Aging and Retirement

Primer on Retirement Income Strategy Design and Evaluation

JANUARY | 2023

SOA Research INSTITUTE

Primer on Retirement Income Strategy Design and Evaluation

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Sponsors Aging and Retirement Strategic Research Program

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Executive Summary

Overview

This Primer addresses strategies to generate income during retirement from the perspectives of their design and evaluation. Design involves specifying a joint investment and withdrawal strategy, which entails deciding how to invest available assets and setting a plan for withdrawing income. Evaluation involves analyzing how well the potential outcomes arising from a strategy satisfy objectives, which provides the foundation for strategy selection. The main focus of this Primer is investment strategy rather than withdrawal strategy. However, investment strategy cannot be examined independently from withdrawal strategy. Income generation is the focal point of a retirement income strategy, and this in turn depends on the rules under which income is drawn. Further, the appropriate investment strategy will itself depend on the preferences of the retiree over the level and variability of income.[1] Thus withdrawal strategies are addressed as necessary, albeit to a limited extent.

A central theme in the design of retirement income strategies is catering to the substantial differences that exist between retirees, i.e., heterogeneity. Characteristics that can make a significant difference to a retirement income strategy include: (a) the type of income objective (e.g., delivering a target income versus maximizing the income extracted); (b) appetite for income risk, including risk tolerance and risk capacity; and (c) personal attributes, such as total available financial assets, homeownership, partnered status, and access to other income sources such as social security. This Primer pays particular attention to how strategies might vary with a retiree's objectives.

This Primer addresses these matters in the following structure:

- Section 1 outlines the ***retiree characteristics*** that can make a difference in designing an appropriate retirement income strategy, including objectives, risk appetite and personal attributes.
- Section 2 considers the ***building blocks*** of retirement income strategies, including the assets and/or products that can be used to construct an investment strategy as well as withdrawal strategies.
- Section 3 briefly overviews the ***modelling of retirement outcomes***, focusing on how stochastic models are structured while highlighting the main modelling choices.
- Section 4 addresses ***methods for evaluating the outcomes*** that are generated by a retirement strategy. These include using a utility function to place a 'score' on the distribution of outcomes

[1] For instance, some retirees may prefer allocating assets to secure the income for life that they need or strongly desire. Meanwhile other retirees may prefer investing in risky assets with the aim of drawing more income, while accepting the risk that income may be variable and/or decline if returns happen to be poor.

that a strategy delivers over time, and various metrics to characterize outcomes so that they can be understood and communicated. An illustrative example is provided.

- Section 5 discusses three approaches to ***strategy selection***, including applying principles and rules, selecting from a menu of candidate strategies and optimization techniques. It also presents illustrative examples.

- Section 6 raises miscellaneous ***other matters*** that are relevant for retirement strategies, including alternative methods for strategy design, the reluctance to purchase annuities, the role of behavioral influences, and business considerations for plan providers.

- Section 7 ***concludes*** by recognizing some of the limitations of this Primer.

- *Appendices* examine selected topics in more depth. Appendix A discusses income risk drivers, including investment, sequencing, longevity and inflation risk. Appendix B presents some technical details of utility functions. Appendix C sets out a broad range of metrics and their calculation. Appendix D contains assumptions and additional outputs for the illustrative examples.

Excel Models

Two Excel models are used to generate the illustrative examples appearing within Section 4, Section 5 and Appendix D of this Primer. One model is directed at an income target objective, and the other model directed at an income optimization objective (to be introduced in Section 1.1). These Excel models are available via the SOA website (see Income Optimizer Model and Income Target Model). The models allow users to adjust the assumptions and investigate how this affects the outputs presented in Section 4.4 or this Primer. These models are provided as a learning resource for educational purposes to supplement this Primer and are not, in any way, intended to be used for financial advice nor any commercial purpose.

Scope

The design and evaluation of retirement strategies is an extensive topic area, creating an unavoidable need to constrain what is covered by this Primer. Notable limitations on scope include:

- **Retirement income lens** – The primary focus is 'retirement income strategies' aimed at the generation of retirement income for spending purposes, noting that retirement strategies more generally might be directed at catering for other objectives such as bequest and precautionary motives. The possibility of a broader set of objectives is recognized, but not examined in depth.

- **Post-retirement phase only** – Strategies for deploying assets available at the point of retirement are addressed. Strategies over the entire life-cycle or the optimal time to retire are not considered.

- **Defined contribution setting** – This Primer can be viewed as operating within a defined contribution setting in the sense that it envisages retirees taking responsibility for managing their own financial affairs. Strategy design for defined benefit pension plans is not considered beyond recognizing defined benefit pensions provide a potential source of income for some retirees.

- **Focus on general principles and concepts** – The main aim is to provide an overview of selected principles and concepts of a general nature, with occasional guidance being offered. Specific jurisdictional elements such as social security and taxation are acknowledged where appropriate.

- **Not a literature review** – Although this Primer contains citations of selected literature that may be of interest, there is no attempt to provide a comprehensive review of what is an extensive body of related research.

Audience

This Primer is written for practitioners who are interested in an overview of the mechanics involved in designing and evaluating retirement income strategies but may not be subject matter experts. Likely readers may work for retirement plan providers, investment managers, asset consultants, actuarial organizations and financial planners.

Section Summaries and Key Messages

The seven sections contained in this Primer are summarized below. Key messages and useful tables or figures are highlighted, with hyperlinks provided for easy access.

Section 1: Retiree Characteristics

The first section sets foundations for this Primer by discussing the many ways in which retirees differ and what these differences may mean for the type of retirement strategy that is suitable. Section 1.1 starts by outlining **three types of income objectives** that are the focus of the discussion and the modelling presented within this Primer:

- An ***income floor objective*** recognizes a minimum level of income that should be secured, if at all possible, before aiming to achieve either of the next two objectives that are more aspirational in nature.
- An ***income target objective*** reflects the desire to achieve a defined level of income until death. This objective aligns with the concepts of income replacement rates and income budgets.
- An ***income optimization objective*** implies maximizing the expected income that is extracted from available assets while managing income risk. It implies a desire to generate the best possible outcome from available resources, rather than aiming to hit any specific income target.

Although this Primer is mainly concerned with generation of retirement income, the potential role for **preferences over assets** needs to be recognized. This issue is raised in Section 1.2. One key reason for considering assets in conjunction with income is that retirees typically value flexible access to funds. Having access to funds caters for any large, unplanned expenditures (e.g., health or aged care) that may not be covered by regular income and delivers bequests where they are valued by the retiree. These aspects suggest that the retirement savings account balance should be reported even where retirement income is the main focus.

Section 1.3 discusses **risk and risk appetite**. The core proposition is that income risk should be the focus of retirees rather than investment risk. Income risk is defined as the possibility of *failing to sustain a sufficient standard of living until death*. Section 1.3.1 raises ***two key trade-offs*** under which the prospect of higher income needs to be balanced against income risk. The first trade-off involves taking greater investment risk in a bid to generate additional wealth that might support higher income or allow a given income to last for a longer period. The second trade-off relates to boosting 'expected' income by withdrawing higher income earlier in retirement when the retiree is more likely to be alive to enjoy it. Pursing higher expected income in either of these ways can increase either the variability of income and/or the chances of exhausting the retirement savings account and hence experiencing lower income upon survival to older ages. Section 1.3.1 also raises the ***underlying drivers of income risk***. The focus is placed on investment risk, sequencing risk, longevity risk and inflation risk, which are discussed in detail within Appendix A. ***Risk appetite*** and its multiple dimensions are discussed in Section 1.3.2. The most relevant dimensions for retirement strategy design are identified as ***risk aversion*** under an income optimization objective, ***loss aversion*** under an income target objective, and ***risk capacity***, which is relevant to the income floor objective. Other aspects such as ***risk perception*** and ***risk composure*** are also recognized, noting that these dimensions can tend to reflect various behavioral influences.

Section 1.4 lists the wide range of **personal attributes** that may be relevant for the design and selection of retirement income strategies. The more influential attributes are highlighted as: age; total financial assets; homeownership; access to 'guaranteed' income streams, such as social security and defined benefit pensions; and partnered (i.e., household) status. Health, taxation, capacity for work, and support from a personal network are noted as attributes that may be important in some situations.

Section 2: Building Blocks of Retirement Income Strategies

The second section discusses the building blocks of retirement income strategies, which might be thought of as comprising a ***joint investment and withdrawal strategy*** directed towards delivering 'income layers' rather than as a 'product solution'. The investment strategy relates to how available assets are deployed to support income generation, while the withdrawal strategy refers to how income is withdrawn from the retirement savings after allowing for other income sources to form the 'top income layer'.

Building blocks that may be used to form the **investment strategy** are addressed in Section 2.1. They may comprise any purchased income streams, assets allocated to a retirement savings account for flexible withdrawal, and any other income sources that are available to the retiree such as guaranteed income streams (e.g., social security, defined benefit pensions). The discussion links these building blocks to their potential role under the three income objectives and their exposure to various risks. Section 2.1.1 describes the major types of income stream products. Section 2.1.2 considers the asset allocation within the retirement savings account, although the discussion is limited to how a growth and defensive portfolio might be combined. Section 2.1.4 categorizes the major investment building blocks in terms of key characteristics, including: expected return or income generation potential; exposure to investment, sequencing, longevity and inflation risk; and access provided to funds (refer to Table 4 for a summary).

Withdrawal strategies are addressed in Section 2.2. Although not the main focus of this Primer, a list of the main types of withdrawal strategy is provided in Section 2.2.1. Two withdrawal strategies are proposed to facilitate the modelling. Under the income target objective, a *'draw-to-target' strategy* is suggested that entails drawing enough income to attain the target until the retirement savings account is exhausted, with provision to draw in excess of the target where justified (see Section 2.2.3). Under the income optimization objective, an '*affordable' withdrawal strategy* is proposed that involves adjusting withdrawals over time based on age, retirement savings account balance and an 'assumed interest rate' (see Section 2.2.4). Under an income floor objective, the purchase of a life annuity to secure the floor is envisaged (see Section 2.2.2).

Section 3: Modelling Retirement Outcomes

The third section asserts that **stochastic modelling** is required to generate a distribution of outcomes that might potentially arise from a retirement income strategy, thus capturing income risk and related trade-offs to inform the design and evaluation of retirement income strategies. Section 3.1 provides an overview of stochastic models and notes how the output from stochastic models might be viewed as a series of potential future outcome 'paths' over a projection horizon. Outcomes may comprise both an income stream and any remaining balance in the retirement savings account.

Section 3.2 discusses the numerous **modelling choices** that can be influential for the outcomes that are projected and how they may be evaluated. Some of the key choices include: how retiree objectives and risk appetite are characterized; the retiree attributes incorporated; the environmental factors considered, such as social security and taxation; investments assumed to be available; the rules governing the withdrawal strategy; and the mathematical relationships between the model variables. A number of choices related to model structure are discussed, including: modelling frequency, i.e., time interval (Section 3.2.1); whether to model in nominal or real terms (Section 3.2.2); projection horizon (Section 3.2.3), specifically choosing between a fixed planning horizon versus applying survival probabilities; and the method by which asset returns are simulated (Section 3.2.4).

Section 3.3 rounds out the modelling section with a discussion of model reliability, in particular raising the issue of model risk. A warning is issued about relying solely on model output. Recommendations to address exposure to model risk include: exercising judgement when making decisions; establishing robustness through applying multiple modelling or decision methods; conducting scenario and sensitivity analysis; and ensuring that the key modelling assumptions be communicated clearly.

Section 4: Strategy Evaluation – Utility and Metrics

The fourth section is based around two main themes. The first is that utility functions and metrics should be used in tandem when evaluating retirement income strategies. The second is that the utility function and metrics applied should be tailored to the income objective.

Section 4.1 commences by outlining some **key concepts**. It is argued that ***utility functions are most useful for strategy selection*** as they condense the entire distribution of outcomes into a single overall score, thus allowing strategies to be ranked or an 'optimal' strategy to be identified. However, a single score conveys little information about the type of outcomes that a strategy might deliver. This issue can be addressed by using ***metrics to characterize outcomes*** that a strategy might deliver. It is further argued that metrics should be selected to convey not only expected outcomes, but also *both the likelihood and magnitude* of failing to achieve the objective. Reporting both dimensions recognizes that investing in growth assets tends to reduce the likelihood of shortfall as horizon lengthens but also simultaneously increases the potential magnitude of any shortfall, see Appendix A.1).

Utility functions are addressed in Section 4.2, with further technical details appearing in Appendix B. It is suggested that a loss aversion (reference dependent) utility function might be used under an income target objective (see Section 4.2.3); while a risk aversion (power) utility function might be applied under an income optimization objective (see Section 4.2.4). Issues around time weighting are addressed in Section 4.2.5, specifically the application of survival weights to capture the likelihood of the retiree being alive to enjoy the income and/or time preference (i.e., the preference for income earlier rather than later). It is suggested that survival weights should be used when conducting utility-based analysis, but that the application of time preference is more debatable and should depend on the situation. Section 4.2.6 acknowledges that parameterizing utility functions is difficult and suggests handling this issue by adopting 'book-end' parameters that span a plausible range of high and low risk appetite. Section 4.2.7 describes how expected utility can be converted into 'certainty equivalents' in the form of risk-adjusted income to make the output more interpretable.

Metrics are addressed in Section 4.3 on a selective basis, with a more complete set of metrics including formulae outlined in Appendix C. A selective set of metrics are recommended for use under each income objective. The recommended set includes metrics to convey the level of income that might be expected, risk of failing to achieve an income objective, 'risk adjusted income' as an overarching utility-based metric, and some basic statistics on retirement savings account balances. The main recommendations include:

- ***Probability of falling below any income floor*** might be reported (Section 4.3.2).
- ***Metrics for an income target objective*** (Section 4.3.3) should aim to convey both the likelihood of the target being sustained, and the magnitude of any shortfall versus the target once the retirement savings account balance is exhausted. These aspects are spanned by reporting the probability of account exhaustion at various ages, and the minimum level of income occurring upon account exhaustion.
- ***Metrics for an income optimization objective*** (Section 4.3.4) should be directed at revealing both the expected level and variability of income over the course of retirement. The recommended metrics include expected income (while noting that median income may be a suitable alternative), while using income percentiles to convey income variability over time.
- ***Statistics on the remaining retirement saving balance*** (Section 4.3.5) should be reported, even if at a rudimentary level, with median balance at various ages suggested.
- Useful charts include a plot of ***median income layers*** to communicate the sources of income, and ***total income percentiles*** at various ages to convey the variability of income over retirement.

Section 4.4 finishes by providing illustrative examples of how modelling output might be presented under the two main income objectives considered in this Primer. Section 4.4.1 presents the example for the income target objective, with suggested metrics reported in Table 7 and charts of income layers and income percentiles appearing as Figure 6 and Figure 7, respectively. Section 4.4.2 presents the example for the income optimization objective, with suggested metrics reported in Table 8 and charts of income layers and income percentile charts appearing as Figure 8 and Figure 9.

Section 5: Strategy Selection

The fifth section addresses the selection of a retirement strategy, supported by illustrative examples for which additional detail appears in Appendix D. **Three approaches** for identifying a suitable strategy are initially outlined in Section 5.1, followed by a discussion of various **factors to consider** when selecting a strategy in Section 5.2. These factors include: how strategies are ranked under quantitative analysis; review of projected outcomes to ensure they seem reasonable; a preference for simplicity and robustness; ensuring that the strategy delivers a sufficient level of flexible access to funds; various behavioral effects that may influence retiree acceptance of the strategy; and business considerations that could impact on the ability of the plan provider to successfully implement the strategy at reasonable cost.

The first strategy selection approach (see Section 5.3) entails specifying a procedure that applies **principles and rules** to translate the objectives and preferences of the retiree into a suitable retirement strategy. Under the example procedure that is presented, the first principle is to secure any income floor, after allowing for any guaranteed income streams such as social security. Subsequently, a decision is made on how much additional lifetime income streams to purchase, either with the aim of attaining the target under an income target objective or to limit downside income risk under an income optimization objective. The principle is that the amount allocated to a lifetime income stream depends on risk appetite. The next step is deciding how to invest the residual assets held within the retirement savings account. Here the rule is to take as much growth exposure as can be tolerated. Finally, a withdrawal strategy is set, with a ‘draw-to-target’ rule applied under the income target objective and an ‘affordable’ withdrawal rule applied under income optimization objective. Flowcharts describing the procedure under each income objective are provided as Figure 10 and Figure 11.

The second approach (see Section 5.4) involves **selecting from a menu of candidate strategies**. This approach might be suitable for a retiree who is choosing from a range of options offered by a plan provider, or perhaps by a plan provider that wants to investigate and compare a set of candidate strategies that are being proposed because they can be implemented at reasonable cost. This second approach is presented through illustrative examples that highlight how the most suitable strategy on the menu can be identified by combining quantitative analysis with consideration of other factors such as those outlined in Section 5.2. It shows how using utility-based metrics can be supplemented by other metrics to assist with consideration of aspects such as flexible access to funds. See Table 9, Figure 12 and Figure 13 for an example of a quantitative comparison of strategies using both utility and metrics.

The third approach (see Section 5.5) entails **identifying the ‘optimal’ strategy**, then potentially making adjustments to that strategy after consideration of other factors as per Section 5.2. Ideally the ‘optimal’ strategy would be identified by maximizing expected utility, as illustrated by Table 10. Consideration of other factors recognizes that strategies that may appear optimal under quantitative modelling may not be feasible or suitable, to the extent that the model excludes relevant aspects such as ability to implement the strategy or desire for flexible access to funds. Although a notionally optimal strategy may not be adopted, it may still act as a point of comparison to guide strategy design and against which alternatives might be evaluated.

Section 6: Other Matters

The sixth section recognizes miscellaneous matters that are relevant for retirement income strategy design, but for which an in-depth investigation is considered beyond the scope of this Primer. Section 6.1 acknowledges alternative methods that may be used to design a retirement income strategy, including goal-based investing and bucketing (Section 6.1.1), analyzing the funding of retirement income as a liability-driven investing problem (Section 6.1.2), and adopting an income-framing lens under which the aim is to consume only the income generated by investments (Section 6.1.3). Section 6.2 acknowledges the ‘annuity puzzle’–the resistance of retirees to purchasing lifetime income streams–which might be considered in both the design and the communication of a retirement income strategy. Section 6.3 discusses behavioral effects that may impact on how retirees engage with retirement decisions and suggests some strategies to help manage these effects. Finally, Section 6.4 lists some ‘business considerations’ that plan providers might wish to take into account.

Section 7: Limitations

This Primer is selective in its scope and does not attempt to fully address all aspects that are important for the design and evaluation of retirement income strategies. This is recognized in Section 7 through a listing of some of the key aspects that are not covered in depth.

SECTION 1: Retiree Characteristics

This section opens by discussing retiree characteristics that impact on retirement income strategies, including objectives, risk appetite and personal attributes. The underlying theme is that retirees can differ in many ways, and it is important to cater for these differences. Section 1.1 identifies three types of income objective. Section 1.2 recognizes the possibility of preferences related to the retirement savings account balance. Section 1.3 focuses on the nature of risk and the trade-offs involved. Section 1.4 outlines the many ways in which personal attributes may vary.

1.1 Three Income Objectives

Three broad types of income objective are discussed in this Primer: (1) income floor, (2) income target, and (3) income optimization. The first objective implies securing a minimum amount of income, if at all possible, whereas the other two objectives relate to income aspirations after addressing the floor.

1. Income floor – This objective envisages some minimum level of income that the retiree strongly desires not to fall below. It might be considered as a subsistence level of income required to cover basic living expenses. This objective implies designing the strategy to secure the income floor as a first priority, unless other income streams such as social security satisfy this need.

2. Income target – This objective presumes that the retiree desires a particular level of income during retirement. Two key forms are replacement rates and budget-based income targets.[2] Replacement rates assume the intent to sustain a similar living standard during retirement to that enjoyed prior to retirement, adjusting for factors such as the absence of work-related expenses and the need to save for retirement. A commonly used replacement rate target is 70% of pre-retirement disposable income.[3] Budget-based income targets are set with reference to the income required to purchase a particular basket of goods and services.[4] An income target needs not be set at a fixed (real) amount through retirement and may change over time. For instance, the target might decline as a retiree ages in recognition of any need or desire to spend less at older ages.[5]

3. Income optimization – This objective presumes the intent to maximize the income extracted from available assets before death, while managing the risk that income falls to undesirably low levels.

The three income objectives suggest a natural way to break down the strategy design problem, i.e., securing any income floor as a first and overriding goal, and having locked-in the floor, then working towards either an income target or income optimization. Figure 1 illustrates.

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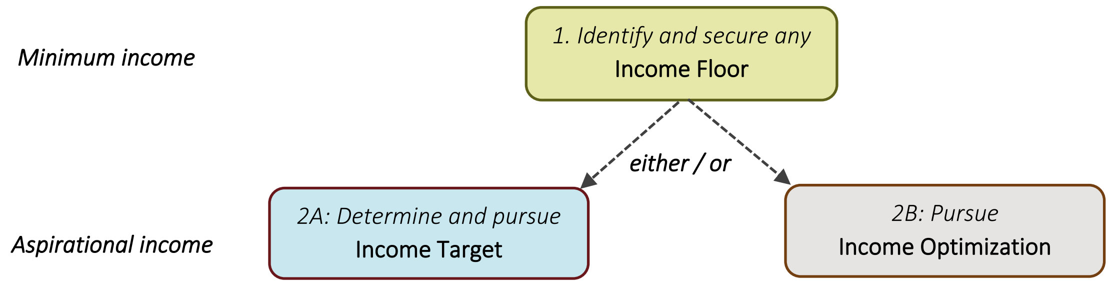
[2] See Chybalski and Marcinkiewicz (2016) for a discussion of various income target objectives.
[3] See MacDonald, Osberg and Moore (2016, pp. 628-9) for a brief discussion of the usage of a 70% replacement rate.
[4] The Retirement Standards issued by the Association of Superannuation Funds of Australia (ASFA) are an example of a budget-based target. They can be found at: https://www.superannuation.asn.au/resources/retirement-standard.
[5] Evidence that spending tends to decline through retirement can be found in Blanchett (2013) and Banerjee (2021). Note the well-cited u-shaped profile in Blanchett (2013) refers to lessening in the *rate* of spending decline later in retirement.

Figure 1:
ORDERING OF THE THREE INCOME OBJECTIVES



1.2 Flexibility and Preferences Related to the Retirement Savings Account Balance

While retirement income is the main focus in this Primer, the possibility needs to be acknowledged that some retirees may hold 'preferences' over accessible retirement savings, which is described in this Primer as the 'retirement savings account'. Three situations are discussed below. The first two relate to access to funds, while the third is more behavioral in nature. Flexible access to funds may be highly valued by many retirees but can be overlooked if potential income is considered in isolation.

- Catering for large yet uncertain expenditures – A key reason to retain some access to funds is to support possible future expenditures that are larger than could be supported by the regular income being drawn. The literature has focused on health and aged care costs,[6] but this concept applies to any large unplanned expense such as home renovations or assisting dependents. One approach to explicitly handle such demands might be through a 'carve out' of assets (i.e., 'rainy day' bucket), with the retirement income strategy then designed around the remaining assets.

- Bequest motives – Some retirees may wish to retain some assets for a bequest when they die. Intentional and strong bequest motives imply a willingness to sacrifice income to provide this bequest. Bequest motives vary considerably across retirees, and there is little evidence that strong bequest motives are pervasive.[7]

- Concern with the account balance – Many retirees are seemingly averse to short-term losses in their retirement savings account. This can make them wary of higher returning yet more volatile assets such as equities, even though these may increase prospects of sustaining higher income. Aversion to volatility in the account balance might stem from behavioral influences such as myopic loss aversion, framing effects or availability biases related to ongoing reporting of account balances. (Section 6.3 discusses behavioral influences.)

Section 4.3.5 presents the case for generating metrics on the remaining retirement savings account balance, even if it is not formally treated as an objective. Retaining some degree of flexible access to funds may also be intentionally incorporated into retirement income strategy design.

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[6] The papers cited in the above footnote also cover these issues.
[7] For instance, see Ameriks et al. (2011), De Nardi, French and Jones (2016, section 7) and Lockwood (2018).

1.3 Risk and Risk Appetite During Retirement

Establishing how much risk is acceptable to a particular retiree is far from straightforward. Risk during retirement is multifaceted, but should be primarily related to income variability, not return volatility. This section discusses the nature of risk during retirement and the trade-offs that are implied, as well as highlighting various risk concepts.

1.3.1 Risk and Trade-offs During Retirement

'*Income risk*' is the primary risk that a retiree might be concerned with during retirement and is defined here as: *the possibility of failing to deliver sufficient income to sustain a desired standard of living until death*. Income risk should ultimately be traded-off against the prospect of higher expected income; and risk appetite should be evaluated in the context of preferences over this trade-off. It is useful to break down this trade-off into two dimensions that tie directly back to the two key building blocks of a retirement income strategy – investments and withdrawals (see Section 2).

(a) **Pursuit of higher expected income through taking investment risk** – Investing in riskier, higher-returning assets (e.g., equities) increases expected income. However, it increases exposure to income risk, in particular the possibility of reduced income if investment returns are poor. Appendix A.1 discusses the nature of investment risk in a retirement setting, emphasizing the need to look beyond volatility of returns and focus on the implications for long-term wealth generation and hence retirement income.

(b) **Pursuit of higher expected income through increasing withdrawals earlier in retirement** – Increasing withdrawals from the retirement savings account earlier in retirement may raise expected income, due to a greater chance of a retiree being alive and active at younger ages. However, drawing more income earlier also lowers the potential to generate income later in retirement, thus heightening exposure to income risk.[8] Conversely, limiting withdrawals earlier in retirement means that income may be sustained for longer, but raises the chance of dying with a larger amount of assets left unutilized and hence income falling short of its potential.

The appetite of a retiree for income risk should be assessed by confronting them with the type of trade-offs described under points (a) and (b) above. That is, risk profiling should seek to establish the willingness to seek higher income through taking investment risk, at the risk that income may turn out lower; as well as the desire to enjoy income earlier, versus hedging against living to an old age. Unfortunately, standard risk profile questionnaires focus on the appetite for investment risk over shorter horizons such as a year,[9] rather than directly addressing income risk in retirement let alone the trade-offs described above. This leaves a potential blind spot in matching retirees to retirement strategies that accord with their appetite for income risk.

Finally, risk in retirement is often addressed through the prism of investment, sequencing, longevity, inflation and other[10] risks. This perspective is useful understanding the underlying 'drivers' of income and income risk. However, they do not gel with the way that individuals might understand risk in retirement, which needs to be framed around potential outcomes such as income. Table 1 summarizes by linking the four primary risk drivers to income risk. Appendix A contains an in-depth discussion of these risk drivers and how they manifest as income risk.

