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ANNUITY MORTALITY

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THE present notes include (*a*) extension to age zero of the 1949 tables by Messrs. Jenkins and Lew, (*b*) modifications at ages 10 to 29 male and 10 to 19 female required by the extension, (*c*) a lowering of the death rates at ages 89 and higher so as not to exceed population and insured life mortality for the same period of time, (*d*) modifications at ages 82–88 male and 83–88 female required by this lowering, (*e*) extension of death rates to age 120, (*f*) consideration of the changes during recent years in the ratio of male to female death rates. The tables here proposed are called the 1950 tables. There are no changes from the 1949 tables in death rate at ages 30–81 male or 20–82 female. The proposed death rates are shown in Table A, and various comparisons with other mortality tables in Tables B, C and D. The ratio of male to female mortality at age 10 is 158% instead of 253%, which seems to have been an erroneous extrapolation.

In Table E there is shown a comparison of the new death rates with those of the 1949 tables. The principal changes from the annuity viewpoint are at ages 90 and beyond. In preparing annuity mortality tables it would seem an axiom that no such death rate should be higher than either (*a*) population or (*b*) insured life death rates for the same age and period of time. The comparative figures shown in Table F indicate that at ages above 95 the Jenkins-Lew 1949 tables violate this axiom. In view of the magnitude of their achievement, it is with reluctance that one recommends any changes. But the extension to age zero is a necessity. And the lowering of death rates at the older ages seems also a necessity due to (*a*) the fact that such a change is conservative in the case of annuitants, (*b*) the impact of both population and insured life material and (*c*) the fact that the old ages are to be much more important in the future than they have been in the past.

The volume of intercompany annuity data was scanty at both these extremes of the life span. The population data, duly projected into the future and modified for class selection, make a firmer basis than an extrapolation from annuity data alone. Where annuity data fail, we use population figures, and where these run out (age 109), we use a carefully prepared extension of insured life experience—the British A1924-29 table.

TABLE A

ANNUITY MORTALITY, 1950 TABLE

Death Rates per 1,000

AGE	MALE				FEMALE			
	$1{,}000q$	Δ	Δ^2	Δ^3	$1{,}000q$	Δ	Δ^2	Δ^3
0	4.040	.....	.....	.....	3.210	.....	.....	.....
1	1.580	.....	.....	.....	1.360	.....	.....	.....
2	.887	− .172	+ .059	− .027	.703	− .182	+ .078	− .045
3	.715	− .113	32	− 22	.521	− .104	33	− 16
4	.602	− 81	10	− 1	.417	− 71	17	− 5
5	521	− 71	9	+ 5	346	− 54	12	− 2
6	450	− 62	14	7	292	− 42	10	+ 4
7	388	− 48	21	− 6	250	− 32	14	− 5
8	340	− 27	15	+ 6	218	− 18	9	+ 2
9	313	− 12	21	− 9	200	− 9	11	− 5
10	301	+ 9	12	+ 6	191	+ 2	6	+ 3
11	310	21	18	3	193	8	9	2
12	331	39	21	− 14	201	17	11	− 7
13	370	60	7	− 18	218	28	4	− 11
14	430	67	− 11	0	246	32	− 7	+ 2
15	497	56	− 11	+ 6	278	25	− 5	3
16	553	45	− 5	− 1	303	20	− 2	2
17	598	40	− 6	− 1	323	18	0	− 1
18	638	34	− 7	+ 1	341	18	− 1	+ 6
19	672	27	− 6	+ 6	359	17	+ 5	− 4
20	699	21	0	1	376	22	1	+ 1
21	720	21	+ 1	5	398	23	2	0
22	741	22	6	− 6	421	25	2	− 1
23	763	28	0	0	446	27	1	+ 1
24	791	28	0	+ 2	473	28	2	0
25	819	28	2	3	501	30	2	1
26	847	30	5	2	531	32	3	0
27	877	35	7	1	563	35	3	0
28	912	42	8	5	598	38	3	0
29	954	50	13	− 7	636	41	3	2
30	1.004	63	6	+ 2	677	44	5	− 2
31	1.067	69	8	− 1	721	49	3	+ 2
32	1.136	77	7	+ 3	770	52	5	1
33	1.213	84	10	− 1	822	57	6	− 1
34	1.297	94	9	+ 1	879	63	5	+ 2
35	1.391	103	10	3	942	68	7	− 2
36	1.494	113	13	0	1.010	75	5	+ 2
37	1.607	126	13	1	1.085	82	7	3
38	1.733	139	14	18	1.167	89	10	0
39	1.872	153	42	24	1.256	99	10	0
40	2.025	195	66	− 4	1.355	109	10	3
41	2.220	261	62	− 2	1.464	119	13	− 1
42	2.481	323	60	− 5	1.583	132	12	+ 4
43	2.804	383	55	− 2	1.715	144	16	1
44	3.187	438	53	− 3	1.859	160	17	1

TABLE A—*Continued*

Age	Male				Female			
	$1{,}000q$	Δ	Δ^2	Δ^3	$1{,}000q$	Δ	Δ^2	Δ^3
45....	3.625	.491	.050	−.002	2.019	.177	.018	.002
46....	4.116	541	48	− 3	2.196	195	20	4
47....	4.657	589	45	− 2	2.391	215	24	1
48....	5.246	634	43	0	2.606	239	25	− 37
49....	5.880	677	43	− 2	2.845	264	− 12	+ 41
50....	6.557	720	41	0	3.109	252	+ 29	+ 5
51....	7.277	761	41	− 1	3.361	281	34	4
52....	8.038	802	40	+ 1	3.642	315	38	4
53....	8.840	842	41	2	3.957	353	42	4
54....	9.682	883	43	0	4.310	395	46	7
55....	10.565	926	43	4	4.705	441	53	6
56....	11.491	969	47	3	5.146	494	59	7
57....	12.460	1.016	50	4	5.640	553	66	7
58....	13.476	1.066	54	33	6.193	619	73	9
59....	14.542	1.120	87	36	6.812	692	82	9
60....	15.662	1.207	123	14	7.504	774	91	12
61....	16.869	1.330	137	13	8.278	865	103	12
62....	18.199	1.467	150	16	9.144	968	115	13
63....	19.666	1.617	166	15	10.112	1.083	128	14
64....	21.283	1.783	181	18	11.195	1.211	142	18
65....	23.066	1.964	199	12	12.406	1.353	160	18
66....	25.030	2.163	211	30	13.759	1.513	178	21
67....	27.193	2.384	241	24	15.272	1.691	199	22
68....	29.577	2.625	265	25	16.963	1.890	221	25
69....	32.202	2.890	290	29	18.853	2.111	246	30
70....	35.092	3.180	319	31	20.964	2.357	276	29
71....	38.272	3.499	350	33	23.321	2.633	305	36
72....	41.771	3.849	383	34	25.954	2.938	341	38
73....	45.620	4.232	417	42	28.892	3.279	379	41
74....	49.852	4.649	459	40	32.171	3.658	420	46
75....	54.501	5.108	499	46	35.829	4.078	466	52
76....	59.609	5.607	545	48	39.907	4.544	518	54
77....	65.216	6.152	593	52	44.451	5.062	572	62
78....	71.368	6.745	645	55	49.513	5.634	634	66
79....	78.113	7.390	700	57	55.147	6.268	700	70
80....	85.503	8.090	.757	.243	61.415	6.968	.770	.235
81....	93.593	8.847	1.000	84	68.383	7.738	1.005	104
82....	102.440	9.847	1.084	− 48	76.121	8.743	1.109	− 4
83....	112.287	10.931	1.036	− 159	84.864	9.852	1.105	− 99
84....	123.218	11.967	877	− 236	94.716	10.957	1.006	− 171
85....	135.185	12.844	641	− 277	105.673	11.963	.835	− 207
86....	148.029	13.485	364	− 268	117.636	12.798	.628	− 210
87....	161.514	13.849	96	− 221	130.434	13.426	.418	− 177
88....	175.363	13.945	− 125	− 149	143.860	13.844	241	− 120
89....	189.308	13.820	− 274	− 69	157.704	14.085	121	− 61

