INV 201 Model Solutions
November 2025

1. **Learning Objectives:**

1. The candidate will understand key types of derivatives.

Learning Outcomes:

(1a) Understand the payoffs of basic derivative instruments including:

- Forwards
- Futures
- Swaps
- Calls
- Puts
- Caps
- Floors
- Swaption
- Currency

Sources:

Hull-Options Futures and other Derivatives-11thEd

Commentary on Question:

This question was meant to test candidate's understanding for fixed floating rate swap and consequent risk management issues. Candidates' performance was overall mixed.

Solution:

(a) Show that this fixed-for-floating rate swap is advantageous for both ABC and XYZ, compared to their own external lending opportunities.

Commentary on Question:

Most candidates approached this part correctly but did not always provide enough detail to earn full credit. Credits are also provided if candidates approached the question through rates comparison directly. Some candidates provided a heuristic argument instead of an algebraic argument and received partial credit.

The net cash flows for ABC and XYZ at each time step without the swap are:

$$ABC\ CF_t = -100 * .25 * (r_t - .001) = .025 - 25r_t$$
$$XYZ\ CF_t = -.061 * 100 * .25 = -1.525$$

1. Continued

The net cash flows for ABC and XYZ at each time step under the swap are:

$$ABC\ CF_t = 100 * (.053 - .05) * .25 - 100 * .25r_t = .075 - 25r_t$$
$$XYZ\ CF_t = 100 * .25r_t - 100 * .25 * .053 - 100 * .25(r_t + .006) = -1.475$$

We can see that the swap transaction results in higher net cash flows for both ABC and XYZ.

(b) Explain the risks to ABC if XYZ credit rating is lowered to CCC.

Commentary on Question:
Most candidates can at least identify 1 risk correctly and elaborate clearly.

- CCC rated entities may no longer be able to refinance at floating + 60bps, which may result in the swap becoming disadvantagous for both parties
- If XYZ defaults, then ABC would stop receiving coupon payments.

(c) Propose two strategies for ABC to manage the risks in part (b).

Commentary on Question:
Most candidates can at least provide 1 strategy.

- Enter into a credit default swap
- Require collateral postings from XYZ
- Make swap terms contingent upon XYZ maintaining their BBB credit rating.

(d) List three differences between LIBOR and SOFR.

Commentary on Question:
Candidates did well in this question. To achieve full marks, candidates are required to provide comparisons instead of to list several key features.

- LIBOR rates are estimated by banks, while overnight SOFR are based on actual transactions between banks.
- LIBOR rates are known at the beginning of the period, while SOFR rates can only be known at the end of the period.
- LIBOR rates incorporates some credit risk, while overnight SOFR rates are considered credit risk-free.

2. Learning Objectives:

2. The candidate will understand the principles and techniques for the valuation of derivatives.

Learning Outcomes:

(2b) Understand Arrow-Debreau security and the distinction between complete and incomplete markets

Sources:

INV201-100-25:Financial Mathematics – A Comprehensive Treatment, Campolieti, Chapter 5.

Commentary on Question:

Thie question was meant to test candidate's conceptual understanding of arbitrage and the first Fundamental Theorem of Asset Pricing (FTAP). Candidates' performance was overall mixed.

Solution:

(a) Assess whether this model is arbitrage-free.

Commentary on Question:

Most candidates approached this part correctly but did not always provide enough detail to earn full credit. A few candidates mistakenly applied the second FTAP instead.

We need to find a strictly positive column vector

$$\Psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix}$$

such that $D\,\Psi = S_0$, or:

$$\begin{cases} \psi_1 + \psi_2 + \psi_3 = 0.95 \\ 120\,\psi_1 + 96\,\psi_2 + 64\,\psi_3 = 100. \end{cases}$$

Formula (5.30) from the formula sheet gives:

$$\begin{cases} \psi_1 = \dfrac{1 - 0.95 \times 0.64}{1.2 - 0.64} - \dfrac{0.96 - 0.64}{1.2 - 0.64} c \\ \psi_2 = c \\ \psi_3 = \dfrac{0.95 \times 1.2 - 1}{1.2 - 0.64} - \dfrac{1.2 - 0.96}{1.2 - 0.64} c \end{cases} = \begin{cases} \psi_1 = 0.7 - \dfrac{4}{7} c \\ \psi_2 = c \\ \psi_3 = 0.25 - \dfrac{3}{7} c \end{cases}.$$

For $0 < c < \frac{7}{12}$, the vector Ψ is strictly positive, therefore by the first FTAP, there are no arbitrage portfolios.

2. Continued

Alternatively: There exists a risk-neutral probability measure because the accumulation factor $1 + r = 0.95^{-1} = 1.0526$ satisfies $d = 0.64 < 1 + r < u = 1.2$. By the first FTAP (second version), there are no arbitrage portfolios.

(b) Verify that the call option's payoff is not attainable.

Commentary on Question:
Most candidates provided a heuristic argument instead of an algebraic argument and received partial credit.

The payoff of a European call option with strike price $K = 90$ is:

$$[30 \quad 6 \quad 0].$$

To decide if the call option is attainable, consider a row vector

$$\Phi = [\varphi_1 \quad \varphi_2]$$

such that:

$$\Phi\, D = [\varphi_1 + 120\varphi_2 \quad \varphi_1 + 96\varphi_2 \quad \varphi_1 + 64\varphi_2] = [30 \quad 6 \quad 0].$$

The third component gives

$$\varphi_1 = -64\varphi_2$$

therefore

$$\begin{cases} 56\,\varphi_2 = 30 \\ 32\,\varphi_2 = 6 \end{cases} = \begin{cases} \varphi_2 = 0.5357 \\ \varphi_2 = 0.1875 \end{cases}$$

which is impossible.