[8] An interaction between withdrawals and longevity risk is implied here.
[9] An example risk profile questionnaire made available by Vanguard is found at https://www.vanguard.com/investorquestionnaire.
[10] Other notable risks include: public policy risk; counterparty risk; large, unplanned expenditures; decline in functionality; and family changes.

Table 1:
LINKING THE FOUR PRIMARY RISK DRIVERS TO INCOME RISK

Risk Driver	Key Features
Investment risk	Taking greater investment risk through investing in higher returning but riskier assets can: • Increase expected income, either through using it to boost withdrawals or improve the likelihood of sustaining a given level of income for longer, but … • Lower income can result if returns turn out less than expected, manifesting either through the need to reduce the income drawn, or assets being exhausted sooner
Sequencing risk	• Poor investment returns earlier in retirement interact with withdrawals to result in retirement savings being depleted faster, thus reducing income later in retirement • A form of investment risk related to the timing rather than average magnitude of returns • Heightened by exposure to risky assets and larger withdrawals, especially when withdrawals are fixed in dollar terms[11]
Longevity risk	• Living longer than planned makes it difficult to sustain income until death • May be mitigated through longevity insurance (e.g., purchase of lifetime income streams) or conservative withdrawals, at the cost of reducing expected income
Inflation (cost of living) risk	• Cost of living increases result in income proving insufficient to sustain a standard of living • May be mitigated if returns earned or available income streams adjust with inflation

1.3.2 Risk Appetite – Concepts

Risk appetite spans a range of concepts, as summarized below.[12] These multiple facets add to the difficulty of establishing a risk profile[13] to feed into the design and evaluation of retirement income strategies.

- **Risk aversion** is perhaps the most familiar risk concept and is typically applied in an investment context. In a retirement income setting, risk aversion might be equated with an aversion to income falling to relatively low levels. It can induce a preference to avoid income variability, and hence a desire for income smoothing over time. Risk aversion aligns with the income optimization objective and relates to how a retiree might view the trade-off between seeking a higher expected level of income against the possibility of income falling to low levels.

- **Loss aversion** implies an aversion to losses relative to gains, determined with respect to some reference point that delineates gains and losses.[14] Loss aversion better aligns with the income target objective, and relates to the risk of being unable to sustain the target until death.

- **Risk capacity** refers to the amount of risk that can be tolerated. It is an objective measure that relates to how much risk the retiree is *able* to bear.[15] In contrast, risk aversion and loss aversion reflect how much risk a retiree is *willing* to bear. Risk capacity aligns with the income floor objective, on the basis that a retiree should be unable to bear any risk that might lead to income declining below the floor.

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[11] Withdrawals of a fixed value result in extraction of a larger percentage of available assets if the account balance has declined.
[12] See Finke and Guillemette (2016) for an overview and review of risk concepts.
[13] See Klement (2018) for a range of papers on the topic of risk profiling.
[14] Loss aversion is typically associated with prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992); although this theory includes other elements such as risk-seeking in the realms of losses, probability distortions and initial editing of prospects to be considered.
[15] Klement (2018, p3) describes risk capacity as applying to “the objective ability of an investor to take on financial risk”.

- **Risk perception** refers to a retiree's subjective evaluation of risk, and may arise from differences in experience, personality or financial literacy. It can be conflated with risk aversion, given that both influence the appetite to (say) invest in risky assets.[16] The relevance is twofold. First, some retirees may hold misguided risk perceptions relative to a more objective assessment of risk, and hence express a preference for strategies that may not be in their own best interest. For instance, they may underinvest in risky assets due to over-focusing on short-term volatility or might be overly wary in drawing down on their assets due to an out-sized fear of outliving their savings. Second, care needs to be taken when soliciting risk appetite that the results are not being distorted or dominated by unreasonable risk perceptions.

- **Risk composure** has been defined as the propensity to behave in a consistent manner.[17] Its main relevance here relates to the ability of a retiree to sustain exposure to risk after an adverse event such as a large market decline.

1.3.3 Incorporating Risk Appetite into Strategy Design

Risk appetite *should* be considered in strategy design, with research showing that risk appetite matters for the strategy that is likely to be appropriate.[18] There are substantial difficulties in accounting for risk appetite in a retirement setting due to a lack of established protocols for gauging appetite for income risk, as well as the problem of untangling the various risk appetite concepts discussed above. One approach to incorporating risk appetite into strategy selection might entail addressing risk capacity as a first priority by securing any income floor; and then consider two 'book-end' strategies respectively designed for high and low risk appetite. Meanwhile, risk perception and risk composure might be addressed through providing information, education and advice, rather than being incorporated directly into strategy design.

1.4 Personal Attributes

Accounting for personal attributes is critical in selecting retirement income strategies, as retirees differ along many dimensions that can impact on how they should invest and what income they should draw. One-size-fits-all strategies are unlikely to suffice during retirement, in contrast to pre-retirement where asset accumulation is the dominant objective. Five personal attributes that can make a significant difference to the retirement income strategy, and hence should ideally *always* be taken into account where possible, include: age; total financial assets; homeownership; access to other 'guaranteed' income streams, such as social security benefits; and partnered status (i.e., a 'household' perspective). Other attributes that may also make a difference but might be considered on a case-by-case basis, include: health, taxation, scope to work, and support from the personal network. Table 2 describes these attributes.

[16] Roszkowski and Davey (2010) discuss the difference between risk perception and risk tolerance (i.e., the inverse of risk aversion), reflecting on the financial crisis of 2008-9.
[17] See Klement (2018, p37)
[18] Studies showing that the optimal retirement income strategy varies significantly with risk aversion or loss aversion include Alserda, Dellaert, Swinkels and van der Lecq (2019) and Butt, Khemka and Warren (2022).

Table 2
INFLUENTIAL PERSONAL ATTRIBUTES FOR A RETIREMENT INCOME STRATEGY

Attributes	Rationale	Link to the Retirement Income Strategy
KEY ATTRIBUTES		
Age	Investments and withdrawals should vary with age	• Adjusting the strategy with realized investment experience • Adjusting the strategy as life expectancy reduces
Total financial assets	Retirement income strategies should be designed considering all available financial assets	• Greater total financial assets imply higher income is affordable • Value of total financial assets and other 'guaranteed' income streams (e.g., social security, defined benefit pensions) interact in determining allocation to defensive exposures such as annuities • Income target objectives indicate increasing risk exposure when assets are either too low or easily sufficient to attain the target
Homeownership	Home is an additional resource, often of significant value	• Renters require higher income to sustain a standard of living, which impacts on income floors and budget-based targets • A home represents an asset that might be deployed to: - Generate income or meet large, unexpected spending needs, through home equity loans, sale or downsizing - Satisfy a bequest motive • Homeownership impacts on social security in some jurisdictions
Other 'guaranteed' income streams, e.g., social security; defined benefit pensions	Provides a baseline level of income, and hence may influence how other assets might be deployed	• Form of defensive asset that can help address longevity risk, and may crowd out need for defensive exposures such as annuities • Might secure the income floor for some retirees • Access varies across jurisdictions and can impact strategy design[19]
Partnered status	Strategies should be designed differently for singles vs. couples	• Greater income required to support a couple, albeit less than double that of single retirees due to shared costs • Income needed until last partner dies, i.e., dual longevity • Joint asset pool: combined assets should be analyzed • Partnered status may impact access to social security
OTHER ATTRIBUTES		
Health	Influences strategies through impacts on longevity risk (i.e., horizon) and income needs	• Poor health implies lower life expectancy and shorter planning horizon, which can impact on the withdrawal strategy • Poor health may increase health-related costs, implying need for higher income target and/or larger precautionary savings
Taxation	May impact strategy by reducing income generation	• Strategies might be designed to minimize tax impacts • Taxation rules vary markedly across jurisdictions[20]
Scope to work	May impact strategy by supplementing income	• Should be considered in strategy design where significant, although this is typically not the case for most retirees
Support from personal network, e.g., transfers from aging parents; assistance from friends, relatives or charities	May impact strategy through expanding total financial assets that are potentially available	• May reduce need to secure income floor • May reduce the requirement for precautionary savings

[19] For example, US Social Security benefits increase with the age they are claimed (see SOA, 2017); while the Age Pension is means-tested in Australia.
[20] OECD (2018) lists 29 countries where benefits from retirement savings are taxed upon withdrawal (including all of the G7 nations), and 13 countries where benefits are tax-exempt. In Australia, retirees have access to imputation tax credits, which increases investment returns and hence potential income, see Butt, Khemka and Warren (2019).

SECTION 2: Building Blocks of Retirement Income Strategies

A retirement income strategy converts the assets that a retiree has accumulated over their working phase into retirement income. It encompasses a joint investment strategy and withdrawal strategy that are referred to here as 'building blocks'. The investment strategy is discussed in Section 2.1 and relates to how assets are allocated across purchased income streams and a retirement savings account, while considering any other income sources. The withdrawal strategy is discussed in Section 2.2 and relates to how income is withdrawn from the retirement savings account. The investment strategy is given more attention as the main focus of this Primer. Withdrawal strategies are covered in general terms; although two withdrawal strategies are framed to support the illustrative examples that appear later in the Primer.

2.1 Investment Strategies

The discussion here outlines the nature of the investment decisions faced by retirees,[21] focusing on selected investment building blocks and how they deal with various risk exposures. Incorporation of investment building blocks into a retirement income strategy is addressed in Section 5. The building blocks fall into three broad categories:

- Purchased lifetime income streams[22] – These products provide regular income for life. The nature of lifetime income streams can vary significantly depending on design, which may combine many features as discussed in Section 2.1.1. In general, lifetime income streams are typically purchased to address longevity risk; although they can also be used to address investment, sequencing risk and inflation risk. However, this comes at a cost of reduced flexibility over withdrawal amounts and sacrifice of access to the capital invested, including reduced capacity for bequests.

- Retirement savings account – Assets not used to purchase an income stream must be invested directly, which will be referred to as occurring via a retirement savings account. Retirees choose how the account is invested, which is commonly via some sort of diversified portfolio as discussed in Section 2.1.2. They also choose withdrawals from the account until it is exhausted: withdrawal strategies are considered in Section 2.2. These accounts are functionally equivalent to a bank account. They provide flexible access to funds to generate income, meet any large and/or unplanned expenditures, and perhaps support a bequest. It is typical for a retirement income strategy to include some direct investment as this flexibility is valuable. The downside is potential exposure to the various risk drivers listed in Section 1.3, i.e., investment, sequencing, longevity and inflation risk.

- Other income sources – These might be considered as part of the investment strategy, to the extent that it is directed at building income layers and are briefly discussed in Section 2.1.3.

Section 2.1.4 provides an overview of the investment building blocks by summarizing their characteristics including risk exposures. Section 2.1.5 briefly introduces dynamic strategies.

[21] Government mandates on use of accumulated savings for making investment decisions in retirement are not considered. Historically, some governments have placed constraints on the use of retirement savings, although this is becoming less common. For example, in 2015 the U.K. removed a requirement for retirees to annuitize by age 75.
[22] It is possible to purchase income streams that have a fixed term (i.e., term annuities), which might be viewed as a type of fixed income asset. These products do not manage longevity risk, and so are not considered in this Primer.

2.1.1 Purchased Lifetime Income Streams

Lifetime income streams may contain many features, the mix of which determine the level of expected income and the extent to which protection is provided against longevity, investment, sequencing and inflation risk. These features are summarized by a simplified product type list in Table 3. Additional description and commentary on the role of each product type in helping to achieve retirement income objectives is provided below the table.[23]

Table 3
PURCHASED LIFETIME INCOME STREAMS – PRODUCT TYPES

Product type	Description & Features
Fixed (Traditional)	• Payments may commence immediately or be deferred • Payments may be fixed for life in either a nominal or real terms
Investment-Linked (Variable)	• Incorporates 'growth' assets, with income varying dependent on portfolio returns • May allow the retiree to choose the portfolio underlying the income stream • Incorporates an 'Assumed Interest Rate' (AIR) to determine income level and variation
Longevity Pools	• Longevity risk pooled across policyholders rather than insured • May provide higher income than traditional lifetime income products • Income varies with the mortality experience of the pool
Guarantees	• Various guarantees or 'riders' can be applied to the above product types • Examples include guarantees over income provided in a given year and/or in total; and return of residual of initial investment not paid out as income upon early death

Fixed Lifetime Income Streams

'Traditional' income stream products eliminate exposure to investment, sequencing and longevity risk by providing a fixed[24] income stream for life. Payments may commence immediately (***immediate life annuity***, shortened hereon to ***life annuity***) or be deferred to commence at a known age in future (***deferred life annuity***). Income for life is generated by the provider utilizing the 'mortality credits' from those who die earlier than expected to fund the income streams of those who die later than expected.[25] These products can potentially hedge inflation risk if purchased with inflation protection (***inflation-protected annuities***).[26] Annuity providers typically charge a 'loading' that reduces the income delivered, to account for the risk that mortality experience may not match expectations, and to cover other costs including a profit margin.

Application to income floor objective
If eliminating longevity risk is essential to meet an income floor, a fixed income stream such as a life annuity might be purchased.

Application to income target objective
Retirees with an income target objective might purchase a fixed lifetime income stream to help sustain the target, especially if there is the opportunity to 'lock-in' the target for life. This may be a life annuity or

[23] See Bär and Gatzert (2022) for a recent review of the literature on products for decumulation of wealth in retirement.
[24] The degree of 'certainty' is subject to the income stream provider being able to continue paying the income stream.
[25] Mortality credits are the 'return' in an income stream generated from the participants who die over a period of measurement. This return is over and above the investment return underlying the income stream. See Fullmer et al. (2019) for a mathematical formulation of mortality credits from an income stream.
[26] Inflation-protected annuities are not readily available in the US but are in other countries such as Australia.

deferred life annuity. In the latter case, income withdrawn from the retirement savings account is used to sustain the target in the intervening period.

Application to income optimization objective
Retirees with an income optimization objective may use fixed income streams to 'lock-in' a level of sustainable income, and hence limit income risk.

Investment-Linked Lifetime Income Streams

Lifetime income stream products called ***investment-linked (or variable) annuities*** are based around underlying assets that are not risk-free (i.e., some growth assets), while still providing longevity protection through mortality credits. They may allow the retiree to choose the portfolio supporting the income stream. These products offer a higher expected income stream than traditional life annuities, but one which varies with the returns of the underlying portfolio and so exposes the owner to investment and sequencing risk.

To determine the initial income generated under an investment-linked annuity, an ***Assumed Interest Rate*** (AIR)[27] is used. Should the return exceed the AIR over a given period, then the income stream is increased. Equivalently, should the return underperform the AIR, the income stream is decreased.[28] The AIR need not equal the expected return on the portfolio. Setting the AIR at less than the expected return would create an income stream that starts at a lower level and is expected to increase over time (and vice versa), depending on actual returns compared to the AIR.[29]

Application to income target objective
For retirees with an income target objective, an investment-linked annuity with an AIR lower than the expected return can be attractive to protect the ability to meet the target in later years, with reliance placed on withdrawals from the retirement savings account to meet the target in earlier years.

Application to income optimization objective
For retirees with an income optimization objective, an investment-linked annuity with an AIR closer to the expected return may be more appropriate so income can be maintained without undue pressure on withdrawals from the retirement savings account early in retirement.

Longevity Pools

Traditional income streams insure against longevity risk to guarantee the income stream throughout life. A recent development in income stream design has been to pool but not insure longevity risk,[30] with the income stream adjusted according to the mortality experience of the pool. For instance, if the mortality experience of the pool is lighter (i.e., less people die) than assumed, then income is reduced for those still alive. As the mortality experience is not insured by the product provider, this reduces the loading and

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[27] Also known as 'assumed investment return' or a 'hurdle rate'.
[28] For a traditional fixed income stream, the AIR effectively equals a guaranteed interest rate and hence the income stream is unchanged over time. For an inflation-protected life annuity, the provider guarantees that the return exceeds the AIR by exactly the inflation rate. This is achieved by the provider investing the underlying assets in a very low-risk portfolio that is inflation-protected.
[29] See Horneff et al. (2010) for an analysis of the impact of different AIR values on annuity payments. See Balter and Werker (2020) for a mathematical treatment of the optimal choice of AIR from a policyholder perspective.
[30] There are a wide variety of actual and theoretical product designs in this space, although no firm naming conventions. The term 'group self-annuitization' has seen use in the actuarial literature. See Donnelly (2015) for more information on naming and actuarially fair and equitable distribution of mortality credits across open and heterogeneous pools. Since this paper, there has been increasing focus on pooled products using the term 'modern tontines' (see Milevsky and Salisbury, 2015).

delivers higher expected income. However, income will vary with the mortality experience of the pool.[31] Since pooled longevity products generate variable income, they are typically combined with the features of investment-linked annuities (as discussed above) to generate a higher expected income, with income variability thus arising from both investment and mortality risk.

Application to income target objective
Retirees with an income target objective might benefit from the additional risk in a pooled longevity product if their asset levels are not sufficient to 'lock-in' the target using a life annuity.[32]

Application to income optimization objective
Retirees with an income optimization objective may choose to enter pooled longevity products to eliminate the loading and hence increase the level of income.

Guarantees (i.e., Riders)

Income stream products may provide various types of guarantees known as 'riders'. Riders are particularly well developed in the US investment-linked income stream market. For example, a common structure is a Guaranteed Minimum Income Benefit (GMIB), which guarantees that an income stream will not decrease below a certain nominal level. This is equivalent to providing a floor on the return equal to the AIR under an investment-linked annuity.[33] Another common benefit across all income streams is a Guaranteed Minimum Withdrawal Benefit (GMWB),[34] also known as a refundable annuity.[35] These products provide a guarantee that total payments to policyholders will be at least equal to the premium paid in nominal terms, at a cost of a lower expected income. GMWBs primarily assist with the behavioral concern of purchasing an income stream and dying before receiving the 'value' of the income stream in return (see Section 6.2 and 6.3), by ensuring the product delivers a bequest upon early death.

The design of riders within income stream products has increased in complexity in recent decades, and it is outside the scope of this Primer to explore them in further depth.[36]

Application to income floor objective
The use of a GMIB product may be particularly attractive under an income floor objective. While the GMIB will likely have a lower guaranteed income than a life annuity, it provides upside potential to meet other objectives.

Application to income target objective
For retirees with an income target objective, a GMIB may be an alternative to a life annuity, particularly for those with assets greater than needed to achieve the target, as it allows them to guarantee the target but also seek income above the target.

Application to income optimization objective
For retirees with an income optimization objective, the use of a GMIB has a similar effect to reducing the growth asset allocation, i.e., it lowers expected income and reduces risk in the portfolio.

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[31] A more detailed treatment of longevity risk pooling can be found in Bauer (2017).
[32] Another response under these circumstances may be to consider reducing the income target.
[33] This return structure can be achieved by utilising options to provide the return floor and/or ceiling. See Blanchett (2021) for discussion in a retirement context. Such a structure could also be used for the balance of the retirement savings account, although this is less common.
[34] This rider may also be called a Guaranteed Lifetime Withdrawal Benefit (GLWB).
[35] See Milevsky and Salisbury (2022) for a detailed description of refundable annuities.
[36] See Morgan Stanley (2021) for additional detail.

2.1.2 Retirement Savings Account

The growth/defensive allocation within the retirement savings account has the largest influence on the portfolio risk profile and is thus the primary investment decision. It is hence the focal point of this Primer. Growth assets include equities, property, infrastructure, etc.; while defensive assets are typically made up of cash, government bonds, and other low-risk assets.[37] Product providers typically offer pre-mixed strategies that allocate between growth and defensive assets across a range of specified weights. Examination of the wide variety of available assets and their characteristics is outside the scope of this Primer.[38]

Growth assets offer higher expected returns and significant volatility. The key implication is growth exposure boosts the potential income that the retiree can expect to enjoy, but this brings greater exposure to income risk arising from the possibility that returns turn out lower than expected (i.e., investment risk) as well as sequencing risk. Defensive assets tend to deliver more certain but lower income. Appendix A discusses how these risks manifest in a retirement income context.

Note that a similar decision must be made in relation to the purchase of an investment-linked annuity that offers a choice of portfolio (see Section 2.1.1).

Application to income target objective
For retirees with an income target objective, the allocation to growth assets depends on loss aversion and their means to sustain the income target. Retirees with assets just sufficient to achieve their income target may have minimal need for growth asset investment and may prefer to secure the target through allocating to defensive assets, in particular lifetime income streams. Retirees with assets less than needed may invest in growth assets to increase potential to meet the target. Retirees with assets that comfortably exceed that needed to sustain the target might invest in growth assets to help boost income further above the target.[39]

Application to income optimization objective
For retirees with an income optimization objective, the allocation to growth assets is dependent upon risk aversion, specifically the willingness to seek higher income and run the risk that income could turn out lower if investment returns are poor. This allocation may be constant over retirement or involve the use of a 'glide path' to adjust exposure to growth assets over time.

2.1.3 Other Income Sources

The following income sources arise from Table 2 in Section 1.4 but are not developed in detail in this Primer. Nevertheless, it is worthwhile recognizing that they can assist in generating income and as such influence the allocation of assets between purchased lifetime income streams and the retirement savings account, as well as potentially the asset mix within the retirement savings account.

- **Guaranteed income streams**, such as social security and/or defined benefit pensions, are effectively forms of defensive assets that act like a life annuity. They can support investing a greater proportion of assets in the retirement savings account, and within a growth portfolio.

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[37] Many assets might be seen as a blend of growth and defensive, e.g., credit securities, infrastructure.
[38] Option strategies directed at limiting investment and hence income risk are also not considered. See Blanchett (2021) for a discussion.
[39] This theme is explored in greater detail in Butt, Khemka and Warren (2022). Allocation to the growth portfolio may adjust over time as the means to meet the income target changes with prior returns.

- **Home ownership** can be used to generate income or provide access to funds through reverse mortgages and downsizing. Owning a home can reduce the level of income required to meet an income floor and income target objectives through removing the need to pay rent. A home can also help meet large expenditures, most notably aged care, and satisfy any bequest motive.

- Support from the retiree's **personal network**, such as intergenerational transfers, can amount to an effective increase in total assets and hence income that is potentially available.

2.1.4 Overview of Investment Building Blocks

Table 4 summarizes the characteristics of the investment building blocks mentioned above. The building blocks listed are representative rather than comprehensive, noting that other products are available including income streams offering combinations of the features discussed.

Table 4
CHARACTERISTICS OF INVESTMENT BUILDING BLOCKS

	Expected Return (Income Generation Potential)	Investment (and Sequencing) Risk Exposure	Inflation Risk Exposure	Longevity Risk Exposure (Asset Exhaustion)	Access to Funds
Lifetime income streams					
Life annuities – nominal, insured	Low				
Life annuities – real (inflation-protected)	Low				
Longevity pooled annuities – nominal	Low, better than insured as no loading				
Investment-linked annuities	Moderate-High *(depends on asset mix)*		*Varies with asset mix*		
Life annuities – nominal, refundable	Lowest				*Limited*
Retirement savings account					
Growth assets (equities, property, etc)	Highest		*Varies with assets*		
Defensive assets (fixed income, cash, etc)	Moderately low				

Legend: *No exposure, i.e., hedged* | *Exposed* | *Depends*

The characteristics examined in Table 4 include the following:

- **Expected returns** – This determines income generation potential. Investments tied to growth assets typically offer higher expected returns and hence income.

- **Investment (and sequencing) risk exposure** – The distinguishing point on this characteristic is whether asset values and/or the income streams are fixed in value, either in nominal or real terms. The general rule is that growth assets give higher exposure to investment risk than defensive assets, although the latter are also exposed to investment risk.[40] Meanwhile, fixed income stream products hedge investment risk, which is not the case for investment-linked annuities. Any investment that is exposed to investment risk also brings exposure to sequencing risk, which reflects the interaction between investment risk and withdrawals (as discussed in Appendix A.2).

- Inflation risk exposure – Exposure to inflation risk depends on the extent to which asset values and/or income adjusts with inflation. Real income streams (e.g., inflation-protected annuities) offer an inflation hedge, while nominal income streams are highly exposed to unexpected increases in inflation. Some growth assets and (as well as inflation-protected bonds and perhaps cash[41]) might also assist in reducing exposure to inflation risk.

- Longevity risk exposure – This reflects the potential for income to cease (or greatly diminish) as assets are exhausted. It largely relates to the retirement savings account and is hedged by lifetime income streams that generate mortality credits through either insurance or pooling.

- Access to funds – This characteristic is important to many retirees, even though it may not be directly connected to income generation. It provides a key point of distinction between using a retirement saving account and purchasing an income stream. Refundable annuities provide some limited access to funds for a bequest upon an early death.

2.1.5 A Note on Dynamic Strategies

The investment strategy need not be fixed at retirement, but may be adjusted over time, i.e., dynamically managed. For instance, the asset mix within the retirement savings account may be varied as the retiree ages and in response to realized investment returns. Also, the purchase of an income stream might be delayed and/or spread over time. Indeed, research finds that dynamic strategies can be more efficient, although findings depend on assumptions and there is no common agreement. [42] This Primer makes reference to dynamic investment strategies where pertinent but does not investigate them in depth.

2.2 Withdrawal Strategies

The withdrawal strategy involves deciding what income is withdrawn from the retirement savings account, after taking into account any guaranteed income streams (e.g., social security) and purchased income stream products. It might be considered as forming the 'top income layer' that sits above the other income stream layers arising from available income streams. This section broadly describes the types of withdrawal strategies, bearing in mind that the primary focus of this Primer is investment strategies (i.e., how available assets are deployed). It also outlines withdrawal strategies that are appropriate for the three income objectives outlined in Section 1.1[43] and are used in the illustrative modelling appearing in Section 4 and Section 5.

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[40] Fixed income assets can be subject to price fluctuations in response to yield changes, and uncertain reinvestment rates with regard to the reinvestment of coupons and principal upon maturity. These exposures reflect duration, with long-term fixed income more exposed to price risk and short-term fixed income more exposed to reinvestment risk. See Warren (2021a) for a detailed discussion.
[41] Cash may provide an inflation hedge if cash rates adjust with inflation over time. However, this depends on the conduct of monetary policy.
[42] There is mixed evidence in the literature for the performance of glide paths in retirement, with results depending on economic conditions and the metrics of evaluation. See Pfau and Kitces (2014) and Blanchett (2015) for examples.
[43] See Macdonald et al (2013) for a far more detailed treatment of income drawdown in retirement. This is a journal-published version of a report initially prepared for the Society of Actuaries Research Institute.

