TRANSACTIONS OF SOCIETY OF ACTUARIES
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DISCUSSION OF PAPERS PRESENTED AT EARLIER REGIONAL MEETING

MORTALITY OF THE AGED

FRANCISCO BAYO

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JOHN E. HEARST:

The flattening of the mortality rates at the very old ages which Mr. Bayo observes in the social security data can be seen at much younger ages in the intercompany group annuity mortality experience. Statistical fluctuations obscure the flattening in some years, but, by combining data for several years, the fluctuations are reduced and the flattening becomes apparent. In Table 1, where exposures and deaths are combined for six-year periods, mortality rates are compared for ages 91–95 with those for ages 96 and over.

The flattening of the rates in the intercompany experience at about age 90 is consistent with the environmental pattern noted by Redington. The fact that flattening occurs later in the social security data may reflect the poorer economic circumstances of the general population.

The falling off of the mortality rates from the exponential can be seen in population tables and in the intercompany mortality experience for group annuities and also for ordinary policies. A comparison is given in Table 2 of $q_x \div q_{x-5}$ for two population tables and for the intercompany group annuity experience from 1964 through 1968 and the intercompany mortality experience on premium-paying policies observed beyond the sixteenth year from 1964 to 1969 anniversaries.

The falling off of the mortality rates from the exponential occurs at an earlier age in the intercompany data than in the social security data. It was unexpected, however, to find that the flattening occurred earlier in the United States Life Tables for 1909–11 than in the United States Life Tables for 1959–61 and in the social security data.

JAMES L. COWEN:

Mr. Bayo's paper gives a very thorough analysis of the mortality of the aged. At the Railroad Retirement Board we also have a significant volume of data regarding the mortality experience of the aged, and this has been presented regularly in our valuation reports.

TABLE 1

INTERCOMPANY GROUP ANNUITY MORTALITY EXPERIENCE
RETIREMENT ON OR AFTER NORMAL RETIREMENT DATE

CALENDAR YEARS	AGES 91–95			AGES 96 AND OVER		
	Exposed	Deaths	Mortality Rate	Exposed	Deaths	Mortality Rate
	Males					
1963–68	3,360.76	741	0.220	378.32	84	0.222
1957–62	1,201.51	315	0.262	128.09	28	0.219
1951–56	464.84	129	0.278	47.17	12	0.254
1951–68	5,027.11	1,185	0.236	553.58	128	0.231
	Females					
1963–68	684.90	144	0.210	92.84	22	0.237
1957–62	214.68	46	0.214	30.67	6	0.196
1951–56	73.00	13	0.178	13.50	2	0.148
1951–68	972.58	203	0.209	137.01	30	0.219

TABLE 2

RATIO OF MORTALITY RATE TO THAT AT THE
PREVIOUS QUINQUENNIAL AGE

	CENTRAL					
	77	82	87	92	97	102
Population experience:						
Social security experience: males, 1968–69	1.45	1.46	1.45	1.40	1.24	1.20
U.S. Life 1959–61: white males	1.47	1.60	1.45	1.44	1.28	1.24
U.S. Life, 1909–11: white males	1.49	1.46	1.36	1.31	1.27	1.35
Social security experience: females, 1968–69	1.67	1.68	1.61	1.49	1.34	1.15
U.S. Life, 1959–61: white females	1.68	1.76	1.65	1.56	1.30	1.24
U.S. Life, 1909–11: white females	1.45	1.55	1.36	1.40	1.30	1.26
Intercompany experience:						
Premium-paying policies, 16th and later years: observed from 1964–69 anniversaries	1.56	1.51	1.49	1.31	.....	.....
Group annuity experience—males—retirement on or after normal retirement: experience from 1963 through 1968	1.56	1.53	1.46	1.44	1.29	1.01

Currently we are in the process of analyzing our most recent mortality experience for use as the basis for the assumptions in the twelfth valuation of the railroad retirement system. These are compared below with the graduated rates shown in Table 11 of Mr. Bayo's paper. Table 3 compares our crude rates for male nondisabled annuitants with Mr. Bayo's male rates; Table 4 compares our rates for aged widows with his female rates; and Table 5 compares our crude mortality rates for wives with

TABLE 3

COMPARISON OF RAILROAD RETIREMENT 1968–70 MORTALITY EXPERIENCE WITH SOCIAL SECURITY GRADUATED RATES

(Male Nondisability Annuitants)

Age* x	RR	SS	Ratio RR/SS	Age* x	RR	SS	Ratio RR/SS
66	.02986	.03509	0.851	85	.14938	.14870	1.005
67	.03255	.03834	0.849	86	.16086	.16015	1.004
68	.03557	.04163	0.854	87	.17214	.17233	0.999
69	.04016	.04499	0.893	88	.18398	.18527	0.993
70	.04400	.04848	0.908	89	.19728	.19898	0.991
71	.04798	.05216	0.920	90	.21082	.21313	0.989
72	.05296	.05609	0.944	91	.23196	.22742	1.020
73	.05800	.06029	0.962	92	.25370	.24143	1.051
74	.06094	.06486	0.940	93	.26548	.25462	1.043
75	.06649	.06983	0.952	94	.28053	.26676	1.052
76	.07382	.07523	0.981	95	.30078	.27813	1.081
77	.07927	.08108	0.978	96	.33248	.28932	1.149
78	.08611	.08742	0.985	97	.35869	.30008	1.195
79	.09284	.09430	0.985	98	.36536	.31070	1.176
80	.10086	.10177	0.991	99	.37033	.32122	1.153
81	.10882	.10984	0.991	100	.43572	.33255	1.310
82	.11810	.11856	0.996	101	.43421	.34563	1.256
83	.12844	.12794	1.004	102	.39254	.35925	1.093
84	.13888	.13797	1.007				

* Approximates interval from age $x - \frac{1}{2}$ to $x + \frac{1}{2}$.

his female rates. The railroad retirement male rates for nondisabled annuitants cover the two-year period from the anniversary of retirement in 1968 to the anniversary of retirement in 1970. They are based on about 622,400 exposure years and 47,200 actual deaths. The aged widow and spouse rates are based on the experience during calendar years 1968–70, with the aged widows having about 730,500 exposure years and 44,100 actual deaths, while the spouses' experience included about 512,700 exposure years and 17,800 actual deaths.

The railroad retirement experience for male nondisability annuitants shows lower rates than the social security experience for ages up to 75, virtually the same rates from ages 76 to 90, and somewhat higher ones

TABLE 4

COMPARISON OF RAILROAD RETIREMENT 1968–70 MORTALITY EXPERIENCE WITH SOCIAL SECURITY GRADUATED RATES

(Aged Widows)

Age* x	RR	SS	Ratio RR/SS	Age* x	RR	SS	Ratio RR/SS
66	.01775	.01630	1.089	85	.11045	.11319	0.976
67	.02137	.01835	1.165	86	.11944	.12436	0.960
68	.02226	.02042	1.090	87	.13773	.13627	1.011
69	.02414	.02254	1.071	88	.15031	.14877	1.010
70	.02616	.02481	1.054	89	.16516	.16177	1.021
71	.02736	.02729	1.003	90	.18073	.17513	1.032
72	.03052	.03009	1.014	91	.18578	.18890	0.983
73	.03382	.03325	1.017	92	.21023	.20318	1.035
74	.03731	.03683	1.013	93	.22188	.21773	1.019
75	.03935	.04083	0.964	94	.24666	.23227	1.062
76	.04715	.04528	1.041	95	.26688	.24642	1.083
77	.04909	.05022	0.977	96	.28253	.25988	1.087
78	.05780	.05575	1.037	97	.28102	.27229	1.032
79	.06215	.06193	1.004	98	.29012	.28337	1.024
80	.06941	.06878	1.009	99	.39378	.29303	1.344
81	.07553	.07627	0.990	100	.37719	.30098	1.253
82	.08426	.08442	0.998	101	.34483	.30754	1.121
83	.09198	.09326	0.986	102	.30000	.31352	0.957
84	.10137	.10282	0.986				

* Age attained in calendar year, so approximates interval from age $x - \frac{1}{2}$ to $x + \frac{1}{2}$.

