SOCIETY OF ACTUARIES

EXAM P PROBABILITY

EXAM P SAMPLE SOLUTIONS

This set of sample questions includes those published on the probability topic for use with previous versions of this examination. Questions from previous versions of this document that are not relevant for the syllabus effective with the September 2022 administration have been deleted. The questions have been renumbered. Unless indicated below, no questions have been added to the version published for use with exams through July 2022.

Some of the questions in this study note are taken from past SOA examinations.

These questions are representative of the types of questions that might be asked of candidates sitting for the Probability (P) Exam. These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on future exams.

For questions involving the normal distribution, answer choices have been calculated using exact values for the normal distribution. Using the normal table provided and rounding to the closest value may give you answers slightly different from the answer choices. As always, choose the best answer provided from the five choices when selecting your answer.

Questions 271-287 were added July 2022.
Questions 288-319 were added August 2022.
Questions 234-236 and 282 were deleted October 2022
Questions 320-446 were added November 2023
Several questions that were duplicates of earlier questions were removed February 2024
Questions 447-485 were added March 2024
Questions 486-570 were added September 2024
Questions 571-627 were added December 2024
Questions 628-680 were added October 2025
Questions 287, 356, and 427were duplicates of earlier questions and removed October 2025
Questions 681-736 were added January 2026

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1. Solution: D

Let G = viewer watched gymnastics, B = viewer watched baseball, S = viewer watched soccer. Then we want to find

$$\Pr\left[\left(G\cup B\cup S\right)^c\right]=1-\Pr\left(G\cup B\cup S\right)$$
$$=1-\left[\Pr\left(G\right)+\Pr\left(B\right)+\Pr\left(S\right)-\Pr\left(G\cap B\right)-\Pr\left(G\cap S\right)-\Pr\left(B\cap S\right)+\Pr\left(G\cap B\cap S\right)\right]$$
$$=1-\left(0.28+0.29+0.19-0.14-0.10-0.12+0.08\right)=1-0.48=0.52$$

2. Solution: A

Let R = referral to a specialist and L = lab work. Then

$$P[R\cap L]=P[R]+P[L]-P[R\cup L]=P[R]+P[L]-1+P[(R\cup L)^c]$$
$$=p[R]+P[L]-1+P[R^c\cap L']=0.30+0.40-1+0.35=0.05.$$

3. Solution: D

First note

$$P\left[A\cup B\right]=P\left[A\right]+P\left[B\right]-P\left[A\cap B\right]$$
$$P\left[A\cup B^c\right]=P\left[A\right]+P\left[B^c\right]-P\left[A\cap B^c\right]$$

Then add these two equations to get

$$P\left[A\cup B\right]+P\left[A\cup B^c\right]=2P\left[A\right]+\left(P\left[B\right]+P\left[B^c\right]\right)-\left(P\left[A\cap B\right]+P\left[A\cap B^c\right]\right)$$
$$0.7+0.9=2P\left[A\right]+1-P\left[\left(A\cap B\right)\cup\left(A\cap B^c\right)\right]$$
$$1.6=2P\left[A\right]+1-P\left[A\right]$$
$$P\left[A\right]=0.6$$

4. Solution: A

For $i = 1,2$, let R_i = event that a red ball is drawn from urn i and let B_i = event that a blue ball is drawn from urn i. Then, if x is the number of blue balls in urn 2,

$$\begin{aligned}
0.44 &= \Pr[(R_1 \cap R_2) \cup (B_1 \cap B_2)] = \Pr[R_1 \cap R_2] + \Pr[B_1 \cap B_2] \\
&= \Pr[R_1]\Pr[R_2] + \Pr[B_1]\Pr[B_2] \\
&= \frac{4}{10}\left(\frac{16}{x+16}\right) + \frac{6}{10}\left(\frac{x}{x+16}\right)
\end{aligned}$$

Therefore,

$$\begin{aligned}
&2.2 = \frac{32}{x+16} + \frac{3x}{x+16} = \frac{3x+32}{x+16} \\
&2.2x + 35.2 = 3x + 32 \\
&0.8x = 3.2 \\
&x = 4
\end{aligned}$$

5. Solution: D

Let $N(C)$ denote the number of policyholders in classification C. Then
N(Young and Female and Single)
= N(Young and Female) – N(Young and Female and Married)
= N(Young) – N(Young and Male) – [N(Young and Married) – N(Young and Married and Male)]
= 3000 – 1320 – (1400 – 600) = 880.

6. Solution: B

Let
H = event that a death is due to heart disease
F = event that at least one parent suffered from heart disease
Then based on the medical records,

$$P\left[H \cap F^c\right] = \frac{210-102}{937} = \frac{108}{937}$$

$$P\left[F^c\right] = \frac{937-312}{937} = \frac{625}{937}$$

and $P\left[H \mid F^c\right] = \dfrac{P\left[H \cap F^c\right]}{P\left[F^c\right]} = \dfrac{108}{937}\Big/\dfrac{625}{937} = \dfrac{108}{625} = 0.173$

7. Solution: D

Let A = event that a policyholder has an auto policy and H = event that a policyholder has a homeowners policy. Then,

$$\Pr\left(A\cap H\right)=0.15$$

$$\Pr\left(A\cap H^{c}\right)=\Pr\left(A\right)-\Pr\left(A\cap H\right)=0.65-0.15=0.50$$

$$\Pr\left(A^{c}\cap H\right)=\Pr\left(H\right)-\Pr\left(A\cap H\right)=0.50-0.15=0.35$$

and the portion of policyholders that will renew at least one policy is given by

$$0.4\ \Pr\left(A\cap H^{c}\right)+0.6\ \Pr\left(A^{c}\cap H\right)+0.8\ \Pr\left(A\cap H\right)$$

$$=\left(0.4\right)\left(0.5\right)+\left(0.6\right)\left(0.35\right)+\left(0.8\right)\left(0.15\right)=0.53\qquad\left(=53\%\right)$$

8. Solution: D

Let C = event that patient visits a chiropractor and T = event that patient visits a physical therapist. We are given that

$$\Pr\left[C\right]=\Pr\left[T\right]+0.14$$

$$\Pr\left(C\cap T\right)=0.22$$

$$\Pr\left(C^{c}\cap T^{c}\right)=0.12$$

Therefore,

$$0.88=1-\Pr\left[C^{c}\cap T^{c}\right]=\Pr\left[C\cup T\right]=\Pr\left[C\right]+\Pr\left[T\right]-\Pr\left[C\cap T\right]$$

$$=\Pr\left[T\right]+0.14+\Pr\left[T\right]-0.22$$

$$=2\Pr\left[T\right]-0.08$$

or

$$P\left[T\right]=\left(0.88+0.08\right)/2=0.48$$

9. Solution: B

Let M = event that customer insures more than one car and S = event that customer insurers a sports car. Then applying DeMorgan's Law, compute the desired probability as:

$$\Pr\left(M^{c}\cap S^{c}\right)=\Pr\left[\left(M\cup S\right)^{c}\right]=1-\Pr\left(M\cup S\right)=1-\left[\Pr\left(M\right)+\Pr\left(S\right)-\Pr\left(M\cap S\right)\right]$$

$$=1-\Pr\left(M\right)-\Pr\left(S\right)+\Pr\left(S|M\right)\Pr\left(M\right)=1-0.70-0.20+\left(0.15\right)\left(0.70\right)=0.205$$

10. Solution: B

Let C = Event that a policyholder buys collision coverage and D = Event that a policyholder buys disability coverage. Then we are given that $P[C]=2P[D]$ and $P[C\cap D]=0.15$.
By the independence of C and D,

$$0.15=[C\cap D]=P[C]P[D]=2P[D]^2$$

$$P[D]^2=0.15/2=0.075$$

$$P[D]=\sqrt{0.075},\ P[C]=2\sqrt{0.075}.$$

Independence of C and D implies independence of C^c and D^c. Then

$$P[C^c\cap D^c]=P[C^c]P[D^c]=(1-2\sqrt{0.075})(1-\sqrt{0.075})=0.33.$$

11. Solution: E

"Boxed" numbers in the table below were computed.

	High BP	Low BP	Norm BP	Total
Regular heartbeat	0.09	0.20	0.56	0.85
Irregular heartbeat	0.05	0.02	0.08	0.15
Total	0.14	0.22	0.64	1.00

From the table, 20% of patients have a regular heartbeat and low blood pressure.

12. Solution: C

Let x be the probability of having all three risk factors.

$$\frac{1}{3}=P\left[A\cap B\cap C\mid A\cap B\right]=\frac{P\left[A\cap B\cap C\right]}{P\left[A\cap B\right]}=\frac{x}{x+0.12}$$

It follows that

$$x=\frac{1}{3}\left(x+0.12\right)=\frac{1}{3}x+0.04$$

$$\frac{2}{3}x=0.04$$

$$x=0.06$$

Now we want to find

$$P\left[\left(A\cup B\cup C\right)^c\mid A^c\right]=\frac{P\left[\left(A\cup B\cup C\right)^c\right]}{P\left[A^c\right]}$$

$$=\frac{1-P\left[A\cup B\cup C\right]}{1-P\left[A\right]}$$

$$=\frac{1-3\left(0.10\right)-3\left(0.12\right)-0.06}{1-0.10-2\left(0.12\right)-0.06}$$

$$=\frac{0.28}{0.60}=0.467$$

13. Solution: A

$$p_k = \frac{1}{5} p_{k-1} = \frac{1}{5}\frac{1}{5} p_{k-2} = \frac{1}{5}\frac{1}{5}\frac{1}{5} p_{k-3} = \cdots = \left(\frac{1}{5}\right)^k p_0 \quad k \geq 0$$

$$1 = 1 = \sum_{k=0}^{\infty} p_k = \sum_{k=0}^{\infty} \left(\frac{1}{5}\right)^k p_0 = \frac{p_0}{1-\frac{1}{5}} = \frac{5}{4} p_0,\ p_0 = 4/5$$

Therefore, P[*N* > 1] = 1 – P[*N* £1] = 1 – (4/5 + 4/5 x 1/5) = 1 – 24/25 = 1/25 = 0.04 .

14. Solution: C

Let x be the probability of choosing A and B, but not C, y the probability of choosing A and C, but not B, z the probability of choosing B and C, but not A.

We want to find $w = 1 - (x + y + z)$.
We have $x + y = 1/4$, $x + z = 1/3$, $y + z = 5/12$.
Adding these three equations gives

$$(x+y)+(x+z)+(y+z) = \frac{1}{4}+\frac{1}{3}+\frac{5}{12}$$

$$2(x+y+z) = 1$$

$$x+y+z = \frac{1}{2}$$

$$w = 1-(x+y+z) = 1-\frac{1}{2} = \frac{1}{2}.$$

Alternatively the three equations can be solved to give $x = 1/12$, $y = 1/6$, $z = 1/4$ again leading to

$$w = 1-\left(\frac{1}{12}+\frac{1}{6}+\frac{1}{4}\right) = \frac{1}{2}$$

15. Solution: D

Let N_1 and N_2 denote the number of claims during weeks one and two, respectively. Then since they are independent,

$$\begin{aligned}\mathrm{P}[N_1 + N_2 = 7] &= \sum_{n=0}^{7} \mathrm{P}[N_1 = n]\,\mathrm{Pr}[N_2 = 7-n] \\ &= \sum_{n=0}^{7} \left(\frac{1}{2^{n+1}}\right)\left(\frac{1}{2^{8-n}}\right) \\ &= \sum_{n=0}^{7} \frac{1}{2^9} \\ &= \frac{8}{2^9} = \frac{1}{2^6} = \frac{1}{64}\end{aligned}$$

16. Solution: D

Let O = event of operating room charges and E = event of emergency room charges. Then

$$0.85 = P(O \cup E) = P(O) + P(E) - P(O \cap E)$$

$$= P(O) + P(E) - P(O)\mathrm{P}(E) \quad (\text{Independence})$$

Because $P(E^c) = 0.25 = 1 - P(E),\ P(E) = 0.75$,

$$0.85 = P(O) + 0.75 - P(O)(0.75)$$

$$P(O)(1-0.75) = 0.85 - 0.75 = 0.10$$

$$P(O) = 0.10 / 0.25 = 0.40.$$

17. Solution: D

Let X_1 and X_2 denote the measurement errors of the less and more accurate instruments, respectively. If $N(\mu, \sigma)$ denotes a normal random variable then $X_1 \sim N(0, 0.0056h),\ X_2 \sim N(0, 0044h)$ and they are independent. It follows that

$$Y = \frac{X_1 + X_2}{2} \sim N(0, \sqrt{\frac{0.0056^2 h^2 + 0.0044^2 h^2}{4}} = 0.00356h)$$. Therefore,

$$P(-0.005h \le Y \le 0.005h) = P\left(-\frac{0.005h - 0}{0.00356h} \le Z \le \frac{0.005h - 0}{0.00356h}\right)$$

$$= P(-1.4 \le Z \le 1.4) = P(Z \le 1.4) - [1 - P(Z \le 1.4)] = 2(0.9192) - 1 = 0.84.$$

18. Solution: B

Apply Bayes' Formula. Let

$A =$ Event of an accident

$B_1 =$ Event the driver's age is in the range 16-20

$B_2 =$ Event the driver's age is in the range 21-30

$B_3 =$ Event the driver's age is in the range 30-65

$B_4 =$ Event the driver's age is in the range 66-99

Then

$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3) + P(A|B_4)P(B_4)}$$

$$= \frac{(0.06)(0.08)}{(0.06)(0.08) + (0.03)(0.15) + (0.02)(0.49) + (0.04)(0.28)} = 0.1584$$

19. Solution: D

Let
S = Event of a standard policy
F = Event of a preferred policy
U = Event of an ultra-preferred policy
D = Event that a policyholder dies
Then

$$P[U \mid D] = \frac{P[D \mid U]P[U]}{P[D \mid S]P[S] + P[D \mid F]P[F] + P[D \mid U]P[U]}$$
$$= \frac{(0.001)(0.10)}{(0.01)(0.50) + (0.005)(0.40) + (0.001)(0.10)}$$
$$= 0.0141$$

20. Solution: B

$$P\left[\text{Seri.} \middle| \text{Surv.}\right]$$
$$= \frac{P\left[\text{Surv.} \middle| \text{Seri.}\right]P[\text{Seri.}]}{P\left[\text{Surv.} \middle| \text{Crit.}\right]P[\text{Crit.}] + P\left[\text{Surv.} \middle| \text{Seri.}\right]P[\text{Seri.}] + P\left[\text{Surv.} \middle| \text{Stab.}\right]P[\text{Stab.}]}$$
$$= \frac{(0.9)(0.3)}{(0.6)(0.1) + (0.9)(0.3) + (0.99)(0.6)} = 0.29$$

21. Solution: D

Let H = heavy smoker, L = light smoker, N = non-smoker, D = death within five-year period.
We are given that $P[D \mid L] = 2P[D \mid N]$ and $P[D \mid L] = \frac{1}{2}P[D \mid H]$
Therefore,

$$P\left[H \middle| D\right] = \frac{P\left[D \middle| H\right]P[H]}{P\left[D \middle| N\right]P[N] + P\left[D \middle| L\right]P[L] + P\left[D \middle| H\right]P[H]}$$
$$= \frac{2P\left[D \middle| L\right](0.2)}{\frac{1}{2}P\left[D \middle| L\right](0.5) + P\left[D \middle| L\right](0.3) + 2P\left[D \middle| L\right](0.2)} = \frac{0.4}{0.25 + 0.3 + 0.4} = 0.42$$

22. Solution: D

Let
C = Event of a collision
T = Event of a teen driver
Y = Event of a young adult driver
M = Event of a midlife driver
S = Event of a senior driver
Then,

$$P[Y \mid C] = \frac{P[C \mid Y]P[Y]}{P[C \mid T]P[T] + P[C \mid Y]P[Y] + P[C \mid M]P[M] + P[C \mid S]P[S]}$$

$$= \frac{(0.08)(0.16)}{(0.15)(0.08) + (0.08)(0.16) + (0.04)(0.45) + (0.05)(0.31)} = 0.22.$$

23. Solution: B

$$P\left[N \geq 1 \middle| N \leq 4\right] = \frac{P\left[1 \leq N \leq 4\right]}{P\left[N \leq 4\right]} = \left[\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}\right] \Big/ \left[\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}\right]$$

$$= \frac{10+5+3+2}{30+10+5+3+2} = \frac{20}{50} = \frac{2}{5}$$

24. Solution: B

Let Y = positive test result
D = disease is present
Then,

$$P[D \mid Y] = \frac{P[Y \mid D]P[D]}{P[Y \mid D]P[D] + P[Y \mid D^c]P[D^c]} = \frac{(0.95)(0.01)}{(0.95)(0.01) + (0.005)(0.99)} = 0.657.$$

25. Solution: C

Let:
S = Event of a smoker
C = Event of a circulation problem
Then we are given that P[C] = 0.25 and P[S½C] = 2 P[S½C^c]
Then,,

$$P[C \mid S] = \frac{P[S \mid C]P[C]}{P[S \mid C]P[C] + P[S \mid C^c]P[C^c]}$$

$$= \frac{2P[S \mid C^c]P[C]}{2P[S \mid C^c]P[C] + P[S \mid C^c](1 - P[C])} = \frac{2(0.25)}{2(0.25) + 0.75} = \frac{2}{2+3} = \frac{2}{5}$$

26. Solution: D

Let B, C, and D be the events of an accident occurring in 2014, 2013, and 2012, respectively.
Let $A = B \cup C \cup D$.

Use Bayes' Theorem

$$P[B \mid A] = \frac{P[A|B]P[B]}{P[A|B][P[B] + P[A|C]P[C] + P[A|D]P[D]}$$

$$= \frac{(0.05)(0.16)}{(0.05)(0.16) + (0.02)(0.18) + (0.03)(0.20)} = 0.45.$$

27. Solution: A

Let
C = Event that shipment came from Company X
I = Event that one of the vaccine vials tested is ineffective

Then, $P[C \mid I] = \dfrac{P[I \mid C]P[C]}{P[I \mid C]P[C] + P[I \mid C^c]P[C^c]}$.

Now

$$P[C] = \frac{1}{5}$$

$$P[C^c] = 1 - P[C] = 1 - \frac{1}{5} = \frac{4}{5}$$

$$P[I \mid C] = \binom{30}{1}(0.10)(0.90)^{29} = 0.141$$

$$P[I \mid C^c] = \binom{30}{1}(0.02)(0.98)^{29} = 0.334$$

Therefore,

$$P[C \mid I] = \frac{(0.141)(1/5)}{(0.141)(1/5) + (0.334)(4/5)} = 0.096.$$

28. Solution: C

Let T denote the number of days that elapse before a high-risk driver is involved in an accident. Then T is exponentially distributed with unknown parameter l . We are given that

$$0.3 = P[T \le 50] = \int_0^{50} \lambda e^{-\lambda t} dt = -e^{-\lambda t} \Big|_0^{50} = 1 - e^{-50\lambda}.$$

Therefore, $e^{=50\lambda} = 0.7$ and $\lambda = -(1/50)\ln(0.7)$.

Then,

$$P[T \le 80] = \int_0^{80} \lambda e^{-\lambda t} dt = -e^{-\lambda t} \Big|_0^{80} = 1 - e^{-80\lambda} \quad = 1 - e^{-80l}$$

$$= 1 - e^{(80/50)\ln(0.7)} = 1 - 0.7^{8/5} = 0.435.$$

29. Solution: D

Let N be the number of claims filed. We are given $P[N=2]=\frac{e^{-\lambda}\lambda^2}{2!}=3P[N=4]=3\frac{e^{-\lambda}\lambda^4}{4!}$.

Then,

$\frac{1}{2}\lambda^2=\frac{3}{24}\lambda^4$ or $\lambda^2=4$ or $\lambda=2$, which is the variance of N.

30. Solution: D

Let X denote the number of employees who achieve the high performance level. Then X follows a binomial distribution with parameters $n = 20$ and $p = 0.02$. Now we want to determine x such that $P[X > x] < 0.01$ or equivalently $0.99 \le P\left[X \le x\right]=\sum_{k=0}^{x}\binom{20}{k}(0.02)^k(0.98)^{20-k}$

The first three probabilities (at 0, 1, and 2) are 0.668, 0.272, and 0.053. The total is 0.993 and so the smallest x that has the probability exceed 0.99 is 2. Thus $C = 120/2 = 60$.

31. Solution: D

Let

X = number of low-risk drivers insured

Y = number of moderate-risk drivers insured

Z = number of high-risk drivers insured

$f(x, y, z)$ = probability function of X, Y, and Z

Then f is a trinomial probability function, so

$$P\left[z \ge x+2\right] = f(0,0,4)+f(1,0,3)+f(0,1,3)+f(0,2,2)$$

$$=(0.20)^4+4(0.50)(0.20)^3+4(0.30)(0.20)^3+\frac{4!}{2!2!}(0.30)^2(0.20)^2$$

$$=0.0488$$

32. Solution: B

$$P\left[X>x\right]=\int_x^{20}0.005(20-t)\,dt=0.005\left(20t-\frac{1}{2}t^2\right)\Big|_x^{20}$$

$$=0.005\left(400-200-20x+\frac{1}{2}x^2\right)=0.005\left(200-20x+\frac{1}{2}x^2\right)$$

where $0 < x < 20$. Therefore,

$$P\left[X>16 \mid X>8\right]=\frac{P\left[X>16\right]}{P\left[X>8\right]}=\frac{200-20(16)+1/2(16)^2}{200-20(8)+1/2(8)^2}=\frac{8}{72}=\frac{1}{9}.$$

33. Solution: C

We know the density has the form $C\left(10+x\right)^{-2}$ for 0 < *x* < 40 (equals zero otherwise). First, determine the proportionality constant *C*.

$$1=\int_0^{40} C\left(10+x\right)^{-2}\,dx=-C(10+x)^{-1}\Big|_0^{40}=\frac{C}{10}-\frac{C}{50}=\frac{2}{25}C$$

So *C* = 25/2 or 12.5. Then, calculate the probability over the interval (0, 6):

$$12.5\int_0^{6}\left(10+x\right)^{-2}\,dx=-12.5\left(10+x\right)^{-1}\Big|_0^{6}=12.5\left(\frac{1}{10}-\frac{1}{16}\right)=0.47\,.$$

34. Solution: B

To determine *k*,

$$1\;=\;1=\int_0^1 k\left(1-y\right)^4\,dy=-\frac{k}{5}\left(1-y\right)^5\Big|_0^1\;=\frac{k}{5}$$, so *k* = 5

We next need to find P[*V* > 10,000] = P[100,000 *Y* > 10,000] = P[*Y* > 0.1], which is

$\int_{0.1}^{1} 5\left(1-y\right)^4 dy=-\left(1-y\right)^5\Big|_{0.1}^{1}\;=0.9^5=0.59$ and P[*V* > 40,000] which is

$\int_{0.4}^{1} 5\left(1-y\right)^4 dy=-\left(1-y\right)^5\Big|_{0.4}^{1}\;=0.6^5=0.078$. Then,

$$P[V>40,000\,|\,V>10,000]=\frac{P[V>40,000\cap V>10,000]}{P[V>10,000]}=\frac{P[V>40,000]}{P[V>10,000]}=\frac{0.078}{0.590}=0.132.$$

35. Solution: D

Let T denote printer lifetime. The distribution function is $F(t)=1-e^{-t/2}$. The probability of failure in the first year is *F*(1) = 0.3935 and the probability of failure in the second year is *F*(2) – *F*(1) = 0.6321 – 0.3935 = 0.2386. Of 100 printers, the expected number of failures is 39.35 and 23.86 for the two periods. The total expected cost is 200(39.35) + 100(23.86) = 10,256.

36. Solution: A

The distribution function is $F(x)=P[X\le x]=\int_1^x 3t^{-4}dt=-t^{-3}\Big|_1^x=1-x^{-3}$. Then,

$$P[X<2\,|\,X\ge 1.5]=\frac{P\left[(X<2)\text{ and }(X\ge 1.5)\right]}{P\left[X\ge 1.5\right]}=\frac{P[X<2]-\Pr[\mathrm{X}<1.5]}{\Pr\left[X\ge 1.5\right]}$$

$$=\frac{F(2)-F(1.5)}{1-F(1.5)}=\frac{(1-2^{-3})-(1-1.5^{-3})}{1-(1-1.5^{-3})}\;=\frac{-1/8+8/27}{8/27}\;=\frac{37}{64}=0.578$$

37. Solution: E

The number of hurricanes has a binomial distribution with *n* = 20 and *p* = 0.05. Then
$P[X < 3] = 0.95^{20} + 20(0.95)^{19}(0.05) + 190(0.95)^{18}(0.05)^2 = 0.9245.$

38. Solution: B

Denote the insurance payment by the random variable *Y*. Then

$$Y = \begin{cases} 0 & \text{if} \quad 0 < X \le C \\ X - C & \text{if} \quad C < X < 1 \end{cases}$$

We are given that

$$0.64 = P[Y < 0.5] = P[0 < X < 0.5 + C] = \int_0^{0.5+C} 2x\,dx = x^2 \Big|_0^{0.5+C} = (0.5 + C)^2 .$$

The quadratic equation has roots at C = 0.3 and –1.3. Because C must be between 0 and 1, the solution is *C* = 0.3.

39. Solution: E

The number completing the study in a single group is binomial (10,0.8). For a single group the probability that at least nine complete the study is $\binom{10}{9}(0.8)^9(0.2) + \binom{10}{10}(0.8)^{10} = 0.376$
The probability that this happens for one group but not the other is 0.376(0.624) + 0.624(0.376) = 0.469.

40. Solution: D

There are two situations where Company B's total exceeds Company A's. First, Company B has at least one claim and Company A has no claims. This probability is 0.3(0.6) = 0.18. Second, both have claims. This probability is 0.3(0.4) = 0.12. Given that both have claims, the distribution of B's claims minus A's claims is normal with mean 9,000 – 10,000 = –1,000 and standard deviation $\sqrt{2,000^2 + 2,000^2} = 2,828.43.$ The probability that the difference exceeds zero is the probability that a standard normal variable exceeds $\frac{0 - (-1,000)}{2,828.43} = 0.354.$ The probability is 1 – 0.638 = 0.362. The probability of the desired event is 0.18 + 0.12(0.362) = 0.223.

41. Solution: D

One way to view this event is that in the first seven months there must be at least four with no accidents. These are binomial probabilities:

$$\binom{7}{4}0.4^4 0.6^3 + \binom{7}{5}0.4^5 0.6^2 + \binom{7}{6}0.4^6 0.6 + \binom{7}{7}0.4^7$$
$$= 0.1935 + 0.0774 + 0.0172 + 0.0016 = 0.2897.$$

Alternatively, consider a negative binomial distribution where K is the number of failures before the fourth success (no accidents). Then

$$P[K < 4] = 0.4^4 + \binom{4}{1}0.4^4 0.6 + \binom{5}{2}0.4^4 0.6^2 + \binom{6}{3}0.4^4 0.6^3 = 0.2898$$

42. Solution: C

The probabilities of 1, 2, 3, 4, and 5 days of hospitalization are 5/15, 4/15, 3/15, 2/15, and 1/15 respectively. The payments are 100, 200, 300, 350, and 400 respectively. The expected value is [100(5) + 200(4) + 300(3) + 350(2) + 400(1)]/15 = 220.

43. Solution: D

$$E(X) = \int_{-2}^{0} x\frac{-x}{10}dx + \int_{0}^{4} x\frac{x}{10}dx = -\frac{x^3}{30}\Big|_{-2}^{0} + \frac{x^3}{30}\Big|_{0}^{4} = -\frac{8}{30} + \frac{64}{30} = \frac{56}{30} = \frac{28}{15}$$

44. Solution: D

The density function of T is

$$f(t) = \frac{1}{3}e^{-t/3}, \quad 0 < t < \infty$$

Therefore,

$$E[X] = E[\max(T,2)] = \int_0^2 \frac{2}{3}e^{-t/3}dt + \int_2^\infty \frac{t}{3}e^{-t/3}dt$$
$$= -2e^{-t/3}\Big|_0^2 - te^{-t/3}\Big|_2^\infty + \int_2^\infty e^{-t/3}dt = -2e^{-2/3} + 2 + 2e^{-2/3} - 3e^{-t/3}\Big|_2^\infty$$
$$= 2 + 3e^{-2/3}$$

Alternatively, with probability $1 - e^{-2/3}$ the device fails in the first two years and contributes 2 to the expected value. With the remaining probability the expected value is 2 + 3 = 5 (employing the memoryless property). The unconditional expected value is $(1 - e^{-2/3})2 + (e^{-2/3})5 = 2 + 3e^{-2/3}$.

45. Solution: D

We want to find x such that

$$1000 = \text{E[P]} = \int_0^1 \frac{x}{10}\, e^{-t/10}\, dt + \int_1^3 \frac{x}{2}\frac{1}{10}\, e^{-t/10}\, dt =$$

$$1000 = \int_0^1 x(0.1)\, e^{-t/10}\, dt + \int_1^3 0.5x(0.1) e^{-t/10} dt$$

$$= -xe^{-t/10}\Big|_0^1 - 0.5xe^{-t/10}\Big|_1^3$$

$$= -xe^{-1/10} + x - 0.5xe^{-3/10} + 0.5xe^{-1/10} = 0.1772x.$$

Thus $x = 5644$.

46. Solution: E

$$E[Y] = 4000(0.4) + 3000(0.6)(0.4) + 2000(0.6)^2(0.4) + 1000(0.6)^3(0.4)$$

$$= 2694$$

47. Solution: C

The expected payment is

$$\sum_{n=1}^{\infty} 10,000(n-1)\frac{(3/2)^n e^{-3/2}}{n!} = \left[\sum_{n=0}^{\infty} 10,000(n-1)\frac{(3/2)^n e^{-3/2}}{n!}\right] - 10,000(-1)e^{-3/2}$$

$$= 10,000(1.5-1) + 10,000e^{-3/2} = 7,231.$$

48. Solution: C

The expected payment is

$$\int_{0.6}^{2} x\left[\frac{2.5(0.6)^{2.5}}{x^{3.5}}\right]dx + \int_{2}^{\infty} 2\left[\frac{2.5(0.6)^{2.5}}{x^{3.5}}\right]dx = 2.5(0.6)^{2.5}\left(\frac{-x^{-1.5}}{1.5}\Bigg|_{0.6}^{2} + \frac{-x^{-2.5}}{2.5}\Bigg|_{2}^{\infty}\right)$$

$$= 2.5(0.6)^{2.5}\left(\frac{-2^{-1.5}}{1.5} + \frac{0.6^{-1.5}}{1.5} + 2\frac{2^{-2.5}}{2.5}\right) = 0.9343.$$

49. Solution: A

First, determine *K*.

$$1 = K\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right) = K\left(\frac{60+30+20+15+12}{60}\right) = K\left(\frac{137}{60}\right)$$

$$K = \frac{60}{137}$$

Then, after applying the deductible, the expected payment is

$$0.05[(3-2)P(N=3)+(4-2)P(N=4)+(5-2)P(N=5)]$$

$$=0.05(60/137)[1(1/3)+2(1/4)+3(1/5)] = 0.0314$$

50. Solution: D

The expected payment is

$$\int_1^{10} y\frac{2}{y^3}\,dy + \int_{10}^{\infty} 10\frac{2}{y^3}\,dy = -\frac{2}{y}\Big|_1^{10} - \frac{20}{2y^2}\Big|_{10}^{\infty} = -\frac{2}{10}+\frac{2}{1}-0+\frac{20}{200} = 1.9.$$

51. Solution: B

The expected payment is (in thousands)

$$(0.94)(0)+(0.02)(15-1)+0.04\int_1^{15}(x-1)0.5003e^{-x/2}dx$$

$$=0.28+(0.020012)\left[-2e^{-x/2}(x-1)\Big|_1^{15}+\int_1^{15}2\,\mathrm{e}^{-x/2}\,dx\right]$$

$$=0.28+(0.020012)\left[-2e^{-7.5}(14)+\left(-4e^{-x/2}\Big|_1^{15}\right)\right]$$

$$=0.28+(0.020012)\left[-2e^{-7.5}(14)-4e^{-7.5}+4e^{-0.5}\right]$$

$$=0.28+(0.020012)(2.408)$$

$$=0.328.$$

52. Solution: C

$1 = \int_0^{\infty}\frac{k}{(1+x)^4}dx = -\frac{k}{3}\frac{1}{(1+x)^3}\Big|_0^{\infty} = \frac{k}{3}$ and so $k = 3$.

The expected value is (where the substitution $u = 1 + x$ is used.

$$\int_0^{\infty} x\frac{3}{(1+x)^4}dx = \int_1^{\infty}3(u-1)u^{-4}du = 3u^{-2}/(-2)-3u^{-3}/(-3)\Big|_1^{\infty} = 3/2-1 = 1/2.$$

Integration by parts may also be used.

53. Solution: C

With no deductible, the expected payment is 500. With the deductible it is to be 125. Let *d* be the deductible. Then,

$$125 = \int_d^{1000} (x-d)(0.001)dx = \frac{(x-d)^2}{2}(0.001)\Big|_d^{1000} = 0.0005\left[(1000-d)^2 - 0\right]$$

$$250,000 = (1000-d)^2$$

$$500 = 1000 - d$$

$$d = 500.$$

54. Solution: B

The distribution function of *X* is

$$F(x) = \int_{200}^{x} \frac{2.5(200)^{2.5}}{t^{3.5}} dt = \frac{-(200)^{2.5}}{t^{2.5}}\Big|_{200}^{x} = 1 - \frac{(200)^{2.5}}{x^{2.5}}, \quad x > 200$$

The *p*th percentile x_p of *X* is given by

$$\frac{p}{100} = F(x_p) = 1 - \frac{(200)^{2.5}}{x_p^{2.5}}$$

$$1 - 0.01p = \frac{(200)^{2.5}}{x_p^{2.5}}$$

$$(1-0.01p)^{0.4} = \frac{200}{x_p}$$

$$x_p = \frac{200}{(1-0.01p)^{0.4}}$$

It follows that $x_{70} - x_{30} = \frac{200}{(0.30)^{0.4}} - \frac{200}{(0.70)^{0.4}} = 93.06$.

55. Solution: E

Let *X* and *Y* denote the annual cost of maintaining and repairing a car before and after the 20% tax, respectively. Then $Y = 1.2X$ and $Var(Y) = Var(1.2X) = 1.44Var(X) = 1.44(260) = 374.4$.

56. Solution: C

First note that the distribution function jumps ½ at $x = 1$, so there is discrete probability at that point. From 1 to 2, the density function is the derivative of the distribution function, $x - 1$. Then,

$$E(X) = \frac{1}{2}(1) + \int_1^2 x(x-1)dx = \frac{1}{2} + \left(\frac{x^3}{3} - \frac{x^2}{2}\right)\Bigg|_1^2 = \frac{1}{2} + \frac{8}{3} - \frac{4}{2} - \frac{1}{3} + \frac{1}{2} = \frac{4}{3}$$

$$E(X^2) = \frac{1}{2}(1)^2 + \int_1^2 x^2(x-1)dx = \frac{1}{2} + \left(\frac{x^4}{4} - \frac{x^3}{3}\right)\Bigg|_1^2 = \frac{1}{2} + \frac{16}{4} - \frac{8}{3} - \frac{1}{4} + \frac{1}{3} = \frac{23}{12}$$

$$Var\left(X\right) = E(X^2) - \left[E(X)\right]^2 = \frac{23}{12} - \left(\frac{4}{3}\right)^2 = \frac{23}{12} - \frac{16}{9} = \frac{5}{36}.$$

57. Solution: C

$$E[Y] = \int_0^4 x(0.2)dx + \int_4^5 4(0.2)dx = 0.1x^2\Big|_0^4 + 0.8 = 2.4$$

$$E[Y^2] = \int_0^4 x^2(0.2)dx + \int_4^5 4^2(0.2)dx = (0.2/3)x^3\Big|_0^4 + 3.2 = 7.46667$$

$$\text{Var[Y]} = E[Y^2] - \left(E[Y]\right)^2 = 7.46667 - 2.4^2 = 1.707.$$

58. Solution: A

The mean is 20(0.15) + 30(0.10) + 40(0.05) + 50(0.20) + 60(0.10) + 70(0.10) + 80(0.30) = 55. The second moment is 400(0.15) + 900(0.10) + 1600(0.05) + 2500(0.20) + 3600(0.10) + 4900(0.10) + 6400(0.30) = 3500. The variance is $3500 - 55^2 = 475$. The standard deviation is 21.79. The range within one standard deviation of the mean is 33.21 to 76.79, which includes the values 40, 50, 60, and 70. The sum of the probabilities for those values is 0.05 + 0.20 + 0.10 + 0.10 = 0.45.

59. Solution: B

Let Y be the amount of the insurance payment.

$$E[Y] = \int_{250}^{1500} \frac{1}{1500}(x-250)dx = \frac{1}{3000}(x-250)^2\Bigg|_{250}^{1500} = \frac{1250^2}{3000} = 521$$

$$E[Y^2] = \int_{250}^{1500} \frac{1}{1500}(x-250)^2 dx = \frac{1}{4500}(x-250)^3\Bigg|_{250}^{1500} = \frac{1250^3}{4500} = 434,028$$

$$\text{Var[Y]} = 434,028 - (521)^2 = 162,587$$

$$SD[Y] = 403.$$

60. Solution: B

The expected amount paid is (where N is the number of consecutive days of rain)

$$1000P[N=1]+2000P[N>1]=1000\frac{e^{-0.6}0.6}{1!}+2000\left(1-e^{-0.6}-e^{-0.6}0.6\right)=573.$$

The second moment is

$$1000^2P[N=1]+2000^2P[N>1]=1000^2\frac{e^{-0.6}0.6}{1!}+2000^2\left(1-e^{-0.6}-e^{-0.6}0.6\right)=816{,}893.$$

The variance is 816,893 – 573^2 = 488,564 and the standard deviation is 699.

61. Solution: C

X has an exponential distribution. Therefore, c = 0.004 and the distribution function is $F(x)=1-e^{-0.004x}$. For the moment, ignore the maximum benefit. The median is the solution to $0.5=F(m)=1-e^{-0.004m}$, which is $m=-250\ln(0.5)=173.29.$ Because this is below the maximum benefit, it is the median regardless of the existence of the maximum. Note that had the question asked for a percentile such that the solution without the maximum exceeds 250, then the answer is 250.

62. Solution: D

The distribution function of an exponential random variable, T, is $F(t)=1-e^{-t/\theta},\ t>0$. With a median of four hours, $0.5=F(4)=1-e^{-4/\theta}$ and so $\theta=-4/\ln(0.5)$. The probability the component works for at least five hours is $P[T\geq 5]=1-F(5)=1-1+e^{5\ln(0.5)/4}=0.5^{5/4}=0.42.$

63. Solution: E

This is a conditional probability. The solution is

$$\begin{aligned}
0.95 &= P[X\leq p\mid X>100]=\frac{P[100\leq X\leq p]}{P[X>100]}=\frac{F(p)-F(100)}{1-F(100)}\\
&=\frac{1-e^{-p/300}-1+e^{-100/300}}{1-1+e^{-100/300}}=\frac{e^{-100/300}-e^{-p/300}}{e^{-100/300}}=1-e^{-(p-100)/300}\\
0.05 &= e^{-(p-100)/300}\\
-2.9957 &= -(p-100)/300\\
p &= 999
\end{aligned}$$

64. Solution: A

The distribution function of X is given by

$$F(x) = \int_1^x \frac{3}{t^4} dt = -t^{-3}\Big|_1^x = 1 - x^{-3}, \quad x > 1$$

Next, let X_1, X_2, and X_3 denote the three claims made that have this distribution. Then if Y denotes the largest of these three claims, it follows that the distribution function of Y is given by

$$G(y) = P[X_1 \le y]P[X_2 \le y]P[X_3 \le y] = (1 - y^{-3})^3.$$

The density function of Y is given by

$$g(y) = G'(y) = 3(1 - y^{-3})^2 (3y^{-4}).$$

Therefore,

$$E[Y] = \int_1^\infty y3(1 - y^{-3})^2 3y^{-4} dy = 9\int_1^\infty y^{-3} - 2y^{-6} + y^{-9} dy$$

$$= 9\left[-y^{-2}/2 + 2y^{-5}/5 - y^{-8}/8\Big|_1^\infty\right] = 9\left[1/2 - 2/5 + 1/8\right]$$

$$= 2.025 \text{ (in thousands).}$$

65. Solution: C

The mean and standard deviation for the 2025 contributions are 2025(3125) = 6,328,125 and 45(250) = 11,250. By the central limit theorem, the total contributions are approximately normally distributed. The 90^{th} percentile is the mean plus 1.282 standard deviations or 6,328,125 + 1.282(11,250) = 6,342,548.

66. Solution: C

The average has the same mean as a single claim, 19,400. The standard deviation is that for a single claim divided by the square root of the sample size, 5,000/5 = 1,000. The probability of exceeding 20,000 is the probability that a standard normal variable exceeds (20,000 – 19,400)/1,000 = 0.6. From the tables, this is 1 – 0.7257 = 0.2743.

67. Solution: B

A single policy has a mean and variance of 2 claims. For 1250 polices the mean and variance of the total are both 2500. The standard deviation is the square root, or 50.
The approximate probability of being between 2450 and 2600 is the same as a standard normal random variable being between (2450 – 2500)/50 = –1 and (2600 – 2500)/50 = 2. From the tables, the probability is 0.9772 – (1 – 0.8413) = 0.8185.

68. Solution: B

Let *n* be the number of bulbs purchased. The mean lifetime is 3*n* and the variance is *n*. From the normal tables, a probability of 0.9772 is 2 standard deviations below the mean. Hence 40 = 3*n* – 2sqrt(*n*). Let *m* be the square root of *n*. The quadratic equation is $3m^2 - 2m - 40$. The roots are 4 and –10/3. So *n* is either 16 or 100/9. At 16 the mean is 48 and the standard deviation is 4, which works. At 100/9 the mean is 100/3 and the standard deviation is 10/3. In this case 40 is two standard deviations above the mean, and so is not appropriate. Thus 16 is the correct choice.

69. Solution: B

Observe that (where *Z* is total hours for a randomly selected person)

$$E[Z] = E[X+Y] = E[X] + E[Y] = 50 + 20 = 70$$

$$Var[Z] = Var[X+Y] = Var[X] + Var[Y] + 2Cov[X,Y] = 50 + 30 + 20 = 100.$$

It then follows from the Central Limit Theorem that *T* is approximately normal with mean 100(70) = 7000 and variance 100(100) = 10,000 and standard deviation 100. The probability of being less than 7100 is the probability that a standard normal variable is less than (7100 – 7000)/100 = 1. From the tables, this is 0.8413.

70. Solution: B

A single policy has an exponential distribution with mean and standard deviation 1000. The premium is then 1000 + 100 = 1100. For 100 policies, the total claims have mean 100(1000) = 100,000 and standard deviation 10(1000) = 10,000. Total premiums are 100(1100) = 110,000. The probability of exceeding this number is the probability that a standard normal variable exceeds (110,000 – 100,000)/10,000 = 1. From the tables this probability is 1 – 0.8413 = 0.1587.

71. Solution: E

For a single recruit, the probability of 0 pensions is 0.6, of 1 pension is 0.4(0.25) = 0.1, and of 2 pensions is 0.4(0.75) = 0.3. The expected number of pensions is 0(0.6) + 1(0.1) + 2(0.3) = 0.7. The second moment is 0(0.6) + 1(0.1) + 4(0.3) = 1.3. The variance is 1.3 – 0.49 = 0.81. For 100 recruits the mean is 70 and the variance is 81. The probability of providing at most 90 pensions is (with a continuity correction) the probability of being below 90.5. This is (90.5 – 70)/9 = 2.28 standard deviations above the mean. From the tables, this probability is 0.9887.

72. Solution: D

For one observation, the mean is 0 and the variance is 25/12 (for a uniform distribution the variance is the square of the range divided by 12). For 48 observations, the average has a mean of 0 and a variance of (25/12)/48 = 0.0434. The standard deviation is 0.2083. 0.25 years is 0.25/0.2083 = 1.2 standard deviations from the mean. From the normal tables the probability of being within 1.2 standard deviations is 0.8849 – (1 – 0.8849) = 0.7698.

73. Solution: C

For a good driver, the probability is $1-e^{-3/6}$ and for a bad driver, the probability is $1-e^{-2/3}$. The probability of both is the product, $(1-e^{-3/6})(1-e^{-2/3}) = 1-e^{-1/2}-e^{-2/3}+e^{-7/6}$.

74. Solution: E

The tour operator collects 21x50 = 1050 for the 21 tickets sold. The probability that all 21 passengers will show up is $\left(1-0.02\right)^{21} = \left(0.98\right)^{21} = 0.65$. Therefore, the tour operator's expected revenue is 1050 – 100(065) = 985.

75. Solution: C

First obtain the covariance of the two variables as (17,000 – 5,000 – 10,000)/2 = 1,000.
The requested variance is

$$\begin{aligned} Var(X+100+1.1Y) &= Var(X)+Var(1.1Y)+2Cov(X,1.1Y) \\ &= Var(X)+1.21Var(Y)+2(1.1)Cov(X,Y) \\ &= 5,000+1.21(10,000)+2.2(1,000) = 19,300. \end{aligned}$$

76. Solution: B

$P(X = 0) = 1/6$
$P(X = 1) = 1/12 + 1/6 = 3/12$
$P(X = 2) = 1/12 + 1/3 + 1/6 = 7/12$.
$E[X] = (0)(1/6) + (1)(3/12) + (2)(7/12) = 17/12$
$E[X^2] = (0)^2(1/6) + (1)^2(3/12) + (2)^2(7/12) = 31/12$
$\text{Var}[X] = 31/12 – (17/12)^2 = 0.58$.

77. Solution: D

Due to the independence of X and Y

$$Var(Z) = Var(3X-Y-5) = 3^2 Var(X) + (-1)^2 Var(Y) = 9(1) + 2 = 11.$$

78. Solution: E

Let X and Y denote the times that the generators can operate. Now the variance of an exponential random variable is the square of them mean, so each generator has a variance of 100. Because they are independent, the variance of the sum is 200.

79. Solution: E

Let S, F, and T be the losses due to storm, fire, and theft respectively. Let Y = max(S,F,T). Then,

$$P[Y > 3] = 1 - P[Y \le 3] = 1 - P[\max(S, F, T) \le 3] = 1 - P[S \le 3]P[F \le 3]P[T \le 3]$$

$$= 1 - (1 - \mathrm{e}^{-3/1})(1 - e^{-3/1.5})(1 - e^{-3/2.4}) = 0.414.$$

80. Solution: A

First obtain Var(X) = 27.4 – 25 = 2.4, Var(Y) = 51.4 – 49 = 2.4, Cov(X,Y) = (8 – 2.4 – 2.4)/2 = 1.6. Then,

$$Cov(C_1, C_2) = Cov(X + Y, X + 1.2Y) = Cov(X, X) + 1.2Cov(X, Y) + Cov(Y, X) + 1.2Cov(Y, Y)$$

$$= Var(X) + 1.2Var(Y) + 2.2Cov(X, Y)$$

$$= 2.4 + 1.2(2.4) + 2.2(1.6) = 8.8.$$

81. Solution: E

Because the husband has survived, the only possible claim payment is to the wife. So we need the probability that the wife dies within ten years given that the husband survives. The numerator of the conditional probability is the unique event that only the husband survives, with probability 0.01. The denominator is the sum of two events, both survive (0.96) and only the husband survives (0.01). The conditional probability is 0.01/(0.96 + 0.01) = 1/97. The expected claim payment is 10,000/97 = 103 and the expected excess is 1,000 – 103 = 897.

82. Solution: C

$$P[Y = 0 \mid X = 1] = \frac{P(X = 1, Y = 0)}{P(X = 1)} = \frac{P(X = 1, Y = 0)}{P(X = 1, Y = 0) + P(X = 1, Y = 1)} = \frac{0.05}{0.05 + 0.125} = 0.286$$

$$P[Y = 1 \mid X = 1] = 1 - 0.286 = 0.714.$$

The conditional variable is Bernoulli with p = 0.714. The variance is (0.714)(0.286) = 0.204.

83. Solution: D

With no tornadoes in County P the probabilities of 0, 1, 2, and 3 tornadoes in County Q are 12/25, 6/25, 5/25, and 2/25 respectively.
The mean is (0 + 6 + 10 + 6)/25 = 22/25.
The second moment is (0 + 6 + 20 + 18)/25 = 44/25.
The variance is 44/25 – $(22/25)^2$ = 0.9856.

