SOA Research INSTITUTE

Aging and Retirement

Variable Uninsured Life (Value) Annuities: Theory, Practice and Country Cases

Appendix 3: Design Guide, Technical Guide and Technical Addendum

November 2021

SOA Research INSTITUTE

Variable Uninsured (Value) Life Annuities

Theory, Practice and Country Cases

Appendix 3: Design Guide, Technical Guide and Technical Addendum

AUTHORS			
	William Price	Evan Inglis	Darren Ryder
	CEO	Associate Director	President
	D3P Global	D3P Global	D3P Global

Caveat and Disclaimer

The opinions expressed and conclusions reached by the authors are their own and do not represent any official position or opinion of the Society of Actuaries or its members. The Society of Actuaries makes no representation or warranty to the accuracy of the information.

Appendix 3: Design Guide, Technical Guide and Technical Addendum

VALUE DESIGN GUIDE

Retirement Income

The purpose of the Variable Uninsured Life Annuity (Value) longevity pool is to provide income for the lifetime of the members of the pool. By paying income to a large group, risk can be shared, and income can last for each member's lifetime without unnecessarily constraining spending in case one lives longer than expected. The objective of the pool is to pay stable and predictable income. This objective is achieved best with larger pools of retirees, good mortality data and accurate assumptions about longevity. Investment income may be volatile and will often be the most important source of uncertainty in future income for retirees. Higher levels of income may be achieved with growth-oriented assets such as equities, but these investments also increase the uncertainty of future income amounts.

The Longevity Pool Concept

Longevity risk is the risk that members in the system live longer than expected thereby increasing the cost. Longevity risk refers to uncertainty about the rates of mortality at each age in a population. There are two types of risk related to a mortality assumption:

- Systematic or population longevity risk—the risk that longevity increases in the broad population. This could occur if living standards and health care improve or there is a cure or new treatment for serious diseases such as cancer or heart disease.
- Individual or idiosyncratic longevity risk—the risk that an individual lives longer than their life expectancy. There are numerous causes for people to live longer than expected. This is the risk that is "pooled" in a longevity pool. When more than one person shares longevity risk, everyone in the pool can receive more income while they are alive. The pool can pay benefit amounts to everyone assuming they live just as long as expected. Then the savings that result from those who happen to die earlier is used to fund the extra income needed by those who live longer than expected.

Annuity products provided by insurance companies are a kind of longevity pool. They provide income based on the members' average lifetimes and thereby provide insurance against the extra financial needs related to living a long life. Annuity products also provide protection against population longevity risk. The insurance company takes on the risk. To make a profit, the insurance company must be conservative in its assumption about longevity, and this reduces the amount of income that can be provided.

A longevity pool like the Value concept pools individual longevity risk but shares the population longevity risk among the members. The members bear the risk as a group rather than paying an insurance company to take on this risk as with insured annuities. That means that income for the members must be reduced if the members of the longevity pool, on average, live longer than expected (longevity loss) or can be increased if the members, on average, die sooner than expected (longevity gain). Because there is no profit and all future longevity gains and losses will be shared by the members (instead of impacting an insurance company's profits), higher levels of income are expected to be provided by the Value concept.

Size of Pool

In small pools, the variability of gains and losses due to mortality experience can be large. There are various aspects of uncertainty and volatility in a longevity pool:

- Lack of experience data to support mortality table assumption—many smaller countries will lack good data over a long enough period to support a solid mortality rate assumption for the population.

- Mismatch between the population included in the longevity pool and the mortality table assumption for the wider population—the retirement system may cover a population that has different characteristics than the country's population as a whole or than the population covered by available mortality tables.
- Uncertainty in future changes to population mortality rates—a table of mortality rates represents the mortality in the recent past, but mortality rates in the future will be different. Mortality rates may decrease if life expectancy goes up, or they may increase if a new disease affects a population. Increases or decreases can be temporary or long-lasting.
- Random variation in the mortality experience of a longevity pool—this is the individual longevity risk that is controlled by creating a longevity pool instead of leaving each individual to try to make their own savings last for their own lifetime. The lifetime for a single individual is highly uncertain, but the average lifetime for a group of 10 people is more predictable, and the average lifetime for a group of 10,000 people is much more predictable.

Because the random variation in longevity is more predictable as the size of a pool increases, bigger pools are preferable. Pools of only 500 or 1,000 people will have a lot of uncertainty, and therefore the amount of income provided to members may increase or decrease by a fair amount. Once pools reach 10,000 or more people relatively stable levels of income can be provided.