TABLE 5

COMPARISON OF RAILROAD RETIREMENT 1968–70 MORTALITY EXPERIENCE WITH SOCIAL SECURITY GRADUATED RATES

(Spouses)

Age* x	RR	SS	Ratio RR/SS	Age* x	RR	SS	Ratio RR/SS
66	.01711	.01630	1.050	81	.07024	.07627	0.921
67	.01704	.01835	0.929	82	.07836	.08442	0.928
68	.02017	.02042	0.988	83	.08838	.09326	0.948
69	.02130	.02254	0.945	84	.09600	.10282	0.934
70	.02419	.02481	0.975	85	.11400	.11319	1.007
71	.02568	.02729	0.941	86	.11463	.12436	0.922
72	.02770	.03009	0.921	87	.13840	.13627	1.016
73	.03043	.03325	0.915	88	.13293	.14877	0.894
74	.03375	.03683	0.916	89	.15426	.16177	0.954
75	.03916	.04083	0.959	90	.15236	.17513	0.870
76	.03985	.04528	0.880	91	.19097	.18890	1.011
77	.04700	.05022	0.936	92	.24862	.20318	1.224
78	.05268	.05575	0.945	93	.20202	.21773	0.928
79	.05394	.06193	0.871	94†	.24561	.23227	1.057
80	.06697	.06878	0.974				

* Age attained in calendar year, so approximates interval from age $x - \frac{1}{2}$ to $x + \frac{1}{2}$.

† Less than 10 actual deaths at each age over 94, hence no rates are shown.

above age 90. Since the railroad retirement data include only individuals whose annuities were awarded because of age and exclude all those who became disabled prior to age 65, in contrast to social security data, which contain the experience of all of the aged, it should be expected that railroad retirement mortality rates prior to age 76 would be lower. The fact that the railroad retirement rates above age 90 are higher than the social security rates is not so easily explained. Perhaps the railroad retirement data contain more accurate dates of birth for these individuals. If more accurate information was not available, the railroads were able to supply dates of birth based on what the employees had told them when first hired.

With respect to the female rates in Tables 4 and 5, the mortality experience of aged widows under the Railroad Retirement Act does not appear to be too different from the social security female experience. On the other hand, the railroad retirement rates for recipients of spouses' annuities appear to be considerably lower than the social security female experience. All the railroad retirement studies in the past have shown that spouse annuitants have generally lower mortality than those receiving aged widows' annuities. At the ages being investigated here, the volume of experience of aged widows is considerably greater than that for spouse annuitants.

In general, it would appear that the railroad retirement experience is not too significantly different from that of the social security beneficiaries. On the other hand, we have noted in our studies that the flow of the railroad retirement experience for male nondisability annuitants parallels (but at about a 20 per cent higher level) the rates shown in the 1971 Group Annuity Mortality Table.

CHARLES L. TROWBRIDGE:

Mr. Bayo's paper will prove to be an extremely valuable contribution to actuarial knowledge concerning high-age mortality in the United States.

The use of the social security data base for ages above 65, made virtually complete by the establishment of Medicare in 1966, is the factor which makes these newly published results so valuable. Using the same data source for the exposures and the deaths eliminates many of the troubles inherent in the traditional way of making mortality tables from population statistics. Moreover, the social security files contain information on year of birth that is much better than the information normally available from vital statistics. Mr. Bayo does not claim that the information as to age is entirely accurate, particularly at the very

high ages; nonetheless it should be better than in any data source previously used.

Mr. Bayo's mortality ratios at the extreme high ages are particularly interesting. Clearly previous ideas as to the magnitude of q_x at ages above 85 have been on the high side. Mr. Bayo's use of the extinct cohort concept applied to death records provides a valuable check and makes one reasonably confident of his results.

The leveling off of q_x at about the 0.3 level after age 100 (as shown in Mr. Bayo's Table 10) is especially intriguing. Our immediate reaction is that there cannot be enough exposure for reliable results; but it should be recognized that there were nearly 5,000 deaths during 1968 and 1969 at ages over 100. When the q_x gets as high as it appears to be at these ages, large exposure is not necessary. It is interesting to note that there were in the neighborhood of 5,700 people over age 100 in the United States on January 1, 1968, and that this result is based on the fairly reliable social security age information.

Note that Mr. Bayo's results, from either the social security data or the extinct cohort calculations, show no discernible tendency for the death rate to rise once a person reaches age 100. This result is, of course, counterintuitive. It is interesting to recognize that a level q_x of about 0.3 means that the complete expectation of life is about three years. On the average, those now 100 could expect to live to about their one hundred and third birthdays, but those already 105 can also expect to live about three years. Perhaps Mr. Bayo can furnish mortality rates split by sex and color at ages over 100. In any case, he has put himself squarely in opposition to the Gompertz and Makeham models at ages above 85.

Mr. Bayo's findings, that mortality differentials by color and by sex are somewhat different from those indicated by National Center for Health Statistics data, seem to be well substantiated. In particular, the fact that there is no sign of a crossover by sex is interesting. The lower female mortality, which persists well beyond age 65 and seemingly all the way to the end of the table, is a phenomenon to which actuaries have become accustomed. The color differentials, particularly the nonwhite's lower mortality at the high ages, is not well understood. One can theorize that the higher mortality among nonwhites at the earlier ages may have left a particularly hardy group surviving at ages above 80, but this explanation is not particularly satisfying. If the concept were sound, we might expect the same phenomenon to show up in male-female comparisons.

There are all kinds of interesting details which can be derived from

Mr. Bayo's results, and a careful study of his tables is well worthwhile. At the very least, I would think that these new data would cause the developers of annuity tables to reduce their q_x's at the very high ages.

(AUTHOR'S REVIEW OF DISCUSSION)

FRANCISCO BAYO:

In the paper I wanted to share with you some of our voluminous data on the Medicare program. I was also trying to find a better fix on the mortality at the older ages; and, in addition, I wanted to encourage some discussion in the somewhat neglected field of demography so as to develop a better understanding of the mortality pattern of older people.

In this last respect, I have been in contact with Mr. Robert J. Myers, our President; Mr. Edward A. Lew; Mr. Abraham Niessen; and Mr. R. E. Beard. Some of these individuals have brought to my attention the fact that the slackening in the rate of increase in mortality at the advanced ages has been observed before. Excellent work in this subject has been done by, among others, Mr. Beard about twenty years ago, Mr. W. Perks some forty years ago, and Mr. G. T. Humphrey very recently, in addition to the analysis by Mr. F. M. Redington cited in the paper. In this respect my paper should be viewed as adding some data to further substantiate previous observations.

I thank the three discussants for their interest in this field, and I hope that we will be able to continue this exchange of data and analysis.

I am very much appreciative of the intercompany group annuity experience brought to our attention by Mr. Hearst and of the railroad retirement annuitant data presented by Mr. Cowen.

I believe that it is important at this time to emphasize that, as indicated in the paper, we are not fully convinced that the social security data at the extreme old ages are of significantly high quality. I suggest that the data at the extreme ages be viewed as one of the best set of data gathered up to now but as still needing substantial further improvement.

I also want to thank Mr. Trowbridge for his discussion of some points in my paper and for the encouragement he offers regarding the presentation of further facts and analysis.

SCHEDULE FOR AMORTIZATION OF ACQUISITION COSTS VERSUS AMORTIZATION OF ACQUISITION COSTS BY USE OF NATURAL RESERVE FACTORS

JOE B. PHARR

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NORMAN E. HILL:

The author describes two methods for deferring and gradually writing off first-year acquisition costs. Under the reserve factor method, the remaining unamortized portion is determined by applying factors to remaining in-force units. Under the scheduling method, amortization is accomplished by applying annual writeoff factors to the unamortized amount at the beginning of the period (or at issue). In either case, the unamortized balance may or may not increase with interest.

I believe that the author deserves credit for describing how either method serves to reduce an asset instead of reducing a negative liability. This analysis should be useful in describing the actuarial process to laymen.

