

CP 351 Model Solutions

November 2025

1. Learning Objectives:

1. The candidate will understand the objectives of Asset Liability Management (ALM).

Learning Outcomes:

- (1a) Explain the principles of Asset Liability Management

Sources:

CP351-100-25: IAA Risk Book: Asset Liability Management Techniques and Practices for Insurance Companies

Commentary on Question:

This question aims to test candidates' understanding of ALM theory, specifically the concept of duration matching, dollar duration, and their limitations in managing interest risks. Candidates did well on this question.

Solution:

- (a) Critique the CRO's statement.

Commentary on Question:

To receive full points, it is important for the candidate to explain why the statement is incomplete or flawed, instead of just stating that it is wrong.

Candidates did fairly well on this question. Some candidates referred to irrelevant risks such as policyholder behavior risks. Some candidates also simply stated that there is convexity without elaboration on how that contributes to interest rate risks.

This statement is incorrect because duration matching alone does not fully eliminate interest rate risk.

1. Continued

- Convexity is ignored:
 - Duration assumes a linear relationship between interest rate changes and asset/liability values, but in reality, the relationship is curved (convexity effect).
 - If interest rates decline, liability duration increases more than asset duration, leading to an asset-liability mismatch.
- Non-parallel interest rate shifts can cause mismatches:
 - Interest rates do not always shift uniformly across maturities.
 - Even if duration is matched, a non-parallel yield curve movement could expose the company to unexpected losses.

(b)

- (i) Calculate the dollar duration mismatch of ABC Life's portfolio.
- (ii) Recommend two ways ABC Life can address the duration mismatch.

Commentary on Question:

To receive the full points, the candidate should conduct appropriate calculation, and specify two approaches for ABC to address the duration mismatch.

Candidates did well on part (i). Some candidates mixed up the concept of dollar duration and DV01 by multiplying their answers by 1%. Partial credits were given to those candidates. Candidates did fairly well on part (ii). Some candidates mixed up receiver and payer swaps which no credits were given. Some candidates recommended multiple types of derivatives as different ways, partial credits were given to those responses.

- (i) Dollar duration of assets: $8.5 \times 500\text{M} = 4.25\text{B}$
Dollar duration of liabilities: $12.0 \times 480\text{M} = 5.76\text{B}$
Dollar duration mismatch: $5.76\text{B} - 4.25\text{B} = 1.51\text{B}$

- (ii) Ways ABC Life can address the duration mismatch:

- Invest in longer duration bonds

ABC Life can reallocate assets to bonds with a longer maturity and higher duration (e.g., 20-year government or corporate bonds). This increases asset duration and reduces the mismatch.

- Use interest rate derivatives such as swaps to extend duration

1. Continued

ABC Life can use an interest rate swap overlay, where it receives fixed and pays floating cashflows, effectively extending the asset duration without having to replace physical assets.

- Capital injection

ABC Life can increase its asset value through capital injection to reduce the mismatch. For example, it could invest in assets with similar duration as the current portfolio to increase the dollar duration.

- Other responses such as using carve out strategy or changing product offerings are also acceptable

(c)

- (i) Describe two risks ABC Life faces in falling interest rate environment.
- (ii) Recommend three approaches for ABC Life to mitigate these risks.

Commentary on Question:

To receive full points, it is important for candidates to provide concise explanations of each risk, showing a clear causal relationship with the given scenario.

Candidates performance was fair on part (i). Many candidates suggested ABC Life faces policyholder risks, but only a few of them correctly described the way policyholders will react in a falling interest rate environment. Candidates performed poorly on part (ii). Only a few candidates were able to correctly recommend and justify three approaches to mitigate the risks. Similar to part b(ii), some candidates recommended multiple derivatives as different ways.

- (i) Two risks ABC Life faces due to the dropping interest rate environment:

- Spread compression and lower investment/reinvestment income:

As interest rates decline, the yield from long-term bonds falls, reducing ABC Life's investment/reinvestment income. This reduces profit margins and increases solvency pressure.

1. Continued

- Higher reserve requirements:

A lower interest rate means future liabilities must be discounted at a lower rate, leading to an increase in required reserves. This forces insurers to allocate more capital toward reserves instead of investments or business growth.

- Other responses like policyholder behaviour risks and ALM mismatch risks are also acceptable

(ii) Three ways on how ABC Life can mitigate these risks:

- Increase allocation to higher-yield assets

ABC Life can diversify into corporate bonds, infrastructure investments, or private debt to improve portfolio returns.

- Use derivatives to hedge against interest rate risk

ABC Life can invest in interest rate swaps or options to protect against further declines in rates. This helps manage exposure without altering the asset portfolio significantly.

- Modify product offerings to reduce guarantees

ABC Life can adjust its product mix by offering more unit-linked or participating policies, which transfer investment risk to policyholders.

Other valid responses such as using reinsurance and investing in longer duration assets can also be accepted.