(c) Verify that there is an arbitrage opportunity by means of a non-strictly positive state vector.

Commentary on Question:
Candidates did well on this part. Many used matrix operations in Excel to derive the vector Ψ, which earned full credit if there was a correct conclusion.

2. **Continued**

We have:

$$S_0 = \begin{bmatrix} 0.95 \\ 100 \\ 21 \end{bmatrix}, \qquad D = \begin{bmatrix} 1 & 1 & 1 \\ 120 & 96 & 64 \\ 30 & 6 & 0 \end{bmatrix}.$$

Find a column vector

$$\Psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix}$$

such that $D\,\Psi = S_0$, or:

$$\begin{cases} \psi_1 + \psi_2 + \psi_3 = 0.95 \\ 120\,\psi_1 + 96\,\psi_2 + 64\,\psi_3 = 100 \end{cases} = \begin{cases} \psi_1 = 0.7 - \dfrac{4}{7}c \\ \psi_2 = c \\ \psi_3 = 0.25 - \dfrac{3}{7}c \end{cases}$$

and

$$30\psi_1 + 6\psi_2 = 21 - \frac{120}{7}c + 6c = 21 \Longrightarrow c = 0 \Longrightarrow \Psi = \begin{bmatrix} 0.7 \\ 0 \\ 0.25 \end{bmatrix}.$$

.

Since the unique Ψ is not strictly positive, there are arbitrage opportunities.

Alternatively: construct an arbitrage portfolio as follows:

- Short 64 units of bonds
- Buy one unit of stock
- Short $\frac{100-0.95\times 64}{21} = 1.87$ units of the call option.

Then demonstrate an arbitrage by computing:

- Initial cost = 0
- Final state = [0, +20.8, 0]

(d) Justify your coworker's claim with an explicit calculation.

Commentary on Question:
Most candidates did not attempt this part, and only a handful earned credit. A common misconception among those who provided a solution was to interpret the superscript as a square.

2. **Continued**

The Arrow-Debreu security $\mathcal{E}^2$ is:

$$\mathcal{E}^2 = [0 \quad 1 \quad 0]\, D^{-1} = \left[-\frac{40}{13} \quad \frac{5}{104} \quad -\frac{7}{78}\right].$$

The initial cost of the portfolio is:

$$\mathcal{E}^2\, S_0 = [0 \quad 1 \quad 0]\, D^{-1} S_0 = [0 \quad 1 \quad 0]\Psi = \psi_2 = 0.$$

The terminal payoff of this portfolio is:

$$\mathcal{E}^2\, D = [0 \quad 1 \quad 0]\, D^{-1} D = [0 \quad 1 \quad 0].$$

Since this vector is positive, it demonstrates an arbitrage.

If the alternative solution is given in part (c), it suffices to compute:

$$\mathcal{E}^2 = \frac{1}{20.8}[-64 \quad 1 \quad -1.87] = \left[-\frac{40}{13} \quad \frac{5}{104} \quad -\frac{7}{78}\right].$$

The arbitrage argument doesn’t need to be repeated.

3. **Learning Objectives:**

2. The candidate will understand the principles and techniques for the valuation of derivatives.

Learning Outcomes:

(2d) Understand Stochastic Calculus theory and technique used in pricing derivatives including: • Stochastic differential equations

- Ito integral
- Ito's Lemma
- Martingales,
- Change of numeraire
- Girsanov's theorem

(2e) Understand and apply the concepts of risk-neutral measure, forward measure, normalization, and the market price of risk

Sources:

John Hull - Options, Futures, and Other Derivatives, Global Edition-Pearson (2021), Chapters 15, 28

Problems and Solutions Chin

INV201-101-25: Introduction to Stochastic Finance with Market Examples by Privault

Commentary on Question:

This question tests candidates' understanding of some key concepts in the valuation of derivatives. Overall, most candidates did well on this question.

Solution:

(a)

(i) Explain what can be measured by the market price of risk of a stock.

(ii) Calculate the market price of risk for this stock.

Commentary on Question:

Most candidates did well on the second subpart, i.e., calculating the market price of risk. However, many candidates failed to interpret the concept correctly.

(i)

The market price of risk of the stock measures the trade-offs between risk and return that are made for securities dependent on this stock.

3. Continued

(ii)

The market price risk of the stock is

$$\lambda = \frac{\mu - r}{\sigma}$$

From the given information, we have $\mu = 0.15$, $r = 0.1$, and $\sigma = 0.2$. Plugging in the numbers gives

$$\frac{0.15 - 0.1}{0.2} = 0.25.$$

(b) Calculate $Var(S_1|S_{0.5} = 100)$.

Commentary on Question:

This part tests candidates' knowledge of the geometric Brownian motion. Many candidates did well on this part.

Solving the stochastic differential equation gives

$$S_t = S_0 e^{\left(0.15 - 0.5\times0.2^2\right)t + 0.2B_t} = S_0 e^{0.13t + 0.2B_t}$$

Using the above equation twice, we obtain

$$S_1 = S_{0.5} e^{0.065 + 0.2(B_1 - B_{0.5})}.$$

Note that $B_1 - B_{0.5}$ is normally distributed with mean 0 and variance 0.5. Hence

$$E[S_1|S_{0.5} = 100] = 100\, E\left[e^{0.065 - 0.2(B_1 - B_{0.5})}\right] = 100\, e^{0.065 + 0.5\times0.5\times0.2^2} = 107.7884$$

$$E[S_1^2|S_{0.5} = 100] = 10{,}000\, E\left[e^{0.13 - 0.4(B_1 - B_{0.5})}\right] = 10{,}000\, e^{0.13 + 0.5\times0.5\times0.4^2} = 11{,}853.05$$

The conditional variance is

$$Var(S_1|S_{0.5} = 100) = E[S_1^2|S_{0.5} = 100] - E[S_1|S_{0.5} = 100]^2 = 11{,}853.05 - 107.7884^2 = 234.71.$$

Alternatively: use formula 15.5 from Hull:

$$Var(S_T|S_t) = S_t^2 e^{2\mu(T-t)}\left(e^{\sigma^2(T-t)} - 1\right).$$

Plugging in the same values for $S_{0.5}, \mu, \sigma$ as above gives

$$Var(S_1|S_{0.5} = 100) = 10{,}000\, e^{0.15}(e^{0.02} - 1) = 234.71.$$

3. Continued

(c) Derive the standard Brownian motion under the risk-neutral measure.