84. Solution: C

From the Law of Total Probability:

$$P[4 < S < 8] = P[4 < S < 8 \mid N = 1]P[N = 1] + P[4 < S < 8 \mid N > 1]P[N > 1]$$

$$= (e^{-4/5} - e^{-8/5})(1/3) + (e^{-4/8} - e^{-8/8})(1/6) = 0.122.$$

85. Solution: C

Due to the equal spacing of probabilities, $p_n = p_0 - nc$ for c = 1, 2, 3, 4, 5. Also, $0.4 = p_0 + p_1 = p_0 + p_0 - c = 2p_0 - c.$ Because the probabilities must sum to 1, $1 = p_0 + p_0 - c + p_0 - 2c + p_0 - 3c + p_0 - 4c + p_0 - 5c = 6p_0 - 15c.$ This provides two equations in two unknowns. Multiplying the first equation by 15 gives $6 = 30p_0 - 15c.$ Subtracting the second equation gives $5 = 24p_0 \Rightarrow p_0 = 5/24$. Inserting this in the first equation gives c = 1/60. The requested probability is $p_4 + p_5 = 5/24 - 4/60 + 5/24 - 5/60 = 32/120 = 0.267.$

86. Solution: D

Because the number of payouts (including payouts of zero when the loss is below the deductible) is large, apply the central limit theorem and assume the total payout S is normal. For one loss, the mean, second moment, and variance of the payout are

$$\int_0^{5,000} 0 \frac{1}{20,000}dx + \int_{5,000}^{20,000} (x - 5,000)\frac{1}{20,000} = 0 + \left.\frac{(x-5,000)^2}{40,000}\right|_{5,000}^{20,000} = 5,625$$

$$\int_0^{5,000} 0^2 \frac{1}{20,000}dx + \int_{5,000}^{20,000} (x - 5,000)^2\frac{1}{20,000} = 0 + \left.\frac{(x-5,000)^3}{60,000}\right|_{5,000}^{20,000} = 56,250,000$$

$$56,250,000 - 5,625^2 = 24,609,375.$$

For 200 losses the mean, variance, and standard deviation are 1,125,000, 4,921,875,000, and 70,156. 1,000,000 is 125,000/70,156 = 1.7817 standard deviations below the mean and 1,200,000 is 75,000/70,156 = 1.0690 standard deviations above the mean. From the standard normal tables, the probability of being between these values is 0.8575 – (1 – 0.9626) = 0.8201.

87. Solution: B

Let H be the percentage of clients with homeowners insurance and R be the percentage of clients with renters insurance.

Because 36% of clients do not have auto insurance and none have both homeowners and renters insurance, we calculate that 8% (36% – 17% – 11%) must have renters insurance, but not auto insurance.

(H – 11)% have both homeowners and auto insurance, (R – 8)% have both renters and auto insurance, and none have both homeowners and renters insurance, so (H + R – 19)% must equal 35%. Because H = 2R, R must be 18%, which implies that 10% have both renters and auto insurance.

88. Solution: B

Let Y be the reimbursement. Then, $G(115) = P[Y < 115 \mid X > 20]$. For Y to be 115, the costs must be above 120 (up to 120 accounts for a reimbursement of 100). The extra 15 requires 30 in additional costs. Therefore, we need

$$P[X \le 150 \mid X > 20] = \frac{P[X \le 150] - P[X \le 20]}{P[X > 20]} = \frac{1 - e^{-150/100} - 1 + e^{-20/100}}{1 - 1 + e^{-20/100}}$$

$$= \frac{-e^{-1.5} + ^{-0.2}}{e^{-0,2}} = 1 - e^{-1.3} = 0.727.$$

89. Solution: E

The conditional probability function given 2 claims in April is

$$p_{N_1}(2) = \sum_{n_2=1}^{\infty} \frac{3}{4}\frac{1}{4} e^{-2}(1-e^{-2})^{n_2-1} = \frac{3e^{-2}}{16}\frac{1}{1-(1-e^{-2})} = \frac{3}{16}$$

$$p(n_2 \mid N_1 = 2) = \frac{p(2, n_2)}{p_{N_1}(2)} = \frac{3}{4}\frac{1}{4} e^{-2}(1-e^{-2})^{n_2-1}\frac{16}{3} = e^{-2}(1-e^{-2})^{n_2-1}.$$

This can be recognized as a geometric probability function and so the mean is $1/e^{-2} = e^2$.

90. Solution: C

The number of defective modems is 20% x 30 + 8% x 50 = 10.

The probability that exactly two of a random sample of five are defective is $\frac{\binom{10}{2}\binom{70}{3}}{\binom{80}{5}} = 0.102$.

91. Solution: B

P(40 year old man dies before age 50) = P(T < 50 | T > 40)

$$= \frac{\Pr(40 < T < 50)}{\Pr(T > 40)} = \frac{F(50) - F(40)}{1 - F(40)}$$

$$= \frac{1 - \exp\left(\frac{1 - 1.1^{50}}{1000}\right) - 1 + \exp\left(\frac{1 - 1.1^{40}}{1000}\right)}{1 - 1 + \exp\left(\frac{1 - 1.1^{40}}{1000}\right)} = \frac{\exp\left(\frac{1 - 1.1^{40}}{1000}\right) - \exp\left(\frac{1 - 1.1^{50}}{1000}\right)}{\exp\left(\frac{1 - 1.1^{40}}{1000}\right)}$$

$$= \frac{0.9567 - 0.8901}{0.9567} = 0.0696$$

Expected Benefit = 5000(0.0696) = 348.

92. Solution: C

Letting t denote the relative frequency with which twin-sized mattresses are sold, we have that the relative frequency with which king-sized mattresses are sold is 3t and the relative frequency with which queen-sized mattresses are sold is (3t+t)/4, or t. Thus, t = 0.2 since t + 3t + t = 1. The probability we seek is 3t + t = 0.80.

93. Solution: E

$Var(N) = E[Var(N \mid \lambda)] + Var[E(N \mid \lambda)] = E[\lambda] + Var(\lambda) = 1.5 + 0.75 = 2.25.$ The variance of a uniform random variable is the square of the range divided by 12, in this case $3^2/12 = 0.75$.

94. Solution: D

X follows a geometric distribution with p = 1/6 and Y = 2 implies the first roll is not a 6 and the second roll is a 6. This means a 5 is obtained for the first time on the first roll (probability = 0.2) or a 5 is obtained for the first time on the third or later roll (probability = 0.8).

$E[X \mid X \geq 3] = \frac{1}{p} + 2 = 6 + 2 = 8.$, The expected value is 0.2(1) + 0.8(8) = 6.6.

95. Solution: B

The unconditional probabilities for the number of people in the car who are hospitalized are 0.49, 0.42 and 0.09 for 0, 1 and 2, respectively. If the number of people hospitalized is 0 or 1, then the total loss will be less than 1. However, if two people are hospitalized, the probability that the total loss will be less than 1 is 0.5. Thus, the expected number of people in the car who are hospitalized, given that the total loss due to hospitalizations from the accident is less than 1 is

$$\frac{0.49}{0.49+0.42+0.09\cdot 0.5}(0)+\frac{0.42}{0.49+0.42+0.09\cdot 0.5}(1)+\frac{0.09\cdot 0.5}{0.49+0.42+0.09\cdot 0.5}(2)=0.534$$

96. Solution: B

Let X equal the number of hurricanes it takes for two losses to occur. Then X is negative binomial with "success" probability $p = 0.4$ and $r = 2$ "successes" needed.

$$P[X=n]=\binom{n-1}{r-1}p^r(1-p)^{n-r}=\binom{n-1}{2-1}(0.4)^2(1-0.4)^{n-2}=(n-1)(0.4)^2(0.6)^{n-2}\text{, for } n\geq 2.$$

We need to maximize $P[X = n]$. Note that the ratio

$$\frac{P[X=n+1]}{P[X=n]}=\frac{n(0.4)^2(0.6)^{n-1}}{(n-1)(0.4)^2(0.6)^{n-2}}=\frac{n}{n-1}(0.6).$$

This ratio of "consecutive" probabilities is greater than 1 when $n = 2$ and less than 1 when $n \geq 3$. Thus, $P[X = n]$ is maximized at $n = 3$; the mode is 3. Alternatively, the first few probabilities could be calculated.

97. Solution: C

There are 10 (5 choose 3) ways to select the three columns in which the three items will appear. The row of the rightmost selected item can be chosen in any of six ways, the row of the leftmost selected item can then be chosen in any of five ways, and the row of the middle selected item can then be chosen in any of four ways. The answer is thus (10)(6)(5)(4) = 1200. Alternatively, there are 30 ways to select the first item. Because there are 10 squares in the row or column of the first selected item, there are 30 − 10 = 20 ways to select the second item. Because there are 18 squares in the rows or columns of the first and second selected items, there are 30 − 18 = 12 ways to select the third item. The number of permutations of three qualifying items is (30)(20)(12). The number of combinations is thus (30)(20)(12)/3! = 1200.

98. Solution: B

The expected bonus for a high-risk driver is 0.8(12)(5) = 48.
The expected bonus for a low-risk driver is 0.9(12)(5) = 54.
The expected bonus payment from the insurer is 600(48) + 400(54) = 50,400.

99. Solution: E

Liability but not property = 0.01 (given)
Liability and property = 0.04 – 0.01 = 0.03.
Property but not liability = 0.10 – 0.03 = 0.07
Probability of neither = 1 – 0.01 – 0.03 – 0.07 = 0.89

100. Solution: B

C = the set of TV watchers who watched CBS over the last year
N = the set of TV watchers who watched NBC over the last year
A = the set of TV watchers who watched ABC over the last year
H = the set of TV watchers who watched HGTV over the last year

The number of TV watchers in the set $\mathrm{C} \cup \mathrm{N} \cup \mathrm{A}$ is 34 + 15 + 10 – 7 – 6 – 5 + 4 = 45.

Because $\mathrm{C} \cup \mathrm{N} \cup \mathrm{A}$ and H are mutually exclusive, the number of TV watchers in the set $\mathrm{C} \cup \mathrm{N} \cup \mathrm{A} \cup \mathrm{H}$ is 45 + 18 = 63.

The number of TV watchers in the complement of $\mathrm{C} \cup \mathrm{N} \cup \mathrm{A} \cup \mathrm{H}$ is thus 100 – 63 = 37.

101. Solution: A

Let *X* denote the amount of a claim before application of the deductible. Let *Y* denote the amount of a claim payment after application of the deductible. Let λ be the mean of *X*, which because *X* is exponential, implies that λ^2 is the variance of *X* and $E(X^2) = 2\lambda^2$.

By the memoryless property of the exponential distribution, the conditional distribution of the portion of a claim above the deductible given that the claim exceeds the deductible is an exponential distribution with mean λ. Given that $E(Y) = 0.9\lambda$, this implies that the probability of a claim exceeding the deductible is 0.9 and thus $E(Y^2) = 0.9(2\lambda^2) = 1.8\lambda^2$. Then, $\mathrm{Var}(Y) = 1.8\lambda^2 - (0.9\lambda)^2 = 0.99\lambda^2$. The percentage reduction is 1%.

102. Solution: C

Let N denote the number of hurricanes, which is Poisson distributed with mean and variance 4.

Let X_i denote the loss due to the i^{th} hurricane, which is exponentially distributed with mean 1,000 and therefore variance $(1{,}000)^2 = 1{,}000{,}000$.

Let X denote the total loss due to the N hurricanes.

This problem can be solved using the conditional variance formula. Note that independence is used to write the variance of a sum as the sum of the variances.

$$\begin{aligned}\text{Var}(X) &= Var[E(X \mid N)] + E[Var(X \mid N)] \\ &= Var[E(X_1 + \cdots + X_N)] + E[Var(X_1 + \cdots + X_N)] \\ &= Var[NE(X_1)] + E[NVar(X_1)] \\ &= Var(1{,}000N) + E(1{,}000{,}000N) \\ &= 1{,}000^2 Var(N) + 1{,}000{,}000E(N) \\ &= 1{,}000{,}000(4) + 1{,}000{,}000(4) = 8{,}000{,}000.\end{aligned}$$

103. Solution: B

Let N denote the number of accidents, which is binomial with parameters 3 and 0.25 and thus has mean 3(0.25) = 0.75 and variance 3(0.25)(0.75) = 0.5625.

Let X_i denote the unreimbursed loss due to the i^{th} accident, which is 0.3 times an exponentially distributed random variable with mean 0.8 and therefore variance $(0.8)^2 = 0.64$. Thus, X_i has mean 0.8(0.3) = 0.24 and variance $0.64(0.3)^2 = 0.0576$.

Let X denote the total unreimbursed loss due to the N accidents.

This problem can be solved using the conditional variance formula. Note that independence is used to write the variance of a sum as the sum of the variances.

$$\begin{aligned}\text{Var}(X) &= Var[E(X \mid N)] + E[Var(X \mid N)] \\ &= Var[E(X_1 + \cdots + X_N)] + E[Var(X_1 + \cdots + X_N)] \\ &= Var[NE(X_1)] + E[NVar(X_1)] \\ &= Var(0.24N) + E(0.0576N) \\ &= 0.24^2 Var(N) + 0.0576E(N) \\ &= 0.0576(0.5625) + 0.0576(0.75) = 0.0756.\end{aligned}$$

104. Solution: B

The 95th percentile is in the range when an accident occurs. It is the 75th percentile of the payout, given that an accident occurs, because (0.95 − 0.80)/(1 − 0.80) = 0.75. Letting x be the 75th percentile of the given exponential distribution, $F(x) = 1 - e^{-\frac{x}{3000}} = 0.75$, so x = 4159. Subtracting the deductible of 500 gives 3659 as the (unconditional) 95th percentile of the insurance company payout.

105. Solution: C

The ratio of the probability that one of the damaged pieces is insured to the probability that none of the damaged pieces are insured is

$$\frac{\dfrac{\binom{r}{1}\binom{27-r}{3}}{\binom{27}{4}}}{\dfrac{\binom{r}{0}\binom{27-r}{4}}{\binom{27}{4}}} = \frac{4r}{24-r},$$

where r is the total number of pieces insured. Setting this ratio equal to 2 and solving yields r = 8.

The probability that two of the damaged pieces are insured is

$$\frac{\binom{r}{2}\binom{27-r}{2}}{\binom{27}{4}} = \frac{\binom{8}{2}\binom{19}{2}}{\binom{27}{4}} = \frac{(8)(7)(19)(18)(4)(3)(2)(1)}{(27)(26)(25)(24)(2)(1)(2)(1)} = \frac{266}{975} = 0.27 .$$

106. Solution: A

The probability that Rahul examines exactly *n* policies is $0.1(0.9)^{n-1}$. The probability that Toby examines more than *n* policies is 0.8^n. The required probability is thus

$$\sum_{n=1}^{\infty} 0.1(0.9)^{n-1}(0.8)^n = \frac{1}{9}\sum_{n=1}^{\infty} 0.72^n = \frac{0.72}{9(1-0.72)} = 0.2857.$$

An alternative solution begins by imagining Rahul and Toby examine policies simultaneously until at least one of the finds a claim. At each examination there are four possible outcomes:

1. Both find a claim. The probability is 0.02.
2. Rahul finds a claim and Toby does not. The probability is 0.08.
3. Toby finds a claim and Rahul does not. The probability is 0.18
4. Neither finds a claim. The probability is 0.72.

Conditioning on the examination at which the process ends, the probability that it ends with Rahul being the first to find a claim (and hence needing to examine fewer policies) is 0.08/(0.02 + 0.08 + 0.18) = 8/28 = 0.2857.

107. Solution: E

Let *a* be the mean and variance of *X* and *b* be the mean and variance of *Y*. The two facts are $a = b - 8$ and $a + a^2 = 0.6(b + b^2)$. Substituting the first equation into the second gives

$$b - 8 + (b-8)^2 = 0.6b + 0.6b^2$$

$$b - 8 + b^2 - 16b + 64 = 0.6b + 0.6b^2$$

$$0.4b^2 - 15.6b + 56 = 0$$

$$b = \frac{15.6 \pm \sqrt{15.6^2 - 4(0.4)(56)}}{2(0.4)} = \frac{15.6 \pm 12.4}{0.8} = 4 \text{ or } 35.$$

At *b* = 4, *a* is negative, so the answer is 35.

108. Solution: C

Suppose there are N red sectors. Let w be the probability of a player winning the game. Then, w = the probability of a player missing all the red sectors and

$$w = 1 - \left[\frac{9}{20} + \left(\frac{9}{20}\right)^2 + \cdots + \left(\frac{9}{20}\right)^N \right]$$

Using the geometric series formula,

$$w = 1 - \frac{\frac{9}{20} - \left(\frac{9}{20}\right)^{N+1}}{1 - \frac{9}{20}} = 1 - \frac{9}{20}\frac{1 - \left(\frac{9}{20}\right)^N}{1 - \frac{9}{20}} = \frac{2}{11} + \frac{9}{11}\left(\frac{9}{20}\right)^N$$

Thus we need

$$0.2 > w = \frac{2}{11} + \frac{9}{11}\left(\frac{9}{20}\right)^N$$

$$2.2 > 2 + 9\left(\frac{9}{20}\right)^N$$

$$0.2 > 9\left(\frac{9}{20}\right)^N$$

$$\frac{2}{90} > \left(\frac{9}{20}\right)^N$$

$$\left(\frac{20}{9}\right)^N > 45$$

$$N > \frac{\ln(45)}{\ln(20/9)} \approx 4.767$$

Thus N must be the first integer greater than 4.767, or 5.

109. Solution: B

The fourth moment of X is

$$\int_0^{10} \frac{x^4}{10} dx = \left.\frac{x^5}{50}\right|_0^{10} = 2000.$$

The Y probabilities are 1/20 for $Y = 0$ and 10, and 1/10 for $Y = 1,2,\ldots,9$.

$$E[Y^4] = (1^4 + 2^4 + \cdots + 9^4)/10 + 10^4/20 = 2033.3.$$

The absolute value of the difference is 33.3.

110. Solution: E

$$P(x=1, y=1) = P(y=1 \mid x=1)P(x=1) = 0.3(0.5)^2 = 0.075$$
$$P(x=2, y=0) = P(y=0 \mid x=2)P(x=2) = 0.25(0.5)^3 = 0.03125$$
$$P(x=0, y=2) = P(y=2 \mid x=0)P(x=0) = 0.05(0.5)^1 = 0.025$$

The total is 0.13125.

111. Solution: C

$$E(X) = \int_1^\infty x \frac{p-1}{x^p} dx = (p-1)\int_1^\infty x^{1-p} dx$$
$$(p-1)\frac{x^{2-p}}{2-p}\Bigg|_1^\infty = \frac{p-1}{p-2} = 2$$
$$p-1 = 2(p-2) = 2p-4$$
$$p = 3$$

112. Solution: D

The distribution function plot shows that *X* has a point mass at 0 with probability 0.5. From 2 to 3 it has a continuous distribution. The density function is the derivative, which is the constant (1 – 0.5)/(3 – 2) = 0.5. The expected value is 0(0.5) plus the integral from 2 to 3 of 0.5*x*. The integral evaluates to 1.25, which is the answer. Alternatively, this is a 50-50 mixture of a point mass at 0 and a uniform(2,3) distribution. The mean is 0.5(0) + 0.5(2.5) = 1.25.

113. Solution: E

The dice rolls that satisfy this event are (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (3,5), (4,2), (4,3), (4,4), (4,5), (4,6), (5,3), (5,4), (5,5), (5,6), (6,4), (6,5), and (6,6). They represent 24 of the 36 equally likely outcomes for a probability of 2/3.

114. Solution: D

$$0.64 = \rho = \frac{Cov(M,N)}{\sqrt{Var(M)Var(N)}}$$
$$Cov(M,N) = 0.64\sqrt{1600(900)} = 768$$
$$Var(M+N) = Var(M) + Var(N) + 2Cov(M,N) = 1600 + 900 + 2(768) = 4036$$

115. Solution: C

$$\int_1^6 (x-1)0.5e^{-0,5x}dx + \int_6^\infty 5(0.5)e^{-0.5x}dx = -(x-1)e^{-0.5x}\Big|_1^6 + \int_1^6 e^{-0.5x}dx - 5e^{-0.5x}\Big|_6^\infty$$
$$= -5e^{-3} + 0 - 2e^{-0.5x}\Big|_1^6 + 5e^{-3} = -2e^{-3} + 2e^{-1/2}$$

116. Solution: A

Let C be the number correct. C has a binomial distribution with $n = 40$ and $p = 0.5$. Then the mean is 40(0.5) = 20 and the variance is 40(0.5)(0.5) = 10. With the exact probability we have

$$0.1 = P(C > N) = \Pr\left(Z > \frac{N+0.5-20}{\sqrt{10}}\right)$$
$$1.282 = \frac{N+0.5-20}{\sqrt{10}}, \quad N = 1.282\sqrt{10} + 19.5 = 23.55.$$

At $N = 23$ the approximate probability will exceed 0.1 ($Z = 1.107$).

117. Solution: B

The months in question have 1, 1, 0.5, 0.5, and 0.5 respectively for their means. The sum of independent Poisson random variables is also Poisson, with the parameters added. So the total number of accidents is Poisson with mean 3.5. The probability of two accidents is

$$\frac{e^{-3.5}3.5^2}{2!} = 0.185.$$

118. Solution: B

The payments are 0 with probability 0.72 (snowfall up to 50 inches), 300 with probability 0.14, 600 with probability 0.06, and 700 with probability 0.08. The mean is 0.72(0) + 0.14(300) + 0.06(600) + 0.08(700) = 134 and the second moment is 0.72(0^2) + 0.14(300^2) + 0.06(600^2) + 0.08(700^2) = 73,400. The variance is 73,400 – 134^2 = 55,444. The standard deviation is the square root, 235.

119. Solution: D

$$1 = \int_0^{20} c(x^2 - 60x + 800)dx = c\,(x^3/3 - 30x^2 + 800x)\Big|_0^{20} = c20,000/3 \Rightarrow c = 3/20,000$$
$$P(X > d) = \int_d^{20} c(x^2 - 60x + 800)dx = c\,(x^3/3 - 30x^2 + 800x)\Big|_d^{20} = 1 - \frac{3}{20,000}(d^3/3 - 30d^2 + 800d)$$
$$P(X > 10 \mid X > 2) = \frac{P(X > 10)}{P(X > 2)} = \frac{0.2}{0.7776} = 0.2572$$

120. Solution: A

Each event has probability 0.5. Each of the three possible intersections of two events has probability 0.25 = (0.5)(0.5), so each pair is independent. The intersection of all three events has probability 0, which does not equal (0.5)(0.5)(0.5) and so the three events are not mutually independent.

121. Solution: C

Let event A be the selection of the die with faces (1,2,3,4,5,6), event B be the selection of the die with faces (2,2,4,4,6,6) and event C be the selection of the die with all 6's. The desired probability is, using the law of total probability,

$$P(6,6) = P(6,6 \mid A)P(A) + P(6,6 \mid B)P(B) + P(6,6 \mid C)P(C)$$

$$= (1/36)(1/2) + (1/9)(1/4) + 1(1/4) = 1/72 + 2/72 + 18/72 = 21/72 = 0.292.$$

122. Solution: D

$$\frac{\binom{6}{2}\binom{4}{2}\binom{2}{2}}{\binom{12}{6}} = \frac{15(6)(1)}{924} = 0.097$$

123. Solution: D

Consider the three cases based on the number of claims.
If there are no claims, the probability of total benefits being 48 or less is 1.
If there is one claim, the probability is 48/60 = 0.8, from the uniform distribution.
If there are two claims, the density is uniform on a 60x60 square. The event where the total is 48 or less is represented by a triangle with base and height equal to 48. The triangle's area is 48x48/2 = 1152. Dividing by the area of the square, the probability is 1152/3600 = 0.32.

Using the law of total probability, the answer is 0.7(1) + 0.2(0.8) + 0.1(0.32) = 0.892.

124. Solution: B

The sum of independent Poisson variables is also Poisson, with the means added. Thus the number of tornadoes in a three week period is Poisson with a mean of 3x2 = 6. Then,

$$P(N < 4) = p(0) + p(1) + p(2) + p(3) = e^{-6}\left(\frac{6^0}{0!} + \frac{6^1}{1!} + \frac{6^2}{2!} + \frac{6^3}{3!}\right) = 0.1512.$$

125. Solution: A

The number of components that fail has a binomial(3, 0.05) distribution. Then,

$$P(N \geq 2) = p(2) + p(3) = \binom{3}{2}(0.05)^2(0.95) + \binom{3}{3}(0.05)^3 = 0.00725.$$

126. Solution: E

The profit variable X is normal with mean 100 and standard deviation 20. Then,

$$P(X \leq 60 \mid X > 0) = \frac{P(0 < X \leq 60)}{P(X > 0)} = \frac{P\left(\frac{0-100}{20} < Z \leq \frac{60-100}{20}\right)}{P\left(Z > \frac{0-100}{20}\right)} = \frac{F(-2) - F(-5)}{1 - F(-5)}.$$

For the normal distribution, $F(-x) = 1 - F(x)$ and so the answer can be rewritten as $[1 - F(2) - 1 + F(5)]/[1 - 1 + F(5)] = [F(5) - F(2)]/F(5)$.

127. Solution: A

Let B be the event that the policyholder has high blood pressure and C be the event that the policyholder has high cholesterol. We are given $P(B) = 0.2$, $P(C) = 0.3$, and $P(C \mid B) = 0.25$. Then,

$$P(B \mid C) = \frac{P(B \cap C)}{P(C)} = \frac{P(C \mid B)P(B)}{P(C)} = \frac{0.25(0.2)}{0.3} = 1/6.$$

128. Solution: D

This is a hypergeometric probability,

$$\frac{\binom{20}{1}\binom{5}{1}}{\binom{25}{2}} = \frac{20(5)}{25(24)/2} = \frac{100}{300} = 0.333\ ,$$

Alternatively, the probability of the first worker being high risk and the second low risk is (5/25)(20/24) = 100/600 and of the first being low risk and the second high risk is (20/25)(5/24) = 100/600 for a total probability of 200/600 = 0.333.

129. Solution: C

$$E\left(\frac{X}{1-X}\right)=60\int_0^1 \frac{x}{1-x}x^3(1-x)^2\,dx=60\int_0^1 x^4(1-x)dx=60(x^5/5-x^6/6)\Big|_0^1=60(1/5-1/6)=2$$

$$E\left[\left(\frac{X}{1-X}\right)^2\right]=60\int_0^1 \frac{x^2}{(1-x)^2}x^3(1-x)^2\,dx=60\int_0^1 x^5dx=60(x^6/6)\Big|_0^1=60(1/6)=10$$

$$Var\left(\frac{X}{1-X}\right)=10-2^2=6$$

130. Solution: B

P(at least one emergency room visit **or** at least one hospital stay) = 1 – 0.61 = 0.39 = *P*(at least one emergency room visit) + *P*(at least one hospital stay) – *P*(at least one emergency room visit **and** at least one hospital stay).

P(at least one emergency room visit **and** at least one hospital stay) = 1 – 0.70 + 1 – 0.85 – 0.39 = 0.060.

131. Solution: A

Let *Y* be the loss and *X* be the reimbursement. If the loss is less than 4, $P(X \le x) = P(Y \le x) = 0.2x$ for $x < 4$ because *Y* has a uniform distribution on [0, 5]. However, the probability of the reimbursement being less than or equal to 4 is 1 because 4 is the maximum reimbursement.

132. Solution: B

The number of males is 0.54(900) = 486 and of females is then 414.
The number of females over age 25 is 0.43(414) = 178.
The number over age 25 is 395. Therefore the number under age 25 is 505. The number of females under age 25 is 414 – 178 = 236. Therefore the number of males under 25 is 505 – 236 = 269 and the probability is 269/505 =0.533.

133. Solution: C

Let *R* be the event the car is red and *G* be the event the car is green. Let *E* be the event that the claim exceeds the deductible. Then,

$$P(R\mid E)=\frac{P(R)P(E\mid R)}{P(R)P(E\mid R)+P(\mathrm{G})P(E\mid \mathrm{G})}=\frac{0.3(0.09)}{0.3(0.09)+0.7(0.04)}=\frac{0.027}{0.055}=0.491.$$

Note that if *A* is the probability of an accident,
$P(E\mid R)=P(E\mid R \text{ and } A)P(A\mid R)=0.1(0.9)=0.09.$

134. Solution: B

Let X and Y be the selected numbers. The probability Paul wins is $P(|X - Y| \le 3)$. Of the 400 pairs, it is easiest to count the number of outcomes that satisfy this event:
If X = 1, then Y can be 1, 2, 3, or 4 (4 total)
If X = 2, then Y can be 1, 2, 3, 4, or 5 (5 total)
For X = 3 there are 6, and for X = 4 through 17 there are 7. For X = 18, 19, and 20 the counts are 6, 5, and 4 respectively. The total is then 4 + 5 + 6 + 14(7) + 6 + 5 + 4 = 128. The probability is 128/400 = 0.32.

135. Solution: C

Let C and K denote respectively the event that the student answers the question correctly and the event that he actually knows the answer. The known probabilities are
$P(C \mid K^c) = 0.5, \quad P(C \mid K) = 1, \quad P(K \mid C) = 0.824, \quad P(K) = N/20.$ Then,

$$0.824 = P(K \mid C) = \frac{P(C \mid K)P(K)}{P(C \mid K)P(K) + P(C \mid K^c)P(K^c)} = \frac{1(N/20)}{1(N/20) + 0.5(20-N)/20} = \frac{N}{N + 0.5(20-N)}$$

$$0.824(0.5N + 10) = N$$

$$8.24 = 0.588N$$

$$N = 14.$$

136. Solution: D

The probability that a randomly selected cable will not break under a force of 12,400 is $P(Y > 12,400) = P[Z > (12,400 - 12,432)/25 = -1.28] = 0.9.$ The number of cables, N, that will not break has the binomial distribution with n = 400 and p = 0.9. This can be approximated by a normal distribution with mean 360 and standard deviation 6. With the continuity correction, $P(N \ge 349) = P[Z \ge (348.5 - 360)/6 = -1.9167] = 0.97.$

137. Solution: D

Because the mode is 2 and 3, the parameter is 3 (when the parameter is a whole number the probabilities at the parameter and at one less than the parameter are always equal). Alternatively, the equation $p(2) = p(3)$ can be solved for the parameter. Then the probability of selling 4 or fewer policies is 0.815 and this is the first such probability that exceeds 0.75. Thus, 4 is the first number for which the probability of selling more than that number of policies is less than 0.25.

138. Solution: E

Of the 36 possible pairs, there are a total of 15 that have the red number larger than the green number. Note that a list is not needed. There are 6 that have equal numbers showing and of the remaining 30 one-half must have red larger than green. Of these 15, 9 have an odd sum for the answer, 9/15 = 3/5. This is best done by counting, with 3 combinations adding to 7, 2 combinations each totaling 5 and 9, and 1 combination each totaling 3 and 11.

139. Solution: B

From the table the exact 93rd percentile comes from a z-score between 1.47 and 1.48. 1.47 implies a test score of 503 + 1.47(98) = 647.1. Similarly, 1.48 implies a score of 648.0. The closest multiple of 10 is 650. Abby's z-score is then (650 – 521)/101 = 1.277. This rounds to the 90th percentile of the standard normal distribution.

140. Solution: C

Let *X*, *Y*, and *Z* be the three lifetimes. We want
$P(X+Y>1.9Z)=P(W=X+Y-1.9Z>0)$.
A linear combination of independent normal variables is also normal. In this case *W* has a mean of 10 + 10 – 1.9(10) = 1 and a variance of 9 + 9 + 1.9(1.9)(9) = 50.49 for a standard deviation of 7.106.
Then the desired probability is that a standard normal variable exceeds (0 – 1)/7.106 = –0.141. Interpolating in the normal tables gives 0.5557 + (0.5596 – 0.5557)(0.1) = 0.5561, which rounds to 0.556.

141. Solution: C

We have

$$0.3=P\left[\text{insurer must pay at least }1.2\right]=P\left[\text{loss}\geq 1.2+d\right]=\frac{2-1.2-d}{2-0}=\frac{0.8-d}{2}$$

$d=0.8-2(0.3)=0.2.$

Then,

$$P\left[\text{insurer must pay at least }1.44\right]=P\left[\text{loss}\geq 1.44+d\right]=\frac{2-1.44-0.2}{2-0}=0.18.$$

142. Solution: E

The cumulative distribution function for the exponential distribution of the lifespan is
$F(x)=1-e^{-\lambda x}$, for positive *x*.
The probability that the lifespan exceeds 4 years is $0.3=1-F(4)=e^{-4\lambda}$. Thus $\lambda=-(\ln 0.3)/4$.
For positive *x*, the probability density function is

$$f(x)=\lambda e^{-\lambda x}=-\frac{\ln 0.3}{4}\mathrm{e}^{(\ln 0.3)x/4}=-\frac{\ln 0.3}{4}(0.3)^{x/4}.$$

143. Solution: C

It is not necessary to determine the constant of proportionality. Let it be *c*. To determine the mode, set the derivative of the density function equal to zero and solve.

$$0 = f'(x) = \frac{d}{dx} cx^2(1+x^3)^{-1} = 2cx(1+x^3)^{-1} + cx^2[-(1+x^3)^{-2}]3x^2$$

$$= 2cx(1+x^3) - 3cx^4 \quad \text{(multiplying by } (1+x^3)^2\text{)}$$

$$= 2cx + 2cx^4 - 3cx^4 = 2cx - cx^4$$

$$= 2 - x^3 \Rightarrow x = 2^{1/3} = 1.26.$$

144. Solution: C

It is not necessary to determine the constant of proportionality. Let it be *c*. To determine the mode, set the derivative of the density function equal to zero and solve.

$$0 = f'(x) = \frac{d}{dx} cxe^{-x^2} = ce^{-x^2} - cx(2x)e^{-x^2} = ce^{-x^2}(1-2x^2)$$

$$= 1 - 2x^2 \quad \text{(multiplying by ce}^{x^2}\text{)}$$

$$\Rightarrow x = (1/2)^{1/2} = 0.71.$$

145. Solution: E

A geometric probability distribution with mean 1.5 will have $p = 2/3$. So Pr(1 visit) = 2/3, P(two visits) = 2/9, etc. There are four disjoint scenarios in which total admissions will be two or less.

Scenario 1: No employees have hospital admissions. Probability = $0.8^5 = 0.32768$.

Scenario 2: One employee has one admission and the other employees have none. Probability = $\binom{5}{1}(0.2)(0.8)^4(2/3) = 0.27307$.

Scenario 3: One employee has two admissions and the other employees have none. Probability = $\binom{5}{1}(0.2)(0.8)^4(2/9) = 0.09102$.

Scenario 4: Two employees each have one admission and the other three employees have none. Probability = $\binom{5}{2}(0.2)^2(0.8)^3(2/3)(2/3) = 0.09102$.

The total probability is 0.78279.

146. Solution: C

The intersection of the two events (third malfunction on the fifth day and not three malfunctions on first three days) is the same as the first of those events. So the numerator of the conditional probability is the negative binomial probability of the third success (malfunction) on the fifth day, which is

$$\binom{4}{2}(0.4)^2(0.6)^2(0.4) = 0.13824.$$

The denominator is the probability of not having three malfunctions in three days, which is $1-(0.4)^3 = 0.936$.

The conditional probability is 0.13824/0.936 = 0.1477.

147. Solution: C

Let p_i represent the probability that the patient's cancer is in stage *i*, for *i* = 0, 1, 2, 3, or 4.

The probabilities must sum to 1. That fact and the three facts given the question produce the following equations.

$$\begin{aligned} p_0 + p_1 + p_2 + p_3 + p_4 &= 1 \\ p_0 + p_1 + p_2 &= 0.75 \\ p_1 + p_2 + p_3 + p_4 &= 0.8 \\ p_0 + p_1 + p_3 + p_4 &= 0.8 \end{aligned}.$$

Therefore, we have

$$\begin{aligned} p_0 &= (p_0 + p_1 + p_2 + p_3 + p_4) - (p_1 + p_2 + p_3 + p_4) = 1 - 0.8 = 0.2 \\ p_2 &= (p_0 + p_1 + p_2 + p_3 + p_4) - (p_0 + p_1 + p_3 + p_4) = 1 - 0.8 = 0.2. \\ p_1 &= (p_0 + p_1 + p_2) - p_0 - p_2 = 0.75 - 0.2 - 0.2 = 0.35. \end{aligned}$$

148. Solution: D

Using the law of total probability, the requested probability is

$$\sum_{k=0}^{\infty} P(k + 0.75 < X \le k+1 \mid k < X \le k+1) P(k < X \le k+1).$$

The first probability is

$$P(k + 0.75 < X \le k+1 \mid k < X \le k+1) = \frac{P(k + 0.75 < X \le k+1)}{P(k < X \le k+1)}$$

$$= \frac{F(k+1) - F(k+0.75)}{F(k+1) - F(k)} = \frac{1 - e^{-(k+1)/2} - 1 + e^{-(k+0.75)/2}}{1 - e^{-(k+1)/2} - 1 + e^{-k/2}} = \frac{e^{-0.375} - e^{-0.5}}{1 - e^{-0.5}} = 0.205.$$

This probability factors out of the sum and the remaining probabilities sum to 1 so the requested probability is 0.205.

149. Solution: B

The requested probability can be determined as
$P(\text{3 of first 11 damaged})P(\text{12th is damaged} \mid \text{3 of first 11 damaged})$

$$= \frac{\binom{7}{3}\binom{13}{8}}{\binom{20}{11}} \frac{4}{9} = \frac{35(1,287)}{167,960} \frac{4}{9} = 0.119.$$

150. Solution: E

Let *M* be the size of a family that visits the park and let *N* be the number of members of that family that ride the roller coaster. We want $P(M = 6 \mid N = 5)$. By Bayes theorem

$$P(M = 6 \mid N = 5)$$
$$= \frac{P(N = 5 \mid M = 6)P(M = 6)}{\sum_{m=1}^{7} P(N = 5 \mid M = m)P(M = m)}$$
$$= \frac{\frac{1}{6}\frac{2}{28}}{0+0+0+0+\frac{1}{5}\frac{3}{28}+\frac{1}{6}\frac{2}{28}+\frac{1}{7}\frac{1}{28}} = \frac{\frac{1}{3}}{\frac{3}{5}+\frac{1}{3}+\frac{1}{7}} = \frac{35}{63+35+15} = \frac{35}{113} \approx 0.3097.$$

151. Solution: C

Let *S* represent the event that the selected borrower defaulted on at least one student loan.
Let *C* represent the event that the selected borrower defaulted on at least one car loan.

We need to find $P(C \mid S) = \dfrac{P(C \cap S)}{P(S)}$.

We are given $P(S) = 0.3, \quad P(S \mid C) = \dfrac{P(C \cap S)}{P(C)} = 0.4, \quad P(C \mid S^c) = \dfrac{P(C \cap S^c)}{P(S^c)} = 0.28$.

Then,
$P(C \cap S^c) = 0.28P(S^c) = 0.28(1-0.3) = 0.196$.

Because
$P(C) = P(C \cap S) + P(C \cap S^c]$ and $P(C) = P(C \cap S)/0.4$ we have
$P(C \cap S)/0.4 = P(C \cap S) + 0.196 \Rightarrow P(C \cap S) = 0.196/1.5 = 0.13067.$

Therefore,
$$P(C \mid S) = \frac{P(C \cap S)}{P(S)} = \frac{0.13067}{0.3} = 0.4356,$$

152. Solution: E

Without the deductible, the standard deviation is, from the uniform distribution, $b/\sqrt{12} = 0.28868b$. Let Y be the random variable representing the payout with the deductible.

$$E(Y) = \int_{0.1b}^{b} (y - 0.1b)\frac{1}{b}dy = \frac{y^2}{2b} - 0.1y\Big|_{0.1b}^{b} = 0.5b - 0.1b - 0.005b + 0.01b = 0.405b$$

$$E(Y^2) = \int_{0.1b}^{b} (y - 0.1b)^2 \frac{1}{b}dy = \frac{y^3}{3b} - 0.1y^2 + 0.01by\Big|_{0.1b}^{b}$$

$$= b^2/3 - 0.1b^2 + 0.01b^2 - 0.001b^2/3 + 0.001b^2 - 0.001b^2 = 0.243b^2$$

$$Var(Y) = 0.243b^2 - (0.405b)^2 = 0.078975b^2$$

$$SD(Y) = 0.28102b.$$

The ratio is 0.28102/0.28868 = 0.97347.

153. Solution: C

i) is false because G includes having one accident in year two.
ii) is false because there could be no accidents in year one.
iii) is true because it connects the descriptions of F and G (noting that "one or more" and "at least one" are identical events) with "and."
iv) is true because given one accident in year one (F), having a total of two or more in two years is the same as one or more in year two (G).
v) is false because it requires year two to have at least two accidents.

154. Solution: B

$$P[D] = P[H]P[D|H] + P[M]P[D|M] + P[L]P[D|L]$$

$$0.009 = P[H]P[D|H] + P[M]\left(\frac{1}{2}P[D|H]\right) + P[L]\left(\frac{1}{2}\frac{1}{3}P[D|H]\right)$$

$$0.009 = 0.20P[D|H] + 0.35\left(\frac{1}{2}P[D|H]\right) + 0.45\left(\frac{1}{6}P[D|H]\right) = 0.45P[D|H]$$

$$P[D|H] = 0.009/0.45 = 0.02$$

155. Solution: C

If the deductible is less than 60 the equation is,
$0.10(60-d) + 0.05(200-d) + 0.01(3000-d) = 30 \Rightarrow d = 100.$.
So this cannot be the answer. If the deductible is between 60 and 200, the equation is $0.05(200-d) + 0.01(3000-d) = 30 \Rightarrow d = 166.67$. This is consistent with the assumption and is the answer.

156. Solution: C

The probability that none of the damaged houses are insured is

$$\frac{1}{120} = \frac{\binom{10-k}{0}\binom{k}{3}}{\binom{10}{3}} = \frac{k(k-1)(k-2)}{720}.$$

$$k(k-1)(k-2) = 6$$

This cubic equation could be solved by expanding, subtracting 6, and refactoring. However, because *k* must be an integer, the three factors must be integers and thus must be 3(2)(1) for *k* = 3.

The probability that at most one of the damaged houses is insured equals

$$\frac{1}{120} + \frac{\binom{10-3}{1}\binom{3}{2}}{\binom{10}{3}} = \frac{1}{120} + \frac{7(3)}{120} = \frac{22}{120} = \frac{11}{60}.$$

157. Solution: B

This question is equivalent to "What is the probability that 9 different chips randomly drawn from a box containing 4 red chips and 8 blues chips will contain the 4 red chips?" The hypergeometric probability is

$$\frac{\binom{4}{4}\binom{8}{5}}{\binom{12}{9}} = \frac{1(56)}{220} = 0.2545.$$

158. Solution: D

Let N be the number of sick days for an employee in three months. The sum of independent Poisson variables is also Poisson and thus *N* is Poisson with a mean of 3.. Then,

$$\mathrm{P}[N \le 2] = e^{-3}\left(\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!}\right) = e^{-3}(1+3+4.5) = 0.423.$$

The answer is the complement, 1 – 0.423 = 0.577.

159. Solution: B

$$A = P(N>3) = 1 - \left[P(N=0)+P(N=1)+P(N=2)+P(N=3)\right]$$
$$= 1 - e^{-3}\left(1+\frac{3}{1}+\frac{9}{2}+\frac{27}{6}\right) = 1-13e^{-3} = 0.3528$$
$$B = P(N>1.5) = 1-\left[P(N=0)+P(N=1)\right]$$
$$= 1-e^{-1.5}\left(1+\frac{1.5}{1}\right) = 1-2.5e^{-1.5} = 0.4422$$

$B - A$ = 0.4422 – 0.3528 = 0.0894.

160. Solution: E

For Policy A, the relevant equation is

$$0.64 = P(L>1.44) = e^{-1.44/\mu}$$
$$\ln(0.64) = -0.44629 = -1.44/\mu$$
$$\mu = 3.2266.$$

For Policy B, the relevant equation is

$$0.512 = P(L>d) = e^{-d/3.2266}$$
$$\ln(0.512) = -0.6694 = -d/3.2266$$
$$d = 2.1599.$$

161. Solution: B

Because the density function must integrate to 1, $1 = \int_0^5 cx^a dx = c\frac{5^{a+1}}{a+1} \Rightarrow c = \frac{a+1}{5^{a+1}}$.

From the given probability,

$$0.4871 = \int_0^{3.75} cx^a dx = c\frac{3.75^{a+1}}{a+1} = \frac{a+1}{5^{a+1}}\frac{3.75^{a+1}}{a+1} = \left(\frac{3.75}{5}\right)^{a+1}$$
$$\ln(0.4871) = -0.71929 = (a+1)\ln(3.75/5) = -0.28768(a+1)$$
$$a = (-0.71929)/(-0.28768) - 1 = 1.5.$$

The probability of a claim exceeding 4 is,

$$\int_4^5 cx^a dx = c\frac{5^{a+1}-4^{a+1}}{a+1} = \frac{a+1}{5^{a+1}}\frac{5^{a+1}-4^{a+1}}{a+1} = 1-\left(\frac{4}{5}\right)^{1.5+1} = 0.42757.$$

162. Solution: A

Let N denote the number of warranty claims received. Then,
$0.6 = P(N=0) = e^{-c} \Rightarrow c = -\ln(0.6) = 0.5108.$

The expected yearly insurance payments are:

$$\begin{aligned}
&5000[P(N=2)+2P(N=3)+3P(N=4)+\cdots] \\
&=5000[P(N=1)+2P(N=2)+3P(N=3)+\cdots]-5000[P(N=1)+P(N=2)+P(N=3)+\cdots] \\
&=5000E(N)-5000[1-P(N=0)]=5000(0.5108)-5000(1-0.6)=554.
\end{aligned}$$

163. Solution: D

If L is the loss, the unreimbursed loss, X is

$$X = \begin{cases} L, & L \le 180 \\ 180, & L > 180. \end{cases}$$

The expected unreimbursed loss is

$$144 = E(X) = \int_0^{180} l[f(l)]dl + 180\,\Pr(L>180) = \int_0^{180} l\frac{1}{b}dl + 180\frac{b-180}{b}$$

$$= \frac{l^2}{2b}\bigg|_0^{180} + 180 - \frac{180^2}{b} = \frac{180^2}{2b} + 180 - \frac{180^2}{b}$$

$$144b = 180^2/2 + 180b - 180^2$$

$$16,200 = 36b$$

$$b = 450.$$

164. Solution: B

Let X be normal with mean 10 and variance 4. Let Z have the standard normal distribution. Let p = 12th percentile. Then

$$0.12 = P(X \le p) = P\left(\frac{X-10}{2} \le \frac{p-10}{2}\right) = P\left(Z \le \frac{p-10}{2}\right).$$

From the tables, $P(Z \le -1.175) = 0.12$. Therefore,

$$\frac{p-10}{2} = -1.175;\ p-10 = -2.35;\ p = 7.65.$$

165. Solution: D

From the normal table, the 14th percentile is associated with a z-score of -1.08. Since the means are equal and the standard deviation of company B's profit is $\sqrt{2.25} = 1.5$ times the standard deviation of company A's profit, a profit that is 1.08 standard deviations below the mean for company A would be1.08/1.5 = 0.72 standard deviations below the mean for company B. From the normal table, a z-score of −0.72 is associated with the 23.6th percentile.

166. Solution: C

The conditional variance is

$$Var(X \mid X \geq 10) = E(X^2 \mid X \geq 10) - E(X \mid X \geq 10)^2$$

$$= \frac{\int_{10}^{\infty} x^2 (0.2) e^{-0.2(x-5)} dx}{\int_{10}^{\infty} 0.2 e^{-0.2(x-5)} dx} - \left[\frac{\int_{10}^{\infty} x (0.2) e^{-0.2(x-5)} dx}{\int_{10}^{\infty} 0.2 e^{-0.2(x-5)} dx} \right]^2 .$$

Performing integration (using integration by parts) produces the answer of 25.

An alternative solution is to first determine the density function for the conditional distribution. It is

$$f(y) = \frac{0.2 e^{-0.2(y-5)}}{\int_{10}^{\infty} 0.2 e^{-0.2(x-5)} dx} = \frac{0.2 e^{-0.2(y-5)}}{-e^{-0.2(x-5)} \Big|_{10}^{\infty}} = \frac{0.2 e^{-0.2(y-5)}}{e^{-0.2(5)}} = 0.2 e^{-0.2(y-10)}, \; y > 10 .$$

Then note that $Y - 10$ has an exponential distraction with mean 5. Subtracting a constant does not change the variance, so the variance of Y is also 25.

167. Solution: C

Let X and Y represent the annual profits for companies A and B, respectively and m represent the common mean and s the standard deviation of Y. Let Z represent the standard normal random variable.

Then because X's standard deviation is one-half its mean,

$$P(X < 0) = P\left(\frac{X - m}{0.5m} < \frac{0 - m}{0.5m} \right) = P(Z < -2) = 0.0228.$$

Therefore company B's probability of a loss is 0.9(0.0228) = 0.02052. Then,

$0.02052 = P(Y < 0) = P\left(\frac{Y - m}{s} < \frac{0 - m}{s} \right) = P(Z < -m/s)$. From the tables, $-2.04 = -m/s$ and therefore $s = m/2.04$. The ratio of the standard deviations is $(m/2.04)/(0.5m) = 0.98$.

168. Solution: B

Y is a normal random variable with mean 1.04(100) + 5 = 109 and standard deviation 1.04(25) = 26. The average of 25 observations has mean 109 and standard deviation 26/5 = 5.2. The requested probability is

$$P(100 < \text{sample mean} < 110) = P\left(\frac{100-109}{5.2} = -1.73 < Z < \frac{110-109}{5.2} = 0.19\right)$$

$$= 0.5753 - (1 - 0.9582) = 0.5335.$$

169. Solution: E

The possible events are (0,0), (0,1), (0,2), (0,3), (1,1), (1,2), (1,3), (2,2), (2,3), and (3,3). The probabilities (without *c*) sum to 0 + 2 + 4 + 6 + 3 + 5 + 7 + 6 + 8 + 9 = 50. Therefor *c* = 1/50. The number of tornadoes with fewer than 50 million in losses is *Y* – *X*. The expected value is (1/50)[0(0) + 1(2) + 2(4) + 3(6) + 0(3) + 1(5) + 2(7) + 0(6) + 1(8) + 0(9)]=55/50 = 1.1.