2. Learning Objectives:

1. The candidate will understand the objectives of Asset Liability Management (ALM).

Learning Outcomes:

- (1b) Understand the lessons learned from ALM failures

Sources:

Financial Enterprise Risk Management - Sweeting_Ch 20_Case Studies, Section 20.6,

CP351-103-25 - Risk Management Lessons Learned From SVB,

What Can Insurers and Pension Funds Learn from Bank Failures

Commentary on Question:

This question tests candidates' understanding of the distinction between true arbitrage and convergence trades, the ability to fact-check and critique simplified explanations of financial failures, and the application of lessons from LTCM and SVB to strengthen ALM practices. Strong answers explained why LTCM's initial strategy qualified as true arbitrage while its later trades did not, accurately described the actual causes of each collapse, and proposed clear, actionable ALM improvements grounded in risk, leverage, and liquidity management.

Solution:

- (a) Assess whether the following fixed income strategies of LTCM are true arbitrage:

- (i) Initial strategy within a single government bond market
- (ii) Expanded strategy across different government bond markets

(i) LTCM's initial strategy is true arbitrage. It involves taking long positions of "on-the-run" government bonds and short positions of "off-the-run" government bonds of a similar term.

On-the-run bonds are generally more liquid than off-the-run bonds, and so are more expensive. This means that if an investor buys an off-the-run bond whilst simultaneously taking a short position in an on-the-run bond, then an arbitrage profit will be realised if both positions are held to maturity.

(ii) Whilst taking positions in on- and off-the-run bonds in a single market can lock an investor into a profit if the position is held until the bonds are redeemed, attempted arbitrage between different markets relies on convergence of markets to some long-term norm, so it's not true arbitrage. Additionally, taking positions across different markets introduces currency, exchange rate, government default, and political risks.

2. Continued

A colleague claims that:

- LTCM's collapse was due to its over-leveraged positions in emerging market bonds, which became unprofitable when the Russian government defaulted on its debt in 1998.
- Silicon Valley Bank's collapse was triggered by its failure to hedge the interest rate risk on its long term Treasury bond portfolio, leaving it exposed to losses as rates rose in 2022-2023.

(b) Critique the colleague's claims.

(1) Partially correct. LTCM's strategy relied on convergence trades across a variety of markets, including U.S. and European government bonds, not specifically emerging market bonds. The Russian default triggered a flight to quality, where investors sold European bonds and bought U.S. Treasuries, causing global bond spreads to diverge unexpectedly. This divergence, not direct losses from emerging market bond spreads, was the primary driver of LTCM's losses. LTCM also had an over-reliance on their models and executive leadership team (2 Nobel Prize winners), which resulted in LTM not vetting their strategies thoroughly.

(2) Your colleague's comment is mostly incomplete. SVB's collapse was triggered by a rapid deposit run due to the burst of the crypto bubble from its tech-sector clients, exacerbated by a duration mismatch between its long-term assets, specifically MBS, and short-term liabilities (unrestricted checking deposits). While hedging could have mitigated some losses, the inability to meet withdrawal demands—forcing SVB to sell devalued assets at a loss—was the decisive factor.

(c) Propose three improvements for the company's ALM practice based on lessons learned from these collapses.

1. **Manage leverage:** Avoid excessive leverage, as seen in LTCM's 25:1 ratio, which amplified losses, and This is critical for insurers to avoid forced asset sales during stress events affecting policyholder obligations.
2. **Manage liquidity risk:** Ensure asset liquidity aligns with liability demands, as SVB's illiquid assets could not meet rapid deposit outflows.
3. **Diversify funding and customer base:** Reduce concentration risk by diversifying the customer base, as SVB's tech-heavy deposit base led to correlated outflows.
4. Use stress tests and scenario analysis to anticipate market changes.
5. Incorporate surrender protections in product design to manage liquidity risks.
6. Assess the true economic value of assets to avoid overvaluation.

2. Continued

7. Limit model reliance, ensuring they capture tail risks and inform rather than dictate decisions.
8. Match asset and liability durations and convexity to one another. Closely monitor the two and ensure they move in tandem.
9. Model policyholder behavior, especially dynamic lapses and surrenders.

3. Learning Objectives:

2. The candidate will understand how to measure risks from assets and liabilities.

Learning Outcomes:

- (2c) Evaluate the difficulties of investing for long-tail liabilities (i.e. beyond 30 years) such as inflation and longevity

Sources:

CP351-110-25: New Frontiers: Backing Long-term Insurance Liabilities with Non-fixed-income Assets, TD Asset Management

CP351-101-25: Asset Liability Management for Life Insurance, Annuities, and Pensions

Commentary on Question:

This question requires the Candidate to understand duration and convexity concepts and apply them to a situation where accounting rules change. The question outlines the accounting rules, so it is not necessary for the Candidate to know anything about the different regimes, but the candidate must use the information provided to think critically about what the impacts would be.

Solution:

- (a) Calculate the IFRS 4 and IFRS 17 liability discount rates for LMN.

Commentary on Question:

This question examines understanding of the illiquidity risk premium. There are two calculations and both calculations needed to be correct for full credit. Most candidates determined the correct answer for IFRS4, but used incorrect spreads for IFRS17.