2.2.1 Types of Withdrawal Strategy

Withdrawal strategies fall into four broad types:

- **Following government mandates on minimum withdrawal rates**. These are typically age-based percentages of the retirement savings account.[44] However, drawing at the minimum level may potentially provide a lower income than the retirement savings account might safely deliver, depending on the withdrawal rates applied.[45]

- **Constant withdrawal amount**. These often reflect a 'safe' amount that can be withdrawn each year with a minimal probability of exhausting the retirement savings account before a chosen age.[46] A key concern with this strategy is that it does not adjust with investment performance, and thus may lead to lower or higher withdrawals than what could safely be made in future years.[47]

- **Dynamic withdrawal amounts**. These strategies adjust withdrawals based on investment experience and age. Like minimum withdrawal rates, these strategies are typically expressed as age-based percentages of the retirement savings account but may also take into account other factors such as the underlying investment strategy and the level of the account balance.[48] A trade-off may exist between a simpler strategy and ensuring withdrawal amounts are appropriate.

- **Draw-to-target**. This strategy draws sufficient income from the retirement savings account to attain an income target after accounting for other income sources such as social security and purchased income streams. This withdrawal strategy delivers stable income but can result in exhaustion of the retirement saving account. It may also result in lower income than might be safely delivered if available assets are more than adequate to sustain the target.

2.2.2 Withdrawal Strategy for an Income Floor

As discussed in Section 2.1, the use of life annuities is appropriate to meet an income floor objective. This does not require an explicit withdrawal strategy. However, if an annuity to guarantee the floor is unable to be purchased due to (say) lack of funds, then the choice becomes either a draw-to-target withdrawal strategy in order to meet the floor for as long as possible; or accepting income below the floor by either purchasing a life annuity as can be afforded or restricting withdrawals.[49]

[44] In the US, a Required Minimum Distribution (RMD) applies after reaching age 72. For the most widely used table, which assumes spouse reversionary payments and makes no assumptions for interest, in 2022 the minimum rate of drawdown is 1/27.4 = 3.65% of the retirement savings account. This rate increases with each year of age. In Australia, the typical minimum drawdown rate of the retirement savings account starts from 4% for those under age 65, increasing to 5% for those aged 65-74, 6% for those aged 75-79, with further increases until eventually reaching 14% for those aged 95 or more. These rates have been adjusted downwards during distressed economic conditions.

[45] There is evidence that many retirees follow minimum withdrawal rates as being the recommended rate (see for example, Sneddon et al., 2016, in an Australian context, and Brown et al., 2017, in a US context). In the US, the RMD are often considered a straightforward and reasonable withdrawal strategy. In an Australian context, Chen et al. (2021) find that the minimum drawdown rules provide an inferior retirement outcome compared to other simple rules with higher drawdown rates.

[46] The most well-known of these is the 4% rule (Bengen, 1994), which states that retirees using a 50/50 equity/bond portfolio can initially withdraw 4% of their retirement savings account, and continue drawing this amount adjusted for inflation, with very high likelihood that the account will last for at least 30 years.

[47] See Scott et al. (2009).

[48] For an Australian example, see De Ravin et al. (2019). This strategy assumes spending your decennial age and adjusts for the influence of the retirement savings account on Age Pension receipts.

[49] The choice between these options will depend on member circumstances and preferences. Further discussion and the analysis in this Primer avoid this issue by assuming sufficient funds to purchase an income stream that guarantees the income floor.

2.2.3 Withdrawal Strategy for an Income Target

The discussion here assumes some need for withdrawals from the retirement savings account, and that life annuities are not used to fully meet this objective. In many situations, the 'draw-to-target' strategy will be appropriate, i.e., withdraw the amount required to meet the income target, after allowing for other income sources. This strategy is applied until the retirement savings account is exhausted, noting that the probability of enjoying income decreases over time due to mortality risk. Intuitively, this is a 'bird in the hand' concept–any income shortfall currently experienced is known, whereas income held back for later might never be enjoyed if death occurs.

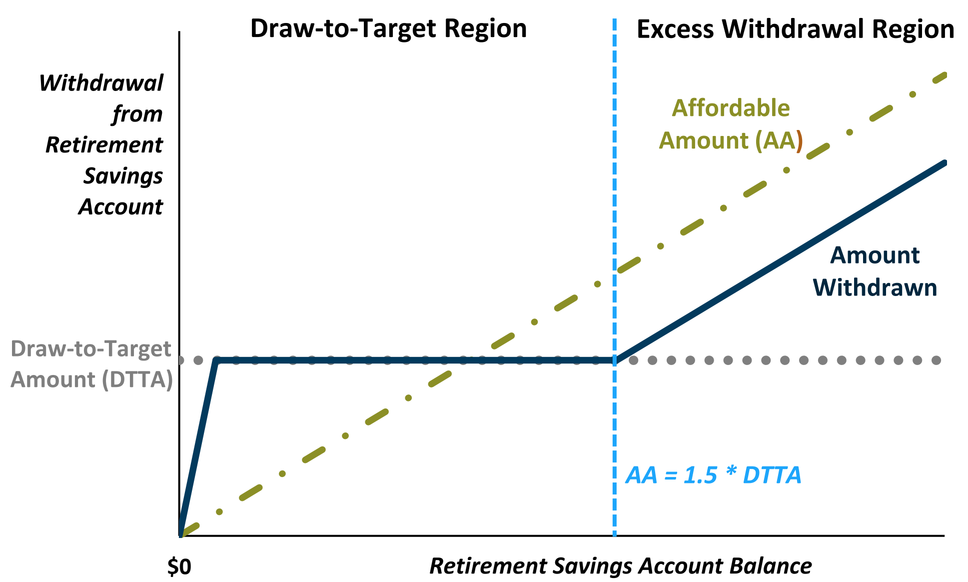
Complicating the draw-to-target strategy is the possibility that withdrawals in excess of the income target might be made where available assets are more than sufficient to meet the target for life: a possibility that should be incorporated into the strategy in some way. With the literature giving no clear guidance, a withdrawal rule is formulated for use in this Primer based on specifying a threshold above which an excess withdrawal amount can safely be taken. The method involves establishing an 'affordable' withdrawal amount, which is estimated in the same way as under the income optimization objective as outlined below in Section 2.3.3. A threshold for excess withdrawal is then set where the affordable withdrawal exceeds a multiple of the draw-to-target amount: testing suggests a *1.5x* multiple. The excess withdrawal is added to the draw-to-target amount. The rule is summarized in Table 5 and plotted in Figure 2.

Table 5
WITHDRAWAL RULE: DRAW-TO-TARGET WITH PROVISION FOR EXCESS

Region	Indicated By	Withdrawal Amount
Draw-to-target	*AA ≤ 1.5 * DTTA*	*DTTA*
Excess withdrawal	*AA > 1.5 * DTTA*	*DTTA + (AA – 1.5 * DTTA)*

DTTA is the draw-to-target amount. AA is the affordable amount of withdrawal.

Figure 2
DRAW-TO-TARGET RULE WITH POTENTIAL FOR EXCESS WITHDRAWALS



2.2.4 Withdrawal Strategy for Income Optimization

An income optimization objective does not provide a natural baseline for choosing a withdrawal strategy. Nevertheless, a dynamic strategy is likely to be best under this objective, given that government mandates and constant withdrawal amounts are likely to be sub-optimal and draw-to-target is inapplicable. Noting that many dynamic withdrawal strategies exist,[50] a representative strategy is formulated for the purpose of the modelling undertaken in this Primer. The strategy estimates an 'affordable' withdrawal amount akin to an annuitization of the retirement savings account at each age. The result is a withdrawal strategy where income adjusts with age[51] and account balance, and thus implicitly accounts for realized investment returns. The strategy accommodates updates to expected returns (through varying the AIR) due to any adjustment to the portfolio mix or shifts in the returns being offered by markets, although this Primer does not delve into this area.

The strategy uses the basic structure recommended by Waring and Siegel (2015) as a starting point. Annuitization each year is conducted assuming a time horizon[52] equal to halfway between the life expectancy[53] and the number of years remaining until age 110.[54] An AIR is needed for the annuitization calculation and is assumed to equal the expected return on the retirement savings account less one percent, subject to a minimum of the risk-free rate.[55] Referring to expected return ensures withdrawal amounts increase, but become more volatile as the proportion invested in the growth portfolio increases. The subtraction of one percent provides some protection to the withdrawal amount against significant downturns due to investment experience.[56] Table 6 summarizes the affordable withdrawal rule by reporting the withdrawal rate as a percentage of account balance at age 65 through to age 105 in 5-year increments for a range of AIRs.

Table 6

WITHDRAWAL RATES UNDER THE 'AFFORDABLE' WITHDRAWAL RULE

AIR	0%	1%	2%	3%	4%	5%	6%	7%	8%
Age 65	2.9%	3.5%	4.0%	4.6%	5.2%	5.9%	6.6%	7.3%	8.0%
Age 70	3.4%	3.9%	4.4%	5.0%	5.6%	6.3%	6.9%	7.6%	8.3%
Age 75	4.0%	4.5%	5.0%	5.6%	6.2%	6.8%	7.4%	8.0%	8.7%
Age 80	4.9%	5.4%	5.9%	6.4%	7.0%	7.5%	8.1%	8.7%	9.3%
Age 85	6.1%	6.6%	7.1%	7.6%	8.1%	8.6%	9.2%	9.7%	10.3%
Age 90	7.8%	8.3%	8.8%	9.3%	9.8%	10.3%	10.8%	11.3%	11.8%
Age 95	10.6%	11.0%	11.5%	11.9%	12.4%	12.9%	13.3%	13.8%	14.3%
Age 100	15.5%	16.0%	16.4%	16.8%	17.2%	17.7%	18.1%	18.5%	19.0%
Age 105	28.0%	28.3%	28.7%	29.0%	29.4%	29.7%	30.1%	30.4%	30.8%

[50] For instance, see Blanchett, Kowara and Chen (2012), Macdonald et al. (2013), Waring and Siegel (2015), and De Ravin et al. (2019).

[51] Adjusting the proportion of the account to be withdrawn at each age is desirable, as a strategy that adjusts in 5-10-year age increments risks having significant lumps in withdrawal amounts at the ages of adjustment. That said, withdrawal amounts can still be quite volatile depending on the asset allocation chosen.

[52] Waring and Siegel (2015) do not explicitly use mortality rates in their annuitization calculations. However, their time horizon adjusts as the retiree ages, reflecting increasing age of life expectancy with survival. Examples using mortality rates can be found in MacDonald et al. (2013).

[53] For simplicity we ignore spousal reversion and treat modelling on an individual basis.

[54] For example, for a 65-year-old with a life expectancy of 20 years, the annuitization is conducted assuming a time horizon of 32.5 years. Waring and Siegel (2015) assume a maximum age of 120 rather than 110.

[55] Waring and Siegel (2015) assume an inflation-protected risk-free rate irrespective of the underlying asset risk profile, in an approach similar to Liability-Driven Investing (see Section 6.1.2). This provides a very conservative withdrawal amount in the early years of retirement.

[56] As discussed in Waring and Siegel (2015), the time horizon is largely chosen to provide a desired withdrawal profile. The same argument can be made for how, and if, to include mortality rates, and the choice of AIR. The withdrawal profile from the strategy used in this Primer can be seen in the illustrations in Section 5.

SECTION 3: Modelling Retirement Outcomes

The design and evaluation of retirement strategies requires modelling the distribution of potential outcomes that they may generate. Stochastic models should be used rather than deterministic models to capture outcome variability, which is necessary to support analysis of income risk and related trade-offs. Stochastic models perform this function by linking the variable nature of drivers such as the economic and market environment to retirement outcomes. Section 3.1 outlines the main components of stochastic models, while Section 3.2 describes the main choices around model structure and Section 3.3 recognizes model reliability and the potential importance of 'model risk'.

3.1 Stochastic Models

The main components included within stochastic models are discussed below with the goal of highlighting the major areas of consideration and the breadth of choice available, rather than give specific guidance on how a stochastic model should be structured. The broad range of potential components that might be included or excluded gives rise to trade-offs between model simplicity and potential insight.

- Retiree attributes – Table 2 in Section 1.4 identifies the key attributes as age, financial assets, homeownership, access to other guaranteed income streams and partnered status; other attributes are also listed. There is considerable scope to choose which attributes to include in a model, and how those attributes are defined. Age, available financial assets and access to guaranteed income streams are fundamental to income generation and needs, and hence are critical inclusions. Nevertheless, even these attributes may need to be excluded due to lack of information. For instance, the strategy designer may need to restrict financial assets to the retirement savings account, omit homeownership or ignore partnered status if these attributes are unknown.

- Objectives and risk appetite – These components were discussed in Sections 1.1 - 1.3 and are needed to evaluate the outcomes arising from the model. Where the model is being used to compare candidate strategies or identify 'optimal' investment and/or withdrawal strategies, an objective function (e.g., expected utility) will need to be built into the model to facilitate the calculations (see Section 4.2 for further discussion).

- Environmental factors – Elements of the environment that have a meaningful impact on income and assets need to be scoped and included. A key element is the investment building blocks that are modelled. Section 2.1 notes the wide palette available, although the focus might be those that are readily available and would be reasonably considered by a retiree. Other factors include any rules governing social security, taxation and minimum withdrawals.

- Modelling investment payoffs, i.e., returns or income streams – Methods for simulating returns on any directly invested retirement savings are discussed in Section 3.2. For purchased income streams such as annuities, either rules are required for determining product prices and income, or these inputs could be obtained from the market.

- Investment strategy – How assets are to be allocated amongst the available building blocks needs to be determined. This entails not only allocating available assets at retirement, but also whether to allow for dynamic shifts in allocations over time.

- Withdrawal strategy – The withdrawal strategy governs how the retirement savings account is utilized to generate income. The model thus needs to determine how much income is drawn in any period, either expressed as predetermined rules or optimization criteria. Section 2.2 discusses withdrawal strategies.

- **Mathematical relationships between the variables** – These should be clearly defined and coded, e.g., the evolution of asset values such as the retirement savings account balance over time; how inflation impacts on other quantities; etc. Allowance should be made for any time or state dependencies between periods, e.g., autoregressive structures; time-varying income targets.

Outcomes from a stochastic model might include income and any remaining assets, for instance in the retirement savings account. The process usually involves projecting or 'simulating' a series of outcome paths on a period-by-period basis until the end of a projection horizon.[57] These paths are then used to estimate utility and/or metrics (see Section 4). Figure 3 portrays such a distribution of outcomes.

Figure 3

DISTRIBUTION OF OUTCOMES GENERATED BY STOCHASTIC MODELS

Period	*Retirement* 0	1	2	3	…	…	*Projection Horizon* T
Path		***Distribution of outcomes O(path,period) from deploying financial resources***					
1	***Financial resources at retirement***	*O(1,1)*	*O(1,2)*	*O(1,3)*	…	…	*O(1,T)*
2		*O(2,1)*	*O(2,2)*	*O(2,3)*	…	…	*O(2,T)*
3		*O(3,1)*	*O(3,2)*	*O(3,3)*	…	…	*O(3,T)*
.		.	.	.			.
.	*(Available assets plus any guaranteed income streams)*	.	.	.			.
.		.	.	.			.
i		*O(i,1)*	*O(i,2)*	*O(i,3)*	…	…	*O(i,T)*
.		.	.	.			.
.		.	.	.			.
.		.	.	.			.
n		*O(n,1)*	*O(n,2)*	*O(n,3)*	…	…	*O(n,T)*

Outcomes evaluated through metrics and utility scores (see Section 4)

Note: 'Outcomes' include income and any remaining assets in (say) the retirement savings account.

3.2 Notable Model Structuring Choices

3.2.1 Modelling Frequency

Modelling frequency defines how often investment and withdrawal decisions are made, as well as the length of time represented by a single simulated stochastic variable. Annual frequency is often used and implies that periodic decisions and simulation of any stochastic variables (e.g., investment returns) are made and outcomes are generated on a yearly basis. Modelling also needs to determine *within period* application of decisions. For example, withdrawals may be made at the start of the period (the case for the examples presented in this Primer), the middle of the period, or the end of the period.

[57] Some very simple stochastic models can be evaluated analytically without the need for simulation; these are not considered in this Primer.

3.2.2 Nominal versus Real

It is appropriate to express retirement outcomes in real terms (i.e., in today's dollars) to be interpretable, given the long horizons involved. Nevertheless, a stochastic model can be set up to either perform calculations entirely in real terms, or in nominal terms with the resulting outcomes then being deflated. Modelling in real terms implicitly embeds the effect of inflation into all model variables and generates outputs in today's dollars that can be easily interpreted. However, the treatment of inflation-related effects become more difficult to incorporate when all components do not inflate in line with a common inflation rate. In some instances, modelling may still be done in real terms by adjusting real growth rates to capture differences in rates of inflation across components. For instance, variables that are tied to wage inflation may be modelled as growing in line with real wages growth. Modelling in real terms may become infeasible where key components are expressed in nominal terms, e.g., annuities offering a nominal income stream. In this case, modelling in nominal terms becomes necessary.[58] The resulting outcomes would then need to be deflated by the general inflation rate within each path before any evaluation tools are applied (see Section 4). Inflation might be modelled on a deterministic basis or treated as stochastic in which case it would be simulated for each path. Treating inflation as stochastic supports the analysis of inflation risk.

3.2.3 Projection Horizon

Uncertainty over time of death raises the issue of the horizon over which analysis is conducted. Two approaches are projecting over a fixed planning horizon, or projecting to an age where there is virtually no chance of survival and applying survival probabilities to outcomes. Each has advantages and disadvantages.

Fixed Planning Horizon
Analysis is conducted as if the retiree expects to live to a particular age. Outcomes at each age are typically equally weighted in estimating utility and any metrics that summarize outcomes over the entire horizon. A common choice is modelling to age 90-95, which is longer than the typical life expectancy at retirement. For instance, a 30-year horizon implies analyzing through to age 95 for a retirement age of 65.

A fixed planning horizon is straightforward to apply but gives limited consideration to the retiree surviving beyond the horizon. Further, the choice of horizon, while somewhat arbitrary, is influential in how strategies are treated in terms of exposure to longevity risk. Shorter horizons tend to favor strategies that may undervalue protection against longevity risk, leaving the retiree exposed to income risk upon living to an old age. Conversely, longer horizons favor strategies that might over-protect against longevity risk, say by restricting withdrawals, thus raising the likelihood of failing to convert available assets into income. Second, the choice of horizon may impact how lifetime income streams are viewed. For instance, a short horizon might attach insufficient value to the protection that lifetime income streams provide against living beyond the planning horizon, whilst a long horizon might overvalue the potential benefits provided by lifetime income streams by failing to allow for the possibility of death before the end of the horizon.

Survival Probabilities
This approach entails projecting to an age with a vanishing chance of survival, e.g., 110. Weighting by survival probability is then applied in estimating utility and metrics that summarize outcomes over the entire horizon. Survival probabilities, and hence life expectancies, may be based on population statistics, or adjusted for retiree health conditions and/or assumed retiree expectations. Meanwhile, metrics at selected

[58] This would include inflating any real payment streams by the rate of inflation, e.g., inflation-protected annuities.

ages need to be interpreted as reflecting the outcomes that may be delivered ***if*** the retiree happens to survive to that age.

The survival probabilities approach directly recognizes the stochastic nature of mortality in a coherent manner. It supports trading-off outcomes spanning the entire potential lifespan based on their likelihood of occurring. Consequently, survival probabilities tend to favor strategies that maximize outcomes over life expectancy, e.g., utility scores will be weighted towards expected age of survival.

Depending on how the survival probabilities are set, this approach may give insufficient credence to the fears of some retirees of outliving their assets, and their desire to hedge longevity risk. (Each retiree travels their own path over retirement and need not die in line with their life expectancy.) This shortcoming may be partly overcome by examining metrics that convey the range of potential outcomes upon survival to various ages to reveal what may happen to income upon survival to older ages. If the level of protection against lower income is deemed inadequate, an alternative strategy might be considered even though it delivers lower expected utility. Survival weighting is also more complex and generates outputs that are a weighted rather than simple average and hence can be harder to interpret.

3.2.4 Methods of Simulating Asset Returns

A method is required to characterize how asset returns (and possibly inflation) fluctuate over time. The method should ideally form 'joint' return observations that embed any asset correlations (and any autocorrelations). It should also reflect expected returns that are available in the market at the time, rather than average historical returns. For example, if historical fixed income returns differ significantly to prevailing interest rates, then simulating future investment returns with reference to unadjusted historical data is questionable. Some methods are briefly outlined below. The choice of method is outside the scope of this Primer but will depend on the purpose of the modelling as well as the data and capabilities available to the modeler.

- **Bootstrap simulations from historical return data** – A series of observations is randomly drawn from historical return data, and these observations can be used to generate potential future return paths. Drawing joint return observations ensures that asset correlations are embedded, although may not adequately account for autocorrelation.[59] If the mean returns reflected in the data are inconsistent with expected returns available in the market, then the geometric mean[60] of the series might be adjusted to realign the implicit expected returns. This technique adjusts the expected returns while retaining the variation around the mean to capture investment risk.

- **Historical experience** – In some situations, it might be helpful to use historical data to create the hypothetical experience that might have occurred in the past under a strategy.[61] For instance, 100 years of returns may be used to create 71 overlapping 30-year periods over which the strategy would be simulated. While this method embeds historical asset correlations and autocorrelations, it does not readily lend itself to aligning expected returns with prevailing market conditions.

[59] Autocorrelation can be captured through using block bootstraps that draw blocks of data spanning multiple periods.
[60] Adjustment is applied to the *ln(1+return)* series, with a view to calibrating the compound return embedded in the time series. Butt, Khemka and Warren (2019) provide an example of mean-adjustment.
[61] Estrada (2013, 2014) uses this method.

- **Simulation from statistical distributions** – Asset returns can be drawn from a joint distribution (e.g., mean and variance/covariance), and can allow for autocorrelation or parameter uncertainty (e.g., stochastic mean). Lognormal return distributions are widely used for this type of analysis.

- **Structural models** – Asset returns can be generated via imposing a structure that links asset prices and income to variation in their underlying drivers. One class is *valuation-based models* where return paths are generated by varying inputs into valuation models, e.g., variations in cash flows and discount rates.[62] Another class is *cascade models* where changes in macroeconomic drivers such as economic activity and inflation cascade down through a tiered structure into estimates for cash flows and interest rates and hence asset prices and returns.[63]

- **Scenario analysis** – Asset return paths may be formed by proposing a range of future scenarios. Scenarios might reflect any dimension deemed relevant, including economic states, market states, political outcomes, themes, etc. Scenarios are typically assigned a probability of occurrence.[64]

3.3 Model Reliability

Quantitative models are useful tools, but their output should not be taken as a 'source of truth'–although this is not unique to stochastic models. Any model is reliant on subjective modelling choices, including the objective function, assumptions, inputs and modelling method. All models are subject to 'model risk', which is the possibility that the model may be incomplete or incorrect, and hence provide results with propensity to mislead. Some relevant factors may not be incorporated within a model, and thus may need to be considered in addition to model output. For instance, flexible access to funds can be particularly important to some retirees but is difficult to incorporate within a model that is designed to generate and evaluate income streams. Other factors may include behavioral influences on retiree preferences (discussed in Section 6.3) and business considerations (discussed in Section 6.4).

Approaches to limit exposure to model risk, and ensure that more robust decisions are made, include:

- Use the model output as a starting point then apply judgement, rather than rely solely on model-based recommendations. This approach is briefly illustrated in Section 5.4.

- Apply multiple models or decision methods, with a view to ensuring that decisions are robust when analyzed under different frameworks.

- Undertake scenario and sensitivity analysis to identify critical variables and understand the extent to which the decision might vary with assumptions about those variables.

- Communicate the key assumptions used in the model to avoid misunderstanding.

[62] Warren (2021a) illustrates the use of valuation-based models.
[63] An example is the model of Wilkie (1984) and extensions as used in actuarial practice.
[64] Gosling (2010) outlines the process of building a scenario set then using it to form portfolios.

SECTION 4: Strategy Evaluation – Utility and Metrics

This section outlines how the outcomes from stochastic modelling of retirement income strategies may be evaluated to gauge the extent to which objectives are being achieved. It discusses two approaches of applying ***utility functions*** to generate a single 'score' as a type of summary statistic and using ***metrics*** to convey aspects of the distribution of outcomes. Key concepts are outlined in Section 4.1, utility functions discussed in Section 4.2, metrics addressed in Section 4.3, and illustrations provided in Section 4.4. Technical details including formulae appear in Appendix B for utility functions and Appendix C for metrics.

4.1 Key Concepts

The overarching theme of this section is that utility functions are effective for strategy selection while metrics can be used to characterize outcomes. Nevertheless, both should be used in tandem.

4.1.1 Two Purposes: Selecting Strategies and Characterizing Outcomes

There are two potential purposes in evaluating retirement income strategies:

(a) **Strategy selection** – Selecting a preferred strategy requires a method for either identifying the 'optimal' strategy, or (more likely) comparing candidate strategies. Any method should address the trade-offs discussed in Section 1.3, especially between expected income and income risk. Strategy selection is addressed in Section 5.

(b) **Characterizing outcomes** – The outcomes that a strategy delivers need to be understood and communicated. Aspects of interest might include the potential level and shape of income, the risk of lower income, the likelihood of the retirement savings account being exhausted, and the remaining account balance available to support either a bequest or any large spending needs.

The criteria for strategy selection and how outcomes are characterized should ***vary with objective***: the focus here is the income target and income optimization objectives as discussed in Section 1.1. A ***toolkit*** of measures is typically required.

4.1.2 Utility, Metrics, or Both?

Outcomes arising from a retirement income strategy may be evaluated using utility functions and metrics:

- **Utility functions** might be seen as 'scoring systems' that place a value on *all* potential outcomes over time, which can then be aggregated into 'expected utility' that amounts to an overall score or summary metric. A variety of utility functions exist, with constant relative risk aversion (i.e., power) utility and the value function from prospect theory perhaps the most notable.[65] Section 4.2 discusses the use of utility functions, focusing on these two functions.

- **Metrics** are statistics that summarize aspects of the distribution of outcomes. Metrics can describe the income that might be expected, and exposure to income risk through reporting measures of the likelihood and magnitude of income below that expected or targeted. Example metrics include measures such as expected values, percentiles, expected shortfall, and the age when the

[65] Warren (2019) discusses the use of utility functions for portfolio formation in some depth.

retirement savings account might become exhausted. Metrics may be presented as numbers in a table or graphical form, such as plots of income over time.