The size of a pool will be impacted by whether the option for payout with a Value longevity pool is mandatory or optional. In addition, it will usually be desirable to create a single longevity pool with one administrator since creating separate pools administered by separate providers will reduce the size of each pool. Note that investments can be managed by more than one entity without creating separate pools.

Account Size

A minimum account size for the longevity pool is typically desirable. Even though any account size can be administered and paid out through a Value longevity pool, the cost of administration relative to additional funds provided to the pool by small accounts decreases the efficiency of the entire pool and reduces income for all participants. This consideration is important when an account balance can be split and paid out through more than one method available in a retirement system.

Actuarial Calculations

The income provided to each member of a longevity pool is calculated using actuarial factors that consider members' expected lifetimes (based on their individual characteristics such as age and gender) and expected investment earnings. Income amounts will also vary based on the type of retirement benefit provided. For example, if a benefit is designed to continue providing income to a surviving spouse after the retiree's death, the income amount must be reduced. The reduction in periodic income is offset by the expected value of the extra funds that would be paid after the retiree's death compared to a benefit that stopped payments upon the retiree's death.

A member's income amount is determined using the following simple equation:

Member's income = Member's savings / Actuarial Factor

The actuarial factor represents the expected value of one (unit of currency) paid for the rest of a member's life. It is the value of a lifetime annuity and is determined based on the member's expected longevity (using mortality rates by age) and expected investment earnings on the account. The higher the expected investment earnings, the lower the actuarial factor will be and the higher the amount of income paid will be. The longer a member is expected to live, the higher the actuarial factor will be and the lower the amount of income will be. Assumptions that are not accurate cause benefit amounts to change more than they would with better assumptions, and benefits may be distributed less equitably between the different members of the pool.

The member's savings is the total amount of money in a members account. It earns investment income and also is allocated a share of amounts forfeited by members who die. However, because the member is receiving payments from their account, the account balance gradually decreases once a member transfers their savings into the pool. As a member ages the actuarial factor will decrease because the expected remaining lifetime is shorter. As the member's savings decreases, the actuarial factor decreases. If the mortality assumption and investment earnings assumption are accurate, then the decrease in the account balance and decrease in actuarial factor will be consistent and the member's income will remain stable.

Building Mortality Experience

Actuarial factors are based on tables of mortality rates that represent the life expectancy of members based on age, gender and potentially other characteristics. The longevity pool functions best when these rates are an accurate representation of the levels of mortality that will actually occur within the pool. If mortality rates are inaccurate, then as members' actual mortality experience materializes, the benefit amounts will need to be adjusted. If mortality rates prove accurate, then income for members will be more stable and predictable.

The mortality experience of the longevity pool itself provides the best information about the likely future mortality rates for the pool. This information is collected from the pool and combined with other mortality information to gradually adapt the tables of mortality rates to be more representative of the pool's population. As that happens, the actuarial factors based on those tables will produce more stable income for the members. Where a country or retirement system lacks good data to support the actuarial factors when the pool is first established, mortality experience from the United Nations or from other countries in the same geographic region can be used as a start and gradually adapted with data as the data emerge from the longevity pool itself.

Measuring Gains and Losses

A Value pool provides variable income meaning that the income amounts change as the experience of the pool develops. When mortality or investment experience is such that income amounts can be raised, it is referred to as a gain. When income amounts need to be lowered, it is referred to as a loss.

As each member enters the pool by transferring their retirement savings into the Value fund, their income amount is determined based on actuarial factors using mortality tables and an assumed rate of investment earnings. When mortality experience in the pool is different than predicted by the tables or when investment earnings are different than the assumed rate, then the income amounts are adjusted. By regularly adjusting the amounts of income paid to each member, the pool is able to constantly remain in financial balance. It will always have money to provide the members with income for the rest of their lives.

Each year, or other period as determined for the pool, income amounts are adjusted. The Value system receives information about members' account balances from a recordkeeping system on a monthly (or other periodic) basis. As deaths happen in the pool of members, their account balances (that remain after any death benefits are paid) are transferred to a longevity fund that will be redistributed to the other members at the end of the year. The amounts in the longevity fund are redistributed based on the mortality risk for a member during the year. Members with higher mortality risk (older individuals) receive a larger portion of the longevity fund. Through this mechanism, the Value fund is kept in financial balance with regard to mortality and is enabled to provide lifelong income to the members.

