969	18.8778	18.6755
17	28.5619	28.1864	20.9405	20.7604
18	30.4514	30.0965	23.0254	22.8681
19	32.3615	32.0242	25.1331	24.9951
20	34.2892	33.9671	27.2601	27.1389
21	36.2321	35.9185	29.4039	29.2921
22	38.1835	37.8758	31.5571	31.4518
23	40.1408	39.8341	33.7168	33.6126
24	42.0991	41.7910	35.8776	35.7718
25	44.0560	43.7417	38.0368	37.9243
26	46.0068	45.6827	40.1893	40.0659
27	47.9477	47.6109	42.3309	42.1935
28	49.8759	49.5220	44.4585	44.3023
29	51.7870	51.4146	46.5673	46.3906
30	53.6796	53.2885	48.6556	48.4582
31	55.5535	55.1405	50.7232	50.5018
32	57.4055	56.9699	52.7668	52.5203
33	59.2349	58.7754	54.7853	54.5126
34	61.0404	60.5576	56.7776	56.4790

35	62.8226	62.3158	58.7440	58.4190
36	64.5808	64.0517	60.6840	60.3344
37	66.3167	65.7690	62.5994	62.2293
38	68.0340	67.4694	64.4943	64.1055
39	69.7344	69.1577	66.3705	65.9684
40	71.4227	70.8446	68.2334	67.8298

Table I.

Zillmer Premium reserve Calculations

Endowment @90, Load 30% NSP

Interest Rate 4.5%

Issue Age 40 (Continued)

Net Premium 2.265013

I= 0 I= 10.3406

-----	-----
Initial Reserves	Terminal Reserves
Initial Reserves	Terminal Reserves

41	73.1096	72.5447	70.0948	69.7056
42	74.8097	74.2814	71.9706	71.6219
43	76.5464	76.0888	73.8869	73.6163
44	78.3538	78.0118	75.8813	75.7381
45	80.2768	80.1181	78.0031	78.0622
46	82.3831	82.5015	80.3272	80.6920
47	84.7665	85.3135	82.9570	83.7948
48	87.5785	88.8086	86.0598	87.6514
49	91.0736	93.4288	89.9164	92.7493
50	95.6938	0.0000	95.0143	0.0000

Table I.
 Zillmer Premium reserve Calculations
 Endowment @90
 Interest Rate 3.5%

Issue Age 50		3.708312		1.25		
Net Premium						
I =	0	I =	1	I =	1.25	
-----	-----	-----	-----	-----	-----	
Initial	Terminal	Initial	Terminal	Initial	Terminal	
Reserves	Reserves	Reserves	Reserves	Reserves	Reserves	
-----	-----	-----	-----	-----	-----	
1	3.7083	2.2806	2.7083	1.2806	2.4583	1.0591
2	5.9889	4.5862	5.0117	3.6098	4.7674	3.3935
3	8.2945	6.9142	7.3404	5.9616	7.1018	5.7506
4	10.6225	9.2618	9.6916	8.3333	9.4589	8.1276
5	12.9701	11.6290	12.0628	10.7247	11.8359	10.5244
6	15.3373	14.0112	14.4536	13.1313	14.2327	12.9364
7	17.7195	16.4065	16.8597	15.5511	16.6447	15.3616
8	20.1148	18.8153	19.2789	17.9845	19.0699	17.8005
9	22.5236	21.2336	21.7117	20.4276	21.5088	20.2490
10	24.9419	23.6585	24.1543	22.8773	23.9573	22.7042
11	27.3668	26.0823	26.6034	25.3259	26.4126	25.1583
12	29.7906	28.5016	29.0514	27.7699	28.8666	27.6079
13	32.2099	30.9108	31.4949	30.2038	31.3162	30.0472
14	34.6191	33.3071	33.9282	32.6246	33.7555	32.4735
15	37.0155	35.6853	36.3485	35.0272	36.1818	34.8814
16	39.3936	38.0411	38.7505	37.4071	38.5897	37.2667
17	41.7495	40.3716	41.1299	39.7614	40.9750	39.6262
18	44.0799	42.6717	43.4836	42.0851	43.3345	41.9551
19	46.3800	44.9404	45.8068	44.3769	45.6634	44.2521
20	48.6487	47.1774	48.0981	46.6369	47.9605	46.5172
21	50.8858	49.3798	50.3575	48.8618	50.2255	48.7471
22	53.0882	51.5469	52.5820	51.0510	52.4554	50.9412
23	55.2552	53.6777	54.7706	53.2036	54.6495	53.0986
24	57.3860	55.7730	56.9227	55.3204	56.8069	55.2202
25	59.4813	57.8327	59.0391	57.4012	58.9285	57.3056
26	61.5410	59.8591	61.1194	59.4483	61.0139	59.3573
27	63.5674	61.8567	63.1660	61.4663	63.0657	61.3799
28	65.5650	63.8277	65.1835	63.4575	65.0882	63.3755
29	67.5360	65.7779	67.1742	65.4276	67.0838	65.3501
30	69.4862	67.7196	69.1439	67.3893	69.0584	67.3161
31	71.4280	69.6693	71.1052	69.3589	71.0245	69.2902
32	73.3776	71.6531	73.0743	71.3630	72.9985	71.2988
33	75.3614	73.7085	75.0780	73.4394	75.0071	73.3798
34	77.4168	75.8841	77.1539	75.6373	77.0881	75.5826