170. Solution: D

Consider three cases, one for each result of the first interview.
Independent (prob 0.5): Expected absolute difference is (4/9)(0) +(5/9)(1) = 5/9
Republican (prob =0.3): Expected absolute difference is (2/9)(0) + (5/9)(1) + (2/9)(2) = 1
Democrat (prob = 0.2): Expected absolute difference is (3/9)(0) + (5/9)(1) + (1/9)(2) = 7/9.
The unconditional expectation is 0.5(5/9) + 0.3(1) + 0.2(7/9) = 6.6/9 = 11/15.

Alternatively, the six possible outcomes can be listed along with their probabilities and absolute differences.

171. Solution: C

Let *Z* = *XY*. Let *a*, *b*, and *c* be the probabilities that *Z* takes on the values 0, 1, and 2, respectively. We have *b* = *p*(1,1) and *c* = *p*(1,,2) and thus 3*b* = *c*. And because the probabilities sum to 1, *a* = 1 – *b* – *c* = 1 – 4*b*. *Then, E(Z) = b + 2c = 7b, E(Z*Z) = b* + 4*c* = 13*b*. Then,

$$Var(Z) = 13b - 49b^2$$

$$(d/db)Var(Z) = 13 - 98b = 0 \Rightarrow b = 13/98.$$

The probability that either *X* or *Y* is zero is the same as the probability that *Z* is 0 which is *a* = 1 – 4*b* = 46/98 = 23/49.

172. Solution: C

Let J and K be the random variables for the number of severe storms in each city.

$$P(J = j \mid K = 5) = \frac{P(K = 5 \mid J = j)P(J = j)}{P(K = 5)}$$

$$P(K = 5 \mid J = 3) = 1/6,\ P(J = 3) = \binom{5}{3} 0.6^3 0.4^2 = 0.3456$$

$$P(K = 5 \mid J = 4) = 1/3,\ P(J = 4) = \binom{5}{4} 0.6^4 0.4^1 = 0.2592$$

$$P(K = 5 \mid J = 5) = 1/2,\ P(J = 5) = \binom{5}{5} 0.6^5 0.4^0 = 0.07776$$

$$P(K = 5) = (1/6)(0.3456) + (1/3)(0.2592) + (1/2)(0.07776) = 0.18288$$

$$P(J = 3 \mid K = 5) = \frac{(1/6)(0.3456)}{0.18288} = 0.31496$$

$$P(J = 4 \mid K = 5) = \frac{(1/3)(0.2592)}{0.18288} = 0.47244$$

$$P(J = 5 \mid K = 5) = \frac{(1/2)(0.07776)}{0.18288} = 0.21260$$

$$E(J \mid K = 5) = 3(0.31496) + 4(0.47244) + 5(0.21260) = 3.89764.$$

173. Solution: B

Given $N + S = 2$, there are 3 possibilities $(N,S) = (2,0), (1,1), (0,2)$ with probabilities 0.12, 0.18, and 0.10 respectively.
The associated conditional probabilities are

$P(N = 0 \mid N + S = 2) = 0.10/0.40 = 0.25$,
$P(N = 1 \mid N + S = 2) = 0.18/0.40 = 0.45$,
$P(N = 2 \mid N + S = 2) = 0.12/0.40 = 0.30$.

The mean is 0.25(0) + 0.45(1) + 0.30(2) = 1.05.
The second moment is 0.25(0) + 0.45(1) + 0.30(4) = 1.65.
The variance is 1.65 – (1.05)(1.05) = 0.5475.

174. Solution: A

Because the territories are evenly distributed, the probabilities can be averaged. Thus the probability of a 100 claim is 0.80, of a 500 claim is 0.13, and of a 1000 claim is 0.07. The mean is 0.80(100) + 0.13(500) + 0.07(1000) = 215. The second moment is 0.80(10,000) + 0.13(250,000) + 0.07(1,000,000) = 110,500. The variance is 110,500 – (215)(215) = 64,275. The standard deviation is 253.53.

175. Solution: D

With each load of coal having mean 1.5 and standard deviation 0.25, twenty loads have a mean of 20(1.5) = 30 and a variance of 20(0.0625) = 1.25. The total amount removed is normal with mean 4(7.25) = 29 and a variance of 4(0.25) = 1. The difference is normal with mean 30 – 29 = 1 and standard deviation sqrt(1.25 + 1) = 1.5. If *D* is that difference,

$$P(D > 0) = P\left(Z > \frac{0-1}{1.5} = -0.67 \right) = 0.7486.$$

176. Solution: C

The probability needs to be calculated for each total number of claims.
0: 0.5(0.2) = 0.10
1: 0.5(0.3) + 0.3(0.2) = 0.21
2: 0.5(0.4) + 0.3(0.3) + 0.2(0.2) = 0.33
3: 0.5(0.1) + 0.3(0.4) + 0.2(0.3) + 0.0(0.2) = 0.23
At this point there is only 0.13 probability remaining, so the mode must be at 2.

177. Solution: B

Let *X* represent the number of policyholders who undergo radiation.
Let *Y* represent the number of policyholders who undergo chemotherapy.
X and *Y* are independent and binomially distributed with 15 trials each and with "success" probabilities 0.9 and 0.4, respectively.

The variances are 15(0.9)(0.1) = 1.35 and 15(0.4)(0.6) = 3.6.

The total paid is 2*X* + 3*Y* and so the variance is 4(1.35) + 9(3.6) = 37.8.

178. Solution: C

Let X and Y represent the number of selected patients with early stage and advanced stage cancer, respectively. We need to calculate $E(Y \mid X \geq 1)$.

From conditioning on whether or not $X \geq 1$, we have
$E(Y) = P[X = 0]E(Y \mid X = 0) + P[X \geq 1]E(Y \mid X \geq 1)$.

Observe that $P[X = 0] = (1-0.2)^6 = (0.8)^6$, $P[X \geq 1] = 1 - P[X = 0] = 1-(0.8)^6$, and
$E(Y) = 6(0.1) = 0.6$. Also, note that if none of the 6 selected patients have early stage cancer, then each of the 6 selected patients would independently have conditional probability
$\frac{0.1}{1-0.2} = \frac{1}{8}$ of having late stage cancer, so $E(Y \mid X = 0) = 6(1/8) = 0.75$.

Therefore,

$$E(Y \mid X \geq 1) = \frac{E(Y) - P[X = 0]E(Y \mid X = 0)}{P[X \geq 1]} = \frac{0.6 - (0.8)^6 (0.75)}{1-(0.8)^6} = 0.547.$$

179. Solution: A

Because there must be two smaller values and one larger value than X, X cannot be 1, 2, or 12. If X is 3, there is one choice for the two smallest of the four integers and nine choices for the largest integer. If X is 4, there are three choices for the two smallest of the four integers and eight choices for the largest integer. In general, if $X = x$, there are (x – 1) choose (2) choices for the two smallest integers and 12 – x choices for the largest integer. The total number of ways of choosing 4 integers from 12 integers is 12 choose 4 which is 12!/(4!8!) = 495. So the probability that $X = x$ is:

$$\frac{\binom{x-1}{2}(12-x)}{495} = \frac{(x-1)(x-2)(12-x)}{990}.$$

180. Solution: A

We have

$$0.95 = P(X < k \mid X > 10,000) = \frac{P(X < k) - P(X \le 10,000)}{1 - P(\mathrm{X} \le 10,000)}$$

$$0.95[1 - P(X \le 10,000)] = 0.9582 - P(X \le 10,000)$$

$$P(X \le 10,000) = \frac{0.9582 - 0.95}{1 - 0.95} = 0.164$$

$$0.164 = \Phi\left(\frac{10,000 - 12,000}{c}\right).$$

The z-value that corresponds to 0.164 is between –0.98 and –0.97. Interpolating leads to z = –0.978. Then,

$$0.164 = \Phi\left(\frac{10,000 - 12,000}{c}\right) \Rightarrow -0.978 = \frac{-2,000}{c} \Rightarrow c = 2045.$$

181. Solution: B

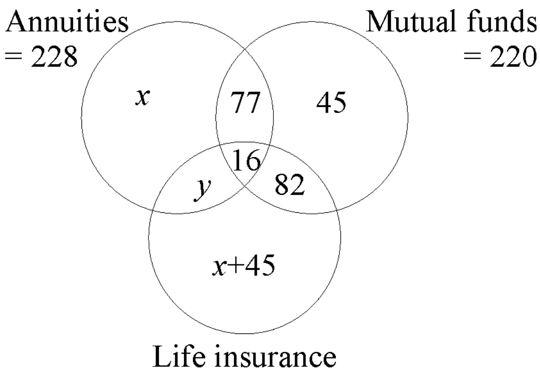
Before applying the deductible, the median is 500 and the 20th percentile is 200. After applying the deductible, the median payment is 500 – 250 = 250 and the 20th percentile is max(0, 200 – 250) = 0. The difference is 250.

182. Solution: B

32 own L/A/H
55 own L/H so 55 – 32 = 23 own L/H/notA
96 own A/H so 96 – 32 = 64 own A/H/notL
207 own H so 207 – 32 – 23 – 64 = 88 own H only
L only = X, A only = X + 76
88 + X + (X + 76) = 270 so X = 53 so L only = 53, A only = 129
129 + 64 + 32 + L/A/notH = 243 so L/A/notH = 18
Total clients = 53 + 129 + 88 + 18 + 64 + 23 + 32 = 407

Alternatively, a Venn diagram could be used to guide the calculations.

183. Solution: D



Then 290 = 45 + 45 + *x* + *x*, thus *x* = 100.
Also 228 = 100 + *y* + 16 + 77, thus *y* = 35.
Total clients = 145 + 82 + 16 + 35 + 100 + 77 + 45 = 500.

184. Solution: D

Let *A*, *B*, and *C* be the sets of policies in the portfolio on three-bedroom homes, one-story homes, and two-bath homes, respectively. We are asked to calculate $1000 - n(A \cup B \cup C)$, where $n(D)$ denotes the number of elements of the set *D*. Then,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$
$$= 130 + 280 + 150 - 40 - 30 - 50 + 10 = 450.$$

The answer is 1000 – 450 = 550.

185. Solution: B

We seek the number of ways to select 4 individuals from 7 and choose one selected member as subcommittee chair. (The existence of a subcommittee secretary is irrelevant.) There are (7 choose 4) = 7(6)(5)(3)/4! = 35 ways to form a collection of 4 individuals from 7. For each of them, there are 4 ways to assign a chair. The product, 140, is the number of different ways to form a subcommittee of 4 individuals and assign a chair and thus is the maximum number without repetition.

186. Solution: D

$$P(2 \text{ red and } 2 \text{ blue transferred} \mid \text{blue drawn}) = \frac{P(2 \text{ red and } 2 \text{ blue transferred and blue drawn})}{P(\text{blue drawn})}$$

$$P(2 \text{ red and } 2 \text{ blue transferred and blue drawn}) = \frac{\binom{8}{2}\binom{6}{2}}{\binom{14}{4}} \times \frac{2}{4} = \frac{28(15)}{1001} \times \frac{2}{4} = \frac{840}{4004}$$

$$P(\text{blue drawn}) = \frac{\binom{8}{0}\binom{6}{4}}{\binom{14}{4}} \times \frac{4}{4} + \frac{\binom{8}{1}\binom{6}{3}}{\binom{14}{4}} \times \frac{3}{4} + \frac{\binom{8}{2}\binom{6}{2}}{\binom{14}{4}} \times \frac{2}{4} + \frac{\binom{8}{3}\binom{6}{1}}{\binom{14}{4}} \times \frac{1}{4} = \frac{60 + 480 + 840 + 336}{4004} = \frac{1716}{4004}$$

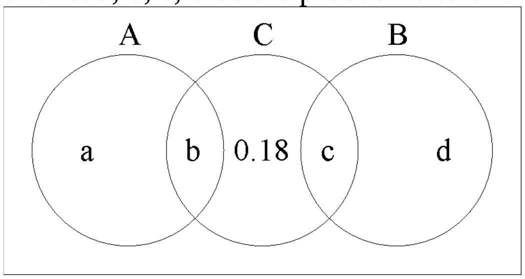
$$P(2 \text{ red and } 2 \text{ blue transferred} \mid \text{blue drawn}) = \frac{840}{1716} = 0.49.$$

187. Solution: D

If a policy is of Type A, the probability that the two claims are equal is (0.4)(0.4) + (0.3)(0.3) + (0.2)(0.2) + (0.1)(0.1) = 0.16 + 0.09 + 0.04 + 0.01 = 0.30.
If a policy is of Type B, the probability that the two claims are equal is 4(0.25)(0.25) = 0.25.
Therefore, the probability that a randomly selected policy has equal claims is 0.70(0.30) + 0.30(0.25) = 0.285.
If four policies are selected, the desired probability is the probability that a binomial random variable with n = 4 and p = 0.285 is 1. This is 4(0.285)(1 – 0.285)^3 = 0.417.

188. Solution: A

Let A = event that person wants life policy P
B = event that person wants life policy Q
C = event that person wants the health policy
and let a, b, c, d be the probabilities of the regions as shown.



i) is reflected by no intersection of A and B
iv) is reflected by the 0.18 in the diagram
ii) implies a + b = 2(c + d)
iii) implies b + c + 0.18 = 0.45 or b + c = 0.27
v) implies P([A or B] and C) = P(A or B)P(C) or b + c = (a + b + c + d)(0.45)

So 0.27 = (a + d + 0.27)(0.45) and then a + d = 0.33.
The desired probability is a + 0.18 + d = 0.33 + 0.18 = 0.51.

189. Solution: B

The state will receive 800,000($1) = $800,000 in revenue, and will lose money if there are 2 or more winning tickets sold. The player's entry can be viewed as fixed. The probability the lottery randomly selects those same six numbers is from a hypergeometric distribution and is

$$\frac{\binom{6}{6}\binom{24}{0}}{\binom{30}{6}} = \frac{1(1)}{\frac{30!}{6!(24!)}} = \frac{6(5)(4)(3)(2)(1)}{30(29)(28)(27)(26)(25)} = \frac{1}{593,775}.$$

The number of winners has a binomial distribution with n = 800,000 and p = 1/593,775. The desired probability is

$$\Pr(2 \text{ or more winners}) = 1 - \Pr(0 \text{ winners}) - \Pr(1 \text{ winner})$$

$$= 1 - \binom{800,000}{0}\left(\frac{1}{593,775}\right)^{0}\left(\frac{593,774}{593,775}\right)^{800,000} - \binom{800,000}{1}\left(\frac{1}{593,775}\right)^{1}\left(\frac{593,774}{593,775}\right)^{799,999}$$

$$= 1 - 0.2599 - 0.3502 = 0.39.$$

190. Solution: E

The number that have errors is a binomial random variable with p = 0.03 and n = 100. Let X be the number that have errors. Then,

$$\Pr(\text{number that are error-free} \leq 95) = \Pr(X \geq 5) = 1 - P(0) - P(1) - P(2) - P(3) - P(4)$$

$$= 1 - \binom{100}{0}(0.03)^0(0.97)^{100} - \binom{100}{1}(0.03)^1(0.97)^{99} - \binom{100}{2}(0.03)^2(0.97)^{98} - \binom{100}{3}(0.03)^3(0.97)^{97}$$

$$-\binom{100}{4}(0.03)^4(0.97)^{96} = 0.1821.$$

Or, the Poisson approximation can be used. Then, $\lambda = 3$ and

$$P(X \geq 5) = 1 - \frac{e^{-3}3^0}{0!} - \frac{e^{-3}3^1}{1!} - \frac{e^{-3}3^2}{2!} - \frac{e^{-3}3^3}{3!} - \frac{e^{-3}3^4}{4!} = 1 - e^{-3}\left(1 + 3 + \frac{9}{2} + \frac{27}{6} + \frac{81}{24}\right) = 0.1847.$$

191. Solution: D

$$P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] - P[A \cap C] - P[B \cap C] + P[A \cap B \cap C]$$
$$= 0.2 + 0.1 + 0.3 - 0.2(0.1) - 0 - 0.1(0.3) + 0 = 0.55.$$

192. Solution: A

The probability a union of three events equals the sum of their probabilities if and only if they are mutually exclusive, that is, no two of them can both occur.

Events A and B cannot both occur since no thefts in the first three years would imply no thefts in the second year, thus precluding the possibility of at least 1 theft in the second year.
Events A and E cannot both occur since no thefts in the first three years would imply no thefts in the third year, thus precluding the possibility of at least 1 theft in the third year.
Events B and E cannot both occur since it is impossible to experience both no thefts and at least 1 theft in the second year.
Thus, events A, B, and E satisfy the desired condition.

193. Solution: D

Consider the two mutually exclusive events "first envelope correct" and "first envelope incorrect." The probability of the first event is 1/4 and meets the requirement of at least one correct. For the 3/4 of the time the first envelope is incorrect, there are now 3 more envelopes to fill. Of the six permutations, three will place one letter correctly. The total probability is 1/4 + 3/4(3/6) = 5/8.

194. Solution: C

The deductible is exceeded for 4, 5 or 6 office visits. Therefore, the requested probability is 0.02/(0.04 + 0.02 + 0.01) = 0.286.

195. Solution: C

Let *A* be the event that part A is working after one year and *B* be the event that part B is working after one year. Then,

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{P(A) + P(B) - P(A \text{ or } B)}{P(A)} = \frac{0.8 + 0.6 - 0.9}{0.8} = 5/8.$$

196. Solution: D

$$\frac{\binom{2}{1}\binom{4}{2}\binom{7}{3}}{\binom{13}{6}} = 0.245.$$

197. Solution: E

The maximum number of draws needed is 5. This can only happen if the first four draws produce four different colors. The first draw can be any sock. The second draw must be one of the 6 (of 7 remaining) that are different. The third draw must be one of the 4 (of 6) that are different from the first two. The fourth draw must be one of the 2 (of 5) that are different. The probability all of this happens is 1(6/7)(4/6)(2/5) = 0.2286.

198. Solution: E

Define the events as follows:
A = applies for a mortgage
S = initially spoke to an attendant
R = call returned the same day
N = call returned the next day
Then, using Bayes' Theorem,

$$P(S \mid A) = \frac{P(A \mid S)P(S)}{P(A \mid S)P(S) + P(A \mid R)P(R) + P(A \mid N)P(N)}$$

$$= \frac{0.8(0.6)}{0.8(0.6) + 0.6(0.4)(0.75) + 0.4(0.4)(0.25)} = 0.69.$$

199. Solution: A

Define the events as follows:
C = files a claim
N = no lifting
M = moderate lifting
H = heavy lifting
Then, using Bayes' Theorem,

$$P(M \text{ or } H \mid C) = 1 - P(N \mid C) = 1 - \frac{P(C \mid N)P(N)}{P(C \mid N)P(N) + P(C \mid \text{M})P(\text{M}) + P(C \mid \text{H})P(\text{H})}$$

$$= 1 - \frac{0.05(0.4)}{0.05(0.4) + 0.08(0.5) + 0.2(0.1)} = 1 - 0.25 = 0.75.$$

200. Solution: E

From the Law of Total Probability, the required probability is

$$\sum_{k=0}^{\infty} P(0 \text{ accidents with an uninsured driver} \mid k \text{ accidents}) P(k \text{ accidents})$$

$$= \sum_{k=0}^{\infty} (0.75)^k \frac{e^{-5}5^k}{k!} = \frac{e^{-5}}{e^{-3.75}} \sum_{k=0}^{\infty} \frac{e^{-3.75}(3.75)^k}{k!} = e^{-1.25} = 0.287.$$

201. Solution: B

From the binomial distribution formula, the probability P that a given patient tests positive for at least 2 of these 3 risk factors is $P = \binom{3}{2} p^2 (1-p)^{3-2} + \binom{3}{3} p^3 (1-p)^{3-3} = 3p^2(1-p) + p^3$.

Using the geometric distribution formula with probability of success $P = 3p^2(1-p) + p^3$, the probability that exactly n patients are tested is

$$(1-P)^{n-1} P = \left[1 - 3p^2(1-p) - p^3\right]^{n-1} \left[3p^2(1-p) + p^3\right].$$

202. Solution: B

For there to be more than three calls before one completed survey all that is required is the first three calls not result in a completed survey. This probability is $(1-0.25)^3 = 0.42$.

203. Solution: B

For a given x, there are x – 1 choices for the smaller of the four integers and 12 – x choices for the two larger integers. Thus, there are $(x-1)\binom{12-x}{2} = \frac{(x-1)(12-x)(11-x)}{2}$ triples that satisfy the event. The total number of possible draws is $\binom{12}{4} = 495$ and the probability is

$$\frac{(x-1)(12-x)(11-x)}{2}\frac{1}{495} = \frac{x-1)(12-x)(11-x)}{990}.$$

204. Solution: D

The cumulative distribution function for the exponential distribution is
$F(x) = 1 - e^{-\lambda x} = 1 - e^{-x/\mu} = 1 - e^{-x/100}$, x > 0.

From the given probability data,

$$\begin{aligned}
&F(50) - F(40) = F(r) - F(60) \\
&1 - e^{-50/100} - \left(1 - e^{-40/100}\right) = 1 - e^{-r/100} - \left(1 - e^{-60/100}\right) \\
&e^{-40/100} - e^{-50/100} = e^{-60/100} - e^{-r/100} \\
&e^{-r/100} = e^{-60/100} - e^{-40/100} + e^{-50/100} = 0.4850 \\
&-r/100 = \ln(0.4850) = -0.7236 \\
&r = 72.36.
\end{aligned}$$

205. Solution: B

The desired event is equivalent to the time of the next accident being between 365 and 730 days from now. The probability is
$F(730) - F(365) = 1 - e^{-730/200} - \left(1 - e^{-365/200}\right) = e^{-1.825} - e^{-3.65} = 0.1352.$
Note that the problem provides no information about the distribution of the time to subsequent accidents, but that information is not needed. With nothing given, anything can be assumed. If the time to subsequent accidents has the same exponential distribution and the times are independent, then the number of accidents in each 365 day period is Poisson with mean 1.825. Then the required probability is $e^{-1.825}(1 - e^{-1.825}) = 0.1352.$

206. Solution: C

$$\Pr(Z \le 0.72) = 0.7642 = \Pr(X \le 2000) = \Pr[Z \le (2000-\mu)/\sigma]$$
$$0.72 = (2000-\mu)/\sigma$$
$$\Pr(Z \le 1.32) = 0.9066 = \Pr(X \le 3000) = \Pr[Z \le (3000-\mu)/\sigma]$$
$$1.32 = (3000-\mu)/\sigma$$
$$1.32/0.72 = (3000-\mu)/(2000-\mu)$$
$$1.8333(2000-\mu) = 3000-\mu$$
$$\mu = [1.8333(2000)-3000]/(1.8333-1) = 800$$
$$\sigma = (3000-\mu)/1.32 = 1666.67$$
$$\Pr(X \le 1000) = \Pr[Z \le (1000-800)/1666.67] = \Pr(Z \le 0.12) = 0.5478.$$

207. Solution: B

$$P(X > V) = 1 - P(X \le V) = 1 - F(V) = 1 - \left(1 - \frac{1}{10}e^{-\frac{V-V}{V}}\right) = 0.10.$$

208. Solution: E

The given mean of 5 years corresponds to the pdf $f(t) = 0.2e^{-0.2t}$ and the cumulative distribution function $F(t) = 1 - e^{-0.2t}$. The conditional pdf is

$$g(t) = \frac{f(t)}{F(10)} = \frac{0.2e^{-0.2t}}{1-e^{-2}}, \ 0 < t < 10.$$

The conditional mean is (using integration by parts)

$$E(T \mid T < 10) = \int_0^{10} tg(t)dt = \int_0^{10} t\frac{0.2e^{-0.2t}}{1-e^{-2}}dt = 0.2313\int_0^{10} te^{-0.2t}dt$$
$$= 0.2313\left[t(-5e^{-0.2t})\Big|_0^{10} - \int_0^{10} -5e^{-0.2t})dt\right] = 0.2323\left[-6.7668 + 0 - 25e^{-0.2t}\Big|_0^{10}\right]$$
$$= 0.2313[-6.7668 - 3.3834 + 25] = 3.435.$$

209. Solution: D

$$F(x) = \int_0^x 2e^{-2y}dy = -e^{-2y}\Big|_0^x = 1 - e^{-2x}$$
$$P[X \le 0.5 \mid X \le 1.0] = \frac{P[X \le 0.5]}{P[X \le 1.0]} = \frac{F(0.5)}{F(1.0)} = \frac{1-e^{-1}}{1-e^{-2}} = 0.731.$$

210. Solution: B

If E and F are independent, so are E and the complement of F. Then,
$P(\text{exactly one}) = P(E \cap F^c) + P(E^c \cap F) = 0.84(0.35) + 0.16(0.65) = 0.398.$

211. Solution: E

Let *M* and *N* be the random variables for the number of claims in the first and second month. Then

$$P[M+N>3 \mid M<2] = 1 - P[M+N \le 3 \mid M<2] = 1 - \frac{P[M+N \le 3, M<2]}{P[M<2]}$$

$$= 1 - \frac{\begin{array}{c} P[M=0,N=0]+P[M=1,N=0]+P[M=0,N=1]+P[M=1,N=1] \\ +P[M=0,N=2]+P[M=1,N=2]+P[M=0,N=3] \end{array}}{P[M=0]+P[M=1]}$$

$$= 1 - \frac{(2/3)(2/3)+(2/9)(2/3)+(2/3)(2/9)+(2/9)(2/9)+(2/3)(2/27)+(2/9)(2/27)+(2/3)(2/81)}{2/3+2/9}$$

$$= 1 - \frac{0.87243}{0.88889} = 0.0185.$$

212. Solution: C

Let *X* = number of patients tested, which is geometrically distributed with constant "success" probability, say *p*.

$$P[X \ge n] = P[\text{first } n-1 \text{ patients do not have apnea}] = (1-p)^{n-1}.$$

Therefore,

$$r = P[X \ge 4] = (1-p)^3$$

$$P[X \ge 12 \mid X \ge 4] = \frac{P[X \ge 12]}{P[X \ge 4]} = \frac{(1-p)^{11}}{(1-p)^3} = (1-p)^8 = \left[(1-p)^3\right]^{\frac{8}{3}} = r^{\frac{8}{3}}$$

213. Solution: D

The number of defects has a binomial distribution with *n* = 100 and *p* = 0.02.

$$P[X=2 \mid X \le 2] = \frac{P[X=2]}{P[X \le 2]} = \frac{\binom{100}{2}(0.02)^2(0.98)^{98}}{\binom{100}{0}(0.02)^0(0.98)^{100} + \binom{100}{1}(0.02)^1(0.98)^{99} + \binom{100}{2}(0.02)^2(0.98)^{98}}$$

$$= \frac{0.27341}{0.13262+0.27065+0.27341} = 0.404.$$

214. Solution: A

The town experiences one tornado every 0.8 years on average, which is the mean of the exponential distribution. The median is found from

$$0.5 = P[X \le m] = 1 - e^{-m/0.8}$$
$$\ln(0.5) = -m / 0.8$$
$$m = -0.8\ln(0.5) = 0.55.$$

215. Solution: A

Let X = the amount of a loss. Ignoring the deductible, the median loss is the solution to

$0.5 = P[X > m] = \int_m^\infty 0.25e^{-0.25x}dx = 0 - (-e^{-0.25m}) = e^{-0.25m}$ which is $m = -4(\ln 0.5) = 2.77$.

Because 2.77 > 1, the loss exceeds 2.77 if and only if the claim payment exceeds 2.77 – 1 = 1.77, which is therefore the median claim payment.

216. Solution: B

The payment random variable is 1000(X – 2) if positive, where X has a Poisson distribution with mean 1. The expected value is

$$1000\sum_{x=3}^{\infty}(x-2)\frac{e^{-1}}{x!} = 1000\left(\sum_{x=0}^{\infty}(x-2)\frac{e^{-1}}{x!} - \sum_{x=0}^{2}(x-2)\frac{e^{-1}}{x!}\right)$$
$$= 1000\left(1 - 2 - [-2e^{-1} - e^{-1}]\right) = 1000(-1 + 3e^{-1}) = 104.$$

Note the first sum splits into the expected value of X, which is 1, and 2 times the sum of the probabilities (also 1).

217. Solution: C

Let X be the number of employees who die. The expected cost to the company is

$$100P[Y = 1] + 200P[Y = 2] + 300P[Y = 3] + 400P[Y > 3]$$
$$= 100(2)e^{-2} + 200(2)e^{-2} + 300(4/3)e^{-2} + 400[1 - (1 + 2 + 2 + 4/3)e^{-2}]$$
$$= 400 - 1533.33e^{-2} = 192.$$

218. Solution: B

Let X be the number of burglaries. Then,

$$E(X \mid X \geq 2) = \frac{\sum_{x=2}^{\infty} xp(x)}{1-p(0)-p(1)} = \frac{\sum_{x=0}^{\infty} xp(x)-(0)p(0)-(1)p(1)}{1-p(0)-p(1)}$$

$$= \frac{1-p(1)}{1-p(0)-p(1)} = \frac{1-e^{-1}}{1-e^{-1}-e^{-1}} = 2.39.$$

219. Solution: A

The expected unreimbursed loss is

$$\int_0^d x\frac{1}{450}dx + \int_d^{450} d\,\frac{1}{450}dx = \frac{d^2}{900} + d\,\frac{450-d}{450} = \frac{1}{900}(900d - d^2) = 56$$

$$d^2 - 900d + 50{,}400 = 0$$

$$d = \frac{900 \pm \sqrt{900^2 - 201{,}600}}{2} = \frac{900-780}{2} = 60.$$

220. Solution: D

Let X represent the loss due to the accident.
From the given information, the probability that X is in[0, b] is 0.75, which is larger than 0.5. So the median, 672, must lie in the interval [0, b].

Note that in a uniform distribution over an interval I, the probability of landing in an interval J is the length of the intersection of J and I, divided by the length of I.
Therefore, we have $0.5 = \mathrm{P}[X \leq 672] = 0.75\left(\frac{672-0}{b-0}\right)$ which gives $b = 672\left(\frac{0.75}{0.5}\right) = 1008$.

From the law of total probability applied to means, the mean loss due to the accident is

$$\mathrm{E}(X) = \mathrm{P}[\text{minor acc.}]\mathrm{E}(X \mid \text{minor acc.}) + \mathrm{P}[\text{major acc.}]\mathrm{E}(X \mid \text{major acc.})$$

$$= 0.75\mathrm{E}(X \mid X \text{ is uniform on } [0,b]) + 0.25\mathrm{E}(X \mid X \text{ is uniform on } [b,3b]).$$

$$= 0.75\left(\frac{0+1008}{2}\right) + 0.25\left(\frac{1008+3(1008)}{2}\right) = 882.$$

221. Solution: C

The claim amount distribution is a mixture distribution with 20% point mass at 0. To obtain the median, the remaining 30% probability is from the case where there is a non-zero payment. This corresponds to the 30/(1 – 0.2) = 37.5 percentile of the unconditional claim amount distribution. The 37.5 percentile of the standard normal distribution is at z = –0.3187 and thus the median is 1000 – 0.3187(400) = 873.

222. Solution: B

First, the *z*-score associated with the deductible is:

$$z = \frac{15,000 - 20,000}{4,500} = -1.11$$

Next, using the normal table, we find that the probability that a loss exceeds the deductible is:

$$P(Z > -1.11) = 0.8665.$$

The 95th percentile of losses that exceed the deductible is the 1 – 0.05(0.8665) = 0.9567 = 95.67th percentile of all losses.

The 95.67th percentile of all losses is between 1.71 and 1.72 standard deviations above the mean. To the nearest hundred, both of these correspond to a loss amount of 27,700.

223. Solution: C

The *z*-score corresponding to the 98th percentile is 2.054. The answer is 20 + 2.054(2) = 24.108.

224. Solution: C

Let *T* be the time of registration. Due to symmetry of the density function about 6.5. The constant of proportionality, *c*, can be solved from

$0.5 = \int_0^{6.5} c \tfrac{1}{t+1} dt = c \ln(t+1)\Big|_0^{6.5} = c \ln(7.5)$, which gives *c* = 0.5/ln(7.5).

Again using the symmetry, if 60th percentile of *T* is at *k*, then $P[T \le 13 - k] = 0.4.$ Thus,

$$0.4 = P[T \le 13 - k] = \int_0^{13-k} \frac{0.5}{\ln(7.5)} \frac{1}{t+1} dt = \frac{0.5}{\ln(7.5)} \ln(14 - k)$$

$$\ln(14 - k) = 0.8 \ln(7.5) = 1.6119$$

$$14 - k = e^{1.6119} = 5.0124$$

$$k = 8.99.$$

225. Solution: E

The distribution function of *L* is $F(x) = 1 - e^{-\lambda x}$ and its variance is $1/\lambda^2$. We are given *F*(2) = 1.9*F*(1) and therefore,

$$1 - e^{-2\lambda} = 1.9(1 - e^{-\lambda})$$

$$(1 - e^{-\lambda})(1 + e^{-\lambda}) = 1.9(1 - e^{-\lambda})$$

$$e^{-\lambda} = 0.9$$

$$\lambda = -\ln(0.9) = 0.10536$$

$$Var(L) = 1/0.10536^2 = 90.1.$$

226. Solution: D

We have $Y = 0$ when $X < d$ and $Y = X - d$ otherwise. Then, noting that the second moment of an exponential random variable is twice the square of the mean,

$$E(Y) = \int_o^d 0(0.1e^{-0.1x})dx + \int_d^\infty (x-d)(0.1e^{-0.1x})dx = 0 + \int_0^\infty x(0.1e^{-0.1(x+d)})dx = e^{-0.1d}(10)$$

$$E(Y^2) = \int_o^d 0^2(0.1e^{-0.1x})dx + \int_d^\infty (x-d)^2(0.1e^{-0.1x})dx = 0 + \int_0^\infty x^2(0.1e^{-0.1(x+d)})dx = e^{-0.1d}(200)$$

$$\text{Var(Y)} = \text{e}^{-0.1d}(200) - [e^{-0.1d}(10)]^2 = 100[2e^{-0.1d} - e^{-0.2d}],$$

227. Solution: C

For the Poisson distribution the variance is equal to the mean and hence the second moment is the mean plus the square of the mean. Then,

$$E[X] = 0.1(1) + 0.5(2) + 0.4(10) = 5.1$$

$$E[X^2] = 0.1(1+1^2) + 0.5(2+2^2) + 0.4(10+10^2) = 47.2$$

$$Var(X) = 47.2 - 5.1^2 = 21.19.$$

228. Solution: C

Let X be the number of tornadoes and Y be the conditional distribution of X given that X is at least one. There are (at least) two ways to solve this problem. The first way is to begin with the probability function for Y and observe that starting the sums at zero adds nothing because that term is zero. Then note that the sums are the first and second moments of a regular Poisson distribution.

$$p(y) = P[Y=y] = P[X=y \mid X>0] = \frac{P[X=y]}{P[X>0]} = \frac{3^y e^{-3}/y!}{1-e^{-3}}, \quad y = 1, 2, \ldots$$

$$E(Y) = \frac{1}{1-e^{-3}}\sum_{y=1}^\infty y\frac{3^y e^{-3}}{y!} = \frac{1}{1-e^{-3}}\sum_{y=0}^\infty y\frac{3^y e^{-3}}{y!} = \frac{3}{1-e^{-3}}$$

$$E(Y^2) = \frac{1}{1-e^{-3}}\sum_{y=1}^\infty y^2\frac{3^y e^{-3}}{y!} = \frac{1}{1-e^{-3}}\sum_{y=0}^\infty y^2\frac{3^y e^{-3}}{y!} = \frac{3+3^2}{1-e^{-3}}$$

$$Var(Y) = \frac{12}{1-e^{-3}} - \left(\frac{3}{1-e^{-3}}\right)^2 = 2.6609.$$

The second way is to use formulas about conditional expectation based on the law of total probability.

$E(X) = E(X \mid X = 0)P[X = 0] + E(X \mid X > 0)P[X > 0]$

$3 = 0(e^{-3}) + E(X \mid X > 0)(1 - e^{-3})$

$E(X \mid X > 0) = \dfrac{3}{1 - e^{-3}} = 3.1572$

$E(X^2) = E(X^2 \mid X = 0)P[X = 0] + E(X^2 \mid X > 0)P[X > 0]$

$3 + 3^2 = 0(e^{-3}) + E(X^2 \mid X > 0)(1 - e^{-3})$

$E(X^2 \mid X > 0) = \dfrac{12}{1 - e^{-3}} = 12.6287$

$Var(X) = 12.6287 - 3.1572^2 = 2.6608.$

229. Solution: C

Let X represent individual expense. Then,

$Y = \begin{cases} 0, & 200 \le X \le 400 \\ X - 400, & 400 < X \le 900 \\ 500, & 900 < X \le 1200 \end{cases}$ and the density function of X is $f(x) = 0.001, \quad 200 \le x \le 1200.$

$$E(Y) = \int_{200}^{400} 0(0.001)dx + \int_{400}^{900} (\mathrm{x} - 400)(0.001)dx + \int_{900}^{1200} 500(0.001)dx$$

$$= 0 + 0.001\frac{(x-400)^2}{2}\bigg|_{400}^{900} + 500(0.001)(1200 - 900) = 0 + 125 + 150 = 275$$

$$E(Y^2) = \int_{200}^{400} 0^2(0.001)dx + \int_{400}^{900} (\mathrm{x} - 400)^2(0.001)dx + \int_{900}^{1200} 500^2(0.001)dx$$

$$= 0 + 0.001\frac{(x-400)^3}{3}\bigg|_{400}^{900} + 500^2(0.001)(1200 - 900) = 0 + 41,666.67 + 75,000 = 116,666.67$$

$Var(Y) = 116,666.67 - 275^2 = 41,041.67.$

230. Solution: D

Let X represent the loss.
The variance for a uniform distribution is the square of the interval length, divided by 12. Thus,
$\text{Var }(X) = \frac{b^2}{12}$.
Let C represent the claim payment from the loss. Then $C = 0$ for $X < b/2$ and $C = X - b/2$, otherwise. Then,

$$E(C) = \int_0^{b/2} 0(1/b)dx + \int_{b/2}^{b} (x-b/2)(1/b)dx = 0 + (x-b/2)^2/(2b)\Big|_{b/2}^{b} = (b/2)^2/(2b) = b/8$$

$$E(C^2) = \int_0^{b/2} 0^2(1/b)dx + \int_{b/2}^{b} (x-b/2)^2(1/b)dx = 0 + (x-b/2)^3/(3b)\Big|_{b/2}^{b} = (b/2)^3/(3b) = b^2/24$$

$Var(C) = b^2/24 - (b/8)^2 = 5b^2/192.$
The ratio is $[5b^2/192]/[b^2/12] = 60/192 = 5/16.$

231. Solution: A
Let X be the profit random variable. Then, $0.05 = P(X < 0) = P(Z < -\mu/\sigma)$ and from the table, $-\mu/\sigma = -1.645.$ From the problem, $\sigma^2 = \mu^3$. Therefore, $-1.645 = -\mu/\mu^{3/2} = -\mu^{-1/2}$ and $\mu = 1/1.645^2 = 0.37.$ in billions, or 370 million.

232. Solution: A

$E[(X-1)^2] = E[X^2] - 2E[X] + 1 = 47$ so $E[X] = (61+1-47)/2 = 7.5$. The standard deviation is $\sqrt{E[X^2] - E[X]^2} = \sqrt{61 - 7.5^2} = 2.18.$

233. Solution: D

Let *D* be the number of diamonds selected and *S* be the number of spades. First obtain the hypergeometric probability *S* = 0:

$$P(S=0)=\frac{\binom{3}{0}\binom{7}{2}}{\binom{10}{2}}=\frac{1(21)}{45}=\frac{7}{15}.$$

The required probability distribution is:

$$P(D=0\,|\,S=0)=\frac{P(D=0,S=0)}{P(S=0)}=\frac{1}{7/15}\frac{\binom{2}{0}\binom{3}{0}\binom{5}{2}}{\binom{10}{2}}=\frac{15}{7}\frac{1(1)(10)}{45}=\frac{10}{21}$$

$$P(D=1\,|\,S=0)=\frac{P(D=1,S=0)}{P(S=0)}=\frac{1}{7/15}\frac{\binom{2}{1}\binom{3}{0}\binom{5}{1}}{\binom{10}{2}}=\frac{15}{7}\frac{2(1)(5)}{45}=\frac{10}{21}$$

$$P(D=2\,|\,S=0)=\frac{P(D=2,S=0)}{P(S=0)}=\frac{1}{7/15}\frac{\binom{2}{2}\binom{3}{0}\binom{5}{0}}{\binom{10}{2}}=\frac{15}{7}\frac{1(1)(1)}{45}=\frac{1}{21}.$$

Then,

$$E(D\,|\,S=0)=0(10/21)+1(10/21)+2(1/21)=12/21=4/7$$

$$E(D^2\,|\,S=0)=0^2(10/21)+1^2(10/21)+2^2(1/21)=14/21=2/3$$

$$Var(D\,|\,S=0)=2/3-(4/7)^2=50/147=0.34.$$

234. – 236. DELETED

237. Solution: A

To be delayed over three minutes, either the car or the bus must arrive between 7:20 and 7:22. The probability for each is 2/15. The probability they both arrive in that interval is (2/15)(2/15). Thus, the probability of at least one being delayed is 2/15 + 2/15 – (2/15)(2/15) = 56/225 = 0.25.

238. Solution: C

The probability that a skateboarder makes no more than two attempts is the probability of being injured on the first or second attempt, which is $p+(1-p)p = 2p - p^2$. Then,

$$0.0441 = F(2,2) = (2p - p^2)^2$$
$$0.21 = 2p - p^2$$
$$p^2 - 2p + 1 = 0.79$$
$$(p-1)^2 = 0.79$$
$$p - 1 = \pm 0.88882$$
$$p = 0.11118.$$

The probability that a skateboarder makes no more than one attempt is *p* while the probability of making no more than five attempts is the complement of having no injuries on the first five attempts. Hence,

$$F(1,5) = p[1-(1-p)^5] = 0.0495.$$

239. Solution: B

$$P(X=3, Y=3)) = F(3,3) - F(2,3) - F(3,2) + F(2,2) = 0.9360 - 0.8736 - 0.9300 + 0.8680 = 0.0004$$

240. Solution: D

Let *X* denote the number of deaths next year, and *S* denote life insurance payments next year. Then *S* = 50,000*X*, where *X* ~ Bin(1000, 0.014). Therefore,

$$E(S) = E(50,000X) = 50,000(1000)(0.014) = 700,000$$
$$Var(S) = Var(50,000X) = 50,000^2(1000)(0.014)(0.986) = 34,510,000,000$$
$$StdDev(S) = 185,769.$$

The 99th percentile is 700,000+185,769(2.326)= 1,132,099, which rounds to 1,150,000.

241. Solution: E

Let X_k be the random change in month *k*. Then $E(X_k) = (0.5)(1.1) + 0.5(-0.9) = 0.1$ and $Var(X_k) = 0.5(1.1)^2 + 0.5(-0.9)^2 - (0.1)^2 = 1$. Let $S = \sum_{k=1}^{100} X_k$. Then, *E*(*S*) = 100(0.1) = 10 and *Var*(*S*) = 100(1) = 100. Finally,

$$P(100 + S > 91) = P(S > -9) \doteq P\left(Z > \frac{-9-10}{\sqrt{100}} = -1.9\right) = 0.9713.$$

242. Solution: E

X has an exponential distribution with mean 8 and variance 64. The second moment is 128. The mean and second moment of *Z* are both 0.45. Then (using the independence of *X* and *Z*),

$$E(ZX) = E(Z)E(X) = 0.45(8)$$

$$E[(ZX)^2] = E(Z^2)E(X^2) = 0.45(128) = 57.6$$

$$Var(ZX) = 57.6 - 3.6^2 = 44.64.$$

243. Solution: B

Each (*x*,*y*) pair has probability 1/25. There are only three possible benefit amounts:
0: Occurs only for the pair (0.0) and so the probability is 1/25.
50: Occurs for the three pairs (0,1), (1,0), and (1,1) and so the probability is 3/25.
100: Occurs in all remaining cases and so the probability is 21/25.
The expected value is 0(1/25) + 50(3/25) + 100(21/25) = 2250/25 = 90.

244. Solution: B

The marginal distribution for the probability of a given number of hospitalizations can be calculated by adding the columns. Then *p*(0) = 0.915, *p*(1) = 0.072, *p*(2) = 0.012, and *p*(3) = 0.001. The expected value is 0.915(0) + 0.072(1) + 0.012(2) + 0.001(3) = 0.099.

245. Solution: E

Let *S* be the speed and *X* be the loss. Given *S*, *X* has an exponential distribution with mean 3*X*. Then, noting that the variance of an exponential random variable is the square of the mean, the variance of a uniform random variable is the square of the range divided by 12, and for any random variable the second moment is the variance plus the square of the mean:

$$\begin{aligned} Var(X) &= Var[E(X \mid S)] + E[Var(X \mid S) \\ &= Var[3S] + E(9S^2) \\ &= 9(20-5)^2/12 + 9[(20-5)^2/12 + 12.5^2] \\ &= 1743.75. \end{aligned}$$

246. Solution: C

The four possible outcomes for which $X + Y = 3$ are given below, with their probabilities.

$$(0,3): e^{-1.7}\frac{2.3^3 e^{-2.3}}{3!} = 2.0278e^{-4}$$

$$(1,2): \frac{1.7e^{-1.7}}{1!}\frac{2.3^2 e^{-2.3}}{2!} = 4.4965e^{-4}$$

$$(2,1): \frac{1.7^2 e^{-1.7}}{2!}\frac{2.3\ e^{-2.3}}{1!} = 3.3235e^{-4}$$

$$(3,0): \frac{1.7^3 e^{-1.7}}{3!}e^{-2.3} = 0.8188e^{-4}.$$

The conditional probabilities are found by dividing the above probabilities by their sum. They are, 0.1901, 0.4215, 0.3116, 0.0768, respectively. These apply to the $X – Y$ values of –3, –1, 1, and 3. The mean is –3(0.1901) –1(0.4215) + 1(0.3116) + 3(0.0768) = –0.4498. The second moment is 9(0.1901) + 1(0.4215) + 1(0.3116) + 9(0.0768) = 3.1352. The variance is 2.9329.

247. Solution: A

The marginal distribution of X has probability $1/5 + a$ at 0, $2a + b$ at 1, and $1/5 + b$ at 2. Due to symmetry, the mean is 1 and so the variance is $(0-1)^2(1/5+a)+(1-0)^2(1/5+a) = 2/5+2a$ which is minimized at $a = 0$. The marginal distribution of Y is the same as that of X and thus has the same variance, 2/5 + 0 = 2/5.

248. Solution: C

$$Cov(X,Y) = E(XY) - E(X)E(Y) = E(X^3) - E(X)E(X^2)$$

$$E(X) = E(X^3) = (1/3)(-1+0+1) = 0$$

$$E(X^2) = (1/3)(1+0+1) = 2/3$$

$$Cov(X,Y) = 0 - 0(2/3) = 0.$$

They are dependent, because

$$\Pr(X=0, Y=0) = \Pr(X=0, X^2=0) = \Pr(X=0) = 1/3$$

$$\Pr(X=0)\Pr(Y=0) = (1/3)(1/3) = 1/9 \neq 1/3.$$

249. Solution: B

Let X and Y be the two independent losses and $Z = \min(X,Y)$. Then,

$$\Pr(Z > z) = \Pr(X > z \text{ and } Y > z) = \Pr(X > z)\Pr(Y > z) = e^{-z}e^{-z} = e^{-2z}$$

$$F_Z(z) = \Pr(Z \le z) = 1 - \Pr(Z > z) = 1 - e^{-2z},$$

which can be recognized as an exponential distribution with mean 1/2.

250. Solution: B

Let X and Y be the miles driven by the two cars. The total cost, is then $C = 3(X/15 + Y/30) = 0.2X + 0.1Y$. C has a normal distribution with mean 0.2(25) + 0.1(25) = 7.5 and variance 0.04(9) + 0.01(9) = 0.45. Then, $\Pr(C < 7) = \Pr(Z < (7 - 7.5)/\sqrt{0.45} = -0.7454) = 0.23$.

251. Solution: B

Let X denote the first estimate and Y the second. Then, $\Pr(X > 1.2Y) = \Pr(X - 1.2Y > 0)$. $W = X - 1.2Y$ has a normal distribution with mean $1(10b) - 1.2(10b) = -2b$ and variance $1^2b^2 + 1.2^2b^2 = 2.44b^2$. Then, $\Pr(W > 0) = \Pr(Z > (0 + 2b)/\sqrt{2.44b^2} = 1.280) = 0.100$.