TABLE A—*Continued*

Age	Male				Female			
	1,000q	Δ	Δ^2	Δ^3	1,000q	Δ	Δ^2	Δ^3
90....	203.128	13.546	− .343	+.004	171.789	14.206	.060	−.009
91....	216.674	13.203	− 339	46	185.995	14.266	51	+ 19
92....	229.877	12.864	− 293	57	200.261	14.317	70	13
93....	242.741	12.571	− 236	44	214.578	14.387	83	− 18
94....	255.312	12.335	− 192	21	228.965	14.470	65	− 56
95....	267.647	12.143	− 171	1	243.435	14.535	9	− 88
96....	279.790	11.972	− 170	− 1	257.970	14.544	− 79	− 90
97....	291.762	11.802	− 171	+ 32	272.514	14.465	− 169	− 53
98....	303.564	11.631	− 139	+ 110	286.979	14.296	− 222	+ 39
99....	315.195	11.492	− 29	244	301.275	14.074	− 183	+ 184
100....	326.687	11.463	+ 215	417	315.349	13.891	+ 1	372
101....	338.150	11.678	632	615	329.240	13.892	373	573
102....	349.828	12.310	1.247	771	343.132	14.265	946	739
103....	362.138	13.557	2.018	833	357.397	15.211	1.685	824
104....	375.695	15.575	2.851	765	372.608	16.896	2.509	784
105....	391.270	18.426	3.616	562	389.504	19.405	3.293	605
106....	409.696	22.042	4.178	255	408.909	22.698	3.898	318
107....	431.738	26.220	4.433	− 86	431.607	26.596	4.216	− 20
108....	457.958	30.653	4.347	− 391	458.203	30.812	4.196	− 327
109....	488.611	35.000	3.956	− 607	489.015	35.008	3.869	− 559
110....	523.611	38.956	3.349	− 738	524.023	38.877	3.310	− 702
111....	562.567	42.305	2.611	− 777	562.900	42.187	2.608	− 756
112....	604.872	44.916	1.834	− 754	605.087	44.795	1.852	− 741
113....	649.788	46.750	1.080	− 667	649.882	46.647	1.111	− 663
114....	696.538	47.830	413	− 540	696.529	47.758	448	− 541
115....	744.368	48.243	− 127	− 373	744.287	48.206	− 93	− 374
116....	792.611	48.116	− 500	− 204	792.493	48.113	− 467	− 206
117....	840.727	47.616	− 704	− 69	840.606	47.646	− 673	− 69
118....	888.343	46.912	− 773	.....	888.252	46.973	− 742	.....
119....	935.255	46.139	.....	.....	935.225	46.231	.....	.....
120....	981.394	.....	.....	.....	981.456	.....	.....	.....

TABLE B

RATIOS % OF MALE TO FEMALE DEATH RATES

Age	U.S. Whites 1939–41	U.S. Whites 1947	1937 Standard Annuity	J-L 1943 Annuity	J-L 1949 Annuity	1950 Table Annuity
0	127	130	126	*	*	126
1	113	110	113	*	*	116
5	125	152	103	*	*	149
10	143	149	102	*	253	158
15	149	180	100	186	193	179
20	146	189	106	159	163	184
30	127	148	132	140	147	147
40	139	157	146	143	149	149
50	152	175	146	198	211	211
60	159	173	146	207	209	209
70	129	142	145	161	167	167
80	115	120	144	134	139	139
90	108	.....	142	117	118	118
100	101	.....	146	105	103	104
108	94	.....	171	99	97	100

* Not given.

TABLE C

MALE DEATH RATES PER 1,000 (ULTIMATE)

Age	U.S. Whites 1939–41	U.S. Whites 1947	CSO Underlying Experience Table*	1937 Standard Annuity	J-L 1943 Annuity	J-L 1949 Annuity	1950 Table Annuity
0	48.12	34.50	21.82	11.31	†	†	4.04
1	4.87	2.39	5.01	5.11	†	†	1.58
5	1.38	.99	1.96	1.23	†	†	.52
10	1.00	.64	1.11	1.26	†	.48	.30
15	1.43	1.15	1.30	1.26	.80	.54	.50
20	2.12	1.78	1.67	1.33	.89	.62	.70
30	2.79	2.11	2.22	2.06	1.32	1.00	1.00
40	5.13	4.40	4.06	4.36	2.59	2.03	2.02
50	11.55	10.98	9.76	9.29	7.99	6.56	6.56
60	25.48	24.39	23.69	19.75	19.01	15.66	15.66
70	54.54	53.60	54.25	41.76	39.76	35.09	35.09
80	124.71	112.72	121.06	87.16	92.70	85.50	85.50
90	248.94	.....	265.23	177.14	220.09	208.49	203.13
100	389.35	.....	1000.00	362.12	487.77	463.42	326.69
108	474.62	.....	.....	833.33	823.34	745.82	457.96

* Includes also some females. See *TASA* XLII, 325; XLIII, 102.

† Not given.

A "model-office" comparison of reserves for immediate single life annuities without refund on the lives of men and women together shows the following relationships: The 1950 table was 6/10 of 1% higher than the 1949 table (unmodified); equal to the 1937 Standard; 3½% lower than the 1937 Standard set back 1 year in age; and 7% lower than that table set back 2 years in age. As many companies are now using the one-year setback, the use of the 1950 table would actually *decrease* the total of immediate annuity reserves to be held by such companies. These results are

TABLE D

FEMALE DEATH RATES PER 1,000 (ULTIMATE)

Age	U.S. Whites 1939–41	U.S. Whites 1947	CSO Underlying Experience Table*	1937 Standard Annuity	J-L 1943 Annuity	J-L 1949 Annuity	1950 Table Annuity
0	37.89	26.50	21.82	9.01	†	†	3.21
1	4.32	2.17	5.01	4.53	†	†	1.36
5	1.10	.65	1.96	1.19	†	†	.35
10	.70	.43	1.11	1.23	†	.19	.19
15	.96	.64	1.30	1.26	.43	.28	.28
20	1.45	.94	1.67	1.26	.56	.38	.38
30	2.20	1.43	2.22	1.56	.94	.68	.68
40	3.68	2.81	4.06	2.98	1.81	1.36	1.36
50	7.62	6.26	9.76	6.36	4.04	3.11	3.11
60	17.14	14.07	23.69	13.55	9.17	7.50	7.50
70	42.33	37.73	54.25	28.75	24.73	20.96	20.96
80	108.19	93.79	121.06	60.46	68.93	61.42	61.42
90	231.41	.....	265.23	124.84	187.65	176.16	171.79
100	387.39	.....	1000.00	248.06	464.14	449.40	315.35
108	507.50	.....	.....	487.28	832.51	770.10	458.20

* Chiefly male. See *TASA* XLII, 325; XLIII, 102.

† Not given.

largely due to the fact that at the important annuity ages 85–95 the 1937 table has rather low death rates, especially on female lives.

In Table G are shown typical annuity values on the 1950 table at 2½% interest and a comparison with the 1949 table and also with the 1937 Standard. It hardly seems desirable to publish the commutation columns until there is general agreement as to the death rates.

Although the above might seem to complete the task in hand, there are a number of most interesting things which we have observed during the course of this work. Some of these will now be set forth as intriguing by-products of the main object of endeavor. These will include also the reasoning behind the proposed changes.