Commentary on Question:
Most candidates did well on this part by correctly identifying the standard Brownian motion under the risk-neutral measure.

The standard Brownian motion under the risk-neutral measure is

$$\tilde{B}_t = \frac{\mu - r}{\sigma} t + B_t.$$

This also follows from equation 28.10 in Hull, after setting

$$d\tilde{z} = \lambda\, dt + dz$$

and integrating.

Plugging in the numbers gives

$$\tilde{B}_t = \frac{0.15 - 0.1}{0.2} t + B_t = 0.25t + B_t.$$

By the Girsanov theorem, $(\tilde{B}_t)_{t\geq 0}$ is a standard Brownian motion under the risk-neutral measure.

(d) Calculate the price of the option at time 0 under the risk-neutral measure.

Commentary on Question:
Most candidates were able to decompose the option price into two components. However, many candidates did not calculate the components correctly, resulting in wrong option prices.

The stock price dynamics are modeled by

$$dS_t = 0.1 S_t dt + 0.2 S_t d\tilde{B}_t$$

where $(\tilde{B}_t)_{t\geq 0}$ is a standard Brownian motion under the risk-neutral measure Q.

By Ito's lemma, we get

$$S_T = S_0 e^{(r - 0.5\sigma^2)T + \sigma \tilde{B}_T} = 100\, e^{(0.1 - 0.5\times 0.2^2)T + 0.2\tilde{B}_T} = 100\, e^{0.08T + 0.2\tilde{B}_T}.$$

By the risk-neutral valuation method, the price of the one-year European option at time 0 is

$$f_0 = e^{-r} E[f_1] = e^{-r} \int_{S_1 < 80 \text{ } or \text{ } S_1 > 120} 1 dQ$$

$$= e^{-0.1} \int_{100e^{0.08 + 0.2\tilde{B}_1} < 80 \text{ } or \text{ } 100e^{0.08+0.2\tilde{B}_1} > 120} 1 dQ$$

3. **Continued**

Note that $\tilde{B}_1$ is normally distributed with mean 0 and variance 1 under the risk-neural measure. Hence

$$f_0 = e^{-0.1}\left[\int_{-\infty}^{\frac{\ln 0.8-0.08}{0.2}} \frac{1}{\sqrt{2\pi}} e^{-0.5x^2} dx + \int_{\frac{\ln 1.2-0.08}{0.2}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-0.5x^2} dx\right]$$

$$= e^{-0.1}[N(-1.5157) + 1 - N(0.5116)]$$

$$\approx 0.9048 \times \big(2 - N(1.52) - N(0.51)\big)$$

$$= 0.9048 \times (2 - 0.9357 - 0.6950) = 0.3342.$$

4. Learning Objectives:

2. The candidate will understand the principles and techniques for the valuation of derivatives.

Learning Outcomes:

(2g) Understand the limitations of the Black-Scholes-Merton model

(2h) Understand and apply numerical discretization methods to price options including Euler-Maruyama discretization and transition density methods

Sources:

https://www.soa.org/resources/research-reports/2023/interest-rate-model-calibration-study/

Commentary on Question:

Part (a):
Performance was mixed. Some indicated difficulty with setup, while a solid group achieved high grading poitns. This part provided moderate discrimination.
Part (b):
This was the strongest-performing part. Most candidates earned full grading points, suggesting the question was routine and well understood.
Part (c):
Results were polarized. Many candidates received very low grading points, while others earned full grading points. Success depended on identifying a key idea, making this part a strong discriminator.
Part (d):
Overall performance was moderate. Partial understanding was common, though execution errors limited full credit.
Part (e):
Performance was weaker overall. Many candidates earned low grading points, with fewer achieving full grading points, indicating higher difficulty or time pressure.
Overall impression of Question 4:

Solution:

(c) Explain how you would use the above data for calibrating

(i) The Vasicek model

(ii) The Hull-White model.

(d) Calculate the prices of one-year and five-year zero-coupon bonds with the Vasicek model.

(c) Calculate the option price using the Vasicek model.

4. Continued

(d) Calculate the option price with the Hull-White model.

(e) Recommend which model you would choose to price and hedge European options on coupon bonds.

For model solution see the excel sheet.

5. **Learning Objectives:**

3. The candidate will understand various applications and risks of derivatives.

Learning Outcomes:

(3a) Understand the Greeks of derivatives

(3b) Understand static and dynamic hedging

(3c) Understand delta hedging, and the interplay between hedging assumptions and hedging outcomes

Sources:

John Hull - Options, Futures, and Other Derivatives, Global Edition-Pearson (2021)
- Ch. 19: The Greek Letters
- Ch. 26: Exotic Options

Commentary on Question:

Most candidates got full credit on part (a), but very few got any credits on part (b) to (e) although some got partial credit. Part (a) is straightforward calculations in Excel, while part (b) to (e) requires mix of calculations and explanations. Most candidates do not seem to understand the questions of part (b) to (e).