IFRS4: 12.96%

IFRS17: 7.5%

The calculation for IFRS4 required determining the discount rate as a weighted average of the rates based on the portfolio allocation. The calculation for IFRS17 is the risk-free rate plus the illiquidity for the investment grade assets in the portfolio: AAA, AA, A, and BBB.

- (b) Explain how the change in discount rate impacts the economic and accounting surplus of LMN.

3. Continued

Commentary on Question:

Changes to the accounting will only impact the liability. So the question is essentially asking to compare the current liability discount rate to what would be the new one and then understanding how that would impact the liabilities versus where they are marked today. The question is not asking for any actual calculations, but just to reason through the directionality. The candidate must supply a valid reason for full credit. Many candidates were unclear in their reasoning.

IFRS 17 will cause an initial surplus loss due to a decrease in the liability discount rate, which won't change the asset value, but will increase the liability value.

- (c) Calculate the interest rate risk on an accounting basis after the change to IFRS 17.

Commentary on Question:

This question is an application of duration and convexity. The candidate needs to arrive at the discount rate to identify the assets backing the liability and then use the table and the asset sensitivities to calculate the risk. Many candidates did not correctly use the table to calculate the risk.

The interest rate risk was -2.70. It is the worst case loss due to a +/-100bps shock to rates.

- (d) Compare the impact on each of the following risk metrics under IFRS 4 vs. IFRS 17:
- (i) Sensitivity of surplus
 - (ii) Duration mismatch

Commentary on Question:

No calculations necessary. This question asks for reasoning to understand the direction of surplus, and interest rate risk as discount rates change. The candidate needs to understand the concepts related to duration and convexity to answer both questions. To get full credit, the candidate must supply rationale in the comparison. Many candidates did not clearly compare the sensitivity to surplus and duration mismatch or did not provide correct reasoning.

- (i) Under IFRS17 this would cause a decrease in surplus. Under IFRS4, there would be no change to surplus since the discount rate of the liabilities would change along with the change in supporting assets.

3. Continued

- (ii) The duration mismatch between assets and liabilities would increase in IFRS 17 in comparison to IFRS 4 as the liability would have lengthened more, both due to having larger convexity, and due to a larger decline in discount rates.

4. Learning Objectives:

2. The candidate will understand how to measure risks from assets and liabilities.
3. The candidate will understand tools and strategies to manage ALM risks

Learning Outcomes:

- (2b) Evaluate a company's or a portfolio's exposures to various risks
- (2c) Evaluate the difficulties of investing for long-tail liabilities (i.e. beyond 30 years) such as inflation and longevity
- (3a) Develop and critique asset allocation strategies appropriate to underlying liability profiles such as pension plans and long-tail insurance liabilities
- (3b) Demonstrate an understanding of and apply tools and techniques for measuring and managing interest rate risk in an ALM context

Sources:

CP351-101-25: ALM for Life, Annuities, and Pensions, Hatfield (2024)

CP351-110-25: New Frontiers: Backing Long-term Insurance Liabilities with Non-fixed-income Assets, TD Asset Management

Commentary on Question:

This question does a good job by distinguishing candidates reasonably well on interest rate management. Average grade is above the mid-point of total grade.

Solution:

- (a) Calculate the following:
 - (i) The liability discount rate
 - (ii) Duration of OPQ's liabilities
 - (iii) Convexity of OPQ's liabilities

Commentary on Question:

Majority candidates performed well on duration and convexity calculation. However, a fair number of candidates did not calculate discount rate correctly by missing the rate of default.

4. Continued

Illiquidity Spread: A weighted average of the market value of assets

$$\frac{(1.25\% - 0.13\%) \times 100 + (2\% - 0.3\%) \times 50 + (3\% - 0.5\%) \times 26 + (4.5\% - 1\%) \times 5}{100 + 50 + 26 + 5} = 1.55\%$$

Discount rate = Risk-free rate + illiquidity spread = 5% + 1.55% = 6.55%

$$\text{Duration: } D_L = \frac{V_- - V_+}{2 \times \Delta r \times V_0} = \frac{31,966 - 31,772}{0.001 \times 31,868} = 6.09$$

$$\text{Convexity: } C_L = \frac{V_- + V_+ - 2V_0}{\Delta r^2 V_0} = \frac{31,772 + 31,966 - 2 \times 31,868}{0.0005^2 \times 31,868} = 251.04$$

- (b) Describe the challenges for OPQ in managing the interest rate risk of the liabilities using only corporate bonds.

Commentary on Question:

Majority candidates performed well. They understand that duration matching is the first step of ALM.

Given the liability has long duration and convexity, assets have to be managed accordingly to ensure the surplus is less volatile upon interest rate changes. Because the liability cashflows extend well beyond the term of the corporate bond market, or scarcity of long duration corporate bonds, OPQ would likely be exposure to key rate duration risk.

- (c) Recommend two methods OPQ could use to manage interest rate risk on the long-term annuity liabilities.