252. Solution: C

Let μ be the common mean. Then the standard deviations of X and Y are 3μ and 4μ respectively. The mean and variance of $(X + Y)/2$ are then $(\mu + \mu)/2 = \mu$ and $[(3\mu)^2 + (4\mu)^2]/4 = 25\mu^2/4$ respectively. The coefficient of variation is $\frac{\sqrt{25\mu^2/4}}{\mu} = 5/2$.

253. Solution: D

$\text{Var}(Z) = \text{Var}(3X + 2Y - 5) = 9\text{Var}(X) + 4\text{Var}(Y) = 9(3) + 4(4) = 43$.

254. Solution: E

The mean is the weighted average of the three means: 0.1(20) + 0.3(15) + 0.6(10) = 12.5. The second moment is the weighted average of the three second moments (each of which is the square of the mean plus the mean, for a Poisson distribution): 0.1(420) + 0.3(240) + 0.6(110) = 180. The variance is the second moment minus the square of the mean, which is 23.75.

255. Solution: B

Let F be the number of fillings and R be the number of root canals. The total claim for a given policyholder, C, in a year is $C = 50F + 0.7(500R) = 50F + 350R$.

We have $E(F) = 0.6(0) + 0.2(1) + 0.15(2) + 0.05(3) = 0.65$ and $E(R) = 0.8(0) + 0.2(1) = 0.2$. Then, $E(C) = 50(0.65) + 350(0.2) = 102.50$.

256. Solution: B

Due to the memoryless property of the exponential distribution, the distribution of the reimbursement given that there is a payment is exponential with the same parameter. Thus $0.5 = F(6000) = 1 - e^{-6000/\lambda}$ which implies that $\lambda = 8656.17$. The solution is $F(9000) - F(3000) = \left(1 - e^{-9000/8656.17}\right) - \left(1 - e^{-3000/8656.17}\right) = 0.35.$

257. Solution: E

Using the formulas for the variance and mean of the uniform distribution:

$$E(X^2) = Var(X) + E(X)^2 = \frac{(100-a)^2}{12} + \left(\frac{100+a}{2}\right)^2 = \frac{100^2 - 200a + a^2 + 3(100)^2 + 600a + 3a^2}{12}$$

$$= \frac{40,000 + 400a + 4a^2}{12} = \frac{19,600}{3}$$

$$0 = 40,000 - 78,400 + 400a + 4a^2$$

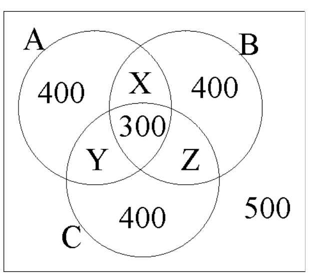
$$0 = a^2 + 100a - 9,600$$

$$0 = (a-60)(a+160)$$

$$a = 60$$

Then, Y is uniform on the interval 1.25(60) = 75 to 100. The 80th percentile is 75 + 0.8(25) = 95.

258. Solution: C



Using a Venn Diagram (calling the risk factors *A, B, C*) we get 400 + 300 + X + Y = 1000 (circle *A*), 400 + 300 + X + Z = 1000 (circle *B*), 400 + 300 + Y + Z= 1000 (circle *C*). Using the first 2 equations we get Y = Z, and using the second 2 equations we get X = Y. Thus X = Y = Z = 150, and so the total number of participants is 3(400) + 3(150) + 300 + 500 = 2450.

259. Solution: C

Consider the following events about a randomly selected auto insurance customer:
A = customer insures more than one car
B = customer insures a sports car
We want to find the probability of the complement of A intersecting the complement of B (exactly one car, non-sports). We have $P(A^c \cap B^c) = 1 - P(A \cup B)$.
By the Additive Law, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
P (A ∪ B) = P (A) + P (B) – P (A ∩ B).
By the Multiplicative Law, $P(A \cap B) = P(B \mid A)P(A) = 0.15(0.64) = 0.096$.
Then, $P(A \cup B) = 0.64 + 0.20 - 0.096 = 0.744$.
Finally, $P(A^c \cap B^c) = 1 - 0.744 = 0.256$.

260. Solution: E

The number of applicants with diabetes has a binomial distribution and thus

$$P(X \le 5) = \sum_{x=0}^{5} \binom{200}{x} (0.01)^x (0.99)^{200-x} = 0.134 + 0.271 + 0.272 + 0.181 + 0.090 + 0.036 = 0.984.$$

A faster solution is to use the Poisson distribution with $\lambda - 200(0.01) = 2$ as an approximation.
Then, $P(X \le 5) = \sum_{x=0}^{5} \frac{e^{-2} 2^x}{x!} = e^{-2}\left(\frac{1}{1} + \frac{2}{1} + \frac{4}{2} + \frac{8}{6} + \frac{16}{24} + \frac{32}{120}\right) = 0.983$.

261. Solution: C

The number of sales has a binomial distribution with n = 5 and p = 0.2. Then,

$$P(X \ge 2) = 1 - P(X \le 1) = 1 - \binom{5}{0}(0.2)^0(0.8)^5 - \binom{5}{1}(0.2)^1(0.8)^4 = 1 - 0.328 - 0.410 = 0.262.$$

262. Solution: C

The number of defective computers has a binomial distribution with n = 100 and p unknown. We have

$$P(X = 3) = 2P(X = 2)$$

$$\binom{100}{3} p^3 (1-p)^{97} = 2\binom{100}{2} p^2 (1-p)^{98}$$

$$161{,}700\, p = 2(4{,}950)(1-p)$$

$$171{,}600\, p = 9{,}900$$

$$p = 9{,}900 / 171{,}600 = 0.058$$

263. Solution: B

P(fire damage and no theft) = 0.2(1 – 0.3) = 0.14
P(no fire damage and theft) = (1 – 0.2)(0.3) = 0.24
P(exactly one) = 0.14 + 0.24 = 0.38

264. Solution: C

Let C be the event that the employee contributes to a supplemental retirement plan and let F be the event that the employee is female. Then, by Bayes' Theorem,

$$P(F \mid C) = \frac{P(C \mid F)P(F)}{P(C \mid F)P(F) + P(C \mid F^c)P(F^c)} = \frac{0.2(0.45)}{0.2(0.45) + 0.3(0.55)} = 0.353.$$

265. Solution: D

Let C be the event that no claim is filed and X be the event that the policyholder is from territory X. Then, by Bayes' Theorem,

$$P(X \mid C) = \frac{P(C \mid X)P(X)}{P(C \mid X)P(X) + P(C \mid X^c)P(X^c)} = \frac{0.15P(X)}{0.15P(X) + 0.4[1 - P(X)]}.$$

From the law of total probability,

$$0.2 = P(C) = P(C \mid X)P(X) + P(C)[1 - P(X)]$$
$$= 0.15P(X) + 0.4[1 - P(X)]$$
$$0.4 - 0.2 = P(X)[0.4 - 0.15]$$
$$P(X) = 0.2 / 0.25 = 0.8.$$

Then,

$$P(X \mid C) = \frac{0.15(0.8)}{0.2} = 0.6.$$

266. Solution: C

The probability that a single claim is less than 25 is

$$P(X < 25) = \int_{10}^{25} 10x^{-2} dx = 3/5.$$

The probability that all three claims are less than 25 is $(3/5)^3 = 27/125.$

267. Solution: D

The distribution function of an exponential distribution with mean 0.5 is $F(x) = 1 - e^{-2x}$.

$\Pr(X > 0.7 \mid X > 0.4) = \dfrac{\Pr(X > 0.7)}{\Pr(X > 0.4)} = \dfrac{e^{-1.4}}{e^{-0.8}} = e^{-0.6} = 0.549.$ This can be more efficiently solved using the memoryless property: $\Pr(X > 0.7 \mid X > 0.4) = \Pr(X > 0.3) = e^{-0.6} = 0.549.$

268. Solution: D

Let X be the number of years in which a payment of 20 is received. X has a binomial distribution with $n = 5$ and $p = 0.5$. Let $p(x)$ be the probability of x payments. The expected payment is

$$0p(0)+20p(1)+40p(2)+60[1-p(0)-p(1)-p(2)]$$
$$=0(1/32)+20(5/32)+40(10/32)+60(16/32)=45.625.$$

269. Solution: E

The mean of the sum is 10 + 12 = 22. The standard deviation of the sum is $\sqrt{3^2+4^2}=5$. The probability the sum is less than 29 is the probability a standard normal random variable is less than (29 – 22)/5 = 1.4, which is 0.9192.

270. Solution: C

The variance of the total is the sum of the variances: 1 + 1 + 2.25 + 4 = 8.25. The standard deviation is the square root, 2.87.

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274. Solution: D

Consider the following events about a randomly selected auto insurance customer:
A = customer insures more than one car
B = customer insures a sports car
We want to find the probability of the complement of A intersecting the complement of B (exactly one car, non-sports).
Then,

$$P(A^c \cap B^c)=1-P(A\cup B)$$
$$=1-[P(A)+P(B)-P(A\cap B)$$
$$P(A\cap B)=P(B\mid A)P(A)=0.2(0.62)=0.124$$
$$P(A^c \cap B^c)=1-[0.62+0.15-0.124]=0.354.$$

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284. Solution: D

X follows an exponential distribution with mean $20\sqrt{5}$ and variance $(20\sqrt{5})^2 = 2000$. Then,
$\text{Cov}[X, Y] = \sqrt{\text{Var}[X]}\sqrt{\text{Var}[Y]}\ \text{Corr}[X, Y] = \sqrt{2000}\sqrt{12,500}(0.2) = 1000$.
It follows that
$\text{Var}[X+Y] = \text{Var}[X] + \text{Var}[Y] + 2\text{Cov}[X, Y] = 2000 + 12,500 + 2(1000) = 16,500$.

285. Solution: D

Let X be the number of appraisals below θ. X is binomial with $n = 4$, $p = 0.75$.
We need exactly 1, 2 or 3 successes for the event $L < \theta < H$ to occur. The probability is
$\text{P}(X = 1) + \text{P}(X = 2) + \text{P}(X = 3)$ =
$\binom{4}{1}(0.75)^1(0.25)^3 + \binom{4}{2}(0.75)^2(0.25)^2 + \binom{4}{3}(0.75)^3(0.25)^1$ = 0.0469 + 0.2109 + 0.4219 = 0.6797.

286. Solution: E

If the first loss is X, then $\text{P}(X > x) = e^{-x}$; the probability that the second loss is more than twice X, would be $\text{P}(X > 2x) = e^{-2x}$. . Thus, the probability that the second loss is more than twice the first loss is $\int_0^\infty e^{-2x}e^{-x}dx = \frac{1}{3}$ due to independence. By symmetry, the probability that the first loss is more than twice the second loss is also 1/3. Thus, the answer is 2/3.

287. Deleted

288. Solution: D

Let WP denote the event "Woman is Pregnant", WNP the event "Woman is Not Pregnant", and TP the event "Test Shows Pregnancy". Using Bayes Theorem:

$$\mathrm{P}(WP \mid TP) = \frac{\mathrm{P}(TP \mid WP) * \mathrm{P}(WP)}{\mathrm{P}(TP \mid WP) * \mathrm{P}(WP) + \mathrm{P}(TP \mid WNP) * \mathrm{P}(WNP)}$$

$$= \frac{(1-0.1)(0.2)}{(1-0.1)(0.2) + (0.2)(1-0.2)} = \frac{0.18}{0.34} = 0.5294$$

289. Solution: A

The probability function of Y, the amount paid in thousands, is

$$p(y) = \begin{cases} 0.72 & y = 0 \\ 0.14 & y = 200 \\ 0.06 & y = 400 \\ 0.08 & y = 500 \end{cases}$$

$\mathrm{E}(Y) = 200 * 0.14 + 400 * 0.06 + 500 * 0.08 = 28 + 24 + 40 = 92$
$\mathrm{E}(Y^2) = 200^2 * 0.14 + 400^2 * 0.06 + 500^2 * 0.08 = 35200$
$\mathrm{Var}(Y) = \mathrm{E}(Y^2) - (\mathrm{E}(Y))^2 = 35200 - 92^2 = 26736$
$\mathrm{SD}(Y) = 26{,}736^{0.5} = 163.5115$

290. Solution: B

$$p(x) = \begin{cases} 0.5 & x = 0 \\ 0.5 & x = 2 \end{cases}$$

$\mathrm{E}(X) = 1$

$\mathrm{Var}(X) = 1$

By the Central Limit Theorem: $\mathrm{E}(S) = 100, \mathrm{Var}(S) = 100.$

$$\mathrm{P}[S > 115] = \mathrm{P}\left[Z > \frac{115 - 100}{\sqrt{100}}\right] = \mathrm{P}[Z > 1.5] = 0.0668.$$

291. Solution: C

$E(X) = 0.25*1 + 0.375*1 = 0.625$
$E(Y) = 0.125*1 + 0.375 = 0.500$
$E(XY) = P[X = 1, Y = 1] = 0.375$
$Var(X) = 0.625 – 0.625^2 = 0.234$
$Var(Y) = 0.5 – 0.5^2 = 0.25$
$Cov(X,Y)=E(XY)-E(X)E(Y) = 0.375 – 0.625*0.500 = 0.0625$
$Corr(X,Y) = 0.0625/(0.234*0.25)^{1/2} = 0.258$

292. Solution: B

$$p(y) = \begin{cases} 6/21 & y = 0 \\ 5/21 & y = 1 \\ 4/21 & y = 2 \\ 3/21 & y = 3 \\ 2/21 & y = 4 \\ 1/21 & y = 5 \end{cases}$$

$E(Y) = 0*6/21 + 1*5/21 + 2*4/21 + 3*3/21 + 4*2/21 + 5*1/21 = 1.67$
$E(Y^2) = 0*6/21 + 1*5/21 + 4*4/21 + 9*3/21 + 16*2/21 + 25*1/21 = 5$
$Var(Y) = 5 – (1.67)^2 = 2.22$

293. Solution: D

Let $W = X + Y$. Then W has the probability function:

w	$p(w)$
0	0.9729
40	0.02
80	0.002
200	0.004
240	0.001
400	0.0001

For instance, the event $\{W = 200\}$ is the union of the events $\{X = 200, Y = 0\}$ and $\{X = 0, Y = 200\}$.
It follows that $E(W) = 2.04$, $E(W^2) = 278.4$, and $Var(W) = 274.24$. Taking the square root gives the standard deviation of 16.56.

294. Solution: B

$E[X] = (0.3)(0.18) + (0.50)(0.08) + (0.20)(-0.13) = 0.068$
$E[Y] = (0.3)(0.15) + (0.50)(0.08) + 0(.20)(-0.06) = 0.068$

$V[X] = 0.3(0.18 - 0.068)^2 + .5(0.08 - 0.068)^2 + 0.2(-0.13 - 0.068)^2 = .01168$
$V[Y] = 0.3(0.15 - 0.068)^2 + .5(0.07 - 0.068)^2 + 0.2(-0.06 - 0.068)^2 = .00529$

So X has a larger variance, but the means are equal.

295. Solution: D

Using the independence of the purchases P(exactly 3 customers purchase)
$= 3(0.5)^2(0.5)(0.1) + (0.5)^3(0.9) = 0.15$, and P(4 purchases $= (0.5)^3(0.1) = 0.0125$.
So P(at most 2) = 1 – P(3) – P(4) = 0.1625 = 0.8375.

296. Solution: A

Let x be the number of policies on male nonsmokers, and let y be the number of policies on female smokers (to be determined). Then, by condition (ii), the number of policies on female nonsmokers is $100 + x$, so that the total number of policies on females is $100 + x + y$. Next, by condition (iii), the number of policies on male smokers is $350 - y$, so that the number of policies on males is $350 - y + x$. Now, by condition (i), we have

$350 - y + x = 150 + (100 + x + y)$.

After simplification, we obtain: $100 = 2y$, which produces $y = 50$, the number of female smokers.

297. Solution: C

Let the random variable X be the future lifetime of a machine part. We know that the density of X has the form $f(x) = C(10 + x)^{-2}$ for $0 < x < 40$ (and it is equal to zero otherwise). First, determine the proportionality constant C from the condition $\int_0^{40} f(x)dx = 1$:

$$1 = \int_0^{40} f(x)dx = -C(10+x)^{-1}\Big|_0^{40} = \frac{2}{25}C$$

so that $C = 25/2 = 12.5$. Then,

$$P(X < 5) = \int_0^5 12.5(10+x)^{-2}dx = -12.5(10+x)^{-1}\Big|_0^5 = -0.8333 + 1.25 = 0.4167.$$

298. Solution: A

E(X) = 0(0.5) + 1(0.2) + 2(0.3) = 0.8
E(X^2) = 0^2(0.5) + 1^2(0.2) + 2^2(0.3) = 1.4,
Var(X) = E(X^2) - (E(X))^2 = 1.4 – (0.8)^2 = 0.76

The premium on a policy is 125% of 0.8, which is 1. The total premium on 76 policies is thus 76. Let Y be the total claim on 76 policies. By the Central Limit Theorem Y has approximately a normal distribution with mean (76)(0.8) = 60.8 and variance 76(0.76) = 57.76. Consequently, P(Y > 76) ≈ P(Z > $P[Y > 76] = P\left(Z > \frac{76-60.8}{\sqrt{57.76}} = 2\right)$, where Z is the standard normal random variable. The answer is 1 – Φ (2) = 0.0228.

299. Solution: C

For Group A: $P[X = 0] = 0.70 = \frac{e^{-\lambda_A}\lambda^0}{0!} \Rightarrow \lambda_A = 0.3567$

For Group B: $P[X = 0] = 0.90 = \frac{e^{-\lambda_B}\lambda^0}{0!} \Rightarrow \lambda_B = 0.1054$

For Group C: $P[X = 0] = 0.50 = \frac{e^{-\lambda_C}\lambda^0}{0!} \Rightarrow \lambda_C = 0.6931$

Then, $20,000 \times 0.3567 + 45,000 \times 0.1054 + 35,000 \times 0.6931 = 36,136$

300. Solution: E

$$\mathrm{P}(0 \text{ claims}) = \frac{e^{-0.288}0.288^0}{0!} = 0.750$$

$$\mathrm{P}(1 \text{ claim}) = \frac{e^{-0.288}0.288^1}{1!} = 0.216$$

For three years the probability of 0 or 1 claim is

$$\mathrm{P}(0 \text{ in 3 years}) + \mathrm{P}(1 \text{ in 3 years}) = (0.75)^3 + \binom{3}{1}(0.75)^2(0.216) = 0.79.$$

Alternatively, the number of claims in three years has a Poisson distribution with mean 3(0.288) = 0. 864. The probability of 0 or 1 claims is

$$\frac{e^{-0.864}0.864^0}{0!} + \frac{e^{-0.864}0.864^1}{1!} = 0.79.$$

301. Solution: E

Var(Total) = Var(Fire) + Var(Theft) + 2Cov(Fire, Theft) = 5 + 8 + 2(3) = 19.

302. Solution: A

Let $X_1,\ X_2,\ X_3$ denote the number of accidents in days 1, 2, and 3, respectively.
Then, each has Poisson distribution with mean 4. Since the X_i s are assumed to be independent, $X = X_1 + X_2 + X_3$ has a Poisson distribution with mean 3(4) = 12. Then,

$$\mathrm{P}\left[X < 2\right] = \mathrm{P}\left[X = 0\right] + \mathrm{P}\left[X = 1\right] = \frac{e^{-12}12^0}{0!} + \frac{e^{-12}12^1}{1!} = 13e^{-12}.$$

303. Solution: C

The probability of success is $p = P(HHH) + P(TTT) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$.
For the first success to be on the third experiment we need two failures followed by one success. The probability is (3/4)(3/4)(1/4) = 9/64 = 0.141.

304. Solution: E

P[At least one Company P truck]:

P[P and (Q or R)]+P[(R or Q) and P]+P[P and P]

$$= 2\left(\frac{4}{9}\right)\left(\frac{5}{8}\right) + \left(\frac{4}{9}\right)\left(\frac{3}{8}\right) = 0.72.$$

Alternatively, take the complement of having no Company P trucks:
1 – (5/9)(4/8) = 0.72.

305. Solution: B

$$P\left(\text{Less than 2 Errors}\right) = \frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} = 4e^{-3} = 0.199$$

$$P\left(\text{Time} < 50\right) = P\left(Z < \frac{50-45}{10}\right) = P\left(Z < 0.5\right) = 0.6915$$

Due to independence of errors and time, the probability is
$0.199 * 0.6915 = 0.1376.$

306. Solution: C

H: Home Renter
A: Apartment Renter

Let T be time from purchase to cancellation. We are looking for $P[H]T > 1]$.

Now $P\left(T > 1 \middle| H\right) = \int_1^{\infty} \frac{1}{4} e^{-\frac{1}{4}t} dt = e^{-1/4}$ and $P\left(T > 1 \middle| A\right) = \int_1^{\infty} \frac{1}{2} e^{-\frac{1}{2}t} dt = e^{-1/2}$..

Using Bayes' Theorem, $P\left(H \middle| T > 1\right) = \dfrac{P(H)P\left(T > 1 \middle| H\right)}{P(A)P\left(T > 1 \middle| A\right) + P(H)P\left(T > 1 \middle| H\right)}$

$$= \frac{0.4e^{-0.25}}{0.6e^{-0.5} + 0.4e^{-0.25}} = 0.4612.$$

307. Solution: B

Let $S_n = \sum_{i=1}^{n} X_i$ and $\overline{S} = \dfrac{S_n}{n}$, where the X_i are the yearly benefit amounts. Then,

$Z = \dfrac{S_n - 2475n}{250\sqrt{n}} = \dfrac{\overline{S} - 2475}{250 / \sqrt{n}}$ is $N(0,1)$ because the sum of independent normal variables is also normal. Then,

$$\mathrm{P}\left(\overline{S} \le 2500\right) = \mathrm{P}\left(Z < \frac{2500 - 2475}{250/\sqrt{n}}\right) = \mathrm{P}\left(Z < 0.1\sqrt{n}\right) = 0.99$$

(So, $0.1\sqrt{n} = 2.32634$

$n = 541.19,$ so use $n = 542.$

The solution must be rounded up to 542 or the probability will be slightly less than 0.99.

308. Solution: D

Let Y be the amount paid out. The amount is 0 if the policyholder lives less than 1 year or more than 5 years. The amount is 1000 otherwise. Then,

$$p(y) = \begin{cases} 0.57, & y = 0 \\ 0.43, & y = 1000 \end{cases}$$

$$\mathrm{E}\left(Y\right) = 0\left(0.57\right) + 1000\left(0.43\right) = 430$$

$$\mathrm{E}\left(Y^2\right) = 0^2\left(0.57\right) + 1000^2\left(0.43\right) = 430{,}000$$

$$\mathrm{Var}\left(Y\right) = 430{,}000 - 430^2 = 245{,}100$$

$$\mathrm{SD}(Y) = 495.$$

309. Solution: E

The solution uses Bayes' Theorem.
Denote the event of Zone A by A, etc.
Denote the event of a fire occurring by F.

$$P(F) = P(F|A)P(A) + P(B|F)P(B) + P(F|C)P(C)$$

$$= 0.015*0.4 + 0.011*0.35 + 0.008*0.25$$

$$= 0.01185.$$

Then $P(A|F) = P(F|A)*P(A)/P(F)$

$$= 0.015*0.4/0.01185 = 0.5063.$$

310. Solution: D

Under plan 1 the expected payment is

$$P_1 = \int_0^4 x \cdot \frac{x}{50}\,dx + \int_4^{10} 4 \cdot \frac{x}{50}\,dx = \left.\frac{x^3}{150}\right|_0^4 + \left.\frac{4x^2}{100}\right|_4^{10}$$

$$= \frac{64}{150} + \left[\frac{400-64}{100}\right] = \frac{1136}{300} = 3.786667$$

Under plan 2,

$$P_2 = \int_0^4 0 \cdot \frac{x}{50}\,dx + \int_4^{10} (x-4) \cdot \frac{x}{50}\,dx$$

$$= \int_4^{10} \frac{x^2}{50} - \frac{4x}{50}\,dx = \left.\frac{x^3}{150} - \frac{x^2}{25}\right|_4^{10} = \left(\frac{1000}{150} - 4\right) - \left(\frac{64}{150} - \frac{16}{25}\right)$$

$$= \frac{8}{3} + \frac{32}{150} = \frac{432}{150} = 2.88$$

So $P_1 - P_2 = 3.78666 - 2.88 = 0.906667$.

311. Solution: C

Let $Z = X - Y$. The probabilities for Z are:
$p(0) = 2/31$ (when $x = 1$ and $y = 1$, that is (1,1))
$p(1) = 6/31$ (for (1,0) and (2,1))
$p(2) = 14/31$ (for (2,0) and (3,1))
$p(3) = 9/31$ (for (3,0))
Then.

$$\mathrm{E}(Z) = \frac{1}{31}\left[2(0) + 6(1) + 14(2) + 9(3)\right] = \frac{61}{31},$$

$$\mathrm{E}(Z^2) = \frac{1}{31}\left[2(0^2) + 6(1^2) + 14(2^2) + 9(3^2)\right] = \frac{143}{31}, \text{ and}$$

and so $\mathrm{Var}(Z) = \frac{143}{31} - \left(\frac{61}{31}\right)^2 = \frac{4433 - 3721}{31^2} = \frac{712}{961} = 0.7409$.

312. Solution: D

M = medical, P = property

$\mathrm{P}[M] = 0.30, \mathrm{P}[P] = 0.42, \mathrm{P}[M \cup P] = 0.60$

Therefore, $\mathrm{P}[M \cap P] = 0.12 \ (0.30 + 0.42 - 0.60 = 0.12)$

$$\mathrm{P}[\text{Exactly 1} \mid \text{Not Both}] = \frac{\mathrm{P}[\text{Exactly 1}]}{\mathrm{P}[\text{Not Both}]} = \frac{0.48}{0.88} = 0.54545.$$

313. Solution: B

P[Reports Property Claim in Year 9]=1.25^8 * P[Reports Property Claim in Year 1]
P[Reports Liability Claim in Year 9]=0.75^8 * P[Reports Liability Claim in Year 1]

$0.01 *(1.25)^8 \text{ x } (0.75)^8 = 0.005967$

314. Solution: B

Define: N = # of claims reported in a year by a policyholder
F = event of a first-year policyholder

$$\mathrm{P}\left[N \ge 1 \mid F\right] = 1 - \mathrm{P}\left[N < 1 \mid F\right] = 1 - \mathrm{P}\left[N = 0 \mid F\right] = 1 - \frac{e^{-0.5}0.5^0}{0!} = 0.39347$$

$$\mathrm{P}\left[N \ge 1 \mid F^C\right] = 1 - \mathrm{P}\left[N < 1 \mid F^C\right] = 1 - \mathrm{P}\left[N = 0 \mid F^C\right] = 1 - \frac{e^{-0.2}0.2^0}{0!} = 0.18127$$

$$\Pr[F \mid N \ge 1] = P[F \mid N \ge 1] = \frac{\mathrm{P}[N \ge 1 \mid F]\mathrm{P}[F]}{\mathrm{P}[N \ge 1 \mid F]\mathrm{P}[F] + \mathrm{P}[N \ge 1 \mid F^C]\mathrm{P}[F^C]}$$

$$= \frac{(0.39347)(0.15)}{(0.39347)(0.15) + (0.18127)(0.85)} = 0.27696$$

315. Solution: B

$P[\min(Y_1, Y_2) > e^{16}] = P[Y_1 > e^{16}, Y_2 > e^{16}] = P[Y_1 > e^{16}]P[Y_2 > e^{16}]$ due to independence.

$$P[Y_1 > e^{16}]P[Y_2 > e^{16}] = P[X_1 > 16]P[X_2 > 16] = P\left[Z > \frac{16-16}{1.5}\right]P\left[Z > \frac{16-15}{2}\right]$$

$$= P[Z > 0]P[Z > 0.5] = 0.5(1-0.6916) = 0.15425.$$

316. Solution: C

$$\mathrm{P}(A) + \mathrm{P}(B) + \mathrm{P}(C) = 1$$
$$5\mathrm{P}(C) + 4\mathrm{P}(C) + \mathrm{P}(C) = 1$$
$$\mathrm{P}(C) = 0.1$$
$$\mathrm{P}(B) = 0.4$$
$$\mathrm{P}(A) = 0.5$$

$$\mathrm{P}\left(C \middle| 0 \text{ Claims}\right) = \frac{\mathrm{P}\left(C \cap 0 \text{ Claims}\right)}{\mathrm{P}\left(0 \text{ Cliaims}\right)} = \frac{\mathrm{P}\left(0 \text{ Claims} \middle| C\right) \cdot \mathrm{P}(C)}{\mathrm{P}\left(0 \text{ Claims}|A\right) \cdot \mathrm{P}(A) + \mathrm{P}\left(0 \text{ Claims} \middle| B\right) \cdot \mathrm{P}(B) + \mathrm{P}\left(0 \text{ Claims}|C\right) \cdot \mathrm{P}(C)}$$

$$= \frac{(0.4)(0.1)}{(0.1)(0.5) + (0.2)(0.4) + (0.4)(0.1)} = \frac{0.04}{0.05 + 0.08 + 0.04} = 0.235$$

317. Solution: B

Let Y be the random variable for the benefit.

$$p(y) = \begin{cases} 0.01, & y = 25{,}000 \\ 0.99(0.01), & y = 20{,}000 \\ 0.99^2(0.01), & y = 15{,}000 \\ 0.99^3(0.01), & y = 10{,}000 \\ 0.99^4(0.01), & y = 5{,}000 \\ 1 - \text{above}, & y = 0 \end{cases}$$

$$\mathrm{E}[Y] = 25000(0.01) + 20000(0.99)(0.01) + 15000(0.99)^2(0.01) + 10000(0.99)^3(0.01) + 5000(0.99)^4(0.01)$$
$$= 740$$

318. Solution: C

Let WP = Woman Pregnant

WNP = Woman Not Pregnant

TP = Test Shows Pregnancy (Positive)

Using Bayes Theorem:

$$\mathrm{P}(WP \mid TP) = \frac{\mathrm{P}(TP \mid WP)\,\mathrm{P}(WP)}{\mathrm{P}(TP \mid WP)\,\mathrm{P}(WP) + \mathrm{P}(TP \mid WNP)\,\mathrm{P}(WNP)}$$
$$= \frac{(0.90)(0.30)}{(0.90)(0.30) + (0.20)(0.70)} = \frac{0.27}{0.27 + 0.14} = \frac{0.27}{0.41} = 0.6585$$

319. Solution: C

$$\mathrm{P}(\text{exactly } 3) = (3)(0.70)^2(0.30)(0.20) + (0.70)^3(0.80)$$
$$= 0.0882 + 0.2744 = 0.3626$$
$$\mathrm{P}(\text{exactly } 4) = (0.70)^3(0.20) = 0.0686$$
$$\mathrm{P}(\text{at most } 2) = 1 - 0.3626 - 0.0686 = 0.5688$$

320. Solution: A

Mean of sum $= 100(1000) = 100,000$

Standard deviation of sum $= \sqrt{100(400)^2} = 4,000$

$$P(X < 92,000) = P\left(Z < \frac{92,000 - 100,000}{4000}\right) = P(Z < -2) = 0.0228$$

321. Solution: A

Let X be the number of non-defective items in a randomly chosen box of 50 items.
Then $X - \text{Binomial}(50, 1-p)$ and P(at least t^* non-defective items in the box of 50 items)
$= P(X \ge t^*) = \sum_{t=t^*}^{50} \binom{50}{t}(1-p)^t(p)^{50-t}$ and we want this probability to be at least $0.95 \Rightarrow$ A is correct.

322. Solution: E

X has a negative binomial probability distribution, where $r = 3$ and $p = \frac{1}{6}$. The probability function is: $p(x) = \binom{x-1}{r-1} p^r q^{y-r}$. The variance for this is $Var(X) = \frac{r(1-p)}{p^2} = \frac{3\left(\frac{5}{6}\right)}{\left(\frac{1}{6}\right)^2} = 90.$

323. Solution: C

$$P(3 \text{ products}) = (0.45)(0.55)(0.60)(0.40) + (0.55)(0.55)(0.60)(0.60)$$
$$+ (0.45)(0.45)(0.60)(0.60) + (0.45)(0.55)(0.40)(0.60) = 0.3006$$
$$P(4 \text{ products}) = (0.45)(0.55)(0.60)(0.60) = 0.0891$$
$$P(3 \text{ or } 4) = 0.3006 + 0.0891 = 0.3897$$

324. Solution: B

The total number of accidents has mean 1600(4) = 6400 and variance 1600(4) = 6400. Using the Central Limit Theorem,

$$P\left(\sum_{i=1}^{1600} X_i \geq 6496\right) \approx P\left(Z \geq \frac{6496 - 6400}{80}\right) = P(Z \geq 1.2) = 1 - 0.8849 = 0.1151 \ .$$

325. Solution: A

Let H – health

L – life

R – retirement

$a = (\text{H} \cap \text{L}) \quad b = (\text{H} \cap \text{R}) \quad c = (\text{L} \cap \text{R})$

$$a + b = 0.625$$
$$a + c = 0.375$$
$$b + c = 0.500$$
$$(b + c) + (a + c) - (a + b) = 2c = 0.500 + 0.375 - 0.625 = 0.25$$
$$c = 0.125$$

326. Solution: B

This is an exponential distribution with mean 20 and standard deviation 20. So, we want:

$$\int_{10}^{30} \frac{1}{20} e^{-\frac{1}{20}x}\, dx = e^{-\frac{1}{20}x} \Big|_{10}^{30} = -e^{-1.5} + e^{-0.5} = 0.3834$$

327. Solution: C

Average of 100 claims is normal with mean 2500 and standard deviation $500/\sqrt{100} = 50$.

$$P\left(\bar{X} < K\right) = 0.99$$
$$P\left(Z < \frac{K-2500}{50}\right) = 0.99$$
$$\frac{K-2500}{50} = 2.32635$$
$$K = 2616$$

328. Solution: B

Var[1.03X + 2.5] = $(1.03)^2$Var[X] = $(1.03)^2(50)$ = 53.045

329. Solution: C

L: left-handed child
0LP: no left-handed parents
1LP: one left-handed parent
2LP: two left-handed parents

$$P(L \mid 2LP) = 1/2$$
$$P(L \mid 1LP) = 1/6$$
$$P(L \mid 0LP) = 1/16$$
$$P(2LP) = 1/50$$
$$P(1LP) = 1/5$$
$$P(0LP) = 1 - 1/50 - 1/5 = 39/50$$
$$P(0LP \mid L) = \frac{P(L \mid 0LP)P(0LP)}{P(L \mid 0LP)P(0LP) + P(L \mid 1LP)P(1LP) + P(L \mid 2LP)P(2LP)}$$
$$= \frac{(1/16)(39/50)}{(1/16)(39/50) + (1/6)(1/5) + (1/2)(1/50)} = \frac{3(39)}{3(39) + 8(10) + (24)(1)} = \frac{117}{221} = 0.529$$

330. Solution: E

$$Var(Z) = Var(4X - Y - 3) = (4)^2 Var(X) + (-1)^2 Var(Y) = (16)(2) + 3 = 35$$

331. Solution: A

Let H = health insurance; D =disability income insurance

Then P(H)= $\frac{x}{100}$; P(D) = $\frac{y}{100}$; P(H and not D)= $\frac{z}{100}$

Let P(H and D) = $\frac{k}{100}$

Since, P(H) = P(H and not D) + P(H and D),

$x = z + k$

$\Rightarrow k = x - z$

Since P(D) = P(D and not H) + P(D and H),

P(D and not H) = P(D) – P(D and H)

P(D and not H) = $\frac{y}{100} - \frac{k}{100} = \frac{y-(x-z)}{100} = \frac{y-x+z}{100}$

332. Solution: B

The first die can be any number, so 6 possibilities. If the second die matches (1 possibility) then the third die must not match (5 possibilities). Hence this outcome can occur 6(1)(5) = 30 ways. A second outcome is the second die does not match (5 possibilities), in which case the third die must match one of the first two (2 possibilities), for an additional 60 ways. The desired probability is then 90/216 = 0.417.

333. Solution: D

Hypergeometric:

$$\frac{\binom{10}{3}\binom{7}{3}}{\binom{17}{6}} = \frac{75}{221} = 0.33937$$

334. Solution: A

P(M) = 0.6
P(F) = 0.4
P(S|M) = 0.20
P(F|S) = 0.20

$$P(F \mid S) = \frac{P(S \mid F)P(F)}{P(S \mid F)P(F) + P(S \mid M)P(M)}$$

$$0.2 = \frac{P(S \mid F)(0.4)}{P(S \mid F)(0.4) + 0.2(0.6)} \Rightarrow P(S \mid F) = \frac{0.2(0.2)(0.6)}{0.4 - 0.2(0.4)} = 0.75$$

335. Solution: E

Hypergeometric:

$$\frac{\binom{8}{2}\binom{2}{1} + \binom{8}{3}\binom{2}{0}}{\binom{10}{3}} = \frac{56 + 56}{120} = 0.9333$$

336. Solution: C

Let *d* be the deductible. Assume *d* < 100 because the expected claim needs to be positive.

The claim is 0 with probability *d*/100 (i.e. when the loss does not exceed *d*), and uniformly distributed over [0, 100 - *d*] with probability (100 - *d*)/100 (i.e. when the loss exceeds *d*).

Note that the probability that a uniformly distributed random variable is in an interval is proportional to the length of the interval; the expected value is halfway between the minimum and maximum possible values of the uniformly distributed random variable.

$$32 = E[Claim] = [\frac{0 + (100 - d)}{2} * \frac{100 - d}{100}] + [0 * \frac{d}{100}]$$

Solving for *d* gives *d* = 20 or *d* = 180; disregard *d* = 180 > 100.

337. Solution: D

Let Y represent the loss and X represent the claim. Note that $X = 0$ if $Y \leq 3$; otherwise, $X = Y - 3$.

$$E[X] = P(Y \leq 3)E[X \mid Y \leq 3) + P(Y > 3)E[X \mid Y > 3)$$
$$= P(Y \leq 3)(0) + P(Y > 3)E[Y - 3 \mid Y > 3]$$
$$= 0 + \int_3^\infty \frac{e^{-\frac{y}{10}}}{10} dy * E[Y] \text{ since exponentials are memoryless.}$$
$$= 10e^{-\frac{3}{10}}.$$

338. Solution: B

R = # of root canals, F = # of fillings, $p(r, f) = P(R = r, F = f)$
$P(R = r \mid F$ is at most 1)
$= P(R = r \mid F = 0$ or 1)
$= P(R = r$ AND $F = 0$ or 1) / $P(F = 0$ or 1)
$= [p(r, 0) + p(r, 1)] / [p(0, 0) + p(0, 1) + p(1, 0) + p(1, 1) + p(2, 0) + p(2, 1)]$

$P(R = 0 \mid F = 0$ or 1) = (0.40 + 0.26) / (0.40 + 0.26 + 0.04 + 0.03 + 0.01 + 0.01) = 22/25
$P(R = 1 \mid F = 0$ or 1) = (0.04 + 0.03) / (0.40 + 0.26 + 0.04 + 0.03 + 0.01 + 0.01) = 7/75
$P(R = 2 \mid F = 0$ or 1) = (0.01 + 0.01) / (0.40 + 0.26 + 0.04 + 0.03 + 0.01 + 0.01) = 2/75

$$E[R \mid F \leq 1] = 1\left(\frac{7}{75}\right) + 2\left(\frac{2}{75}\right) = \frac{11}{75} = 0.14667 \ .$$

339. Solution: A

N is Binomial, n=500, p=0.12, q=0.88

Standard deviation = $\sqrt{500(0.12)(0.88)} = 7.26636$

340. Solution: E

$$F(x) = \int_1^x 2t^{-3} dt = -t^{-2}\Big|_1^x = 1 - x^{-2}$$
$$F(4) = 15/16$$
$$F(3) = 8/9$$
$$\Pr(X < 4 \mid X \geq 3) = \frac{\Pr(3 \leq X < 4)}{\Pr(X \geq 3)} = \frac{F(4) - F(3)}{1 - F(3)} = \frac{15/16 - 8/9}{1 - 8/9} = \frac{135 - 128}{144 - 128} = \frac{7}{16} = 0.4375$$

341. Solution: B

$$1 = \int_0^{30} k(x+5)^2 dx$$
$$1 = -k(x+5)^{-1}\Big|_0^{30}$$
$$1 = -k\left[\frac{1}{35} - \frac{1}{5}\right]$$
$$1 = k(0.171429)$$
$$k = 5.8333$$
$$\int_5^{10} \frac{5.8333}{(x+5)^2} dx = 0.1944$$

342. Solution: B

$$F(1) = \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!}$$
$$F(2) = \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!} + \frac{e^{-\lambda}\lambda^2}{2!}$$
$$\frac{F(2)}{F(1)} = \frac{1+\lambda+\frac{\lambda^2}{2}}{1+\lambda} = \frac{13}{5} = \frac{26}{10}$$
$$10 + 10\lambda + 5\lambda^2 = 26 + 26\lambda$$
$$5\lambda^2 - 16\lambda - 16 = 0$$
$$(5\lambda+4)(\lambda-4) = 0$$
$$\lambda = 4 = E(X).$$

Ignore negative solution.

343. Solution: B

P[$X \ge 0$] = 1
P[$X \ge 1$] = 0.5
P[$X \ge 2$] = 0.27639
P[$X \ge 3$] = 0.18377
P[$X \ge 4$] = 0.11270
P[$X \ge 5$] = 0.05279

$$p(x) = \begin{cases} 0.5, & x = 0 \\ 0.22361, & x = 1 \\ 0.09262, & x = 2 \\ 0.07107, & x = 3 \\ 0.05991, & x = 4 \\ 0.05279, & x = 5 \end{cases}$$

$$E(X) = \sum_{i=0}^{5} x_i p(x_i) = 1.12565$$

344. Solution: B

Let $g(y)$ represent the density function for the median.

$$g(y) = 6(y^2)(1 - y^2)(2y)$$
$$= 12(y^3 - y^5)$$
$$E(Y) = \int_0^1 yg(y)dy = 12(1/5 - 1/7) = 24/35$$
$$E(Y^2) = \int_0^1 y^2 g(y)dy = 12(1/6 - 1/8) = 1/2$$
$$Var(Y) = (1/2) - (24/35)^2 = 0.030$$

345. Solution: A

$W - T$ has a normal distribution with mean 0 and variance 4 + 12 = 16.

$$P\left(\left|W - T\right| < 1\right) = P(-1 < W - T < 1)$$

$$= P(-1 < N(0,16) < 1) = P(-.25 < N(0,1) < .25) = 0.197$$

346. Solution: B

The only way exactly two of the events can occur is if A and B both occur. Hence $P[A \cap B] = 0.06.$

Because the intersections of A and C and of B and C are empty,
$0.90 = P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B].$

The probability of exactly one of A and B is $P[A] + P[B] - 2P[A \cap B] = 0.38.$

Therefore, $P[A] + P[B] - P[A \cap B] = 0.38 + 0.06 = 0.44.$

Then, P[C] = 0.90 – 0.44 = 0.46.

347. Solution: D

$$p(y) = \begin{cases} 0.91, & y = 0 \\ 0.09, & y = 1 \end{cases}$$

$$E(Y) = (0)(0.91) + (1)(0.09) = 0.09$$

$$E(Y^2) = (0^2)(0.91) + (1^2)(0.09) = 0.09$$

$$V(Y) = 0.09 - 0.09^2 = 0.0819$$

$$SD(Y) = \sqrt{0.0819} = 0.28618$$

$$CV(Y) = \frac{0.28618}{.09} = 3.179797$$

348. Solution: E

The policyholder will receive benefits if and only if at least 1 of the 3 losses exceeds 30.
The probability that a given loss exceeds 30 is (100 - 30)/100 = 0.7.

The probability that no losses exceed 30 is $(1 - 0.7)^3 = 0.027.$

The probability that at least one loss exceeds 30 is 1 – 0.027 = 0.973.

349. Solution: B

$$P[J] = 0.70$$

$$P[F] = 0.50$$

$$P[F \mid J] = 2P[F \mid J^c]$$

$$P[F] = 0.50 = P[F \mid J]P[J] + P[F \mid J^c]P[J^c]$$

$$0.50 = P[F \mid J]0.70 + 0.50P[F \mid J]0.30$$

$$P[F \mid J] = 0.50 / (0.70 + 0.25) = 0.58824$$

$$P[F \cap J] = P[F \mid J]P[J]$$

$$P[F \cap J] = 0.58824(0.70) = 0.41176.$$

350. Solution: B

$$P[Y \geq 3000] = e^{-\frac{3000}{2000}} = 0.223.$$

351. Solution: E

P(0 employees have accidents) = $(0.6)^2(0.9)^2$ = 0.2916

P(1 employee has accidents)
= P(1 high-risk employee has accidents and 0 low-risk employees have accidents
OR 0 high-risk employees have accidents and1 low-risk employee has accidents)

= P(1 high-risk employee has accidents)P(0 low-risk employees have accidents)
+ P(0 high-risk employees have accidents)P(1 low-risk employee has accidents)
[combining the addition rule for a union of mutually exclusive events and the multiplication rule for independent events]

= $(2)(0.4)(0.6)(0.9)^2 + (0.6)^2(2)(0.1)(0.9) = 0.4536$

P(at most 1 employee has accidents) = 0.2916 + 0.4536 = 0.7452.

352. Solution: D

Amt Paid	**Prob**
0	0.70
5000	0.11
10000	0.08
15000	0.07
18000	0.04

E(X) = 3120
E(X^2) =39,460,000
V(X) =39,460,000 – $3{,}120^2$ = 29,725,600
SD(X) = 5452

353. Solution: B

$$F(x) = \int_2^x 8t^{-3}dt = -4t^{-2}\Big|_2^x = 1 - 4x^{-2}.$$

$$P\left(2.5 \leq x \leq 3 \mid x \geq 2.5\right)$$

$$= \frac{F(3) - F(2.5)}{1 - F(2.5)} = \frac{1 - 4/9 - 1 + 4/6.25}{1 - 1 + 4/6.25} = 0.3056.$$

354. Solution: B

M = Male
F = Female
A = Accident
$P[M \cap A] = 0.30$
$P[A \mid M] = 0.50 = \frac{P[M \cap A]}{P[M]}$
$P[M] = \frac{0.30}{0.50} = 0.60$
$P[F] = 1 - P[M] = 1 - 0.60 = 0.40.$

355. Solution: E

$$p(x) = \frac{e^{-4}4^x}{x!}, x = 0,1,2,...;1 - \left[p(0) + p(1)\right] = 1 - 0.091578 = 0.908$$

356. Deleted

357. Solution: E

X follows a geometric distribution with $p = \frac{1}{6}$

Y = 1 implies the first roll is a 6. Counting from the second roll $E\left[X\right] = \frac{1}{p} = 6.$ So $E\left[X \middle| Y = 1\right] = 1 + 6 = 7$.

358. Solution: C

$$\text{P}[X = x] = \frac{e^{-2.5}2.5^x}{x!}$$

P[0] = 0.082, P[1] = 0.205, P[2] = 0.257, P[3] = 0.214.
Probabilities for values greater than 3 are less than the probability of 3. The largest probability occurs at the value of 2.

359. Solution: E

$$\text{P}[M \mid X] = 0.3 \Rightarrow \text{P}[F \mid X] = 0.7$$
$$\text{P}[F \mid Y] = 0.4 \Rightarrow \text{P}[M \mid Y] = 0.6$$
$$\text{P}[Y] = 0.6 \Rightarrow \text{P}[D] = 0.4$$
$$\text{P}[Y \mid M] = \frac{\text{P}[M \mid Y]\text{P}[Y]}{\text{P}[M \mid Y]\text{P}[Y] + \text{P}[M \mid X]\text{P}[X]} = \frac{(0.6)(0.6)}{(0.6)(0.6)+(0.3)(0.4)} = 0.75$$

360. Solution: A

The probability of a 20% surcharge is 0.1, the probability of a 15% surcharge is 0.9(0.1), the probability of a 10% surcharge is 0.9(0.9)(0.1) and the probability of a 5% surcharge is 0.9(0.9)(0.9)(0.1).
Expected surcharge = $0.20(0.1)+0.15(0.1)(0.9)+0.10(0.1)(0.9)^2+0.05(0.1)(0.9)^3 = 0.045245$

361. Solution: C

Let Y be the expected payout. Then
$$E[Y] = 0(0.75)+(1000-500)(0.12)+(5000-500)(0.08)$$
$$+(10000-500)(0.04)+(15000-500)(0.01) = 945$$
The premium sis $945+75=1020$.