The account balances for members are credited with investment earnings daily or otherwise periodically. If income from the investments exceeds the assumed investment earnings used in the actuarial factors, then when income amounts are redetermined at the end of the year, there will be a gain related to the expected investment return, and income amounts will be higher because of the positive investment experience.

The final way that gains or losses can develop is through evolving mortality experience. As the pool gathers more data and adjusts the tables of mortality to represent actual experience and any new expectations about how mortality rates will change in the future, the actuarial factors will be adjusted. If mortality rates evolve to be lower than expected (people are living longer) than when the last set of actuarial factors was determined, then using the new set of actuarial factors will result in a gain. Income amounts will be higher once they are redetermined with the new actuarial factors (ignoring any impact of investment gains or losses).

Assumed Investment Return

Actuarial factors are developed using an assumed rate of return on investment. If actual returns are lower than the assumption, then income amounts will decrease. This outcome may not be desirable since decreasing income may lead to dissatisfaction with the system. The potential for this kind of development can be reduced by using a conservative estimate of future returns. However, conservative return assumptions will increase the actuarial factors and decrease the initial income amounts. Thus, choosing the level of return to be assumed is a balance between minimizing the potential for income to decrease over time and maximizing the initial income received at retirement.

Adverse Selection (Antiselection)

Entering a longevity pool may be an option for receiving retirement income or it may be mandatory, depending on the system. If there is an option to take the full amount of retirement savings in a lump sum, then people who have reason to expect not to live a long time are not likely to enter the pool. If they do enter the pool, they are likely to be one of the members that dies early and thereby funds the retirement of members who live longer. That means that, on average, members in a pool where the system provides the option of a lump-sum payment may be expected to have longer lives than the general population that participates in the retirement system. Actuarial factors need to be adjusted upwards for this effect, and it lowers the amount of income that can be paid to the members. This effect is called “adverse selection” or “antiselection.” If adverse selection is not accounted for, it will create benefits that are too high to start and gradual losses as the pool progresses.

Similarly, once a member has become part of a longevity pool, they cannot be provided with the option to leave the pool or increase any optional death benefit. If that option were available, then members would leave the pool or increase their death benefit whenever they got an indication that their life expectancy was shortened, for example, by contracting a serious illness. By moving their money out of the system prior to their predictable death, they would reduce the amount in the longevity fund and ultimately reduce the amount of income available for the members. While it is possible to design a pool with limited exceptions to this rule, any exceptions will reduce the level of income payable, and the benefits of any flexibility or options available to members must be weighed against the antiselection impact.

Cost-of-Living Adjustments and Inflation-Adjusted Benefits

While the system may choose to offer inflation-indexed annuities, individual members should not be given this choice. If individual members are allowed to choose this option, then the risk of inflation being higher would be borne by the rest of the pool, and their benefit amounts would need to be adjusted to fund the inflation adjustments.

Benefit amounts can be indexed with a fixed percent increase each year, and different levels of increase can be included in the same pool. Death benefits may also be indexed with a fixed percent increase. It is possible to offer the option to change indexing of either annuity payments or death benefits each year. The death benefit amount for the current year determines the allocation of forfeitures at the end of the year.

Equity in Actuarial Factors

Policymakers may want to reflect the well-established relationship between wealth and longevity in the actuarial factors used in the system. A class of members with higher actual mortality (shorter lives) than the rest of the pool will be disadvantaged if they are included in a pool using the same assumed mortality rates as the members that have lower expected mortality (longer lives). Generally lower income populations are expected to have higher rates of mortality. For this reason, it may be viewed as inequitable to use the same assumed mortality rates for lower income members and higher income members. To address this issue, the assumed mortality rates can be adjusted based on some measure of wealth or income.

Classes of members with higher actual mortality rates will be disadvantaged in a pooled approach because mortality gains will go disproportionately to those who are allocated more forfeitures because they live longer. By using higher rates of mortality to determine the annuity factors, higher income amounts will be generated for those with lower wealth or income. Higher assumed rates of mortality also translate into a larger allocation from the longevity fund for those who survive. Through these mechanisms a pool may be designed to be more equitable. Tiers of wealth/income can be used, but a continuous linear interpolation eliminates the problem of small changes in wealth/income pushing a member into another category.