Commentary on Question:

Most candidates performed well by identifying using derivatives, however, only a handful of candidates recommended investing in long-term government bonds.

OPQ can invest in long-term government bonds to reduce duration mismatch, but this would reduce the overall yield of the portfolio.

OPQ may be able to enter into a derivative contract, such as a long-term swap. Due to the lack of need for an initial investment, this can add long-term duration without impacting spread (i.e. it will add leverage to the portfolio). However, this may come with accounting and liquidity implications.

4. Continued

(d)

- (i) Explain what a carve-out strategy is.
- (ii) Describe two advantages of a carve-out strategy for OPQ.

Commentary on Question:

Most candidates received full credit.

- (i) A carve-out strategy is when a portion of the liabilities (the long-tail) is separated from the short-term portion of the liabilities. The short-term portion is managed to a tight interest rate risk match, while the long-term is managed more loosely with a significant portion of the assets invested in higher yielding assets such as equity, which can benefit from the long-term and illiquid nature of the cashflows and can provide ability to diversify into alternative asset classes.
- (ii) OPQ's liabilities have a significant amount of long-term cashflows, and they are illiquid, which allows the segmentation to work well by tightly managing the interest rate risk in the short-term while having flexibility to manage the assets in long-term by investing assets in Non-Fixed-Income assets such as equity, real estate and infrastructure. That can lead to significantly high expected yields as well as other benefits.

5. Learning Objectives:

2. The candidate will understand how to measure risks from assets and liabilities.

Learning Outcomes:

- (2a) Demonstrate an understanding of various risk identification tools
- (2b) Evaluate a company's or a portfolio's exposures to various risks

Sources:

Fixed Income Securities: Tools for Today's Markets, Fourth Edition. Tuckman and Serrat, Ch. 4

CP351-101-25: ALM for Life, Annuities, and Pensions, Hatfield (2024), section 4

CP351-107-25: Key Rate Durations: Measures of Interest Rate Risk

CP351-109-25: IAIS Application Paper on Liquidity Risk Management

Commentary on Question:

Commentary listed underneath question component.

Solution:

- (a) Estimate the value of the assets under Scenario A based on the results from the simulations.

Commentary on Question:

The purpose of this question is to test the candidate's understanding of the formula for effective convexity. Overall, candidates did well on this part of the question. Candidates who were able to identify the correct formula generally could identify all the correct inputs, although some candidates incorrectly used the full -1.5% rate change.

Recall the formula for effective convexity:

$$Conv_L \approx \frac{\hat{L}_+ - 2\hat{L}_0 + \hat{L}_-}{\delta^2 \hat{L}_0}.$$

L_+ corresponds to Scenario B

L_0 corresponds to Scenario A

L_- corresponds to Scenario C

δ is 0.001, since the interest rate shock is 10 basis points up and down

Rearrange the formula to solve for L_0 , which represents the value of Company A's assets as of 12:01 PM yesterday:

$$C = (L_+ - 2L_0 + L_-)/(0.001^2 * L_0)$$

$$C * 0.001^2 * L_0 + 2L_0 = L_+ + L_-$$

5. Continued

$$L_0(C*0.001^2 + 2) = L_+ + L_-$$
$$L_0 = (L_+ + L_-)/(C*0.001^2 + 2)$$

Extract the relevant information from the question

$$L_+ = 2,583,800$$

$$L_- = 2,637,300$$

$$C = 25$$

Calculate L_0

$$L_0 = (2,583,800 + 2,637,300)/(25*0.001^2 + 2)$$

$$L_0 = 2,610,517$$

- (b) Estimate the following as of yesterday at 11:59 AM (before the drop in interest rates):
- (i) DV01 of XYZ's assets
 - (ii) DV01 of XYZ's liabilities

Commentary on Question:

The purpose of this question is to test the candidate's understanding of DV01 and the relationship between assets, liabilities, and equity. Candidates generally performed as expected on this question. To receive full credit, candidates needed to include duration and convexity in their calculation.

Part (b)(i)

Extract relevant information from the question (before interest rate drop):

$$\text{Assets} = 2,500,000$$

$$\text{Equity} = 450,000$$

$$\text{Change in interest rates} = -150 \text{ bps}$$

Calculate change in asset value after interest rate change:

$$\text{From part (a), the value of assets after the interest rate change is } 2,610,517$$

$$\text{Change in asset value} = 2,610,517 - 2,500,000 = 110,517$$

Use the formula for approximating the change in value of a portfolio using duration and convexity:

$$\Delta P/P \approx -D*y + 0.5*C*y^2$$

Rearrange the formula to solve for duration:

$$D \approx -(0.5*C*y^2 - \Delta P/P)/y$$

5. Continued

Substitute the known values to calculate duration:

$$D \approx -(0.5 \cdot 25 \cdot 0.015^2 - 110,517 / 2,500,000) / 0.015 = 2.76$$

Calculate DV01:

$$DV01 = \text{Duration} \cdot \text{Price} \cdot 0.0001$$

$$DV01 = 2.76 \cdot 2,500,000 \cdot 0.0001 = 690$$

Part (b)(ii)

Calculate the value of the liabilities before the interest rate drop:

$$\text{Liabilities} = \text{Assets} - \text{Equity}$$

$$\text{Liabilities} = 2,500,000 - 450,000 = 2,050,000$$

Since the asset and liability durations were matched, the liability duration is 2.76, as calculated in part (b)(i).