There is a case for using utility functions and metrics in tandem, noting both have advantages and disadvantages. Utility can act as the 'engine' for strategy selection through providing an overarching score, while metrics operate as a 'dashboard' to characterize the potential outcomes.[66] While using a utility function to distill the entire distribution of outcomes into a single score is a strength for strategy selection, a single score conveys minimal information about the outcomes that a strategy might deliver. Further, selecting a utility function is difficult, and parameterization is tricky. For many people, utility is a mysterious black box that seems overly technical. Meanwhile, metrics can convey the nature of outcomes and tend to be more familiar, but also raise some issues. Metrics need to be chosen from a wide range of possible options. Their effectiveness for selecting strategies is hampered by the complexity of retirement outcomes that are spread over multiple periods. No single metric comprehensively summarizes all outcomes in the same way that utility functions can, and how various metrics might be traded-off or weighted is typically unclear.[67] Metrics have particular difficulty in addressing trade-offs that span time, including the possibility that outcomes occurring sooner in retirement may be of higher value due to either the probability of survival decreasing with age and/or time preference. While utility functions are often seen as too subjective, this charge can equally be directed at the application of metrics.

4.1.3 Capturing Likelihood *and* Magnitude

Evaluation of retirement outcomes should consider both the *likelihood* <u>and</u> *potential magnitude* of any shortfall versus some reference point, which might be expected or target income. It matters not just *whether* a retiree fails to achieve their aspirations, but also by *how much*. Utility functions naturally consider both likelihood and magnitude of failure (and success) through scoring all potential outcomes. Metrics should also be designed to span both dimensions. For example, as well as reporting the probability of falling below an income target, it is useful to convey how severe the deficit might be. The probability of shortfall might thus be coupled with metrics that provide information about the lower tail of the income distribution, including the minimum income that occurs if the retirement savings account is exhausted.

A fundamental reason for considering magnitude as well as the likelihood of shortfall relates to the nature of investing over long time periods, as discussed in Appendix A.1. Risky assets offering higher compound returns deliver an increasing probability of outperforming more defensive assets as investment horizon lengthens. However, the potential magnitude of losses also gets increasingly larger. This sets up a trade-off between lower likelihood but greater potential magnitude of shortfall. Presenting metrics to conveying the possible degree of income loss as well as its probability helps reveal this trade-off.

4.2 Utility Functions

This subsection describes use of utility function for evaluating retirement income in general terms, with Appendix B providing further technical details including formulae and alternative utility functions.[68]

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[66] Warren (2022) sets out how utility functions can be used in designing of retirement strategies while communicating outcomes with metrics.
[67] While it may be possible to combine metrics into a single score, it is debatable whether this approach is more effective. This matter is discussed in Section 5.1.
[68] Warren (2019) details the selection and parameterization of utility functions and their use for portfolio construction.

4.2.1 Applying Utility Functions

Utility functions provide a method for assigning an overall score to a distribution of outcomes, taken here to comprise a series of income 'paths' across time arising from stochastic modelling (see Section 3). Utility is essentially a ranking tool, and it is the *relative values* that utility functions place on various outcomes that matter. The discussion below hence focuses on the overall shape of the function and the weights assigned to differing values.

The main implementation challenges include selecting a utility function, choosing the parameters, and deciding the approach to time weighting. In many situations, a simple 'adding up' of utility scores across time is feasible, with the steps involved outlined in Appendix B. A simple adding up across time is not sufficient in some situations, such as when applying dynamic optimization or under certain utility functions that require recursive estimation.[69] This Primer skirts over these complex cases.[70]

Utility-based analysis can assist with strategy selection in two ways. First is locating the 'optimal' strategy as the one offering the greatest expected utility. Second is comparing candidate strategies based on the relative value of their overall utility scores. In the latter instance, utility can be used both to rank strategies and to gauge the utility loss in pursuing one strategy over another, e.g., the utility loss versus a baseline strategy, or 'optimal' strategy. The ability to compare candidate strategies can be particularly useful when the financially optimal strategy may not be the most appropriate due to considerations such as a desire to avoid complexity or implementation difficulties. These matters are discussed in more detail in Section 5.

Application of utility functions under each of the income objectives identified in Section 1.1 is now discussed.

4.2.2 Income Floor Objective

The income floor objective can be tricky to directly include in utility-based analysis, as utility is conceptually 'undefined' below the floor. The income floor is better addressed by assuming the initial purchase of a lifetime income stream (e.g., annuity) that secures the floor if possible, then directing the utility analysis towards strategies applied to the remaining assets. This approach is adopted in this Primer.[71]

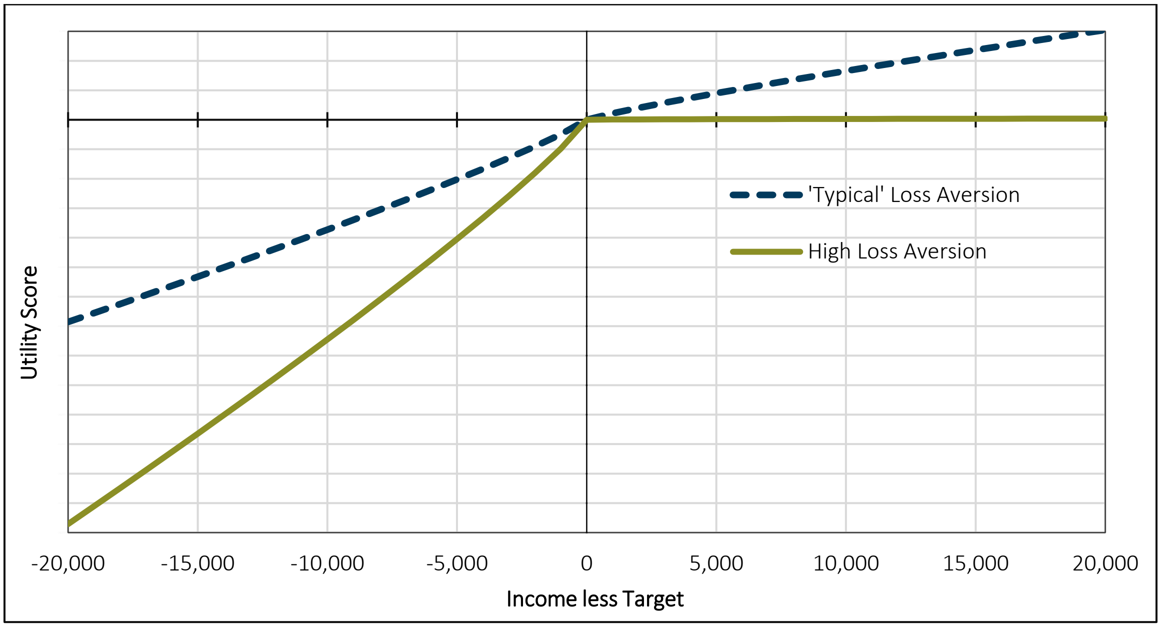
4.2.3 Income Target Objective

The income target objective naturally fits with reference-dependent or loss aversion utility functions, where the income target provides the reference point delineating gains and losses. The 'value function' from prospect theory[72] is most widely used. This function generates positive scores above the target and

[69] Such complications can be avoided provided that dynamic effects can be handled in a simplified manner without much loss of insight, e.g., it suffices to reduce dynamic responses to pre-determined rules that are established at the beginning of the analysis period. See Warren (2019, pp. 64-65) for a discussion of static versus dynamic analysis.
[70] Butt and Khemka (2015) provide an overview and example of the application of dynamic programming to retirement problems.
[71] Allowance for potentially falling below the income floor can be accounted for within a utility function through imposing an outsized penalty for breaching the floor, with the effect that staying above the floor is addressed as a first priority for the purpose of strategy selection. Assigning a very large negative utility score to any income below the floor ensures that utility remains defined while imposing a dominating penalty that induces a preference for any strategy that minimizes the probability of income falling below the floor. The ranking function is retained; although a consistent utility function is no longer being applied, with the implication being that certainty equivalents can no longer be estimated if the floor is breached in any situation.
[72] Prospect theory (see Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) includes other elements that make it a behavioral theory, rather than based around expected utility. Specifically, prospect theory allows for distortion of probabilities that alter how gambles are evaluated, and the initial editing of the prospects to be considered.

negative scores below the target. Figure 4 plots two loss aversion utility functions, revealing the hallmark kink at the reference point. (Utility scores are meaningless in isolation and hence not shown in the charts.)

Figure 4
LOSS AVERSION UTILITY FUNCTION



In Figure 4, the 'typical' loss aversion function reflects the parameters of Tversky and Kahneman (1992).[73] This function might be suitable for a retiree with a clear preference to avoid income falling below the target but is willing to accept some income risk to pursue higher-than-target income. The high loss aversion function[74] might be suitable for a retiree who places minimal value on above-target income, which they might seek only if there was a very small probability of income falling below the target. Such a retiree would have a high propensity to draw the target income.

4.2.4 Income Optimization Objective

A natural pairing with the income optimization objective is what may be called 'risk aversion' utility.[75] This function effectively trades off the prospect of higher overall expected income against the risk that income could fall towards lower levels. Figure 5 plots the risk aversion utility function for both a 'typical' and high risk aversion.[76] (Again, the utility scores are meaningless in isolation and hence not shown in the charts.) The chart indicates how variation around a notional expected income would be scored, where expected income is represented by 0% on the x-axis.

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[73] The parameters include curvature coefficients of 0.88 on both gains and losses, and a loss aversion coefficient of 2.25. See Appendix B.3 for the mathematical formulation. Subsequent experiments by researchers confirm that the Tversky and Kahneman (1992) estimates are broadly confirmed for average individuals, e.g., see Brown et al. (2021). However, these studies also reveal considerable variation across individuals.
[74] These parameters follow Blake, Wright and Zhang (2013), who halve the curvature coefficient on gains to 0.44, retain the same curvature function on losses of 0.88, and double the loss aversion coefficient to 4.5, relative to Tversky and Kahneman (1992). See Appendix B.3 for the mathematical formulation.
[75] The function being used here is the constant relative risk aversion (CRRA) or 'power' utility function.
[76] The coefficient of relative risk aversion is two for 'typical' and five for high. For example, Alserda et al. (2019) estimate an average risk aversion coefficient of 1.926 in a pensions context. See Appendix B.4 for the mathematical formulation.

Figure 5
RISK AVERSION UTILITY FUNCTION

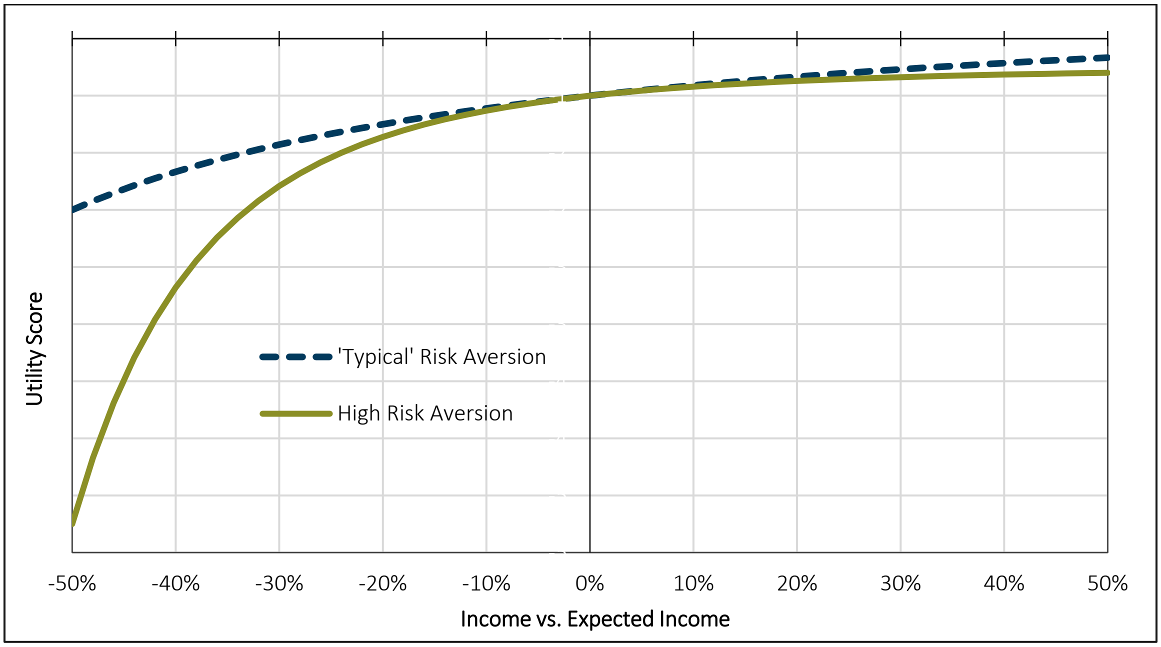

Figure 5 illustrates the hallmark of risk aversion utility that the scores tend to drop away at an increasing rate[77] as income declines to lower levels: a tendency that becomes more marked as risk aversion increases. This type of utility function might be suitable for a retiree who prefers higher overall income, with their level of risk aversion determining their willingness to take the risk that income falls to lower levels in pursuit of higher expected income. A retiree with high risk aversion will have a stronger dislike of income decreasing below expected levels, and hence is likely to have a preference for smoother income over time.

4.2.5 Time Weighting – Survival Probabilities Risk and Time Preference

Utility scores from each period need to be added up in determining the total utility from an income path. Time weighting may be incorporated in two ways. First is weighting by the probability of survival and thus enjoying the income at each age. Second is time preference, which applies a discount rate to recognize that a retiree may prefer income earlier rather than later (if alive). The stance taken is that survival probabilities but not time preference should be applied when calculating utility. Section 3.2.3 notes that whether to apply time weighting or a fixed planning horizon is debatable. Nevertheless, applying survival probabilities better captures the trade-offs related to uncertainty around longevity under utility analysis. Meanwhile, time preference relates to personal preferences. Some retirees may place higher value on a dollar of income enjoyed sooner rather than later, while others may be as concerned with having income available later in retirement *if* they happen to survive.[78] Hence, the inclusion of time preferences might be restricted to situations where there is a clear case. It is worth noting that applying both survival probabilities and time preference acts to heavily discount outcomes later in retirement and may not give adequate recognition to

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[77] This arises due to the function applying an exponent to the outcomes being evaluated, hence the name power utility.
[78] In a similar vein, it might be argued that plan providers have an obligation to provide consistent retirement income while a retiree remains alive, in which case allowing for time preference might be questioned as it induces a bias towards delivering income earlier rather than later.

retiree concerns of being unable to sustain income upon surviving to older ages. Appendix B.4 discusses time weighting in further depth, including illustrating how the weightings can change with age.

4.2.6 Parameterization

There is a lack of solid guidance from the literature on parameterizing utility functions for the purpose of evaluating retirement income streams. The research is rarely done in a retirement income context, and studies deliver a wide range of parameter estimates that vary across individuals and methods. Experimental studies asking individuals (often students) to evaluate gambles tend to come up with estimates reflecting the 'typical' parameters presented in Figure 4 and Figure 5 above. On the other hand, higher loss aversion or risk aversion is generally needed to explain observed behaviors such as investment or insurance choices. There are also reasons to suspect that many retirees may have low appetite for risk in a setting where retirement income is supporting living expenses; and surveys suggest that retirees are averse to taking risk. The recommended solution is to cover the bases by *choosing parameters that span low and high risk appetite*, such as the parameter ranges that underpin Figure 4 and Figure 5.

4.2.7 Certainty Equivalents

Raw expected utility scores are uninterpretable. However, they can be better understood by conversion into 'certainty equivalents', which is the known amount that generates the same utility as a risky bet. In a retirement income context, the natural choice is to calculate the certain income stream that yields the same expected utility as the stochastic income distribution, denoted as risk-adjusted income (*RAI)* in this Primer. *RAI* will typically fall below the expected income delivered by a strategy, with the gap reflecting the degree of income risk and assumed risk aversion or loss aversion. For the income optimization objective, the estimation of *RAI* is straightforward. For an income target objective, complications emerge where the income target varies over time. To account for this possibility, a constant risk-adjusted difference from target (*CRADT*) may be estimated as constant difference between income and the target that generates the same utility as the stochastic income stream. If the income target is constant, then *CRADT* may be added to the target to generate a *RAI*. Appendix B.2 and B.3 sets out how these certainty equivalents are calculated.

4.3 Metrics

This section describes how metrics that characterize outcomes arising from retirement income strategies might be used and described to retirees, focusing on primary metrics for the three income objectives outlined in Section 1.1. Appendix C identifies a wider range of metrics and provides detail on calculations.

4.3.1 Classes of Metrics

Most metrics fit into one of five classes:

1. **Distribution of income** – These metrics characterize the expected level and variability of income over time. They are more relevant to an income optimization objective and may be summarized across all ages or presented at specific ages (assuming survival until that age).

2. **Likelihood of income shortfall** – These metrics convey the likelihood of failing to attain an income objective. They are more relevant to an income floor and income target objective and may be summarized across all ages or presented at specific ages.

3. **Magnitude of income shortfall** – These metrics convey the potential extent of any failure to attain an income objective, including portraying how low income might fall. They are more relevant to an

income floor and income target objective and may be summarized across all ages or presented at specific ages.

4. **Remaining assets** – These metrics reveal the remaining assets at various ages. Ideally this should reflect all available financial assets, although the retirement savings account is often the focus. These metrics are relevant irrespective of income objective.

5. **Utility-based** – Expected utility can be used as an overarching metric that summarizes the value of entire distribution of outcomes and may be expressed as a certainty equivalent. Section 4.2 and Appendix B discuss utility functions under the income target and income optimization objectives.

4.3.2 Primary Metric for an Income Floor Objective

Given the overarching concern under this objective is to avoid income falling below the floor, a 'likelihood' measure such as a **probability of shortfall** versus the floor may provide a sufficient metric. If the probability of failing to attain the income floor meaningfully exceeds zero, the strategy may require reformulation.

Example description for retirees: *The estimated probability of income falling below the $FF,FFF floor at any time during retirement is x%.*[79]

4.3.3. Primary Metrics for an Income Target Objective

The central concern under a target income objective is how long the target is sustained. This is often synonymous with exhaustion of the retirement savings account, most notably under a withdrawal strategy where sufficient income is drawn to achieve the target until assets run out (see Section 2.2.3). A further concern is how far income could fall once the retirement savings account is exhausted, after which income typically drops to the level underpinned by guaranteed income (i.e., social security, defined benefit pensions, etc.) and any purchased income streams. This situation may be adequately captured through the following set of metrics:

- **Expected age of account exhaustion –** This statistic summarizes the typical age to which the retiree can expect their retirement savings account to be exhausted, and by implication the age at which the target is no longer sustainable and income falls to the minimum income. It needs to be calculated as a simple average over the projection horizon, noting that weighting by survival probabilities if projecting to an unlikely survival age produces inconsistent estimates.[80]

- **Probability of shortfall at various ages** – This metric conveys information about the 'likelihood' of failing to sustain the income target as the retiree ages.

- **Minimum income** – This 'magnitude' metric describes the level to which income falls upon exhaustion of the retirement savings account and reflects the sum of any guaranteed income and purchased lifetime income streams. A single number suffices if the minimum is constant over time; otherwise, it may be better reported as minimum income at selected ages.

- **Constant risk-adjusted difference from target (*CRADT*)** – This 'utility-based' metric (outlined in Section 4.2.7 and Appendix B.2) is a type of certainty equivalent. It is more likely to be a negative

[79] This description may be reformulated to highlight probability of shortfall at various ages where more appropriate.
[80] Under survival probabilities, sustaining the balance to older ages acts to reduce the expected age of exhaustion as older ages are attributed lower probability. An alternative summary metric might be expected years of shortfall or expected shortfall, but these tend to be less intuitive.

number on the basis that income should either be in line or below target. Under a constant target, adding *CRADT* to the target to generate a *RAI* may aid interpretability. This metric would be reported to assist strategy selection,[81] but probably not presented to retirees to avoid confusion.

Example description for retirees: *The expected age to which you can expect your retirement savings account to last while drawing your targeted level of income of $TT,TTT in today's dollars is XX. However, returns on your retirement savings account will determine if you are able to sustain the targeted income for a shorter or longer period. As a guide, the estimated probability of sustaining the income target (and surviving) to age 75 is x% (a%), to age 85 is y% (b%), and to age 95 is z% (c%). If your retirement savings do happen to run out, then your income will fall to the level delivered by social security and the lifetime income stream that was purchased. This minimum income is estimated at $LL,LLL in today's dollars.*

4.3.4 Primary Metrics for an Income Optimization Objective

The income optimization objective implies a concern with both the overall level and variability of income. Two focal points might be expected income and how low income could fall if investment returns are poor, i.e., the lower part of the income distribution. Metrics that indicate potential for income variability may also be informative.[82] These aspects might be adequately captured by the following set of metrics, with reliance placed on income percentiles to convey the lower tail and variation in income over time:

- Expected income – This 'distribution' metric provides a summary statistic for the overall level of income that a strategy can be expected to deliver over the course of retirement. It could be calculated either as a simple average for a fixed planning horizon or weighted by survival probabilities if projecting through to an unlikely survival age (e.g., 110). One issue with expected income is that, as an average, it may be boosted by the tendency for positive skewness in the income distribution, and hence overstate the 'typical' income that might be anticipated. The alternative is to report overall ***median income***. However, the latter is made available if income percentiles are reported, as per the next point.

- Income percentiles at various ages – Reporting percentiles is an efficient way of presenting the distribution of income, including its spread over time. It might be reported as either deciles or selected percentiles such as 1st/5th/25th/50th/75th/95th/99th percentiles (or some subset). Percentiles might be presented as a matrix in a table format, or as graphs of percentiles over time.

- Minimum income – This metric describes the level to which income falls upon exhaustion of the retirement savings account and reflects the sum of any guaranteed income sources (e.g., social security, defined benefit pensions) and purchased income streams. A single number is required if the minimum is a constant value across time; otherwise, it may be better reported as minimum income at selected ages. It is a useful metric to report even though income may never reach the minimum under some withdrawal strategies, e.g., withdrawal strategies based on withdrawing a percentage of account balance rather than a fixed amount.

[81] If the deviation from target is significantly different from zero, consideration might be given to adjusting the target as it may be set either too low or too high relative to what is comfortably affordable.
[82] While this Primer does not recommend fixed withdrawals under the income optimization objective, if they were applied then metrics such as average age and probability of exhaustion of assets would become relevant.

- **Risk-adjusted income (*RAI*)** – This 'utility-based' metric (outlined in Section 4.2.7 and Appendix B.3) is a type of certainty equivalent. It would be reported to assist strategy selection, but probably not presented to retirees to avoid confusion.

Example description for retirees: *The income that you can expect averages $MM,MMM in today's dollars. This is just a general guide, and income in any year will depend on the amount that is intended to be drawn at each age under the withdrawal strategy, and any adjustments to income due to returns on your retirement savings account being different to expected. The (percentile) chart shows the possible range of income at various ages. As a guide to downside risk, there is an estimated 10% chance of generating income (and surviving) at age 75 of at least $PP,PPP (a%), at age 85 of at least $QQ,QQQ (b%), and to age 95 of at least $RR,RRR (c%). The bottom end of the income range is supported by income from social security and the lifetime income stream that was purchased and is estimated at $LL,LLL in today's dollars.*

4.3.5 Metrics for Remaining Assets

Even where the focus is retirement income, it is useful to report supplementary metrics on remaining assets for four reasons. First, it is useful to know the assets available for flexible access at each age, including to support either large expenditures or a bequest even if the strategy is not designed towards meeting these objectives. Second, when modelling over a fixed planning horizon, the assets at the end of the horizon is a useful indicator of capacity to generate future income if the retiree happens to survive beyond the horizon. Third, very high remaining assets can flag situations where a strategy is not efficiently converting available assets into income, and hence needs reviewing. **Median remaining balance at various ages** should be a sufficient metric where retirement income is the focus; although a wider range of metrics for remaining assets might be reported where other objectives such as bequest motives are important.

Example description for retirees: *If investment returns turn out as expected, you can anticipate the assets remaining (in your retirement savings account) in the order of $XX,XXX at age 75, $YY,YYY at age 85 and $ZZ,ZZZ at age 95. These amounts could support either subsequent future spending, or perhaps a bequest.*

4.4 Presentation of Metrics - Illustrative Examples

Metrics may be provided for internal use by strategy providers and to supplement communication with retirees using summary tables, charts and data visualization methods. This Primer presents traditional tables and charts as examples. Tables are used to report the primary metrics under each income objective, as discussed in Section 4.3. Charts are used to display how income and the retirement savings account develop over the course of retirement. Three useful types of charts are illustrated and include:

- Median[83] income over time, built up as 'income layers' to reveal sources of income.
- Income percentiles, to display the overall distribution and reveal the downside risk to income.
- The trajectory of the median retirement savings account balance.

Simplified examples are now presented to demonstrate the presentation and interpretation of the primary metrics proposed in Section 4.3 for both income target and income optimization objectives. Application of

[83] Medians are useful for illustrating general trends but need to be interpreted with care as they do not present a single path but rather the median outcome across multiple paths at each age. Under the income optimization objective in particular, they do not capture the year-to-year income variability that would be encountered along a single path under the dynamic withdrawal strategy being used (see Section 2.2).

metrics to strategy selection is illustrated in Section 5. The examples involve a retiree with low risk appetite that retires at age 65 with assets of $500,000 and constant real social security income of $20,000. Metrics are presented for two investment strategies. Strategy A allocates 25% of assets at retirement to a (inflation-protected) life annuity to secure the income floor of $25,000, and 75% to a retirement savings account with a 75/25 growth/defensive mix. Strategy B allocates 75% of assets at retirement to the life annuity and 25% to a retirement savings account with 25/75 growth/defensive mix. These strategies are crafted to create a point of distinction, and do not necessarily represent reasonable choices. Appendix D outlines main assumptions underpinning the analysis in both this section and in Section 5.4 and Section 5.5. The Excel models used to generate the example may be accessed on the SOA website at (see Income Optimizer Model and Income Target Model).

4.4.1 Income Target Objective

The example assumes a real income target of $50,000, and a 'draw-to-target' withdrawal strategy with provision for excess withdrawals as outlined in Section 2.2. Table 7 reports the primary metrics, along with explanatory comments on the right. Figure 6 presents the median income built up as 'income layers', and median account balance. Figure 7 presents selected income percentiles at each age for each strategy.

Metrics for the income target objective are directed at conveying both how long the target income might be sustained (i.e., the likelihood of shortfall), and minimum income once the account balance is exhausted (i.e., the potential magnitude of shortfall). These elements are communicated in Table 7 through reporting the expected age of account exhaustion, the probability of income shortfall versus target for ages 70 through to 105 at 5-year intervals, and minimum income as the sum of social security income and the life annuity that was purchased at retirement. Table 7 also reports the utility-based measures of *CRADT* and *RAI*, as well as the median account balance for ages 70 through to 105 at 5-year intervals. Overall, Table 7 reveals that Strategy A is superior on most metrics including the likelihood of sustaining both the target and retaining some account balance for longer, as well as the utility-based measures that summarize the entire income distribution into a single number. However, Strategy B delivers higher minimum income once the account balance is exhausted.