Assuming wealth and income data from outside the system is not used, there are four approaches to adjusting mortality rates:

- Account balance at the beginning of each year—as an account gets paid down this may not represent the wealth of a member as well as the account balance at retirement
- Account balance at retirement—this may not work well if partial lump sums can be paid, or retirement can be phased
- Income each year—this may underrepresent the income of early retirees
- Income at retirement—this may underrepresent the income of early retirees

If the account balance is used, a floor account balance, e.g., 20,000, and a ceiling account balance, e.g., 200,000, would be chosen. Actuarial factors would be determined using higher mortality rates for all account balances below 20,000 and using lower mortality rates for all account balances above 200,000. For account balances between these levels the actuarial factor would be determined by an interpolation between the actuarial factor at 20,000 and the actuarial factor at 200,000.

Death Benefits (Return of Premium, Life Insurance)

A system may want to allow members to direct some of their savings to a beneficiary after their death. A percentage of account balance or a fixed amount can be chosen. A fixed amount should be limited to the amount of the account balance and adjusted in the future as the account balance changes. Members who choose a death benefit participate to a lesser degree in the mortality gains from the system. A member who chooses to have their entire account balance paid to a beneficiary on death will not participate in mortality gains at all, and other levels of death benefit result in proportional adjustment of the allocation of mortality gains to the member's account. The account balance for a member who chooses a death benefit will decrease faster than if no death benefit had been chosen such that the annuity paid will be expected to gradually decrease. It is possible for a system to include members with different levels of death benefit. It is also possible for members to decrease their election of death benefit each year or at such times as desired by the system. Members should not be allowed to increase their death benefit as this will lead to adverse selection when members become ill or otherwise gain information that indicates that their mortality risk may be higher. This will lead to losses for the rest of the pool.

To gain the benefits from pooling longevity risk, there should be a substantial amount of longevity risk being taken. That means that it is not desirable for participants to elect a full death benefit (100% return of retirement savings). Death benefits must always be limited to no higher the value of the account balance, but ideally should be substantially lower.

Fixed dollar or percentage of account balance amounts can be used to define death benefits, but fixed dollar amounts will need to be adjusted if the account balance falls below the fixed dollar amount. This is another reason that it is best to limit death benefit amounts to levels significantly below the account balance—e.g., 75% of the account balance. Participants can be free to decrease their death benefit.

Investment Strategy/Policy

Investment and longevity experience are the two drivers of gains and losses that result in changes to members' account balances. In a system where investments are in diversified portfolios with equities and other riskier, growth-oriented assets, investment gains and losses are likely to be much larger than longevity gains and losses.

Generally, longevity pools will invest in a single pool of assets rather than allowing each member to choose their own investments. The collective nature of the longevity pool lends itself to a single pool of assets, and the investment strategy can take advantage of a longer time horizon when assets are not separately invested for each individual.

Fixed income assets are well suited to delivering retirement income since they deliver a steady flow of cash that aligns with the requirement to pay members a steady flow of income. However, equity and other more growth-oriented assets such as real estate can provide higher returns and some protection against the erosion of purchasing power over time due to inflation. Even if a portfolio of assets that includes growth-oriented assets delivers higher returns and ultimately higher income, declines in income from any level create an issue relative to the expectation that the current level of income will continue. The problems related to uncertain returns are a key concern, and expectations need to be managed with appropriate communication and educational materials.

Returns may be smoothed as described in the next section, and this can help a system achieve one of its main objectives, which is to provide steady, predictable income streams. However, smoothing returns must be managed carefully, and problems can arise. The number one issue is that if returns are lower than expected for an extended period and losses are built up in a reserve account, then benefit amounts may need to be adjusted downwards for an extended period or essentially permanently. The other main issue is that reserves give rise to concern about equity between cohorts of system members. If reserves are built up by holding back returns for an extended period, members may be unhappy that returns they might view as belonging to them are now being shared with other cohorts of system members.

Smoothing can be accomplished by averaging returns over a period, three or five years, for example, or smoothing can be accomplished by deferring gains and losses on individual investments. If gains and losses on investments are deferred, then the timing of trading investments will be constrained by the objectives of the smoothing approach. These methods must be coordinated with accounting requirements as explained in the next section.

Smoothing may be applied selectively. For example, members of an advanced age, with lower income amounts or in other special categories can be provided with smoothed returns. Again, care must be taken in designing and communicating these kinds of provisions since members in or out of the special categories to which smoothing applies may be disadvantaged during some periods by the smoothing approach.