Calculate DV01:

$$DV01 = \text{Duration} \cdot \text{Price} \cdot 0.0001$$

$$DV01 = 2.76 \cdot 2,050,000 \cdot 0.0001 = 566$$

- (c) After the shock in interest rates, your manager is concerned about the company's liquidity and calls for a meeting. During the meeting, your coworkers make the following statements.

Coworker A: "Our company just recently completed a capital management framework, which showed that it has no issues maintaining its status as a going concern. Why do we need to implement a liquidity risk management framework?"

Coworker B: "Our company needs to conduct a liquidity stress test. However, assets from separate accounts and closed blocks should be excluded."

Critique your coworkers' statements.

Commentary on Question:

The purpose of this question is to test the candidate's understanding of Asset-Liability Management concerns. Candidates generally performed well on this question although some candidates did not recognize the legal requirements of the separate account.

5. Continued

Coworker A:

Liquidity fundamentally differs from capital: while both are essential to remaining a going concern, liquidity has a “real time” dimension that capital does not. Insufficient liquidity can cause sudden distress and/or default in insurers that are otherwise well-capitalized. As a result, the insurer’s capital management framework may be inappropriate or inapplicable to liquidity risk management.

Coworker B:

The coworker raised a valid concern. As part of its stress testing, where material, the insurer should appropriately address legally or operationally ring-fenced assets. Such assets could include legally insulated separate accounts and closed blocks. These blocks of assets, therefore, should only be included as cash flow sources to back cash flow needs arising from these same accounts.

6. Learning Objectives:

3. The candidate will understand tools and strategies to manage ALM risks

Learning Outcomes:

- (3b) Demonstrate an understanding of and apply tools and techniques for measuring and managing interest rate risk in an ALM context

- (3d) Understand and evaluate model and parameter risks

Sources:

CP351-113-25: Chapter 4 of ALM for Banks and Insurance Companies, Habart (excluding 4.3.6)

Commentary on Question:

This question aims to test Candidates on their knowledge of applying ALM model to insurance portfolio. Candidates need to understand how the financial return, crediting rate and profit sharing of insurance portfolio is derived, and how the efficient frontier of a portfolio is determined. Candidates need to demonstrate full understanding in their answer to receive full credit.

Solution:

- (a) Calculate the financial return of the asset portfolio for the product in year 2024 based on the information above.

Commentary on Question:

Most Candidates got the answer for Bond-Weighted Return, yet for Equity-Weighted Return, Candidates struggled with the concept of equity turnover ratio. Equity turnover is the assumed realized profit and loss on equity's Average Return each year. Only Candidates who calculate both equity and bond return correctly can receive full credit.

Bond-Weighted Return:

Bonds Weighted Allocation x Average coupon rate of Bonds:

$$60\% \times 5\% = 3\%$$

Equity-Weighted Return:

Equity Weighted Allocation x (Average Return of equity x Equity Turnover Ratio + Equity Dividend Rate)

$$40\% \times (8\% \times 20\% + 2\%) = 40\% \times 3.6\% = 1.44\%$$

Financial Return of Asset Portfolio:

$$3\% + 1.44\% = 4.44\%$$

6. Continued

(b)

- (i) Identify the efficient frontier on the sample plot above.
- (ii) Describe the steps to generate the plot.

Commentary on Question:

i) This part was answered fairly. Partial credit is given if the candidates defined efficient frontier correctly even if they did not identify the correct allocation percentage in their answer.

ii) The key in the question is to identify generating the plot requires different “Asset Allocations” under “Numerous Scenarios/Simulations”. In addition, Candidates need to explain how the X and Y axis of the plot is calculated through the model result. Most candidates were unable to explain how to obtain the risk and return of the plot from the model and the overall performance in this part was poor.

(i) The efficient frontier is the inflection of the risk-expected return plot of the portfolio, which is the portfolio with 10% equity allocation in the plot.

(ii) We need to run the ALM model with different asset allocation under numerous asset scenarios to obtain the expected return and risks for each asset allocation. The expected return/profit of each asset allocation is the mean of the Present Value of Future Profit (PVFP) of all scenarios from the ALM Model. The risk of each asset allocation can be measured through Value-at-risk (VaR) of PVFP at a level defined by the company. Then we plot the result for each asset allocation.

(c) Calculate the crediting rate and profit sharing rate for the following two scenarios:

- (i) The expected financial return of the asset in year 2025 is 6%.
- (ii) The expected financial return of the asset in year 2025 is 1%.

Commentary on Question:

This part was answered fairly. This question requires Candidate to apply the crediting rate and profit-sharing rate equations correctly under different financial return scenarios. Candidates need to recognize the formula is different if the asset financial return is above or below the target crediting rate. Partial credit was given for Candidates who did not use the correct formula, but they demonstrated some knowledge in their response.