TABLE E

1,000 q_x (TO 2 DECIMAL PLACES)

AGE	MALE				FEMALE			
	(1) 1950 Table A	(2) 1949 J-L	(1)−(2) +	(1)−(2) −	(1) 1950 Table A	(2) 1949 J-L	(1)−(2) +	(1)−(2) −
0	4.04	.....	.....	.....	3.21	.....	.....	.....
1	1.58	.....	.....	.....	1.36	.....	.....	.....
2	.89	.....	.....	.....	.70	.....	.....	.....
3	.72	.....	.....	.....	.52	.....	.....	.....
4	.60	.....	.....	.....	.42	.....	.....	.....
5	.52	.....	.....	.....	.35	.....	.....	.....
6	.45	.....	.....	.....	.29	.....	.....	.....
7	.39	.....	.....	.....	.25	.....	.....	.....
8	.34	.....	.....	.....	.22	.....	.....	.....
9	.31	.....	.....	.....	.20	.....	.....	.....
10	.30	.48	.....	.18	.19	.19	.....	.....
11	.31	.49	.....	.18	.19	.21	.....	.02
12	.33	.50	.....	.17	.20	.22	.....	.02
13	.37	.51	.....	.14	.22	.24	.....	.02
14	.43	.52	.....	.09	.25	.26	.....	.01
15	.50	.54	.....	.04	.28	.28	.....	.....
16	.55	.55	.....	.....	.30	.30	.....	.....
17	.60	.57	.03	.....	.32	.32	.....	.....
18	.64	.58	.06	.....	.34	.33	.01	.....
19	.67	.60	.07	.....	.36	.35	.01	.....
20	.70	.62	.08	.....	.38	.38	.....	.....
21	.72	.65	.07	.....	.....	.....	.....	.....
22	.74	.67	.07	.....	.....	.....	.....	.....
23	.76	.70	.06	.....	.....	.....	.....	.....
24	.79	.73	.06	.....	.....	.....	.....	.....
25	.82	.77	.05	.....	.....	.....	.....	.....
26	.85	.81	.04	.....	.....	.....	.....	.....
27	.88	.85	.03	.....	.....	.....	.....	.....
28	.91	.90	.01	.....	.....	.....	.....	.....
29	.95	.95	.....	.....	.....	.....	.....	.....
30	1.00	1.00	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....
82	102.44	102.44	.....	.....	.....	.....	.....	.....
83	112.29	112.11	.18	.....	84.86	84.71	.15	.....
84	123.22	122.67	.55	.....	94.72	94.22	.50	.....
85	135.18	134.18	1.00	.....	105.67	104.76	.91	.....
86	148.03	146.71	1.32	.....	117.64	116.41	1.23	.....
87	161.51	160.33	1.18	.....	130.43	129.27	1.16	.....
88	175.36	175.12	.24	.....	143.86	143.44	.42	.....
89	189.31	191.15	.....	1.84	157.70	159.04	.....	1.34

TABLE E—*Continued*

Age	Male				Female			
	(1) 1950 Table A	(2) 1949 J-L	(1)−(2)		(1) 1950 Table A	(2) 1949 J-L	(1)−(2)	
			+	−			+	−
90	203.13	208.48	.....	5.35	171.79	176.16	.....	4.37
91	216.67	227.19	.....	10.52	186.00	194.91	.....	8.91
92	229.88	247.33	.....	17.45	200.26	215.40	.....	15.14
93	242.74	268.96	.....	26.22	214.58	237.71	.....	23.13
94	255.31	292.12	.....	36.81	228.96	261.94	.....	32.98
95	267.65	316.83	.....	49.18	243.44	288.15	.....	44.71
96	279.79	343.12	.....	63.33	257.97	316.39	.....	58.42
97	291.76	370.97	.....	79.21	272.51	346.67	.....	74.16
98	303.56	400.35	.....	96.79	286.98	378.99	.....	92.01
99	315.20	431.20	.....	116.00	301.28	413.27	.....	111.99
100	326.69	463.42	.....	136.73	315.35	449.40	.....	134.05
101	338.15	496.87	.....	158.72	329.24	487.22	.....	157.98
102	349.83	531.39	.....	181.56	343.13	526.48	.....	183.35
103	362.14	566.76	.....	204.62	357.40	566.87	.....	209.47
104	375.70	602.71	.....	227.01	372.61	608.02	.....	235.41
105	391.27	638.96	.....	247.69	389.50	649.46	.....	259.96
106	409.70	675.14	.....	265.44	408.91	690.67	.....	281.76
107	431.74	710.90	.....	279.16	431.61	731.09	.....	299.48
108	457.96	745.82	.....	287.86	458.20	770.10	.....	311.90
109	488.61	1000.00	.....	511.39	489.02	1000.00	.....	510.98
110	523.61	.....	.....	.....	524.02	.....	.....	.....
111	562.57	.....	.....	.....	562.90	.....	.....	.....
112	604.87	.....	.....	.....	605.09	.....	.....	.....
113	649.79	.....	.....	.....	649.88	.....	.....	.....
114	696.54	.....	.....	.....	696.53	.....	.....	.....
115	744.37	.....	.....	.....	744.29	.....	.....	.....
116	792.61	.....	.....	.....	792.49	.....	.....	.....
117	840.73	.....	.....	.....	840.61	.....	.....	.....
118	888.34	.....	.....	.....	888.25	.....	.....	.....
119	935.26	.....	.....	.....	935.22	.....	.....	.....
120	981.39	.....	.....	.....	981.46	.....	.....	.....

MORTALITY DIFFERENTIAL BY SEX

Until 1928 it was the custom to prepare annuity mortality tables separately for each sex, as also population tables of mortality. Then James D. Craig presented (*TASA* XXIX, 123) what later came to be called the Combined Annuity table. It was for male lives, but was used for female lives by taking an age four years lower. This convenient principle was quickly seized upon by other actuaries and applied to other annuity

tables. In May 1938 the Standard Annuity Table had a five year differential. In all of these tables, however, it was recognized that the rule would not apply in the first few years of life. Separate extensions were presented later for the ages below ten.

A survey of this matter may be obtained in Table B, showing ratios percent of male to female death rates at representative ages throughout

TABLE F

1,000 q_x (NEAREST UNIT PER 1,000)

AGE	MALE					FEMALE				
	U.S. White 1939–41	J-L 1949	British Insured A1924–29 (M & F)	1950	1937 Stand. Ann'y	U.S. White 1939–41	J-L 1949	British Insured A1924–29 (M & F)	1950	1937 Stand. Ann'y
85	181	134	187	135	125	163	105	187	106	87
90	249	208	256	203	177	231	176	256	172	125
95	321	317	337	267	248	308	288	337	243	177
96	335	*343*	354	280	265	324	316	354	258	190
97	349	*371*	372	292	284	340	*347*	372	273	203
98	363	**400**	391	304	306	356	*379*	391	287	217
99	376	**431**	410	315	332	372	**413**	410	301	232
100	389	**463**	430	327	362	387	**449**	430	315	248
105	448	**639**	537	391	**610**	464	**649**	537	390	362
108	475	**746**	609	458	**833**	508	**770**	609	458	487
109	*	*1000*	635	489	**1000**	521	**1000**	635	489	*542*
110	.....	.....	661	524	.....	.....	.....	661	524	610
114	.....	.....	.....	.....	.....	.....	.....	.....	.....	1000
115	.....	.....	805	744	.....	.....	.....	805	744	.....
120	.....	.....	971	981	.....	.....	.....	971	981	.....

REMARKS: The J-L 1949 death rates at ages 100 and over, for each sex, violate the rules by being ***higher*** than both (*a*) population and (*b*) insured life death rates. Also note that this is true even though (*a*) and (*b*) are for much earlier years of experience. If allowance is made for this factor, then age 95 would probably come under the same ban. Higher J-L death rates are indicated by italics or bold type.

* Not given.

life. Here there are two population tables for white lives in the United States (1939–41 and 1947); the 1937 Standard Annuity basis; two by Jenkins and Lew (1943 and 1949); and finally the 1950 basis (Table A above). In passing, it may be mentioned that the 1959 and 1979 tables had the same sex differentials as the 1949 table. Probably very few actuaries or biostatisticians have observed the marked differences in recent mortality, when set forth for each sex. As will appear in the charts below, these differentials have been increasing every decade for at least thirty years. The changes during the seven years from 1940 to 1947 have been

especially extensive. The peak at ages 15 and 20 has come into being along with the widespread use of the motor vehicle. Perhaps the boys were playing in the streets while their sisters were indoors or in the yard near to the home. The second and less high peak appears at ages 50 to 60, when heart diseases have their toll, or more accurately the cardio-vascular-renal diseases. Ever since 1920 this peak has been increasing in