Mark-to-Market Versus Book Value Accounting for Investments

In a system invested exclusively or primarily in government bonds, book value accounting may be preferred to mark-to-market accounting by some entities, but it is not typical in OECD economies. Fixed income investments are ideally suited to delivering retirement income because they provide a steady flow of payments that can be matched with members' benefit payments. When fixed income investments lose market value, yield increases, such that the level of income remains steady. However, when market value accounting methods are used, the change in market value creates a change in income amounts. Changes in income amounts are unavoidable but also undesirable and generally should be minimized. Book value accounting for fixed income properly represents the investment as a steady flow of cash rather than an instrument with fluctuating value.

Equity, and other similar investments, have volatile returns that can drive significant changes in income from year to year. It is possible to smooth these returns using book value methods and an investment return reserve. An investment return reserve is increased in years of high returns when the full amount of return will not be added to member accounts. The reserve is then utilized to increase the return allocated to accounts in years when the investments themselves provide than expected returns. This approach smooths the return pattern of the system and minimizes the changes in members income. The approach to smoothing needs to be coordinated with accounting standards that the system is required to follow.

Investments in funds may present different issues than investments in individual securities. Investments in individual equity securities can be adapted for smoothing by deferring gains and losses on securities until the security is sold. This can facilitate smoothing when accounting standards are an issue. It is especially valuable for equity investments, which are likely to be traded from time to time. Fixed income in a retirement system is often held to maturity, in which case accounting using book value works well and will be consistent with accounting standards. Smoothing returns from investments in collective funds may present more challenges since the fund itself usually accounts for returns using a mark-to-market approach to allow investors to buy and sell their share at any time.

Return Guarantees

While the accounting method offers one approach to smoothing returns, a more explicit guarantee may take the form of a minimum return for any year. For example, the guarantee may be for a minimum return of 0%, i.e., no investment losses. This kind of provision may be desirable to enhance confidence and increase participation in the payout system. These kinds of guarantees require government subsidy in years when there are significant losses on risky assets.

Another concept sometimes used is to guarantee that there will be no loss of principal, which means that the account balance will never drop below the original investment in the pool adjusted for income paid from the account. This approach can be used with or without adjustment for longevity credits. The requirement for government subsidy with this method is much lower than the with annual floor on investment returns described above.

VALUE TECHNICAL GUIDE

The essential feature for a longevity pool is the need to allocate gains and losses from mortality and investment experience to members' accounts. This brief technical guide focuses on mortality since methods for allocating investment returns are more commonly understood and practiced. Determining investment returns is done outside the Value system structure, and the investment returns to be allocated are assumed to be entered or read into the Value system.

TERMINOLOGY

The following terms are used.

- Longevity Fund—the account holding amounts forfeited by members who have died that accumulates investment earnings and is allocated to members remaining in the pool periodically.
- Forfeiture to Longevity Fund—the amount of account balance forfeited, less any death benefit, that is allocated to the Longevity Fund on a member's death.
- Longevity Credit—the total amount of the Longevity Fund allocated to a surviving member's account. The Longevity Credit is driven by forfeitures to the Longevity Fund.
- Longevity Gain—the actual Longevity Credit minus the expected amount of Longevity Credit, which is related to Mortality Gains and Losses.
- Longevity Allocation Factor—the factor, representing mortality risk, that determines the proportion of the Longevity Fund to be received by a surviving member.
- Longevity Risk—the risk of living a long life and needing additional income.
- Mortality Risk—the risk that a member dies during the next year and forfeits their account balance.
- Mortality Gains and Losses—the general concept, as opposed to the specific amount, of mortality experience being higher or lower than the mortality rate assumption.

ALLOCATING LONGEVITY CREDITS

Mortality gains and losses are allocated to members' accounts based on the size of the account balance and the amount of mortality risk being taken. The amount of mortality risk is related to the size of the account balance, age, gender and other characteristics that drive mortality rates and the size of any benefit amounts paid on death.

In a system that allows death benefits, the amount of mortality risk is lower for any member who is provided with or that elects to receive a benefit on death. Therefore, that member will receive a lower allocation of mortality credits when the longevity fund is reallocated to member accounts.

The Value system process is for the account balances of members who die, net of any death benefits, to be allocated to the longevity fund. The longevity fund is allocated investment earnings just like a member account and then is reallocated to the living members, typically at the end of each year.