6. Continued

- If the financial return of the assets in year 2025 is 6%

As the Financial Return of Asset > Target Crediting Rate

Crediting Rate:

$$CR_{t2025} = CR_{2025}^{Target} + \max (PS_{2024} + FRA_{2025} - CR_{2025}^{Target} - PS^{Max}; 0) \\ = 5\%$$

Profit Sharing Rate:

$$PS_{2025} = PS_{2024} + FRA_{2025} - CR_{2025} \\ = 3\%$$

- If the financial return of the assets in year 2025 is 1%

As the Financial Return of Asset < Target Crediting Rate

Crediting Rate:

$$CR_{t2025} = FRA_{2025} + \min (CR_{2025}^{Target} - FRA_{2025}; PS_{2024} - PS^{min}) \\ = 3\%$$

Profit Sharing Rate:

$$PS_{2025} = PS_{2024} + FRA_{2025} - CR_{2025} \\ = 0\%$$

- (d) Describe the impact on policyholder behavior if UVW decides to:

- (i) Increase the crediting rate.
- (ii) Keep the crediting rate at its current level.

Commentary on Question:

Most Candidates did well in this question. Many Candidates were able to explain the policyholders' behaviors under different UVW crediting rate decisions.

For part I, Candidates receive full credit only if they recognize the difference in policyholders' behavior when UVW's crediting rate increase is aligned or below the overall market.

For part II, Candidates receive full credit if they manage to recognize policyholders will be likely to surrender under UVW's decision and are able to provide the explanation clearly.

- (i) If UVW increases the crediting rate, the policyholders are likely to persist if the scale of increased rate aligns with the overall market; yet if the scale of increased crediting rate by UVW is below the market competitors, policyholders are still likely to surrender their existing contracts to seek higher return product in the market.

6. Continued

(ii) If UVW keeps its crediting rate unchanged, some policyholders will surrender their existing contracts and move to other funds in the market that better reflect current market conditions.

7. Learning Objectives:

1. The candidate will understand the objectives of Asset Liability Management (ALM).
3. The candidate will understand tools and strategies to manage ALM risks

Learning Outcomes:

- (1d) Describe how different pension and insurance contracts generate embedded options.
- (3e) Explain and implement techniques used to mitigate market risks
- (3f) Understand interest rate derivatives and use them to mitigate interest rate risk

Sources:

CP351-105-25: Chapter 16 of Asset/Liability Management of Financial Institutions, Tilman 2003

CP351-101-25: ALM for Life, Annuities, and Pensions (section 5)

Quantitative Enterprise Risk Management, Hardy, Mary and Saunders, David, 2022, Ch. 15: Risk mitigation using options and derivatives

Commentary on Question:

This question tests candidates on their knowledge of derivatives and assessment of the hedging strategy's effectiveness based on their analysis. Candidates are asked to identify the derivatives embedded in the annuity products from the perspective of the insurance company. Candidates are then tested to suggest uses of derivatives for risk mitigation. Particular data are provided and candidates are asked to figure out the units of future and put option required for hedging purposes. Based on the analysis of the P&L, assessment of the hedging strategy's effectiveness is to be made.

Solution:

- (a) Describe the embedded options in each product from the perspective of the insurance company.

Commentary on Question:

This question tests the candidates on their knowledge of derivatives. Candidates get full credits if they can identify that the insurance company offers an interest rate floor of 1.5% to the policyholders in product A, and a put option on the account value with strike at the principal invested in product B. Those with incomplete or vague descriptions receive partial credit only. Quite many candidates refer to the generic features of annuity products in general and gain no credits.

7. Continued

For product A, the insurance company is providing (writing) an interest rate floor of 1.5% to the policyholder. For product B, the insurance company is providing (writing) a put option on the account value with the strike equal to the principal amount invested.

- (b) Explain how two distinct interest rate derivatives can be used to mitigate the interest rate risk in Product A.

Commentary on Question:

This question tests the candidates' knowledge utilization with respect to the use of derivatives to mitigate interest rate risks. *Those candidates who are able to come up with two recommendations to mitigate the falling interest rate risk (inherent to the particular product features) get full credit. Partial credit is granted to those who suggest measures to mitigate rising interest rate risks (which is not related to the particular embedded derivatives offered).*

ANSWER: For Product A, the insurance company can enter into a receive fixed/pay floating swap/swaption contract with a strike rate of 1.5%. This would shield the company from the risk of falling rates. Also, the company could purchase an interest rate floor contract at the strike rate of 1.5% to protect against the risk of falling rates.

- (c) Calculate the number of units for each instrument and the total initial cost of setup.

Commentary on Question:

To get the units of future and units of put option correctly, candidates have to set up the two equations which result in 0 gamma and 0 delta. A number of candidates get full credits by setting up clear and simple equations. However, quite many candidates have the components of the equations mixed up, or make a mistake with the plus and minus signs in the equations. Some end up shorting the put options and longing the future, which is opposite to what should be done for risk mitigation. Partial credits are granted to those who have the components right but make mistakes in the equations one way or the other.