362. Solution: E

$$\mu = 180\left(\frac{1}{6}\right) = 30$$
$$\sigma = \sqrt{180\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)} = 5$$
$$P(X \geq 40) \rightarrow P(Z > \frac{39.5-30}{5}) = P(Z > 1.90) = 0.0287.$$

363. Solution: E

Let H be the event that the policyholder undergoes hospitalization this year.
Let A be the event that the policyholder is in class A.
P(A) = 12,000/30,000 = 0.4; P(B) = 18,000/30,000 = 0.6
P(H|A) = 1 - 0.98 = 0.02; P(H|B) = 1 - 0.995 = 0.005.
P(A|H) = P(A)P(H|A) / [P(A)P(H|A) + P(B)P(H|B)] by Bayes' Theorem
= (0.4)(0.02) / [(0.4)(0.02) + (0.6)(0.005)]
= 0.008 / 0.011 = 0.7272

364. Solution: D

Let X be the random variable that a part works. Then:

$P(X > x) = e^{-0.2x}$

$P(X < x) = 1 - e^{-0.2x}$

One year from now,

P[both parts will work | at least 1 part will work]

$$= \frac{P[\text{both parts will work}]}{P[\text{at least 1 part will work}]}$$

$$\frac{e^{-0.2}e^{-0.2}}{1 - P(\text{neither part works})}$$

$$= \frac{e^{-0.2}e^{-0.2}}{1-(1-e^{-0.2})(1-e^{-0.2})}$$

$$= \frac{0.67032}{0.96714} = 0.69310$$

365. Solution: E

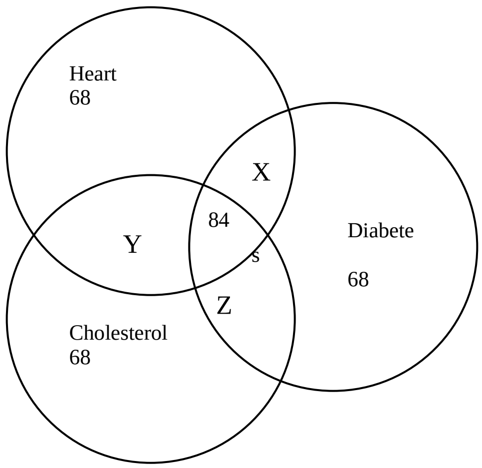
P[no root canal | no fillings] = P[no root canals and no fillings] / P[no fillings]
= {1 – P[at least 1 root canal or filling]} / P[no fillings]
= (1 – 0.35)/0.7 = 13/14 = 0.92857.

366. Solution: D

This is a hypergeometric probability:

$$\frac{{}_7C_3\,{}_3C_2}{{}_{10}C_5} = \frac{35(3)}{252} = 0.41667$$

367. Solution: C



68 + 84 + X + Z = 268
68 + 84 + X + Y = 268
68 + 84 + Y + Z = 268

X, Y and Z must all be equal, X = Y = Z = 58.

Total = 68 + 68 + 68 + 84 + 58 + 58 + 58 + 155 = 617

368. Solution: C

A: The event that a person is a smoker
B: The event that a person has below normal lung function.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(B \mid A)P(A)$$

$$0.40 = 0.25 + P(B) - (0.70)(0.25)$$

$$P(B) = 0.325$$

369. Solution: E

$P[H \cap W] = P[H]P[W]$ where H is the husband's survival and W is the wife's survival.

$P[H] = 1 - 0.10 = 0.90$, $P[H \cap W] = 0.70$. Therefore, $P[W] = \dfrac{0.70}{0.90} = 77.8\%$ and the probability of the wife dying is $1 - P[W] = 22.2\%$.

370. Solution: C

$$P[S \mid B] = \frac{P[B \mid S]P[S]}{P[B \mid S]P[S] + P[B \mid R]P[R] + P[B \mid T]P[T] + P[B \mid O]P[O]}$$
$$= \frac{0.08(0.25)}{0.08(0.25) + 0.12(0.60) + 0.06(0.10) + 0(0.05)} = 0.20408$$

371. Solution: C

Using the quadratic equation and being careful with inequalities,
$P\left[8X - X^2 < 1\right] = P\left[X < 0.127016654\right] + P\left[X > 7.872983346\right]$. For the standard normal variable $Z = \frac{X-5}{2}$, this is:
$$P\left[Z < -2.436491673\right] + P\left[Z > 1.436491673\right]$$
$$= 0.0073 + 0.0749 = 0.0822.$$

372. Solution: A

Since the normal and uniform are symmetric the mean and median are the same and doubling one doubles the other. For the exponential distribution the median is ln2 times the mean. Thus doubling the mean doubles the median.

373. Solution: B

The marginal distribution of X is: $P(X = 0) = 3/12$; $P(X = 1) = 4/12$; $P(X = 2) = 5/12$. Thus $E(X^2) = 0^2 \times \frac{3}{12} + 1^2 \times \frac{4}{12} + 2^2 \times \frac{5}{12} = 2$ and $E(X) = 0 \times \frac{3}{12} + 1 \times \frac{4}{12} + 2 \times \frac{5}{12} = \frac{14}{12}$ so the variance is $Var(X) = 2 - \frac{196}{144} = \frac{23}{36} = 0.639.$

374. Solution: B

To determine the mode, set $f'(x) = \frac{5}{72}\left[6(x-2) - 4(x-2)^3\right] = 0$. This solves for $6(x-2) - 4(x-2)^3 = 0$ so either $x - 2 = 0$ and $x = 2$ or $6 - 4(x-2)^2 = 0$. In this case, $(x-2)^2 = \frac{6}{4}$ giving $x = 2 \pm 1.2247$, so $x = 0.7753$ or $x = 3.2247$ which is outside the domain.
Check these two values and the endpoints:
$f(0) = 0$
$f(3) = 0.4167$
$f(2) = 0.2778$
$f(0.7753) = 0.4340$
The mode occurs at 0.775.

375. Solution: A

$$F(t) = 4\beta^4 \int_\beta^t r^{-5} dr = 1 - \left(\frac{\beta}{t}\right)^4, \text{ for } 0 < \beta \le t$$

$$P\left[T > 4 + 3 \mid T > 3\right] = \frac{P\left[T > 7\right]}{P\left[T > 3\right]} = \frac{1 - F(7)}{1 - F(3)} = \frac{\left(\frac{\beta}{7}\right)^4}{\left(\frac{\beta}{3}\right)^4} = \frac{81}{2401}$$

376. Solution: C

The number of senior employees *X* included in the sample is distributed per the hypergeometric distribution. The required probability is *P*[*X* = 3 or 4], which is:

$$\frac{\binom{5}{3}\binom{10}{1} + \binom{5}{4}\binom{10}{0}}{\binom{15}{4}} = \frac{10(10) + 5(1)}{1365} = 0.07692.$$

377. Solution: C

$$P(H)=\frac{1100}{3000}, P(L)=\frac{1900}{3000}, P(S\,|\,H)=\frac{60}{1100}, P(S\,|\,L)=\frac{28}{1900}$$

$$P(L\,|\,S)=\frac{P(S\,|\,L)P(L)}{P(S\,|\,L)P(L)+P(S\,|\,H)P(H)}$$

$$=\frac{\frac{28}{1900}\frac{1900}{3000}}{\frac{28}{1900}\frac{1900}{3000}+\frac{60}{1100}\frac{1100}{3000}}=\frac{28}{28+60}=0.31818$$

378. Solution: C

Let X = the company's profit in a given year, which is normally distributed with mean $\mu = 6.72$, 80 percentile of 8.4, and standard deviation σ. Then,

$$P(X<8.4)=0.80$$

$$P(Z<\frac{8.4-6.72}{\sigma})=0.80$$

$$\frac{8.4-6.72}{\sigma}=0.84162$$

$$\sigma=2$$

$$P(Z<\frac{P-6.72}{2})=0.90$$

$$\frac{P-6.72}{2}=1.28155$$

$$P=9.28$$

379. Solution: B

Let X = the length of the power failure in days.
We need to find the value of m for which the cumulative probability = 0.5

$$0.5=\int_0^m\frac{(4-x)^3}{64}dx=-\frac{(4-x)^4}{256}\bigg|_0^m=\frac{1}{256}\left(256-(4-m)^4\right)=1-\frac{(4-m)^4}{256}$$

$\frac{(4-m)^4}{256}=0.5$; $(4-m)^4 = 128$; $4-m=\sqrt[4]{128}$; $m=4-\sqrt[4]{128}$

380. Solution: B

L has a normal distribution with mean 1205 – 2 – 2 + 2 = 1203 and variance 5.0 + 0.5 + 0.5 = 6.0. Then, $P[L \ge 1200] = P[Z \ge -3/\sqrt{6}\,] = 0.8897$.

381. Solution: C

This is an exponential distribution with $P[X < x] = 1 - e^{-x/16}$.

$$P[X < 20 \mid X > 5] = \frac{P[5 < X < 20]}{P[X > 5]}$$

$$= \frac{P[X < 20] - P[X < 5]}{P[X > 5]}$$

$$= \frac{(1 - e^{-\frac{20}{16}}) - (1 - e^{-\frac{5}{16}})}{e^{-\frac{5}{16}}} = 0.60839$$

382. Solution: A

X = 3 implies *Y* = 0 or 1

Let *W* be the number of pages with low graphical content. Then,

$$P(Y = 0 \mid X = 3) = \frac{P(Y = 0, X = 3)}{P(X = 3)} = \frac{P(Y = 0, W = 1, X = 3)}{P(X = 3)} = \frac{\dfrac{\binom{15}{0}\binom{5}{1}\binom{10}{3}}{\binom{30}{4}}}{\dfrac{\binom{20}{1}\binom{10}{3}}{\binom{30}{4}}} = \frac{\dfrac{600}{27,405}}{\dfrac{2,400}{27,405}} = 0.25.$$

The denominator follows because the distribution of *X* is hypergeometric with categories of *X* and not *X*. The conditional distribution of *Y* is Bernoulli with success probability 0.75. The variance is 0.75(0.25) = 0.1875.

383. Solution: D

The density function is $f(x) = cx^{1/n}$. Then

$$1 = \int_0^1 cx^{1/n}dx = \left.\frac{cx^{1/n+1}}{(1/n)+1}\right|_0^1 = \frac{c}{(1/n)+1} = \frac{cn}{1+n} \Rightarrow c = \frac{1+n}{n}.$$

Let x_p represent the *p*th percentile. Then,

$$p = F(x_p) = \int_0^{x_p} \frac{(1+n)x^{1/n}}{n} dx = \left. x^{1+1/n} \right|_0^{x_p} = (x_p)^{1+1/n} \Rightarrow x_p = p^{n/(n+1)}.$$

The desired ratio is

$$\frac{x_{0.3}}{x_{0.2}} = \frac{0.3^{n/(n+1)}}{0.2^{n/(n+1)}} = \left(1.5^n\right)^{1/(n+1)} = \sqrt[n+1]{1.5^n}.$$

384. Solution: B

$$c\int_0^1 x^2 dx = 1 \Rightarrow c = 3$$

$$F(x) = 3\int_0^x t^2 dt = x^3$$

The (100*p*) percentile of a claim under the first plan is the value of *x* satisfying $x^3 = p$.
Since this value of *x* is also the (100p^2) percentile of a claim under the second plan, the cumulative distribution function *F* of a claim under the second plan is $F(x) = p^2 = \left(x^3\right)^2 = x^6$.

Then differentiating yields its probability density function $f(x) = F'(x) = 6x^5$.

385. Solution: D

Let β be the mean of *X*. We know that

$$0.2 = \Pr\left[X \le 2\right] = 1 - e^{-2/\beta}$$

$$\beta = 8.96284$$

Let *t* be the time at which 80% of its computers have failed. Then,

$$0.8 = \Pr\left[X \le t\right] = 1 - e^{-t/8.96284}$$

$$0.2 = e^{-t/8.9628}$$

$$t = 14.42513$$

386. Solution: D

Let *N* = number of tickets the driver receives, which has a Poisson distribution with mean 4.
Note that the total fine is the sum of the integers from 1 to *N*, which equals $\frac{N(N+1)}{2}$.

So the expected total fine is

$$\text{E}\left[\frac{N(N+1)}{2}\right] = 0.5E(N^2) + 0.5E(N) = 0.5[Var(N) + E(N)^2 + E(N)]$$

$$= 0.5(4 + 16 + 4) = 12.$$

387. Solution: C

The probability that the 2 most recent files are both intact is $\frac{40}{51} = \frac{\binom{2}{0}\binom{d-2}{2}}{\binom{d}{2}} = \frac{(d-2)(d-3)}{d(d-1)}$.

This leads to $40d^2 - 40d = 51d^2 - 255d + 306 \Rightarrow 0 = 11d^2 - 215d + 306 = (11d - 17)(d - 18)$.
Since d is an integer that is at least 3, the unique solution is $d = 18$.
The probability that the 3 most recent files are all intact is

$$\frac{\binom{3}{0}\binom{d-3}{2}}{\binom{d}{2}} = \frac{(d-3)(d-4)}{d(d-1)} = \frac{15(14)}{18(17)} = \frac{35}{51} = 0.686..$$

388. Solution: D

Let N = number of claims; let X = the benefit paid.
P(N = 0) = 0.75; P(N = 1) = 0.25
E(X | N = 0) = 0; E(X | N = 1) = 8
Var(X | N = 0) = 0; Var(X | N = 1) = 8^2 = 64
Var(E(X | N)) = 8^2(0.25)(0.75) = 12 because i) a Bernoulli (0-1) random variable has variance $p(1 - p)$ if p = P(Bernoulli variable = 1), and ii)the variance of 8 times the Bernoulli is 8^2 times the variance of the Bernoulli
E(Var(X | N)) = 0(0.75) + 64(0.25) = 16
Var(X) = Var(E(X | N)) + E(Var(X | N)) = 12 + 16 = 28.

389. Solution: D

Let the hats (and men) be A, B, C, and D. There are 24 orderings of the hats drawn. Consider the complement, where at least one man draws his hat.
If A is drawn first, all 6 subsequent combinations meet this event.
If B is drawn first, combinations ACD,CAD, and DCA meet this event.
If C or D is drawn first, each also have three combinations that meet this event.
Hence there are 6 + 3 + 3 + 3 = 15 ways to meet the complementary event.
The desired probability is (24-15)/24 = 0.375.

390. Solution: E

Using the probability density function for gamma distribution and integration by parts we have

$$P(X > 4) = \int_4^\infty \frac{1}{3^2\Gamma(2)} x^{2-1} e^{-x/3} = \frac{1}{9}\int_4^\infty x e^{-x/3} dx = -\frac{1}{9} 3x e^{-x/3}\Big|_4^\infty + \frac{1}{9}\int_4^\infty 3e^{-x/3} dx$$

$$= \frac{4}{3}e^{-4/3} - \frac{1}{9} 9 e^{-x/3}\Big|_4^\infty = \frac{4}{3}e^{-4/3} + e^{-4/3} = \frac{7}{3}e^{-4/3} = 0.6151.$$

391. Solution: D

$$P[A < 214] = 0.96$$
$$P\left[Z < \frac{214-30}{\sigma}\right] = 0.96$$
$$\frac{214-30}{\sigma} = 1.75$$
$$\sigma = 105.143$$
$$P[B < 214] = 0.90$$
$$P\left[Z < \frac{214-\mu}{105.143}\right] = 0.90$$
$$\frac{214-\mu}{105.143} = 1.2816$$
$$\mu = 79.25$$

The value for 0.96 was obtained using the closest entry in the normal table (1.75) while the value for 0.90 (1.2816) was obtained from the list of percentiles at the bottom of the table.

392. Solution: E

$$c\left[\int_5^8 (x-5)\,dx + \int_8^{11}(11-x)dx\right] = c\left[\left.\frac{(x-5)^2}{2}\right|_5^8 - \left.\frac{(11-x)^2}{2}\right|_8^{11}\right] = c(4.5+4.5) = 1$$
$$c = \frac{1}{9}$$
$$\frac{1}{9}\int_6^8 (x-5)\,dx = \left.\frac{(x-5)^2}{9(2)}\right|_6^8 = \frac{9-1}{18} = 0.444.$$

393. Solution: C

Since both events are Poisson distributed, the combination is also Poisson distributed with $\lambda = 0.1 + 0.02 = 0.12$ per year. The claims in forty years are also Poisson distributed with $\lambda = 40(0.12) = 4.8$. To find the mode, First note that

$$p(n) = \frac{e^{-4.8}4.8^n}{n!} = \frac{4.8}{n}\frac{e^{-4.8}4.8^{n-1}}{(n-1)!} = \frac{4.8}{n}p(n-1).$$

This means that the probability increases when n < 4.8 and decreases thereafter. We have $p(4) = 1.2p(3)$ while $p(5) = 0.96p(4)$ and hence the mode must be at 4. The total payments are 4000.

394. Solution: B

Since $f'(x) = -\frac{1}{\beta^2} e^{-\frac{x-d}{\beta}} < 0$, $f(x)$ is monotonically decreasing for $x \geq d$. Thus, the mode of X is at its lowest possible value, that being $x = d$.

Let p be the 10th percentile of X.

$$0.10 = F(p) = \int_d^p \beta^{-1} e^{-(x-d)/\beta} dx = -e^{-(x-d)/\beta}\Big|_d^p = 1 - e^{-(p-d)/\beta}$$

$$0.90 = e^{-(p-d)/\beta}$$

$$\ln(0.9) = -(p-d)/\beta$$

$$p = d - \beta\ln(0.9) = d + \beta\ln(10/9).$$

So, $|p-d| = \left|\beta\ln(10/9) + d - d\right| = \beta\ln(10/9).$

395. Solution: D

$$1 = c\int_{100}^{\infty} x^{-2}dx = -cx^{-1}\Big|_{100}^{\infty} = c100^{-1} \Rightarrow c = 100$$

$$0.5 = 100\int_{100}^{M} x^{-2}dx = -100x^{-1}\Big|_{100}^{M} = 1 - 100M^{-1} \Rightarrow M = 200$$

396. Solution: E

First, $\int_0^{100} kx^{0.25}dx = k\frac{x^{1.25}}{1.25}\Big|_0^{100} = \frac{k}{1.25}100^{1.25} = 1 \Rightarrow k = \frac{1.25}{100^{1.25}}$ and $F(x) = \frac{k}{1.25}x^{1.25} = \frac{x^{1.25}}{100^{1.25}}$

The desired percentile is

$$0.9 = \Pr(X \leq p \mid X > 20) = \frac{\Pr(20 \leq X \leq p)}{\Pr(X > 20)} = \frac{F(p) - F(20)}{1 - F(20)} = \frac{p^{1.25} - 20^{1.25}}{100^{1.25} - 20^{1.25}}$$

$$p = [0.9(100^{1.25} - 20^{1.25})]^{1/1.25} + 20^{1.25} = [0.9(316.23 - 42.29) + 42.29]^{1/1.25} = 93.01.$$

397. Solution: B

The three modes for A must be at 0, 2, and 5. Therefore, the frequencies for B at these values must be 1, 4, and 1.

For values of 1, 3 and 4 the frequencies for A are at least 4. For B they are then at most 2 for each of these values.

There is one frequency of 4 and all other frequencies are 2 or less. Hence there is one mode, at a value of 2.

398. Solution: E

Let X = event of no fires the first 2 years; Y = event of no fires the following 2 years.
Let H = event of homeowner being high-risk; L = event of homeowner being low risk.

$$P[H] = 0.1; P[L] = 0.9$$

$$P[X \mid H] = 0.8^2; P[X \mid L] = 0.99^2$$

$$P[X\text{and}Y \mid H] = 0.8^4; P[X\text{and}Y \mid L] = 0.99^4$$

$$P[Y \mid X] = \frac{P[X\text{and}Y]}{P[X]} = \frac{P[X\text{and}Y \mid H]P[H] + P[X\text{and}Y \mid L]P[L]}{P[X \mid H]P[H] + P[X \mid L]P[L]}$$

$$= \frac{0.8^4(0.1) + 0.99^4(0.9)}{0.8^2(0.1) + 0.99^2(0.9)} = 0.95709.$$

399. Solution: A

A given person in this population has no cancer with probability $1 - 0.2 = 0.8$, cancer other than stage IV with probability $0.2(1-0.08) = 0.184$, and stage IV cancer with probability $0.2(0.08) = 0.016$.

We need to find the probability that i) out of the first $n-1$ patients, $n-c$ have no cancer, $c-5$ have cancer other than stage IV, and 4 have stage IV cancer, <u>and</u> ii) the nth patient has stage IV cancer.

Keeping in mind that we are given that the patients are independent, we multiply the individual probabilities together. We then multiply this product by the number of possible orders in which these results need to occur, to get $p_{N,C}(n,c) = \frac{(n-1)!}{(n-c)!(c-5)!4!}(0.8)^{n-c}(0.184)^{c-5}(0.016)^5$.

400. Solution: E

Let W denote the total claims received.

$$P(W \geq 2) = 1 - P(W < 2) = 1 - [P(X=0, Y=0) + P(X=0, Y=1) + P(X=1, Y=0)]$$

$$= 1 - \left(\frac{8}{54} + \frac{7}{54} + \frac{6}{54}\right) = \frac{33}{54} = 0.61$$

401. Solution: C

$$P\left(Z < \frac{1000-\mu}{500}\right) = 0.3446$$
$$\frac{1000-\mu}{500} = -0.40$$
$$\mu = 1200$$
$$P\left(Z < \frac{P_{0.9}-1200}{500}\right) = 0.90$$
$$\frac{P_{0.9}-1200}{500} = 1.2816$$
$$P_{0.9} = 1840.8$$

Median equals the mean or 1200, the difference is $1840.8 - 1200 = 640.8$.

402. Solution: A

$$Var(X) = \lambda = E(X^2) - E(X)^2$$
$$\lambda = 0.2756 - \lambda^2$$
$$\lambda^2 + \lambda - 0.2756 = 0$$

By quadratic formula: $\lambda = 0.2250$
Then: $15\lambda = 3.37474$
Then,

$$P(Y > 2) = 1 - P(Y \le 2)$$
$$= 1 - \left[P(0) + P(1) + P(2)\right]$$
$$= 1 - e^{-3.37474}\left[\frac{3.37474^0}{0!} + \frac{3.37474^1}{1!} + \frac{3.37474^2}{2!}\right]$$
$$= 0.65536.$$

403. Solution: C

Hypergeometric:

$$\frac{\binom{5}{1}\binom{4}{3}\binom{6}{1}}{\binom{15}{5}} = 0.040$$

404. Solution: B

The marginal distribution of *Y* is:

$$p_Y(0) = p(0,0) + p(1,0) + p(2,0) = \frac{24}{126} + \frac{17}{126} + \frac{10}{126} = \frac{51}{126}$$

$$p_Y(1) = p(0,1) + p(1,1) + p(2,1) = \frac{21}{126} + \frac{14}{126} + \frac{7}{126} = \frac{42}{126}$$

$$p_Y(2) = p(0,2) + p(1,2) + p(2,2) = \frac{18}{126} + \frac{11}{126} + \frac{4}{126} = \frac{33}{126}$$

$$E(Y) = 0\frac{51}{126} + 1\frac{42}{126} + 2\frac{33}{126} = \frac{108}{126}$$

$$E(Y^2) = 0^2\frac{51}{126} + 1^2\frac{42}{126} + 2^2\frac{33}{126} = \frac{174}{126}$$

$$V(Y) = \frac{174}{126} - \left(\frac{108}{126}\right)^2 = 0.64626.$$

405. Solution: A

The expected monthly payout is:
100,000 * [300,000 * P(match all 5 and bonus number) + 50,000 * P(match all 5 but no bonus number) + 0 * P(match fewer than 5 numbers)].

$$P(\text{Match all five and Bonus}) = \frac{1}{\binom{30}{5}} * \frac{1}{5} = \frac{1}{712{,}530}$$

$$P(\text{Match all five and No Bonus}) = \frac{1}{\binom{30}{5}} * \frac{4}{5} = \frac{4}{712{,}530}$$

Thus, expected monthly payout equals:

$$100{,}000\left[300{,}000\frac{1}{712{,}530} + 50{,}000\frac{4}{712{,}530}\right] = 70{,}172.$$

406. Solution: E

Y is a binomial random variable with n = 5 and probability of success equal to $s + t$.
Next, the probability that Y = 5 is:

$$P(Y=5)=\binom{5}{5}(s+t)^5=c \Rightarrow s+t=c^{1/5} \Rightarrow s=c^{1/5}-t$$

Then,

$$s+0.75s+t=1.75s+t=1$$

$$t=1-1.75(c^{1/5}-t)$$

$$t=1-1.75c^{1/5}+1.75t$$

$$0.75t=1.75c^{1/5}-1$$

$$t=\frac{1.75c^{1/5}-1}{0.75}=\frac{7c^{1/5}-4}{3}.$$

407. Solution: E

Let X denote the time until failure. Because the median of X is 3, we have:

$$P(X>3)=0.5=e^{-3/\beta}$$

$$\ln 0.5=-\frac{3}{\beta}$$

$$\beta=4.32809$$

The variance of an exponential random variable is $\beta^2=18.73232$.
Because the exponential distribution is memoryless, the variance of the future lifetime from t = 0.5 is also $\beta^2=18.73232$.

408. Solution: E

Let H be the event a single head and F be the event the fair coin is selected. Then,

$$P(HHH \mid HH)=\frac{P(HHH)}{P(HH)}$$

$$P(HH)=P(HH \mid F)P(F)+P(HH \mid F')P(F')=[(1/2)(1/2)](1/2)+[(1)(1)](1/2)=5/8$$

$$P(HHH)=P(HHH \mid F)P(F)+P(HHH \mid F')P(F')=[(1/2)(1/2)(1/2)](1/2)+[(1)(1)(1)](1/2)=9/16$$

$$P(HHH \mid HH)=\frac{9/16}{5/8}=\frac{9}{10}.$$

409. Solution: A

Let X represent the loss. Since it is given that losses never exceed 10, we have

$$1 = \mathrm{P}[X \le 10] = F(10) = c\left(\frac{10}{15}\right)^{\frac{4}{3}} \Rightarrow c = \left(\frac{15}{10}\right)^{\frac{4}{3}}.$$

Since the probability that a given loss is partially reimbursed is 0.56, we have

$$0.56 = \mathrm{P}[X > m] = 1 - \mathrm{P}[X \le m] = 1 - F(m) = 1 - c\left(\frac{m}{15}\right)^{\frac{4}{3}} = 1 - \left(\frac{15}{10}\right)^{\frac{4}{3}}\left(\frac{m}{15}\right)^{\frac{4}{3}} = 1 - \left(\frac{m}{10}\right)^{\frac{4}{3}}.$$

Solving for m yields $m = 10\left(1-0.56\right)^{\frac{3}{4}} = 5.40$.

410. Solution: E

We need to find $F_{X,Y}(2,3) = \mathrm{P}[X \le 2 \text{ and } Y \le 3]$, which equals the probability of at most 2 tornadoes and a total loss of at most 3.
The probability of no tornadoes or exactly one tornado is 0.8 + 0.12 = 0.92, and in this situation, the total loss is no more than 2 and is therefore no more than 3.
The probability of exactly two tornadoes is 0.05, and in this situation, the total loss is at most 3 as long as both losses are not 2, which occurs with probability $1-(0.5)^2 = 0.75$.
Therefore, $F_{X,Y}(2,3) = 0.92 + 0.05(0.75) = 0.9575$.

411. Solution: A

Let the marginal pdf for be $p_Y(y)$, for y = 0,1,2.

$$p_Y(0) = p(0,0) + p(1,0) + p(2,0) + p(3,0) = (12+9+6+3)/60 = 30/60$$

$$p_Y(1) = p(0,1) + p(1,1) + p(2,1) + p(3,1) = (8+6+4+2)/60 = 20/60$$

$$p_Y(2) = 1-(30+20)/60 = 10/60$$

$$E(Y) = 0(3/6) + 1(2/6) + 2(1/6) = 4/6$$

$$Var(Y) = 0^2(3/6) + 1^2(2/6) + 2^2(1/6) - (4/6)^2 = (36-16)/36 = 20/36 = 0.56$$

412. Solution: D

This is the same as the probability that all three claims are less than 7, which is $(0.7)^3 = 0.343$.

413. Solution: B

This is a negative binomial distribution. The probability of finding fraud is 0.9(0.2) = 0.18. Let $p(x)$ be the probability of examining x policies. Then,

$$p(1) = 0$$
$$p(2) = 0.18^2 = 0.0324$$
$$p(3) = 2(0.82)(0.18)^2 = 0.0531$$
$$p(4) = 3(0.82)^2(0.18)^2 = 0.0654$$
$$p(5) = 1 - 0.0324 - 0.0531 - 0.0654 = 0.8491$$
$$E(X) = 2(0.0324) + 3(0.0531) + 4(0.0654) + 5(0.8491) = 4.7312.$$

414. Solution: E

This is a hypergeometric problem.

$$P[X = 3] = \frac{\binom{6}{3}\binom{3}{1}}{\binom{9}{4}} + P[X = 4] = \frac{\binom{6}{4}\binom{3}{0}}{\binom{9}{4}}$$
$$= 0.4762 + 0.1190 = 0.5952.$$

415. Solution: B

Let C be the expected number buying the carriage only
S be the expected number buying the seat only
B be the expected number buying both

Then C + S + B + 0.60(200) = 200
C + B = 0.20(200) = 40
S + B = 0.35(200) = 70
40 + 70 – B + 120 = 200
B = 30, C = 10, S = 40
Revenue = 10(300) + 40(100) + 30(360) = 17,800

416. Solution: C

Let X_i be the number of claims for policyholder *i*. We have $1 = Var(X_i) = E(X_i)$. The total amount paid is then $S = 100(X_1 + \cdots + X_{1000})$. Note that the solution employs the continuity correction.

$$E(S) = 100[E(X_1) + \cdots + E(X_{1000})] = 100(1,000)(1) = 100,000$$

$$Var(S) = 100^2[Var(X_1) + \cdots + Var(X_{1000})] = 100^2(1,000)(1) = 10,000,000$$

$$StDev(S) = 10,000,000^{1/2} = 3,162.28$$

$$\text{Premium} = 1.03(100,000) = 103,000$$

$$P(S > 103,000) = P\left(Z > \frac{103,050 - 100,000}{3,162.28} = 0.9645\right) = 0.167$$

417. Solution: B

This is a negative binomial distribution.

P(Second fatal crash occurs on the fourth save) = $\binom{3}{1}(0.02)^2(0.98)^2$ = 0.001152.

418. Solution: C

Let M = # of accidents in first year, N = # of accidents in second year.

$$P(M = 1, N = 1 \mid M + N = 2) = \frac{P(M = 1, N = 1)}{P(M + N = 2)} = \frac{0.08(0.08)}{0.9(0.02) + 0.08(0.08) + 0.02(0.9)}$$

$$= \frac{0.0064}{0.0424} = 0.151$$

419. Solution: B

X = amount of time (years) before repair is needed, which is uniform on [0, 5]

$$P[X > 4.5 \mid X > 2]$$

$$= \frac{P[X > 4.5]}{P[X > 2]}$$

$$= \frac{\frac{5 - 4.5}{5 - 0}}{\frac{5 - 2}{5 - 0}} = \frac{1}{6} = 0.167$$

420. Solution: C

Using integration by parts

$$\int_1^2 xe^{-x}dx = -xe^{-x}\Big|_1^2 - \int_1^2 -e^{-x}dx = (-2e^{-2} + e^{-1}) - e^{-x}\Big|_1^2 = -3e^{-2} + 2e^{-1} = 0.329753.$$

421. Solution: C

Let X = lifetime of a television of this brand:

$$0.5 = P[X < 2.7] = 1 - e^{-2/7/\beta}$$
$$2.7/\beta = 0.6931$$
$$\beta = 3.8953$$
$$0.875 = P[X < P] = 1 - e^{-p/3.8953}$$
$$p/3.8953 = 2.0794$$
$$p = 8.10.$$

422. Solution: A

Since one of C or D is guaranteed to occur, and they are mutually exclusive:
$P[A] = P[A \cap C] + P[A \cap D]$ so,
$P[A \cap D] = 0.75 - 0.50 = 0.20$.
Then, since one of A and B is guaranteed to occur:
$P[D] = P[A \cap D] + P[B \cap D]$,
so $P[B \cap D] = 0.20 - 0.20 = 0.00$

423. Solution: C

We are given the following:

	Overstated	Didn't overstate	Row Total
Claimed at least 1000	0.45		0.70
Claimed less than 1000			
Column Total	0.50		

This gives

	Overstated	Didn't overstate	Row Total
Claimed at least 1000	0.45	0.25	0.70
Claimed less than 1000	0.05	0.25	0.30
Column Total	0.50	0.50	1.00

Then, P(Overstated | Less than 1000) = P(Overstated and Less than 1000)/P(Less than 1000) = 0.05/0.30 = 1/6.

424. Solution: E

There are $({}_{26}C_3)(3)$ = 7,800 ways to select the 3 distinct upper case letters and choose one for the leading position of the password. For each of those ways, there are ${}_4P_2$ = 12 ways to allocate the other two letters among the 4 open positions. So there are 93,600 ways to allocate the letters. For each, there are 10^2 ways to select and place the two digits in the remaining two open positions. Thus, there are 9,360,000 possibilities.

425. Solution: B

Let X = number of fillings, which has a Poisson distribution with mean λ

$$0.18 = P[X = 0] = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda}$$

$\lambda = 1.7147$

Since the mean is 1.7147, the mode will be 1 as it can be shown that when the mean is not a whole number, the mode is the largest whole number less than the mean. When the mean is a whole number, there are two modes, the mean and one less than the mean.

426. Solution:B

$$1=\int_0^{50} ktdt = 1250k \Rightarrow k = \frac{1}{1250}$$

$$P[T \le 25 \,|\, T \ge 20] = \frac{P[20 < T < 25]}{P[T > 20]} = \frac{\int_{20}^{25} \frac{t}{1250}dt}{\int_{20}^{50} \frac{t}{1250}dt} = \frac{(625-400)/2500}{(2500-400)/2500} = \frac{225}{2100} = 0.10714$$

427. Deleted

428. Solution: B

Let X = working lifetime of chips, in years. Then

$$0.05 = P[X \le t] = 1 - e^{-t/7.2}$$

$$e^{-t/7.2} = 0.95$$

$$-t/7.2 = -0.05129$$

$$t = 0.3693$$

429. Solution: E

$$1=\int_0^2 f(x)dx = \int_0^2 cxdx = 2c \Rightarrow c = 0.5$$

$$0.8 = \int_0^p 0.5xdx = 0.25p^2 \Rightarrow p = \sqrt{3.2} = 1.7889$$

430. Solution: E

Let X and Y represent the exponentially distributed life spans of televisions A and B, respectively, and let λ_1 and λ_2 represent the means for X and Y, respectively.

$$\text{Var}(X) = \lambda_1^2 = 5.6 \Rightarrow \lambda_1 = 2.3664$$

$$0.49 = P(X > T) = 1 - \left(1 - e^{-T/\lambda_1}\right) = e^{-T/2.3664} \Rightarrow -T/2.3664 = -0.7133 \Rightarrow T = 1.6881$$

$$0.7 = P(Y > T) = e^{-T/\lambda_2} = e^{-1.6881/\lambda_2} \Rightarrow -1.6882/\lambda_2 = -0.3567 \Rightarrow \lambda_2 = 4.7332.$$

$$\text{Var }(Y) = \lambda_2^2 = 4.7332^2 = 22.403.$$

431. Solution: D

Let X_i be the profit earned in quarter *i*. Then

$$0.8 = P[X_i > 0] = P\left[Z > \frac{0-\mu}{\sigma}\right] \Rightarrow \mu/\sigma = 0.84$$

Let $S = X_1 + X_2 + X_3 + X_4$ be the total profit for the year. Then *S* has a normal distribution with mean 4μ and variance $4\sigma^2$. Then,

$$P[S > 0] = P\left[Z > \frac{0-4\mu}{2\sigma} = -2\mu/\sigma = -1.68\right] = 0.9535.$$

432. Solution: B

$$0.1056 = P[N_{past} < 400] = P\left[Z < \frac{400-500}{\sigma}\right] \Rightarrow \frac{400-500}{\sigma} = -1.25 \Rightarrow \sigma = 80$$

$$P\left[370 < N_{future} < 730\right] = P\left[\frac{370-550}{100} < Z < \frac{730-550}{100}\right] = P\left[-1.8 < Z < 1.8\right] = 0.9641 - 0.0359 = 0.9282$$

433. Solution: A

Let *X* be the profit from flood insurance and *Y* be the profit from fire insurance and let *S* = *X* + *Y* be the total profit. Let μ and σ be the mean and standard deviation of *X*.

$$P[X > 0] = P\left[Z > \frac{0-\mu}{\sigma}\right] = 0.67 \Rightarrow \frac{0-\mu}{\sigma} = -0.44 \Rightarrow 0.44\sigma = \mu$$

S has a normal distribution with mean $E[X] + E[Y] = \mu + 3\mu = 4\mu$ and standard deviation

$$\sqrt{Var(X) + Var(Y)} = \sqrt{\sigma^2 + 9\sigma^2} = \sqrt{10}\sigma .$$

$$P[S > 0] = P\left[Z > \frac{0-4\mu}{\sqrt{10}\sigma} = \frac{-4(0.44\sigma)}{\sqrt{10}\sigma} = -0.56\right] = 0.71$$

434. Solution: D

Let X_j = number of accidents occurring on the *j*th roller coaster, for *j* = 1, 2, which is Poisson distributed with mean λ_j.

The probability that at least one accident occurs on roller coaster *j* equals

$$1 - \mathrm{P}\left[X_j = 0\right] = 1 - \frac{e^{-\lambda_j}\left(\lambda_j\right)^0}{0!} = 1 - e^{-\lambda_j}.$$

Since the probability that at least one accident occurs on the second roller coaster is twice that for the first roller coaster, $1 - e^{-\lambda_2} = 2\left(1 - e^{-\lambda_1}\right)$. Then

$$e^{-\lambda_2} = 2e^{-\lambda_1} - 1 \Rightarrow \lambda_2 = -\ln\left(2e^{-\lambda_1} - 1\right) = -\ln\left(2e^{-0.5} - 1\right) = 1.546 .$$

435. Solution: B

Standard deviation equals 85 implies mean = 85.
Let *M* equal the median

$P[X > M] = 0.5$

$e^{-M/85} = 0.5$

$\frac{-M}{85} = \ln 0.5$

$M = 58.92$

So median minus mean = 58.92 - 85 = –26.08

436. Solution: D

The mean equals the standard deviation, so is also 1000. By the memoryless property of the exponential distribution,

$$P[X > 1500 \mid X > 1000] = P[X > 500] = \int_{500}^{\infty} 0.001e^{-0.001x}dx = -e^{-0.001x}\Big|_{500}^{\infty} = e^{-0.5} = 0.6065.$$

437. Solution: D

P[exactly 2 injuries]
= P[1 or 2 injuries] + P[2 or 3 injuries] – P[at least 1 injury] + P[4 or more injuries]
= 0.25 + 0.036 – 0.26 + 0.002
= 0.028

438. Solution: E

The event that the first 5 items fill the truck to its capacity is equivalent to the event that the first 5 items include 3 items of Type A and 2 items of Type B. This is a hypergeometric probability:

$$\frac{\binom{6}{3}\binom{4}{2}}{\binom{10}{5}} = \frac{20(6)}{252} = \frac{10}{21}$$

439. Solution: D

Let X and Y be the profits in years 1 and 2 respectively. Let $S = X + Y$ be the total profit for the two years.

$$0.8531 = P[X > 0] = P\left[Z > \frac{0-\mu}{\sigma_X}\right] \Rightarrow -\mu/\sigma_X = -1.05 \Rightarrow \sigma_X = \mu/1.05$$

$$0.9192 = P[Y > 0] = P\left[Z > \frac{0-\mu}{\sigma_Y}\right] \Rightarrow -\mu/\sigma_Y = -1.40 \Rightarrow \sigma_X = \mu/1.40$$

$$E[S] = \mu + \mu = 2\mu, Var(S) = \sigma_X^2 + \sigma_Y^2 = \mu^2(1/1.05^2 + 1/1.40^2), \sigma_S = 1.1905\mu$$

$$P[S > 0] = P\left[Z > \frac{0-2\mu}{1.1905\mu} = -1.68\right] = 0.9535$$

440. Solution: E

The distribution function for an exponential distribution is $F(x) = 1 - e^{-x/\lambda}$.

$$1 - e^{-400/\lambda} = 0.5$$

$$e^{-400/\lambda} = 0.5$$

$$-400/\lambda = \ln 0.5$$

$$\lambda = 577.08.$$

441. Solution: C

The variance of a uniform distribution is 1/12 of the square of the interval length. The standard deviation of the hospitalization charge is $\frac{b}{\sqrt{12}} = 9.6 \Rightarrow b = 33.255$. The standard deviation of the surgery charge is $\frac{2b-6}{\sqrt{12}} = \frac{2(33.255)-6}{\sqrt{12}} = 17.4677.$

442. Solution: B

$$P[X > 11] = 0.2 = \frac{b-11}{b-a}$$
$$b - a = 5(b-11)$$
$$P[X > 8] = 0.6 = \frac{b-8}{b-a}$$
$$0.6[5(b-11)] = b-8$$
$$3b - 33 = b - 8$$
$$b = 12.5$$
$$a = 5$$
$$Var(X) = \frac{(12.5-5)^2}{12} = 4.6875.$$

443. Solution: E

$$E[|X-2|] = \int_1^5 |x-2| f(x)dx = \int_1^2 (2-x) f(x)dx \int_2^3 (x-2) f(x)dx \int_3^5 (x-2) f(x)dx$$
$$= \int_1^2 (2-x)\frac{x-1}{4}dx \int_2^3 (x-2)\frac{x-1}{4}dx \int_3^5 (x-2)\frac{5-x}{4}dx$$

444. Solution: C

The question requires calculation of E$\left(\frac{Y}{X}\right)$**.** The joint density of *X* and *Y* is given by

			X	
		1	2	3
Y	0	6/18	4/18	2/18
	1	3/18	2/18	1/18

The six cells yield the quotients $\frac{Y}{X}$ = 0, 0, 0, 1, 1/2, 1/3. The expected value of $\frac{Y}{X}$ is thus $\left(\frac{3}{18}\right)\left(\frac{1}{1}\right)+\left(\frac{2}{18}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{18}\right)\left(\frac{1}{3}\right)$, which is $\frac{13}{54}$.

445. Solution: C

There are 3(420) = 1260 drivers who have exactly one risk factor. There are 320 with all three factors. Look at the 1200 drivers with the first risk factor. 420 + 320 = 740 are accounted for. The remining 460 drivers have exactly one other risk factor. This is 230 for each and hence 3(230) = 690 with exactly two risk factors. Also adding the 480 with no risk factors gives a total of 1260 + 320 + 690 + 480 = 2750.

446. Solution: A

We have that 11.0 = 0*y* + 5(2*x* + 2*x*) + 30(3*x*), so *x* =0.10. Thus *y* = 0.30. The variance we seek is therefore $(0–11)^2(0.3) + (5–11)^2(0.4) + (30–11)^2(0.3) = 159.0$.

447. Solution: D

Let *X* be loss and *Y* be the amount paid by the insurance. Then *Y* = 0 if *X* < 400 and *Y* = *p*(*X* – 400) otherwise. Then,

$$90 = \int_{400}^{1000} p(x-400)(0.001)dx = p[(x-400)^2/2](0.001)\Big|_{400}^{1000} = 180p \Rightarrow p = 0.50.$$

448. Solution: D

Let *X* and *Y* be the two claims; *X* and *Y* are independent and identically distributed. Let *U* and *V* the amount paid on each claim. *U* and *V* are also independent and identically distributed.
We want

$P(\max(U,V) \le t) = P(U \le t, V \le t) = P(U \le t)^2$. We have

$P(U \le t) = P(X - 5 \le t) = P(X \le t+5) = (t+5)/10.$

Thus, $P(\max(U,V) \le t) = \left(\frac{5+t}{10}\right)^2$.

449. Solution: E

$$\begin{aligned} p(4,9) + p(5,9) &= P(3 < X \le 5, 8 < Y \le 9) = F(5,9) - F(3,9) - F(5,8) + F(3,8) \\ &= 0.84 - 0.65 - 0.67 + 0.53 = 0.05 \end{aligned}$$

450. Solution: A

$$E(U) = 100\lambda + 150\lambda + 200\lambda = 450\lambda$$

$$Var(U) = 100^2\lambda + 150^2\lambda + 200^2\lambda = 72,500\lambda$$

$$0.9 = \frac{\sqrt{72,500\lambda}}{450\lambda} \Rightarrow \lambda = 72,500/(0.9 \times 450)^2 = 0.442$$

451. **Solution: B**

The distribution function is $F(y)=\int_5^y e^{-(t-5)}dt=-e^{-(t-5)}\Big|_5^y=1-e^{-(y-5)}$. The density function for the first order statistic is: $g_1(y)=3f(y)[1-F(y)]^2=3e^{-(y-5)}[e^{-(y-5)}]^2=3e^{-3(y-5)}$. The expected value is:

$$\int_5^\infty 3ye^{-3(y-5)}dy=-ye^{-3(y-5)}-(1/3)e^{-(y-5)}\Big|_5^\infty=5+1/3=5.33.$$

Alternatively, recognize that the first order statistics is 5 plus an exponentially distributed random variable with mean 1/3.

452. Solution: D

The group has a normal distribution with mean 25 x 1000 = 25,000 and standard deviation of 625 x $(25)^{1/2}$ = 3,125.

P[Lose Money] = P[Claims $>$ 27,500] $= P\left[Z>\frac{27,500-25,000}{3125}\right]=P[Z>0.80]$ = 0.2119.

453. Solution: C

Let p represent the percentage, expressed as a decimal; let Y represent the loss; let X represent the claim. Then X = 0 when Y < 240 and $X = p(Y - 240)$ otherwise. Then X has a mixed distribution with $P(X=0)=P(Y<240)=0.5$ and for $0 < x < 240p$,

$F_X(x)=P(X\le x)=P(p(Y-240)\le x)=P(Y\le 240+x/p)=0.5+x/480p$ and $f_X(x)=1/480p$

.

Then,

$$E(Y)=0(0.5)+\int_0^{240p} x/(480p)dx=(240p)^2/(960p)=60p$$

$$E(Y^2)=0^2(0.5)+\int_0^{240p} x^2/(480p)dx=(240p)^3/(1440p)=9600p^2$$

$$2000=9600p^2-(60p)^2=6000p^2 \Rightarrow p=0.577.$$

So $p=\sqrt{\frac{2000}{6000}}=0.57735.$

454. Solution: B

P[claim > 300 | claim < 400] = P[loss > 700 | loss < 800] = P[700 < loss < 800] / P[loss < 800] = [(800 - 700)/(1000 - 0)] / [(800 - 0)/(1000 - 0)] = 0.125.

455. **Solution: D**

L = event that policyholder is low risk
H = event that policyholder is high risk
N = event that policyholder undergoes no hospitalizations

$$\text{P}[L]=0.7,\ \text{P}[H]=0.3$$
$$\text{P}[N|L] = e^{-0.05},\ \text{P}[N|H] = e^{-0.3}$$
$$\text{P}[L|N] = \frac{P[L]P[N \mid L]}{P[L]P[N \mid L] + P[H]P[N \mid H]}$$
$$= \frac{0.7e^{-0.05}}{0.7e^{-0.05} + 0.3e^{-0.3}} = 0.749756.$$

456. Solution: E

This is a hypergeometric probability:

$$\frac{\binom{6}{2}\binom{4}{2}}{\binom{10}{4}} = \frac{(15)(6)}{210} = 0.42857.$$

457. Solution: C

The probability of at least one visit is one minus the probability of no visits. The probability of no visits is $0.28 + 0.12\frac{e^{-1}1^0}{0!} + 0.42\frac{e^{-2}2^0}{0!} + 0.18\frac{e^{-3}3^0}{0!} = 0.39$, so the answer is 1-0.39 = 0.61.

458. Solution: C

Let u be the number of units of A purchased, and hence 10 – u is the number of units of B purchased.

The payoff will be $uX + (10 - u)Y$. Because of independence,

$Var(uX + (10-u)Y) = u^2 30 + (10-u)^2 20 = 50u^2 - 400u + 2000.$ Differentiating and setting the derivative to 0 to yields $u = 4$. The second derivative is positive, indicating it is a minimum.

459. Solution: D

Poisson with mean of 16 means the variance is also 16 and the standard deviation is 4. Then for samples of size 64, the mean for the average is 16 and the standard deviation for the average is 4/8 = 0.5 Then using the normal approximation from the Central Limit Theorem:

$$P(15 < \bar{X} < 18) = P\left(\frac{15-16}{0.5} < Z < \frac{18-16}{0.5}\right) = P(-2 < Z < 4) = 0.9772.$$

The continuity correction is not needed. If it were, the adjustment would be 1/128, not 1/2.