TABLE G

ANNUITY VALUES (a_x) @ 2½% INTEREST

AGE	MALE			FEMALE		
	1950 Table	Ratio % to 1949 Table	Ratio % to 1937 Table	1950 Table	Ratio % to 1949 Table	Ratio % to 1937 Table
0	32.699	.....	104.5	33.679	.....	104.7
5	32.021	.....	103.4	33.079	.....	103.9
10	31.042	100.0	103.5	32.216	100.0	104.0
15	29.922	100.0	103.6	31.229	100.0	104.1
20	28.691	100.0	103.9	30.130	100.0	104.4
25	27.318	100.0	104.3	28.900	100.0	104.7
30	25.778	100.0	104.8	27.528	100.0	105.1
35	24.063	100.0	105.1	26.001	100.1	105.7
40	22.171	100.0	105.3	24.310	100.1	106.2
45	20.119	100.0	105.2	22.450	100.1	106.6
50	17.993	100.1	105.1	20.424	100.1	106.8
55	15.847	100.1	105.2	18.237	100.1	106.6
60	13.688	100.1	105.2	15.908	100.2	105.6
65	11.511	100.1	104.5	13.486	100.2	103.6
70	9.370	100.2	102.9	11.048	100.3	100.3
75	7.350	100.4	100.1	8.691	100.6	95.4
80	5.536	100.8	96.1	6.530	101.1	88.9
85	4.012	102.3	91.5	4.683	102.7	81.3
90	2.922	110.2	90.5	3.321	110.3	75.7
95	2.233	133.2	98.8	2.421	131.7	75.0
100	1.754	179.5	129.4	1.810	178.9	80.1
105	1.315	261.4	252.4	1.319	275.9	97.3
110	.776	.....	.....	.775	.....	148.8
115	.309	.....	.....	.309	.....	.....
119	.063	.....	.....	.063	.....	.....
"Model Office"	.....	100.5%	101.9%	.....	100.6%	99.3%

size. Also it has moved from age 60 to 50. A more detailed and extended review of these phenomena appears below.

By contrast to the population tables the 1937 Standard Annuity data seem artificial and at certain ages quite out of line, namely, ages 10–20, 50–60 and 90–108. At age 15 this table shows equal death rates for both sexes as against an 80% excess for males in other tables; at age 108 it has

a 70% excess for males as against relative equality in other tables. At certain other ages also it is quite unrealistic. The annuity tables have sharper peaks at ages 50–60 than do the population tables, and even reach 210% as ratio of death rates of men to those of women. If these facts have not been brought to attention in the literature of either mortality or sex, it is because they are of relatively recent occurrence. There is nothing very startling in the ratios for 1900–02, where the maximum ratio, 121%, is for the first year of life, and the next highest is 115% at age 50.

A friend of mine, who is head of the astronomy department of a great university, upon seeing the figures of Table B, reacted like the farmer who saw a giraffe for the first time, saying, "I just don't believe it!" The question is: How can the numbers of men and women be approximately equal in the population, when the former have 30% and 40% higher mortality than the latter? This question may be answered by considering a somewhat typical situation. In a normal year there are 1,056 boys born to every 1,000 girls. In a recent mortality table it is not until age 50 that the female population has gained equality in numbers. Assuming a male death rate of two per 1,000 for 40 years, divide this by 1.30 to obtain 1.54 per 1,000 as the female death rate. At age 40 there are 1,056 times .92304 or 974.73 men and 940.22 women. This gives a 3.7% excess of men. For the next 50 years of life, assume a yearly death rate of 15 per 1,000 for men and (dividing by 1.40) of 10.714 per 1,000 for women. Thus for a unit starting at age 40 we have at age 90, .46969 men and .58357 women, a ratio (F/M) of 1.2424. This is 19.8% greater than 1.037, and thus at age 90 we would expect 19.8% excess of women. Hence, if at age zero there is a 5.6% excess of males, at age 40 a 3.7% excess of males, equal numbers at age 50, and a 19.8% excess of women at age 90, then the actual population figures as a whole are not so inconsistent with these hypotheses. It is because of (a) the excess of male births and (b) the extremely low death rates, that these figures stand as representative and consistent.

The U.S. population in April 1950, at the decennial census, is expected to be about 1,008 females to 1,000 males as against a corresponding figure of 994 in 1940. As this is the first U.S. census with such an excess, the above discussion is doubly appropriate at this time.

The matter of "secular trend" in mortality has assumed considerable importance in recent decades, due partly to the fact that death rates have decreased proportionately so much more at the younger ages and so very little at the oldest ages. When we consider the ratios of male to female mortality, there has also been a remarkable "secular trend." During the last 45 years the principal increases in the M/F ratio of death rates have

been at ages 5 to 70. They have ranged from 30% to 75%, averaging at these ages, 48%. Thus these ratios have increased at the very ages where death rates themselves have declined. The females have benefited more than the males.

These figures have paralleled five events in particular: (1) a vast decline in deaths from tuberculosis and the infectious and parasitic diseases, (2) an increased urbanization of the people, (3) a continued decrease in the size of the family, (4) a marked increase in the use of machinery in commerce, industry and the home and (5) freer dress and more athletic life of women. The decrease in the size of families has run parallel with the increase in urbanization. It is mentioned here because tuberculosis, for example, has so often been related to childbirth and matters incident thereto. The substitution of mechanical developments in place of manual labor would be expected to decrease heart diseases, but they have increased relatively, due to the decline in so many other causes of death. The transformation has probably been more complete in the home than in either office, factory, mine or transportation. The washing machine, vacuum cleaner, waxer, sewing machine and several other conveniences have replaced many fatiguing and often "back-breaking" jobs of a generation earlier. Women have perhaps adapted themselves better than have men to the atomization, rationalization and artificialities of modern life. The disappearance of the whalebone corsets and the appearance of women in light athletics have no doubt had an influence favorable to the longevity of women. These are among the many changes which have taken place during the first half of the twentieth century, now under brief review.

In a distinguished book, *Exercise and Health*, Dr. Woods Hutchinson has used the key phrase, "Muscle makes the man." And able biologists have pointed out that the muscular system is in general of greater moment to men than to women. Thus as machinery takes over more and more of the operations which people used to do, it may be that the men have been affected the most. The shorter hours of work in recent years have been an endeavor to remedy the situation. But so often the sedentary occupation is supplemented by sedentary recreations—watching the other people play (100,000 watching a few score at play), sitting at the movies, the radio and now the two combined in the television set, sitting in the automobile as it rolls along! These have so often become our recreations rather than doing things ourselves. We so often buy what the Young Men's Christian Association calls "canned amusements," rather than coordinating brain and hand in some form of play or work which is more personal and instinctive, and therefore creative. This is the problem of the adult education movement in our cities and suburbs, and it is also the

problem of every citizen. Benjamin Franklin two centuries ago summed it up in a single sentence: "Dost thou love life—then make the most of the minutes, for life is made up of minutes!"

Of course, the slogan of the city has no relation to muscles and often seems to depart from the ancient ideal, "healthy mind in healthy body." It is to develop the forebrain, which most distinguishes man from the animals. This is a task to which many are called, but few are chosen. Those who have permanently fallen by the wayside in one direction or another are so many, that one can appreciate Arnold Toynbee's view (*A Study of History*) that our so-called civilization is already on the decline, like that of ancient Rome and so many others of past centuries.

The remarkable data which have come to attention regarding mortality differentials by sex would warrant a paper by themselves. In fact, one was published in the April 1938 Milbank Memorial Fund Quarterly, "Sex Differences in Mortality in the United States" by Miss Dorothy G. Wiehl (vol. 16, pp. 145–155). The events of the succeeding twelve years make it desirable that a sequel to that work be published at an early date. She showed data for England, Canada and New Zealand, as well as for the United States, and it would be of interest to see similar material for other lands and at various decades of experience. In India, for example, and in some of the Latin countries the facts may be very different. If so, it would throw light on causation.

Such material as has come to my attention would lead to the following tentative inferences, now set down for the record:

1. Urban life today in the United States is generally *relatively* harder on men than on women (exceptions, yes).

2. White *men* are relatively better off in the Northern States, next best in the Southern States and worst in the Western States—as compared with the corresponding white women's death rates.

3. Canadian men make out *relatively* better than men in the Northern States of the U.S.A. At ages 10 to 60 the difference is about 10% in the M/F ratio; at other ages, not much difference. But, compared with the M/F ratios for the entire U.S.A. (whites) the Canadian ratios are lower at ages 10–70 by an average of 30%.

4. English ratios M/F are higher than those of the U.S.A. at ages over 50; at younger ages they are definitely lower. The first of these facts may be due to the greater urbanization in England; the second fact, to the larger share of deaths from accidents and violence in the U.S.A.