7. Continued

Let units of future = A and units of put = B

Units of P * put gamma = dollar gamma

$$0.0027 * B = 1060$$

$$B = 392,593 \text{ (long)}$$

units of future * delta F = dollar delta - units of p * delta P

$$A = -57,750 - (-0.093) * B$$

$$A = -21,239 \text{ (short)}$$

$$\begin{aligned} \text{Total initial setup cost} &= 392,593 * \$2.45 \\ &= \$961,852 \end{aligned}$$

Units of put option: 392,593

Units of future: -21,239

Total initial set up cost: \$961,852

See Excel for details

(d)

- (i) Calculate the profit and loss after two months for the hedged portfolio in Part (c), including the initial setup cost.
- (ii) Assess whether the hedging strategy enhances the profitability for the company in this situation.

Commentary on Question:

This part is relatively easy. Wrong answers from part (c) are not penalized as long as the components to derive the P&L are right and the formula is correct. Still many candidates do not do well. Some candidates omit the P&L from the futures or put options, or make a mistake in the calculation of the P&L from the future (profit turning into loss). Some others leave this part empty or just write down a couple things. It could be due to a shortage of time as it gets close to the end of the paper. [Note: double-counting the put option setup cost is not penalized due to the way the question is asked. In such case, a loss of 867,945 is not regarded as a wrong answer.]

$$\begin{aligned} \text{(i) P\&L from the put options (after setup cost)} \\ &= 392,593 * (\$6.67 - \$2.45) = \$1,656,741 \end{aligned}$$

$$\begin{aligned} \text{P\&L from the futures} \\ &= (-21,239) * (\$270 - \$300) = \$637,167 \end{aligned}$$

7. Continued

$$\begin{aligned} &\text{Change in liability} \\ &= -4,400,000 - (-2,200,000) = -2,200,000 \end{aligned}$$

$$\text{P\&L after two months} = \$1,656,741 + \$637,167 + -2,200,000 = \$93,907 \text{ (profit)}$$

See Excel for details

Commentary on Question:

Candidates who either know to compare the result in part (d)(i) with the 2.2 M increase in liability if unhedged, or able to point out that shorting the futures and longing the put options make profits (from part (d)(i)) and draw a conclusion that the hedging strategy enhances profits get full credits. For those candidates who have the wrong figures for units of put or units of future, or have the hedging strategy turned the other way around (due to errors in the equations), as long as their conclusion is consistent with the previous work and is sensible, partial credit is granted.

(ii) It certainly enhanced the profitability. The declining index unhedged would have caused an increase in liability value of 2.2M. The long Put and the short futures position increased in value, offsetting the change in value of the liability.

- (e) The ALM team anticipates the upcoming economic landscape to be more volatile in the next few quarters. Your colleague suggests using a dynamic delta hedging strategy for Product B.

Commentary on Question:

This question tests the candidate's understanding of dynamic delta hedging strategy. Candidates do not do well in this question. Almost all candidates mention only one to two merits/limitations of dynamic delta hedging strategy, or none.

Under more volatile economic landscape, the portfolio's delta is more volatile and change more. Dynamic hedging which adjusts hedged portfolio more frequently will reduce hedging error. However, dynamic delta hedging only considers first order impact (delta hedging) and don't adjust portfolios based on changed gamma (can be more material under market volatility). Also, dynamic hedging requires frequent rebalancing, which is costly and operationally complex.

8. Learning Objectives:

3. The candidate will understand tools and strategies to manage ALM risks

Learning Outcomes:

- (3c) Apply stress testing and scenario analysis to assess extreme ALM events

Sources:

Quantitative Enterprise Risk Management, Hardy, Mary and Saunders, David, 2022, Ch. 7 Stress Testing

Commentary on Question:

This question tests the candidate's understanding of stress scenarios, in particular how to evaluate when and how certain frameworks are best applied to a specific environment.

Solution:

- (a) You ask your colleagues for inputs on stress testing scenarios.

- Colleague A recommends focusing on interest rates as GHI's balance sheet has significant exposures to interest rates.
- Colleague B recommends using the stressed scenarios prescribed by the regulator.
- Colleague C recommends using only historical scenarios for stress testing.

Critique each colleague's recommendation.

Commentary on Question:

Full points are awarded by identifying key advantages and disadvantages of each approach, including ways to improve or supplement the approach.

Employee A: It is recommended that a company stresses factors which are most pertinent to the organization, which includes interest rates. The company should stress additional variables to create a wide-ranging, comprehensive scenario; for an insurance company, this includes also includes mortality.

Employee B: Prescribed scenarios are advantageous as the results can be compared across institutions. The disadvantage is that risks most applicable to the firm may be overlooked and the company may try to work around the prescribed scenarios rather than use them to inform risk management activities.

Employee C: Historical scenarios represent extreme events that happened and are easily understood; they are plausible by definition. The effects of the scenario are known and the risk factors are observable which is advantageous. However, historical scenarios are constrained to be no more severe than the most extreme historical event.