35	79.5924	78.2532	79.3512	78.0307	79.2909	77.9814
36	81.9615	80.9162	81.7441	80.7209	81.6897	80.6777
37	84.6245	84.0343	84.4337	83.8709	84.3860	83.8348
38	87.7426	87.8770	87.5830	87.7530	87.5431	87.7255
39	91.5853	92.9100	91.4641	92.8375	91.4338	92.8214
40	96.6184	0.0000	96.5475	0.0000	96.5297	0.0000

Table I.
Zillmer Premium reserve Calculations
Endowment @90, Load 30% NSP
Interest Rate 4.5%
Issue Age 50

Net Premium		3.4654		
I=	0	I=	13.3771	
-----		-----		
Initial	Terminal	Initial	Terminal	
Reserves	Reserves	Reserves	Reserves	

1	3.4654	2.0603	-9.9117	-11.0412
2	5.5257	4.1548	-7.5758	-8.6666
3	7.6202	6.2811	-5.2012	-6.2558
4	9.7465	8.4369	-2.7904	-3.8116
5	11.9023	10.6223	-0.3462	-1.3338
6	14.0877	12.8333	2.1316	1.1729
7	16.2987	15.0680	4.6383	3.7065
8	18.5334	17.3270	7.1719	6.2678
9	20.7924	19.6069	9.7332	8.8526
10	23.0723	21.9046	12.3180	11.4577
11	25.3700	24.2126	14.9231	14.0744
12	27.6780	26.5274	17.5398	16.6989
13	29.9928	28.8434	20.1643	19.3248
14	32.3088	31.1578	22.7902	21.9487
15	34.6232	33.4649	25.4141	24.5644
16	36.9303	35.7603	28.0298	27.1669
17	39.2257	38.0408	30.6323	29.7524
18	41.5062	40.3010	33.2178	32.3150
19	43.7664	42.5394	35.7804	34.8528
20	46.0048	44.7555	38.3182	37.3654
21	48.2209	46.9459	40.8308	39.8488
22	50.4113	49.1094	43.3142	42.3017
23	52.5748	51.2448	45.7671	44.7228
24	54.7102	53.3525	48.1882	47.1124
25	56.8179	55.4319	50.5778	49.4699
26	58.8973	57.4849	52.9353	51.7976
27	60.9503	59.5159	55.2630	54.1003
28	62.9813	61.5269	57.5657	56.3803

29	64.9923	63.5236	59.8457	58.6441
30	66.9890	65.5187	62.1095	60.9061
31	68.9841	67.5293	64.3715	63.1857
32	70.9947	69.5833	66.6511	65.5144
33	73.0487	71.7209	68.9798	67.9380
34	75.1863	73.9951	71.4034	70.5164
35	77.4605	76.4862	73.9818	73.3407
36	79.9516	79.3049	76.8061	76.5365
37	82.7703	82.6307	80.0019	80.3072
38	86.0961	86.7643	83.7726	84.9937
39	90.2297	92.2284	88.4591	91.1888
40	95.6938	0.0000	94.6542	0.0000

Table II

Overview of the business outcomes which result when a life insurance company insures a cohort of 30 year olds, each one for the same sum, in a total of \$500,000. The annual premium is percent.