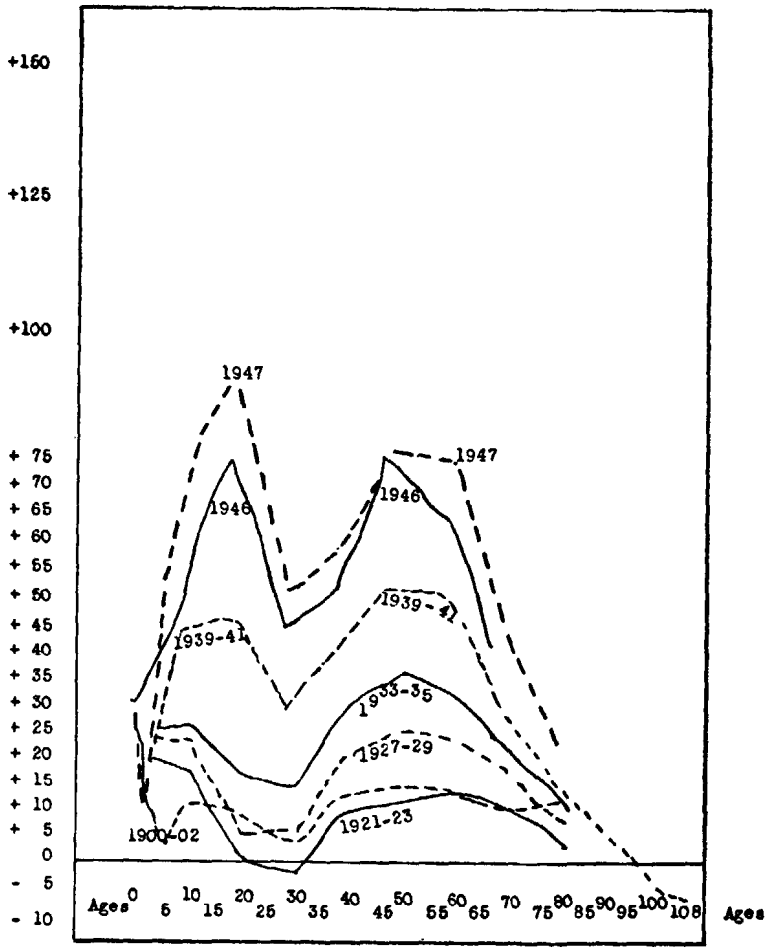
5. Negro ratios M/F are lower than those of whites at ages 5 to 70 and the reverse at older ages. At each group of ages the difference is about 15%.

6. The Canadian results above accord with the more rural character of Canada, and the British data with the greater degree of urbanization there. These both are in harmony with the fact that the U.S.A. used to be more rural than it is now.

7. The excess mortality of foreign-born people (in the U.S.A.) may be considered, especially as they have more often been men than women. But the proportion of foreign-born has been decreasing in those very

CHART I

PERCENTAGE EXCESS OF MALE DEATH RATE OVER FEMALE, U.S. WHITES, FOR YEARS STATED

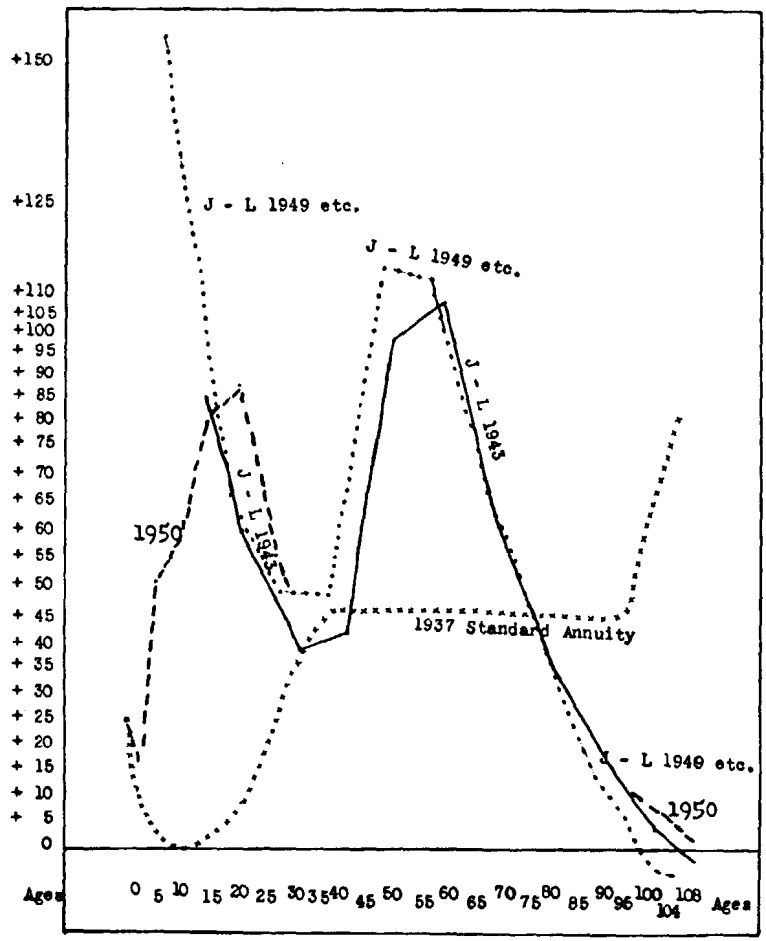


years, when the spread between mortality of men and women has been increasing. Furthermore, the death rates of foreign-born do not now differ nearly so much as formerly from those of native Americans (see *Length of Life* by Dublin, Lotka and Spiegelman, 1949 edition, pp. 55–58). Thus this factor can be of only minor significance; and more especially so since similar differentials have been found in other Anglo-Saxon countries which have not had the infusion of foreign-born migrants.

8. In making these comparisons, one feels safer with tables which have

CHART II

PERCENTAGE EXCESS OF MALE DEATH RATE OVER FEMALE, INTERCOMPANY, ANNUITANTS

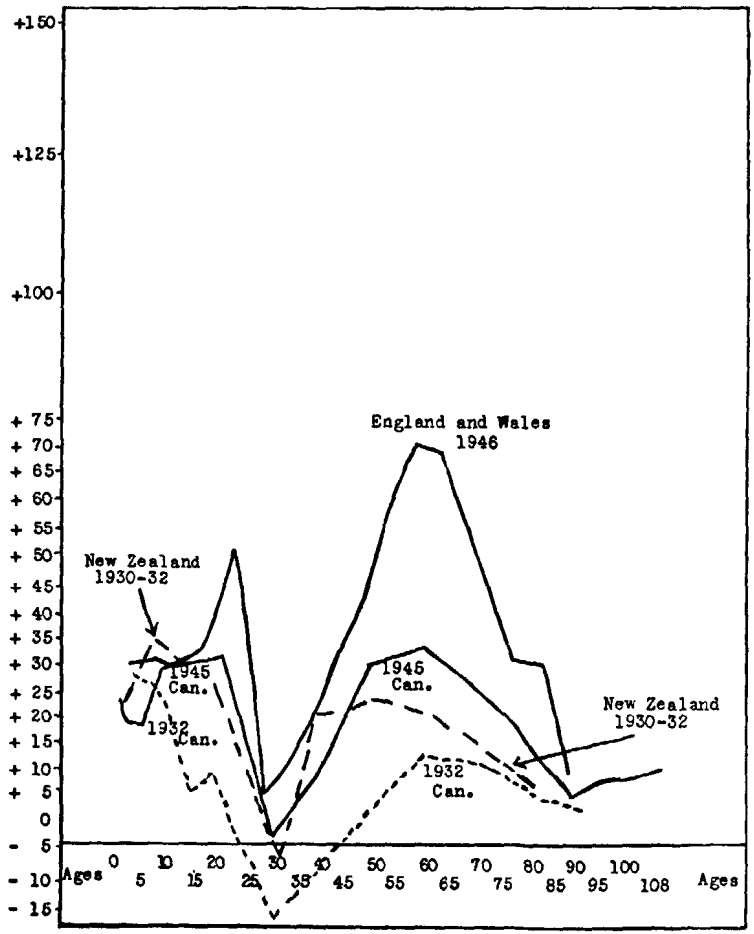


not been forced into the Makeham mold. In general, the population tables have *not* been so forced or distorted.