Mr. Pharr specified that he was not attempting to describe advantages and disadvantages of either method. I believe, however, that a few considerations about these methods should be mentioned. Although the pattern of writeoff can be the same under either method, there should be an initial test for recoverability of acquisition cost. One test is to compare a "break-even" premium, sufficient to cover benefits plus all expenses for all years against gross premiums. This former premium can be provided as part of the calculation of expense reserve factors.

The scheduling method or "accountants' worksheet" method ensures that the actual amount of acquisition cost will always be the amount involved in writeoffs. On the other hand, if actual persistency is different from expectations, the pattern of writeoff may be distorted.

The expense factor method does not automatically provide that unit costs included in factors correspond to original actual acquisition expenditures. However, since factors are applied to actual remaining in-force, there is some degree of correction of incorrect persistency assumptions.

Finally, the question of heaped commissions should also be considered. A possible commission scale can run 50 per cent the first year, 20 per cent the second year, 10 per cent in years 3–10, and 3 per cent thereafter. In this case the deferred amounts are 47 per cent the first year, 17 per

cent the second year, and 7 per cent in years 3–10. In other words, any excess over the ultimate 3 per cent rate is deferrable.

Normally, the scheduling method would begin amortization of each of these commissions from the policy year paid. Technically, however, to level expense charges over the premium-paying period, amortization of each of these ten amounts should begin in the first policy year. The expense factor method accomplishes this objective, by calculation of a level annual premium which serves as the amortizing element. In other words, the amount of expense writeoff is greater in the first policy year than it is later, because of the existence of deferrable commissions in years 2–10. This is synonymous with stating that the unamortized deferred acquisition cost for year 1 is less than for other years because of deferrable amounts in years 2–10.

(AUTHOR'S REVIEW OF DISCUSSION)

JOE B. PHARR:

Norman Hill's comments add greatly to the rather limited scope of the paper. I wish to thank Norman publicly for his interest and for taking the time to present a written discussion. The following comments on the points raised by Norman may be helpful.

The expense factor approach does provide *only* "some degree of correction of incorrect persistency assumptions." However, the scheduling approach can also be adapted to provide some degree of correction for variation of expected (assumed) persistency from actual by expressing the results of the schedule approach as a function of some measure (premium or units in force) of assumed in-force with the results per measure of in-force applied to the actual measures. Unless there is significant variation of actual from expected persistency, there seem to be strong arguments in consistency and materiality areas which suggest retention of any initially assumed amortization scheduling pattern—especially if this amortization pattern is likely eventually to prove conservative.

A scheduling method is also readily adaptable to the "question of heaped commissions," so that there is a level expense charge over the expected premium-paying period. Norman has raised an excellent point which deserves emphasis: deferrable acquisition expenses of the heaped commission type should be spread on a level basis over all expected premium revenue—which includes revenue expected prior to the payment of the heaped commissions as well as subsequent expected premium revenue.

ADJUSTED EARNINGS FOR MUTUAL LIFE INSURANCE COMPANIES

DONALD D. CODY

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BERT A. WINTER:

Mr. Cody has presented in this paper a very penetrating analysis of the pervasive role played by statutory reserves in the calculation by mutual companies of dividends to holders of individual life insurance policies that are required by the standard nonforfeiture laws to provide guaranteed cash surrender values. This discussion is from the point of view of an actuary associated with a mutual company using prospective asset shares directly in the calculation of annual dividend scales for such policies. I do not agree with Mr. Cody's suggestion that an adjustment for unamortized variable acquisition expense may be required by generally accepted accounting principles (GAAP) for a company using the dividend calculation methods described; I am, however, wholly in accord with his conclusion that no other adjustment from statutory valuation assumptions is required.

When what is deemed current experience as to interest, mortality, persistency, and expense has changed sufficiently from the experience underlying the annual dividend scale currently in effect to require a change in annual dividend scales, we calculate prospective asset shares based on the new experience for representative dividend classes (issue age and plan) of the policy series currently being issued. The new scale (for the annual dividend payable at the end of the tth year of a policy issued at age x, or as a mortuary dividend upon an earlier death during that year) is derived from these asset shares by the following three-term formula:

$$ {}_1\phi_{x+t-1}(P + {}_{t-1}V_x) + {}_2\phi_{x+t-1}F + {}_3\phi_{[x]+t-1} , $$

where the values of the ϕ's are given in the Appendix to this discussion, using Mr. Cody's notation to the extent feasible.

For older policy series, the same formula is employed, introducing the i', q', ${}_dE'$, and ${}_pE'$ of the new scale for current issues but retaining everything else from the old scale for the older series. (It will be observed that, at the time that new premium rates are adopted for current issues, a dominant part of the actual experience of the policy series just closed is on hand as to such relationships as ${}_pE''_t$ to ${}_pE''_{\text{ult}t}$, $q''_{[x]+t}$ to q''_{x+t}, A''_t to

$[(1 + a)V + bF]$, for $t \leq n_c$, the highest duration then attained by the policy series just closed. Thus it is then appropriate to adopt $\Lambda'_{[x]+t}$ of the new dividend scale for the old series, so that D_{n_c} is the same on both new and old scales, and, at as early a later duration n_d as possible, A'_{n_d} will be the same for the earliest and latest issues of the old series, assuming that the experience underlying the new scale applies from issue on the latest issues and from n_c on the earliest. This permits subsequent changes in the dividend scale for this series to be confined to i', q', ${}_aE'$, and ${}_pE'$, so long as the changes in i' and q' are not such as to require a change in the interest or mortality basis of the statutory reserve for the series and there have occurred no adverse fluctuations in asset value or claim rates requiring a change in a, b, α, or β.)

For a company calculating annual dividends on individual life insurance policies by this method, aggregate statutory surplus of this line of business may be considered as made up of

$$(A'' - A') + (A' - V) ,$$

where

A'' = Aggregate statutory assets less liabilities other than policy reserves;

A' = Aggregate asset share, after dividends according to the current scale, for all dividend classes containing insurance now in force, plus

$$\sum_{z=0}^{49} A'_{TZ} e^{\delta'(z+1/2)} ,$$

where A'_{TZ} is the aggregate asset share at expiry of term insurance dividend classes expiring in the zth calendar year before the statement year and $e^{\delta'(z+1/2)}$ indicates the accumulation of A'_{TZ} from the middle of the calendar year of expiry to statement date at the i' used in each of the involved years to calculate the annual dividends on cash-value policies of the same series as the expired term policies; and

V = Aggregate statutory reserve for all valuation groups containing insurance now in force.

Considering in turn the two terms (indicated by parentheses) in the above expression for aggregate statutory surplus, the following will be noted of the first $(A'' - A')$:

1. It does not involve V, and hence is not material to whether V, or some other "revenue reserve" factor, should be applied to units of insurance in force on statement date so as best to match costs and revenues according to GAAP.
2. It will not be material in aggregate amount, if dividend scales have been

changed frequently enough to reflect emerging trends in *unit* rates of interest, mortality, and expense (that is, regardless of changes in the *aggregate amount* of sales in relation to in-force) and if there have occurred no adverse fluctuations in claim rates or asset values that have not already been absorbed by changes in dividend scales.

As to the second term $(A' - V)$, the following will be noted:

1. When the level of recent issues is essentially equal to the level that produced the present in-force of earlier issues, the effect of the aggregate tests imposed on $\Lambda'_{[x]+t}$ (and the nonreturnable charge against term insurance dividends) is to make this second term of statutory surplus, *not* adjusted for unamortized variable acquisition expense, essentially equal to $aV + bF$.
2. When the level of current issues rises, the immediate effect is for statutory surplus to increase less rapidly than $aV + bF$, but, if the experience factors used for the current dividend scale and the scale itself remain unchanged and issues remain constant at the new level, statutory surplus will gradually return to $aV + bF$. The return will be more rapid if issues decline from the new level; this may result from such management action as introduction of war clauses or other protective changes in the underwriting of new issues, in the event of adverse claim fluctuations, and has often happened without management action when the general economy is such as to produce adverse asset value fluctuations. (Of course, a severe adverse fluctuation may require, in addition to any reduction occurring in "issue strain," emergency reductions in annual and termination dividend scales. Annual and termination dividend scales adopted thereafter might still be calculated by the methods described, but based on "postfluctuation" experience factors and the reduced statutory surplus then on hand.)