Solution:

(a) Calculate the cost of purchasing European put options to achieve the downside protection against a decline of more than 10%.

5. **Continued**

Fill in your final answers here:		
The cost of purchasing European put options		12,657,387.69
Show your work here, for part a):		
Assumptions		
Portfolio Value	500,000,000	
buffer%	0.1	
S&P 500 index (S0)	5,000	
Strike (K)	4,500	
Dividend yield (d)	0	
Risk-free rate (r)	0.04	
Volatility	0.2	
Expiry (TTM)	1	
d1	0.8268	
d2	0.6268	
N(-d1)	0.2042	N(d1) 0.79582552
N(-d2)	0.2654	N(d2) 0.73460567
Put option	126.57	
Cost of purchasing put option	12,657,387.69	

The portfolio is worth 100,000 times the value of the S&P index.
When the portfolio value falls by 10%, the value of the S&P 500 index also falls by 10%.
Therefore, you require European put options on the 100,000 times the S&P 500 with exercise price of 4,500.

$$S_0 = 5{,}000$$
$$K = 4{,}500$$
$$r = 0.04$$
$$q = 0.00$$
$$T = 1.0$$
$$\sigma = 0.2$$

$$d_1 = \frac{\ln\left(\frac{5000}{4500}\right) + \left(0.04 - 0.00 + \frac{0.04}{2}\right) * 1.0}{0.2 * \sqrt{1.0}}$$
$$d_2 = d_1 - 0.2 * \sqrt{1.0}$$
$$N(d_1) = 0.7958; N(d_2) = 0.7346$$
$$N(-d_1) = 0.2042; N(-d_2) = 0.2654$$

5. **Continued**

The put option value is, by Black-Scholes
$K * e^{-rT}N(-d_2) - S_0 * e^{-qT}N(-d_1)$
$= 4500 * e^{-0.04}N(-d_2) - 5000 * e^{-0.00}N(-d_1) = \$126.57$
Total cost the portfolio insurance is therefore
=100,000 * 126.57 = $12,657,000

(b) Describe an alternative strategy using traded European call options to achieve the same protection.

From the put-call parity formula,

$$S_0e^{-qT} + p = c + Ke^{-rT}$$

or

$$p = c - S_0e^{-qT} + Ke^{-rT}$$

This demonstrates that a put option can be synthetically made by shorting $e^{-qT}$the S&P 500 index, buying a call option and investing the rest at risk-free rate securities. Under this scenario,

- Sell 500$e^{-0.00}$=500 million of the portfolio
- Buy 1,000 call options (100x) on the S&P 500 index with the strike price of 4,500 and 1-year maturity
- Invest the remaining cash at the risk-free rate earning 4%

(c) Calculate the portion of the portfolio to be sold initially and invested in risk-free securities to achieve the same protection.

The delta of one put option is $e^{-qT}[N(d_1) - 1]$
= $e^{-0.00} * (-0.2042) = -0.2042$
This shows that 20.42% of the portfolio, about $102.10 million should be initially sold and invested in risk-free securities.

(d) Calculate the initial position in one-year S&P 500 futures to provide the same protection.

The delta of a one-year index futures contract is $e^{(r-q)T} = 1.0408$
The spot short position required is
$\frac{126{,}570{,}000}{5{,}000}$=25,314
times the index, and therefore the short position in term of the S&P 500 futures is

$$\frac{25{,}314}{1.0408 * 250} = 97.29$$

5. **Continued**

(e) Explain whether a forward contract on the S&P 500 index has the same delta as the corresponding futures contract.

The value of a forward contract on the asset is $S_0e^{-qT} - Ke^{-rT}$. When there is a small change ΔS in S_0, the value of the forward contract changes by ΔSe^{-qT}. Therefore, the delta of the forward contract is e^{-qT}.
The futures price is $S_0e^{(r-q)T}$. When there is a small change ΔS in S_0, the value of the futures contract changes by $\Delta Se^{(r-q)T}$. Given the required daily settlement of the futures contracts, the delta of futures contract is $e^{(r-q)T}$.
The deltas of a futures and a forward contract are not the same. The delta of the futures is greater than the delta of the corresponding forward by a factor of e^{rT}.

6. Learning Objectives:

1. The candidate will understand key types of derivatives.

3. The candidate will understand various applications and risks of derivatives.

Learning Outcomes:

(1c) Be able to compare European, American, Bermudan, Asian options, and various exotic options

(1d) Understand the mechanics of derivatives trading including:
- Exchange traded vs OTC
- Central Clearing
- Daily margin variation and daily settle

(3a) Understand the Greeks of derivatives

(3b) Understand static and dynamic hedging

(3c) Understand delta hedging, and the interplay between hedging assumptions and hedging outcomes

(3e) Understand how hedge strategies may fail

Sources:
John Hull - *Options, Futures, and Other Derivatives*, Global Edition-Pearson (2021)
Chap 26: Exotic Options;

Chap 37: Derivatives Mishaps and What We Can Learn from Them

Commentary on Question:
The question tests candidate's understanding of various types of vanilla and exotic options; differences between static and dynamic hedging methods and their pros and cons; as well as practical considerations related to"inception profit".
To receive maximum points for parts (a) and (b), candidates are expected to explain why the recommended replicating strategy would work by comparing the option payoffs, in addition to listing all the components of the replicating portfolio.

Solution:

(a) Recommend a strategy using plain-vanilla European call options to statically and approximately replicate the cash-or-nothing European call option with the strike price $K_1 = 50$.