460. Solution: E

$$P(N=1, X \le 2000) = 0.08P(X \le 2000 \mid N=1) = 0.08(3/4) = 0.06$$
$$P(N=2, X \le 2000) = 0.08P(X \le 2000 \mid N=2) = 0.02(1/2) = 0.01$$
$$P(N>0, X \le 2000) = 0.06 + 0.01 = 0.07$$
$$P(N=1 \mid N>0, X \le 2000) = \frac{P(N=1, X \le 2000)}{P(N>0, X \le 2000)} = \frac{0.06}{0.07} = 6/7$$

461. Solution: B

Let λ represent the mean number of tickets. Then,

$$e^{-1.5} = P(0 \text{ tickets}) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda}, \text{ so } \lambda = 1.5$$
$$P(X \ge 4 \mid X \ge 1) = \frac{P(X \ge 4)}{P(X \ge 1)} =$$
$$\frac{1-(\mathrm{P}(0)+\mathrm{P}(1)+\mathrm{P}(2)+\mathrm{P}(3))}{1-\mathrm{P}(0)} = \frac{0.06564}{0.77687} = 0.8449.$$

462. Solution: E

Let X = number of people tested until a diabetic is found
Then X is geometric with success parameter p.

$$P(X \le m \mid X \le n) = \frac{P(X \le m)}{P(X \le n)} = \frac{\sum_{k=1}^{m} P(X=k)}{\sum_{k=1}^{n} P(X=k)} = \frac{\sum_{k=1}^{m} p(1-p)^{k-1}}{\sum_{k=1}^{n} p(1-p)^{k-1}} = \frac{1-(1-p)^m}{1-(1-p)^n}.$$

463. Solution: A

Let X = number of brake jobs needed. Then, $P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$.

$$\mathrm{P}(X = 0) = 0.9 = e^{-\lambda}; \lambda = -\ln 0.9$$
$$\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X = 0) - \mathrm{P}(X = 1) = 1 - e^{-\lambda} - e^{-\lambda}\lambda = 0.00518.$$

464. Solution: A

The variance for a uniform distribution is the square of the interval length, divided by 12. The standard deviation is the interval length, divided by $\sqrt{12}$.

Since the distribution is uniform and 75% – 25% = 50%, the interquartile range is half the interval length.

The interval length cancels in the ratio of standard deviation to interquartile range, so this ratio is

$$\frac{\frac{1}{\sqrt{12}}}{\frac{1}{2}} = \frac{1}{\sqrt{3}}, \text{ or } 1 : \sqrt{3}.$$

465. Solution: A

For an exponential distribution, the mean is equal to the standard deviation. Hence, the distribution function is $F(x) = 1 - e^{-x/\sigma}$. Then,

$$P(L > d + \sigma) = e^{-(d+\sigma)/\sigma} = e^{-d/\sigma} e^{-1}$$
$$P(L > d + 0.5\sigma) = e^{-(d+0.5\sigma)/\sigma} = e^{-d/\sigma} e^{-.5} = e^{-d/\sigma} e^{-1} e^{0.5} = (0.20)e^{0.5} = 0.3297.$$

466. Solution: C

Let X = automobile claims and Y = homeowners claims. X and Y are binomial with n = 1.

Note that $\mathrm{E}(XY) = 1(1)\,\Pr(X = 1, Y = 1)$ and thus E(XY) is the solution.

$$\rho = 0.30 = \frac{\mathrm{E}(XY) - \mathrm{E}(X)\mathrm{E}(Y)}{\sigma_X \sigma_Y}$$

$$\sigma_X = \sqrt{0.1(1-0.1)} = 0.30$$

$$\sigma_Y = \sqrt{0.05(1-0.05)} = 0.218$$

Then $0.30 = \dfrac{\mathrm{E}(XY) - (0.1)(0.05)}{(0.30)(0.218)} \Rightarrow \mathrm{E}(XY) = 0.025$

And $\mathrm{E}(XY) = 0.025$

467. Solution: B

The solution is the sum of (y/x)p(x,y) over the six combinations of x and y. Because the three cases where y = 0 contribute nothing to the total, the solution is

(1/1)(1/18) + (1/2)(1/18) + (1/3)(1/18) = (6 + 3 + 2)/108 = 11/108.

468. Solution: D

Let H be the event that a driver is high-risk and L be the event that a driver is low-risk.
Let T be the event that a driver has an accident this year and N be the event that the same driver has an accident next year. Then,

$$P[N \mid T] = \frac{P[N \cap T]}{P[T]} = \frac{P[N \cap T \mid H]P[H] + P[N \cap T \mid L]P[L]}{P[T \mid H]P[H] + P[T \mid L]P[L]}$$
$$= \frac{P[N \mid H]P[T \mid H]P[H] + P[N \mid L]P[T \mid L]P[L]}{P[T \mid H]P[H] + P[T \mid L]P[L]}$$
$$= \frac{(0.12)^2(0.10) + (0.05)^2(0.90)}{(0.12)(0.10) + (0.05)(0.90)} = 0.6474.$$

469. Solution: C

Let a and b be the end points of the range for this uniform distribution. Then,

$$12 = \frac{a+b}{2} \Rightarrow a+b=24$$

$$18 = 0.75b + 0.25a \Rightarrow 3b + a = 72$$

$$2b = 48$$

$$b = 24$$

$$a = 0$$

$$Var = (24-0)^2/12 = 48.$$

470. **Solution: C**

$$\text{P}\left(X > 1\right) = 1 - P(X = 0) - P(X = 1) = 1 - 1/5 - (1/5)(4/5) = 16/25 = (4/5)^2,$$

so the conditional probability function is $\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^{x-2}$, for $x = 2, 3, 4, ...$

This is the unconditional probability function shifted to the right by 2. Thus, the conditional probability distribution has the same variance as the unconditional probability distribution, which is a geometric distribution with variance (4/5)/(1/5) = 20..

471. Solution: A

	x= 0	1	2
y = 0	1/36	3/36	5/36
1	2/36	4/36	6/36
2	3/36	5/36	7/36
$p(x)$	6/36	12/36	18/36

$$E(X) = 0(6/36) + 1(12/36) + 2(18/36) = 4/3$$

$$E(X^2) = 0(6/36) + 1(12/36) + 4(18/36) = 7/3$$

$$V(X) = (7/3) - (4/3)^2 = 5/9 = 0.556$$

472. Solution: D

$$\binom{n}{3}(0.6)^3(0.4)^{n-3} = 5\binom{n}{2}(0.6)^2(0.4)^{n-2}$$

$$\frac{n!}{(n-3)!3!}(0.6)^3(0.4)^{n-3} = 5\frac{n!}{(n-2)!2!}(0.6)^2(0.4)^{n-2}$$

$$\frac{1}{6}(0.6) = 5\frac{1}{(n-2)2}(0.4)$$

$$0.1 = \frac{1}{n-2}$$

$$n = 12$$

473. Solution: B

Let X represent the loss, which is uniformly distributed on the interval [0, 1], let d represent the deductible, and let Y represent the claim on this loss. Note that we are given that $0 < d < 1$.

Note that $Y = \max(0, X - d)$. So $Y = 0$ with probability d, and is uniformly distributed on the interval $[0, 1 - d]$ with probability $1 - d$. Thus the expected value of the claim payment is

$$0.245 = \mathrm{E}(Y) = d(0) + (1-d)\left(\frac{0+1-d}{2}\right) = \frac{(1-d)^2}{2}.$$

Keeping in mind that $0 < d < 1$, $1-d = \sqrt{0.49} = 0.7$.

The expected value of the square of the claim payment is

$$\mathrm{E}\left(Y^2\right) = d(0)^2 + (1-d)\int_0^{1-d}\frac{y^2dy}{1-d} = \frac{(1-d)^3}{3}.$$

The variance of the claim payment on a given loss is

$$\mathrm{Var}\ (Y) = \mathrm{E}\left(Y^2\right) - \left[\mathrm{E}(Y)\right]^2 = \frac{(1-d)^3}{3} - \left[\frac{(1-d)^2}{2}\right]^2 = \frac{0.7^3}{3} - \frac{0.7^4}{4} = 0.0543.$$

474. Solution: C

Let X represent the amount of time (in years) between now and the first ticket.
Let Y represent the amount of time (in years) between the first ticket and the second ticket.
X and Y are independent and exponentially distributed with means 0.8 and $\mu > 0.8$, respectively.

The variance of an exponential distribution is the square of its mean, and the variance of a sum of independent random variables is the sum of the variances. Therefore, the variance of the number of years from now to the second ticket is

$2.65 = Var(X+Y) = Var(X) + Var(Y) = 0.8^2 + \mu^2$. Thus, $\mu = \sqrt{2.65-(0.8)^2} = 1.418$..

475. Solution: A

$$p(x)=\begin{cases}1-P(X\geq 1)=1-0.444=0.556, \text{ for } x=0\\ P(X\geq 1)-P(X\geq 2)=0.444-0.250=0.194, \text{ for } x=1\\ P(X\geq 2)-P(X\geq 3)=0.250-0.111=0.139, \text{ for } x=2\\ P(X\geq 3)-P(X\geq 4)=0.111-0.028=0.083, \text{ for } x=3\\ P(X\geq 4)=0.028, \text{ for } x=4\end{cases}$$

$$\mathrm{E}(X)=0(0.556)+1(0.194)+2(0.139)+3(0.083)+4(0.028)=0.833.$$

476. Solution: D

Add the probabilities for number of deaths less than 2. Add the probabilities for number of deaths less than 2 and number of disabilities at least 2. Divide second sum by first sum.

$$\frac{0.18}{0.86}=0.20930.$$

477. Solution: B

$$P[T\leq 10\,|\,T\geq 5]=\mathrm{P}[5\leq T\leq 10]\,/\,\mathrm{P}[T\leq 5]$$

$$\frac{\int_5^{10} ktdt}{\int_5^{20} ktdt}=\frac{t^2\big|_5^{10}}{t^2\big|_5^{20}}=\frac{75}{375}=0.20$$

478. Solution: B

The variance of the sum of independent random variables is the sum of the individual variances. So the variance of the total health insurance cost is $1^2 + 2^2 + 3^2 + 4^2 = 30$. The standard deviation of the total cost is the square root: $\sqrt{30}=5.4772.$

479. Solution: D

The <u>variance</u> of a uniform distribution is $\frac{1}{12}$ of the square of the interval length. For the hospitalization charge: $\frac{c}{\sqrt{12}}=4\sqrt{3}$, so that $c = 24$.

The standard deviation of the surgery charge is $\frac{3c-18}{\sqrt{12}}=\left(\frac{3(24)-18}{\sqrt{12}}\right)=\frac{54}{\sqrt{12}}=15.5885.$

480. Solution: C

For one policyholder

$E(X) = 0.8(0) + 0.16(1) + 0.04(2) = 0.24$

$E(X^2) = 0.08(0) + 0.16(1) + 0.04(4) = 0.32$

$Var(X) = 0.32 - 0.24^2 = 0.2624.$

Because they are independent, the variance of the sum is the sum of the variances: 144(0.2624) = 37.7856.

481. Solution: E

Each value has probability 0.2.

$E(N) = 1(0.2) + 2(0.2) + 3(0.2) + 4(0.2) + 5(0.2) = 3$

$E(N^2) = 1(0.2) + 4(0.2) + 9(0.2) + 16(0.2) + 25(0.2) = 11$

$E(\text{Cost}) = E(N^2 + N + 1) = E(N^2) + E(N) + 1 = 11 + 3 + 1 = 15.$

482. Solution: B

$$E(X) = \frac{a+b}{2} = 3$$

$a + b = 6$

$b = 6 - a$

$$V(X) = \frac{(b-a)^2}{12} = 1^2 = 1$$

$$\frac{(6-a-a)^2}{12} = 1$$

$a = 1.26795.$

483. Solution: A

The number who test positive for disease A has a binomial distribution with n = 100 and p. Then 9 = 100p(1 – p) which implies that p = 0.1 (0.9 is also a solution but we are given that p is between 0 and 0,5.

The probability of having at least one disease is 1 minus the probability of neither having the disease. Thus, this probability is 1 – 0.9(0.9) = 0.19. The variance for 100 patients is 100(0.19)(0.81) = 15.29.

484. Solution: B

The 40th percentile for Denver is at 0.6(25) + 0.4(90) = 51.
The 20th percentile for Philadelphia is at 0.8(45) + 0.2x = 36 + 0.2x.
Equating them yields x = 75.
Therefore, Salt Lake City salaries are uniform from 10 to 25 and the median is (10 + 25)/2 = 17.5.

485. Solution: B

The 40th percentile is at 0.6a + 0.4(2a) = 1.4a.
The 80th percentile is at 0.2a + 0.8(2a) = 1.8a.
The pth percentile (treating p is a decimal) is at (1 – p)a + p(2a) = a + pa = (1 + p)a.
Then,

$$\frac{1.4a}{(1+p)a}=\frac{(1+p)a}{1.8a}$$

$$1.4(1.8)=(1+p)^2$$

$$1.58745=1+p$$

$$p=0.58745=58.7\%.$$

486. Solution: B

The number of such claims, N, is binomial with n = 3 and p = 0.2.

$$P(N>0)=1-P(N=0)=1-0.8^3=0.488$$

487. Solution: B

The denominator is the probability of not passing all three exams, which is 1 – 0.36 = 0.64. The numerator is the probability of passing Exam 2 and not passing Exam 3. This is 0.48 – 0.36 = 0.12. The desired conditional probability is 0.12/0.64 = 0.1875.

488. Solution: B

Let N denote the number of the phone call that produced the third completed survey. For N = 8, the preceding seven surveys must produce exactly two completed surveys. This is a binomial probability, $\binom{7}{2}(0.25)^2(0.75)^5=0.31146$. The eighth call must result in a completed survey, so the total probability is 0.31146(0.25) = 0.078. A direct solution can be obtained by recognizing this is a negative binomial random variable and remembering the probability function.

489. Solution: C

Let b = maximum benefit. Then,

$$31=\int_{10}^{b} x/50dx+\int_{b}^{60} b/50dx=\frac{b^2-10^2}{100}+b\frac{60-b}{50}=\frac{-b^2+120b-100}{100}$$

$$3100=-b^2+120b-100$$

$$b^2-120b+3200=0$$

$$(b-40)(b-80)=0.$$

The only feasible solution is b = 40.

490. Solution: D

Let X = # of fragile packages that break; Y = # of non-fragile packages that break. For the denominator,

P(X + Y = 2) = P(X = 2)P(Y = 0) + P(X = 1)P(Y = 1) + P(X = 0)(Y = 2)

= [(0.2)(0.2)][(0.9)(0.9)] + [2(0.8)(0.2)][2(0.1)(0.9)] + [(0.8)(0.8)][(0.1)(0.1)] = 0.0964.

For the numerator,

P(X = 2 and X + Y = 2) = P(X = 2 and Y = 0) = (0.2)(0.2)(0.9)(0.9) = 0.0324.

The conditional probability is 0.0324/0.0964 = 0.336.

491. Solution: E

The density function is $f(x)=\begin{cases}0, & x<0\\ \frac{3}{2}x^2, & 0\le x<1\\ 2-x, & 1\le x\le 2\\ 0, & x>2\end{cases}$.

The expected value is

$$E(X)=\int_0^1\frac{3}{2}x^3dx+\int_1^2 x(2-x)dx=\frac{3}{8}+\frac{2}{3}=\frac{25}{24}.$$

492. Solution: C

Let X = machine's lifetime.

$$0.80=P[X>1]=e^{-\lambda}$$

$$F(x)=P[X\le x]=1-e^{-\lambda x}=1-0.80^x.$$

493. Solution: B

$$\text{P}[\text{Dis}\ge 2\,|\,\text{Death}\le 1]=\frac{0.04+0.03+0.03+0.02}{0.51+0.09+0.08+0.06+0.04+0.03+0.03+0.02}=\frac{0.12}{0.86}=0.1395.$$

494. Solution: C

Let X represent the number of accident-free employees; X is binomially distributed.
Let n = number of employees = 12.
Let p = the probability that a given employee is accident-free this year.
$9 = E(X) = np = 12p$, so $p = 0.75$.
$\text{Var}(X) = np(1-p) = 9(1-0.75) = 2.25$.

495. Solution: B

$$E[X] = \int_5^7 xf(x)dx = \frac{3}{2}\int_5^7 (x^3 - 12x^2 + 36x)dx = \frac{3}{2}\left(\frac{x^4}{4} - 4x^3 + 18x^2\right)\Bigg|_5^7$$
$$= 1.5[(7^4 - 5^4)/4 - 4(7^3 - 5^3) + 18(7^2 - 5^2)] = 6$$
$$E[X^2] = \int_5^7 x^2 f(x)dx = \frac{3}{2}\int_5^7 (x^4 - 12x^3 + 36x^2)dx = \frac{3}{2}\left(\frac{x^5}{5} - 3x^4 + 12x^3\right)\Bigg|_5^7$$
$$= 1.5[(7^5 - 5^5)/5 - 3(7^4 - 5^4) + 12(7^3 - 5^3)] = 36.6$$
$$Var(X) = 36.6 - 6^2 = 0.60$$

496. Solution: C

The probability that a machine fails in the second year is $F(2) = 1 - (1/2)^{1/4} = 0.1591.$
The probability that at least one machine fails is the complement of the probability of no machines failing, $1 - (1 - 0.1591)^3 = 0.4054.$ The expected payment is 1000(0.4054) = 405.

497. Solution: D

The distribution function is $F(x) = 1 - e^{-x/6}$. For a set of three independent and identically distributed random variable, the distribution function of the minimum is
$F_{\min}(x) = 1 - [1 - F(x)]^3 = 1 - (e^{-x/6})^3 = 1 - e^{-x/2}$.
This is an exponential distribution with a mean of 2.

498. Solution: C

P(suburban) = 0.55(1/5) + 0.30(3/5) + 0.15(7/20) = 0.3425.
P(suburban and under 25) = 0.55(1/5) = 0.11.
P(under 25 | suburban) = 0.11/0.3425 = 0.3212.

499. Solution: B

Let X be the number of policies sold random variable. Then,

$$P[X>3\,|\,X\geq 1]=\frac{P[X>3]}{P[X\geq 1]}=\frac{1-e^{-4}\left(\frac{4^0}{0!}+\frac{4^1}{1!}+\frac{4^2}{2!}+\frac{4^3}{3!}\right)}{1-e^{-4}\left(\frac{4^0}{0!}\right)}=\frac{0.5665}{0.9817}=0.5771.$$

500. Solution: D

Let p be the probability of being injury-free in a given year. Using the binomial distribution,
$x = P$(injury-free for exactly 1 of next 2 seasons) = $2p(1 - p)$
Then, P(injury-free for exactly 2 of next 4 seasons) = $6p^2(1 - p)^2 = 1.5[2p(1 - p)]^2 = 1.5x^2$.

501. Solution: E

The key is that the probability of experiencing at least one loss is not needed. The population of homeowners is split into two groups, eight with insurance and two without. Three homeowners with losses are randomly sampled from these ten. We want the probability that at least two are from the eight that are insured. This is a hypergeometric probability:

$$\frac{\binom{8}{2}\binom{2}{1}}{\binom{10}{3}}+\frac{\binom{8}{3}\binom{2}{0}}{\binom{10}{3}}=\frac{28(2)}{120}+\frac{56(1)}{120}=0.9333.$$

502. Solution: D

Let X = loss due to hospitalization, which is exponentially distributed with parameter λ.

$$0.15=P(X\leq 2)=\int_0^2 \lambda e^{-\lambda x}dx=1-e^{-2\lambda}$$

$$0.85=e^{-2\lambda}.$$

$$P(X\leq 10)=1-e^{-10\lambda}=1-0.85^5=0.55629.$$

503. Solution: D

Let p be the probability of an accident in a given performance.
Let X = the number of performances it takes to have the first accident, which is geometrically distributed with "success" probability p; $p < 0.5$.

$$P[X = i] = p(1-p)^{i-1}$$

$$P[X = 2] = 0.16 = p(1-p) \Rightarrow p = 0.2$$

$$P[X = 4] = 0.2(0.8)^3 = 0.1024$$

504. Solution: A

Let X be the loss, which is uniform [2, 10].
Let Y be the unreimbursed loss. Then, $Y = 0.4X$. Then,

$$0.3 = P[Y \le p] = P[0.4X \le p] = P[X \le 2.5p] = \frac{2.5p - 2}{10 - 2} = 5p/16 - 1/4$$

$$5p/16 = 3/10 + 1/4 = 11/20$$

$$p = \frac{16(11)}{5(20)} = 1.76.$$

505. Solution: C

If one day has a Poisson(0.2) distribution, then seven days has a Poisson(1.4) distribution.

$$P[X = 3] = \frac{e^{-1.4}(1.4)^3}{3!} = 0.1128$$

506. Solution: C

The probability of getting one of each type when Bob is selecting is $\frac{3}{4} \times \frac{1}{4} \times 2 = \frac{3}{8}$. The same probability when Ann is selecting is $\frac{3}{4} \times \frac{1}{3} \times 2 = \frac{1}{2}$. Thus, the desired probability is $\frac{\frac{3}{8}}{\frac{1}{2} + \frac{3}{8}} = \frac{3}{7}$.

507. Solution: E

This is a gamma distribution with parameters $\alpha = 6, \ \beta = 100$. $\mathrm{E}(X) = \alpha\beta = 600$.
If the distribution type is not recognized integration by parts is needed to get c and then to get the expected value.

508. Solution: D

Let a represent the 2nd percentile; keep in mind that $a > 0$. We have the following equation:

$$0.02 = \frac{\int_0^a x(1+x^2)^{-9}\,dx}{\int_0^\infty x(1+x^2)^{-9}\,dx} = \frac{-(1+x^2)^{-8}/16\Big|_0^a}{-(1+x^2)^{-8}/16\Big|_0^\infty} = \frac{1-(1+a^2)^{-8}}{1}$$

$$0.98 = (1+a^2)^{-8}$$

$$1.0025285 = 1+a^2$$

$$a = 0.050$$

509. Solution: B

Let μ and σ be the monthly mean and standard deviation. Then the yearly mean is 12μ and the standard deviation is $\sqrt{12}\sigma$. Let X be the annual profit. Then,

$$0.6 = P[X > 0] = P[Z > -12\mu/(\sqrt{12}\sigma) = -\sqrt{12}(\mu/\sigma)].$$

From the normal table,

$$0.253347 = \sqrt{12}(\mu/\sigma)$$

$$\mu/\sigma = 0.073135.$$

Let Y be the monthly profit. Then,

$$P[Y > 0] = P[Z > -\mu/\sigma = -0.73135] = 0.52915.$$

510. Solution: C

The maximum is 2 when either X or Y is 2, the probability is 0.02+0.03+0.05+0.09+0.06 = 0.25.
The maximum is 0 when both X and Y are 0, the probability is 0.38.
Thus the maximum is 1 with probability 1 – 0.25 – 0.38 = 0.37.
The expected value is 0(0.38) + 1(0.37) + 2(0.25) = 0.87.

511. Solution: E

Let p = the probability that the insurer pays nothing for a given loss.

$$p = \mathrm{P}\left[\text{loss} \le d\right] = \frac{\int_0^d x^5\,dx}{\int_0^1 x^5\,dx} = \frac{\frac{d^6}{6}}{\frac{1}{6}} = d^6.$$

P = P[insurer pays for at least 1 of 2 losses] = 1 – P[insurer pays nothing for 2 losses]

$= 1 - p^2 = 1 - d^{12}.$

Then, $d = (1-P)^{1/12}$.

512. Solution: D

$$\mathrm{E}(X) = \mathrm{V}(X) = \lambda = 20$$
$$\mathrm{V}(X) = \mathrm{E}(X^2) - \mathrm{E}(X)^2$$
$$\mathrm{E}(X^2) = 20 + 20^2 = 420$$
$$\mathrm{E}(X^2) + \mathrm{E}(X) = 420 + 20 = 440$$

513. Solution: D

Let X be the distance between P and the origin. Then X has the uniform distribution on [0, 3]. We seek $\mathrm{Var}(\pi X^2) = \pi^2 \mathrm{Var}(X^2)$, which is:

$$\pi^2 \left[E(X^4) - E(X^2)^2 \right]$$
$$E(X^4) = \int_0^3 \frac{x^4}{3} dx = \frac{243}{15} = \frac{81}{5}$$
$$E(X^2) = \int_0^3 \frac{x^2}{3} dx = 3$$
$$\pi^2 \left[E(X^4) - E(X^2)^2 \right] = \pi^2 \frac{81 - 45}{5} = 36\pi^2 / 5.$$

514. Solution: A

We seek E(XY), which is (10)(1)p(1,10) + (12)(1)p(1,12) + (14)(1)p(1,14) + (10)(2)p(2,10) + (12)(2)p(2,12) + (14)(2)p(2,14).

$$= 10(1)\frac{2}{18} + 12(1)\frac{4}{18} + 14(1)\frac{6}{18} + 10(2)\frac{4}{18} + 12(2)\frac{2}{18} + 14(2)\frac{0}{18} = \frac{280}{18} = \frac{140}{9}.$$

515. Solution: A

First, $\int_0^1 f(x)dx = 1 \Rightarrow k = 4$

and $F(x) = 1 - (1 - x)^4$

Because $1 - (1 - x)^4 = 0.5 \Rightarrow x = 0.1591$ and this result is less than the benefit maximum, the median benefit payment is 0.1591(1,000,000)= 159,000.

516. Solution: C

Since $f(r) = kr^{\alpha-1}(1-r)^{\beta-1} = kr^{12}(1-r) = k(r^{12} - r^{13})$

$$1 = \int_0^1 f(r)dr = \int_0^1 kr^{12}(1-r)dr = k\int_0^1 (r^{12} - r^{13})dr$$

$$= k\left[\frac{r^{13}}{13} - \frac{r^{14}}{14}\right]_{r=0}^{r=1} = k\left(\frac{1}{13} - \frac{1}{14}\right) = \frac{k}{182}.$$

Thus, k = 182, and $f(r) = 182(r^{12} - r^{13})$

Then,

$$P[R > 0.9] = \int_{0.9}^1 f(r)dr = \int_{0.9}^1 182r^{12}(1-r)dr$$

$$= 182\left[\frac{r^{13}}{13} - \frac{r^{14}}{14}\right]_{r=0.9}^{r=1} = \left[14r^{13} - 13r^{14}\right]_{r=0.9}^{r=1}$$

$$= 14(1-0.9^{13}) - 13(1-0.9^{14}) = 0.415.$$

517. Solution: B

First, note that $p + q = 0.3$.

Next, the expected Type I claim costs are:

$$(0.21 + 0.13 + q)750 = 345 \Rightarrow$$

$$0.34 + q = 0.46 \Rightarrow q = 0.12 \Rightarrow p = 0.18.$$

Next,

$$P(Y = 0) = 0.52$$

$$P(Y = 1) = 0.31$$

$$P(Y = 2) = 0.17$$

Finally,

$$E(Y^2) = 0.31 + 0.17(4) = 0.99$$

$$E(Y) = 0.31 + 0.17(2) = 0.65$$

$$Var(Y) = 0.99 - 0.65^2 = 0.5675.$$

518. Solution: D

Let X represent the number of earthquakes, which is Poisson distributed with
$\text{P}[X = i] = \dfrac{e^{-\lambda}\lambda^i}{i!}$, for integer $i \geq 0$.

Since we are given $\text{P}[X = 2] = 0.43\text{P}[X = 1]$, we have $\dfrac{e^{-\lambda}\lambda^2}{2!} = 0.43\left(\dfrac{e^{-\lambda}\lambda^1}{1!}\right)$, which gives

$\dfrac{\lambda}{2} = 0.43$ and then $\lambda = 0.86$.

The probability of at least 3 earthquakes is

$$\text{P}[X \geq 3] = 1 - \text{P}[X = 0] - \text{P}[X = 1] - \text{P}[X = 2]$$

$$= 1 - \frac{e^{-\lambda}\lambda^0}{0!} - \frac{e^{-\lambda}\lambda^1}{1!} - \frac{e^{-\lambda}\lambda^2}{2!} = 1 - e^{-0.86}\left[1 + \frac{0.86}{1!} + \frac{(0.86)^2}{2!}\right] = 0.056.$$

519. Solution: C

Let X represent the number of minutes that city A's tornado lasts.
Let Y represent the number of minutes that city B's tornado lasts.
X and Y are independent and exponentially distributed with means 5 and 4, respectively.

The variance of an exponential distribution is the square of its mean, the variance of a sum of independent random variables is the sum of the variances and multiplying a random variable by a constant results in multiplying the variance by the square of the constant. Therefore, the variance of the monetary value of the damage is

$$(5.2)^2 = \text{Var}(X + cY) = \text{Var}(X) + \text{Var}(cY) = \text{Var}(X) + c^2\text{Var}(Y) = 5^2 + c^2(4)^2 = 25 + 16c^2.$$

Solving for c gives $c = \sqrt{\dfrac{(5.2)^2 - 25}{16}} = 0.35707$.

520. Solution: B

Here are the possible vectors with their probabilities:

(w, x, y)	$x + y - w$	probability
(0,1,1)	2	2/20
(0,1,2)	3	3/20
(0,2,1)	3	3/20
(0,2,2)	4	4/20
(1,1,1)	1	1/20
(1,1,2)	2	2/20
(1,2,1)	2	2/20
(1,2,2)	3	3/20

Given that $Y = 1$, W is 0 with probability (2 + 3)/(2 + 3 + 1 + 2) = 5/8 and 1 with probability 3/8. Then,

$$E(W) = \frac{3}{8}$$

$$E(W^2) = \frac{3}{8}$$

$$V(W) = \frac{3}{8} - \left(\frac{3}{8}\right)^2 = \frac{15}{64}.$$

521. Solution: D

We want $P(X + Y < 45)$.

The random variable $X + Y$ is normal with mean 15 + 20 = 35 and variance $4^2 + 5^2 = 41$.
The standard deviation of $X + Y$ = 6.40.
Then $P\left(X + Y < 45\right) = P\left(Z < 1.5617\right) = 0.9408$.

522. Solution: A

Let X = number of months until the motorist is charged, which is exponentially distributed with mean β. Then,

$$0.5 = e^{-1.733/\beta}$$

$$\beta = 2.5$$

$$P(1.733 < X < 2.5) = \left(1 - e^{-2.5/2.5}\right) - \left(1 - e^{-1.733/2.5}\right) = 1 - e^{-1} - 1 + 0.5 = 0.5 - e^{-1}.$$

523. Solution: D

From the standard normal table, the 81st percentile is 0.87790 standard deviations above the mean. Therefore, the 81st percentile of losses covered by Policy II is $y = 8000 + 0.87790(1.5k)$.

A loss of *y* covered by Policy I would be $\dfrac{y-8000}{k} = 0.87790(1.5) = 1.3168$ standard deviations above the mean. Using the standard normal table, this corresponds to the 90.6th percentile. (Using the table provided gives 90.7.)

524. Solution: C

There are four subsets here:
Neither high blood pressure or high cholesterol: 35 – 15 = 20
Both high blood pressure and high cholesterol: 9 + 9 -15 = 3
High blood pressure only: 9 – 3 = 6
High cholesterol only: 9 – 3 = 6.

There are 6 choose 4 = 15 ways to select four with high blood pressure only.
There are 6 choose 1 = 6 ways to select one with high cholesterol only.
There are 3 chose 2 = 3 ways to select two with both high blood pressure and high cholesterol.
There are 20 choose 1 = 20 ways to select one with neither high blood pressure or high cholesterol.

The total number of groups is 15(6)(3)(20) = 5400.

525. Solution: B

The probability of regularly watching News is 0.35 + 0.08 + 0.22 + 0.05 = 0.70.
Of those probabilities, 0.05 watch neither Movies or Sports, 0.08 + 0.22 = 0.30 watch exactly one of Movies and Sports, and 0.35 watch both Movies and Sports.
The conditional distribution of the number of entertainment categories watched by those watch News is then:
0 – 0.05/0.70 = 1/14
1 – 0.30/0.70 = 6/14
2 – 0.35/0.70 = 7/14.

The mean is 0(1/14) + 1(6/14) + 2(7/14) = 20/14 = 10/7.
The second moment is 0(1/14) + 1(6/14) + 4(7/14) = 34/14 = 17/7.
The variance is 17/7 – 100/49 = 0.388.

526. Solution: C

Let T denote the total losses due to tornadoes. For each territory, the mean is 0.1 million and the standard deviation is also 0.1 million. Then, for the sum of 50 such independent random variables,

$$E(T) = 50(0.1) = 5$$

$$SD(T) = \sqrt{50}(0.1) = 0.7071$$

$$\Pr(T > 5.5) = \Pr\left(Z > \frac{5.5-5}{0.7071} = 0.7071\right) = 0.2397.$$

527. Solution: D

$$10 = Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y) = 2Var(X) + 2Cov(X,Y)$$

$$16 = Var(X-2Y) = Var(X) + 4Var(Y) - 4Cov(X,Y) = 5Var(X) - 4Cov(X,Y)$$

From the first equation, $Var(X) = 5 - Cov(X,Y)$. Substituting in the second equation,

$$16 = 5[5 - Cov(X,Y)] - 4Cov(X,Y) = 25 - 9Cov(X,Y)$$

$$Cov(X,Y) = 1.$$

528. Solution: A

Let A be the event that the fire policy is purchased and B be the event that the flood policy is purchased. Also let x be the probability that neither is purchased:

$x = 1 - \mathrm{P}(A \cup B) = 0.2 + \mathrm{P}(A \cap B)$, so $\mathrm{P}(A \cap B) = x - 0.2$.

First from statement iii): $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = 2P(B|A) = \dfrac{2P(A \cap B)}{P(A)}$.

From the second and fourth expressions in the above line we get $P(B) = \frac{1}{2}P(A)$, since

$P(A \cap B) \neq 0$.

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.Substituting, we get

$1 - x = P(A) + \frac{1}{2}P(A) - (x - 0.2)$, and so $1 - 0.2 = \frac{3}{2}P(A)$. Thus $P(A) = \frac{2}{3}(0.8) = \frac{8}{15}$, and

$P(B) = \frac{1}{2}P(A) = \frac{4}{15} = 0.2666$.

529. Solution: B

This is a Poisson Random Variable. with λ = 0.25. Since we are talking about two applications,

$$\mathrm{P}(x > 1) = 1 - p(0) - p(1)$$
$$= 1 - \frac{e^{-0.5}(0.5)^0}{0!} - \frac{e^{-0.5}(0.5)^1}{1!}$$
$$= 1 - 0.6065 - 0.3033$$
$$= 0.0902.$$

530. Solution: C

Label the three dice, A, B, and C in the order presented.
If A and B are selected, an 11 can be obtained only with a 5 from A and a 6 from B. The probability is (1/6)(1/3) = 1/18
If A and C are selected, an 11 can be obtained only with a 5 from A and a 6 from C. The probability is (1/6)(1) = 1/6.
If B and C are selected, an 11 is not possible.
The total probability is (1/3)(1/18) + (1/3)(1/6) = 4/54 = 0.0741.

531. Solution: D

Assuming independence:

$$\mathrm{P}(F \cap T) = 0.10(0.30) = 0.03$$
$$p = 1 - \left(\mathrm{P}(F) + \mathrm{P}(T) - \mathrm{P}(F \cap T)\right) = 1 - \left(0.10 + 0.30 - 0.03\right) = 0.63.$$

Assuming mutually exclusive:

$$r = 1 - \mathrm{P}(F) - \mathrm{P}(T) = 1 - 0.10 - 0.30 = 0.60$$
$$p - r = 0.63 - 0.60 = 0.03.$$

532. Solution: D

$$\mathrm{P}(F) = 0.05, \mathrm{P}(FDIC) = 0.80$$
$$\mathrm{P}(F \mid FDIC) = 0.03 = \frac{\mathrm{P}(F \cap FDIC)}{\mathrm{P}(FDIC)} = \frac{\mathrm{P}(F \cap FDIC)}{0.80} \Rightarrow \mathrm{P}(F \cap FDIC) = 0.024.$$

We want: $\mathrm{P}(FDIC \mid F) = \frac{\mathrm{P}(F \cap FDIC)}{\mathrm{P}(F)} = \frac{0.024}{0.05} = 0.48.$

533. Solution: E

Since all employees are independent and have the same accident risk, all combinations of three employees with accidents are equally likely. Therefore,

P[at least 1 accident in dept. B] = 1 – P[the 3 "accident" employees are all in dept. A]:

$$1-\frac{\binom{6}{3}\binom{4}{0}}{\binom{10}{3}}=1-\frac{20\times 1}{120}=\frac{5}{6}=0.8333.$$

534. Solution: C

P[at least 1 accident during a given year] = $1-\mathrm{P}[0]=1-\frac{e^{-0.10}0.10^0}{0!}=1-e^{-0.10}$.

P[first accident in sixth year | no accidents in first two years]
= P[first accident in sixth year and no accidents in first two years] / P[no accidents in first two years]
= P[first accident in sixth year]/ P[no accidents in first two years]

$$=\frac{\mathrm{P}[0]^5\,\mathrm{P}[\geq 1]}{\mathrm{P}[0]^2}, \text{by independence.}$$

$$\frac{\left(e^{-0.10}\right)^5\left(1-e^{-0.10}\right)}{\left(e^{-0.10}\right)^2}=0.070498.$$

535. Solution: A

Let:
X = loss under health insurance policy
Y = loss under dental insurance policy
Var(X) = 40,000; Var(Y) = 10,000; X and Y are independent
U = total unreimbursed loss = $0.2X + 0.1Y$

$$\mathrm{Var}(U)=\mathrm{Var}(0.2X+0.1Y)$$
$$=\mathrm{Var}(0.2X)+\mathrm{Var}(0.1Y), \text{ since independent}$$
$$=0.2^2\,\mathrm{Var}(X)+0.1^2\,\mathrm{Var}(Y)$$
$$=0.2^2(40{,}000)+0.1^2(10{,}000)$$
$$=1700.$$

536. Solution: C

$$P(A \cup B) = P(<5) = 0.3$$
$$P(A \cup C) = P(<5 \text{ or } 6) = 0.3 + 0.7 = 1.0$$
$$P(B \cup C) = P(<4 \text{ or } 6) = 0.1 + 0.7 = 0.8$$
$$0.3 + 1.0 + 0.8 = 2.1$$

537. Solution: C

In a given year, the probability of evacuating is the probability of evacuating given there is at least one hurricane times the probability of at least one hurricane, 0.35(0.25) = 0.0875.
The probability of never evacuating in three years is $0.9125^3 = 0.7598$.
The probability of being evacuated at least once is 1 – 0.7598 = 0.2402.

538. Solution: A

Let X = number of tests; X is geometric with success probability $p = 0.0625$;
$P[X = n] = (1 - 0.0625)^{n-1}(0.0625)$ for all values of $n \geq 1$.
P[$X = n$] is maximized when the exponent $n - 1$ is minimized, which occurs at $n = 1$. The mode of X is 1.

539. Solution: D

Let $Y = \frac{1}{3}(X_1 + X_2 + X_3)$. Then Y is normal, with mean 100 and variance
$\left(\frac{1}{9}\right)\left(\frac{27}{2}\right)(1+2+3) = 9.$

Thus $\text{P}[X > 106] = \text{P}\left[\text{Z} > \frac{106-100}{\sqrt{9}}\right] = \text{P}\left[Z > 2\right] = 1 - P[Z < 2] = 1 - \Phi(2).$

540. Solution: B

$$\text{P[Female | 5 Years]} = \frac{\text{P[5|F]P[F]}}{\text{P[5|F]P[F]+P[5|M]P[M]}} = \frac{(0.2)(0.4)}{(0.2)(0.4)+(0.4)(0.6)} = 0.25.$$

541. Solution: A

Let X = machine's lifetime.

Let β equal the mean. Then the cumulative distribution function is: $\mathrm{P}[X < x] = 1 - e^{-x/\beta}$ and $\mathrm{P}[X > x] = e^{-x/\beta}$.

Since the exponential distribution is memoryless,

$0.027 = \mathrm{P}[X > 15 \mid X > 5] = \mathrm{P}[X > 10] = e^{-10/\beta}$

$\beta = 2.76861.$

Then, $\mathrm{P}[X > 25 \mid X > 5] = \mathrm{P}[X > 20] = e^{-20/2.76861} = 0.000729.$

542. Solution: B

Let l = number of low-risk employees; therefore, there are 76 – l high-risk employees.
For independent random variables, the variance of the sum equals the sum of the variances.
Therefore:

$43^2 = 1849 = \text{ Var[Total \# of Accidents]}$

$= l \times \text{Var[\# of Accidents of Low Risk Ee]} + (76\text{-}l) \times \text{Var[\# of Accidents of High Risk Ee]}$

$= l(0.50)^2 + (76 - l)(5.50)^2$

$= 2299 - 30l$

$450 = 30l$

$l = 15.$

543. Solution: B

Let X represent the reported loss; note that X always exceeds d and Var[X] = v.
The claim payment is $p(X - d)$ since X always exceeds d.

Var[claim payment] = Var[$p(X - d)$] = p^2Var[$X - d$] = p^2Var[X] = p^2v.

544. Solution: D

Let A be the probability that total claim size is at least 2000. Let B be the probability of at least two claims.

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]}.$$

$P[A \cap B]$ is the probability of exactly two claims, both of them for 1000: $(0.2)(0.5)(0.5) = 0.05$, plus the probability of exactly three claims, at least one of which is for more than 500, which is the complement of all being 500: $(0.1)[1-(0.5)^3] = 0.0875$, for a total of 0.1375. $P[B]$ is the probability of two or more claims: $0.2 + 0.1 = 0.3.$ Therefore, the answer is: $\frac{0.1375}{0.3} = 0.4583.$

545. Solution: D

This is an exponential distribution with mean 1,000,000.

$F(x) == 1 - e^{-x/1,000,000}$. For the fifteenth percentile:

$\mathrm{P}[X < a] = 1 - e^{-a/1,000,000} = 0.15$

$0.85 = e^{-a/1,000,000}$

$a = 162,519.$

For the ninety-fifth percentile:

$\mathrm{P}[X < b] = 1 - e^{-b/1,000,000} = 0.95$

$0.05 = e^{-b/1,000,000}$

$b = 2,995,732.$

$2,995,732 - 162,519 = 2,833,213.$

546. Solution: D

Let D be the event that the board is defective and let T be the event that the test indicates that the board is defective. We want:

$$\mathrm{P}[D \mid T] = \frac{\mathrm{P}[T \mid D]P[D]}{\mathrm{P}[T \mid D]P[D] + \mathrm{P}[T \mid \bar{D}]P[\bar{D}]} = \frac{(0.8)(0.1)}{(0.8)(0.1) + (0.6)(0.9)} = \frac{4}{31}.$$

547. Solution: C

Let B_i denote the event "i was sent," $i = 0,1$, and let A_i denote the event "i is received," $i = 0,1$. Need to determine $P[B_0 \mid A_1]$.

$P[B_0] = 0.7 \quad \Rightarrow \quad P[B_1] = 0.3$, since B_0 and B_1 are mutually exclusive.

$P[A_0 \mid B_0] = 0.8 \quad \Rightarrow \quad P[A_1 \mid B_0] = 0.2$, since A_0 and A_1 are mutually exclusive.

$P[A_1 \mid B_1] = 0.8 \quad \Rightarrow \quad P[A_0 \mid B_1] = 0.2$.

$$\begin{aligned} P[B_0 \mid A_1] &= \frac{P[A_1 \mid B_0]P[B_0]}{P[A_1 \mid B_0]P[B_0] + P[A_1 \mid B_1]P[B_1]} \\ &= \frac{(0.2)(0.7)}{(0.2)(0.7)+(0.8)(0.3)} \\ &= 0.37. \end{aligned}$$

548. Solution: B

Let W be the event "white chip is drawn from urn B." Let 0WT denote zero white chips transferred. Let 1WT denote one white chip transferred. Let 2WT denote two white chips transferred. Therefore,

$$\mathrm{P}[W] = \mathrm{P}[W \mid 0WT]\mathrm{P}[0WT] + \mathrm{P}[W \mid 1WT]\mathrm{P}[1WT] + \mathrm{P}[W \mid 2WT]\mathrm{P}[2WT]$$

$$\mathrm{P}[W] = \frac{5}{11}\frac{\binom{4}{0}\binom{5}{2}}{\binom{9}{2}} + \frac{6}{11}\frac{\binom{4}{1}\binom{5}{1}}{\binom{9}{2}} + \frac{7}{11}\frac{\binom{4}{2}\binom{5}{0}}{\binom{9}{2}} = \frac{212}{396} = 0.5354.$$

549. Solution: D

The number of ways these 6 people can be seated at a round table is 5!. Let A and B be two left-handed people. The number of ways 6 people can seat in a round table with A and B seating next to each other is (2)(4!). Therefore, the probability that both A and B sit together is $\frac{(2)(4!)}{5!} = \frac{2}{5}$.

Alternatively, consider having the first left-handed person take a seat at random. There are two of five seats remaining for the other left-hander to sit in.

550. Solution: C

There are two tasks: choose two people from the five high-blood-sugar people and choose three people from the remaining four people. Therefore,

Number of combinations = $\binom{5}{2}\binom{4}{3} = 10(4) = 40$.

551. Solution: B

Let X = number of accidents, which is binomial with "success" probability 0.3 and 4 trials. We want to find $P[X > 2]$.

$$P[X > 2] = \sum_{k=3}^{4} \binom{4}{k} (0.3)^k (0.7)^{4-k} = 4(0.3)^3(0.7) + (0.3)^4 = 0.0837.$$

552. Solution: B

Since the lifetime has constant probability density in the interval [a, b], the 80th percentile is 80% of the way from a to b, which equals: $a + 0.8(b-a) = a + 0.8b - 0.8a = 0.2a + 0.8b.$

553. Solution: B

$$0.4 = P\left[X < 4\ln\frac{5}{3}\right] = 1 - \exp\left[-4\ln(5/3)/\beta\right]$$

$$0.6 = \exp\left[-4\ln(5/3)/\beta\right] = \exp\left\{\ln\left[(5/3)^{-4/\beta}\right]\right\} = (5/3)^{-4/\beta} = (0.6)^{4/\beta}$$

$$4/\beta = 1$$

$$\beta = 4$$

Then,

$$0.5 = 1 - \exp(-M/4)$$

$$0.5 = \exp(-M/4)$$

$$\ln 0.5 = -M/4$$

$$M = 4(-\ln 0.5) = 4\ln 2.$$

554. Solution: D

Let X be uniformly distributed on [100, 225]. Let C be the total charge:

$$C = 15 + 1.3X$$

$$\text{Var}(C) = 1.3^2\,\text{Var}(X) = 1.3^2\frac{(225-100)^2}{12} = 2200.52$$

$$\text{SD}(C) = \sqrt{2200.52} = 46.91.$$

555. Solution: C

Let X_i amount of beverage in one bottle *i*, in ounces: $X_i \sim N(12, \sigma^2), i = 1, \ldots, 24.$

Let $Y = \sum_{i=1}^{24} X_i$ = amount of beverage in one case, in ounces. $Y \sim N(288, 24\sigma^2)$

$P[Y > 290] = 0.20$

$P\left[Z > \frac{290 - 288}{\sqrt{24\sigma^2}}\right] = 0.20$

$\frac{290 - 288}{\sqrt{24\sigma^2}} = 0.84162$

$\sigma = 0.4851.$

556. Solution: D

$\text{P}\left[Z < \frac{1000 - \mu}{\sigma}\right] = 0.60$

$\frac{1000 - \mu}{\sigma} = 0.253347$

$\text{P}\left[Z < \frac{2000 - \mu}{\sigma}\right] = 0.80$

$\frac{2000 - \mu}{\sigma} = 0.841621$

Divide the first equation by the second equation:

$\frac{1000 - \mu}{2000 - \mu} = \frac{0.253347}{0.841621} = 0.301023$

$1000 - \mu = 602.046 - 0.301023\mu$

$\mu = (1000 - 602.046) / (1 - 0.301023) = 569.338$

Then,

$\sigma = (1000 - 569.338) / 0.253347 = 1699.89$

Solve for the 95th percentile:

$\text{P}\left[Z < \frac{N - 569.338}{1699.89}\right] = 0.95$

$\frac{N - 569.338}{1699.89} = 1.644854$

$N = 3365$

557. Solution: D

Let X = # of years before malfunction, which is exponentially distributed with mean β. Then,

$$\mathrm{P}\left[X < x\right] = 1 - e^{-x/\beta}$$

$$\mathrm{P}\left[X < 5\right] = 1 - e^{-5/\beta} = 0.40$$

$$e^{-5/\beta} = 0.60$$

$$\beta = \frac{-5}{\ln(0.60)} = 9.78808$$

which equals the standard deviation.

558. Solution: B

The random variable X, defined as # of mice exposed to such disease on which the third mouse catches it, follows a negative binomial distribution. The probability for the third success on the tenth try is:

$$\binom{9}{2}(0.45)^3(0.55)^7 = 0.049943.$$

559. Solution: D

Let X_1 and X_2 represent the lifespans of the first and second units, respectively. Both are modeled by an exponential distribution. The mean for Brand A is β. The mean for Brand B is 0.5β. For Brand A,

$$\mathrm{P}\left[X_1 > 15\right] = e^{-15/\beta} = 0.046656$$

$$\beta = 4.894038.$$

For Brand B: $\mathrm{P}\left[X_2 > 5\right] = e^{-5/(0.5\beta)} = e^{-5/2.447019} = 0.1296.$

560. Solution: B

For an exponential distribution with random variable X and with mean 6: $\mathrm{P}[X > x] = e^{-x/6}$.

$p_A = e^{-m/6}$ and $p_B = e^{-(m+2)/6}$

$$\frac{p_B}{p_A} = \frac{e^{-(m+2)/6}}{e^{-m/6}} = e^{-[(m+2)-m]/6} = e^{-1/3}.$$

561. Solution: E

There are $\binom{8}{4}$ = 70 ways that 4 keys can be selected. Of those 70 ways, $\binom{2}{1}\binom{6}{3}$ = 40 contain exactly one fitting key. The solution is thus $\frac{40}{70} = 0.5714$.