It appears to be the consensus of both the AICPA audit guide committee and the ALC-LIAA Joint Committee on Financial Reporting Principles that, for a stock company, a "paper asset" of sums already expended on recently issued nonparticipating policies may, if recoverable from future receipts of premiums and investment income, properly be taken into account in arriving at stockholders' equity. This seems reasonable to me as well, since, to the extent that the securities market, as informed by such financial reporting, takes the "paper asset" into account in the value of the company's stock, a stockholder may realize his share of it by selling his stock. On the other hand, in a mutual company using these dividend methods, while sums already expended on recently issued policies will be restored to statutory surplus over the future through the dividends declared later on these policies, thereby reducing the future surplus requirements, and hence increasing the future dividend potential, of earlier issued policies, the analogous "paper asset" does not seem to me to represent a currently realizable right of anyone.

In summary, for a mutual company using the methods I have described in this discussion to calculate dividends on its participating policies, I am of the opinion that statutory balance sheets and income statements, without adjustment for either unamortized variable acquisition expense or differences from statutory valuation assumptions, are in accord with GAAP.

APPENDIX

$$
{}_1\phi_{x+t-1} = \left\{(1+a)\left[(i'-i) - \frac{1+i}{p_{x+t-1}}\,(q_{x+t-1} - q'_{x+t-1})\right]\right.
$$

$$
\left. - \frac{a(1+i)q'_{x+t-1}i'}{p_{x+t-1}\delta'}\right\} \Big/ \left[1 + q'_{x+t-1}\left(\frac{i'}{\delta'} - 1\right)\right],
$$

$$
{}_2\phi_{x+t-1} = \left\{\frac{q_{x+t-1}i}{p_{x+t-1}\delta}\left[(1+a)p'_{x+t-1} + aq'_{x+t-1}\frac{i'}{\delta'}\right]\right.
$$

$$
\left. - (1+\beta+{}_aE)q'_{x+t-1}\frac{i'}{\delta'} + b(i' + q'_{x+t-1})\right\} \Big/
$$

$$
\left[1 + q'_{x+t-1}\left(\frac{i'}{\delta'} - 1\right)\right],
$$

$$
{}_3\phi_{[x]+t-1} = \frac{[\pi - {}_pE' - (1+a)P](1+i') + \Lambda'_{[x]+t-1}}{1 + q'_{x+t-1}(i'/\delta' - 1)},
$$

where

i' = Portfolio average rate of net investment income (after investment expense and federal income tax) applicable to the policy series (taking account of i and its guaranteed policy loan rate);

q'_{x+t-1} = Ultimate rate of mortality used in the asset share accumulations;

$(1+a)V + bF$ = A' at the end of the premium-paying period (and during the fully-paid-up period);

$aV + \beta F$ = Terminal dividend payable at death near the end of the premium-paying period and during the fully-paid-up period;

${}_dE'$ = Functional expense of termination by death (noncontestable);

${}_pE'$ = "Ultimate" annual administrative expense of a premium-paying policy (other than functional expenses of termination);

${}_wE'$ = Functional expense of termination by cash surrender;

Demand liability

$$= C + {}_wE' + \lambda_t(a_{n_2}V + \beta F)$$

$$\left(\lambda_t = 0 \text{ for } t \leq n_1\,; \quad \frac{t - n_1}{n_2 - n_1} \text{ for } n_1 < t < n_2\right)$$

$$= V + {}_wE' + aV + \beta F$$

$$(\text{for} \quad t \geq n_2\,, \quad \text{when} \quad C = V)\,;$$

$A'_{[x]+t}$ is determined so that A' approximates the demand liability when annual dividends begin, equals it at the end of the premium-paying period (and thereafter), and exceeds it by enough in between so that

$$\sum_{t=1}^{w-x} l'_{[x]+t-1}[A'_{t-1} + \tfrac{1}{2}(\pi - {}_pE'_{t-1})]$$

$$\geq \sum_{t=1}^{w-x} l'_{[x]+t-1}[(1 + a)({}_{t-1}V + \tfrac{1}{2}P) + bF]$$

when summed for all durations (including those before annual dividends are payable) for most dividend classes (all but those with short premium-paying or endowment periods); $l'_{[x]+t-1}$ is the insurance in force (according to the asset share service table) at the beginning of the tth policy year out of $l'_{[x]}$ paid-for issue.

For term insurance (policies and riders) not requiring cash surrender values, termination dividends are not payable, and annual dividends are calculated directly from the asset shares, without using a three-term formula. A nonreturnable annual charge for mortality catastrophes is, however, imposed. This charge has the effect of producing funds accumulated by fifty years of level issues (including those retained from issues no longer in force) which exceed in the aggregate $(1 + a)V + bF$ summed for all term insurance currently in force.

EDWARD H. COLTON:

In his paper Mr. Cody concludes that statutory net level premium reserves, together with a prepaid expense asset equal to unamortized variable acquisition expenses, provide a basis of reporting that should qualify under GAAP for mutual companies. In addition, recognition of a reserve for terminal dividends is suggested.

In my discussion, I conclude that nothing has been advanced that would indicate that mutual life insurance companies should be denied a clean auditor's opinion on their statutory statements, for the following reasons:

1. The basic concept of GAAP as applied to mutual life insurance companies is yet to be defined.

2. The various procedures that have been advanced for consideration as being in agreement with GAAP as applied to mutual life insurance companies present interesting theoretical analyses, but in the light of the needs of interested user groups they do not represent improvements over the statutory statement.

In support of this conclusion, the discussion briefly summarizes the distinctive aspects of mutual life insurance operations, explores the principal areas in which Mr. Cody and I have taken different points of view (with particular emphasis on the concept of a reserve for policyholder dividends), and reviews two other proposals that have been advanced as possible approaches to GAAP as applied to mutual life insurance companies. An Appendix includes a set of tables designed to illustrate the fundamental effect of these approaches on financial reports.

The principal thrust of the following discussion deals with individual life insurance, as does the thrust of most general discussions of life insurance accounting. Annuities, health insurance, and group lines each have special characteristics that would require separate consideration.

The most distinctive feature of life insurance operations, for both mutual and stock companies, is probably the long duration of the service guarantee that is provided to policyholders. The characteristic of mutual life insurance companies that distinguishes them from all other companies is that, for this long term during which the service guarantee is provided, management is also responsible for the maintenance of equity among policyholders.

Equity among policyholders of a mutual life insurance company implies that the cost of insurance to each policyholder class (e.g., same policy series, plan, age, year of issue, and so on) will be that cost that actually emerges; that each policyholder class will be self-supporting; and that no policyholder class will have to subsidize others. The unique mechanism used by mutual life insurance companies to provide long-term service "at cost" is to apportion dividends to policyholder classes that reflect the extent to which the company has been "released from risks" that had been assumed.

The responsibility for maintaining equity among policyholder classes can be fulfilled through the dividend mechanism if and only if, for each class, adequate premiums are established and an appropriate level of funds is accumulated. A variety of techniques are used by mutual life insurance companies to determine the total fund necessary to fulfill all obligations, or the accumulated statutory surplus deemed advisable. While these techniques vary in practical application, they have the common characteristic of recognizing both the reality of high acquisition ex-

penses and the need to accumulate amounts in excess of statutory reserves. Consequently, when the total fund held by a mutual life insurance company is analyzed by policyholder classes, the following can be seen:

1. There is amortization of acquisition expenses causing funds to be less than statutory reserves at the early durations.
2. There is recognition of the need to accumulate amounts in excess of statutory reserves, but the fund does not exceed the reserve until several durations have elapsed.

The AICPA Committee on Insurance Accounting and Auditing has not yet made a proposal for mutual life companies. In its exposed guide the committee reserved comment on the applicability of the key chapters to mutual companies. The proposals which follow have been advanced by various other persons or groups. The fundamental effect on financial reports of these proposals is illustrated in the Appendix to this discussion.