Business Insurance Year	Inforce	Claims	Premium Income	Life Reserves on the Basis 3 1/2 % Interest &		
				I = 0%	I = 1%	I=11/4%
---	0	500000				
1	495800	4200	10458	5100	181	-1045
2	491500	4300	10370	10200	5422	4219
3	487200	4300	10281	15400	10719	9540
4	482900	4300	10191	20700	16074	14919
5	478500	4400	10101	26000	21486	20355
6	474100	4400	10009	31400	26954	25847
7	469600	4500	9917	36800	32473	31391
8	465100	4500	9822	42300	38052	36995
9	460500	4600	9728	47800	43678	42646
10	455800	4700	9632	53400	49350	48344
11	451100	4700	9534	59000	55080	54100
12	446300	4800	9436	64700	60852	59898
13	441400	4900	9335	70400	66655	65728
14	436400	5000	9233	76100	72464	71563
15	431300	5100	9128	81700	78248	77374
16	426000	5300	9021	87400	83967	83120
17	420500	5500	8910	92900	89583	88763
18	414800	5700	8795	98200	95078	94287
19	408900	5900	8676	103500	100433	99669
20	402700	6200	8553	108500	105604	104869
21	396300	6400	8423	113400	110594	109886
22	389600	6700	8289	118100	115351	114672

23	382600	7000	8149	122500	119850	119199
24	375300	7300	8003	126600	124063	123441
25	367700	7600	7850	130300	127972	127379
26	359700	8000	7691	133800	131510	130945
27	351400	8300	7524	136800	134685	134149
28	342700	8700	7350	139500	137440	136932
29	333700	9000	7168	141700	139785	139305
30	324300	9400	6980	143500	141649	141197
31	314500	9800	6783	144700	142992	142568
32	304200	10300	6578	145300	143739	143341
33	293500	10700	6363	145400	143899	143529
34	282400	11100	6139	144800	143450	143106
35	270900	11500	5907	143600	142361	142043
36	259000	11900	5666	141800	140609	140316
37	246700	12300	5417	139200	138173	137904
38	234000	12700	5160	136000	135031	134786
39	221000	13000	4894	132100	131228	131006
40	207700	13300	4623	127600	126759	126558

Table II (Continued)

Business Insurance Year	Inforce	Claims	Premium Income	Life Reserves on the Basis 3 1/2 % Interest &		
				I = 0%	I = 1%	I=11/4%
41	194200	13500	4344	122400	121675	121495
42	180600	13600	4062	116700	116041	115882
43	166900	13700	3778	110400	109862	109721
44	153200	13700	3491	103700	103213	103089
45	139600	13600	3204	96600	96171	96064
46	126300	13300	2920	89300	88897	88805
47	113300	13000	2642	81700	81417	81338
48	100700	12600	2370	74100	73827	73760
49	88600	12100	2106	66500	66231	66175
50	77100	11500	1853	58900	58739	58693
51	66300	10800	1613	51600	51464	51427
52	56300	10000	1387	44600	44526	44497
53	47100	9200	1178	38100	37964	37941
54	38800	8300	985	32000	31897	31880
55	31400	7400	812	26400	26362	26350
56	25000	6400	657	21500	21480	21471
57	19400	5600	523	17100	17115	17109
58	14700	4700	406	13400	13385	13382
59	10800	3900	307	10200	10235	10234
60	0	10800	226	0	0	0

Table II (Continued)