562. Solution: B

$$0.9 = P\left[X > 4 \mid X > 3\right] = \frac{P\left[X > 4\right]}{P\left[X > 3\right]} = \frac{\frac{b-4}{b-a}}{\frac{b-3}{b-a}} = \frac{b-4}{b-3}$$

$$b - 4 = (0.9)(b-3)$$

$$b = 1.3/0.1 = 13$$

$$0.7 = P\left[X > 4 \mid X < 11\right] = \frac{P\left[4 < X < 11\right]}{P\left[X < 11\right]} = \frac{\frac{7}{13-a}}{\frac{11-a}{13-a}} = \frac{7}{11-a} = 0.7$$

$$0.7(11-a) = 7$$

$$a = 1$$

$$\text{P}[X > 4] = \frac{13-4}{13-1} = \frac{9}{12} = 0.75$$

563. Solution: D

Let $\beta = 400/\ln 2$ be the mean. The expected claim payment is

$$\int_0^\infty \min\left(x,1000\right)\beta^{-1}e^{-x/\beta}dx = \int_0^{1000} \beta^{-1}xe^{-x/\beta}dx + \int_{1000}^\infty 1000\beta^{-1}e^{-x/\beta}dx = -\left(x+\beta\right)e^{-x/\beta}\Big|_0^{1000} - 1000e^{-x/\beta}\Big|_{1000}^\infty$$

$$= \beta - \left(1000+\beta\right)e^{-1000/\beta} + 1000e^{-1000/\beta} = \beta\left(1-e^{-1000/\beta}\right) = \frac{400}{\ln 2}\left(1-e^{-2.5\ln 2}\right) = 475.06.$$

564. Solution: A

Domestic:

$$P[1 < X < 3] = \left(1-e^{-3/0.75}\right) - \left(1-e^{-1/0.75}\right) = 0.245281$$

International:

$$P[1 < X < 3] = \left(1-e^{-3/0.5}\right) - \left(1-e^{-1/0.5}\right) = 0.132857$$

The desired probability is

$$0.4(0.245281) + 0.6(0.132857) = 0.17783.$$

565. Solution: B

This is a beta distribution with $\alpha = 4$ and β=2.

The variance of a Beta distribution is: $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{4\times 2}{6^2\times 7} = \frac{2}{63} = 0.031746$

So, the standard deviation is: $\sqrt{0.031746} = 0.178174.$

The variance can also be calculated directly. The mean is

$$\int_0^1 20x^4(1-x)dx = \int_0^1 20x^4 - 20x^5)dx = 20x^5/5 - 20x^6/6\Big|_0^1 = 20/5 - 20/6 = 2/3.$$

The second moment is

$$\int_0^1 20x^5(1-x)dx = \int_0^1 20x^5 - 20x^6)dx = 20x^6/6 - 20x^7/7\Big|_0^1 = 20/6 - 20/7 = 10/21.$$

The variance is $10/21 - (2/3)^2 = 30/63 - 28/63 = 2/63.$

566. Solution: D

Let *X* denote the number of claims in the past year for a randomly selected policy. We are looking to find:

$$P(X\geq 3 \mid X\geq 1) = \frac{P(X\geq 3)}{P(X\geq 1)} = \frac{1-\left(\frac{e^{-1.2}1.2^0}{0!}+\frac{e^{-1.2}1.2^1}{1!}+\frac{e^{-1.2}1.2^2}{2!}\right)}{1-\frac{e^{-1.2}1.2^0}{0!}} = \frac{0.120513}{0.698806} = 0.17246.$$

567. Solution: A

The second moment is the variance plus the square of the mean. For a binomial distribution, we have (knowing that *p* must be positive)

$$1.80 = 5p(1-p)+(5p)^2 = 5p+20p^2$$

$$0 = 20p^2+5p-1.80$$

$$p = \frac{-5\pm\sqrt{25+144}}{40} = \frac{-5+13}{40} = 0.20.$$

The variance is $5p(1-p) = 5(0.2)(1-0.2) = 0.80$.

568. Solution: D

The probability a randomly selected policyholder had a claim exceeding 55,000 is given by:

$$\text{P[claim]P[claim} > 55{,}000 \mid \text{claim]} = 0.8\text{P}\left[Z > \frac{55{,}000 - 50{,}000}{c} \right] = 0.1837$$

$$\text{P}\left[Z > \frac{55{,}000 - 50{,}000}{c} \right] = 0.229625$$

$$\frac{55{,}000 - 50{,}000}{c} = 0.74008$$

$$c = 6756.$$

569. Solution: A

Let T be the total number of theft claims, T has a binomial distribution with n = 10 and p = 0.1, so:

$$P[T = 0] = \binom{10}{0}(0.1)^0(0.9)^{10} = 0.348678$$

$$P[T = 1] = \binom{10}{1}(0.1)^1(0.9)^9 = 0.387420$$

Let H be the total number of hailstorm claims, H has a binomial distribution with n = 10 and p = 0.2, so:

$$P[H = 0] = \binom{10}{0}(0.2)^0(0.8)^{10} = 0.107374$$

$$P[H = 1] = \binom{10}{1}(0.2)^1(0.8)^9 = 0.268435$$

$$\text{P}[T + H < 2] = P[T = 0]P[H = 0] + P[T = 0]P[H = 1] + P[T = 1]P[H = 0]$$

$$= (0.348678)(0.107374) + (0.348678)(0.268435) + (0.387420)(0.107374)$$

$$= 0.172635.$$

570. Solution: C

There are ${}_{10}C_3$ = 120 possible and equally likely combinations of three damaged pieces.

There are three events to add up, with the indicated number of possibilities.

i) 1 damaged piece insured, 1 damaged piece partially insured, 1 damaged piece uninsured;

$${}_2C_1({}_3C_1)({}_5C_1) = 2(3)(5) = 30.$$

ii) 1 damaged piece insured, 2 damaged pieces partially insured, 0 damaged pieces uninsured;

$$({}_2C_1)({}_3C_2)({}_5C_0) = 2(3)(1) = 6.$$

iii) 2 damaged pieces insured, 1 damaged piece partially insured, 0 damaged pieces uninsured;

$$({}_2C_2)({}_3C_1)({}_5C_0) = 1(3)(1) = 3.$$

The solution is (30 + 6 + 3)/120 = 39/120 = 0.325.

571. Solution: C

$$\mathrm{E}(X) = \frac{1}{10}\int_{-2}^{0} -x^2 dx + \frac{1}{10}\int_{0}^{4} x^2 dx = -\frac{8}{30} + \frac{64}{30} = \frac{56}{30}$$

$$\mathrm{E}(X^2) = \frac{1}{10}\int_{-2}^{0} -x^3 dx + \frac{1}{10}\int_{0}^{4} x^3 dx = \frac{16}{40} + \frac{256}{40} = \frac{272}{40}$$

$$\mathrm{Var}(X) = \frac{272}{40} - \left(\frac{56}{30}\right)^2 = 3.3156.$$

572. Solution: D

$$\mathrm{P[safe|no\,accidents]} = \frac{\mathrm{P[no\,accidents\mid safe]P[safe]}}{\mathrm{P[no\,accidents\mid safe]P[safe]+P[no\,accidents\mid dang]P[dang]}}$$

$$= \frac{e^{-0.5}(0.4)}{e^{-0.5}(0.4) + e^{-1}(0.6)} = 0.52.$$

573. Solution: A

The expected number of accidents for a single driver is 0.16.
The expected number of accidents for 40,000 drivers is 40,000(0.16) = 6,400
The variance of the number of accidents for a single driver is 0.16.
The variance of the number of accidents for 40,000 drivers is 40.000(0.16) = 6,400 (the variance of the sum of independent random variables is the sum of their variances).

$$\mathrm{CV}(X) = \frac{\sqrt{6400}}{6400} = 0.0125.$$

574. Solution: D

$$p(s) = \begin{cases} 0.25, & \text{for } s = 1 \\ 0.33, & \text{for } s = 2 \\ 0.24, & \text{for } s = 3 \\ 0.18, & \text{for } s = 4 \end{cases}$$

$$\mathrm{E}(S) = 0.25(1) + 0.33(2) + 0.24(3) + 0.18(4) = 2.35$$

$$\mathrm{E}(S^2) = 0.25(1^2) + 0.33(2^2) + 0.24(3^2) + 0.18(4^2) = 6.61$$

$$\mathrm{Var}(S) = 6.61 - 2.35^2 = 1.0875.$$

575. Solution: C

The yearly mean = $4 \times 8 = 32$
The yearly variance = $4 \times 24^2 = 2304$
Yearly standard deviation = $\sqrt{2304}$
Answer = $P[X > 0] = P\left[Z > \frac{0-32}{\sqrt{2304}}\right] = P\left[Z > -0.67\right]$

576. Solution: C

$$0.6 = \text{P[lifetime < 30 | lifetime > } a]$$
$$= \frac{\text{P[}a \text{ < lifetime < 30]}}{\text{P[lifetime > } a]}$$
$$= \frac{\frac{30-a}{40-0}}{\frac{40-a}{40-0}} = \frac{30-a}{40-a}$$
$$0.60(40-a) = 30-a$$
$$24 - 0.60a = 30 - a$$
$$0.40a = 6$$
$$a = 15.$$

577. Solution: A

X = total claim for thefts
Y = total claim for fires
$$E(X) = 100, E(Y) = 150$$
$$E(X+Y) = 250$$
$$Var(X) = 40^2, Var(Y) = 30^2$$
$$Var(X+Y) = 40^2 + 30^2 = 2500$$
$$SD(X+Y) = 50$$
$$CV = \frac{50}{250} = 0.20.$$

578. Solution: B

Let X – automobile Insurance
Let Y – homeowners insurance

$$\mathrm{E}(X+Y)=200+400=600$$

$$\mathrm{Var}(X+Y)=400^2+300^2=250,000$$

$$\mathrm{SD}(X+Y)=\sqrt{250,000}=500$$

$$\mathrm{P}[X+Y>0]=\mathrm{P}\left[Z>\frac{0-600}{500}\right]=\mathrm{P}[Z>-1.2].$$

579. Solution: C

$$\mathrm{P}[T<2\,|\,T>1]=\frac{\mathrm{P}[1<T<2]}{\mathrm{P}[T>1]}$$

$$=\int_1^2 te^{-t}dt\Big/\int_1^\infty te^{-t}dt=\left[-te^{-t}-e^{-t}\,\big|_1^2\right]/\left[-te^{-t}-e^{-t}\,\big|_1^\infty\right]$$

$$=\left[2e^{-1}-3e^{-2}\right]/\left[2e^{-1}\right]=1-\frac{3}{2}e^{-1}=0.4482$$

580. Solution: E

$$Var(3X_1-X_2-X_3-X_4)$$
$$=9Var(X_1)+Var(X_2)+Var(X_3)+Var(X_4)$$
$$=9(9)+9+9+9=108$$

581. Solution: A

$$\mathrm{E}(X)=0\times\frac{1}{6}+1\times\frac{1}{3}+2\times\frac{1}{2}=\frac{4}{3}$$

$$\mathrm{E}(Y)=0\times\frac{2}{3}+1\times\frac{1}{3}=\frac{1}{3}$$

$$\mathrm{E}(XY)=\left(0\times0\times\frac{1}{6}\right)+\left(1\times0\times0\right)+\left(2\times0\times\frac{1}{2}\right)+\left(0\times1\times0\right)+\left(1\times1\times\frac{1}{3}\right)+\left(2\times1\times0\right)=\frac{1}{3}$$

$$\mathrm{Cov}(X,Y)=\frac{1}{3}-\frac{4}{3}\times\frac{1}{3}=\frac{3-4}{9}=-\frac{1}{9}.$$

582. Solution: D

For independent random variables, the variance of the sum equals the sum of the variances. Therefore:

$$\text{Var(Annual Claim Payment)} = 3^2 = 9$$

$$9 = 52 \times \text{Var(Weekly Claim Payment)}$$

$$\text{Var(Weekly Claim Payment)} = \frac{9}{52}$$

$$\text{SD(Weekly Claim Payment)} = \sqrt{\frac{9}{52}} = \frac{3}{\sqrt{52}}.$$

583. Solution: C

Let *X* represent the loss. *X* is uniformly distributed on [*a*, *b*].

$$0.5 = P(X < 12) = \frac{12-a}{b-a}$$

$$0.875 = P(X > 6) \Rightarrow 0.125 = P(X < 6) = \frac{6-a}{b-a}$$

$$\frac{0.5}{0.125} = 4 = \frac{12-a}{6-a}$$

$$24 - 4a = 12 - a \Rightarrow a = 4$$

$$0.5 = \frac{12-4}{b-4} \Rightarrow b = 20$$

$$P(X < 10 \mid X > 6) = \frac{P(6 < X < 10)}{P(X > 6)} = \frac{4/16}{14/16} = 2/7$$

584. Solution: E

Let *X* be number of thefts in one year and *S* be the total number of thefts in 15 years. The sum of 15 independent Poisson distributions with mean λ is Poisson with mean 15λ.

$$0.10 = \text{P}[X \ge 1] = 1 - \text{P}[X = 0] = 1 - \frac{e^{-\lambda}\lambda^0}{0!} = 1 - e^{-\lambda}$$

$$0.9 = e^{-\lambda}$$

$$\lambda = 0.105361$$

$$\text{Var}(S) = 15\lambda = 1.580408.$$

585. Solution: D

$$P[1 < Y < 3 \mid 2 < Y < 4] = \frac{P[2 < Y < 3]}{P[2 < Y < 4]} = \frac{\int_2^3 \frac{y}{6} - \frac{y^2}{36}\,dy}{\int_2^4 \frac{y}{6} - \frac{y^2}{36}\,dy}$$

$$= \frac{\left.\frac{y^2}{12} - \frac{y^3}{108}\right|_2^3}{\left.\frac{y^2}{12} - \frac{y^3}{108}\right|_2^4} = \frac{0.240741}{0.481481} = 0.5.$$

586. Solution: C

Let L represent the size (in thousands) of a single loss and let Y represent the amount paid (in thousands) on a loss by the insurance company. Then, $Y = 0.80(L - 0.5)$ for all losses $L > 0.5$ while $Y = 0$ for smaller losses. Then,

$$P(Y > 5) = P[0.8(L - 0.5) > 5] = P[L > 6.75] = e^{-6.75/6} = 0.3247.$$

587. Solution: B

$$1 = 2p + 3p + 2p + q = 7p + q$$
$$6 = 0(q) + 5(4p) + 10(3p) = 50p$$
$$p = 0.12$$
$$q = 1 - 7p = 1 - 7(0.12) = 0.16$$
$$\text{Var[Afternoon Routes]} = (0-6)^2(0.16) + (5-6)^2(0.48) + (10-6)^2(0.36) = 12$$

588. Solution: D

A single bulb's lifetime has a variance of 1,000,000 (for an exponential distribution the variance is the square of the mean).
The variance of the sum of five independent light bulb's lifetimes is 5,000,000. The standard deviation is the square root, 2,236.

589. Solution: B

$$\text{E(Total Payment)} = 12\int_2^8 (x-2)\frac{8-x}{32}\,dx$$

$$= \frac{12}{32}\int_2^8 10x - x^2 - 16\,dx$$

$$= \frac{12}{32}\left(5x^2 - \frac{x^3}{3} - 16x\right)\Bigg|_2^8$$

$$= \frac{12}{32}(21.333 + 14.667) = 13.50.$$

590. Solution: C

Probability that the event does not end in any one round:
$P[HHH] + P[TTT] = 0.25^3 + 0.75^3 = 0.4375$.
Probability that the event ends in any one round:
$1 - 0.4375 = 0.5625.$
Probability that the event ends on the fifth round

$= \text{Probability that the event does not end in the first four rounds and ends in the } 5^{\text{th}} \text{ round}$

$$= (0.4375)^4(0.5625) = 0.020608.$$

591. Solution: E

Let X be the profit and Z be a standard normal random variable.
$0.25 = P(\mu - c < X < \mu + c)$

$$= P\left(\frac{\mu - c - \mu}{\sigma} < Z < \frac{\mu - c - \mu}{\sigma}\right)$$

$$= P(-0.25c < Z < 0.25c) = P(Z < 0.25c) - P(Z < -0.25c)$$

$$= P(Z < 0.25c) - 1 + P(Z < 0.25c)$$

$$P(Z < 0.25c) = 0.625$$

$$0.25c = 0.3186$$

$$c = 1.2744$$

The interval runs from 20 – 1.2744 = 18.7256 to 20 + 1.2744 = 21.2744.

592. Solution: C

Let X = number of patients who test positive for diabetes, which has a binomial distribution with n = 18 trials and success probability p = 0.15.

$$P[X = i] = \binom{n}{i} p^i (1-p)^{n-i} = \binom{18}{i}(0.15)^i (0.85)^{18-i} = \frac{18!}{i!(18-i)!}(0.15)^i (0.85)^{18-i}$$

We need to maximize $P[X = i]$.
The mean of this distribution is np =18(0.15) = 2.7. Calculate various probabilities near the mean:

$P[X = 1] = 0.1704$

$P[X = 2] = 0.2556$

$P[X = 3] = 0.2406$

Since binomial probabilities increase, then decrease, the mode is at X = 2.

593. Solution: E

Let X = the company's profit in a given year.

$$0.28 = \text{P}[X < 21] = \text{P}\left[Z < \frac{21-39}{\sigma}\right]$$

$$\frac{-18}{\sigma} = -0.5828$$

$$\sigma = 30.885$$

$$\text{P}\left[Z < \frac{3-39}{30.885}\right] = P[Z < -1.1656] = 0.122.$$

594. Solution: C

This is a hypergeometric probability:
P = Probability of 2 kings and no aces or jacks + Probability of 3 kings and no aces or jacks + Probability of 4 kings and no aces or jacks

$$P = \frac{\binom{4}{2}\binom{8}{0}\binom{40}{3}}{\binom{52}{5}} + \frac{\binom{4}{3}\binom{8}{0}\binom{40}{2}}{\binom{52}{5}} + \frac{\binom{4}{4}\binom{8}{0}\binom{40}{1}}{\binom{52}{5}}$$

$$= 0.024025.$$

595. Solution: D

Let X = checkout time, in minutes. Then X has a Uniform distribution on the interval [1.1, 8.6].

$$\mathrm{P}[X < y] = \frac{y - 1.1}{8.6 - 1.1} = 0.95$$
$$y = 8.225.$$

596. Solution: B

Let X be the random variable for annual profit.

$$\mathrm{P}[X > 3.50] = 0.3264$$
$$\mathrm{P}\left[Z > \frac{3.50 - \mu}{\sigma}\right] = 0.3264$$
$$\frac{3.50 - \mu}{\sigma} = 0.449876$$
$$\mathrm{P}[X > 3.62] = 0.2743$$
$$\mathrm{P}\left[Z > \frac{3.62 - \mu}{\sigma}\right] = 0.2743$$
$$\frac{3.62 - \mu}{\sigma} = 0.599860$$
$$\frac{3.62 - \mu}{3.50 - \mu} = \frac{0.599860}{0.449876} = 1.33339$$
$$\mu = [3.50(1.33339) - 3.62]/(1.33339 - 1) = 3.140061$$
$$\sigma = (3.62 - 3.140061)/0.599860 = 0.800085$$
$$\sigma^2 = 0.640136.$$

597. Solution: B

A given loss under each policy is uniformly distributed on [a, 19]. Since the expected claim payment on Policy A is 10 when there is no deductible; $(a + 19)/2 = 10,\ a = 1$

For policy B, the expected value is
$$\int_4^{19} (x - 4)(1/18)dx = \left.\frac{(x-4)^2}{36}\right|_4^{19} = \frac{15^2 - 0^2}{36} = 6.25.$$

598. Solution: A

First, $\mathrm{E}(X^2) = \mu^2 + \sigma^2 = 74$.
Second:

$$\mathrm{P}[X > 0] = 0.92$$

$$P\left[Z > \frac{0-\mu}{\sigma}\right] = 0.92$$

$$\frac{0-\mu}{\sigma} = -1.405072$$

$$\mu = 1.405072\sigma$$

$$(1.405072\sigma)^2 + \sigma^2 = 74$$

$$2.974227\sigma^2 = 74$$

$$\sigma^2 = 24.88$$

599. Solution: D

Let *a* be the 75th percentile and *b* be the 25th percentile.

$$\mathrm{P}[X < x] = 1 - e^{-x/\mu}$$

$$0.75 = 1 - e^{-a/\mu}$$

$$0.25 = e^{-a/\mu}$$

$$\ln(0.25) = -\frac{a}{\mu}$$

$$a = -\mu \ln(0.25)$$

Similarly,

$$b = -\mu \ln(0.75)$$

$$a - b = -\mu \ln(0.25) - \left[-\mu \ln(0.75)\right]$$

$$= \mu\left[\ln(0.75) - \ln(0.25)\right]$$

$$= \mu \ln 3$$

600. Solution: D

Let *p* = the probability that a given hospitalization loss is fully reimbursed.

Then, $p = \mathrm{P}\left[\text{loss} < 0.8\right] = F(0.8) = \dfrac{0.8 + 15(0.8)^2}{16} = 0.65$.

So, the probability that the first partially unreimbursed loss occurs on the third hospitalization is

$$p^2\left(1-p\right) = \left(0.65\right)^2(0.35) = 0.147875.$$

601. Solution: C

$$\int_{1100}^{\infty}(x-1100)\frac{3}{1000}\left(\frac{1000}{x}\right)^4 dx$$

$$= \int_{1100}^{\infty} 3(1000)^3 x^{-3}dx - \int_{1100}^{\infty} 3.3(1000)^4 x^{-4}dx$$

$$= \left.\frac{3(1000)^3 x^{-2}}{-2}\right|_{1100}^{\infty} - \left.\frac{3.3(1000)^4 x^{-3}}{-3}\right|_{1100}^{\infty}$$

$$= \frac{3(1000)^3}{2(1100)^2} - \frac{3.3(1000)^4}{3(1100)^3}$$

$$= 1239.67 - 826.45 = 413.22.$$

602. Solution: D

Let p represent the probability of exactly three accidents. Then,

$$0.95 = \text{P[three or fewer accidents]}$$

$$= \text{P[3 accidents]} + \text{P[2 accidents]} + \text{P[1 accident]} + \text{P[0 accidents]}$$

$$= p + 5p + 4\times 5p + 3\times 4\times 5p = 86p$$

$$p = \frac{0.95}{86}.$$

Then,

$$\text{P[two or fewer accidents]} = \text{P[2 accidents]} + \text{P[1 accident]} + \text{P[0 accidents]}$$

$$= 5p + 4\times 5p + 3\times 4\times 5p = 85p$$

$$= 85\frac{0.95}{86} = 0.93895.$$

603. Solution: D

Let S be the number of break-ins with a security system and N be the number without. Let $E(S)=\lambda.$ Then,

$$P(S=0)=e^{-\lambda}=3P(N=0)=3e^{-3}$$

$$-\lambda=\ln(3e^{-3})=(\ln 3)-3$$

$$\lambda=3-\ln 3=1.90139.$$

Let A be the event that the house has a security system and X be the number of break-ins. Then,

$$P(A\,|\,X=2)=\frac{P(X=2\,|\,A)P(A)}{P(X=2\,|\,A)P(A)+P(X=2\,|\,A^c)P(A^c)}$$

$$=\frac{P(S=2)P(A)}{P(S=2)P(A)+P(N=2)P(A^c)}=\frac{\dfrac{1.90139^2e^{-1.90139}}{2!}(0.4)}{\dfrac{1.90139^2e^{-1.90139}}{2!}(0.4)+\dfrac{3^2e^{-3}}{2!}(0.6)}$$

$$=\frac{0.10800}{0.10800+0.13443}=0.44549.$$

604. Solution: A

Let X be the number of tornadoes in a month where $\text{P}[X=x]=\dfrac{e^{-\lambda}\lambda^x}{x!}$, with λ equaling the mean of the distribution.

The number of tornados in six months follows a Poisson distribution with parameter 6λ.

$$\text{P[none in six months]}=\frac{e^{-6\lambda}(6\lambda)^0}{0!}=0.008$$

$$e^{-6\lambda}=0.008$$

$$e^{-2\lambda}=(0.008)^{\frac{1}{3}}=0.20$$

$$\text{P[none in two months]}=\frac{e^{-2\lambda}(2\lambda)^0}{0!}=0.20.$$

Due to independence, the fact that there were no tornados in June is not relevant.

605. Solution: B

Let T be the time until failure. Then,

$$\Pr(A\,|\,T<3)=\frac{\Pr(T<3\,|\,A)\Pr(A)}{\Pr(T<3\,|\,A)\Pr(A)+\Pr(T<3\,|\,B)\Pr(AB)}$$

$$=\frac{(1-e^{-3/2})0.1}{(1-e^{-3/2})0.1+(1-e^{-3/4})0.9}=0.1406.$$

606. Solution: B

The joint density is:

			y		
		0	1	2	Total
	0	36/116	12/116	4/116	52/116
x	1	15/116	20/116	5/116	40/116
	2	8/116	8/116	8/116	24/116
	Total	59/116	40/116	17/116	116/116

$$p(x\,|\,y=1)\begin{cases} 12/40, & \text{for } x=0 \\ 20/40, & \text{for } x=1 \\ 8/40, & \text{for } x=2 \end{cases}$$

Variance equals

$$\left(0^2\times\frac{12}{40}+1^2\times\frac{20}{40}+2^2\times\frac{8}{40}\right)-\left(0\times\frac{12}{40}+1\times\frac{20}{40}+2\times\frac{8}{40}\right)^2$$

$$=\frac{52}{40}-\left(\frac{36}{40}\right)^2=49/100.$$

607. Solution: B

Due to mutual exclusivity, 20% + 42% + 28% = 90% of the calls last at least 2 minutes and less than 8 minutes. Also due to mutual exclusivity, 34% + 38% = 72% of the calls last at least 3 minutes and less than 7 minutes. The solution is the difference, 18%.

608. Solution: E

Let p represent the probability that a given patient has the disease.
Using the binomial formula, we have:

$$10=\frac{\binom{6}{0}p^0(1-p)^{6-0}}{\binom{6}{1}p^1(1-p)^{6-1}}=\frac{1}{6}\left(\frac{1-p}{p}\right) \text{ and } x=\frac{\binom{6}{0}p^0(1-p)^{6-0}}{\binom{6}{3}p^3(1-p)^{6-3}}=\frac{1}{20}\left(\frac{1-p}{p}\right)^3.$$

From the first equation: $60=\left(\frac{1-p}{p}\right)$. Plug into the second equation:

$$x=\frac{1}{20}60^3=10{,}800.$$

609. Solution: D

Let X represent the loss. Since it is given that losses never exceed 5, we have

$$1 = \mathrm{P}[X \le 5] = F(5) = c\left(5^2 + 5\right) = 30c \Rightarrow c = \frac{1}{30}.$$

Therefore, the probability that the insurer pays something on a given loss is

$$\mathrm{P}[X > 3.2] = 1 - \mathrm{P}[X \le 3.2] = 1 - F(3.2) = 1 - c\left[\left(3.2\right)^2 + 3.2\right] = 1 - \frac{1}{30}\left(13.44\right) = 0.552$$

610. Solution: C

Let X represent the loss. We have

$$910 = \int_0^m \frac{x}{2000}\,dx + \int_m^{2000} \frac{m}{2000}\,dx = \frac{m^2}{4000} + \frac{m(2000-m)}{2000} = m - \frac{m^2}{4000}.$$

Solving the resulting equation $m^2 - 4,000m + 3,640,000 = 0$ for m using the quadratic formula yields m = 1400, since the other solution, m = 2600, exceeds 2000.

611. Solution: E

First find the constant c of the density function.

$$c\left[\int_5^8 (x-5)dx + \int_8^{11} (11-x)dx\right] = 1$$

$$c\left[4.5 + 4.5\right] = 1$$

$$c = \frac{1}{9}.$$

The cumulative distribution function from 5 to 8 is:

$$F(x) = \int_5^x \frac{y-5}{9}\,dy = \left.\frac{(y-5)^2}{18}\right|_5^x = \frac{(x-5)^2}{18}.$$

Because $F(8) = 0.5$, the 30th percentile must be less than 8. Then,

$$0.3 = F(m) = \frac{(m-5)^2}{18}$$

$$5.4 = (m-5)^2$$

$$2.324 = m - 5$$

$$m = 7.324$$

612. Solution: A

First, solve for c for this exponential distribution:

$$\int_0^{10} ce^{-cx}dx = 1 - e^{-10c} = 0.5 => c = \frac{\ln 2}{10} = 0.069315$$

Next, the expected cost of the warranty is:

$$35F(5) + 25[F(7.5) - F(5)]$$
$$= 35(1 - e^{-5c}) + 25(e^{-5c} - e^{-7.5c})$$
$$= 35(1 - 0.70711) + 25(0.70711 - 0.5946) = 13.06.$$

613. Solution: D

Let λ_1 and λ_2 represent the means of the numbers of accidents at the first and second construction sites, respectively. From the given data, we have $\frac{e^{-\lambda_2}\lambda_2^{\,0}}{0!} = 1.1\left(\frac{e^{-\lambda_1}\lambda_1^{\,0}}{0!}\right)$ which simplifies to $e^{-\lambda_2} = 1.1e^{-\lambda_1}$. and $\sqrt{\lambda_1} = 1.5$. So $\lambda_1 = (1.5)^2$ and then,

$$e^{-\lambda_2} = 1.1e^{-(1.5)^2} \Rightarrow -\lambda_2 = (\ln 1.1) - (1.5)^2 \Rightarrow \lambda_2 = (1.5)^2 - \ln 1.1.$$

The standard deviation for the number of accidents at the second construction site is

$$\sqrt{\lambda_2} = \sqrt{(1.5)^2 - \ln 1.1} = 1.46789.$$

614. Solution: C

For independent random variables, the variance of the sum is the sum of the variances. Therefore, the standard deviation of the company's two-year profit is $\sqrt{1100^2 + 2640^2} = 2860$. Let *X* represent the two-year profit.

$$\mathrm{P}[X > 0] = \mathrm{P}\left[Z > \frac{0 - (660 + \mu)}{2860}\right] = 0.8643$$

$$\mathrm{P}\left[Z < \frac{0 - (660 + \mu)}{2860}\right] = 0.1357$$

$$\frac{0 - (660 + \mu)}{2860} = -1.099844$$

$$\mu = 2485.55.$$

615. Solution: B

Let D be the number of accidents in one day. Due to independence the expected number of accidents in one day is E(D) = 63/7 = 9. For a Poisson distribution the variance is equal to the mean. Thus, $\text{Var}(D) = \text{E}(D) = 9, \text{SD}(D) = 3.$

The probability of being two more standard deviations below the mean is

$$\text{P}[D \le 9 - 2\times 3] = \text{P}[D \le 3]$$
$$= \text{P}\left[D = 0\right] + \text{P}\left[D = 1\right] + \text{P}\left[D = 2\right] + \text{P}\left[D = 3\right]$$
$$= \frac{e^{-9}9^0}{0!} + \frac{e^{-9}9^1}{1!} + \frac{e^{-9}9^2}{2!} + \frac{e^{-9}9^3}{3!}$$
$$= 0.000123 + 0.001111 + 0.004998 + 0.014994 = 0.021226.$$

616. Solution: C

For Random Number Generator C:

$$\text{Var}(C) = \frac{(10-0)^2}{12} = 8.3333, \text{SD}(C) = \sqrt{8.3333} = 2.8868$$

For Random Number Generator D:

$$\text{E}(D) = \frac{1}{11}\left[0+1+2+3+4+5+6+7+8+9+10\right] = 5$$
$$\text{E}(D^2) = \frac{1}{11}\left[0^2+1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2\right] = 35$$
$$\text{Var}(D) = 35 - 5^2 = 10, SD(D) = \sqrt{10} = 3.1623.$$

The absolute difference is: $|2.8868 - 3.1623| = 0.2755.$

617. Solution: D

Var($T|F$ = 1) = E($T^2|F$ = 1) – {E($T|F$ = 1)}^{2}. To calculate these two expected values, we must first find the conditional probability mass function $p(t|F = 1)$, where t = 0,1,2.

$$p(t \mid F = 1) = \begin{cases} 14/25 = 0.56, & t = 0 \\ 8/25 = 0.32, & t = 1 \\ 3/25 = 0.12, & t = 2 \end{cases}$$
$$\text{E}(T \mid F = 1) = 0(0.56) + 1(0.32) + 2(0.12) = 0.56$$
$$\text{E}(T^2 \mid F = 1) = 0^2(0.56) + 1^2(0.32) + 2^2(0.12) = 0.80$$
$$\text{Var}(T \mid F = 1) = 0.80 - 0.56^2 = 0.4864.$$

618. Solution: B

$$\frac{a+b}{2}=8.5,\ a=17-b$$

$$\frac{(b-a)^2}{12}=0.75, (b-a)^2=9,\ b-a=3$$

$$b-(17-b)=3,\ 2b=20,\ b=10$$

$$a=17-10=7$$

$$\mathrm{P}[X>9 \mid X>7.5]=\frac{\frac{10-9}{10-7}}{\frac{10-7.5}{10-7}}=\frac{1.0}{2.5}=0.40.$$

619. Solution: E

$$\frac{e^{-\lambda}\lambda^4}{4!}=\frac{54e^{-\lambda}\lambda^0}{0!}$$

$$\frac{\lambda^4}{4!}=54$$

$$\lambda^4=1296, \lambda=6$$

The expected number of hits in a 60-minute period is 60(6) = 360.

620. Solution: B

$$\mathrm{P}[3\text{ products}]=(0.55)(0.45)(0.50)(0.40)+(0.45)(0.45)(0.50)(0.60)$$
$$+(0.55)(0.55)(0.50)(0.60)+(0.55)(0.45)(0.50)(0.60)=0.27525$$

$$\mathrm{P}[4\text{ products}]=(0.55)(0.45)(0.50)(0.60)=0.07425$$

So $P(3\text{ or }4)=0.27525+0.07425=0.3495$.

621. Solution: D

Since exponential distributions are memoryless,

$$\mathrm{P}[15<X<35 \mid X>10]=\mathrm{P}[5<X<25]=\mathrm{P}[X<25]-\mathrm{P}[X<5]$$
$$=1-e^{-25/15}-\left(1-e^{-5/15}\right)$$
$$=e^{-5/15}-e^{-25/15}=0.52766.$$

622. Solution: E

$$\frac{a+b}{2}=16.36,\ a=32.72-b$$

$$\frac{(b-a)^2}{12}=7.63^2,\ \frac{b-a}{\sqrt{12}}=7.63,\ b-a=26.431095$$

Substitute the first equation into the second,

$$b-(32.72-b)=26.431095$$

$$b=29.5755$$

Plug this value of *b* back into the first equation:

$$a=32.72-29.5755=3.1445$$

$$\frac{b}{a}=\frac{29.5755}{3.1445}=9.4056.$$

623. Solution: B

Let X be the normal random variable. For a normal distribution, the mode equals the mean, so μ= 56.

$$\mathrm{P}[X<52.20]=0.40$$

$$\mathrm{P}\left[Z<\frac{52.20-56}{\sigma}\right]=0.40$$

$$\frac{52.20-56}{\sigma}=-0.253347$$

$$\sigma=14.999185$$

$$\text{Now, } P[X<65.50]=P\left[Z<\frac{65.50-56}{14.999185}\right]$$

$$=P\left[Z<0.6334\right]=0.7368.$$

624. Solution: E

Let X – automobile Insurance
Let Y – homeowners insurance

$$\mathrm{E}(X+Y)=400-100=300$$

$$\mathrm{Var}(X+Y)=200^2+500^2=290,000$$

$$\mathrm{SD}(X+Y)=\sqrt{290,000}=538.5165$$

$$\mathrm{P}[X+Y>0]=\mathrm{P}\left[Z>\frac{0-300}{538.5165}\right]=\mathrm{P}[Z>-0.5571]=0.7113.$$

625. Solution: E

The number of hurricanes in 3 years is Poisson distributed with mean 3. For 10 hurricanes to occur in 10 years given that 8 have occurred in the first 7 years, 2 hurricanes must occur in the last 3 years.
Let *E* be the event that 2 hurricanes occur in the last 3 years.

Then, $\text{P}[E] = \dfrac{e^{-3} \cdot 3^2}{2!} = 0.2240.$

626. Solution: A

$V_i = \lambda_i$ for $i = 1, 2$
From the given info regarding the probability of no claims, we have:

$$\frac{e^{-\lambda_1}\lambda_1^{\ 0}}{0!} = 0.5\frac{e^{-\lambda_2}\lambda_2^{\ 0}}{0!}$$

$$2e^{-\lambda_1} = e^{-\lambda_2}$$

$$2 = e^{-(\lambda_2 - \lambda_1)}$$

$$\lambda_1 - \lambda_2 = \ln 2 = 0.6931.$$

627. Solution: C

$$p(y \mid X < 2) = \begin{cases} 0.14/0.70 = 0.2000, & y = 0 \\ 0.21/0.70 = 0.3000, & y = 1 \\ 0.27/0.70 = 0.3857, & y = 2 \\ 0.08/0.70 = 0.1143, & y = 3 \end{cases}$$

$$E(Y \mid X < 2) = 0(0.2000) + 1(0.3000) + 2(0.3857) + 3(0.1143) = 1.4143.$$

628. Solution: E

$$0.75 = \int_0^1 (\alpha+1) r^{\alpha+1} dr = \frac{\alpha+1}{\alpha+2} r^{\alpha+2}\Big|_0^1 = \frac{\alpha+1}{\alpha+2}$$

$$\alpha = 2$$

$$\Pr(R > 0.5) = \int_{0.5}^1 3r^2 dr = r^3\Big|_{0.5}^1 = 0.875.$$

629. Solution: D

$$47.8 = E(Y^2) = a^2(0.2) + 1^2(0.1) + 3^2(0.3) + 10^2(0.4) = 0.2a^2 + 42.8$$
$$5 = 0.2a^2$$
$$a = -\sqrt{25} = -5$$
$$E(Y) = -5(0.2) + 1(0.1) + 3(0.3) + 10(0.4) = 4 \ .$$

Consequently, $\sigma^2 = E(Y^2) - [E(Y)]^2 = 47.8 - 4^2 = 47.8 - 16 = 31.8$.

630. Solution: A

$$\int_2^\infty (x-2)e^{-x}dx = -(x-2)\,e^{-x}\Big|_2^\infty + \int_2^\infty e^{-x}dx = e^{-2} \ .$$

631. Solution: A

$$P[Y=0 \mid X=1] = \frac{P[Y=0, X=1]}{P[X=1]} = \frac{1/8}{1/8+1/2} = 1/5$$
$$P[Y=1 \mid X=1] = 1 - P[Y=0 \mid X=1] = 4/5$$
$$E[Y \mid X=1] = 0(1/5) + 1(4/5) = 4/5$$
$$E[Y^2 \mid X=1] = 0^2(1/5) + 1^2(4/5) = 4/5$$
$$Var[Y \mid X=1] = 4/5 - (4/5)^2 = 4/25$$

Or, recognize this is a binomial distribution with $n = 1$ and $p = 4/5$ and thus the variance is 1(4/5)(1/5) = 4/25.

632. Solution: D

Let X be the number of years in which his crops are destroyed. Then X is binomial with $n = 5$ and $p = 0.5$ and E (benefit) is

$$30\big[0P(X=0) + 1P(X=1) + 2P(X=2) + 3P(X=3,4,5)\big]$$

$P(X=0) = 0.03125$ $\qquad$ $P(X=3) = 0.31250$

$P(X=1) = 0.15625$ $\qquad$ $P(X=4) = 0.15625$

$P(X=2) = 0.31250$ $\qquad$ $P(X=5) = 0.03125$

The answer is 30[0.15625 + 2(0.31250) + 3(0.5)] = 68.4375.

633. Solution: C

N_1: Claims filed by trucking company, N_2: Claims filed by shuttle service
Note: Sum of Poissons is also Poisson.

$$\left.\begin{aligned} P[N_1=6] &= \frac{2^6 e^{-2}}{6!} \\ P[N_2=2] &= \frac{3^2 e^{-3}}{2!} \\ P[N_1+N_2=8] &= \frac{5^8 e^{-5}}{8!} \end{aligned}\right\} P[N_1=6 \mid N_1+N_2=8] = \frac{\frac{2^6 e^{-2}}{6!} \cdot \frac{3^2 e^{-3}}{2!}}{\frac{5^8 e^{-5}}{8!}} = \frac{(0.01203)(0.22404)}{(0.06528)} = 0.041288.$$

634. Solution: E

From the given information:

$$2 = 1.2 + \mathrm{E}(Z)$$
$$\mathrm{E}(Z) = 0.8$$
$$2.25 = 0.9 + \mathrm{V}(Z)$$
$$\mathrm{V}(Z) = 1.35$$
$$\mathrm{CV}(Z) = \frac{\sqrt{1.35}}{0.8} = 1.45237.$$

635. Solution: D

From i) and ii),

$E(U) = \alpha + \beta + 4$

$Var(U) = \alpha^2 + \beta^2 + 16$

$E(V) = \alpha - \beta$

$Var(V) = \alpha^2 + \beta^2$

From iii),

$\alpha + \beta + 4 = \alpha^2 + \beta^2$

From iv),

$$\alpha + \beta + 4 - (\alpha - \beta) = \frac{\alpha^2 + \beta^2 + 16 - (\alpha^2 + \beta^2)}{2}$$

$2\beta + 4 = 8$

$\beta = 2$

Then iii) becomes

$\alpha + 2 + 4 = \alpha^2 + 4$

$\alpha^2 - \alpha - 2 = 0$

$(\alpha - 2)(\alpha + 1) = 0$

$\alpha = 2,$ because the value must be positive

636. Solution: A

Let *X* denote total hail losses. We will apply the Central Limit Theorem, due to independence. For each rating zone, the mean is 50,000 and the standard deviation is also 50,000 so the variance is 2,500,000,000. Then,

E(*X*) =60(50,000) = 3,000,000
Var(*X*) = 60(2,500,000,000) = 150,000,000,000
Std Dev(*X*) = =387,298.3346
Then P [*X* > 3,500,000]
= P [*Z* > (3,500,000-3,000,000)/387,298.3346] = P[*Z* > 1.2910] = 0.098

637. Solution: B

Let S be the random variable that is the number of claims processed during the year.

S is an approximate normal distribution with mean $3300(303) = 999{,}900$ and standard deviation $\frac{130(303)}{\sqrt{303}} = 2262.8964$.

$$P[S > 1{,}000{,}000] = P\left[Z > \frac{1{,}000{,}000 - 999{,}900}{2{,}262.8964}\right]$$
$$= P\left[Z > 0.0442\right] = 0.4840.$$

638. Solution: B

$$P(B \mid S = 2) = \begin{cases} 0.07/0.33 = 0.2121, & b = 1 \\ 0.10/0.22 = 0.3030, & b = 2 \\ 0.08/0.33 = 0.2424, & b = 3 \\ 0.05/0.33 = 0.1515, & b = 4 \\ 0.03/0.33 = 0.0909, & b = 5 \end{cases}$$
$$E(B \mid S = 2) = 1(0.2121) + 2(0.3030) + 3(0.2424) + 4(0.1515) + 5(0.0909) = 2.60606$$
$$E(B^2 \mid S = 2) = 1^2(0.2121) + 2^2(0.3030) + 3^2(0.2424) + 4^2(0.1515) + 5^2(0.0909) = 8.30303$$
$$Var(B \mid S = 2) = 8.30303 - 2.60606^2 = 1.51148.$$

639. Solution: B

Let X = # of fillings; Y = # of root canals

$$P[X = x] = \frac{e^{-1}1^x}{x!},\ P[Y = y] = \frac{e^{-0.3}0.3^y}{y!}$$
$$P[X \le 2] = e^{-1}\left[\frac{1^0}{0!} + \frac{1^1}{1!} + \frac{1^2}{2!}\right] = 2.5e^{-1}$$
$$P[Y \le 1] = e^{-0.3}\left[\frac{0.3^0}{0!} + \frac{0.3^1}{1!}\right] = 1.3e^{-0.3}$$
$$P[X \le 2]P[Y \le 1] = 2.5e^{-1}1.3e^{-0.3}, \text{by independence.}$$

640. Solution: E

$$\mathrm{P}\left[\text{At least one package lost}\right] = 1 - \mathrm{P}\left[\text{No packages lost}\right] = 1 - (1-p)^3$$

$$\mathrm{P}\left[\text{Exactly 1 package lost and at least 1 package lost}\right]$$

$$= \mathrm{P}\left[\text{Exactly 1 package lost}\right] = 3p(1-p)^2$$

$$\mathrm{P}\left[\frac{\text{Exactly 1 package lost}}{\text{At least 1 package lost}}\right] = \frac{3p(1-p)^2}{1-(1-p)^3}$$

641. Solution: B

P(exactly one type of loss) = 0.1(0.9)(0.8) + 0.9(0.1)(0.8) + 0.9(0.9)(0.2) = 0.306.
P(exactly one type of loss and a loss due to fire) = 0.1(0.9)(0.8) =0.072.
The conditional probability is 0.072/0.306 = 0.235.

642. Solution: C

Let X represent the car's lifetime.

$\mathrm{Var}[X \mid X \geq 5] = \mathrm{Var}[X - 5 \mid X \geq 5]$ since adding a constant does not affect variance

$= \mathrm{Var}[X]$ since X is memoryless.

$= 9^2 = 81.$

643. Solution: E

$$P[X \leq 4] = e^{-2}\left[\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!}\right] = 7e^{-2}.$$

644. Solution: D

Each computer has the same chance of breaking, so all combinations of 3 broken computers are equally likely. 4 computers are insured; 2 are not. Using the hypergeometric distribution:

$$\text{P[Uninsured} = 0 \mid \text{Broken} = 3] = \frac{\binom{2}{0}\binom{4}{3}}{\binom{6}{3}} = \frac{1(4)}{20} = 0.20.$$

Alternatively, using the binomial distribution the probability of three being damaged is $\binom{6}{3}(0.1)^3(0.9)^3 = 0.01458$. Assume computers 1 and 2 are not insured. The probability of three being damaged and all insured is $\binom{2}{0}(0.1)^0(0.9)^2\binom{4}{3}(0.1)^3(0.9)^1 = 0.002916$. The ratio is 0.2.

645. Solution: C

$$\text{P}\left[\text{First reimbursed theft in 2nd year} \mid \text{Exactly 2 reimbursed thefts in 5 years}\right]$$

$$= \frac{\text{P}\left[\text{First reimbursed theft in 2nd year and exactly 1 reimbursed theft in years 3,4,5}\right]}{\text{P}\left[\text{Exactly 2 reimbursed thefts in 5 years}\right]}$$

$$= \frac{(0.8)(0.2)\times\binom{3}{1}(0.8)^2(0.2)}{\binom{5}{2}(0.8)^3(0.2)^2} = 0.30.$$

646. Solution: D

P[at least 1 claim in 7 days] = 1 – P[no claims in 7 days] = 1 – {P[no claims on a given day]}^{7}
= 1 – r^7.