6. **Continued**

Commentary on Question:
Most candidates understand the technique of using a tight callspread to approximately replicate the payoff of a cash-or-nothing call. Many choices of strikes are acceptable for the callspread, as long as the strikes of the two legs are reasonably close to each other and cover the original strike price of 50.

Payoff of the cash-or-nothing European call option at maturity =

- 1, if $S_T > 50$
- 0, if $S_T \leq 50$

Can buy 1 European call with strike price K of 50 and sell 1 European call with strike price K of 51, both with maturity of 6 months.
Payoff of this portfolio =

- 1, if $S_T > 51$
- S_T - 50, if $50 < S_T \leq 51$
- 0, if $S_T \leq 50$

This payoff can approximately replicate the payoff of the cash-or-nothing European call.

(b) Recommend a strategy using the 6-month cash-or-nothing call options and plain-vanilla European call options to statically and approximately replicate the up-and-out call option.

Commentary on Question:
Most candidates struggled with this question. About half of the candidates understand the payoff structure of the up-and-out call option correctly, and only a few candidates know how to use a portfolio of a callspread and a cash-or-nothing call to replicate the payoff structure. A common mistake among candidates is missing the short leg of the callspread in the replicating portfolio.

Payoff of the up-and-out European call with strike 50 and barrier 60 at maturity (assuming the barrier of 60 was not triggered before maturity) =

- 0, if $S_T < 50$
- S_T - 50, if $50 \leq S_T < 60$
- 0 if $S_T \geq 60$

Can buy 1 European call with strike price 50, sell 1 European call with strike price 60, and sell 10 units of cash-or-nothing call with strike price 60, all with maturity of 6 months.
The payoff of this portfolio is

- 0, if $S_T < 50$
- S_T - 50, if $50 \leq S_T < 60$
- 0 if $S_T \geq 60$

This exactly matches the payoff of the up-and-out call at maturity.

6. Continued

(c)

(i) Describe two advantages of static replication of an exotic option over dynamic delta hedging.

(ii) Describe two limitations of static replication of an exotic option with respect to dynamic delta hedging.

Commentary on Question:
Candidates generally did well on part (i). Most candidates understand the primary difference between static replication and dynamic delta hedging (i.e.. whether continuous rebalancing is required.)
For part (ii), the majority of candidates can touch on at least one relevant point. Some other points that received partial credits are: often times static replication can only be approximate; often there is a large initial set-up cost to purchase the replicating portfolio for static hedging; there could be increased hedge error for static replication if real-world market dynamics significantly deviate from model assumptions used in constructing the static portfolio.

(i)
Advantage 1: Static replication is often simpler to construct and implement than dynamic delta hedging, which requires continuous rebalancing. This can be especially hard for delta hedging near the barrier where the option's delta can change rapidly or jump discontinuously.

Advantage 2: In addition to being simpler, no need for rebalancing also means reduced transaction costs and model risk for static replication.

(ii)
Limitation 1: The options with the exact strikes and maturities needed for static replication of the exotic option may not be available in the market. Even if the options exist in the market, they may be illiquid and expensive to trade.

Limitation 2: The barrier option could have path dependency that is hard to capture. If the stock price jumps across the barrier, static hedges using vanilla payoffs can't capture the sudden loss of option value perfectly.

(d)

(i) Explain how the recognition of "inception profit" could lead to an increase in the marking-to-model risk.

(ii) Recommend two mitigation strategies to reduce the potential conflict of interest.

6. Continued

Commentary on Question:
For part (i), most candidates could not present a coherent argument as to why traders tend to favor models that produce higher "inception profit" when it is recognized early on and why this practice could lead to increased marking-to-model risk.
For part (ii), only a minority of candidates can point to at least one relevant mitigating strategy. Other points that received credits include: separate duties of front, middle and back offices; and closely monitor or audit hedging activities.

(i)
The magnitude of the "inception profit" when marking-to-model is heavily dependent on the model being used and the parameters being calibrated. Traders are incentivized to choose models that produce high "inception profit" if the profit can be recognized early on. However, if the models/parameters turn out to be incorrect/inaccurate representation of the reality, the final/actual profit could be very different from the initial estimate, and the "inception profit" could evaporate or even turn into losses. Thus, recognition of "inception profit" could give rise to increased marking-to-model risk.

(ii)
1. Align incentives with the goal of hedge effectiveness; defer bonus payouts to ensure long-term hedge performance; "claw back" bonuses if hedge performance is not as good as originally thought.
2. Have clear mandates to hedge company's risks instead of to speculate.

7. **Learning Objectives:**

2. The candidate will understand the principles and techniques for the valuation of derivatives.

Learning Outcomes:
Understand option pricing techniques including:

- Calculating an expectation
- By solving an PDE

Sources:
INV201-107-25: It's RILA time: An introduction to registered index-linked annuities

Hull – Options, Futures and Other Derivatives, Ch. 15, 26

Commentary on Question:
Candidates generally performed well on parts (a) and (c) and showed average performance on parts (e) and (f). A significant number of candidates skipped parts (b) and (d), which resulted in poor performance in those sections. Some candidates skipped key steps and jumped directly to the final results, which meant full credit was not awarded for those steps. For question (f), there were two valid interpretations of the cap: one following Equation 3 on page 350 of INV201-107-25 in the syllabus text, and the other following Equations 1.39 and 1.41 on page 34 of INV201-105-25. Either approach was awarded credit if presented correctly. The following answers (ii) and (iii) should be consistent with what the candidates proposed in (i).

Solution:

(a) Verify that $V(\alpha, g, T) = e^{-rT}\mathbb{E}\left[\max\left((1-\alpha) + \alpha e^{\left(r-\frac{1}{2}\sigma^2\right)T+\sigma W_T}, (1+g)^T\right)\right]$.