The author presents evidence that this system (use of statutory net level premium reserves and a prepaid expense asset) matches revenues with the costs of claims and expenses. However, such a matching is incomplete because it ignores the "cost" of maintaining self-supporting policyholder classes. The complete matching of revenues and costs would require the recognition of the additional funds which, together with statutory reserves, are necessary to provide insurance at cost.

Under GAAP applying to a service contract (and a life insurance contract is a service contract) financial results must recognize the extent to which services have been completed—all of the services. The additional funds that must be accumulated in order to provide full service cannot be ignored. Neither can they be permitted to flow into income until the insurer is released from risk, and, on that occasion, the funds released will be reflected in dividends to policyholders.

Moreover, it is not sufficient to draw conclusions from the income statement alone. For mutual companies it is most important to examine the balance that results after the statutory balance sheet is altered by the amount of unamortized acquisition expenses. Apart from the propriety of a balance sheet that carries amortized invested asset values together with unamortized expense assets on one side, and statutory net level reserves at least as large as surrender values on the other, it is doubtful that a prepaid expense asset deserves the prominence of separate balance-sheet identification in determining whether the interests of policyholders have been adequately secured.

Turning away from the mainstream of thought for a moment, the paper introduces several concepts which must be examined from a

different point of view. In several instances the author speaks of dividends to policyholders, both annual and terminal, as amounts which are provided for in pricing and in fund accumulation objectives.

It is more meaningful to focus on the need for a given level of premiums and funds (a need generated by management's responsibility to maintain equity among policyholder classes) rather than to focus on the manner in which amounts are released at the time that the company is released from risk (the payment of annual and/or terminal dividends).

From this point of view, holding an accumulation or a contingency reserve for terminal dividends is just as improper as holding an accumulation for annual dividends. Similarly, reserving for terminal dividends as an adjustment to benefit amounts is no different, in theory, from reserving for annual dividends as a benefit. Both miss the point that premiums are charged and funds are accumulated in order to fulfill a package of services—insurance protection at cost.

Assuming that the level of funds deemed appropriate to have on hand is related to the current volume of business, statutory reserves and the balance of fund amounts are released upon termination. Whether these amounts are paid in the form of terminal dividends or are paid to the persisting policyholders in the form of increased annual dividends is a matter that reflects the management's philosophy of equity. It does not reflect a difference in accounting, and it should not result in a difference in reporting.

The system of policyholder reserves and entity surplus recognizes that, in spite of the variety of methods in which fund objectives are determined, the largest amount of the total fund held by a mutual life insurance company is associated with particular policyholder classes. The balance, if any, is associated with policyholders in broader groups. The policyholder reserve would be a liability of the entity, consisting of funds associated with particular policy classes. The balance would be entity surplus under GAAP, and changes therein would be GAAP income.

The policyholder reserve concept presents a theoretically sound financial expression of the manner in which mutual life insurance companies operate, if the reserve reflects not only the statutory reserve but also the relative deficiency in the fund at early durations and the additional fund in the later durations. On the other hand, the reporting of GAAP income equal to changes in entity surplus focuses attention on a relatively minor financial activity. Entity surplus would consist, largely if not entirely, of an accumulation of small "risk" charges against particular policyholder classes. The accumulation represents a provision for adverse events that are almost certain to occur over a long period of time but that are unpredictable as to incidence during that period. Examples

of such events are wars, epidemics, investment losses, and unforeseen expenses and taxes. Consequently, this system would tend to show GAAP income when these events do not occur and GAAP losses when they do occur. This would emphasize a relatively minor aspect of the total mutual life insurance company operation.

The point of view underlying the no-profit proposal is that, since the objective of mutual companies is to provide insurance protection at cost, the "profit" will ultimately be zero. Consequently, when the revenue is "reserved" to allow the total "profit" to emerge in proportion to the service rendered, zero "profit" should emerge each year. Looked at from another point of view, this philosophy finds expression in the exposure draft of "Audits of Life Insurance Companies." There this philosophy is expressed in the rule that earnings that will never inure to the benefit of stockholders cannot be reported in net income. The translation of this philosophy to the case of mutual life insurance companies suggests that such companies cannot report an amount of net income other than "zero."

Under the "no-profit" approach, the reporting format would replace the income statement by a statement showing the sources of income and an equal amount of applications of income. The latter would combine amounts currently reported as "net gain from operations" with "increases in reserve." (Other changes in statutory surplus might also be included.) Several variations have been suggested, including: (1) a single "increase in reserves" item; (2) a separation of the increase in statutory reserves from increases in other reserves; and (3) a separation of the increases in funds associated with particular policyholder classes from increases in other funds.

There is general agreement (be it tacit or expressed) that the approach that the AICPA committee has taken for stock companies is not applicable to mutual life insurance companies. The approach suggested by Mr. Cody, and the other approaches for mutual companies that have been discussed, have not been advanced as being more meaningful to any interested user group than is the statutory statement.

Consequently, nothing has been advanced that would indicate that mutual life insurance companies should be denied a clean auditor's opinion on their statutory statements. Perhaps what would generally serve best would be an appropriate definition of the concept of GAAP as applied to mutual companies, which, together with the doctrine of materiality, would encompass the statutory statement. Then, when the accountant considers it to be appropriate, he would be permitted to certify that for mutual life insurance companies the statutory statements were in all material respects in conformity with generally accepted accounting principles.

APPENDIX

TABLE 1

CONDENSED HYPOTHETICAL STATUTORY FINANCIAL STATEMENTS

	Current Year	Prior Year
Assets:		
Cash and invested assets	$ 945	$910
Premiums	20	20
Interest	10	10
Other	25	24
	$1,000	$964
Obligations:		
Liabilities:		
Policy reserves	$ 800	$765
Claims	5	5
Dividends	25	25
Deposits	50	45
Expenses	3	3
Federal taxes	2	2
Miscellaneous	13	17
MSVR	20	21
	$ 918	$883
Surplus funds	82	81
	$1,000	$964
Summary of operations (current year):		
Premiums and considerations	$ 100	
Investment income	50	
Net deposits	2	
	$ 152	
Benefits incurred	$ 50	
Increase in policy reserves	35	
Increase in amounts on deposit	5	
Operating expenses	25	
Federal taxes	10	
Dividends	25	
	$ 150	
Gain from operations	$ 2	
Surplus account (current year):		
Balance, January 1	$ 81	
Gain from operations	2	
Realized capital gains	1	
Unrealized capital losses	(3)	
MSVR	1	
Balance, December 31	$ 82	

TABLE 2

CONDENSED HYPOTHETICAL FINANCIAL STATEMENTS
STATUTORY BASIS AUGMENTED BY PREPAID EXPENSE ASSET

	Current Year	Prior Year
Assets:		
Cash and invested assets	$ 945	$910
Premiums	20	20
Interest	10	10
Other admitted assets	25	24
	$1,000	$964
Prepaid expenses	40	35
	$1,040	$999
Obligations:		
Liabilities:		
Policy reserves	$ 800	$765
Claims	5	5
Dividends	25	25
Deposits	50	45
Expenses	3	3
Federal taxes	2	2
Miscellaneous	13	17
MSVR	20	21
	$ 918	$883
Surplus funds:		
Statutory	$ 82	$ 81
Other	40	35
	$ 122	$116
	$1,040	$999
Summary of operations (current year):		
Premiums and considerations	$ 100	
Investment income	50	
Net deposits	2	
	$ 152	
Benefits incurred	$ 50	
Increase in policy reserves	35	
Increase in amounts on deposit	5	
Operating expenses	20	
Federal taxes	10	
Dividends	25	
	$ 145	
Gain from operations	$ 7	
Surplus account (current year):		
Balance, January 1	$ 116	
Gain from operations	7	
Realized capital gains	1	
Unrealized capital losses	(3)	
MSVR	1	
Balance, December 31	$ 122	