9. Many sociological forces are at work which should lead these M/F ratios to level off after a while. One is reminded of an extreme analogy in a section about Russian men and women in Ellsworth Huntington's great work, *Mainsprings of Civilization* (1945), pages 410-415. During the long, cold winters the women kept busy caring for the children and livestock, cooking and other household duties. These kept them in condition. But the long continued idleness of the men led them to fall into dissipation and

CHART III

PERCENTAGE EXCESS OF MALE DEATH RATE OVER FEMALE, FOREIGN, FOR YEARS STATED

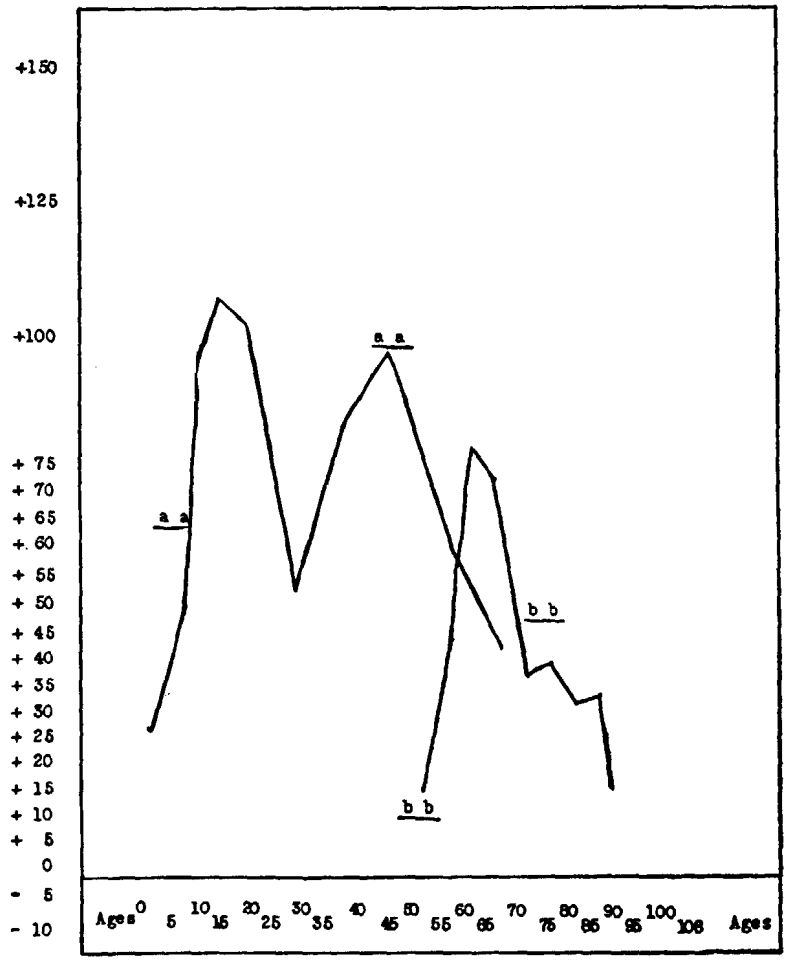


personal dissolution. For these reasons Russian locomotive engineers were usually women—they were more reliable! In recent decades the Russian climate has been much warmer, and the men have been working in factories during the winter months. Thus the vicious circle has been broken!

Charts I to IV show the relative death rates throughout life for the

CHART IV

PERCENTAGE EXCESS OF MALE DEATH RATE OVER FEMALE
MISCELLANEOUS TABLES



a a Metropolitan Life Industrial 1st 9 months 1949 (whites)
b b Canadian Government Annuities 1944–1948

respective sexes. The ratio of male to female death rate as an excess over 100% is the indicated quantity. There are four charts—one for United States population data, one for Canadians, British and New Zealanders, and a third for annuitants, while a fourth shows miscellaneous recent data. The visual appeal of the charts is more impressive than the rows of figures, such as those in Table B. There are also certain additional tables included in charts, so that the evolutionary aspect stands out more clearly. The death rates for each sex are shown numerically in Tables C and D.

1937 STANDARD ANNUITY TABLE

The 1937 Standard Annuity table is evidently outmoded in two particulars at least: (1) Its ratios of male to female death rates are at many ages at wide variance from those of (*a*) the general population and (*b*) annuitants. Thus I would agree with Messrs. Jenkins and Lew that the 5 year setback in age is not close enough to realities, and must (reluctantly) be discarded for future tables of annuity mortality. (2) At ages 1 to 40 its death rates are higher than those of the U.S. 1946 tables, and this should not occur in an up-to-date annuity table. It seems axiomatic that annuity death rates should not be higher than either (*a*) population or (*b*) insured life death rates for comparable years of experience.

CENTENARIANS

Question may be raised as to the use of age 120 as the final age in the proposed (1950) mortality table (age 121 as omega, the age not reached). This goes back to 1934, when British actuaries produced the A1924–29 table. This is the most recent such table at these ages. The literature was reviewed in the paper "Centenarians" in September 1939 (*TASA* XL, 372-378). For convenience of the present reader, some extracts and brief simplifications will now be presented:

(1) The trend in recent years has been rather steadily toward a higher limiting age in mortality tables. McClintock (1898) had 109; the Standard (1938) female had 115 and the British A1924–29 had 121 as its limiting age (as shown on p. xxix of the introduction to the monetary tables). In the case of insurance policies the Company may pay the face amount when a certain age is reached (as 96 in American Experience and 100 in CSO Table). Thus there is no mix-up or embarrassment when anyone lives beyond the "limiting age." But in annuity matters the situation is different. If the terminal age is 107 (as in American Annuitants female table) and someone lives to 109, what reserve is to be held? The conservative procedure is to determine the oldest (authenticated) age that anyone has lived, and then extend the annuity table a few years beyond that age.

(2) The oldest authenticated age in the "Centenarians" paper was 113.

The use of 120 as arbitrary final age would seem adequate and appropriate for annuitants, though one may feel that no one ever has lived or ever will live beyond age 115. Since the 1939 paper was published I have found another case of age 113, a woman in central N.Y. State who apparently lived a few weeks longer than the French Canadian man mentioned in the paper.

(3) There are three alternative ways of deriving columns of l_x and d_x (see p. 375 of 1939 paper). The best way is to use decimals rather than only integers. The preferred method was used in Spurgeon's textbook and also in the A1924–29 table and in the Jenkins-Lew paper. The method used in the American Experience and CSO tables and in the 1937 Standard Annuity table is definitely outmoded, especially for annuitants.

(4) It is not necessary to reach eventually a death rate of unity (examples on p. 376 include U.S. 1939–41 table). The rationale seems to be, if you are in doubt as to the limiting age, don't show any! Leave the matter open for future research!

(5) The optimistic school believes that human life can attain to such ages as 150 and 200 years. The example of C. J. Drakenberg is cited (145 years, 325 days) and Prof. Steffensen said "The principal facts of his life have, as far as I know, never been disputed" (*JIA* LXII, 103). Perhaps father and son had the same name and the date of birth of the father was associated with the date of death of the son! These dates were 1626 and 1772, long before modern methods of vital statistics were adopted!

The pessimistic school believes that 106 or 107 years is the longest period of human life in the past (based on life annuity experience in insurance companies) and that "history repeats itself" here as elsewhere.

The use of 121 is between these extremes—an endeavor to walk on the middle path of safety and conservatism!

