

**Formula Sheet Used on Exam ALTAM**  
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**ALTAM Formula Sheet**

**Interest Functions**

$$\alpha(m) = \frac{id}{i^{(m)}d^{(m)}} \quad \text{and} \quad \beta(m) = \frac{i - i^{(m)}}{i^{(m)}d^{(m)}}$$

**Makeham's Law**

$$\mu_x = A + Bc^x \quad \text{and} \quad {}_t p_x = \exp\left(-At - \frac{B}{\log c} c^x (c^t - 1)\right)$$

**Three-term Woolhouse's Formula in a single decrement context**

$$\ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m} - \frac{m^2-1}{12m^2}(\delta + \mu_x)$$

**Three-term Woolhouse's Formula in a multiple state context**

$$\ddot{a}_x^{(m)ii} \approx \bar{a}_x^{ii} + \frac{1}{2m} + \frac{\mu_x^{i\bullet} + \delta}{12m^2} \quad \text{where} \quad \mu_x^{i\bullet} = \sum_{j \neq i} \mu_x^{ij}$$

$$\ddot{a}_x^{(m)ij} = \bar{a}_x^{ij} - \frac{\mu_x^{ij}}{12m^2} \quad i \neq j$$

Note that in the "three-term" equation for  $\ddot{a}_x^{(m)ij}$ , where  $i \neq j$ , the second term is equal to zero.

**Black-Scholes put option formula**

$$p(t) = Ke^{-r(T-t)}\Phi(-d_2(t)) - S_t\Phi(-d_1(t))$$

$$\text{Where } d_1(t) = \frac{\log(S_t/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \quad \text{and} \quad d_2(t) = d_1(t) - \sigma\sqrt{T-t}$$