Liability NPV	Age 30 Business Insurance	Reserves	Asset	NPV		
Year	Inforce	Claims	Income	Premium	4 1/2% Premiums	30% Load
Benefits						
0	500000					
1	495800	4200	10458	-35900	173800	137900
2	491500	4300	10370	-31000	170800	139900
3	487200	4300	10281	-25900	167700	141900
4	482900	4300	10191	-20800	164600	143900
5	478500	4400	10101	-15500	161500	146000
6	474100	4400	10009	-10200	158300	148200
7	469600	4500	9917	-4800	155100	150300
8	465100	4500	9822	700	151800	152600
9	460500	4600	9728	6300	148500	154800
10	455800	4700	9632	12000	145100	157100
11	451100	4700	9534	17800	141700	159500
12	446300	4800	9436	23600	138200	161800
13	441400	4900	9335	29600	134600	164300
14	436400	5000	9233	35600	131000	166700
15	431300	5100	9128	41600	127400	169100
16	426000	5300	9021	47700	123700	171400
17	420500	5500	8910	53700	119900	173600
18	414800	5700	8795	59600	116100	175700
19	408900	5900	8676	65400	112300	177700
20	402700	6200	8553	71200	108400	179600
21	396300	6400	8423	76700	104500	181200
22	389600	6700	8289	82200	100500	182700
23	382600	7000	8149	87400	96500	183900
24	375300	7300	8003	92400	92500	184900
25	367700	7600	7850	97100	88500	185600
26	359700	8000	7691	101600	84400	186000
27	351400	8300	7524	105700	80300	186000
28	342700	8700	7350	109400	76300	185700
29	333700	9000	7168	112800	72200	185100
30	324300	9400	6980	115800	68200	184000
31	314500	9800	6783	118300	64200	182400
32	304200	10300	6578	120200	60200	180400
33	293500	10700	6363	121600	56200	177800
34	282400	11100	6139	122300	52300	174700
35	270900	11500	5907	122500	48500	171000
36	259000	11900	5666	122000	44800	166800
37	246700	12300	5417	120900	41100	162000

38	234000	12700	5160	119000	37600	156600
39	221000	13000	4894	116400	34200	150600
40	207700	13300	4623	113200	30900	144100
41	194200	13500	4344	109400	27700	137100
42	180600	13600	4062	104900	24700	129700
43	166900	13700	3778	99900	21900	121800
44	153200	13700	3491	94400	19200	113600
45	139600	13600	3204	88400	16700	105100

Table II (Continued)

Liability NPV Year Benefits	Age 30 Business Insurance			Reserves	Asset	NPV
	Inforce	Claims	Income	Premium	4 1/2% Premiums	
46	126300	13300	2920	82100	14500	96500
47	113300	13000	2642	75500	12300	87900
48	100700	12600	2370	68800	10400	79200
49	88600	12100	2106	62000	8700	70700
50	77100	11500	1853	55200	7200	62400
51	66300	10800	1613	48600	5800	54400
52	56300	10000	1387	42200	4600	46800
53	47100	9200	1178	36100	3600	39700
54	38800	8300	985	30500	2700	33200
55	31400	7400	812	25300	2000	27300
56	25000	6400	657	20700	1400	22100
57	19400	5600	523	16600	900	17500
58	14700	4700	406	13100	500	13600
59	10800	3900	307	10100	200	10300
60	0	10800	226	0	0	0

Table III

Overview of the business outcomes which result when a life insurance company insures a cohort of 40 year olds, each one for the same sum, in a total of \$750,000. The annual premium is percent.

Business Insurance Year	Inforce	Claims	Premium Income	Life Reserves on the Basis		
				3 1/2 % Interest & I = 0%	I = 1%	I = 1 1/4%
	0	750000				
1	742200	7800	22062	11500	4204	2378
2	734300	7900	21833	23100	16008	14230
3	726300	8000	21601	34800	27894	26165
4	718100	8200	21365	46500	39812	38133
5	709700	8400	21124	58200	51704	50076
6	701000	8700	20877	69800	63522	61944
7	692000	9000	20621	81300	75194	73667
8	682600	9400	20356	92600	86679	85204
9	672900	9700	20080	103600	97949	96526
10	662800	10100	19794	114400	108957	107586
11	652200	10600	19497	124900	119644	118326
12	641200	11000	19186	135000	129979	128714
13	629700	11500	18862	144700	139897	138685
14	617700	12000	18524	154000	149349	148190
15	605200	12500	18171	162700	158298	157191
16	592100	13100	17803	170900	166658	165605
17	578400	13700	17418	178400	174378	173378
18	564100	14300	17015	185200	181421	180474
19	549200	14900	16594	191300	187727	186833
20	533700	15500	16156	196600	193243	192401
21	517500	16200	15700	201000	197859	197068
22	500600	16900	15223	204500	201517	200777
23	483000	17600	14726	206900	204156	203466
24	464700	18300	14208	208300	205726	205085
25	445700	19000	13670	208500	206172	205579
26	426100	19600	13111	207700	205493	204947
27	405800	20300	12534	205600	203605	203105
28	384900	20900	11937	202300	200517	200061
29	363500	21400	11322	197900	196260	195846
30	341700	21800	10693	192400	190877	190503
31	319500	22200	10052	185700	184356	184021
32	297100	22400	9399	178000	176810	176513
33	274600	22500	8740	169400	168309	168046
34	252100	22500	8078	159900	158933	158702