Figure 6 and Figure 7 display these themes graphically. (Note: All charts in this section use the same scales to aid visual comparisons.) The income layer charts in Figure 6 show how the two strategies differ and the underlying income sources. They depict the tendency for Strategy A to retain a positive account balance and sustain the income target for a considerably longer period than Strategy B; but that income falls to a noticeably lower level once the account balance is exhausted under Strategy A because it allocates less to the life annuity.[84] The income percentile charts of Figure 7 reveal the income distribution around the median, which appear in the form of probability distribution of ages at which income drops to the minimum level. These charts reveal that Strategy A has about a 50% probability of sustaining the target to age 90, whereas Strategy B is unlikely to sustain the income target beyond age 78. Strategy A also provides a small chance of being able to withdraw some excess over the target, which appears at the 95th percentile. However, Strategy A also faces some risk of declining to (lower) minimum income relatively early into retirement, e.g., the probability of being unable to sustain the target beyond age 78 is 5%, and beyond age 83 is 25%. Thus, Strategy A does not guarantee a better outcome relative to Strategy B. In this way, the metrics help to reveal the trade-offs being made across the two strategies.

[84] Income falls to near the income floor under Strategy A.

Table 7

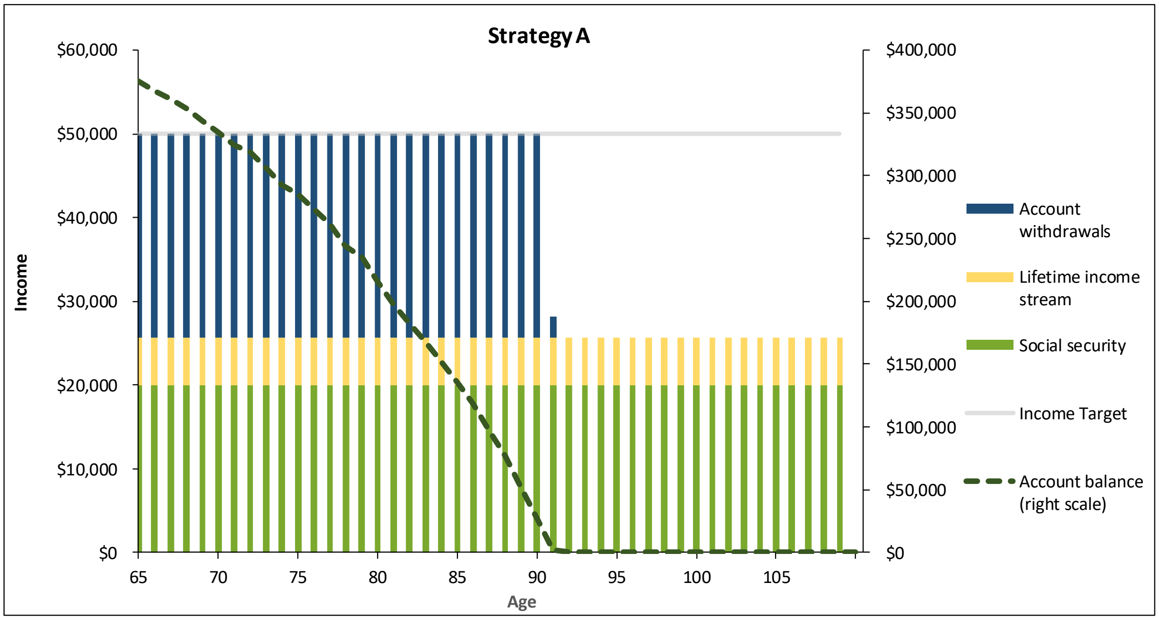
ILLUSTRATIVE METRICS FOR INCOME TARGET OBJECTIVE

	A: Low Annuities / High Growth Weighting	B: High Annuities / Low Growth Weighting	Comments
Assumptions About Retiree			
Income floor	$25,000	$25,000	
Income target	$50,000	$50,000	
Risk appetite	Low	Low	
Savings at retirement (age 65)	$500,000	$500,000	
Allocation of savings at retirement			
Lifetime income stream (annuity)	$125,000 25%	$375,000 75%	23% allocation needed to secure income floor
Retirement savings account	$375,000 75%	$125,000 25%	
Total	$500,000 100%	$500,000 100%	
Growth component: retirement account	$281,250 75%	$31,250 25%	
Income streams:			
Social security	$20,000	$20,000	
Lifetime income stream	$5,648	$16,944	
Primary Metrics			
Expected age of account exhaustion	94	75	Average age at which account is exhausted, and income falls to 'minimum' income
Probability of shortfall vs. income target			Indicates likelihood of sustaining the income target to various ages. Strategy A performs much better on this metric, with Strategy B providing little to no chance of sustaining the target beyond age 80.
Age 70 (Survival probability 97%)	0%	0%	
Age 75 (Survival probability 91%)	0%	63%	
Age 80 (Survival probability 81%)	10%	100%	
Age 85 (Survival probability 66%)	33%	100%	
Age 90 (Survival probability 42%)	50%	100%	
Age 95 (Survival probability 18%)	61%	100%	
Age 100 (Survival probability 4%)	69%	100%	
Age 105 (Survival probability 0.5%)	73%	100%	
Minimum income (Social security + lifetime income stream)	$25,648	$36,944	Shows how far income falls once account is exhausted. Strategy B delivers higher minimum income due to larger annuity purchase.
Probability of shortfall vs. income floor	0%	0%	Annuity purchase secures the income floor
Utility-based metrics: #			Overarching metric that provides a summary score across all outcomes. Strategy A offers more than $4,000 in additional risk-adjusted income.
Constant risk-adjusted difference from target (*CRADT*)	($2,476)	($6,604)	
Risk-adjusted income (Target + *CRADT*)	$47,524	$43,396	
Median account balance			Provides a general indication of the potential remaining assets at each age. Strategy A is more likely to retain a meaningful account balance, potentially into the retiree's nineties, suggesting it may offer benefits in terms of possible bequests or precautionary assets. *Note*: If account balance is of particular relevance to the retiree, a percentile distribution might be reported or charted.
Age 70 (Survival probability 97%)	$334,832	$70,701	
Age 75 (Survival probability 91%)	$285,590	$9,490	
Age 80 (Survival probability 81%)	$216,136	-	
Age 85 (Survival probability 66%)	$135,610	-	
Age 90 (Survival probability 42%)	$26,712	-	
Age 95 (Survival probability 18%)	-	-	
Age 100 (Survival probability 4%)	-	-	
Age 105 (Survival probability 0.5%)	-	-	

Survival probabilities are applied in estimating utility-based metrics

Figure 6

MEDIAN INCOME LAYERS AND ACCOUNT BALANCE - INCOME TARGET OBJECTIVE



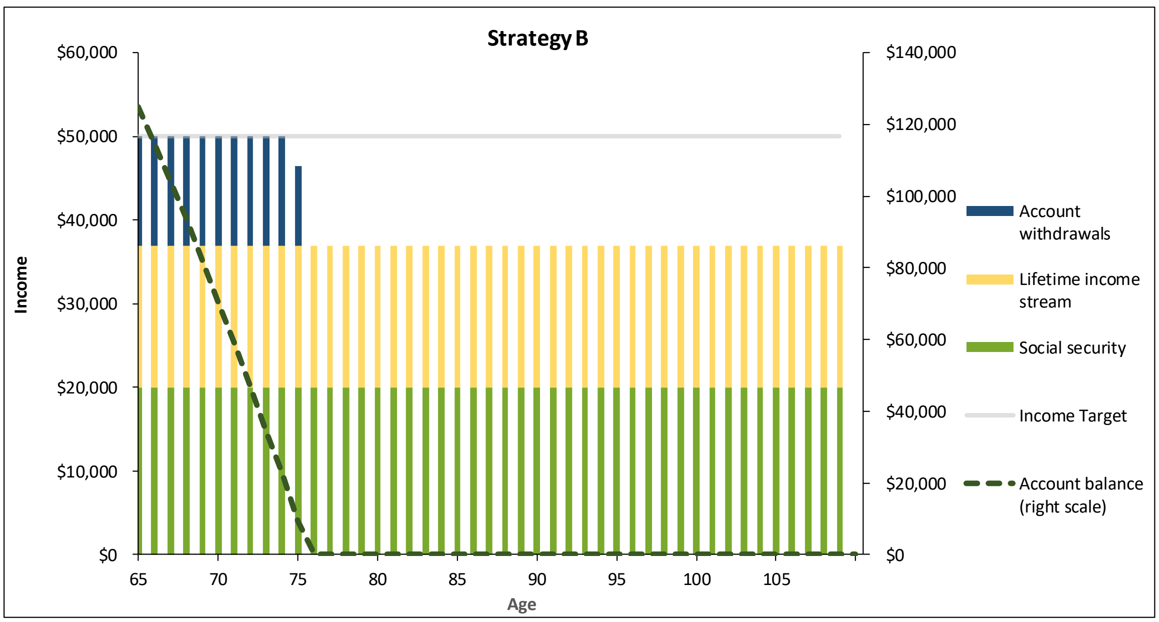
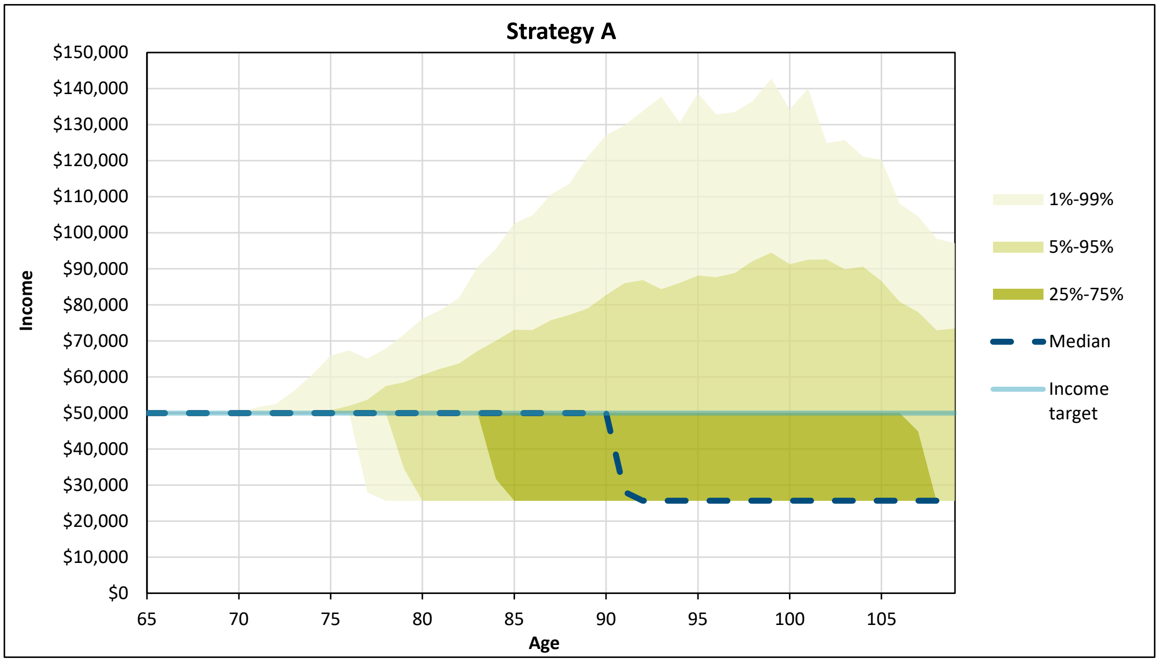
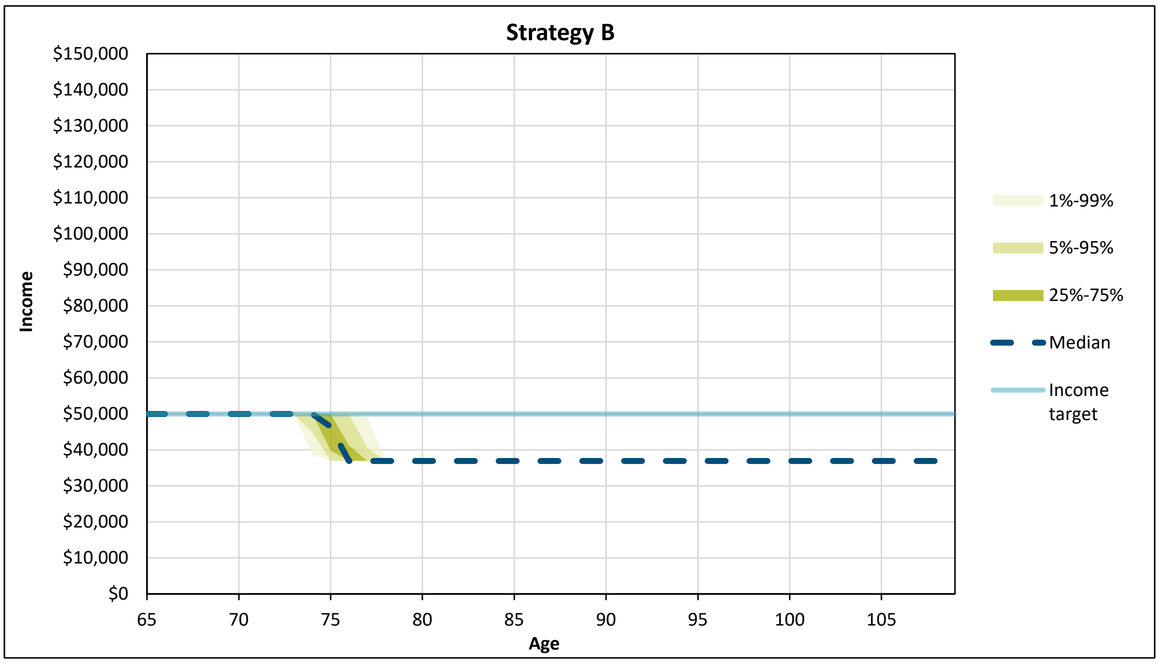


Figure 7

INCOME PERCENTILE CHARTS - INCOME TARGET OBJECTIVE





4.4.2 Income Optimization Objective

This example combines the same two investment strategies with an 'affordable' withdrawal strategy as outlined in Section 2.2. Table 8 reports the metrics, again with explanatory comments to the right. Figure 8 plots median income layers and account balance. Figure 9 plots selected income percentiles at each age under each strategy.

Table 8 indicates that expected income is higher under Strategy A relative to Strategy B ($48,391 versus $41,865), which arises from its higher allocation to a retirement savings account that is invested mainly in growth assets. Examining the median outcomes reported in Table 8 reveals that Strategy A delivers median income that is relatively stable over retirement, while always retaining some assets within the retirement savings account. Strategy A also delivers higher utility as indicated by risk-adjusted income ($44,722 versus $41,831).

Nevertheless, the charts reveal that the income distribution differs considerably from the target income objective, as well as between Strategy A and Strategy B. The income layer charts in Figure 8 show that total income tends to be hump-shaped. Further, while median income reaches a higher level under Strategy A than Strategy B, as underpinned by withdrawals from the retirement savings account, this higher income tends to occur earlier in retirement with lower income being delivered later in retirement. The percentile charts of Figure 9 indicate that Strategy A offers much higher income variability than Strategy B. Nevertheless, *RAI* suggests that Strategy A offers higher utility, implying that the higher income risk is adequately compensated. The stark difference in income variability is worth noting and potentially communicating to retirees.

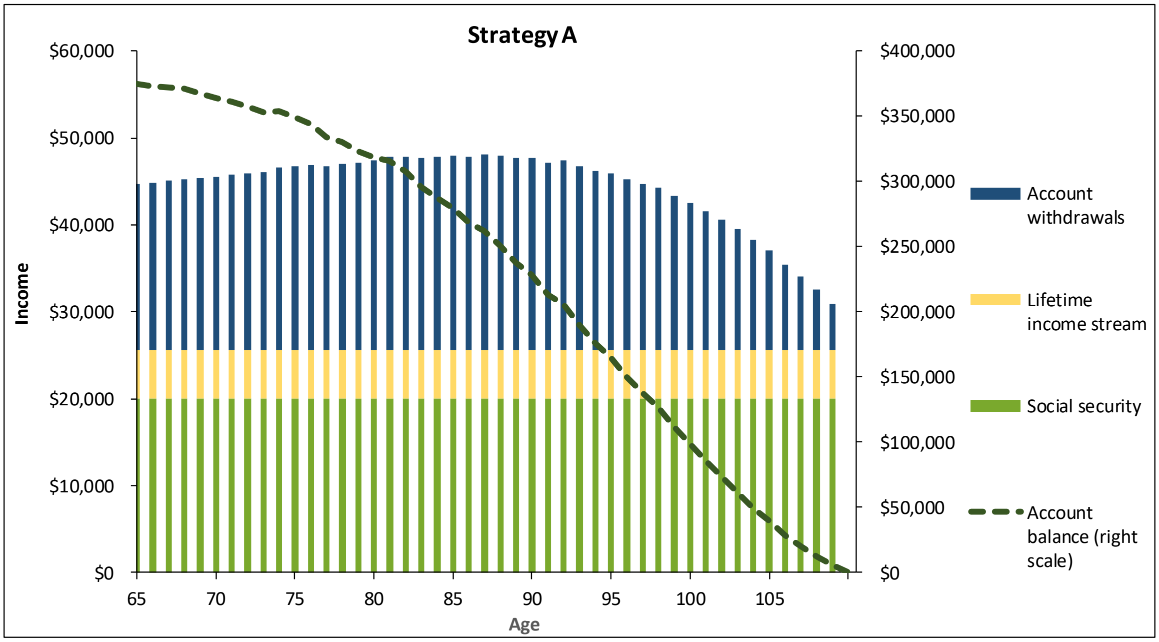
Table 8
ILLUSTRATIVE METRICS FOR INCOME OPTIMIZATION OBJECTIVE

	A: Low Annuities / High Growth Weighting	B: High Annuities / Low Growth Weighting	Interpretation
Assumptions About Retiree			
Income floor	$25,000	$25,000	
Risk appetite	Low	Low	
Savings at retirement (age 65)	$500,000	$500,000	
Allocation of assets at retirement			
Lifetime income stream (annuity)	$125,000 25%	$375,000 75%	25% allocation needed to secure income floor
Retirement savings account	$375,000 75%	$125,000 25%	
Total	$500,000 100%	$500,000 100%	
Growth weighting (retirement account)	$280,000 75%	$31,250 25%	
Income streams:			
Social security	$20,000	$20,000	
Lifetime income stream	$5,648	$16,944	
Primary Metrics			
Expected income#	$48,391	$41,865	Summary indicator for overall level of income. Strategy A in particular is impacted by positive skewness (i.e., some high incomes) in the income distribution.
Median income			Indicates trajectory of the 'typical' level of income as retiree ages. The withdrawal strategy is designed to generate an element of income stability on average. Income percentiles should be reported to indicate spread of income. Charts might be preferred (see below.)
Age 65 (Survival probability 97%)	$44,636	$41,425	
Age 70 (Survival probability 97%)	$45,566	$41,645	
Age 75 (Survival probability 91%)	$46,746	$41,876	
Age 80 (Survival probability 81%)	$47,398	$42,023	
Age 85 (Survival probability 66%)	$47,910	$42,102	
Age 90 (Survival probability 42%)	$47,670	$41,969	
Age 95 (Survival probability 18%)	$45,918	$41,503	
Age 100 (Survival probability 4%)	$42,571	$40,680	
Age 105 (Survival probability 0.5%)	$37,059	$39,400	
Minimum income (Social security + lifetime income stream)	$25,648	$36,944	Shows base layer of income. Strategy B base is higher due to larger annuity purchase.
Probability of shortfall vs. income floor	0%	0%	Annuity purchase secures the income floor
Utility-based metrics: #			
Risk-adjusted income (*RAI*)	$44,722	$41,831	Overarching metric that provides a summary score across all outcomes. Strategy A offers almost $3,000 in additional RAI.
Median account balance			Provides a general indication of the potential remaining assets at each age. Strategy A is more likely to retain a meaningful account balance, suggesting it offers benefits in terms of possible bequests or precautionary assets. *Note*: If account balance is of particular relevance to the retiree, a percentile distribution might be reported or charted.
Age 70 (Survival probability 97%)	$363,950	$116,290	
Age 75 (Survival probability 91%)	$349,693	$106,019	
Age 80 (Survival probability 81%)	$318,775	$92,454	
Age 85 (Survival probability 66%)	$279,591	$77,154	
Age 90 (Survival probability 42%)	$228,221	$59,608	
Age 95 (Survival probability 18%)	$165,072	$40,983	
Age 100 (Survival probability 4%)	$98,804	$23,261	
Age 105 (Survival probability 0.5%)	$38,942	$8,644	

Survival probabilities are applied in estimating these metrics

Figure 8

MEDIAN INCOME LAYERS AND ACCOUNT BALANCE - INCOME OPTIMIZATION OBJECTIVE



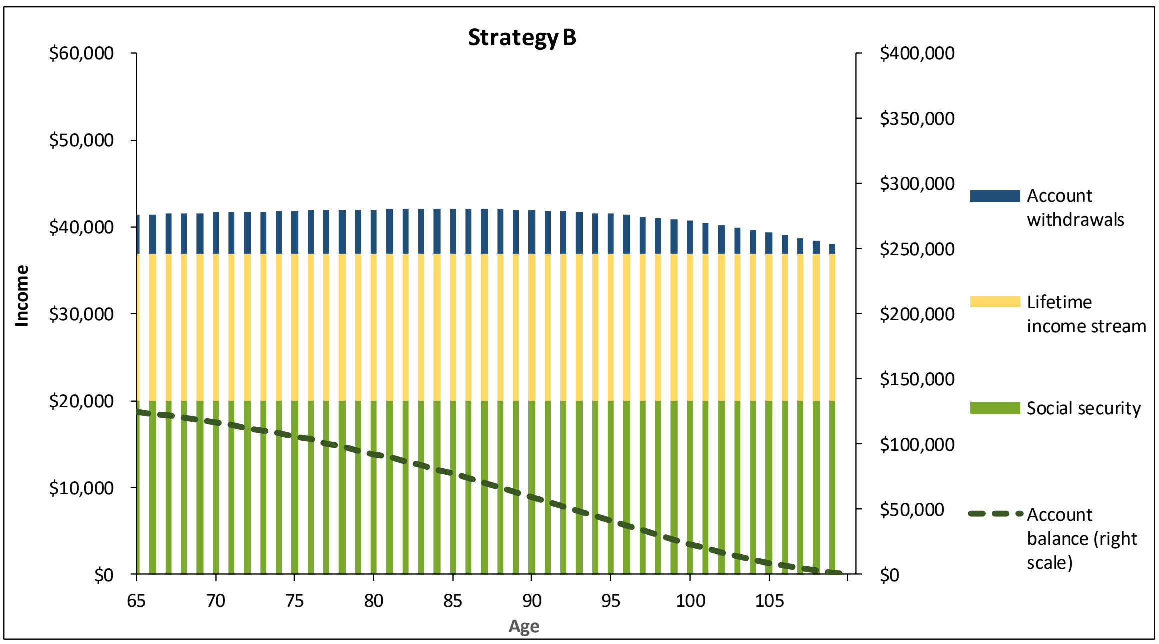
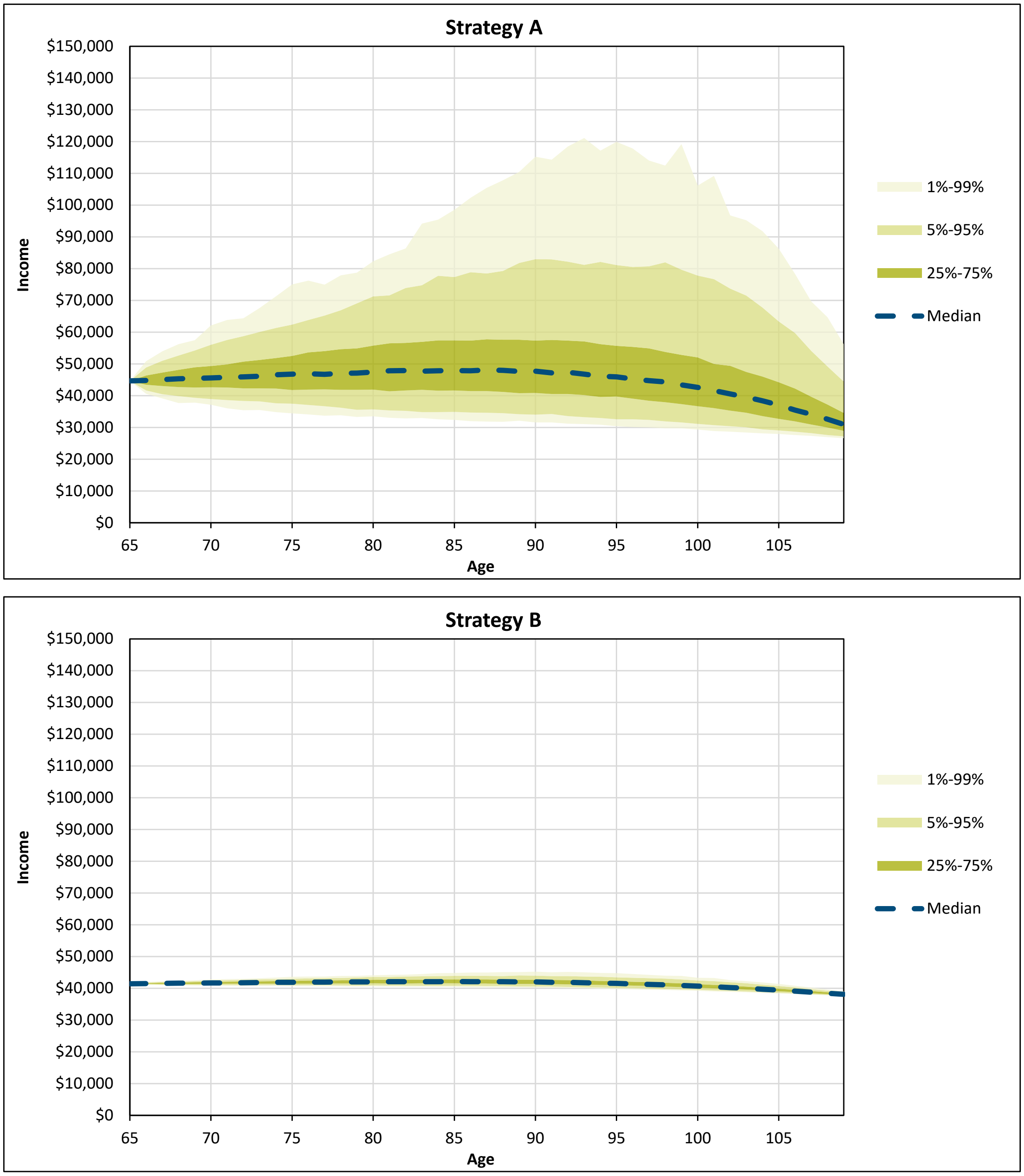


Figure 9

INCOME PERCENTILE CHARTS - INCOME OPTIMIZATION OBJECTIVE



SECTION 5: Strategy Selection

This section discusses selecting an appropriate retirement income strategy for a retiree. Section 5.1 outlines three approaches: applying principles and rules; selecting from candidate strategies; and identifying an 'optimal' strategy. Section 5.2 identifies factors that might be considered in choosing a strategy. Section 5.3 set out a procedure for identifying strategies based on applying principles and rules. Section 5.4 presents an example where four retiree types choose from an illustrative menu of candidate strategies. Section 5.5 demonstrates the use of modelling to identify optimal strategies for the same four retiree types.