AGES ZERO AND ONE

In the 1937 Standard Annuity table the death rates at age zero were 11.31 (male) and 9.01 (female). These were 90% of the rates by policies for intercompany juvenile ordinary issues of 1925–1934, after adjustment for sex. The ratio of male to female death rates was made the same as in the U.S. 1929–1931 Life Tables for white lives, namely, 126% (*TASA* XL, 77–87). Fifteen years have elapsed since the latest year included in this experience. Thus it must be regarded as outdated, provided that anything more recent can be found. In October 1948 (*TASA* XLIX, 437–442) mortality experience at ages zero and one on insured lives was presented from each of several companies. These were for all or some of the years 1940–1946. The death rate at age zero was about 4 per 1,000, and

that for age one was about 1.65 per 1,000. This includes approximately 40% girls and 60% boys, there being a 20% differential in mortality by sex. These data were confirmed by unpublished material from another large company. The proposed death rates for the 1950 annuity table have been obtained by using 90% of these insured life death rates (1940–1946), modifying the *male* figures so as to have M/F ratio of 126% at age zero and 116% at age one. This procedure gives the rates shown in Table A at these two ages, namely:

DEATH RATES PER 1,000

Age	Male	Female	M/F
0	4.04	3.21	126%
1	1.58	1.36	116

These conform to the rule that annuity death rates should never exceed insured life figures for the same sex, age and period of exposure. The male death rate at age zero is about one-fifth of that in the CSO underlying experience table and one-ninth of that of U.S. whites in 1947. The female death rate at age zero bears a similar relation to the U.S. 1947 female table. In each case the ratio to the 1937 Standard Annuity table (age 0) is about one-third.

At age one the male death rate (1.58) is about one-third of that in the CSO underlying experience and in the 1937 Standard table; while it is two-thirds of the 1947 figure. The female death rate (1.36) is a somewhat smaller proportion of the corresponding female tables. These facts are in accordance with the secular trend as to mortality and sex.

Incidentally, the death rates at age one are about two-fifths of those at age zero, instead of one-half as in the 1937 Standard table, one-fourth in CSO underlying experience and one-ninth in several recent population tables. This may be defended on the grounds of class-selection as well as of the secular trend.

If the *female* death rates were to be modified, instead of the male as above, the death rates at age zero would be 5% less and those at age one the same as above. The figures for males would be 3.86 and for females 3.06 at age zero. There is very little annuity business at these early ages, and the necessary choice of death rates may seem somewhat academic. This is particularly so, because a return of premium death benefit often accompanies deferred annuities, especially at young ages. The issuance of an immediate annuity without return is probably very rare in these first two years of life—but there are some.

AGES TWO TO TWENTY-FIVE

The female death rates of the 1949 table were modified at ages 11–19, so as to permit a reasonable extension below age 10. At ages 2 to 9 the female death rates were obtained from a projection of U.S. 1939–41 and U.S. 1947 tables, arranged so that the figure at age ten is the same as in the 1949 table. The male death rates were obtained by multiplying by the M/F ratios at these ages. These ratios were lowered so as to be 158% at age ten instead of 253%. The M/F ratio in the U.S. 1946 (white) table is 154%. Other M/F ratios at these juvenile ages were obtained by interpolation.

AGES 86 AND OVER

At ages 86–94 I have used the "curve of sines" (*TASA* XXXIV, 9) to merge the 1949 female death rates with those derived from the U.S. 1939–41 and U.S. 1946 tables projected into the future. The projection at these ages was prepared by deducting 20% from the U.S. 1939–41 death rates and multiplying the results by the respective ratios of J-L 1949 to J-L 1943 death rates.

At age 90 this result was 173.44, quite close to the 1949 female death rate of 176.16. At ages 86–90 the 1949 table had the greater influence, and at ages 91–94 the population projection had the larger weight upon the merged results. At ages 95–109 a similar use was made of the U.S. 1939–41 female white table. The percentage deductions graded down from 18% at age 95 to 6% at age 109. In each case the net death rate was further reduced in the ratio of J-L 1949 to J-L 1943 death rates.

For ages 110–120 a merger was made into the insured life tables of England, A1924–29. This is the most recent available at these ages. At age 109 the ratio of female death rate to this table was .771 and at ages 118–120 it was taken as unity. The intermediate death rates (ages 110–117) were inserted by interpolation in these ratios. (1.000 − .771 = .229 and .229 ÷ 9 = .0254.)

After the female death rates had been thus obtained, the male rates were derived by multiplying by the respective M/F ratios. The latter were as derived by Jenkins and Lew, but modified so as to be never less than 100%. At ages 107–120 the male and female death rates are nearly equal to each other.

GRADUATION

Before the resulting mortality tables could be offered for general use, they had to be subjected to a fourth-difference smoothing process near the extremities by Robert Henderson's formula A, with $n = a = 1$, $k = e = 1/3$ and $z = 2$. At ages 1 − 19 after grading the male table, we graded the ratio M/F rather than the female rates. The graded female rates were

obtained by dividing the graded male rates by these graded M/F ratios. The same procedure was followed at ages 80 – 120. In order to secure greater smoothness at these older ages, use was made of the sixth-difference formula A with $n = a = 3$, $k = e = .009$ and $z = 3$. This gave excellent results.

SETTLEMENT OPTIONS—PAYEE OPTION DIFFERENTIAL

There is at present considerable difference in practice among the companies as to how rates for settlement options differ from those for new annuities. Furthermore, a few companies make use of a different rate when the selection is made by the payee than by someone else. Most companies make no such differential charge.

TABLE H

DEATHS (PAYEE SELECTION AND ALL OTHERS)

	Actual	Expected	Ratio
Female	3,607	3,403	106%
Male	964	1,015	95
Total	4,571	4,418	103%

TABLE I

	DEATHS (PAYEE SELECTION)			DEATHS (SELECTION BY NON-PAYEE AND UNKNOWN)		
	Actual	Expected	Ratio	Actual	Expected	Ratio
Female	1,010	1,068	95%	2,597	2,335	111%
Male	631	675	94	333	340	98
Total	1,641	1,743	95%	2,930	2,675	109%

The latest mortality experience under settlement options (Table 25 in paper by Jenkins and Lew) shows on male lives a 5% lower mortality than for immediate annuities, while for female lives it is 6% the other way (higher). This includes all options, both by payee and by others. The figures are shown in Table H, where the "expected" mortality is that of non-refund immediate annuities. When the picture is thus viewed as a whole, it is evident that settlement option mortality is not far different from that of immediate annuities. Table I shows the figures for (*a*) payee selection and (*b*) all others.

Here is evidence of a 14% differential in mortality as to payee selection, but it is chiefly on female lives (16% against 4% for males). Only one-fourth of the payees are male. It is interesting to observe, as to the deaths, that males form 40% of those with payee selection, but only 11% of the others.

In the book *Settlement Options* by Flitcraft and Company one finds that seven companies make an age differential as to the payee option. These companies are about equally divided as to the manner in which this is done. There are four which word their contracts so that a benefit is given where the election is made by the insured and recorded during his lifetime. The other three penalize the election of option by the payee. The former method has the advantage of being diplomatic and positive. Under the former method, the policy provides that if the life income option is elected during the insured's lifetime, the age used will be ——— years older than the true age of the payee (the number of years of differential is one, two and three among the *seven* companies). The other method provides that the age is set back ——— years if the sum applied under the option is the cash value or endowment maturity value, or any part thereof. In such case the beneficiary receives less than the value printed in the table opposite her age, and the effect is negative and not so pleasing. Of the seven companies, there are two with one year differential in age, two with two years and three with three years. The average number is 2.1, and when weighted by assets, 1.6.

The last bit of information is doubly interesting, because a 14% differential in mortality corresponds to about 1½ years in age, and the use of two years in the contract is a conservative practice to correspond to this fact of experience.

The percentage adjustment in mortality which is equivalent to a one-year setback in age has been referred to several times in the *Transactions* (*TASA* XL, 243 and XLI, 229). The ratio has usually been closely related to the value of c in the formula of graduation. (Gompertz or Makeham's 1st or 2d law). Thus in the 1937 Standard Annuity Table, $c = 1.079$, and the death rate increases by nearly 7.9% at each age. (*TASA* XL, 244.) In the 1949 Annuity Table (Jenkins & Lew) the value of c is 1.104 for males and 1.120 for females. Thus one would look for approximately a 10% yearly increase in mortality for males and 12% for females. The actual rates of increase were as follows (Table J), as may be seen by simply dividing one death rate by its predecessor in the ultimate mortality table.

It will be observed from Table J that at the principal annuity ages the yearly increase in death rate is a little less than would have occurred if the tables had followed the Gompertz formulas.