8. Continued

- (b) Identify three attributes that any stress scenario should possess.

Commentary on Question:

Full credit is awarded for the name of each characteristic and a brief description.

Scenarios should be comprehensive and a single scenario should stress multiple risk factors.

Scenarios should be extreme (or severe) so they do not understate risks facing the portfolio.

Scenarios should be plausible meaning the scenarios could actually occur.

- (c) Compare top-down stress testing and reverse stress testing.

Commentary on Question:

This question addresses the procedural difference in determining reverse stress scenarios.

A reverse stress test starts from a known stress test outcome for a particular portfolio (e.g. breaching regulatory capital), rather than the top-down stress test which starts with the scenario. A reverse stress test seeks to identify those scenarios that are most adverse for that portfolio. The top-down method considers the impact on the portfolio after the scenario has been defined.

- (d)

- (i) Describe a scenario for top-down stress testing.
- (ii) Describe key inputs to GHI's risk models based on the scenario in Part (i).
- (iii) Describe a scenario for reverse stress testing.

Commentary on Question:

This question tests candidates' ability to utilize the above definitions to develop scenarios in the context of a life insurance company.

- (i) A global pandemic, causing both increased mortality and an economic downturn.
- (ii) Key inputs to the firm's models would include mortality and interest rates. Mortality rates would increase due to the pandemic which means that the company would have higher claims on their term life insurance product. Interest rates would decrease as governments try to help the economy recover.
- (iii) Interest rates are stressed downwards until the minimum regulatory capital ratio is breached.

9. Learning Objectives:

3. The candidate will understand tools and strategies to manage ALM risks

Learning Outcomes:

- (3c) Apply stress testing and scenario analysis to assess extreme ALM events

Sources:

Quantitative Enterprise Risk Management, Hardy, Mary and Saunders, David, 2022, Ch. 6 Copulas

Quantitative Enterprise Risk Management, Hardy, Mary and Saunders, David, 2022, Ch. 7 Stress Testing

Commentary on Question:

This question tests the candidates understanding of dependency and related tail risks, and, in particular, how these impact and interact with risk measures such as Expected Shortfall (ES).

Solution:

- (a) Calculate the marginal probability of default for each firm.

Commentary on Question:

Candidates performed above average on this straightforward calculation question.

$$P(X_1 \leq 1) = 1 - e^{-0.05} = 0.04877$$

$$P(X_2 \leq 1) = 1 - e^{-0.1} = 0.09516$$

- (b) Calculate 99% Expected Shortfall of the loss given default, assuming X_1 and X_2 are independent.

Commentary on Question:

Candidates performed below average on this question. Candidates need to understand the distribution of losses for the two assets to structure the equation to determine the expected shortfall.

$$C_{\text{default}} = 0.04877 * 0.09516 = 0.004641$$

Since this value is less than 1% but the probability of X_1 defaulting is above 1% at 4.877%, the VaR at the 99% level will be the loss from only X_1 defaulting.

$$ES(99) = \frac{0.004641 * \$1,700 + (0.01 - 0.004641) * \$600}{.01} = \$1,111$$

9. Continued

- (c) Calculate the probability of both firms defaulting, assuming the dependency between X_1 and X_2 follows the Frank Copula with $\alpha = 2$.

Commentary on Question:

Candidates performed above average on this question. The formula was provided and a surprising number of candidates made calculation mistakes, such as incorrect placement of parentheses. Partial credits were awarded as appropriate.

$$\begin{aligned} C_{\text{Frank}}(0.04877, 0.09516) \\ = -2^{-1} \ln \left[1 + \frac{(e^{-2 \cdot 0.04877} - 1)(e^{-2 \cdot 0.09516} - 1)}{(e^{-2} - 1)} \right] = 0.0094016 \end{aligned}$$

- (d) Calculate the 99% Expected Shortfall of the loss given default, assuming the dependency between X_1 and X_2 follows the Frank Copula with $\alpha = 2$.

Commentary on Question:

Candidates performed below average on this question. Candidates will recognize the formula has as the same used in part (b), given the joint probability is still below 1%.

Because 0.941016% is less than 1% but the probability of X_1 defaulting is above 1% at 4.877%, the VaR at the 99% level will be the loss from only X_1 defaulting, which is 600.

$$ES(99) = \frac{0.0094016 * \$1,700 + (0.01 - 0.0094016) * \$600}{.01} = \$1,634$$

- (e) Explain the difference between the 99% Expected Shortfall computed in Part (b) and Part (d).

Commentary on Question:

Candidates performed as expected. Candidates should note that dependence is more reasonable in the context of these two assets and accordingly leads to the higher Expected Shortfall (ES). Candidates that described correlation instead of dependence were awarded partial credits.

The Expected Shortfall (ES) in part (b) reflects an assumption of independence of probability of default (PD), which is not reasonable for these assets, especially given the conditional loss amounts are not independent. The Copula-derived joint default probability utilized in part (d) reflects the expected tail dependence between these default events, and provides a correspondingly larger ES.