647. Solution: C

Let P[Heads] = p and P[Tails] = $(1-p)$

P[X = 1] = $p(1/9) + (1-p)(1/6) = 1/6 - p/18$

P$[X = 2] = p(2/9) + (1-p)(1/6) = 1/6 + p/18$

P$[X = 3] = p(1/9) + (1-p)(1/6) = 1/6 - p/18$

$F(3) = P[X = 1] + P[X = 2] + P[X = 3] = 1/2 - p/18$

$$0.463 = \frac{1}{2} - \frac{p}{18}$$

$p = 0.666$

$P[X = 4] = 0.666(2/9) + (1-0.666)(1/6) = 0.204.$

648. Solution: D

$$f(x) = \begin{cases} \frac{2x}{25}, & \text{for } 0 \le x \le 5 \\ 0, & \text{otherwise} \end{cases}$$

$$E[X^2] = \int_0^5 \frac{2x^3}{25} dx = \left.\frac{2x^4}{100}\right|_0^5 = 12.50.$$

649. Solution: C

Let m be the median.

$$\int_0^m 8xe^{-4x^2} dx = 0.5$$

$$\left.-e^{-4x^2}\right|_0^m = 0.5$$

$$-e^{-4m^2} + 1 = 0.5$$

$$e^{-4m^2} = 0.5$$

$$m = \sqrt{\frac{-\ln 0.5}{4}} = 0.4163.$$

650. Solution: E

$$\mathrm{E}\left(\text{Total Loss}\right)=\sum_{0\le i+j\le 2}(2i+20j)\frac{2!}{i!\,j!(2-i-j)!}\left(0.2\right)^{i}\left(0.1\right)^{j}\left(0.7\right)^{2-i-j}$$
$$=(0+0)0.49+(0+20)0.14+(0+40)0.01+(2+0)0.28+(2+20)0.04+(4+0)0.04$$
$$=4.80.$$

651. Solution: A

Let Y = loss in Year 2.

$$p(y)=\begin{cases}0.80, & y=0\\ 0.10, & y=2\\ 0.06, & y=5\\ 0.04, & y=10\end{cases}$$
$$\mathrm{E}\left(\text{Unreimbursed Loss}\right) = \mathrm{E}\left(0.4Y\right)$$
$$=0.4(0.80)(0)+0.4(0.10)(2)+0.4(0.06)(5)+0.4(0.04)(10)=0.36.$$

652. Solution: A

The marginal probability function for X is:

$$p(x)=\begin{cases}0, & x=0\\ \frac{1}{40}+\frac{3}{40}=\frac{4}{40}, & x=1\\ \frac{2}{40}+\frac{4}{40}+\frac{6}{40}=\frac{12}{40}, & x=2\\ \frac{3}{40}+\frac{5}{40}+\frac{7}{40}+\frac{9}{40}=\frac{24}{40}, & x=3\end{cases}$$
$$\mathrm{E}(X)=0(0)+\frac{4}{40}(1)+\frac{12}{40}(2)+\frac{24}{40}(3)=2.5$$
$$\mathrm{E}(X^2)=0(0^2)+\frac{4}{40}(1^2)+\frac{12}{40}(2^2)+\frac{24}{40}(3^2)=6.7$$
$$\mathrm{Var}(X)=6.7-2.5^2=0.45.$$

653. Solution: C

Let X be the value showing on a face and let S be the sum of a roll.

$$\mathrm{P}[X=1]=\frac{1}{3},\mathrm{P}[X=2]=\frac{2}{3}$$

$$\mathrm{P}[S=2]=\frac{1}{9},\mathrm{P}[X=3]=\frac{4}{9},\mathrm{P}[X=4]=\frac{4}{9}$$

P[two sums are different] = 1 – P[two sums are the same] =

$$1-\left[\frac{1}{9}\frac{1}{9}+\frac{4}{9}\frac{4}{9}+\frac{4}{9}\frac{4}{9}\right]=1-\frac{33}{81}=\frac{48}{81}=\frac{16}{27}.$$

654. Solution: B

Let X_1, X_2, X_3 be random variables with uniform probability density on [0,10].

$$X_{(3)}=\mathrm{Max}(X_1, X_2, X_3)$$

$$\mathrm{P}(X_{(3)}\le x)=\mathrm{P}(X_1\le x, X_2\le x, X_3\le x)=\mathrm{P}(X_1\le x)\mathrm{P}(X_2\le x)\mathrm{P}(X_3\le x)$$

$$=\left[\mathrm{P}(X\le x)\right]^3=F(x)^3=\left[\frac{x}{10}\right]^3=\frac{x^3}{1000}.$$

$$f_{(3)}(x)=\frac{d}{dx}\left[\frac{x^3}{1000}\right]=\frac{3x^2}{1000}$$

$$\mathrm{E}\left[X_{(3)}\right]=\int_0^{10}\frac{3x^3}{1000}dx=\frac{3x^4}{4000}\Bigg|_0^{10}=7.5.$$

655. Solution: D

Let X = number of earthquakes in next 10 years, so

$$\lambda=\frac{2}{30}(10)=\frac{2}{3}.$$

Then the mean number of earthquakes is $\frac{2}{3}$.

The variance is also $\frac{2}{3}$.

Standard Deviation = $\sqrt{\frac{2}{3}}=0.816.$

656. Solution: D

Then the probability sought is:
$p(0,1)+p(0,2)+p(1,1)=0.14+0.01+0.04=0.19.$

657. Solution: C

Let *i* be the event that the randomly chosen individual donates items to the auction, and let *c* be the event that the randomly chosen individual donates cash. Events *i* and *c* could be mutually exclusive, in which case the answer is p = 1 – 0.3 – 0.6 = 0.1. At the other extreme, event *c* could be entirely contained in event *i*. In this case the answer is p = 1 – 0.6 = 0.4. Henc, p will lie in the interval [0.1, 0.4].

658. Solution: D

$$\mathrm{P}[A\cup B\cup C]=\mathrm{P}[A]+\mathrm{P}[B]+\mathrm{P}[C]-\mathrm{P}[A\cap B]-\mathrm{P}[A\cap C]-\mathrm{P}[B\cap C]+\mathrm{P}[A\cap B\cap C]$$
$$\mathrm{P}[A\cup B\cup C]=\mathrm{P}[A]+\mathrm{P}[B]+\mathrm{P}[C]-\{\mathrm{P}[B]\mathrm{P}[A\mid B]\}-\{\mathrm{P}[A]+\mathrm{P}[C]-\mathrm{P}[A\cup C]\}$$
$$\mathrm{P}[A\cup B\cup C]=\mathrm{P}[B]-\mathrm{P}[B]\mathrm{P}[A\mid B]+\mathrm{P}[A\cup C]$$
$$1=0.80-(0.80)(0.15r)+(0.17r)$$
$$r=4.00$$

659. Solution: D

Let X_r denote the trial on which the r^{th} success occurs in a sequence of Bernoulli trials. Then X_r has the negative binomial distribution:

$$\mathrm{P}[X_r=x]=\binom{x-1}{r-1}p^r(1-p)^{x-r}, \quad where \quad x=r,r+1,r+2,\ldots$$

$$\mathrm{P}[10\le X_2\le 12]=\sum_{x=10}^{12}\binom{x-1}{1}\left(\frac{3}{10}\right)^2\left(\frac{7}{10}\right)^{x-2}=0.0467+0.0363+0.0280=0.1110.$$

660. Solution: B

Let X = number of months until the motorist is charged, which is exponentially distributed with parameter $\lambda=\frac{1}{2.5}=0.4$. We want to find $P(X>2.5)=e^{-\frac{1}{2.5}2.5}=e^{-1}$.

661. Solution: A

Let X = number of buildings in the group damaged by high tides
We want to find P[at least one damaged | at least one undamaged]

$$\mathrm{P}[X \geq 1 \mid X \leq 3] = \frac{\mathrm{P}[1 \leq X \leq 3]}{\mathrm{P}[X \leq 3]} = \frac{0.015 + 0.01 + 0.02}{0.92 + 0.015 + 0.01 + 0.02} = \frac{9}{193} = 0.0466.$$

662. Solution: E

At least eight problems are resolved if and only if the two technicians achieve one of the following joint records of problem resolution:

Problems Solved by Technician #1	Problems Solved by Technician #2	Probability
5	5	(2/20)(2/20) = 4/400
5	4	(2/20)(3/20) = 6/400
4	5	(3/20)(2/20) = 6/400
4	4	(3/20)(3/20) = 9/400
5	3	(2/20)(8/20) = 16/400
3	5	(8/20)(2/20) = 16/400

The solution is the sum, 57/400.

663. Solution: B

Let X denote the time it takes to complete the 5 reports. We want:

$$\mathrm{P}[X \leq 8 \mid X \geq 7.5] = \frac{\mathrm{P}[7.5 \leq X \leq 8]}{\mathrm{P}\left[X \geq 7.5\right]}$$

$$= \frac{\mathrm{P}\left[\frac{7.5-7.5}{1.5} \leq Z \leq \frac{8-7.5}{1.5}\right]}{\mathrm{P}\left[Z \geq \frac{7.5-7.5}{1.5}\right]} = \frac{\mathrm{P}\left[0 \leq Z \leq 0.333\right]}{\mathrm{P}[Z \geq 0]}$$

$$= \frac{0.13056}{0.5} = 0.2611.$$

664. Solution: B

$\mathrm{P}[X > 20000] = e^{\frac{-1}{20,000}20,000} = e^{-1} = \frac{1}{e}.$ The answer is binomial for number of losses:

$$\binom{10}{9}\left(\frac{1}{e}\right)^9\left(1-\frac{1}{e}\right) + \binom{10}{10}\left(\frac{1}{e}\right)^{10} = \frac{10(e-1)+1}{e^{10}} = \frac{10e-9}{e^{10}}.$$

665. Solution: C

The mean for a 100 page document is 100(0.10) = 10.

$P[X > 4] = 1 - P[X \le 4] = 1 - (P[X = 0] + P[X = 1] + P[X = 2] + P[X = 3] + P[X = 4]$

$= 1 - \frac{e^{-10}10^0}{0!} - \frac{e^{-10}10^1}{1!} - \frac{e^{-10}10^2}{2!} - \frac{e^{-10}10^3}{3!} - \frac{e^{-10}10^4}{4!}$

$= 1 - (.000045 + .00045 + .00227 + .00757 + .01892) = 0.97075.$

666. Solution: B

$P[10.5 < X < 11.5] = 0.80$

By symmetry of the normal distribution:

$P[X < 11.5] = 0.90$

$P\left[Z < \frac{11.5 - 11}{\sigma} \right] = 0.90$

$\frac{11.5 - 11}{\sigma} = 1.28155$

$\sigma = 0.39015.$

667. Solution: C

For the exponential model, the 80th percentile is $1 - e^{\frac{-p}{500}} = 0.8, \quad p = 804.7190$.

For the normal distribution, the *z*-value of the 80th percentile is 0.84162. Hence,

$\frac{804.7190 - 500}{\sigma} = 0.84162, \quad \sigma = 362.06245.$. Then the second moment is

$E(X^2) = 362.06245^2 + 500^2 = 381,089.$

668. Solution: E

Let the interval be from *a* to *b*. The median is 6 = (*a* + *b*)/2 leading to the equation *a* + *b* = 12. The 90th percentile is 13.2 =0.1*a* +-0.9*b* leading to the equation *a* +9*b* = 132. Subtracting the first equation from the second yields 8*b* = 120 for *b* = 15 and then *a* = –3. The second moment is

$\int_{-3}^{15} x^2 (1/18) dx = x^3 / 54 \Big|_{-3}^{15} = (3375 + 27) / 54 = 63.$

Alternatively, using the formulas for the variance and mean of the uniform distribution the second moment is $[15 - (-3)]^2 / 12 + [(15 + (-3)) / 2]^2 = 27 + 36 = 63.$

669. Solution: C

Let p = probability of nonnegative annual profit in a given year.

Then 0.36 = P[negative annual profit in at least one of the next two years] = $1 - p^2$.
So, $p = 0.80$.

Since the mean is 100, then by symmetry the probability that a given annual profit is less than 200 is also 0.80.

From the normal table, a cumulative probability 0.80 yields an approximate z-score of 0.84162. So annual profits of 100 and 200 correspond to z-scores of 0 and approximately 0.84162, respectively.

This means that a profit difference of 200 – 100 = 100 represents approximately 0.84162 – 0 = 0.84162 standard deviations. Thus the standard deviation of the annual profit is
$$\frac{100}{0.84162} = 118.82.$$

670. Solution: A

The 95th percentile of the loss is 2500+1.6449(500) = 3322.45. Since the median and the mean are the same, the median is 2500. Therefore, the excess is 3322.45 – 2500 = 822.45.

671. Solution: C

The mean of the sum is $70 + 80 = 150$. The standard deviation of the sum is $\left[14^2 + 20^2\right]^{0.5} = 24.4131$. The mean of the average is $\frac{150}{2} = 75$. The standard deviation is $\frac{24.4131}{2} = 12.2066$. The z-score we are looking for is $\frac{80-75}{12.2066} = 0.4096$. So, we want $P[X > 80] = P\left[Z > 0.4096\right] = 0.34105$.

672. Solution: D

Let λ be the mean of X, the time to failure. Let m be the median of X. Since

$$0.5 = \mathrm{P}[X \ge m] = e^{-m/\lambda}$$
$$\lambda \ln 2 = m$$
$$3.8 = \lambda - \lambda \ln 2$$
$$\lambda = 12.38379$$
$$\mathrm{Var}(X) = \lambda^2 = 153.36.$$

673. Solution: B

$$Cov(X,Y) = \rho\sigma_X\sigma_Y = 0.5(1)(2) = 1$$
$$0 = Cov[(X+Y),\ (cX+Y)] = E\left[\left(X+Y\right)\left(cX+Y\right)\right] - \mu_{X+Y}\mu_{cX+Y}$$
$$0 = cE\left(X^2\right) + \left(1+c\right)E\left(XY\right) + E\left(Y^2\right) - \mu_{X+Y}\mu_{cX+Y}$$
$$0 = c(\sigma_X{}^2 + \mu_X{}^2) + \left(1+c\right)(Cov(X,Y) + \mu_X\mu_Y) + \sigma_Y^2 + \mu_Y^2 -\ (\mu_X + \mu_Y)(c\mu_X + \mu_Y)$$
$$0 = c\sigma_X{}^2 + \left(1+c\right)(Cov(X,Y) + \mu_X\mu_Y) + \sigma_Y^2$$
$$0 = c + (1+c)(1+2(0)) + 4$$
$$0 = 2c + 5$$
$$c = -\frac{5}{2}.$$

674. Solution: C

Let X = number of fillings this year, which is Poisson distributed with mean 3.

We need to find the smallest integer n such that $P[X \le n] \ge 0.75$. Thus, we need to find the smallest integer n such that $\sum_{i=0}^{n} \frac{e^{-3}3^i}{i!} \ge 0.75$.

$P[X \le 3] = 0.6472$

$P[X \le 4] = 0.8153$

So, $n = 4$

The patient should choose the policy that reimburses a maximum of 4 fillings, policy C.

675. Solution: D

$$P[X \ge 1] = 0.368$$
$$P[X = 0] = 0.632 = \frac{e^{-\lambda}\lambda^0}{0!}$$
$$\lambda = -\ln(0.632) = 0.4589$$
$$\mathrm{E}[X^2] = \mathrm{Var}(X) + \mathrm{E}(X)^2 = \lambda + \lambda^2 = 0.6694.$$

676. Solution: B

For a three-month period, the combined total number of such traffic violations is the sum of monthly violations:

$S = X_1 + X_2 + X_3 + Y_1 + Y_2 + Y_3$.

Assuming independence, we have

$$Var(S) = \mathrm{Var}(X_1 + X_2 + X_3 + Y_1 + Y_2 + Y_3) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \cdots + \mathrm{Var}(Y_3)$$
$$= 15 + 15 + 15 + 30 + 30 + 30 = 135.$$

677. Solution: B

$$0.5 = F(m) = \int_0^m 0.4\left(e^{-0.6x} + e^{-1.2x}\right)dx = 0.4\left(-\frac{e^{-0.6x}}{0.6} - \frac{e^{-1.2x}}{1.2}\right)\Bigg|_0^m = 0.4\left(2.5 - \frac{e^{-0.6m}}{0.6} - \frac{e^{-1.2m}}{1.2}\right)$$

Let $y = e^{-0.6m}$.

$$1.25 = 2.5 - \frac{y}{0.6} - \frac{y^2}{1.2}$$

$$1.5 = 3 - 2y - y^2$$

$$y^2 + 2y - 1.5 = 0 \qquad .$$

$$y = \frac{-2 + \sqrt{4+6}}{2} = 0.58114$$

$$m = -\ln(0.58114)/0.6 = 0.9046.$$

(Note that the negative solution to the quadratic equation does not work since *e* to any power must be positive.)

678. Solution: C

$$\mathrm{E}(X) = \frac{a+b}{2} = \frac{65+95}{2} = 80$$

$$\mathrm{Var}(X) = \frac{(b-a)^2}{12} = \frac{(95-65)^2}{12} = 75$$

$$SD(X) = \sqrt{75} = 8.660254$$

$$P\left[80 - \sqrt{75} < X < 80 + \sqrt{75}\right] = \frac{88.6603 - 71.3397}{30} = 0.5774.$$

679. Solution: E

14% is the probability that a claim exceeds 500 + *d*. Hence,

$$0.14 = P(X > 500 + d) = P\left(Z > \frac{500 + d - 525}{100} = 0.01d - 0.25\right)$$

$$1.0803 = 0.01d - 0.25$$

$$d = 133.03.$$

680. Solution: D

Let p be the 10th percentile of X. Then, we wish to solve for p such that
$P[X < p] = F(p) = 0.10.$

Now, $F(x) = 1 - e^{\frac{-(x-d)}{\beta}}$

So, $1 - e^{\frac{-(p-d)}{\beta}} = 0.10$

$$e^{\frac{-(p-d)}{\beta}} = 0.90$$

$$\frac{-(p-d)}{\beta} = \ln 0.90$$

$$\frac{(p-d)}{\beta} = \ln\frac{10}{9}$$

$$p = \beta \ln\frac{10}{9} + d$$

681. Solution: D

For a single policyholder, the variance is

$$E(X) = 0.9(0) + 0.08(1) + 0.02(2) = 0.12$$
$$E(X^2) = 0.9(0) + 0.08(1) + 0.02(4) = 0.16$$
$$Var(X) = 0.16 - 0.12^2 = 0.1456$$

Since the policyholders are independent, the variance of the sum is the sum of the variance, or 64(0.1456) = 9.3184.

682. Solution: C

Let X be the number with the blood disorder. It has a binomial distribution with n = 3 and p = 0.01. We then want:

$$P[X = 2 \mid X \geq 1] = \frac{P[X = 2 \text{ and } X \geq 1]}{P[X \geq 1]}$$

$$= \frac{P[X = 2]}{1 - P[X = 0]} = \frac{\binom{3}{2}(0.01)^2(0.99)}{1 - 0.99^3}$$

$$= \frac{0.000297}{0.029701} = 0.00999966.$$

683. Solution: D

There are four possibilities, with the following probabilities
Neither has a theft: 0.7
A has a theft and B does not: *x*
B has a theft and A does not: *y*
Both have a theft: *z*
We have
$x + 0.7 = 0.8$, $x = 0.1$
$y + 0.7 = 0.8$, $y = 0.1$
$0.7 + x + y + z = 1.0$, $z = 0.1$

The desired probability is *P*(A and B both have thefts)/*P*(B has a theft) = 0.1/0.2 = 0.5.

684. Solution: B

The benefit paid is: $\begin{cases} y-1000, & 1000 \le y \le 15000 \\ 0, & \text{otherwise} \end{cases}$

$$\text{E}(Y) = \int_{1000}^{15000} (y-1000)\frac{1}{15000}dy = 6533.33$$

$$\text{E}(Y^2) = \int_{1000}^{15000} (y-1000)^2 \frac{1}{15000}dy = 60{,}977{,}777.78$$

$$\text{Var}\left(Y\right) = 60{,}977{,}777.78\text{-}6533.33^2 = 18{,}293{,}333.34$$

$$\text{SD}\left(Y\right) = \sqrt{18{,}293{,}333.34} = 4277$$

685. Solution: B

A: The husband is alive after ten years
B: The wife is alive after ten years

$$\text{P}[A] = 0.85$$

$$\text{P}[B] = 0.95$$

$$\text{P}[A \mid A \cup B] = \frac{\text{P}[A]}{\text{P}[A \cup B]} = \frac{0.85}{0.85 + 0.95 - (0.85)(0.95)} = \frac{0.85}{0.9925} = 0.8564.$$

686. Solution: E

Let X = number of fires a policyholder experiences this year; E(X) = 0.01.
Let Y = number of floods a policyholder experiences this year; E(Y) = 0.05.
Let p = percentage of each loss the company reimburses (expressed as a decimal).
Profit company gains for a policyholder = $1 - p(30X + 10Y)$.

$$\begin{aligned}
&\text{E(profit)} = 0.25 = E(1 - p(30X + 10Y)) \\
&= 1 - p(30 \times 0.01 + 10 \times 0.05) \\
&= 1 - p(0.80) \\
&p = 0.9375.
\end{aligned}$$

687. Solution: A

Let X_i = number of candidates in one center. The mean and variance are both 50.

$$\mathrm{P}\left(\sum X_i > 5100\right) = \mathrm{P}\left(Z > \frac{5100 - 100 \times 50}{\sqrt{100 \times 50}}\right) = \mathrm{P}\left(Z > 1.4142\right) = 0.07865.$$

688. Solution : E

This is a hypergeometric probability.

$$\frac{\binom{5}{2}\binom{3}{1}\binom{2}{1}}{\binom{10}{4}} + \frac{\binom{5}{1}\binom{3}{2}\binom{2}{1}}{\binom{10}{4}} + \frac{\binom{5}{1}\binom{3}{1}\binom{2}{2}}{\binom{10}{4}} = \frac{60 + 30 + 15}{210} = \frac{1}{2}.$$

689. Solution: A

Let X denote the number of hours that the operator works in the morning and let Y denote the number of hours that the operator works in the afternoon. The possibilities and probabilities are:
(2,1) – 1/3
(4,1), (4,2), and (4,3) – 1/9 each
(5,1), (5,2), (5,3). And (5,4) – 1/12 each

The marginal probabilities for Y are:
1: 1/3 + 1/9 + 1/12 = 19/36
2: 1/9 + 1/12 = 7/36
3: 1/9 + 1/12 = 7/36
4: 1/12 = 3/36

The mean of Y is (1)(19/36) + (2)(7/36) + (3)(7/36) + (4)(3/36) =66/36 = $1\frac{5}{6}$.

690. Solution: E

$$\text{P}\left[\text{Two policyholders have 1claim each \& third policyholder has 0 claims}\right]$$
$$= 3(0.0875)^2(0.8000) = 0.018375$$
$$\text{P}\left[\text{Two policyholders have no claims and third policyholder has 1claim}\right]$$
$$= 3(0.8000)^2(0.0625) = 0.12$$
$$\text{P}\left[\text{Three independent policyholders have exactly two claims among them}\right]$$
$$= 0.018375 + 0.12 = 0.138375$$

691. Solution: E

Sixty percent of all contract sales are for 3-year terms and thirty percent of all service contract sales cover both parts and labor. Since no contract provides for both of these, the required probability is the sum of 0.3 and 0.6 = 0.90.

692. Solution: E

The total has mean 10(625) = 6250. Since the claims are independent, the variances add. So, the standard deviation of the total is $8\sqrt{625}$, which equals 200.

The approximate probability that the total payment exceeds 5750 is:

$$P\left[Z > \frac{5750 - 6250}{200}\right] = P(Z > -2.5) = P(Z < 2.5) = 0.9938.$$

693. Solution: B

The given conditional statement, in terms of the cumulative distribution function reads:
$0.4 = \frac{1-F(25)}{1-F(10)} = \frac{e^{-25k}}{e^{-10k}} = e^{-15k}$. So $k = (-1/15)\ln(0.4)$. Then the probability that a pole will last 50 years is given by $e^{-50k} = e^{(50/15)\ln(.4)} = 0.0472$.

694. Solution: E

Let X = number of missiles that will not fire. Then X is hypergeometric and

$$P(X=3) = \frac{\binom{3}{3}\binom{7}{1}}{\binom{10}{4}} = \frac{1(7)}{210} = \frac{1}{30}.$$

Then $P(X \le 2) = 1 - P(X=3) = 1 - 1/30 = 29/30 = 0.9667$.

695. Solution: C

The second moment is the variance plus the square of the mean. Then

$12 = \lambda + \lambda^2$

$\lambda^2 + \lambda - 12 = 0$

$(\lambda - 3)(\lambda + 4) = 0$

$\lambda = 3$ (because the value cannot be negative).

Then,

$$P(X \ge 2 \mid X \ge 1) = \frac{P(X \ge 2)}{P(X \ge 1)} = \frac{1 - P(X=0) - P(X=1)}{1 - P(X=0)}$$

$$= \frac{1 - e^{-3} - 3e^{-3}}{1 - e^{-3}} = \frac{1 - 0.0498 - 0.1494}{1 - 0.0498} = 0.8428.$$

696. Solution: B

Let p = probability of positive annual profit in a given year.

Then 0.9025 = P[positive annual profits in both of the next two years] = p^2.
So, p = 0.95.

Since the mean is 250, then by symmetry the probability that a given annual profit is less than 500 is also 0.95.

From the normal table, a cumulative probability 0.95 yields a z-score of 1.6449. This means that a profit difference of 500 – 250 = 250 represents 1.6449 standard deviations. Hence, the standard deviation is 250/1.6449 = 151.98.

The z-score for the 75th percentile is 0.6745 and hence the 75th percentile is 250 + 0.6745(151.98) = 352.51 and the 25th percentile is 250 – 0.6745(151.98) = 147.49. The interquartile range is 352.51 – 147.49 = 205.02.

697. Solution: B

Let X = lifetime of primary blade with mean μ_X = 3 years.
Let Y = lifetime of secondary blade with mean μ_Y = 4 years.
P[lathe works for at least five years] $= P[X \geq 5, Y \geq 5]$
$= P[X \geq 5] \times P[Y \geq 5]$ by independence
$= e^{-5/3} e^{-5/4} = e^{-35/12} = 0.541.$

698. Solution: E

$$P(N=1) = 2P(N=0) \Rightarrow \frac{\lambda^1 e^{-\lambda}}{1!} = 2\frac{\lambda^0 e^{-\lambda}}{0!} \Rightarrow \lambda = 2.$$

The total number of accidents X in one day at the three intersections follows a Poisson distribution with $\lambda = 2(3) = 6$. Then,

$$P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1 - \frac{6^0 e^{-6}}{0!} - \frac{6^1 e^{-6}}{1!} = 0.9826.$$

Alternatively, the probability of zero accidents is $\left(e^{-2}\right)^3 = e^{-6}$. One accident means two intersections with zero and one with one, which can happen in three ways. The probability is $3(e^{-2})^2(2e^{-2}) = 6e^{-6}$. The desired probability is $1 - e^{-6} - 6e^{-6} = 0.9826.$

699. Solution: B

$P(\text{ABS} = \text{Yes}) = 200/1000 = 0.2 \qquad P(\text{ABS} = \text{No}) = 1 - \text{P}(\text{ABS} = \text{Yes}) = 0.8$

$P(N = 2 \mid \text{ABS} = \text{Yes}) = \dfrac{1^2 \text{e}^{-1}}{2!} = 0.1839 \quad P(N = 2 \mid \text{ABS} = \text{No}) = \dfrac{3^2 \text{e}^{-3}}{2!} = 0.2240$

$$\begin{aligned}
&P(\text{ABS} = \text{Yes} \mid N = 2) \\
&= \frac{P(N = 2 \mid \text{ABS} = \text{Yes})P(\text{ABS} = \text{Yes})}{P(N = 2 \mid \text{ABS} = \text{Yes})P(\text{ABS} = \text{Yes}) + P(N = 2 \mid \text{ABS} = \text{No})P(\text{ABS} = \text{No})} \\
&= \frac{0.1839(0.2)}{0.1839(0.2) + 0.2240(0.8)} = 0.1703.
\end{aligned}$$

700. Solution: E

Per page the number of errors is Poisson(0.2). For 25 pages it is also Poisson, with mean 25(0.2) = 5. Then,

$$\begin{aligned}
&P(N \le 3) = e^{-5} + 5e^{-5} + 25e^{-5}/2 + 125e^{-5}/6 \\
&= (1 + 5 + 12.5 + 20.833)e^{-5} = 39.333(0.006738) = 0.2650.
\end{aligned}$$

701. Solution: B

The probabilities must sum to 1, so

$1 = k(3^{-1} + 3^{-2} + \cdots) = k\dfrac{3^{-1}}{1 - 3^{-1}} = k(1/2) \Rightarrow k = 2.$ Then,

$P(O_2, O_4, \ldots) = 2(3^{-2} + 3^{-4} + \cdots) = 2\dfrac{3^{-2}}{1 - 3^{-2}} = 2(1/8) = 1/4.$

702. Solution: B

The marginal distribution of *X* is zero with probability 0.85 and one with probability 0.15. The mean is 0.15 and the variance is 1(0.15)(0.85) = 0.1275.
The marginal distribution of *Y* is zero with probability 0.45 and one with probability 0.55. The mean is 0.55 and the variance is 1(0.55)(0.45) = 0.2475.
The covariance is 0.40(0)(0) + 0.45(0)(1) + 0.05(1)(0) + 0.10(1)(1) – 0.15(0.55) = 0.0175.
The correlation coefficient is

$$\rho = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} = \frac{0.0175}{\sqrt{0.1275(0.2475)}} = 0.0985.$$

703. Solution: C

The conditions of the problem imply:

$$(1-p)^5 p = \binom{7}{1}(1-p)^6 p^2$$

$$1 = 7(1-p)p$$

$$1 = 7p - 7p^2$$

$$7p^2 - 7p + 1 = 0$$

$$p = \frac{7 \pm \sqrt{49 - 4(7)(1)}}{14}$$

$$p = 0.17267, 0.82733$$

$$p = 0.173.$$

704. Solution: D

Let p represent the probability that both spouses of a given married couple survive another year. So we have $p = (0.79)^2$. Using the parameters $p = (0.79)^2$ and n = 7 in the binomial distribution formula, we find that the probability that both spouses of each of at least 3 couples survive for another year is

$$1 - \sum_{k=0}^{2} \binom{7}{k} \left[(0.79)^2\right]^k \left[1-(0.79)^2\right]^{7-k}$$

$$= 1 - \left[1-(0.79)^2\right]^7 - 7(0.79)^2\left[1-(0.79)^2\right]^6 - 21(0.79)^4\left[1-(0.79)^2\right]^5 = 0.925226.$$

705. Solution: E

Let X represent the loss. Note that the unreimbursed loss is 40% of the loss, so the policyholder's unreimbursed loss is less than 1.2 if and only if the loss is less than $\frac{1.2}{0.4} = 3$.

The probability that the unreimbursed loss is less than 1.2 is

$$\mathrm{P}[X < 3] = \int_0^3 f(x)dx = \int_0^3 \frac{1}{42}\left(\frac{x}{2}+1\right)^2 dx = \frac{1}{42} \left. \frac{\left(\frac{x}{2}+1\right)^3}{3\left(\frac{1}{2}\right)} \right|_0^3 = \frac{1}{63}\left[\left(\frac{3}{2}+1\right)^3 - 1\right] = \frac{1}{63}\left(\frac{117}{8}\right) = \frac{13}{56}.$$

706. Solution: A

Let Z represent the lifespan in years, which is exponentially distributed with rate parameter $\lambda = \frac{1}{10}$, since the rate parameter is the reciprocal of the mean. Using the cumulative distribution function, we have

$$r = \mathrm{P}[Z \le x] = 1 - e^{-\lambda x} = 1 - e^{-x/10}$$
$$1 - r = \mathrm{P}[Z \le y] = 1 - e^{-\lambda y} = 1 - e^{-y/10}$$

One way to eliminate r is to add the two equations to get
$1 = 2 - \left(e^{-x/10} + e^{-y/10}\right)$.
Solving this equation for y in terms of x yields

$$e^{-x/10} + e^{-y/10} = 1 \Rightarrow e^{-y/10} = 1 - e^{-x/10} \Rightarrow -\frac{y}{10} = \ln\left(1 - e^{-x/10}\right) \Rightarrow y = -10\ln\left(1 - e^{-x/10}\right).$$

707. Solution: B

Let X and d represent the loss and the deductible respectively. We have

$$90 = \mathrm{E}\left(\max(0, X - d)\right) = \int_0^{500} \frac{\max(0, x-d)}{500}\,dx = \int_d^{500} \frac{x-d}{500}\,dx = \left.\frac{(x-d)^2}{1000}\right|_{x=d}^{x=500} = \frac{(500-d)^2}{1000}.$$

Solving the quadratic equation for d yields $d = 200$ only, since the other solution $d = 800$ exceeds 500 (in which case, the above equation does not even apply).

708. Solution: B

The cumulative distribution $F(x)$ is equal to

$$\int_{-\infty}^{x} f(y)dy = \begin{cases} 0, & x \le 0 \\ 0.8x, & 0 < x \le 0.5 \\ 0.4 + 0.3(x - 0.5), & 0.5 < x \le 1.5 \\ 0.4 + 0.3 + 0.1(x - 1.5), & 1.5 < x \le 3.5 \\ 0.4 + 0.3 + 0.2 + 0.04(x - 3.5), & 3.5 < x \le 6 \\ 1, & x > 6. \end{cases}.$$

We need to find the value of m such that $F(m) = 0.5$. Since 0.4 < 0.5 < 0.4 + 0.3, we must have $0.5 < m \le 1.5$. Therefore, $0.4 + 0.3(m - 0.5) = 0.5 \Rightarrow m - 0.5 = \frac{1}{3} \Rightarrow m = \frac{5}{6} = 0.8333.$

709. Solution: D

$$1 = c\int_0^3 \frac{x}{1+x^2}\,dx = \frac{c}{2}\int_0^3 \frac{2x}{1+x^2}\,dx = \frac{c}{2}\ln(1+x^2)\Big|_0^3 = 1.15129c$$

$$c = 0.86859$$

$$P[X > 1] = 0.86859\int_1^3 \frac{x}{1+x^2}\,dx = 0.43429\ln(1+x^2)\Big|_1^3$$

$$= 0.69896.$$

710. Solution: B

The mean and median are equal for the uniform distribtion. and equal $\frac{a+b}{2}$. The 75th percentile is $\frac{3}{4}(b-a)+a = \frac{3b+a}{4}$.

$$\frac{a+b}{2}+\frac{a+b}{2}+\frac{3b+a}{4} = \frac{2a+2b+2a+2b+3b+a}{4} = \frac{5a+7b}{4}.$$

711. Solution: C

Let T be a random variable representing the time until the next claim. Then,

$$\mathrm{P}[5 < T < 8 \mid T > 2] = \frac{\mathrm{P}[5 < T < 8]}{\mathrm{P}[T > 2]} = \frac{\mathrm{P}[T < 8] - \mathrm{P}[T < 5]}{\mathrm{P}[T > 2]}$$

$$= \frac{(1-e^{-8/4}) - (1-e^{-5/4})}{e^{-2/4}} = \frac{e^{-5/4} - e^{-8/4}}{e^{-2/4}} = e^{-3/4} - e^{-6/4} = 0.24924.$$

712. Solution: C

The cumulative distribution function is $F(x) = 1 - e^{-\beta x}$, for positive *x*. We have,

$$0.25 = F(10) - F(5) = \left(1 - e^{-10\beta}\right) - \left(1 - e^{-5\beta}\right) = e^{-5\beta} - e^{-10\beta} = e^{-5\beta} - \left(e^{-5\beta}\right)^2$$
$$\left(e^{-5\beta}\right)^2 - e^{-5\beta} + 0.25 = 0$$
$$\left(e^{-5\beta} - 0.5\right)^2 = 0$$
$$e^{-5\beta} = 0.5.$$

Then,
$$F(20) - F(10) = \left(1 - e^{-20\beta}\right) - \left(1 - e^{-10\beta}\right) = e^{-10\beta} - e^{-20\beta} = \left(e^{-5\beta}\right)^2 - \left(e^{-5\beta}\right)^4$$
$$= (0.5)^2 - (0.5)^4 = 0.1875.$$

713. Solution: A

$$1 = c\int_0^5 \frac{x}{1+x^2}\,dx = \frac{c}{2}\int_0^5 \frac{2x}{1+x^2}\,dx = \frac{c}{2}\ln(1+x^2)\Big|_0^5 = 1.62905c$$
$$c = 0.61386$$
$$0.5 = c\int_1^m \frac{x}{1+x^2}\,dx = \frac{c}{2}\ln(1+x^2)\Big|_0^m = 0.30693\ln(1+m^2)$$
$$\ln(1+m^2) = 1.62905$$
$$1+m^2 = 5.09902$$
$$m = 2.02460.$$

714. Solution: C

$$0.75 = \frac{2}{(\ln 80)^2}\int_1^s \frac{\ln x}{x}\,dx$$
$$\int_1^s \frac{\ln x}{x}\,dx = 7.20081$$
$$\frac{(\ln x)^2}{2}\Big|_1^s = 7.20081$$
$$\frac{(\ln s)^2}{2} = 7.20081$$
$$s = 44.47586.$$

715. Solution: B

For each selling price, the probability for the combined offices is 0.7 times that for Office #1 plus 0.3 times that for Office #2. The individual and cumulative probabilities are (stopping when the cumulative probability exceeds 0.500.

150: 0.215 and 0.215
155: 0.115 and 0.330
160: 0.080 and 0.410
165: 0.100 and 0.510

The median is the smallest value such that the probability of being less than or equal to it is greater than or equal to 0.5. The first value with cumulative probability greater than or equal to 0.5 is 165.

716. Solution: D

The marginal probability of a $5 million cost of recall is (0.05)(0.10) + (0.10)(0.25) + (0.25)(0.30) + (0.50)(0.25) + (0.90)(0.10) = 0.32. The marginal probability of a $10 million cost of recall is 1 – 0.32 = 0.68. The expected cost of recall is 0.32 x 5 + 0.68 x 10 = 8.4 million.

717. Solution: A

Conditional on $X \le 25{,}000$, we update the table as follows:

		x		
		0	25,000	50,000
y	0	0.00	0.125	0
	25,000	0.250	0.375	0
	50,000	0.125	0.125	0

Conditionally,

$$\text{P}[Y = 0] = 0.125$$
$$\text{P}[Y = 25{,}000] = 0.625$$
$$\text{P}[Y = 50{,}000] = 0.250$$

$$\text{E}(Y \mid X \le 25{,}000) = 25{,}000(0.625) + 50{,}000(0.250) = 28{,}125$$
$$\text{E}(Y^2 \mid X \le 25{,}000) = 25{,}000^2(0.625) + 50{,}000^2(0.250) = 1{,}015{,}625{,}000$$
$$\text{Var}(Y \mid X \le 25{,}000) = 1{,}015{,}625{,}000 - 28{,}125^2 = 224{,}609{,}375.$$

718. Solution: D

For any Poisson distribution, consider the ratio

$$\frac{\mathrm{P}[X=i+1]}{\mathrm{P}[X=i]}=\frac{\frac{e^{-\lambda}\lambda^{i+1}}{(i+1)!}}{\frac{e^{-\lambda}\lambda^{i}}{i!}}=\frac{\lambda}{i+1}.$$

The probabilities increase as long as $i+1<\lambda$ and decrease when $i+1>\lambda$. The mode is the smallest value of *i* such that $i>\lambda-1$ and if λ is an integer, both λ and $\lambda-1$ are modes.

For city B the mode is 2 and 3 and hence the mean is 3. For city A the mean is 4(3) = 12 and hence the modes are 11 and 12.

Alternatively, if the modes are both 2 and 3, then the probabilities of these two values must be equal. Hence,

$$\frac{\lambda^2 e^{-\lambda}}{2}=\frac{\lambda^3 e^{-\lambda}}{6}\Rightarrow\frac{1}{2}=\frac{\lambda}{6}\Rightarrow\lambda=3.$$

The mean for city A is then 4(3) = 12. Inspection of the first equality reveals that if the mean is an integer, the Poisson probabilities are equal at the mean and the mean minus one. Hence the modes for city A are 11 and 12.

719. Solution: D

For a Poisson distribution the mean and variance are both equal to λ. Then,

$$3=E(X-Y)=\lambda_X-\lambda_Y\Rightarrow\lambda_X=3+\lambda_Y$$

$$Var(X)=\lambda_X=2Var(Y)=2\lambda_Y$$

$$3+\lambda_Y=2\lambda_Y$$

$$\lambda_y=3$$

$$E(Y^2)=Var(Y)+[E(Y)]^2=3+3^2=12.$$

720. Solution: E

The 5th and 99th percentiles from the Normal table are -1.64485 and 2.32635.

$$\frac{-25-\mu}{\sigma}=-1.64485,\quad\frac{83-\mu}{\sigma}=2.32635$$

Simultaneously solve and get

$$\sigma=27.19580,\mu=19.73311$$

$$\frac{\sigma}{\mu}=1.37818.$$

721. Solution: D

The sum of the individual normal claims is also normal with mean 122,500 and standard deviation $[49(700)]^{1/2} = 4900$. The z-value for the 90th percentile is 1.28155. The percentile is 122,500 + 1.28155(4900) = 1,28,780.

722. Solution: B

Snowstorms	Cost	Probability
0	0	0.13534
1	200	0.27067
2	500	0.27067
3+	1000	0.32332

$\text{E(Cost)} = 512.79$

$\text{E(Cost}^2) = 401{,}818.05$

$\text{Var(Cost)} = 401{,}818.05 - 512.79^2 = 138{,}861.$

723. Solution: B

Let the subscripts *L*, *M*, and *H* refer to the three types of policyholder. From the given information (where the third equation uses the law of total probability):

$p_M = 2p_L$

$p_H = 4p_M$

$0.15p_L + 0.55p_M + 0.30p_H = 0.018.$

Then,

$0.15(0.5p_M) + 0.55p_M + 0.30(4p_M) = 0.018$

$1.825p_M = 0.018$

$p_M = 0.018/1.825 = 0.00986$

724. Solution: D

Let *n* be the number of employees. The mean and variance of total claims are then 2500*n* and 490,000*n*. The 90th percentile of the standard normal distribution is 1.28155. Then,

$167{,}168 = 2500n + 1.28155(490{,}000n)^{0.5}$.

Letting *x* be the square root of *n* produces the quadratic equation and solution

$2500x^2 + 897.085x - 167{,}168 = 0$

$$x = \frac{-897.085 + \sqrt{897.085^2 + 4(2500)(167{,}168)}}{5000} = 8.$$

Then *n* =8(8) = 64.

725. Solution: E

Let M = the number of mergers occurring during a single calendar year.

$$\mathrm{P}[M=3]=\mathrm{P}[M=4]$$
$$\frac{e^{-\lambda}\lambda^3}{3!}=\frac{e^{-\lambda}\lambda^4}{4!}$$
$$1=\frac{\lambda}{4}$$
$$\lambda=4$$
$$\mathrm{P}[M\ge 3]=1-\mathrm{P}[M=0]-\mathrm{P}[M=1]-\mathrm{P}[M=2]$$
$$=1-\frac{e^{-4}4^0}{0!}-\frac{e^{-4}4^1}{1!}-\frac{e^{-4}4^2}{2!}=1-0.23810=0.76190.$$

726. Solution: D

First find the marginal probability function for Y:

y	probability
0	6/28
1	8/28
2	8/28
3	6/28

$$\mathrm{E}(Y)=\frac{42}{28},\mathrm{E}(Y^2)=\frac{94}{28},\mathrm{Var}(Y)=\frac{94}{28}-\left(\frac{42}{28}\right)^2=1.10714.$$

Next find the conditional probability function for $Y|X$=0:

y	probability
0	3/6
1	2/6
2	1/6
3	0

$$\mathrm{E}(Y\mid X=0)=\frac{4}{6},\mathrm{E}(Y^2\mid X=0)=\frac{6}{6},\mathrm{Var}(Y\mid X=0)=1-\left(\frac{4}{6}\right)^2=0.55556.$$
$$\frac{\mathrm{Var}(Y)}{\mathrm{Var}(Y\mid X=0)}=\frac{1.10714}{0.55556}=1.99286.$$

727. Solution: B

Let X represent the loss, which is exponentially distributed with mean β.

Let Y represent the amount that the insurer pays the policyholder, which has mean 1200.
Note that $Y = 0$ when $X < 0.4\beta$; otherwise, $Y = X - 0.4\beta$.

Keeping in mind that the exponential distribution is memoryless, we have

$$\begin{aligned} 1200 = \mathrm{E}(Y) &= \mathrm{P}[X < 0.4\beta]\mathrm{E}(Y \mid X < 0.4\beta) + \mathrm{P}[X \geq 0.4\beta]\mathrm{E}(Y \mid X \geq 0.4\beta) \\ &= \mathrm{P}[X < 0.4\beta]\mathrm{E}(0 \mid X < 0.4\beta) + \mathrm{P}[X \geq 0.4\beta]\mathrm{E}(X - 0.4\beta \mid X \geq 0.4\beta) \\ &= \mathrm{P}[X < 0.4\beta](0) + \mathrm{P}[X \geq 0.4\beta]\mathrm{E}(X) \\ &= \left(e^{\frac{-0.4\beta}{\beta}} \right)\beta = e^{-0.4}\beta \end{aligned}.$$

Thus $\beta = 1200e^{0.4} = 1790$.

728. Solution: A

For zero visitors in a one-hour period:
$0.60653 = e^{-\lambda}$
$\lambda = 0.5$

For a two-hour period:
$\lambda = 2(0.5) = 1.$
$\mathrm{P}[\text{Fewer than two}] = \mathrm{P}[0] + \mathrm{P}[1]$
$= \frac{e^{-1}1^0}{0!} + \frac{e^{-1}1^1}{1!} = 0.73576.$

729. Solution: E

$$6\left[\left(\frac{1}{3}\right)^{c}+\left(\frac{1}{3}\right)^{c+1}+\left(\frac{1}{3}\right)^{c+2}+\ldots\right]=1$$

$$\left(\frac{1}{3}\right)^{c}\left[1+\frac{1}{3}+\left(\frac{1}{3}\right)^{2}+\ldots\right]=\frac{1}{6}$$

$$\left(\frac{1}{3}\right)^{c}\left[\frac{1}{1-\frac{1}{3}}\right]=\frac{1}{6}$$

$$\left(\frac{1}{3}\right)^{c}=\frac{1}{9}$$

$$c=2$$

$$\mathrm{P}[N<5\,|\,N\geq 3]=\frac{\mathrm{P}[N=3]+P[N=4]}{1-P[N\leq 2]}$$

$$\frac{\mathrm{P}[N=3]+P[N=4]}{1-\mathrm{P}[N=0]-P[N=1]-P[N=2]}$$

$$=\frac{0.02469+0.00823}{1-0.66667-0.22222-0.07407}=\frac{0.03292}{0.03704}=0.88889.$$

730. Solution: A

Let m represent the median and let s represent the 75th percentile.

$$s=m+1.3863$$

$$e^{-m/\beta}=0.5,\ e^{-s/\beta}=0.25$$

$$m/\beta=0.69315,\ s/\beta=1.38629$$

$$s/m=2$$

$$m+1.3863=2m$$

$$m=1.3863$$

$$\beta=1.3863/0.69315=2$$

$$P[X<3]=1-e^{-3/2}=0.77687$$

731. Solution: D

Since 24.2 is the 80th percentile of the distribution of *X*:

$$\frac{24.2-20}{\sigma_X}=0.84162$$

$$\sigma_X=5$$

$$25=\sigma_X{}^2=\text{Var}(X)=0.5\text{E}(Y),\text{E}(Y)=50$$

$$\text{Var}(Y)=25=a^2\text{Var}(X)=a^2 25,\ a=1$$

$$50=\text{E}(Y)=\text{E}(aX+b)=a\text{E}(X)+b=20a+b=20+b$$

$$b=30.$$

732. Solution: C

The mean μ of the distribution is 5 and the variance is $\sigma^2=25$.

Let $X_1,X_2,\ldots,X_{64}$ be a random sample from this distribution and $\overline{X}=\frac{\sum_{i=1}^{64}X_i}{64}$ the corresponding sample mean. The desired probability is

$$P\left[\overline{X}>6\right]=P\left[\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}>\frac{6-\mu}{\sigma/\sqrt{n}}\right]$$

$$=P\left[Z>\frac{6-5}{\sqrt{25}/\sqrt{64}}\right]\quad (Z\text{ approx. stand. normal by CLT})$$

$$=\text{P}\left[\text{Z}>1.6\right]=0.0548.$$

733. Solution: A

Let A represent the event that the selected policyholders has medical insurance.
Let B represent the event that the selected policyholders has dental insurance.

From the given data, we have

$$\frac{P[A \cap B]}{P[A]} = 0.25 \Rightarrow P[A \cap B] = 0.25P[A] \text{ and}$$
$$\frac{P[A \cap B]}{P[B]} = 0.4 \Rightarrow P[A \cap B] = 0.4P[B] = 0.25P[A] \Rightarrow \frac{P[B]}{P[A]} = \frac{0.25}{0.4}.$$

Therefore, the desired ratio is $\frac{s}{t} = \frac{P[A \cup B]}{P[A]} = \frac{P[A] + P[B] - P[A \cap B]}{P[A]} = 1 + \frac{0.25}{0.4} - 0.25 = 1.375$.

An alternative way to think about it is as follows:
Assume 200 policyholders have medical. Then 50 also have dental and thus 150 have medical only.
Assume x have dental only. We have 0.4 = 50/(x + 50) for x = 75.
Thus there are 150 with medical only, 75 with dental only, and 50 with both, for a total of 275.
The number with medical is 200. The desired ratio is 275/200 = 1.375.

734. Solution: E

Let X denote the time until failure.

The pdf of X is $f(x) = \frac{1}{\theta} e^{-x/\theta}$ and the variance of X is θ^2.

Next,

$$\int_0^{\theta^2} f(x)dx = \frac{k}{100}$$
$$\int_0^{\theta^2} \frac{1}{\theta} e^{-x/\theta} dx = 1 - e^{-\frac{\theta^2}{\theta}} = 1 - e^{-\theta} = \frac{k}{100}$$
$$\Rightarrow k = 100\left(1 - e^{-\theta}\right)$$

735. Solution: C

Set up a joint probability table and then a marginal function for X.

		x		
		1	2	3
y	1	2/60	3/60	4/60
	2	5/60	6/60	7/60
	3	10/60	11/60	12/60
$f_X(x)$		17/60	20/60	23/60

$$0.44\text{E}(X) = 0.44\left[\frac{17}{60}(1) + \frac{20}{60}(2) + \frac{23}{60}(3)\right] = 0.44(2.1) = 0.924.$$

736. Solution: C

$$\text{E}(X^2) = 6$$
$$\text{E}(X) = \lambda, \text{Var}(X) = \lambda$$
$$\text{E}(X^2) - \text{E}(X)^2 = \lambda$$
$$6 - \lambda^2 = \lambda$$
$$0 = \lambda^2 + \lambda - 6$$
$$0 = (\lambda - 2)(\lambda + 3)$$
$$\lambda = 2$$

$$\text{P}(X \geq 3) = 1 - \text{P}(X = 0) - \text{P}(X = 1) - \text{P}(X = 2)$$
$$= 1 - \frac{e^{-2}2^0}{0!} - \frac{e^{-2}2^1}{1!} - \frac{e^{-2}2^2}{2!}$$
$$= 1 - 0.13534 - 0.27067 - 0.27067 = 0.32332.$$