Commentary on Question:
Candidates generally performed well on this part/section. The main issue observed was that some candidates misunderstood how the participation rate is applied in the EIA. Therefore, they can't demonstrate the EIA payoff.

Payoff of the EIA is $max[1 + \alpha(S(T) - 1), (1+g)^T]$

$$= max[1 + \alpha S(T) - \alpha, (1+g)^T]$$

$$= max[(1-\alpha) + \alpha S(T), (1+g)^T]$$

$= max\left[(1-\alpha) + \alpha e^{\left(r-\frac{1}{2}\sigma^2\right)T+\sigma W_T}, (1+g)^T\right]$, *given that S(T) is lognormal under* $\mathbb{Q}$

The value today of the option is the present value of the expected payoff, therefore:

$$V(\alpha, g, T) = e^{-rT}\mathbb{E}\left[\max\left((1-\alpha) + \alpha e^{\left(r-\frac{1}{2}\sigma^2\right)T+\sigma W_T}, (1+g)^T\right)\right]$$

7. Continued

(b) Construct a hedging strategy that validates your colleague's claim and could be implemented at $t = 0$.

Commentary on Question:
Candidates performed poorly on this question. Many did not attempt it. Only a few successful candidates explained the hedging strategy using call options and zero-coupon bonds correctly. Some candidates only use words to explain what option or/and bond should be used, without showing any steps to derive the strategy.

$$Payoff = max\left((1-\alpha) + \alpha S(T), (1+g)^T\right)$$
$$= (1+g)^T + max(\alpha S(T) + (1-\alpha) - (1+g)^T, 0)$$
$$= (1+g)^T + \alpha\, max\left(S(T) - \frac{(1+g)^T - (1-\alpha)}{\alpha}, 0\right)$$

The first term is simply a constant. The second has the same payoff as a vanilla European call option on $S(T)$. This call has a strike price of $\frac{(1+g)^T-(1-\alpha)}{\alpha}$. On a present value basis, the expected value of the equity indexed annuity is

$$(1+g)^T e^{-rT} + \alpha Call\left(S(0), 0, T, \frac{(1+g)^T - (1-\alpha)}{\alpha}\right)$$

To hedge the EIA, you can invest $(1+g)^T e^{-rT}$ into a risk-free bond maturing at time T, and purchase α units of European call options on the index at the strike price indicated.

(c)

(i) (*1.5 points*) Verify that:

$$\frac{\partial V}{\partial \alpha} = C(S,K) - \frac{\partial C}{\partial K} * (K-1)$$

(ii) (*1 point*) Verify that:

$$\frac{\partial V}{\partial \alpha} = \frac{\partial C}{\partial S} + \frac{\partial C}{\partial K}.$$

7. Continued

Commentary on Question:
Candidates performed well on part (i) and average on part (ii).
(i) To receive full credit, candidates needed to show the steps in accurately applying the chain rule and deriving the (K - 1) terms. Sometime, the is sign is not correct in the steps, which is considered a failure in that part of the solution.
*(ii) Candidates did not receive full marks if they jumped directly to the final statement without showing intermediate steps, or if the intermediate steps did not lead to the required result. A few candidates used an alternative approach by identifying C=S*dS/dC+K*dC/dK from the call option formula; this approach also received full credit when done correctly.*

C(i)

From part (b), we saw $V(\alpha,g,T) = (1+g)^T e^{-rT} + \alpha Call\left(S(0), 0, T, \frac{(1+g)^T-(1-\alpha)}{\alpha}\right)$.

For simplicity, let's refer to the call as $Call(K = K_0)$, *where* $K_0 = \frac{(1+g)^T-(1-\alpha)}{\alpha}$

Taking the partial with respect to α *on both sides, requires use of product rule and chain rule:*

$$\begin{aligned}\frac{\partial}{\partial\alpha}[V(\alpha,g,T)] &= \frac{\partial}{\partial\alpha}[(1+g)^T e^{-rT} + \alpha Call(K=K_0)] \\ &= 0 + \frac{\partial}{\partial\alpha}[\alpha] * Call(K=K_0) + \alpha * \frac{\partial}{\partial\alpha}[Call(K=K_0)] \\ &= Call(K=K_0) + \alpha\frac{\partial Call(K=K_0)}{\partial K} * \frac{\partial K}{\partial\alpha}\end{aligned}$$

Now, the partial of the call price with respect to strike is provided in the hint. For the derivative of the strike with respect to α, *we find:*

$$\frac{\partial K}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left[\frac{(1+g)^T-(1-\alpha)}{\alpha}\right] = \frac{\partial}{\partial\alpha}\left[\frac{(1+g)^T-1}{\alpha}+1\right] = -\frac{(1+g)^T-1}{\alpha^2}$$

Substituting into our previous work, we find:

$$\begin{aligned}\frac{\partial}{\partial\alpha}[V(\alpha,g,T)] &= Call(K=K_0) + \alpha\frac{\partial Call(K=K_0)}{\partial K} * \frac{\partial K}{\partial\alpha} \\ &= Call(K=K_0) + \alpha\left(-e^{-rT}N(d_2)\right) * \left(-\frac{(1+g)^T-1}{\alpha^2}\right) \\ &= Call(K=K_0) + \frac{(1+g)^T-1}{\alpha}e^{-rT}N(d_2)\end{aligned}$$

7. Continued

This does not yet look like the desired result. Notice that part of the 2nd term almost matches our strike. It is actually equal to $K_0 - 1$.