5.1 Three Approaches to Selecting a Retirement Income Strategy

Three approaches for identifying a suitable retirement income strategy for a retiree are as follows:

- **Applying principles and rules** – Retiree objectives and preferences might be translated into a suitable retirement income strategy through devising and applying principles and rules. The advantage of this approach is that it is flexible and may be received as more intuitive, especially by those who struggle with models and their output. The disadvantage is that rules provide imprecise prescriptions and rely on judgement, and applying them effectively may rely on understanding of the principles. Section 5.3 sets out a procedure that is directed towards delivering income streams that satisfy the three income objectives outlined in Section 1.1, while taking income risk appetite into account and allowing scope to consider preferences for flexible access to funds and bequests.

- **Selecting from a menu of candidate strategies** – This approach envisages forming a menu of candidate strategies for consideration from which a strategy is selected. This approach aligns with a plan provider designing a menu of retirement income strategies to offer, leading to the task of matching retirees to the strategy on the menu that is most suitable for their needs. An advantage of this approach is that candidate strategies can be proposed that are feasible to deliver at reasonable cost, and likely to be accepted by retirees. The disadvantage is that some retirees may find none of the strategies to be a good fit for their needs.

- **Identifying an 'optimal' strategy** – This approach envisages initially applying a quantitative model to locate the strategy that maximizes some objective function, e.g., delivers greatest expected utility. This strategy might then be adjusted for other qualitative factors, perhaps using the quantitative model to help gauge the impact of proposed amendments. The aim is to arrive at a strategy that is considered appropriate after considering both the model output and other relevant factors. This approach offers the potential to deliver strategies to retirees that are well-tailored. The disadvantage is that the process can be analysis and information intensive, noting that quantitative optimization techniques (e.g., dynamic programming) are technically difficult and rely on correctly specifying the characteristics of the retiree and the stochastic model (see Section 3). Optimization may also generate strategies that neither a retiree would accept nor a provider could easily deliver,[85] and if significant qualitative adjustment is then required, this defeats the purpose of optimizing in the first place.

[85] The optimal solution might still be useful as a point of comparison, nevertheless.

5.2 Factors to Consider in Strategy Selection

Factors that might be considered in selecting a retirement income strategy are outlined below.

- Quantitative analysis – Modelling can be used to form an initial ranking of candidate strategies or identify a quantitatively optimal strategy. As discussed in Section 4.1, utility functions may be most effective for this task, ideally through converting expected utility into a certainty equivalent (denoted as *CRADT* or *RAI* in this Primer). It may also be possible to rank strategies through combining selected metrics into an overall score,[86] but it is debatable whether this approach is more effective than utility-based methods.[87] A quantitative score that provides an order-of-magnitude difference between candidate strategies can also assist strategy selection by indicating the potential value gain or loss under the model from choosing one strategy over another. For instance, a strategy with a lower *RAI* might be selected if there are sizable benefits on other factors, on the provision that the reduction in *RAI* is not too large. However, quantitative models should not be solely relied on. Not only are they unable to account for other important factors as listed immediately below, but they reflect subjective modelling choices and can provide a false sense of precision.

- Subjective review of projected outcomes – Examination of projected outcomes arising from the modelling and related metrics can be used to understand what a candidate strategy delivers and whether it is fit for purpose. For example, metrics might be used to gauge whether the strategy delivers income variability that seems undesirably high, gives inadequate access to funds, or results in excessively large account balances later in retirement that could be a signal that assets are not being efficiently converted into income.

- Simplicity and robustness – Simpler and more robust strategies might be preferred over those that are model or assumption dependent. For example, a strategy that is dependent on certainty of taxation or social security rules over the coming decades should be viewed warily.

- Access to funds – Flexible access to funds is valuable to retirees, but difficult to incorporate in modelling focused around income. It hence may need to be considered subjectively as an additional criterion.

- Behavioral considerations – How retirees might react to strategies should be considered. Behavioral effects are important in this regard and are discussed in Section 6.3. For instance, a strategy might be preferred if it reduces retiree aversion to investing in growth assets or provides confidence to draw a higher but affordable amount of income. The reluctance to purchase lifetime income steams (i.e., annuities, see Section 6.2) and accept investment risk might also be considered.

[86] See Shang and Jiang (2016) for an example in a retirement context.

[87] Utility functions directly evaluate each outcome individually and then aggregate. Combining metrics into a single score entails extra levels of aggregation, i.e., constructing the metrics, and then aggregating them. This implicitly requires imposing assumptions about preferences over the trade-off between expected outcomes and risk, which will be embedded in the way that the metrics are weighed or combined. Both approaches amount to imposing some form of objective function on the data and involve modelling choices that will be partly subjective. Arguably utility functions achieve this within a more coherent framework. One advantage of the metric approach is that it may be better received by some as less of a 'black box' than utility, as the components that make up the overall score are more visible and interpretable.

- **Business considerations** – Strategies need to be feasible for the plan provider to implement at a reasonable cost, as well as meet a market need. Various business considerations that might impact on strategy selection are outlined in Section 6.4.

5.3 Applying Principles and Rules

The translation of objectives and preferences into retirement income strategies through applying principles and rules is demonstrated by outlining a procedure for designing strategies under the income target and income optimization objectives, while taking into account any income floor. The procedure commences by ***setting withdrawal rules*** linked to the income objective, and then proposing ***investment strategies aimed at building income layers*** to maximize income while managing income risk in line with risk appetite. The manner in which preferences over assets including flexible access to funds and bequest motives may be incorporated in the procedure is also discussed. The presentation implicitly assumes that lifetime income streams take a basic form such as a life annuity.[88] This approach might be used to either design a retirement income strategy for an individual retiree or guide the formation of a menu of retirement income strategies through helping to narrow down the range to those likely to be suitable.

5.3.1 Income Floor Objective

This objective implies securing a base layer of income to ensure the floor is delivered as a first priority. The first step is to determine the availability of any guaranteed income streams, such as social security or defined benefit pensions. The second step is to estimate the additional lifetime income stream that needs to be purchased to secure the income floor. Having established a base layer of income in line with the income floor, the remaining assets are then deployed towards building income layers to meet one of the two 'aspirational' income objectives.

5.3.2 Income Target Objective

This objective implies a *draw-to-target withdrawal strategy* where sufficient income is drawn from the retirement savings account to attain the income target, unless more income can be safely taken.[89, 90] The *investment strategy is then directed at building income layers to maximize the chances of sustaining the income target until death*. For example, lifetime income streams might feature prominently if they can lock-in the income target, where sustaining the target would otherwise be tenuous. Meanwhile, the use of lifetime income streams might be limited in favor of risky asset exposure if taking risk is required to attain the target. After having established the desired allocation to lifetime income streams, the remaining assets are invested in a retirement savings account in line with risk appetite. Here, risky asset exposure might be set as high as can be tolerated if lifetime income streams have been applied to help lock-in the target, and hence fulfilled the defensive role within the portfolio.[91] Figure 10 presents a flowchart of the procedure.

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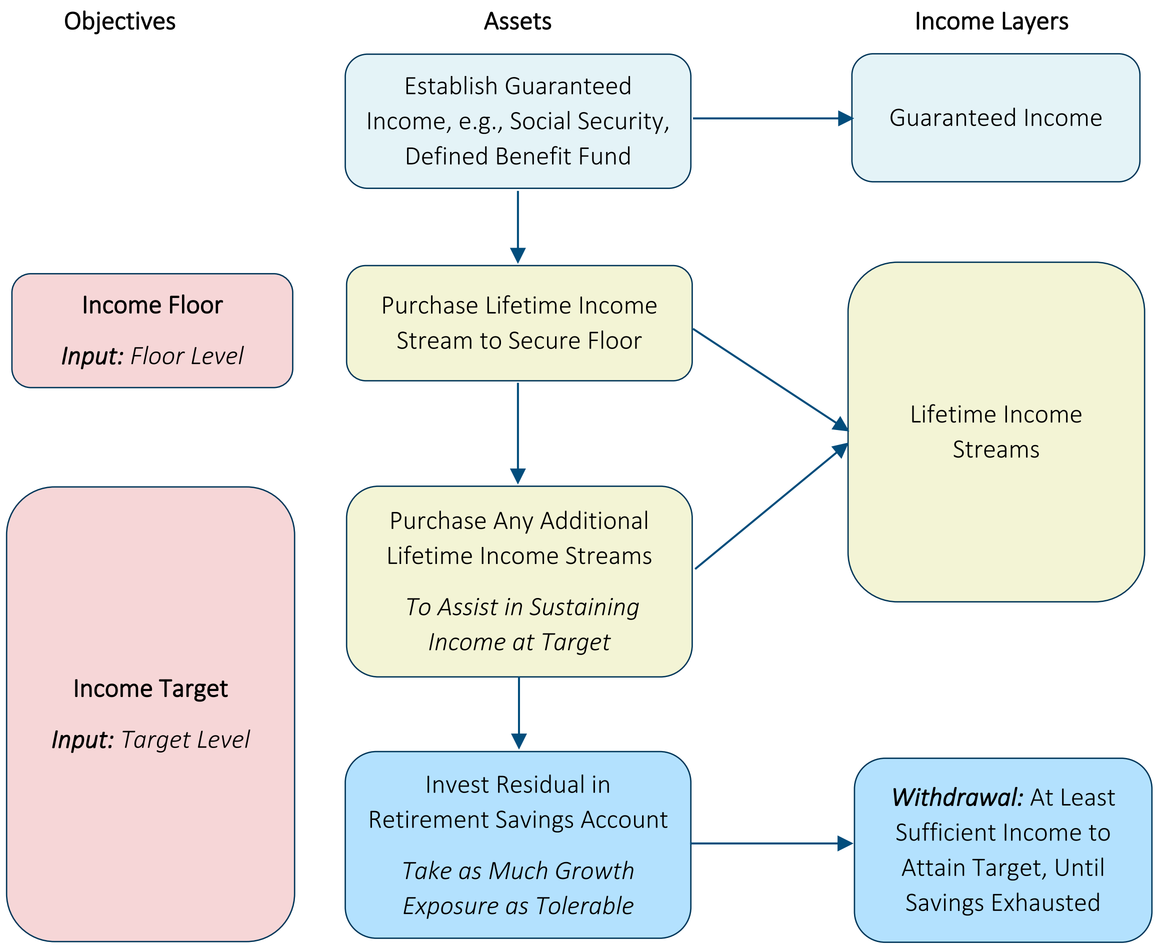
[88] This facilitates drawing out the main concepts, while avoiding complexities that arise if deferred or investment-linked annuities are considered.
[89] Rules for drawing income in excess of the target are discussed in Section 2.2.
[90] If the deviations from target are significantly different from zero, consideration might be given to adjusting the target as it may be set either too low or too high relative to what is comfortably affordable.
[91] Where the lifetime income stream is a deferred annuity, defensive exposure might be used to help ensure that the retirement savings account is not exhausted and hence income can be sustained until annuity payments commence.

Figure 10
IDENTIFYING CANDIDATE STRATEGIES UNDER AN INCOME TARGET OBJECTIVE



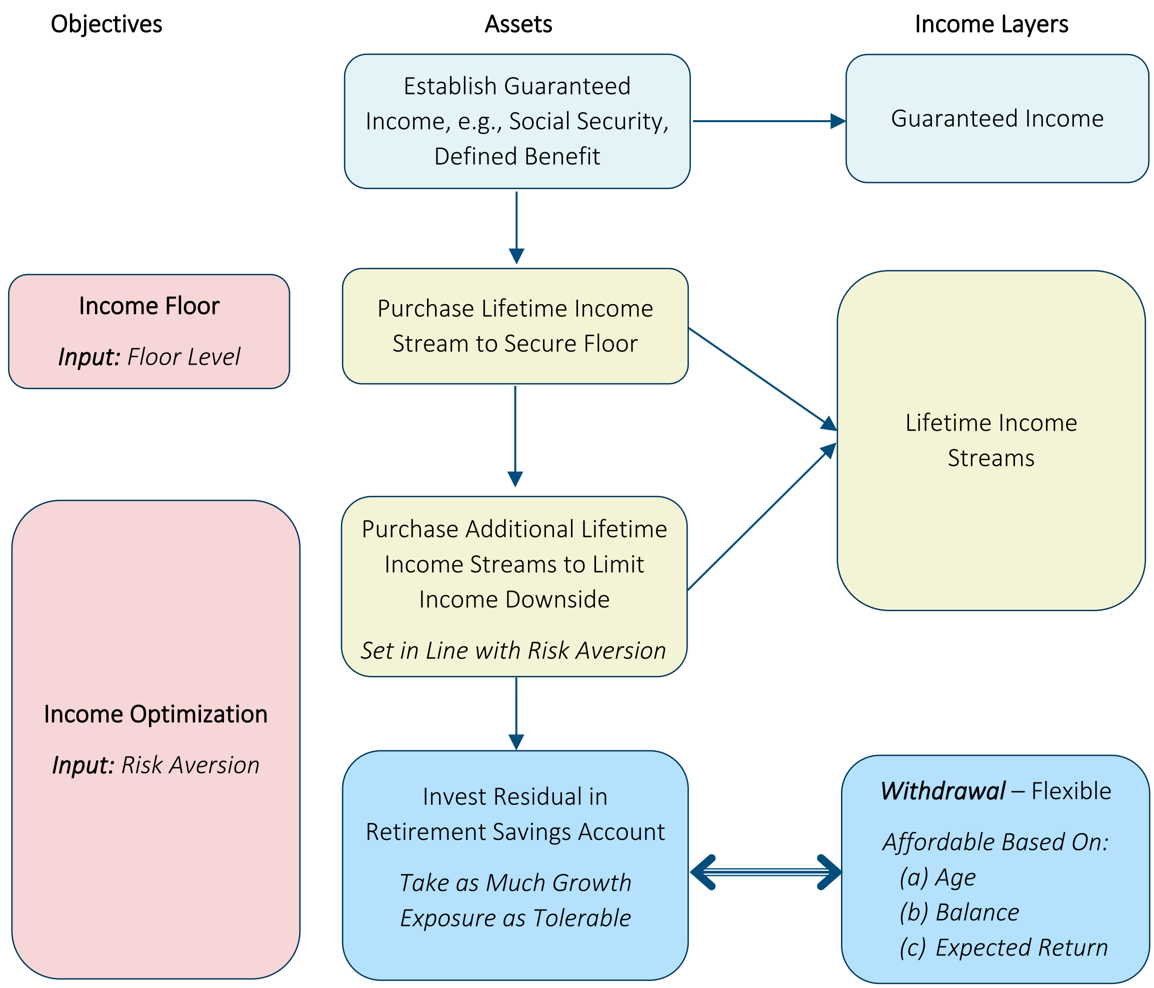
5.3.3 Income Optimization Objective

Separating the withdrawal strategy from the investment strategy is difficult under this objective as the ‘affordable’ withdrawal is itself a function of the returns that are expected under the investment strategy. This challenge can be solved through setting *pre-determined withdrawal rules that depend on age, account balance and expected future investment returns*. Such a strategy was outlined in Section 2.2. The result is a pre-determined yet flexible withdrawal plan that adjusts with both age and realized investment experience, with the latter impacting via the changes in the retirement savings account balance. The rule might be expressed as a scheduled percentage of the account balance to be withdrawn at each age, as per Table 6 appearing at the end of Section 2.2.[92]

[92] The US required minimum drawdown (RMD) rules specify the withdrawal percentage required to avoid additional tax. However, this is a universal rule that is not calibrated to retiree’s circumstances. For instance, the RMD percentages may be too low for retirees who are willing to take income risk by investing in higher returning but riskier assets, who could be targeting higher withdrawals.

With a pre-determined withdrawal plan in place, the *investment strategy is then set with reference to risk appetite, i.e., risk aversion*. High risk aversion would imply allocating more to lifetime income streams to limit downside in income. High risk aversion might also imply allocating less to risky assets within the retirement savings account. Nevertheless, risky asset exposure within the retirement savings account should be as high as can be tolerated to boost expected return and hence increase long-term income potential and scheduled withdrawal rates. One principle is that it is better to rely on lifetime income streams to play the primary defensive role within the portfolio rather than defensive assets within the retirement savings account, as indicated by the analysis in Section 5.5. Figure 11 presents a flowchart of the procedure.

Figure 11
IDENTIFYING CANDIDATE STRATEGIES UNDER AN INCOME OPTIMIZATION OBJECTIVE



5.3.4 Access to Funds and Preferences over Remaining Assets

The above discussion addresses strategies directed at converting assets into income. Additional consideration might be given to whether some access to funds is desired to address unplanned large spending needs and/or support a bequest. The value of flexible access to funds may be recognized through placing a limit on the allocation to lifetime income streams, or perhaps creating a 'carve-out' of assets that is considered unavailable to support regular income. The latter may be approached as a ***precautionary***

balance to cater for large unplanned spending, which might be defensively invested. A ***strong bequest motive*** implies willingness to sacrifice income to accumulate a bequest, suggesting adjusting the strategy by: (a) restricting withdrawals, (b) limiting purchases of lifetime income streams to that required to secure the income floor, and (c) increasing growth asset exposure to the maximum that can be tolerated.

5.4 Selecting from a Menu of Candidate Strategies

This section illustrates how selections might be made from a menu of retirement income strategies through a simple example involving four types of retirees that choose from five candidate strategies. The task of identifying candidate strategies to include on the menu is beyond the scope of this Primer, although the considerations raised in Section 5.2 and the procedure outlined in Section 5.3 may assist.[93] Strategies on the illustrative menu are graded from Strategy 1 to Strategy 5 according to potential for income variability, and include the following allocation of assets at the point of retirement:

Strategy 1: 100% life annuity[94] (included for a point of comparison)

Strategy 2: 50% life annuity; 50% retirement savings account with a 25/75 growth/defensive mix

Strategy 3: 100% retirement savings account with a 25/75 growth/defensive mix

Strategy 4: 25% life annuity; 75% retirement savings account with a 75/25 growth/defensive mix

Strategy 5: 100% retirement savings account with a 75/25 growth/defensive mix

The four retiree types are distinguished by income objective (*income target* and *income optimization*) and available financial resources. A *low wealth* retiree has $300,000 in savings, access to $20,000 in guaranteed income stream in the form of social security, and an income floor of $20,000.[95] In the case of the income target objective, the low wealth retiree aspires to an income of $40,000. A *high wealth* retiree has $700,000 in savings, access to $30,000 in guaranteed income streams in the form of social security and a defined benefit pension, and an income floor of $30,000. In the case of the income target objective, the high wealth retiree aspires to income of $60,000. Appendix D outlines the main modelling assumptions.

Table 9 reports selected outputs from the quantitative modelling, with a full set of metrics appearing in Appendix D. The strategies that are selected from the menu for the four retiree types are identified by **bolded** metrics within the table. Following Table 9 are chart sets for the selected strategies: Figure 12 plots medians for the income layers and account balance, and Figure 13 plots income percentiles. The selection process starts by considering *RAI* (highlighted in red font within Table 9), before considering other factors. The strategy selection process is discussed after Figure 13.

[93] Another approach is to segment or cohort the customer base, and design strategies tailored for each segment. Customer segmentation might aim to span the key retiree characteristics that were discussed in Section 1. A cohorting approach is envisaged under the Australian Retirement Income Covenant, see https://treasury.gov.au/sites/default/files/2021-09/c2021-209553-explan_memorandum.pdf.
[94] An inflation-protected annuity is used in these examples.
[95] The income floor is assumed to be covered by guaranteed income in these examples, in order to facilitate inclusion of strategies with zero allocation to the life annuity to create a point of comparison.

Table 9

STRATEGY COMPARISONS AND SELECTIONS ACROSS FOUR RETIREE TYPES

Strategies	1	2	3	4	5	1	2	3	4	5
Life annuity	100%	50%	Nil	25%	Nil	100%	50%	Nil	25%	Nil
Retirement savings - growth/defensive	Nil	25/75	25/75	75/25	75/25	Nil	25/75	25/75	75/25	75/25
Representative metrics*										
Income target objective	High wealth ($700,000)					Low wealth ($300,000)				
Target Income	*$60,000*					*$40,000*				
Withdrawal strategy	***Draw-to-target*** *(with excess)*					***Draw-to-target*** *(with excess)*				
Expected age of account exhaustion	n.a.	**102**	99	106	106	n.a.	78	83	89	**93**
Probability of shortfall vs. income target:										
Age 75 (Survival probability 91%)	0%	**0%**	0%	0%	0%	100%	2%	0%	2%	**0%**
Age 85 (Survival probability 66%)	0%	**0%**	0%	3%	3%	100%	100%	86%	52%	**36%**
Age 95 (Survival probability 18%)	0%	**16%**	31%	13%	14%	100%	100%	100%	77%	**65%**
Age 105 (Survival probability 0.5%)	0%	**69%**	83%	24%	26%	100%	100%	100%	86%	**76%**
Minimum income ($'000)	61.6	**45.8**	30.0	37.9	30.0	33.6	26.8	20.0	23.4	**20.0**
Risk-adjusted income, i.e., utility ($'000) #	61.6	**59.9**	59.6	59.8	59.7	33.6	34.6	35.4	37.2	**37.7**
Median account balance ($'000):										
Age 75 (Survival probability 91%)	0	**284.2**	549.7	555.0	734.9	0	42.3	159.7	140.3	**215.6**
Age 85 (Survival probability 66%)	0	**196.6**	350.8	521.9	689.0	0	0	0	0	**74.6**
Age 95 (Survival probability 18%)	0	**89.7**	106.3	382.2	509.0	0	0	0	0	**0**
Age 105 (Survival probability 0.5%)	0	**0.0**	0	142.7	190.6	0	0	0	0	**0**
Income optimization objective	High wealth ($700,000)					Low wealth ($300,000)				
Withdrawal	***Affordable*** *(annuitization at AIR-1%)*					***Affordable*** *(annuitization at AIR-1%)*				
Expected income#, +	61.6	59.6	57.6	**69.7**	72.5	33.6	32.7	31.8	37.0	**38.2**
Median income at selected ages ($'000):										
Age 75 (Survival probability 91%)	61.6	59.6	57.6	**67.4**	69.4	33.6	32.7	31.8	36.0	**36.9**
Age 85 (Survival probability 66%)	61.6	60.3	58.9	**69.1**	71.6	33.6	33.0	32.4	36.7	**37.8**
Age 95 (Survival probability 18%)	61.6	58.6	55.5	**66.3**	67.8	33.6	32.2	30.9	35.6	**36.2**
Age 105 (Survival probability 0.5%)	61.6	52.7	43.8	**53.9**	51.3	33.6	29.7	25.9	30.2	**29.1**
Minimum income ($'000)	61.6	45.8	30.0	**37.9**	30.0	33.6	26.8	20.0	23.4	**20.0**
Risk-adjusted income, i.e., utility ($'000)#	61.6	59.4	56.8	**64.7**	64.2	33.6	32.6	31.6	35.2	**35.2**
Median account balance ($'000):										
Age 75 (Survival probability 91%)	0	296.9	593.7	**489.6**	652.8	0	127.2	254.4	209.8	**279.8**
Age 85 (Survival probability 66%)	0	216.0	432.1	**391.4**	521.9	0	92.6	185.2	167.8	**223.7**
Age 95 (Survival probability 18%)	0	114.8	229.5	**231.1**	308.1	0	49.2	98.4	99.0	**132.1**
Age 105 (Survival probability 0.5%)	0	24.2	48.4	**54.5**	72.7	0	10.4	20.7	23.4	**31.2**

* Full details found in Appendix D.

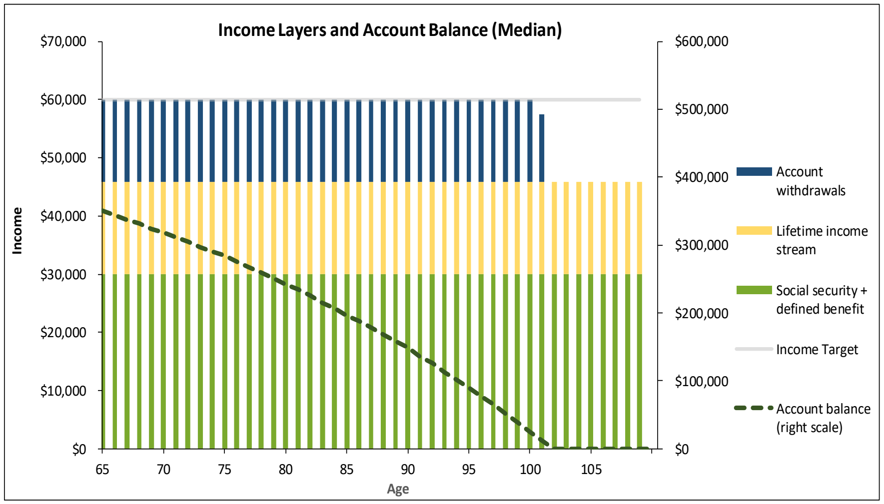
Survival probabilities are applied in estimating utility-based metrics.

+ Expected income is higher than median income due to income being positively skewed.