TABLE 3

CONDENSED HYPOTHETICAL FINANCIAL STATEMENTS STATUTORY BASIS ADJUSTED TO REFLECT POLICYHOLDER RESERVES AND ENTITY SURPLUS

	Current Year	Prior Year
Assets:		
Cash and invested assets	$ 945	$910
Premiums	20	20
Interest	10	10
Other	25	24
	$1,000	$964
Obligations:		
Liabilities:		
Policyholder reserves*	$ 860	$824
Claims	5	5
Dividends	25	25
Deposits	50	45
Expenses	3	3
Federal taxes	2	2
Miscellaneous	13	17
MSVR	20	21
	$ 978	$942
Surplus funds	22	22
	$1,000	$964
Summary of operations (current year):		
Premiums and considerations	$ 100	
Investment income	50	
Net deposits	2	
	$ 152	
Benefits incurred	$ 50	
Increase in policyholder reserves	36	
Increase in amounts on deposit	5	
Operating expenses	25	
Federal taxes	10	
Dividends	25	
	$ 151	
Gain from operations	$ 1	
Surplus accounts (current year):		
Balance, January 1	$ 22	
Gain from operations	1	
Realized capital gains	1	
Unrealized capital losses	(3)	
MSVR	1	
Balance, December 31	$ 22	

* Current year includes statutory policy reserves, $800, plus part of statutory surplus associated with particular policyholder classes, $60. Respective prior-year amounts are $765 plus $59.

TABLE 4

CONDENSED HYPOTHETICAL FINANCIAL STATEMENTS
SOURCE AND APPLICATION OF INCOME BASIS

	Current Year	Prior Year
Assets:		
Cash and invested assets	$ 945	$910
Premiums	20	20
Interest	10	10
Other	25	24
	$1,000	$964
Obligations:		
Liabilities:		
Policy reserves and funds*	$ 882	$846
Claims	5	5
Dividends	25	25
Deposits	50	45
Expenses	3	3
Federal taxes	2	2
Miscellaneous	13	17
MSVR	20	21
	$1,000	$964
Summary of operations (current year):		
Sources of income:		
Premiums and considerations	$ 100	
Investment income	50	
Net deposits	2	
	$ 152	
Applications of income:		
Benefits incurred	$ 50	
Increase in policy reserves and funds	36	
Increase in amounts on deposit	5	
Operating expenses	25	
Federal taxes	10	
Dividends	25	
Net asset adjustments and change in MSVR	1	
	$ 152	

* Current year includes statutory policy reserves, $800, plus statutory surplus, $82. Respective prior-year amounts are $765 plus $81.

Tables 1–4 are designed to illustrate the fundamental effect on financial reports of various approaches to GAAP as applied to mutual life insurance companies. Consequently, the statutory format has been preserved to the extent possible, and the mandatory security valuation reserve (MSVR) is included among the liability items. Furthermore, not-admitted items such as furniture and fixtures have been excluded (they would probably be lost in proportion to $1,000 of total assets), and deferred taxes have been ignored.

In addition to the hypothetical statutory data in Table 1, the supplemental data shown in the accompanying tabulation have been assumed.

Statutory Surplus	Current Year	Prior Year
Associated with particular policyholder classes:		
Excess of amounts held over statutory reserves at the longer durations	$100	$94
Unamortized acquisition expenses—excess of statutory reserves over amounts held at the shorter durations	40	35
Net	$ 60	$59
Other	22	22
Total	$ 82	$81

NORMAN E. HILL:

The entire question of adjusted earnings for life insurance companies is controversial, and doubly so for mutual life insurance companies. With this in mind, I emphasize that my comments do not stem from the premise that mutuals should adjust.

I believe that the principal problem with mutual companies is that the customers are the owners, since policyholders own the company. Financial statements using GAAP are supposed to be for the owners' benefit.

For several reasons I believe it can be argued that the approach outlined in the paper, of using statutory net level reserves together with amortization of deferred acquisition cost, may not correspond to GAAP. Even if this approach produces a pattern of earnings emergence in a rational, controlled manner, this is not enough to satisfy GAAP requirements.

The paper did not specify the amortization period for expenses. Such a period might be the life of the policy, the premium-paying period, or the number of years in which the expected net cost of insurance to the policyholder is positive. Some periods of amortization could be quite different from the premium-paying period mentioned in the audit guide exposure draft for stock life companies.

It seems that, regardless of the type of financial concern, generally accepted accounting requires a matching of revenues with expense. In all the controversy over adjusted earnings for life insurance companies, two theories of proper matching have been introduced. The first theory is that, for a closed block, the incidence of expenses and hence earnings should follow the pattern of revenues. This theory produces at least two curves of earnings:

1. If revenues are defined as gross premiums, and earnings exclude investment income on prior profits, then earnings are matched with revenues under a "ski slope."
2. If revenues are defined as gross premiums, plus investment income on prior profits for this block, then earnings are (at least substantially) matched with revenues under a "reverse bell curve."

Under the second theory, the pattern of revenues and hence earnings follows the pattern of expenses. If expenses are defined as cash payout to policyholders (either mortality alone or mortality plus cash surrenders), then closed block earnings are matched with expenses under a bell-shaped curve. In either case, of course, earnings would also be affected by annual deviations between actual and expected experience flowing through or against earnings. At present, the "ski slope" approach in the first theory seems to prevail.

I believe that a justification for the first theory is that the primary service provided by life insurance companies is pooling of risk and that, for a closed group, the service is greatest when units in force are greatest, that is, in early durations. Under such an argument the first theory does not involve anticipation of profit.

In any event, I believe that the approach outlined in Mr. Cody's paper would have to include a demonstration of how this matching requirement is satisfied, before it can be considered in agreement with GAAP.

If an approach for adjusted earnings for mutual life companies follows the wording of the stock life audit guide, the question may arise whether a requirement for "reasonably conservative" reserve assumptions is satisfied by net level premium reserves based on the Commissioners 1958 Standard Ordinary Mortality Table with interest at $2\frac{1}{2}$ per cent (even with dividends considered).

In any audit guide for mutual companies the status of policyholder dividends must be resolved. Mr. Cody suggests that they can be either benefits or return of premiums. A third possibility is that, for these

purposes, they are akin to stockholder dividends, that is, payments to the owners. In the latter case it might be necessary to exclude dividends from revenues in making the matching test.

Concepts in any mutual life audit guide will probably be measured against those in the counterpart for stock life companies. If this is so, I do not believe that "occasional massive adjustments" should be viewed as inherent in adjusting earnings of stock life companies. If the actuary is watching experience closely, the number and amount of writeoffs of nonrecoverable deferred acquisition cost should be minimized, and assumptions for later issue years can be modified to preclude later writeoffs.

If an audit guide exposure draft for mutual life insurance companies is ever developed, it will undoubtedly be a controversial document. During any discussions about such a guide, I hope that actuaries will assume leading roles in presenting issues with papers such as this one by Mr. Cody.

CLAYTON A. CARDINAL:

Mr. Cody's paper advances further the development of the theory of the application of GAAP to life insurance companies. His area of concern is the application of the principles to mutual life insurance companies. It is difficult for me to envision to what audience a GAAP statement of a mutual life insurance company is directed and, therefore, "what the fuss is all about"; however, since Mr. Cody does pursue a method of application of GAAP to mutual life insurance company accounting, several questions are raised.

Does the admission that certain acquisition costs may be capitalized and amortized for GAAP purposes give greater credence to the modified statutory valuation methods adopted by a minority of mutual life insurance companies? This is an interesting question, inasmuch as the modified valuation methods were developed primarily to give effect to the high incidence of initial acquisition costs incurred by companies and thus provide them with some measure of surplus drain relief.

The paper sets out to demonstrate three things, which essentially are (1) the impropriety of natural reserves for adjusting earnings of mutual companies; (2) the propriety of net level statutory reserves colligated with capitalization and amortization of certain acquisition costs; and (3) the propriety of other approaches similar to the second. I fail to see that the first objective has been achieved.