The portion of the longevity fund allocated to a member is determined using the Longevity Allocation Factor:

Longevity Allocation Factor (LAF) = $q / p = q / (1 - q)$, where
q is the actuarial probability of dying in the next period
$p = (1 - q)$ is the actuarial probability of surviving to the next period

The LAF for an individual member divided by the sum of all LAFs for all members determines the share of the longevity fund when it is reallocated.

It may not be immediately obvious why the LAF should work this way, and the following example may help. Consider a coin-tossing game where many players participate, and each player risks the same amount. The coins have a 75% chance of heads in which case the player wins (corresponds to surviving to the next year in the longevity pool) and a

25% chance of tails in which case the player forfeits their coin (corresponds to dying and forfeiting one's account balance). In this game, the expected winnings in each round are 25% (the amount that is expected to be forfeited) divided by 75% (the remaining players to which the forfeited amounts will be allocated) or 25% / 75% = 1/3 of the amount being risked by each player. This is the same q / (1 – q) structure as the LAF. In both the coin-tossing game and the longevity pool, if actual experience turns out as expected, then each player or member will increase their playing amount or account balance by the factor 1 + [probability of forfeiture / (1 – probability of forfeiture)]. Of course, in the longevity pool income is also being paid out.

In the coin-tossing game, envision that each player decides how much to put at risk (corresponds to members' account balance). Since a player is risking more and will contribute more to the winnings of other players if they lose and forfeit, then they correspondingly will be awarded a higher percentage of the forfeited amounts if they win and survive to the next round. Next, envision that players are allowed to bring their own coins that have different probabilities of ending up heads or tails. Since a player with a lower probability of winning with heads (corresponds to a lower mortality rate q) has a higher likelihood of contributing forfeited amounts, the proportion of the forfeited amounts that they win on surviving is adjusted upwards accordingly.

Similar to the adjustments in the coin-tossing game, it is straightforward to adapt the longevity pool to the different size account balances, different probabilities of dying and different amounts that may be returned at death. The LAF for each member is based on their own age and gender. It is multiplied by the member's account balance, and the probability of dying (forfeiting) is adjusted directly for the amount of death benefit being provided. For example, a member with a US$500,000 account and a US$100,000 death benefit would have their LAF weighted by US$400,000 instead of US$500,000. The weighted LAFs are added up for all members, and each member receives a share of the longevity fund equal to the size of their individual weighted LAF to the total of all weighted LAFs.

$$\text{Proportion of Longevity Fund allocated to individual member} = \text{weighted LAF} / \Sigma \text{ weighted LAFs}$$
$$= [(\text{AcctBal} - \text{DeathBen}) \times q / (1 - q)] / \Sigma\, [(\text{AcctBal} - \text{DeathBen}) \times q / (1 - q)]$$

BENEFIT AMOUNTS

Each member in the longevity pool receives benefit payments, and the stability and predictability of these amounts are presumed to be important to the members. Therefore, the stability and predictability of benefits are a key objective for the system.

Periodically, usually each year, members' benefit payment amounts will be recalculated. The change in amount will represent changes in the account balance due to investment earnings and longevity credits, a new actuarial factor at the current age and changes in assumptions to determine the actuarial factor. The most likely assumption to be adjusted is the mortality rates.

At the end of each year and prior to the beginning of the next year:

1. Account balances are determined with final reconciliation of investment accounts and allocation of investment earnings to members' account balances
2. The Longevity Fund is allocated to members' account balances
3. Actuarial factors may be redetermined based on new mortality experience
4. The new benefit amount to be paid during the next year is determined by dividing the updated account balance with the new actuarial factor

The change in account balance can be attributed to various aspects of gain or loss. The Value Design Guide includes a discussion of these factors. The Technical Addendum that follows shows details of how these factors are calculated and the formulas that can be used to analyze what factors are contributing to changes in income.