$\log_{10} c$

"Gompertz's law may be stated as follows: The vital force or recuperative power of each individual loses equal proportions in equal times; and the proportion of vital force so lost by each is *universally* the same, being approximately represented by $\log_{10} c = .04$" (W. M. Makeham in 1890) see *JIA* XXVIII, 320. As an amendment to this "universal" law we read the following written fifty years later: "There is a tendency for the value

TABLE J

RATE OF INCREASE IN DEATH RATE

Age	1949 Table		1950 Table Where Different	
	Male	Female	Male	Female
10	2%	9%	3%	1%
20	4	6	3	6
30	6	6	.....	.....
40	10	8	.....	.....
45	14	9	.....	.....
50	12	9	.....	.....
55	9	9	.....	.....
60	8	10	.....	.....
65	8	11	.....	.....
70	9	11	.....	.....
75	9	11	.....	.....
80	9	11	.....	.....
85	9	11	10	12
90	9	11	7	8
95	8	10	5	6
100	7	8	4	4
Mean at Chief Annuity Ages.	9%	10½%	.....	.....
Expected by Use of c	10	12	.....	.....

of c to decrease with the progressive decline in mortality, which has been so marked during recent decades" (*TASA* XLI, 229). This latter remark appeared in a report on annuity mortality by the Joint Committee. It was based chiefly upon observation of experiences prepared in the United States, such as American Experience in 1868 (.046), Thirty American Offices in 1881 (.041), Canadian Men in 1918 (.040), American Men in 1918 (.034), American Annuitants in 1920 (.035), Combined Annuity in 1928 (.035) and the Standard Annuity in 1938 (.033). It may be observed that the first of these values was about 40% higher than the last listed and the progression was rather steadily downward. The comment of the Joint

Committee came a year prior to publication of The Commissioners' Standard Ordinary Table (.039), which tended to reverse the trend by use of a higher value of $\log_{10}c$. The rate of disability in the Inter-Company report of 1926 had a grading formula with $\log_{10}c = .034$, the same as in the AM(5) table with which it was combined.

When British tables are reviewed, there are some evidences of a parallel tendency. The Actuaries' table in 1843 and the Healthy Males in 1869, both used .040; the British Offices Assurances in 1893 used .039 and the corresponding Annuities .038; the A1924-29 insurance tables in 1934 were not Makehamized, but a value of .035 seems to best fit the picture. When we come to the British annuitants of the years 1900–1920 a more complex pattern presents itself, involving both higher and lower values of $\log_{10}c$. In the Government Annuities we find for males .052 based on ages 44–70 and .040 based on ages 64–89, while for females the value is .046 at both sets of ages. The Company male annuities, $a^{(m)}$, used .034 at ages 50–70 and .035 at ages 81 and over in two Gompertz formulas. The corresponding female table, $a^{(f)}$, had two Makeham formulas which overlapped, ages 40–85 .053 and ages 80-104 .030. Thus the highest and the lowest values so far recorded above occurred in this last experience. These two sets of British annuitant data seem to be the only tables so far, which had different values of c for the respective sexes. They also went further and had two different values of c for males and two others for females. This seems to overwork the Makeham hypothesis, without any advantages of simplification in joint life calculations. In these tables the values of $\log_{10}c$ for females are both higher and lower than those for males.

Authors Jenkins and Lew have continued the difficult British custom just observed, by having different values of $\log_{10}c$ for the respective sexes. In each set of their tables the value is higher for females than for males, the difference being uniformly .006, but they have used a single value of c for each table. Instead of continuing the trend toward lower values of $\log_{10}c$, they have definitely reversed that trend—first, by having high values (greater in each instance than .040) and, secondly, by increasing those values as they project into the future, as 1959 and 1979. Their proposals for $\log_{10}c$ are as follows:

LOG $_{10}c$ (JENKINS AND LEW, NOVEMBER 1949)

	1949 Tables	1959 Tables	1979 Tables
Men	.043	.045	.049
Women	.049	.051	.055

In these figures, we may observe an *increase* of .002 for each ten years of advance into the future. The figure of .055 for women in 1979 is higher than any previously recorded value, so far as I have observed, although not much higher than the $a^{(f)}$ figure (.053) at ages 40–85 produced in 1923 in England.

As W. P. Elderton and A. E. King showed in 1910 (*JIA* XLIV, 293–301), and V. Howell in 1920 (*TASA* XXI, 178–206), it is usually better to determine a value of log c, and use the table as though it were Makehamized for joint life calculations. This gives better results than to force the data into a Makeham mold. The values of log c to use in the 1950 tables are the same as in the 1949 tables, .043 for males and .049 for females.

One further remark may be made as to the value of log c. The death rate at or about age ten is nowadays not very far from zero. It is a fraction of unity per 1,000. Thus the function of c is to advance the death rate through most of the range from zero to unity. This it does by more or less equal geometric intervals. If the last age in a table is 95, as in the American Experience table, one would expect a high value of log c. There are only 85 years during which it can have its effect. But if the final age is 120, the value of log c would be expected to be lower, as there is a further quarter-century in its path. A logical extension of this thought would be that when the calculated value of log c is much above .040, then the final age is lower than it should be. But this thought should wait upon experimental verification. If it were to prove true, it would be a splendid vindication of foresight and penetrating observation by Messrs. Gompertz and Makeham.

A number of writers have expressed the opinion that Makeham's first law can hardly be expected to hold at other ages than 30 to 80. Thus to follow Vaihinger's doctrine of "as if" for all ages, in setting up a table of uniform seniority (addition to younger age to obtain equal ages), requires experimental verification. One has to balance the great convenience of the method against the degree of fidelity to the original data.

SELECTED BIBLIOGRAPHY

1. "A New Mortality Basis for Annuities" by W. A. Jenkins and E. A. Lew, *TSA* I, 369.
2. "Continuous Investigation into the Mortality of Assured Lives—Monetary Tables A1924–29, vol. I"; University Press, Cambridge, Eng., 1934.
3. "Henderson's Mechanico-Graphic Method of Graduation" by W. G. Bowerman, *TASA* XXXVIII, 7–11.
4. "The Combined Annuity Table" by J. D. Craig, *TASA* XXIX, 123.

5. *Exercise and Health* by Woods Hutchinson, M.D.; 1911.
6. *A Study of History* by Arnold Toynbee; Oxford Univ. Press, 1947.
7. "Sex Differences in Mortality in the U.S." by Dorothy G. Wiehl, Milbank Memorial Fund Quar., 16, pp. 145–155; April, 1938.
8. *Length of Life* by Dublin, Lotka and Spiegelman; Ronald Press, N.Y.C., 1949.
9. *Mainsprings of Civilization* by Ellsworth Huntington; J. Wiley and Sons, N.Y.C., 1945.
10. "Centenarians" by W. G. Bowerman, *TASA* XL, 360–378.
11. "Extension of 1937 Standard Annuity Table to Age Zero" by B. F. Blair, *TASA* XL, 77–87.
12. "Mortality Experience at Ages Zero and One" (discussion), *TASA* XLIX, 437–442.
13. *Settlement Options;* Flitcraft & Company, N.Y.C., 1948.
14. "Mortality under Annuities" by Joint Committee on Mortality, *TASA* XL, 243; XLI, 229.
15. "Development of Gompertz's Law" by W. M. Makeham, *JIA* XXVIII, 316–332.
16. "Joint Life Annuity Approximations" by W. P. Elderton and A. E. King, *JIA* XLIV, 293–301.
17. "Calculation of Rates of Mortality" by A. R. Davidson and A. R. Reid, *TFA* XI, 183–213.
18. "Variations in Masculinity" by J. N. and C. J. Lewis, *JIA* XL, 154–181.
19. *Man and Woman* by Havelock Ellis; 6th edition in 1926.
20. *Men and Women* by Abram Scheinfeld.
21. *Studies in Statistics* by Dr. G. B. Longstaff (England).
22. "Sex Differences in Morbidity and Mortality" by Antonio Ciocco, Quar. Rev. of Biology, 15, p. 203; 1940.
23. "Role of Testicular Secretions in the Short Life Span of Males" by Dr. J. B. Hamilton, Proc, Laurentian Hormone Conference, vol. 3; 1948.
24. *Cybernetics* by Norbert Wiener; 1944.
25. *The Limitations of Science* by J. W. N. Sullivan; 1933.
26. "The Differences between Men and Women" by Jacques Bacal, *Look Magazine*, February 28, 1950.