What the paper demonstrates is the resulting adjusted earnings and the pattern thereof that follow and result from adoption of a specified statutory valuation system. The paper then concludes, in observing this

demonstration, that since the statutory reserves can now be perceived to be an inherent part of the adjusted earnings formula, the statutory reserves must then be a proper method of valuation for mutual companies in determining adjusted earnings.

It is set forth in the paper that revenue is comprised of the excess of premiums over dividends and of investment income. A basic accounting principle is concerned with the determination of the results from operations. Related to this are the determination of revenue, the determination of expenses, matching of expenses and revenue, and the continuity of methods. The paper proposes one method of amortization of the acquisition cost, which is known as the "sum of the premiums method." That is, the amount of each year's amortization is obtained as the product of the capitalized value and $\pi/\Sigma\pi$; however, if $\pi - D + \delta' A'$ represents revenue, then the amortization appears to be more properly determined by $(\pi - D + \delta' A')/\Sigma(\pi - D + \delta' A')$. If, however, dividends are viewed as benefits, a view proscribed by the paper, then the basis for amortization would be $(\pi + \delta' A')/\Sigma(\pi + \delta' A')$.

Since statutory reserves are a real, determining limit on the dividend practices of mutual companies, the paper ably demonstrates the propriety of the statutory valuation method, whether net level or modified, as the basis for determining dividends. In the determination of adjusted earnings, it does become necessary to fragment the statutory reserve if natural reserves are used as a basis in determining those adjusted earnings. Mr. Cody refers to the two elements or fragments of the statutory reserves as the natural reserve and the balance. Elsewhere this balance has been referred to as a reserve for future dividends. There seems to be an aversion on the part of mutual company actuaries toward making such a reference to the balance as a reserve for future dividends.

I would be indebted to Mr. Cody if he would give me an explanation based on general reasoning of the term $E'_0(1 - \omega' a'_{\overline{n-t}|})/a'_{\overline{n}|}$.

CECIL J. NESBITT:

On reading Mr. Cody's paper, I was motivated to compare it with Richard G. Horn's paper, "Life Insurance Earnings and the Release from Risk Policy Reserve System." The latter is in terms of discrete functions and uses different notation. I shall follow Mr. Cody's notation as much as is feasible and shall use continuous functions. On this basis, I shall first develop and expand some of the ideas of Mr. Horn's paper, with nonparticipating insurance in mind, and shall then suggest an application to Mr. Cody's case of participating insurance.

A release from risk policy reserve system will be defined in terms of primed symbols by the equation

$$\frac{dV'_t}{dt} = P' - E'_t + \delta'_t V'_t - \mu'_t(F - V'_t) - \omega'_t(C_t - V'_t) , \qquad (1)$$

and the actual asset accumulation by the relation

$$\frac{dA''_t}{dt} = \pi - E''_t + \delta''_t A''_t - \mu''_t(F - A''_t) - \omega''_t(C_t - A''_t) . \qquad (2)$$

(I have modified Mr. Cody's notation by attaching right subscripts to V', A'', C, δ, μ, and ω; also, the insurance is nonparticipating.) On subtracting expression (1) from expression (2), we obtain, for the rate of growth of surplus (relative to V' reserves),

$$\frac{d(A''_t - V'_t)}{dt} = g_t , \qquad (3)$$

where g_t may be expressed either as the sum of

$$h_t = \pi - P' + E'_t - E''_t + (\delta''_t - \delta'_t)V'_t + (\mu'_t - \mu''_t)(F - V'_t) + (\omega'_t - \omega''_t)(C_t - V'_t) , \qquad (4)$$

$$i_t = (\delta''_t + \mu''_t + \omega''_t)(A''_t - V'_t) , \qquad (5)$$

or as the sum of

$$j_t = \pi - P' + E'_t - E''_t + (\delta''_t - \delta'_t)A''_t + (\mu'_t - \mu''_t)(F - A''_t) + (\omega'_t - \omega''_t)(C_t - A''_t) , \qquad (6)$$

$$k_t = (\delta'_t + \mu'_t + \omega'_t)(A''_t - V'_t) . \qquad (7)$$

Here g_t represents a rate of net earnings (in relation to the reserve system) per existing policy at duration t. It consists of h_t, the sum of rates of gain in regard to loading, expenses, investment, mortality, and surrenders, where the latter three are calculated on the basis of V'_t, plus the accrual rate, i_t, of existing surplus in terms of the actual experience forces. Alternatively, g_t consists of j_t, which includes gains from investment, mortality, and surrenders calculated on the basis of A''_t, plus the accrual rate, k_t, of existing surplus in terms of the forces assumed for the reserve system. This latter way of looking at g_t is suggested by methods used by A. Loewy

in a 1917 paper, "On the Theory and Application of Forces in Insurance Mathematics."[1]

By setting $V_0' = -E_0'$, $V_n' = M$ (maturity or other predetermined value), $A_0'' = -E_0''$, and integrating over the range 0 to n, we obtain

$$A_n'' - M = E_0' - E_0'' + \int_0^n g_t dt \,. \tag{8}$$

This tempts one to say that over the n years total earnings are the same no matter what reserve system is utilized. This is true if the choice of reserve system does not affect the expenses considered in the calculation of A_n'', but it is possible that, by reason of taxes, A_n'' might be affected.

If now we define the discount function

$$H_t'' = \exp\left[-\int_0^t (\delta_h'' + \mu_h'' + \omega_h'')dh\right] \tag{9}$$

and rewrite equation (3) above in the form

$$d(A_t'' - V_t') - (\delta_t'' + \mu_t'' + \omega_t'')(A_t'' - V_t')dt = h_t dt \,,$$

then multiplication by H_t'' yields

$$d[H_t''(A_t'' - V_t')] = H_t'' h_t dt \,.$$

From this, one obtains

$$A_n'' - M = \left(E_0' - E_0'' + \int_0^n H_t'' h_t dt\right) \Big/ H_n'' \,, \tag{10}$$

which shows that the total earnings by the end of n years are equal to the accrual of $E_0' - E_0''$ and the momentary gains $h_t dt$ under the force $(\delta_t'' + \mu_t'' + \omega_t'')$. Alternatively, one may show that the total earnings by the end of n years are equal to the accrual of $E_0' - E_0''$ and the momentary gains $j_t dt$ under the force $(\delta_t' + \mu_t' + \omega_t')$.

We note from equation (10) above that if, under two different reserve systems, the actual accumulation A_n'' has the same value at the end of n years, then the gains under the one reserve system are equivalent to the gains under the other system, where equivalence is calculated in regard to the total force $(\delta_t'' + \mu_t'' + \omega_t'')$. A similar statement can be made for the equivalence of gains under the total force $(\delta_t' + \mu_t' + \omega_t')$, if gains are calculated by the form j_t rather than the form h_t.

As Mr. Horn has pointed out in his paper, P' and V_t' may be on a basis which incorporates risk margins for each of the factors of expense, inter-

[1] *Sitzungsberichte heidelberger Akademie der Wissenschaften*, 1917, VIII, A.

est, mortality, and withdrawal. These margins will be released as gains (compare eq. [4]) unless they are absorbed by unfavorable experience. Another possibility is that P' and V' are based on realistic factors without deliberate risk margins, in which case the gain rate h_t will consist of $\pi - P'$, modified by such deviations as have occurred between the experience and the assumed realistic factors.

One way to apply equations (3)–(10) above to participating insurance is to make the following replacements in equation (1):

V'_t replaced by A'_t (see the author's formula [1]);

P' replaced by $\pi - D_t$ (where D_t is rate of dividend at time t and is to be determined by the author's eq. [4]);

C_t replaced by A'_t (see the author's formula [3]).