35	229800	22300	7416	149600	148829	148629
36	207800	22000	6760	138800	138099	137927
37	186400	21400	6113	127600	126989	126842
38	165600	20800	5483	116000	115546	115422
39	145700	19900	4871	104400	104035	103932
40	126700	19000	4286	92900	92524	92440

Table III (Continued)

Business Insurance Year	Inforce	Claims	Premium Income	Life Reserves on the Basis 3 1/2 % Interest &		
				I = 0%	I = 1%	I = 1 1/4%
41	108900	17800	3727	81600	81300	81231
42	92400	16500	3203	70700	70513	70459
43	77300	15100	2718	60500	60318	60276
44	63700	13600	2274	51000	50863	50832
45	51600	12100	1874	42300	42223	42200
46	41000	10600	1518	34500	34462	34446
47	31900	9100	1206	27700	27644	27634
48	24200	7700	938	21800	21749	21742
49	17800	6400	712	16800	16745	16743
50	0	5200	0	6400	0	0

Table III (Continued)

Business Year	Age 40 Insurance Inforce	Claims	Premium Income	Reserves 4 1/2% 30% Load	Asset NPV Premiums	Liability NPV Benefits
0	750000					
1	742200	7800	22062	-65700	327800	262400
2	734300	7900	21833	-53700	319700	266300
3	726300	8000	21601	-41600	311500	270300
4	718100	8200	21365	-29200	303200	274300
5	709700	8400	21124	-16900	294800	278200
6	701000	8700	20877	-4500	286200	282000
7	692000	9000	20621	7900	277500	285700
8	682600	9400	20356	20200	268700	289200
9	672900	9700	20080	32400	259900	292500
10	662800	10100	19794	44400	250900	295500
11	652200	10600	19497	56200	241800	298300
12	641200	11000	19186	67800	232600	300700
13	629700	11500	18862	79100	223400	302700
14	617700	12000	18524	90000	214100	304300
15	605200	12500	18171	100500	204700	305500
16	592100	13100	17803	110600	195400	306100
17	578400	13700	17418	120100	185900	306200
18	564100	14300	17015	129000	176500	305700
19	549200	14900	16594	137300	167100	304600
20	533700	15500	16156	144800	157800	302800
21	517500	16200	15700	151600	148500	300200
22	500600	16900	15223	157400	139200	296800
23	483000	17600	14726	162300	130100	292600
24	464700	18300	14208	166200	121100	287400
25	445700	19000	13670	169000	112200	281400
26	426100	19600	13111	170700	103600	274400
27	405800	20300	12534	171200	95200	266500
28	384900	20900	11937	170500	87000	257600
29	363500	21400	11322	168600	79100	247800
30	341700	21800	10693	165600	71500	237100
31	319500	22200	10052	161400	64200	225600
32	297100	22400	9399	156000	57200	213300
33	274600	22500	8740	149700	50700	200400
34	252100	22500	8078	142400	44500	186900
35	229800	22300	7416	134200	38800	173100
36	207800	22000	6760	125400	33400	158800
37	186400	21400	6113	116000	28600	144600
38	165600	20800	5483	106200	24100	130300
39	145700	19900	4871	96100	20100	116300
40	126700	19000	4286	85900	16500	102500

41	108900	17800	3727	75900	13400	89300
42	92400	16500	3203	66200	10600	76800
43	77300	15100	2718	56900	8300	65200
44	63700	13600	2274	48200	6300	54500
45	51600	12100	1874	40300	4600	44900

Table III (Continued)

Business Year	Age 40 Insurance Inforce	Claims	Premium Income	Reserves 4 1/2% 30% Load	Asset NPV Premiums	Liability NPV Benefits
46	41000	10600	1518	33100	3200	36300
47	31900	9100	1206	26700	2100	28800
48	24200	7700	938	21200	1200	22400
49	17800	6400	712	16500	500	17000
50	0	17800	524	0	0	0

Table IV

Overview of the business outcomes which result when a life insurance company insures a cohort of 50 year olds, each one for the same sum, in a total of \$250,000. The annual premium is percent.