Note

$$\frac{(1+g)^T - 1}{\alpha} = \frac{(1+g)^T - 1}{\alpha} + \frac{\alpha}{\alpha} - \frac{\alpha}{\alpha}$$
$$= \frac{(1+g)^T - 1 + \alpha}{\alpha} + \frac{\alpha}{\alpha} - 1$$
$$= \frac{(1+g)^T - (1-\alpha)}{\alpha} - 1$$
$$= K_0 - 1$$

Applying the hint, our final result is

$$\frac{\partial V}{\partial \alpha} = C(S,K) - \frac{\partial C}{\partial K} * (K - 1)$$

C(ii)

From part (i) we have,

$$\frac{\partial V}{\partial \alpha} = C(S,K) - \frac{\partial C}{\partial K} * (K - 1)$$

Distributing the partial derivative, we find:

$$\frac{\partial V}{\partial \alpha} = C(S,K) - K * \frac{\partial C}{\partial K} + \frac{\partial C}{\partial K}$$

If we can show the first two terms simplify to $\frac{\partial C}{\partial S}$*, we have verified the target equation.*

Recall $C(S,K) = SN(d_1) - Ke^{-rT}N(d_2)$. *Combined with the Hint provided in the question, we find:*

$$\frac{\partial V}{\partial \alpha} = S(0)N(d_1) - Ke^{-rT}N(d_2) - K\left(-e^{-rT}N(d_2)\right) + \frac{\partial C}{\partial K}$$
$$= N(d_1) - Ke^{-rT}N(d_2) + Ke^{-rT}N(d_2) + \frac{\partial C}{\partial K}$$
$$= N(d_1) + \frac{\partial C}{\partial K}$$

Since we are given $S(0) = 1$.

$N(d_1)$ *is the delta (i.e.,* $\frac{\partial C}{\partial S}$*) of a European call on a stock with no dividends.*

Therefore,

$$\frac{\partial V}{\partial \alpha} = \frac{\partial C}{\partial S} + \frac{\partial C}{\partial K}.$$

7. Continued

(d) Derive the approximate participation rate that aligns with the break-even point for a 3.0% guarantee.

Commentary on Question:
Many candidates did not attempt this question, but those who did generally performed well. The candidates are supposed to leverage the existing information, $V(75\%, 3.0\%, 5) = 1.0265$, *to derive the new participation rate. Partial credit was awarded to candidates using a goal-seek approach, provided that V was correctly set and they arrived at the correct final answer. Quite a few candidates assume strike price be fixed (which is wrong), as it will change with the participation rate.*

We are given that a 3.0% guarantee with a 75% participation rate yields a price of 1.0265. To maintain the break-even nature, we need to adjust the participation rate to bring this price back down to 1.

In other words, we want to find a $d\alpha$ *such that* $\frac{\partial V}{\partial \alpha} d\alpha = -0.0265$.

From part (c), we can approximate $\frac{\partial V}{\partial \alpha}$ *around* $g = 3.0\%, \alpha = 75\%$.

$$\frac{\partial}{\partial \alpha}[V(75\%, 3\%, 5)] = N(d_1) - e^{-.06*5}N(d_2)$$

$$d_1 = \frac{-\ln\left(\frac{(1.03)^5 - 1}{0.75} + 1\right) + \left(.06 + \frac{0.2^2}{2}\right)5}{0.2\sqrt{5}} = 0.464$$

$$d_2 = d_1 - 0.2\sqrt{5} = 0.017$$

Using the provided normal distribution tables,

$$\frac{\partial}{\partial \alpha}[V(75\%, 3\%, 5)] = N(0.46) - e^{-.06*5}N(0.02) = 0.6772 - e^{-.3}(0.5080) = 0.3009$$

Therefore, $d\alpha = \frac{-0.0265}{\frac{\partial V}{\partial \alpha}} = \frac{-0.0265}{0.3009} = -0.088$.

Alternative solution: (Partial marks will be given only)

*Setting up the V(75%, 3%, 5) = 1.0265, and goal seek the α such that V (α, 3%, 5) = 1
Candidate is expected to be able to set the V (75%, 3% ,5) = 1.0265 to demonstrate the understanding of the underlying EIA.*

7. Continued

(e) Compare RILAs and EIAs.

Commentary on Question:
Candidate performed average on this question. Most candidates could get some points right. However, some candidates provide contradictory statements.

- *Structurally similar, where account value is credited based on performance of a common index over a given term*
- *Both usually subject to minimum guarantees*
- *Both may have caps on potential gains or limited participation*
- *RILAs are regulated as securities, unlike EIAs*
- *RILAs offer increased equity exposure, and may credit negative returns*
- *EIAs don't charge fees, and neither do most RILA products*
- *EIAs are general account products, while RILAs may not be*

(f)

(i) Sketch the payoff structure of proposed design, clearly labeling how the characteristics are reflected.

(ii) Develop a hedge strategy for this structure using vanilla European options on the underlying stock.

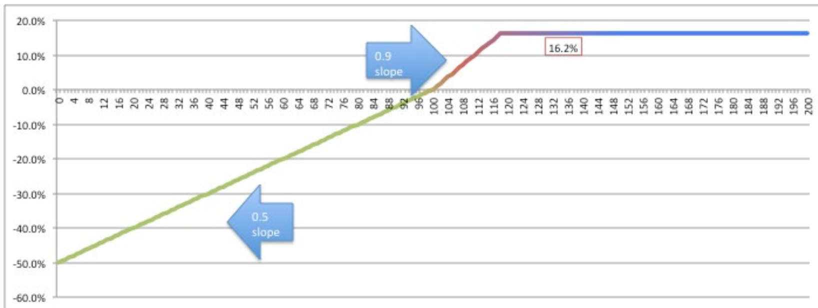
(iii) Verify that your strategy is appropriate.