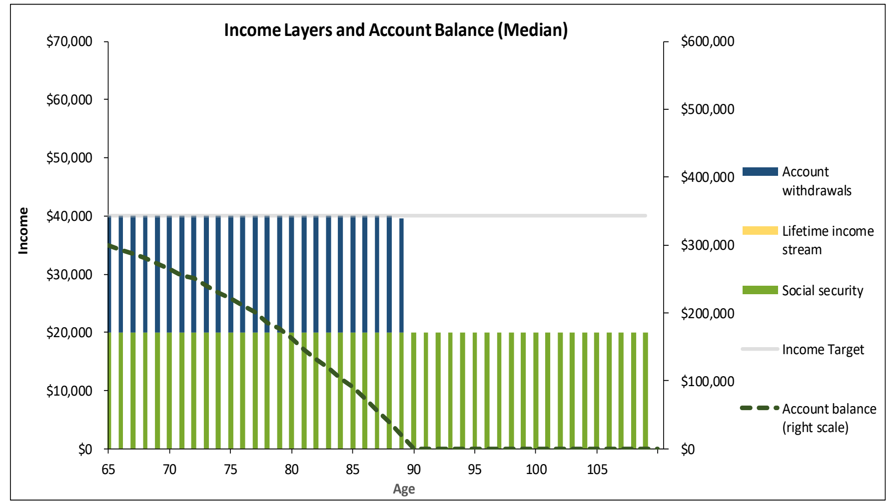
Figure 12

MEDIAN INCOME LAYERS AND ACCOUNT BALANCE FOR THE SELECTED STRATEGIES

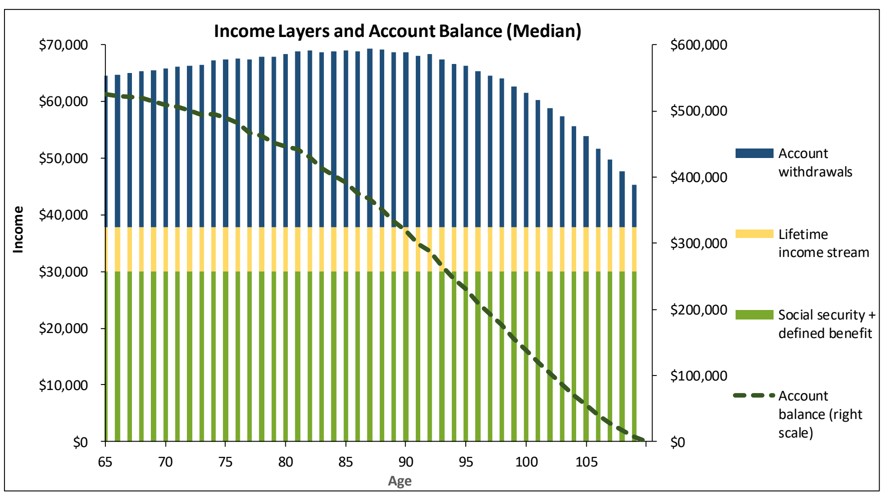
Income Target Objective, High Wealth



Income Target Objective, Low Wealth



Income Optimization Objective, High Wealth



Income Optimization Objective, Low Wealth

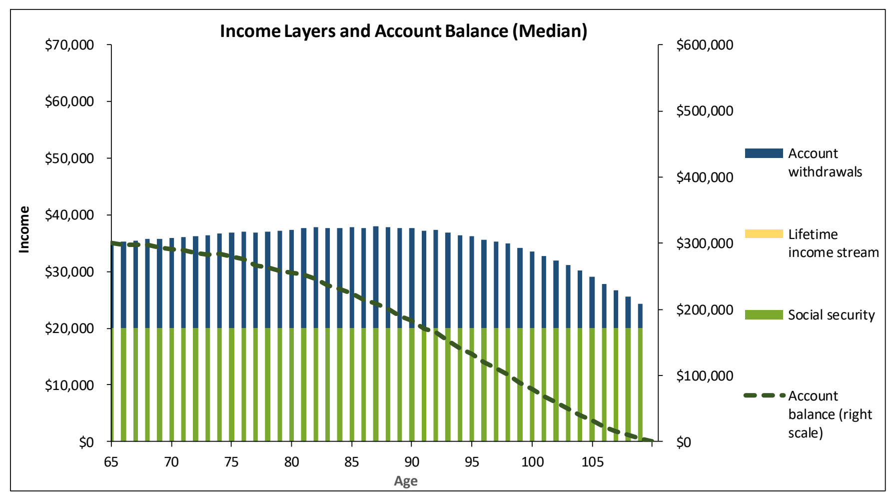
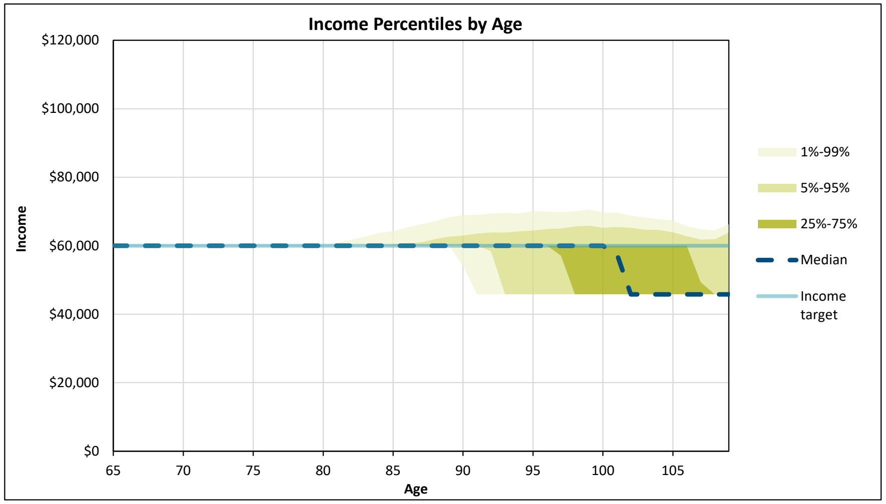


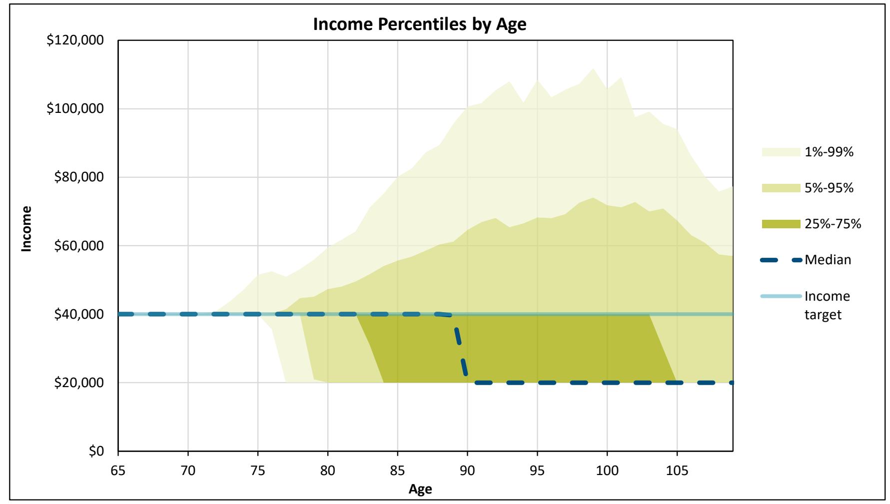
Figure 13

INCOME PERCENTILES FOR SELECTED STRATEGIES

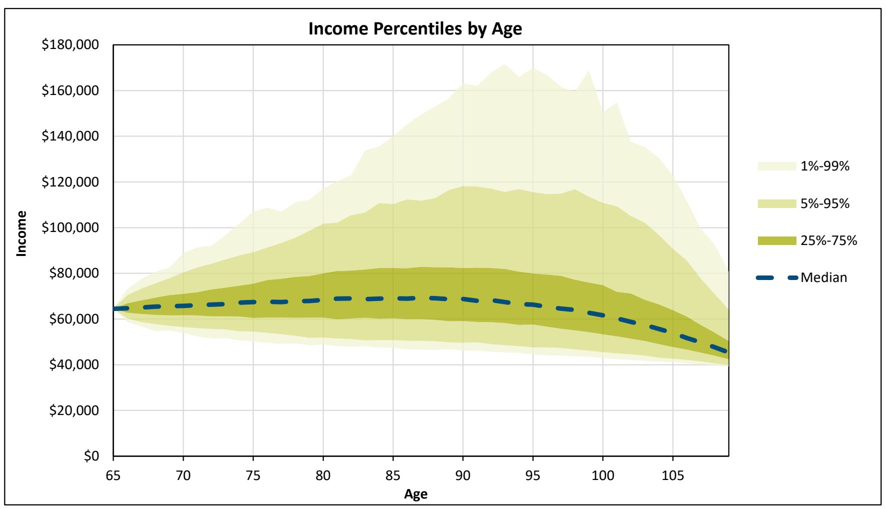
Income Target Objective, High Wealth



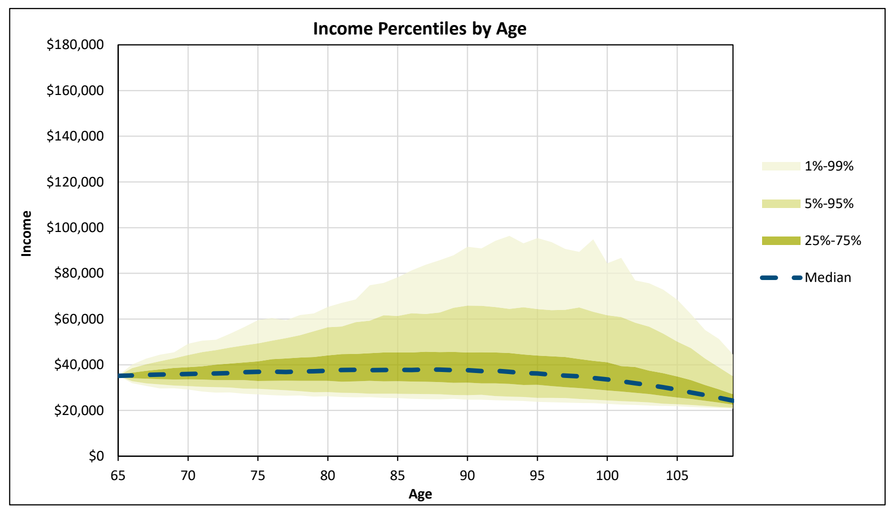
Income Target Objective, Low Wealth



Income Optimization Objective, High Wealth



Income Optimization Objective, Low Wealth



The reasoning behind the strategy selection for each retiree type is described below. In addition to the quantitative analysis, the value of flexible access to funds and potential acceptability to the retiree is also considered, meaning that the strategy with the highest *RAI* is not always selected.

- Income target, high wealth – Strategy 2 is selected with an allocation of 50% to the life annuity and 50% to a 25/75 growth/defensive retirement savings account, although Strategy 1 with a 100% allocation to a life annuity generates a higher *RAI* at $61,600 versus $59,900 for Strategy 2. While Strategy 1 locks in income that exceeds the target of $60,000 for life, it amounts to full annuitization and hence leaves no flexible access to funds (the value of which is not captured in *RAI*). Strategy 2 provides access to some funds, although at the risk that income falls to the minimum of $45,800 after account exhaustion. Nevertheless, account exhaustion occurs at an expected and median age of 102 (Table 9, Figure 12), to which there is less than 4% probability of survival. The income percentile chart (Figure 13) also reveals only a 5% probability of being unable to sustain the income target beyond age 92. A further consideration is that the $45,800 of minimum income delivered by Strategy 2 is comfortably above the income floor of $30,000.

- Income target, low wealth – Strategy 5 with 100% allocation to a 75/25 growth/defensive retirement savings account generates the highest *RAI* and is selected as the preferred strategy. Allocating entirely to a retirement savings account and excluding annuities is suggested for this retiree for two reasons. First, the income floor is covered by social security. Second, the retiree has insufficient assets to purchase annuities and secure the income target. Meanwhile, allocating towards growth assets maximizes the chances of attaining the target for a longer period. This can be seen in Table 9, which shows that Strategy 5 has the highest expected age of account exhaustion of 93 and the lowest probability of income shortfall across all ages shown. Further, Strategy 5 provides greater access to funds over a longer period than other strategies.

- Income optimization, high wealth – Strategy 4 is selected, and entails a 25% allocation to the life annuity and a 75% allocation to a 75/25 growth/defensive retirement savings account. Strategy 4 generates a modestly higher RAI than Strategy 5 ($64,700 versus $64,200), assisted by the reduction in income risk provided by the annuity. For instance, Figure 13 reveals that the 1st and 5th percentile incomes remain above $40,000 relative to the income floor of $30,000, in part because minimum income is augmented by $7,907 in annuity income (see Appendix D.2). Strategy 4 also provides flexible access to funds, although median account balances are lower than under Strategy 5 (which hence might be preferred under a strong bequest motive.)

- Income optimization, low wealth – Strategy 4 and Strategy 5 deliver the highest but identical RAI of $35,200 for this retiree. Strategy 5 is selected for its higher account balances and thus better flexible access to funds, noting that this benefit is not accounted for in *RAI*. However, Strategy 4 might be preferred by a retiree with a strong desire to limit income downside risk.

5.5 Identifying an 'Optimal' Strategy

Identification of 'optimal' retirement income strategies is illustrated for the four retiree types introduced in Section 5.4 through solving for the allocation at retirement to life annuities, growth assets and defensive assets that maximizes *RAI* (i.e., expected utility). The analysis optimizes along the limited dimension of an initial allocation of available assets assuming that the growth/defensive mix in the retirement savings account is held constant through retirement, which is combined with pre-determined withdrawal strategies (i.e., either draw-to-target with provision for excess withdrawal, or affordable withdrawal). Table 10 reports the resulting 'optimal' strategies and their representative metrics, with each compared to the strategies that were selected from the menu in Section 5.4. See Appendix D for a full set of metrics.

Table 10

OPTIMAL STRATEGIES FOR FOUR RETIREE TYPES

Income Target Objective	High Wealth ($700,000)				Low Wealth ($300,000)		
Strategy	Optimal	Selected from Menu	Difference		Optimal	Selected from Menu	Difference
Life annuity allocation	**100%**	50%	50%		**0%**	0%	0%
Retirement savings account (growth/defensive)	**Nil**	25/75	-25/+25		**100/0**	75/25	25/-25
Target income	$60,000				$40,000		
Withdrawal	Draw-to-target (with excess)				Draw-to-target (with excess)		
Representative metrics*							
Expected age of account exhaustion	**n.a.**	102			**96**	93	
Probability of shortfall vs. income target:							
Age 75 (Survival probability 91%)	**0%**	0%	0%		**2%**	0%	2%
Age 85 (Survival probability 66%)	**0%**	0%	0%		**31%**	36%	-5%
Age 95 (Survival probability 18%)	**0%**	16%	-16%		**51%**	65%	-14%
Age 105 (Survival probability 0.5%)	**0%**	69%	-69%		**62%**	76%	-14%
Minimum income	**61.6**	45.8	15.8		**20.0**	20.0	0.0
Risk-adjusted income, i.e., utility ($'000)	**61.6**	59.9	1.7		**38.0**	37.7	0.3
Median account balance $'000):							
Age 75 (Survival probability 91%)	**-**	284.2	(284.2)		**254.6**	221.6	33.0
Age 85 (Survival probability 66%)	**-**	196.6	(196.6)		**162.7**	90.6	72.1
Age 95 (Survival probability 18%)	**-**	89.7	(89.7)		**5.8**	-	5.8
Age 105 (Survival probability 0.5%)	**-**	-	-		**-**	-	-
Income Optimization Objective	High Wealth ($700,000)				Low Wealth ($300,000)		
Strategy	Optimal	Selected from Menu	Difference		Optimal	Selected from Menu	Difference
Life annuity allocation	**34.6%**	25%	9.6%		**16.9%**	0%	16.9%
Retirement savings account (growth/defensive)	**100/0**	25/75	-25/+25		**100/0**	75/25	25/-25
Withdrawal	Affordable (annuitization at AIR-1%)				Affordable (annuitization at AIR-1%)		
Representative metrics*							
Expected income ($'000)#	**74.6**	69.7	4.9		**40.6**	38.2	2.4
Median income at selected ages ($'000):							
Age 75 (Survival probability 91%)	**70.4**	67.4	2.9		**38.3**	36.9	1.4
Age 85 (Survival probability 66%)	**71.4**	69.1	2.4		**38.9**	37.8	1.1
Age 95 (Survival probability 18%)	**68.5**	66.3	2.3		**37.3**	36.2	1.1
Age 105 (Survival probability 0.5%)	**56.5**	53.9	2.6		**30.8**	29.1	1.6
Minimum income	**40.9**	37.9	3.0		**32.3**	30.0	2.3
Risk-adjusted income, i.e., utility ($'000)	**66.6**	64.7	1.9		**36.3**	35.2	1.0
Median account balance ($'000):							
Age 75 (Survival probability 91%)	**434.2**	489.6	(55.4)		**236.4**	279.8	(43.3)
Age 85 (Survival probability 66%)	**353.3**	391.4	(38.2)		**192.4**	223.7	(31.3)
Age 95 (Survival probability 18%)	**214.4**	231.1	(16.7)		**116.8**	132.1	(15.3)
Age 105 (Survival probability 0.5%)	52.4	54.5	(2.1)		28.5	31.2	(2.6)

* Full details found in Appendix D.

Expected income is higher than median income due to income being positively skewed.

The analysis helps highlight some features of the use of models to identify 'optimal' strategies:

- ***Models can provide useful insights into the kind of strategy that may be suitable.*** For instance, Table 10 reveals optimal allocations to the life annuity, thus giving an indication of what might be an appropriate level of annuitization for each retiree. Under the income target objective, the annuity allocation is 100% for the high wealth retiree and 0% for the low wealth retiree, while under the income optimization objective it is 34.6% for the high wealth retiree and 16.9% for the low wealth retiree. In addition, all optimal strategies hold 100% in growth assets within the retirement savings account. This is consistent with the rule suggested in Section 5.3 that retirees should hold the maximum growth asset exposure they can tolerate within the retirement savings account, while using lifetime income streams for defensive exposure.

- ***Models can generate strategies that may not be acceptable in practice.*** The 'optimal' strategy for the high wealth retiree with an income target objective suggests 100% annuitization. However, this partly reflects the analysis placing no value on access to capital and ignores the likely resistance of retirees to full annuitization. For the low wealth retiree with an income target objective, the 'optimal' allocation of 100% to growth assets in the retirement savings account might also be met with resistance. In both cases, the strategy selected for the retiree might reasonably differ from the optimal strategy indicated by the model.

- ***The optimal strategy depends on the model set-up.*** The optimal strategies presented in Table 10 reflect a highly simplified set-up with limited decision variables. The optimal strategy could differ if the model allowed for a wider range of investment products, scope to defer annuity purchase until later in retirement, and the ability to dynamically adjust the growth-defensive mix and the withdrawal strategy over time.[96] Modelling assumptions can also be influential to the optimal strategy that emerges from the model, e.g., allocation to the life annuity might depend on the interest rate or loading assumptions.

As a general rule, modelling of optimal strategies often indicates that life annuities should be used for defensive exposure as they hedge both investment risk and longevity risk, with the remainder invested in 100% growth assets to maximize expected income.[97] In this setting, lower appetite for income risk is accommodated by increasing the allocation to annuities, which reduces expected income but lowers income variability. Meanwhile, defensive assets such as fixed income tend to be crowded out by annuities as the primary defensive assets, except in limited situations.[98] However, such strategies are often resisted by retirees due to reluctance to annuitize and adopt 100% growth exposure within their retirement savings account. Narrow framing (see Section 6.3) may also play a role, to the extent that retirees view annuities and the retirement savings account allocation as separate rather than integrated decisions.

In summary, optimal strategies derived from quantitative modelling can be useful as a point of comparison, and for delivering insights into the type of strategy that might be suitable. However, output from any model is subject to the limitations of the model and model risk (see Section 3.3). Hence judgement also needs to be applied to account for this and other relevant factors, such as those discussed in Section 5.2.

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[96] Allowing for adjustment over time requires techniques such as dynamic programming, which significantly increases technical difficulty.
[97] For instance, see Butt, Khemka and Warren (2021, 2022).
[98] An example would be where a deferred life annuity is purchased, and defensive assets assist in managing income risk in the interim.

SECTION 6: Other Matters

This section acknowledges other matters not covered in previous sections that may be relevant for retirement strategy formulation more broadly. Brief overviews are provided, noting that in-depth coverage of these matters is beyond the scope of this Primer. Section 6.1 describes alternative methods for designing retirement strategies, including goal-based investing, liability-driven investing and income framing. Section 6.2 discusses the 'annuity puzzle', i.e., the reluctance to purchase retirement income stream products. Section 6.3 outlines behavioral influences that may affect retiree decisions. Section 6.4 lists various business considerations that can impact on retirement strategy design and selection.

6.1 Alternative Methods

This Primer discusses designing retirement income strategies by allocating assets to build income layers in pursuit of income objectives. This section describes other methods for designing retirement strategies in a broader sense. Some of these methods may result in similar strategies but differ in how decisions are structured and choices are framed.

6.1.1 Goal-based Investing and Bucketing

Goal-based investing (GBI) refers to explicitly designing strategies towards achieving particular goals or outcomes, rather than aiming to maximize the risk-return trade-off for a portfolio viewed in isolation, e.g., conducting mean-variance analysis. Indeed, the discussion in this Primer around achieving income objectives through building income layers might be seen as a form of GBI. Much of the GBI literature adopts a 'bucketing' approach, where assets are spread across pots directed towards achieving specific goals.[99] Bucketing might be applied within a retirement context as follows:

- **Minimum income bucket** – Some assets are allocated towards purchasing a retirement income stream to achieve the goal of securing a minimal (i.e., floor) level of income.

- **Regular income bucket** – An amount is set aside and invested in defensive assets to satisfy the goal of drawing regular income over the next (say) 3-5 years. This bucket would be occasionally topped up by transfers from the growth bucket, under a set of rules where no transfers are made when the growth bucket has declined in value.[100]

- **Growth bucket** – This bucket is invested in risky assets offering higher returns and is directed towards the goal of sustaining income (or building wealth) over the long run.

- **Other buckets** – Some assets might be allocated towards a bequest bucket, or perhaps a precautionary 'rainy day' bucket to support any large, unplanned expenditures.

Decomposing the portfolio into loosely connected buckets doesn't change the fact that there is one pot of assets funding all goals and runs some risk that the decomposition leads to a sub-optimal strategy in aggregate. Nevertheless, bucketing approaches offer advantages related to how retirees may engage with

[99] Brunel (2011) and Parker (2020) are a good starting point to access this literature.
[100] This rule acts as a mechanism to adjust overall portfolio weights towards risky assets when the latter have performed poorly and will be most effective if markets mean-revert. It is often motivated to address sequencing risk by avoiding selling assets that have declined in value.

retirement decisions. Requiring a retiree to explicitly consider their goals and priorities can be beneficial. Certain behavioral influences are also being exploited, such as narrow framing (see Section 6.3). For instance, the comfort that minimum income and regular income needs are 'locked-in' can boost confidence to invest in risky assets.

6.1.2 Liability-Driven Investing

Liability-driven investing (LDI) focuses on the use of assets to service some liability. It is often applied by first identifying the 'risk-free' asset that hedges the liability, and then deciding how much to invest in risky assets in pursuit of better outcomes. Nobel Prize winner Robert Merton has championed an LDI-based approach in a retirement income context based around cash flow matching (or immunization).[101] Merton identifies the baseline risk-free asset as one that delivers guaranteed real income during retirement, which might be a real life annuity or a ladder of inflation-protected bonds. Consideration is then given to how much additional risk to take in pursuit of higher income. An alternative LDI approach is to translate the retirement problem into asset and liability space and managing the 'funding ratio'.[102] Assets include the retirement savings account and the present value of any lifetime income streams, social security and defined benefit pensions. Liabilities are defined as the present value of future income needs.[103]

6.1.3 Income Framing

Some retirees invest to generate investment income (i.e., dividends, interest, etc.), with the intent of spending that income while leaving assets intact. This strategy makes a distinction between capital gains and other forms of investment income and will result in failure to convert assets into an income stream that is affordable given that assets are not being run down. It may even lead to assets growing in value and hence large bequests. It can also generate increasing income over retirement to the extent that dividends grow, at odds with the observed spending patterns of most retirees. An advantage is that focusing on long-term income generation encourages investing for the long run and helps mitigate the risk of over-reacting to market volatility. This approach might be suitable for retirees with strong bequest motives but is likely to be an inappropriate strategy for maximizing potential retirement income.

6.2 Annuity Puzzle

Many of the strategies outlined in this Primer employ lifetime income streams such as life annuities. However, retirees can be quite reluctant to purchase lifetime income streams,[104] notwithstanding that they often play a role within the 'optimal' strategies that appear in the academic literature. Table 11 summarizes many of the explanations offered for what is known as the 'annuity puzzle'. The key implication is the willingness of retirees to purchase lifetime income streams should be considered when designing and offering retirement income strategies, including how strategies might be presented to encourage acceptance. A brief discussion on addressing behavioral effects in the context of retirement strategies appears in Section 6.3.

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[101] See Merton (2014).
[102] See Waring and Whitney (2009) for discussion of the implementation of this style of analysis.
[103] This method requires the 'income' liability to be well-defined to establish its present value. This is the case under an income target objective, but not under an income optimization objective. Also, the relation between fluctuations in asset and liability values can be hard to characterize.
[104] For instance, only 31,689 (about 10%) of the 306,443 plans accessed in the UK in 2020-21 were used to buy an annuity according to The Financial Conduct Authority (see https://www.fca.org.uk/data/retirement-income-market-data-2020-21#:~:text=fully%20encashed%20plans.-,Key%20findings,fell%20by%209%25%20to%20341%2C404). The percentage in US is less than 10% (see Ramsay and Oguledo, 2018, p624).