Then equation (1) above is equivalent to the author's equation (1) (with $C = A'$), namely,

$$\frac{dA'_t}{dt} = \pi - D_t - E'_t + \delta'_t A'_t - \mu'_t(F - A'_t) \,. \tag{11}$$

If we also replace π by $\pi - D_t$ in our formula (2), then equation (3) above becomes

$$\begin{aligned}\frac{d(A''_t - A'_t)}{dt} = {} & E'_t - E''_t + (\delta''_t - \delta'_t)A'_t + (\mu'_t - \mu''_t)(F - A'_t) \\ & + (\delta''_t + \mu''_t + \omega''_t)(A''_t - A'_t) \,,\end{aligned} \tag{12}$$

which is comparable to the author's equation (13). However, in our formula (12) there is no surrender profit term, since the present approach has adhered consistently to the relation

$$C_t = A'_t = V_t - E'_0 a'_{\overline{n-t}|} / a'_{\overline{n}|} \,. \tag{13}$$

It follows from equation (13) that

$$\frac{d\,(A''_t - A'_t)}{dt} = \frac{d}{dt}\left[A''_t - \left(V_t - E'_0 \frac{a'_{\overline{n-t}|}}{a'_{\overline{n}|}}\right)\right], \tag{14}$$

while the author, in his formulas (8)–(13), calculated the adjusted rate of profit as

$$\frac{d({}^{a}A''_t - V_t)}{dt} = \frac{d}{dt}\left[A''_t - \left(V_t - E''_0 \frac{a''_{\overline{n-t}|}}{a''_{\overline{n}|}}\right)\right]. \tag{15}$$

It might be argued that equations (11)–(13) above define a release from risk policy reserve system directly in terms of the assumptions underlying the dividend scale, while the author introduces a variation therefrom by amortizing E_0' in terms of experience factors instead of amortizing E_0' in terms of dividend factors.

This may all be academic if the mutual life insurance companies are not required to adjust earnings. However, the author's paper, and variations of it such as are indicated in this discussion, should contribute to the basic understanding and control of surplus distribution.

(AUTHOR'S REVIEW OF DISCUSSION)

DONALD D. CODY:

Those offering discussions have added much to the background, utility, and understanding of my paper. Since such an important subject should be examined against the broadest variety of backgrounds, the discussions by Mr. Bert Winter and Mr. Edward Colton are especially welcome. Mr. Winter has provided a description of Prudential's prospective asset share dividend approach, and Mr. Colton has described the zero profit and the policyholder fund–entity surplus approaches to adjusted earnings. Not only is the description of the Prudential's approach valuable to actuarial literature, but it extends the list of dividend formula approaches, since the paper itself treated the classical contribution formula and the historical asset share formula. The paper analyzes very briefly the policyholder fund approach, but it makes no reference to the zero profit philosophy or the entity surplus philosophy.

My paper was intended to set forth a simple mathematical analysis of the mechanism of the individual life insurance operation of the mutual life insurance company, which can be applied to any of the dividend systems in use today and to any of the philosophies of emergence of earnings in mutual life insurance companies. The paper suggests that net level premium reserves adjusted by a not-admitted asset for unamortized variable acquisition expenses and, if material, a reserve for terminal dividends (which might be a segregation of surplus) appear to be acceptable under GAAP. I would emphasize, however, that this approach is stated as not being exclusive.

Within my own philosophy, I would raise no objections to the suggestion by Mr. Winter and Mr. Colton that unadjusted net level premium reserves should be recognized as conforming to GAAP. Indeed, the discussions between the mutual life insurance industry and the AICPA may very well develop this result, since the adjustments under discussion

between the stock companies and the AICPA are aimed primarily at reflecting emerging earnings on a going-concern basis for the information of stockholders, a motivation entirely absent in connection with mutual life insurance companies. The paper was written because of my concern as to what adjustments mutual companies may reasonably make if they are forced by the AICPA to adjust general-purpose annual statements.

I am pleased that both Mr. Colton and Mr. Winter agree with the most important conclusion of my paper, namely, that any form of natural reserves, including those using conservative parameters, are impractical, unnecessary, and undesirable in mutual life insurance company annual statements.

The analysis in my paper treats the dividend as a reduction in premium revenue. The dividend is the primary release from risk mechanism of the mutual life insurance company and reflects the intent of management as to the conservation and development of surplus and the realization of earnings.

If mutual life insurance company annual statements are adjusted by holding a not-admitted asset for unamortized variable acquisition expenses, then it seems appropriate to consider a reserve for terminal dividends. I was led to this by the thought that, if the variable acquisition costs are spread, thereby affecting the net operating gain in the early policy years, it would be entirely appropriate to recognize the excess of assets over reserves in the later policy years, when the opposite effect on earnings develops. The paper shows that the result is similar to holding adjusted reserves equal to historical asset shares of policyholder funds. The terminal dividend is designed to be somewhat less than the asset share surplus in the later policy durations but is nevertheless not an unreasonable measure of that asset share surplus. A reserve for terminal dividends in some companies will go far toward offsetting the effects of spreading variable acquisition expenses.

The reserve for terminal dividends is a disturbing thought to Mr. Colton, since he implies that conceptually there is little difference between annual dividends and terminal dividends. The distinction which I have made arises, of course, because I regard the annual dividend as a reduction in premium revenue, the terminal dividend being in the nature of a benefit in my treatment.

Mr. Norman Hill and Mr. Clayton Cardinal have raised questions as to allowable definitions for matching revenue and costs in time intervals under GAAP. Matching revenues to costs is no new concept to an actuary. Indeed, all actuarial premium and reserve equations are based on this concept. The formulas in the paper for gain after dividends are inter-

esting. They show how revenues are matched to costs day by day after the dividend is netted out. Surprisingly, premiums and loading disappear explicitly. Revenues are the charges in the dividend for expense and for mortality, the investment income on assets, and the asset released on surrender. Costs are the actual renewal expenses, any amortization charge for variable acquisition expenses, the dividend credit for investment income, the actual mortality cost, and the actual surrender cost. This matching day by day strikes me as entirely consistent with GAAP. Additionally, the matching provides a direct reflection of the intent of management as to the realization of operating gains and the development and conservation of surplus, which I understand is one of the fundamentals of proper accounting. Acceptable theories of appropriate matching of revenues and costs under GAAP ought to comprehend this approach.

Professor Nesbitt's discussion has added a valuable theoretical dimension to my paper. Since my paper used mathematics only to demonstrate the operations of a mutual life insurance company, it is fortunate that Professor Nesbitt has provided deeper insight into the mathematics. He has applied the generalized equation of equilibrium to the stock company situation, using natural reserves, and has outlined the procedure for integrating the differential equations by the use of a discount function involving the three forces of investment income, mortality, and termination.

Professor Nesbitt's relation (13) represents an ideal but rarely realized relationship between asset share and cash value; in practice many mutual companies guarantee cash values higher than asset shares in early policy years and lower than asset shares in later policy years. An early version of my paper factored his expression (13) into the differential equations, but because of its approximate nature and for reasons of brevity I omitted this interesting treatment.

I noted in my oral presentation that there is one area needing further attention, namely, the question of which acquisition expenses can be appropriately deferred. A case can be made for deferring only those acquisition expenses arising directly and variably as the result of the issue of a particular policy. These would include such items as medical exams, credit reports, commissions, overrides, expense allowances, and agency office expense related directly to the issue of the policy. Excluded would be such items as advertising, financing of new agents, opening of new agencies, ongoing nonvariable expenses of agency, underwriting and issue operations, and all overhead expenses. These latter expenses are either highly speculative as to direct effects on new business or are ongoing

overhead not susceptible to easy change. There is a difference between the acquisition expenses used by actuaries in asset shares for determination of cash values and dividend scales and the acquisition expenses appropriately eligible for deferment in the annual statement.

Mr. Cardinal asks for an explanation of the term $E'_0(1 - \omega' a'_{\overline{n-t}|})/a'_{\overline{n}|}$ in my equation (4), the derived formula for the three-factor dividend, expressed in continuous form. The term is the acquisition expense amortization rate, containing the explicit effect of terminations at time t. The corresponding effects from mortality and interest factored out when A' was removed by use of the expression $A' = V - E'_0 a'_{\overline{n-t}|}/a'_{\overline{n}|}$. Had A' been left in equation (4), the acquisition expense amortization rate would have been $(E'_0/a'_{\overline{n}|})(1 - \omega' - \mu' - \delta')a'_{\overline{n-t}|}$, a form which is more easily comprehended.