Commentary on Question:
Candidates performed below average on this question. Quite a lot of candidates skipped the question, especially (iii).
(i) To receive full credit, candidates needed to clearly label the slope, cap, and floor on the graph. Some candidates did not explicitly indicate the slopes, but clearly showed the values on the x-axis and y-axis, from which the slopes could be derived.
(ii) Some candidates misunderstood the hedge position, incorrectly taking the opposite direction of the intended hedging strategy. And, candidates should clearly provide correct units of each of the options in the strategy.
(iii) Successful candidates applied the strategy from (ii) across the price range to demonstrate a hedging strategy that is consistent with part (i). Some candidates selected specific points within the range and calculated the payoff accordingly. Partial credit was given in these cases. Candidates should calculate the total payoff, rather than only listing the payoff of each option across the price range.

7. Continued

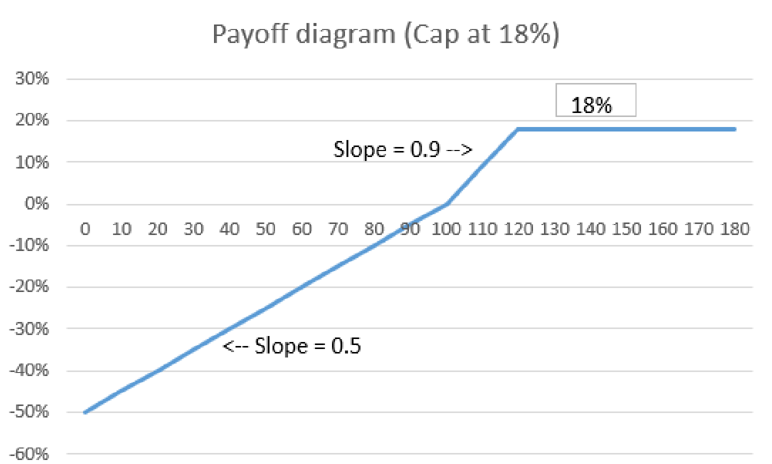
<u>*f(i)*</u>

Students should indicate the 0.5 and 0.9 slopes under and above current value, as well as the cap value. If results reflected in terms of crediting % or in currency are acceptable.



Slope = 0.5 when $S_T < 100$
Slope = 0.9 when $100 \leq S_T \leq 118$
Slope = 0 when $S_T > 118$
Clear label with return = - 50% when $S_T = 0$
Clear label with return = 16.2% when $S_T \geq 118$

Alternative solution



Slope = 0.5 when $S_T < 100$
Slope = 0.9 when $100 \leq S_T \leq 120$
Slope = 0 when $S_T > 120$
Clear label with return = - 50% when $S_T = 0$
Clear label with return = 18% when $S_T \geq 120$

<u>*f(ii)*</u>

Buy 0.9 units of at-the-money calls.
Sell 0.9 units of out of the money calls with K =118
Sell 0.5 units of at-the-money puts

7. **Continued**

Alternative solution:

(see the proof/verification in part f) iii) below)
Buy 0.9 units of at-the-money calls.
Sell 0.9 units of out of the money calls with K =120
Sell 0.5 units of at-the-money puts

(For those candidates using cap at 16.2%, K needs to be 118)
(For those candidates using cap at 18%, K needs to be 120)

f(iii)
If at end of period, the stock is above $118:
Long 0.9 At-the-money calls = 0.9*max(S-100,0) = 0.9*S-90*
*Short 0.9 Calls with 118 strike = -0.9*max(S-118, 0) = -0.9*S+106.2*
*Short 0.5 at-the-money puts = -0.5*max(100-S, 0) = 0*

*Total payoff = 0.9*S-90-0.9*S+106.2+0=16.2*

If at end of period, the stock is above $100 and below $118:
Long 0.9 At-the-money calls = 0.9*max(S-100,0) = 0.9*(S-100)*
*Short 0.9 Calls with 118 strike = -0.9*max(S-118, 0) = 0*
*Short 0.5 at-the-money puts = -0.5*max(100-S, 0) = 0*

Total payoff = 0.9(S-100)+0+0, i.e. 90% of the growth*

If at end of period, the stock is below $100:
Long 0.9 At-the-money calls = 0.9*max(S-100,0) = 0*
*Short 0.9 Calls with 118 strike = -0.9*max(S-118, 0) = 0*
*Short 0.5 at-the-money puts = -0.5*max(100-S, 0) = -0.5*(100-S)*

Total payoff = -0.5(100-S), i.e. 50% of stock loss*

Alternative solution:

If at end of period, the stock is above $120:
Long 0.9 At-the-money calls = 0.9*max(S-100,0) = 0.9*S-90*
*Short 0.9 Calls with 120 strike = -0.9*max(S-120, 0) = -0.9*S+108*
*Short 0.5 at-the-money puts = -0.5*max(100-S, 0) = 0*

*Total payoff = 0.9*S-90-0.9*S+108+0=18*

If at end of period, the stock is above $100 and below $120:
Long 0.9 At-the-money calls = 0.9*max(S-100,0) = 0.9*(S-100)*
*Short 0.9 Calls with 120 strike = -0.9*max(S-120, 0) = 0*
*Short 0.5 at-the-money puts = -0.5*max(100-S, 0) = 0*

7. **Continued**

Total payoff = 0.9(S-100)+0+0, i.e. 90% of the growth*

If at end of period, the stock is below $100:
Long 0.9 At-the-money calls = 0.9*max(S-100,0) = 0*
*Short 0.9 Calls with 120 strike = -0.9*max(S-120, 0) = 0*
*Short 0.5 at-the-money puts = -0.5*max(100-S, 0) = -0.5*(100-S)*

Total payoff = -0.5(100-S), i.e. 50% of stock loss*