Business Insurance Year	Inforce	Claims	Premium Income	Life Reserves on the Basis 3 1/2 % Interest &		
				I = 0%	I = 1%	I = 1 1/4%
0	250000					
1	246000	4000	11271	5600	3150	2605
2	241800	4200	11090	11100	8728	8206
3	237500	4300	10901	16400	14159	13658
4	233000	4500	10707	21600	19417	18937
5	228300	4700	10504	26500	24484	24027
6	223400	4900	10293	31300	29335	28900
7	218200	5200	10072	35800	33932	33519
8	212800	5400	9837	40000	38271	37879
9	207200	5600	9594	44000	42326	41956
10	201300	5900	9341	47600	46052	45704
11	195200	6100	9075	50900	49436	49109
12	188800	6400	8800	53800	52430	52124
13	182200	6600	8512	56300	55031	54746
14	175300	6900	8214	58400	57191	56926
15	168100	7200	7903	60000	58881	58636
16	160700	7400	7579	61100	60113	59888
17	153000	7700	7245	61800	60835	60628
18	145100	7900	6898	61900	61065	60877
19	137000	8100	6542	61600	60796	60625
20	128800	8200	6176	60800	60068	59914
21	120400	8400	5807	59500	58830	58692
22	112000	8400	5428	57700	57177	57054
23	103500	8500	5049	55600	55066	54957
24	95000	8500	4666	53000	52554	52459
25	86600	8400	4283	50100	49709	49627
26	78300	8300	3904	46900	46548	46477
27	70200	8100	3530	43400	43149	43089
28	62400	7800	3165	39800	39597	39546
29	54900	7500	2813	36100	35920	35877
30	47800	7100	2475	32400	32212	32177
31	41100	6700	2155	28600	28507	28478
32	34900	6200	1853	25000	24906	24883
33	29200	5700	1573	21500	21444	21427

34	24100	5100	1316	18300	18229	18215
35	19500	4600	1087	15300	15216	15206
36	15500	4000	879	12500	12512	12505
37	12100	3400	699	10200	10148	10144
38	9200	2900	546	8100	8073	8071
39	6800	2400	415	6300	6313	6312
40	0	6800	307	0	0	0

Table IV (Continued)

Business Year	Age 50 Insurance Inforce	Claims	Premium Income	Reserves 4 1/2% 30% Load	Asset NPV Premiums	Liability NPV Benefits
0	250000					
1	246000	4000	11271	-27200	139800	112500
2	241800	4200	11090	-21000	134400	113400
3	237500	4300	10901	-14900	129100	114200
4	233000	4500	10707	-8900	123800	114800
5	228300	4700	10504	-3000	118400	115200
6	223400	4900	10293	2600	113000	115500
7	218200	5200	10072	8100	107500	115500
8	212800	5400	9837	13300	102100	115300
9	207200	5600	9594	18300	96600	114900
10	201300	5900	9341	23100	91200	114200
11	195200	6100	9075	27500	85800	113200
12	188800	6400	8800	31500	80500	111900
13	182200	6600	8512	35200	75200	110400
14	175300	6900	8214	38500	70000	108400
15	168100	7200	7903	41300	64900	106100
16	160700	7400	7579	43700	59900	103500
17	153000	7700	7245	45500	55000	100500
18	145100	7900	6898	46900	50300	97100
19	137000	8100	6542	47700	45700	93400
20	128800	8200	6176	48100	41300	89400
21	120400	8400	5807	48000	37100	85000
22	112000	8400	5428	47400	33100	80400
23	103500	8500	5049	46300	29300	75500
24	95000	8500	4666	44800	25700	70400
25	86600	8400	4283	42800	22400	65200
26	78300	8300	3904	40600	19300	59900
27	70200	8100	3530	38000	16500	54500
28	62400	7800	3165	35200	13900	49100
29	54900	7500	2813	32200	11600	43800
30	47800	7100	2475	29100	9600	38700
31	41100	6700	2155	26000	7700	33700
32	34900	6200	1853	22900	6200	29000

33	29200	5700	1573	19800	4800	24600
34	24100	5100	1316	17000	3600	20600
35	19500	4600	1087	14300	2700	17000
36	15500	4000	879	11900	1900	13700
37	12100	3400	699	9700	1200	10900
38	9200	2900	546	7800	700	8500
39	6800	2400	415	6200	300	6500
40	0	6800	307	0	0	0

Table V

Overview of the business results achieved by a life insurance company when it closes annually insurance policies in the amount of \$1.5 million [There are annually insured 30 year olds with \$500,000, 40 year olds with \$750,000, and 50 year olds with \$250,000].