VALUE TECHNICAL GUIDE ADDENDUM

VARIABLE LABELS

AcctBal = Account Balance

AnnFact = Actuarial value of $1 lifetime annuity

AnnFactAdj = Annuity factor after adjustment for value of death benefits

AnnAmt = Amount of annuity to be paid, after adjustment for death benefit

LifeFact = Actuarial value of US$1 of life insurance (for death benefit)

LifeAmt = Amount of benefit to be paid on death (death benefit)

LongGain = Unexpected increase in account balance due to longevity credits

LongForf = Forfeitures of account balances to longevity fund on death

InvRet = Investment return

InvRetRate = Rate of return on investments

i = interest rate (inflation + real rate)

q = probability of death

p = 1 – q = probability of survival

ANNUITY AMOUNT ADJUSTED FOR DEATH BENEFIT (LIFE INSURANCE)

AcctBal = AnnFact × AnnAmt + LifeFact × LifeAmt

AcctBal = AnnFact × (AcctBal/AnnFactAdj) + LifeFact × LifeAmt

1 = AnnFact / AnnFactAdj + LifeFact × LifeAmt/AcctBal

AnnFact / AnnFactAdj = 1 – LifeFact × LifeAmt/AcctBal

AnnFactAdj = AnnFact / (1 – LifeFact × LifeAmt/AcctBal)

ALLOCATION OF MORTALITY FORFEITURES AND LONGEVITY GAIN

LongGain = LongForfAct – LongForfExp

LongForfAct = LongForfTotal × qx/(1 – qx) / Σ qx/(1 – qx)

LongForfExp = AcctBalAvg × qx/(1 – qx)

INVESTMENT GAIN

InvRetExp = AcctBalAvg x InvRetRateExp

InvRetGain = InvRetAct - InvRetEx

ESTIMATED GAIN/LOSS IN ANNUITY AMOUNT DUE TO CHANGE IN ASSUMPTIONS—CHANGES IN ANNFACT AND LIFEFACT

$AnnAmt_1 – AnnAmt_e$ = AnnAmtGainAcctBal + AnnFactAdjGain

Change in annuity amount is made up of a gain in the amount of the account balance (items shown above) plus a gain due to a change in the annuity factor

AnnAmtGainAcctBal = (LongGain + InvRetGain)/$AnnFact_1$

Gains in the account balance increase the annuity amount and do not affect the death benefit

AnnFactAdjGain = $AcctBal_1$ / $AnnFact_1$ – $AcctBal_1$ / $AnnFact_e$

AnnFactAdjGain = (AnnFactGain + LifeFactGain)

AnnFactGain = $AnnAmt_1$ - $AcctBal_1$ / (($AnnFact_0$ – p_0 / (1+i/2)) × (1+i) / p_0) / (1-$LifeFact_1$ × $LifeAmt_1$/$AcctBal_1$))

LifeFactGain = $AnntAmt_1$ – $AcctBal_1$ / $AnnFactAdj_e$ = $AnntAmt_1$ – $AcctBal_1$ / ($AnnFact_1$ / (1 - $LifeFact_e$ × LifeAmt/$AcctBal_1$)

LifeFactGain = $AnntAmt_1$ – $AcctBal_1$ / ($AnnFact_1$ / (1 - ((($LifeFact_0$ – q_0) x (1 + i)) / p_0) × LifeAmt/$AcctBal_1$)

About The Society of Actuaries Research Institute

Serving as the research arm of the Society of Actuaries (SOA), the SOA Research Institute provides objective, data-driven research bringing together tried and true practices and future-focused approaches to address societal challenges and your business needs. The Institute provides trusted knowledge, extensive experience and new technologies to help effectively identify, predict and manage risks.

Representing the thousands of actuaries who help conduct critical research, the SOA Research Institute provides clarity and solutions on risks and societal challenges. The Institute connects actuaries, academics, employers, the insurance industry, regulators, research partners, foundations and research institutions, sponsors and non-governmental organizations, building an effective network which provides support, knowledge and expertise regarding the management of risk to benefit the industry and the public.

Managed by experienced actuaries and research experts from a broad range of industries, the SOA Research Institute creates, funds, develops and distributes research to elevate actuaries as leaders in measuring and managing risk. These efforts include studies, essay collections, webcasts, research papers, survey reports, and original research on topics impacting society.

Harnessing its peer-reviewed research, leading-edge technologies, new data tools and innovative practices, the Institute seeks to understand the underlying causes of risk and the possible outcomes. The Institute develops objective research spanning a variety of topics with its strategic research programs: aging and retirement; actuarial innovation and technology; mortality and longevity; diversity, equity and inclusion; health care cost trends; and catastrophe and climate risk. The Institute has a large volume of topical research available, including an expanding collection of international and market-specific research, experience studies, models and timely research.

Society of Actuaries Research Institute
475 N. Martingale Road, Suite 600
Schaumburg, Illinois 60173
www.SOA.org