Business Year	Insurance		Premium Income	Life Reserves: Closing Exp		
	Inforce	Claims		0 %	1 %	1 1/4 %
1	1484000	16000	43800	22200	7500	3900
2	2951600	32400	87100	66600	37700	30600
3	4402600	49000	129900	133200	90500	80000
4	5836600	66000	172100	222000	165800	151900
5	7253100	83500	213900	332700	263400	246400
6	8651600	101500	255000	465200	383300	363100
7	10031400	120200	295600	619100	524900	501700
8	11391900	139500	335700	794000	687900	661700
9	12732500	159400	375100	989400	871800	842900
10	14052400	180100	413800	1204800	1076200	1044500
11	15350900	201500	451900	1439600	1300300	1266000
12	16627200	223700	489400	1693100	1543600	1506800
13	17880500	246700	526100	1964500	1805200	1765900
14	19109900	270600	562000	2253000	2084200	2042600
15	20314500	295400	597200	2557400	2379600	2335800
16	21493300	321200	631600	2876800	2690300	2644400
17	22645200	348100	665200	3209900	3015100	2967200
18	23769200	376000	697900	3555200	3352700	3302800
19	24864300	404900	729700	3911600	3701700	3650000
20	25929500	434800	760600	4277500	4060600	4007200
21	26963700	465800	790600	4651400	4427900	4372800
22	27965900	497800	819500	5031700	4801900	4745300
23	28935000	530900	847400	5416700	5181000	5122900
24	29870000	565000	874300	5804600	5563300	5503900
25	30770000	600000	900100	6193500	5947200	5886500
26	31634100	635900	924800	6581900	6330700	6268900
27	32461500	672600	948400	6967700	6712200	6649200
28	33251500	710000	970800	7349300	7089700	7025700
29	34003600	747900	992100	7725000	7461700	7396800
30	34717400	786200	1012300	8093300	7826400	7760600
31	35392500	824900	1031300	8452300	8182300	8115700
32	36028700	863800	1049100	8800600	8527700	8460500
33	36626000	902700	1065800	9136900	8861400	8793500
34	37184600	941400	1081300	9459900	9182000	9113500
35	37704800	979800	1095700	9768400	9488400	9419400

36	38187100	1017700	1109000	10061500	9779600	9710100
37	38632300	1054800	1121300	10338500	10054900	9985000
38	39041100	1091200	1132500	10598600	10313600	10243300
39	39414600	1126500	1142600	10841400	10555100	10484500
40	39749000	1165600	1151900	11061900	10774400	10703500

Table V (Continued)

Business Year	Insurance Inforce	Claims	Premium Income	Life Reserves: Closing Exp		
				0 %	1 %	1 1/4 %
41	40052100	1196900	1159900	11265900	10977400	10906200
42	40325100	1227000	1167200	11453300	11164000	11092600
43	40569300	1255800	1173700	11624200	11334100	11262600
44	40786200	1283100	1179400	11778900	11488200	11416500
45	40977400	1308800	1184500	11917800	11626600	11554800
46	41144700	1332700	1189000	12041600	11750000	11678000
47	41289900	1354800	1192800	12151000	11859000	11787000
48	41414800	1375100	1196100	12246900	11954600	11882500
49	41521200	1393600	1198900	12330200	12037600	11965400
50	41598300	1410300	1200800	12395500	12096300	12024100
51	41664600	1421100	1202400	12447100	12147800	12075500
52	41720900	1431100	1203800	12491700	12192300	12120000
53	41768000	1440300	1205000	12529800	12230300	12158000
54	41806800	1448600	1206000	12561800	12262200	12189900
55	41838200	1456000	1206800	12588200	12288500	12216200
56	41863200	1462400	1207400	12609700	12310000	12237700
57	41882600	1468000	1207900	12626800	12327100	12254800
58	41897300	1472700	1208400	12640200	12340500	12268200
59	41908100	1476600	1208700	12650400	12350800	12278400
60	41908100	1487400	1208900	12650400	12350800	12278400

Table V (Continued)

Business Year	Premium reserves	
	Insurance@4 Inforce	Load 30% NSP
1	1484000	-128800
2	2951600	-234500
3	4402600	-316900
4	5836600	-375800
5	7253100	-411200
6	8651600	-423300
7	10031400	-412100
8	11391900	-377900
9	12732500	-320900
10	14052400	-241400
11	15350900	-139900
12	16627200	-17000
13	17880500	126900
14	19109900	291000
15	20314500	474400
16	21493300	676400
17	22645200	895700
18	23769200	1131200
19	24864300	1381600
20	25929500	1645700
21	26963700	1922000
22	27965900	2209000
23	28935000	2505000
24	29870000	2808400
25	30770000	3117300
26	31634100	3430200
27	32461500	3745100
28	33251500	4060200
29	34003600	4373800
30	34717400	4684300
31	35392500	4990000
32	36028700	5289100
33	36626000	5580200
34	37184600	5861900
35	37704800	6132900
36	38187100	6392200
37	38632300	6638800
38	39041100	6871800
39	39414600	7090500

40 39749000 7289600

Table V (Continued)

Business Year	Premium reserves	
	Insurance@4 Inforce	Load 30% NSP
41	40052100	7474900
42	40325100	7646000
43	40569300	7802800
44	40786200	7945400
45	40977400	8074100
46	41144700	8189300
47	41289900	8291500
48	41414800	8381500
49	41521200	8460000
50	41598300	8515200
51	41664600	8563800
52	41720900	8606000
53	41768000	8642100
54	41806800	8672600
55	41838200	8697900
56	41863200	8718600
57	41882600	8735200
58	41897300	8748300
59	41908100	8758400
60	41908100	8758400

Table VI

Overview of the business results when a life insurance company closes insurance policies in the amount of \$1.5 million in the first year and in each of the following years \$150,000 more than in the preceding year [The new insurance policies are distributed over the ages 30, 40, and 50 in the same relationship as Table V].

Business Year	Insurance Inforce	Premium Income	Life Reserves:		
			0 %	1 %	Closing Cost 1 1/4 %
0	1500000				
1	1484000	43800	22200	7500	3900
2	3100000	91500	68800	38500	31000
3	4846200	143000	142100	95000	83500
4	6720400	198200	244200	179400	163300
5	8720600	257200	377100	293500	273000
6	10844400	319700	542900	439800	414400
7	13089400	385700	743300	619700	589300
8	15453000	455500	980100	835200	799400
9	17932800	528400	1254900	1087900	1046900
10	20525900	604600	1569200	1379500	1332700
11	23229700	684100	1924500	1711200	1658600
12	26041100	766800	2322000	2084500	2026100
13	28957100	852500	2762700	2500500	2435800
14	31974500	940900	3247600	2960000	2889100
15	35090100	1032300	3777300	3463800	3386600
16	38300400	1126400	4352500	4012400	3928700
17	41601600	1223200	4973300	4606200	4516000
18	44990100	1322400	5639600	5245400	5148300
19	48462100	1424000	6351500	5929800	5825900
20	52013800	1527900	7108500	6658800	6548100
21	55640900	1634100	7910200	7432200	7314400
22	59339500	1741900	8755600	8248900	8124100
23	63105200	1851700	9643800	9108300	8976200
24	66933700	1963400	10573400	10008600	9869500
25	70820700	2076700	11542700	10948900	10802600
26	74761800	2191400	12550500	11927000	11773700
27	78752600	2307500	13594500	12941700	12780800
28	82788700	2424600	14672800	13990300	13822100
29	86866000	2542900	15783500	15071400	14896000
30	90980200	2662500	16924300	16182100	15999200

Table VI (Continued)

Business Year	Insurance Inforce	Premium reserve 4.5% Load 30% NSP
1	1484000	-128800
2	3100000	-247400
3	4846200	-353200
4	6720400	-443800
5	8720600	-516800
6	10844400	-570000
7	13089400	-601200
8	15453000	-608200
9	17932800	-589000
10	20525900	-541500
11	23229700	-464200
12	26041100	-355300
13	28957100	-213100
14	31974500	-36300
15	35090100	176200
16	38300400	425700
17	41601600	712600
18	44990100	1037700
19	48462100	1401200
20	52013800	1803500
21	55640900	2244300
22	59339500	2723500
23	63105200	3240400
24	66933700	3794300
25	70820700	4384100
26	74761800	5008700
27	78752600	5666600
28	82788700	6356200
29	86866000	7075800
30	90980200	7823700