Table 11

EXPLANATIONS FOR RELUCTANCE TO PURCHASE ANNUITIES[105]

Explanation	Notes
Loss of flexibility and control	• Annuity purchase sacrifices capital, and is generally irreversible • Loss of access to assets when needed, e.g., for large, unplanned expenditures such as health costs or aged care • Importance of this explanation is diluted by partial annuitization, i.e., some assets are invested in a retirement savings account with flexible access to the funds
Bequest motives	• Assets used for annuity purchase are unavailable for a bequest • Relevance of this explanation depends on a strength of any bequest motive • Partially addressed by refundable annuities that refund a residual nominal value of the purchase price not yet paid out upon early death
Other income sources	• Guaranteed income streams such as social security and defined income pensions may reduce the need for annuities, especially for retirees with modest assets • Family arrangements may provide additional income support and risk sharing
Cost (i.e., loadings)	Value can be eroded by various influences that may be built into any 'loading': • Adverse selection, i.e., healthy retirees are more likely to purchase annuities • Inability to fully hedge longevity and investment risk • Provider profit margins
Low interest rates	• Low interest rates make annuities appear poor value • This explanation has two shortcomings: - Views annuities in isolation: other assets may also offer lower expected returns - Value of longevity protection is largely independent of interest rates
Inflation risk	• Nominal annuities leave a retiree highly exposed to inflation risk • While mitigated by inflation-protected annuities, these are not readily available in all markets, including the US.
Low financial literacy	• Retirees could be unaware of annuities • Annuities might be seen as complex and hard to understand financial products
Incomplete annuity market	• Income streams might not match the income needs of a retiree, e.g., nominal annuities offer declining real income over time
Low retirement assets	• Some retirees have assets that leave them below purchase thresholds, or make the income stream provided seem paltry and not worth pursuing
Institutional influences	• Plan provider or financial adviser resistance to recommending annuities, as it may cede control over client assets and any associated fees (i.e., agency effects) • Processes, traditions and public policy may not encourage annuity purchase
Behavioral influences *(see Section 6.3 for expanded discussion)*	Various behavioral influences could discourage annuity purchase, including: • *Mental accounting* – annuities evaluated as a stand-alone purchase, rather than as a component of a broader retirement income strategy • *Framing* – annuities may be framed in ways that emphasize costs over benefits: - Annuities viewed as an asset purchase rather than an income stream - Focus placed on low income, lack of flexibility and loss of access to funds, rather than the longevity insurance benefits - Retiree focuses attention on possible loss of transfer of value upon early death • *Mortality salience* – aversion to annuities due to prompting thoughts of death • *Loss of control* – aversion of some retirees to conceding control over their assets

[105] This table draws on Ramsay and Oguledo (2018), who provide a detailed overview of the annuity puzzle, albeit with adjustments.

6.3 Behavioral Effects

Although not the primary focus of this Primer, behavioral effects are important as how retires engage with retirement decisions is highly relevant for the strategies they are likely to accept. Listed below are some relevant behavioral considerations in a retirement context,[106] followed by some mitigation techniques.

6.3.1 Behavioral Considerations for Retirees

Shu and Shu (2018) describe decumulation decisions as "*choice problems with large financial stakes and limited learning opportunities, difficult consumption trade-offs, multiple sources of uncertainty, issues of trust and branding, and long time periods.*" This provides fertile ground for behavioral effects to impact on retiree choices.[107] Discussed below are notable aspects that might influence the capacity of retirees to identify a retirement income strategy that is in their best interests.

- Distorted risk perceptions – The risks associated with retirement are prone to misinterpretation given long time horizons and complex trade-offs. Circumstantial evidence hints that some risks are over-estimated and result in retirees making what appear to be poor choices. Fears of running out of money seem overstated by some retirees, resulting in them dying with substantial assets intact after failing to draw the income they could have afforded. Pre-retirees tend to have heightened concern over the adequacy of retirement savings according to surveys,[108] which might contribute to over-saving in the first instance. Over-concern with short-term investment risk[109] may contribute to a propensity to limit exposure to risky assets, notwithstanding the enhanced possibility of sustaining higher income over the long run. Lack of risk composure[110] may lead to distorted risk perceptions, resulting in a tendency to invest more defensively after markets have declined.

- Narrow framing – Narrow framing emerges where retirees consider strategy building blocks in isolation, rather than as integrated. Table 11 listed some framing effects that might apply in the context of retirement income streams. For instance, a retiree viewing allocations to lifetime income streams and the retirement savings account as separate decisions may avoid income stream products due to focusing on lack of flexibility and access to capital, despite this being available through the retirement savings account. While narrow framing can be useful to help simplify decisions, it can also lead to choices that are counter to achieving overall objectives.

- Difficulty understanding mortality risk – Observed inconsistencies suggest that many retirees have trouble understanding uncertainty over the time of death and its implications for retirement income strategies. For instance, life expectancy tends to be underestimated earlier in retirement and over-estimated at older ages.[111] Further, the propensity for many retirees to over-insure

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[106] Statman (2019) provides an excellent overview of behavioral effects in an investment context. In particular, Chapter 2 presents an interesting discussion of needs and wants beyond increasing wealth, many of which are relevant for retirement. Shu and Shu (2018) discuss the psychology of decumulation decisions and provide an overview of the state of the literature.
[107] MacDonald (2013, Section 2) summarizes some of the findings from surveys of retirees on how they make retirement decisions.
[108] See Figure 1 in SOA (2020), for instance.
[109] The tendency to limit exposure to risky assets could stem from myopic loss aversion, which can induce the desire to avoid losses in the retirement savings account relative to its most recent value.
[110] Lack of risk composure (see Section 1.3) could arise from recency bias.
[111] See Wu, Stevens and Thorp (2015) for a study and discussion of distorted perceptions of life expectancy.

against longevity risk by under-spending is difficult to reconcile with widespread reluctance to allocate to lifetime income streams that offer a longevity hedge.

- **Impaired decision-making capacity** – The complexity of retirement heightens the importance of capacity for decision-making, particularly for retirees who do not seek financial advice. Unfortunately, low financial literacy is widespread. Further, most retirees will suffer cognitive decline as they age. This leaves many retirees at risk of making poor decisions, especially if presented with intricate strategies.

6.3.2 Addressing Behavioral Effects

Strategies should be designed to minimize adverse behavioral effects; and some techniques are outlined below. Communication of strategies is of equal importance but is outside the scope of this Primer.

- **Degree of paternalism** – Whether strategies should be designed to cater for the perceived needs and wants of retirees even if they are distorted, versus taking a paternalistic approach aimed at addressing unhelpful behavioral effects, is a debatable matter. Paternalism usually appears in the form of defaults that are imposed in the absence of action. Libertarian paternalism[112] presents a compromise under which strategies designed to mitigate adverse behavioral influences are presented as a nudge,[113] coupled with the choice to pursue another option. This could be achieved by offering a menu of products while recommending one as most suitable.

- **Simple and relatable** – Simpler strategies that are easier to understand are more likely to be understood and accepted by retirees. They also tend to be straightforward to implement.

- **Income framing** – Framing the design and presentation of candidate strategies around 'building income layers' places attention squarely on generating income,[114] and is preferable to focusing on asset allocation. In this context, lifetime income streams might be better presented as a way of locking-in a basic amount of income for life, rather than as an investment choice.

- **Provide confidence to take risk** – Allocating to lifetime income streams to secure a minimal level of income may be used as a way of providing extra confidence to take on risk within the retirement savings account. Similarly, additional confidence might be supplied by any carve-out of precautionary savings through providing comfort that contingencies have been covered.

- **Gradual implementation –** The strategy of gradual implementation may assist retirees to become comfortable with a strategy, and thus make it more likely that they will pursue it. For instance, annuitization might be put in place over a number of years to provide the retiree with an appreciation for how annuities work and help calibrate an appropriate amount to annuitize.

[112] See Thaler and Sunstein (2003).
[113] For example, both nudges and careful framing can be used where appropriate. Recommendations or tailored defaults combined with the ability to opt-out could be offered as a nudge, with the aim of directing retirees towards more optimal strategies. Such an approach might lead to better strategy choices than providing menu of options from which the retiree is left to choose.
[114] Bucketing approaches (see Section 6.1) also amount to a framing technique.

- **Robustness to cognitive decline** – Basing retirement income strategies on pre-determined rules puts in place an 'automatic pilot' in the event of cognitive decline. For instance, an account might be set up from which daily expenses are automatically paid, thus providing a safety net. More complex strategies that rely on retirees making explicit decisions as they age might be avoided.

6.4 Business Considerations for Plan Providers

Strategies ultimately need to be converted into *solutions* or *products* that both meet the needs of retirees and can be readily implemented at a reasonable cost by plan providers or financial planners. There is no point in designing strategies that no-one will want, do not add value, or a provider is not able to offer. Table 12 lists some of the business considerations that might be taken into account by plan providers.

Table 12
BUSINESS CONSIDERATIONS FOR RETIREMENT INCOME STRATEGY DESIGN AND SELECTION

Consideration	Notes
Organizational capability	• Governance, i.e., ability to successfully oversee the solutions included on the menu • Staff capability, e.g., complex strategies will require access to technical skills • System capability, e.g., ability of the customer management system to combine strategy building blocks and administer dynamic strategies for individual retirees • Capital requirements, e.g., may be required if offering lifetime income streams • Investment building blocks that are accessible, e.g., potential lack or ready access to inflation-protected annuities[115] • Outsourcing as an alternative: requires capability to select and monitor suppliers
Cost	• Strategy design should consider cost (relative to benefit)
Competitive Situation	• Whether a solution competes effectively with those offered by other providers • Whether a solution fills a gap in the market
Customer Acceptance	• Whether retirees will want the solution, i.e., it meets a need, and will not be rejected due to behavioral influences • Whether retirees can sufficiently understand the strategy underpinning the solution, i.e., communicable in simple terms • Whether financial planners will recommend the solution • Risk of creating a legacy product with low acceptance that is difficult to wind-up
Access to Personal Information	• Sufficient information is available on personal characteristics to support design and tailoring of strategies, and consequent matching of retirees to suitable solutions
Equity	• Implications for equity across retirees, e.g., unit pricing; potential for unreasonable 'inter-generational' transfers of value under group pooling
Regulation	• Consistency with regulatory requirements

[115] Inflation-protected annuities are not readily available in the US but are available in Australia, for instance.

SECTION 7: Limitations

This Primer has outlined the issues and methods for the design and evaluation of retirement income strategies. It focuses on the use of stochastic models to analyze income streams but recommends that models should not be relied on in isolation when selecting strategies, and that various qualitative factors should also be considered. Some concepts have been raised that are somewhat novel. The central idea that there are three main types of income objectives, and that strategy design and evaluation tools should be tailored accordingly, does not seem to be widely recognized. The proposition that utility functions and metrics both have a role to play and should be used in tandem is also notable as an approach that is currently not widely used in practice, which tends to focus analysis around a selection of metrics.

Retirement income strategies are an extensive and complex area, making it necessary to limit the scope and depth of the discussion. Some of the more important topics that have been mentioned but were not investigated in any depth include:

- Wide variety of withdrawal strategies that are available, which forms an extensive area in the literature;
- Income target objectives that are not fixed, but vary with age;
- The wide range of utility functions that might be applied;
- Role of bequest motives;
- Role of precautionary balances to allow for large expenditures that are not covered by regular income, such as health and aged care expenses;
- Investments beyond the general growth and defensive categories, such as individual asset classes and option-based strategies;
- Full range of retirement income products that are available;
- Dynamic investment and withdrawal strategies;
- Ongoing monitoring and review of strategies, to ensure they remain appropriate;
- Importance of housing, including the prospect of using home equity as a source of capital;
- Customer segmentation or cohorting;
- Process of identifying candidate retirement income strategies for further consideration; and,
- Communication of retirement income strategies to retirees.

The Primer has still covered a lot of territory, notwithstanding not having done justice to the above topics. It is hoped that readers will find considerable value in what has been addressed.

APPENDIX A: Underlying Drivers of Income Risk

Section 1.3 defines income risk as the "*possibility of failing to deliver sufficient income to sustain a desired standard of living until death*". Addressing income risk requires understanding the distribution of income, including what factors may result in failing to generate sufficient income. This Appendix investigates four key drivers of income risk: investment, sequencing, longevity and inflation risk. Other possible risk sources are briefly described. While the risk drivers are discussed separately, there are interactions and trade-offs. Key trade-offs are between investment risk and sequencing risk, and investment risk and inflation risk.

A.1 Investment Risk[116]

Retirement involves investing to generate income over a long horizon, possibly two-to-three decades or more. Long-term investment risk does not directly equate with the more familiar concept of return volatility, which only reveals the possibility of poor investment outcomes over a single period. From the standpoint of retirement income, *investment risk relates to compound returns being insufficient to support the desired standard of living.* A retiree should be concerned with the return achieved over the course of retirement, or perhaps more appropriately, the wealth that is generated from which income may be drawn (or bequests paid). Examining the *distribution of real wealth*[117] at the end of an appropriate investment horizon is a better way to evaluate investment risk over the long run than focusing on return volatility.

Any discussion of investment risk over long horizons ultimately runs into the issue of whether equities are less risky over the long run, and the debates on 'time diversification' and 'Kelly investing'.[118] The upshot is that higher returning but more volatile assets become increasingly *more likely* to generate higher wealth as the investment horizon lengthens. However, this comes with the ever-present possibility of even worse wealth outcomes in the lower tail. This strand of literature ultimately concluded that investor preferences are pivotal for the extent to which equities should be preferred over long horizons, namely how the investor might view the trade-off between the higher likelihood of better outcomes and a small possibility of landing up in the lower tail where risky assets deliver much worse outcomes.

Analysis by Warren (2021a) is drawn on to illustrate this trade-off and how it evolves with horizon. Real wealth paths arising from investing in equities and fixed income are projected over a 30-year horizon and compared with a target wealth trajectory implied by a real wealth accumulation objective of 'inflation (CPI) plus 3.5%' per annum. (The latter might be interpreted here as the real return required to deliver an income target to a desired level of confidence.) Return assumptions in this analysis include compound real expected returns of 6.0% per annum for equities with standard deviation of 18%, and 1.5% per annum for fixed income with standard deviation of 4.5%.[119] Figure 14 reveals the wealth paths implied by the expected returns. Not only does projected wealth generated by equities exceed the CPI+3.5% objective while fixed income falls short, but the gaps widen over time due to compounding effects. Of course, this is just an expected outcome around which there is uncertainty. Figure 15 conveys the distribution of the expected wealth paths relative to the target. The chart is formed by simulating 10,000 wealth paths

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[116] Warren (2021a) discusses investment risk over the long term in some depth.
[117] While the ultimate focus is the distribution of income, examining end-period wealth whilst ignoring inflows and outflows helps isolate the role of investment risk.
[118] Warren (2019, p46) provides a brief overview and cites some key papers.
[119] The assumptions used by Warren (2021a) differ to those used in the modelling used in this Primer.

assuming annual returns follow a lognormal distribution, estimating the ratio between the simulated wealth outcomes and the implied wealth target, and plotting the 1st, 10th, 90th and 99th percentiles.

Figure 14
ACCUMULATED EXPECTED REAL WEALTH VS. TARGET OF CPI+3.5%

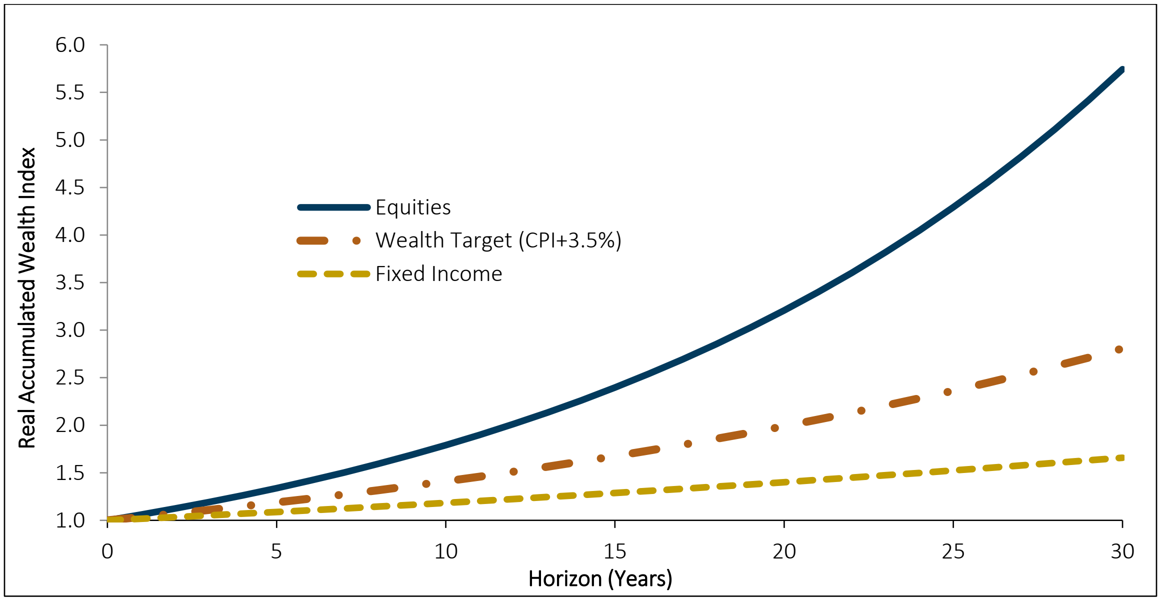


Figure 15
DISTRIBUTION OF EXPECTED WEALTH VS. TARGET

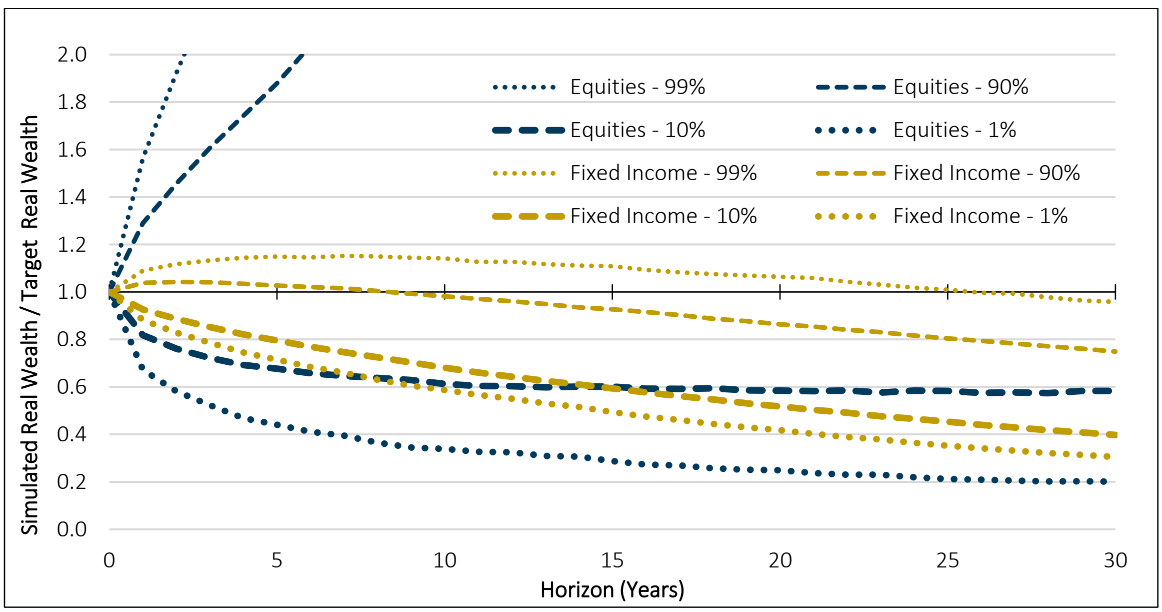


Figure 15 confirms that fixed income provides far less variable wealth outcomes than equities, noting that the percentiles are much more tightly clustered than equities. However, fixed income also delivers a probability of shortfall versus the CPI+3.5% target that is both higher than equities and increasing with horizon. For example, the 99th percentile outcome for fixed income crosses zero after 25 years, indicating a 99% probability of failing to achieve the target beyond year 25. Essentially, fixed income is almost certain to fall short of the target over long horizons. Equities not only offer a much higher chance of achieving and exceeding the target, but the probability that they will outperform fixed income also increases with time. For example, the 10th percentile outcome for equities is higher than the 10th percentile outcome for fixed income after year 15. However, equities always have some chance of generating a worse outcome than fixed income in the lower tail. This is represented by the 1st percentile outcome for equities standing persistently below the 1st percentile outcome for fixed income.

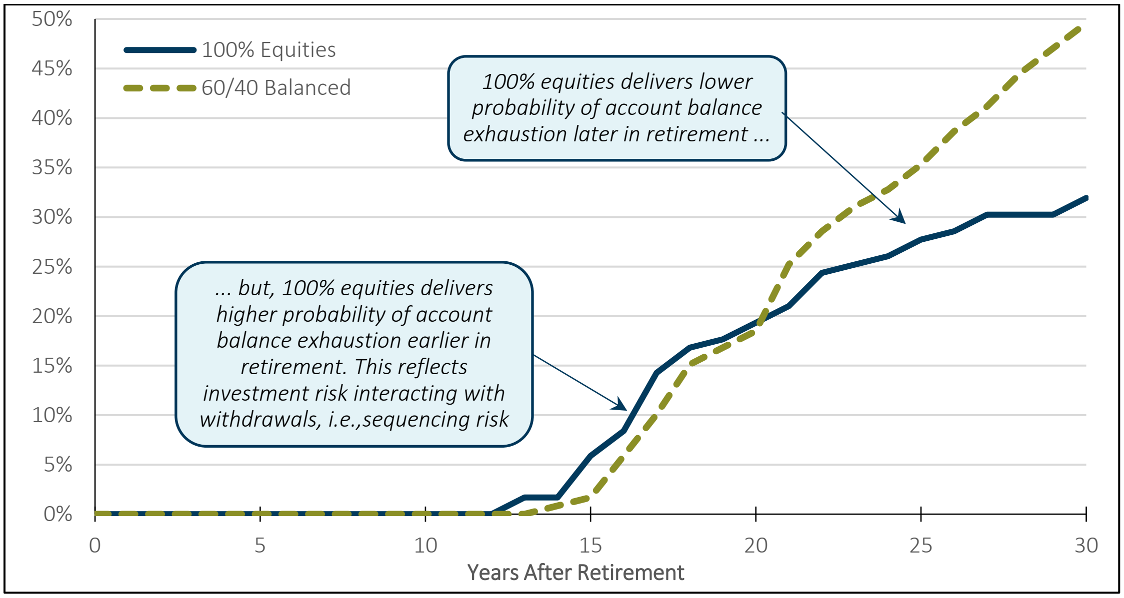
The implications for income risk during retirement are as follows:

- Investing more in risky assets increases the probability of being able to either:
  (a) generate greater income, or
  (b) sustain a given level of income for longer
- Investing more in risky assets with higher expected returns heightens the risk of substantially worse outcomes if returns are poor, resulting in either:
  (a) need to sharply reduce income, or
  (b) finding that a given level of income is sustainable over a shorter period.

A sense for the implications is provided by drawing on additional analysis by Warren (2021b), who estimates the probability of exhaustion of a retirement savings account over a 30-year horizon under a 6% initial withdrawal rate.[120] The analysis assumes a fixed drawdown, and hence effectively illustrates the two points listed as '(b)' immediately above.[121] The analysis is based around 'historical experience' simulations drawing on overlapping 30-year return periods observed over the period 1873 to 2020.[122] The analysis assumes that income is drawn until the retirement savings account balance is exhausted over each 30-year period appearing in the data. The percentage of simulations where the account balance is exhausted is then estimated. Analysis is undertaken for a balanced portfolio comprising 60% in equities and 40% in fixed income,[123] and a portfolio 100% invested in equities. Figure 16 plots the probability of account exhaustion for each year following retirement.[124] The heightened investment risk under 100% equities appears in the form of a higher probability of account balance exhaustion between year 13 and year 20, after which the probability of account balance exhaustion is increasingly larger for the 60/40 balanced portfolio. The higher probability of account balance exhaustion in the middle part of the period reflects poor investment returns interacting with withdrawals for a meaningful portion of the historical 30-year period (see sequencing risk discussion in Section A.2).

[120] This implies drawing a constant $6,000 in real terms for each $100,000 in initial balance. This is a variation on the 4% 'safe' withdrawal rule proposed by Bengen (1994), recalibrated towards a more affordable withdrawal rate for the purpose of illustration.
[121] The alternative is investment risk manifests as income variability rather than exhaustion of income, as implied by the points denoted (a).
[122] The series of yearly real returns for US equities (S&P 500 Index), 10-year bonds and 1-year bonds (mixture of government and commercial paper) over the period 1873-2020 are constructed by combining data from Robert Shiller's website (http://www.econ.yale.edu/~shiller/data.htm) and the Federal Reserve Board of St. Louis (see https://fred.stlouisfed.org/). Nominal returns are deflated by CPI.
[123] The fixed income component is made up of 30% in 10-year bonds and 10% in 1-year bonds.
[124] Under a deterministic analysis assuming the compound returns reflected in the data are realized, the balanced portfolio would be exhausted in year 40, whereas the 100% equity portfolio continues to grow in size, noting that the compound return on equities of 6.9% exceeds the 6% withdrawal rate. A portfolio 100% invested in fixed income would be exhausted during year 22.

Figure 16
PROBABILITY OF RETIREMENT SAVINGS ACCOUNT BALANCE EXHAUSTION – HISTORICAL SIMULATIONS



The key message is that accepting more investment risk through greater exposure to higher returning but more volatile assets has nuanced implications. While the likelihood of better income outcomes over the very long run is increased, some risk of even worse outcomes remains.[125]

Finally, investment risk may be further unpacked into the components related to cash flows, discount rates and reinvestment; all of which have differing implications for wealth accumulated over the short versus the long run. See Warren (2021a) for an in-depth discussion.

A.2 Sequencing Risk

Sequencing risk[126] relates to the sequence of returns (rather than their overall level) and how they interact with withdrawals. Investing in riskier assets and drawing more income can combine to increase the scope for faster depletion of the retirement savings account if poor investment returns are incurred, hence potentially reducing the ability to generate income moving forward. This is particularly the case when withdrawals are fixed in real dollar terms, as poor investment returns can result in a higher percentage of remaining assets being drawn, thus accelerating the depletion of assets. The impact of a sequence of poor returns also has greater impact when the account balance is higher, which is typically earlier in retirement.

Against this background, Figure 17 illustrates how sequencing risk translates into income risk. One 30-year return series is generated that delivers a retirement savings account that is exhausted in year 30 under an initial withdrawal rate of 6%. This return series has a compound real return of 2.85% per annum with standard deviation of 12.0%, similar to what might be expected for a typical balanced portfolio. Sequencing effects are gauged by reordering the return series and recalculating the trajectory of the account balance, and by implication the risk of being unable to sustain the income target implied by the 6% withdrawal rate.

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[125] This risk appears in Figure 16 as an increased likelihood of account balance exhaustion in the middle-years of the projection horizon, but could manifest as reduced income if a more flexible withdrawal strategy is pursued.
[126] Clare et al. (2020) provide an overview and analysis of sequencing risk.

One series is created where the returns are re-sequenced from lowest in year one to highest in year 30, which represents the worst possible return ordering. The result is that the account balance is exhausted in year nine. A 'best order' series is also created, where returns are sequenced from highest to lowest. This results in a real residual (real) account balance at year 30 of 109% of the initial investment. Ten random re-sequencings are then generated for illustration, resulting in a spread of outcomes that sit between the worst order and best order outcomes. The wide variation seen in Figure 17 confirms that sequencing risk has potential to translate into a meaningful level of income risk.

Figure 17

RETIREMENT SAVINGS ACCOUNT BALANCE UNDER RE-ORDERED RETURN SEQUENCES

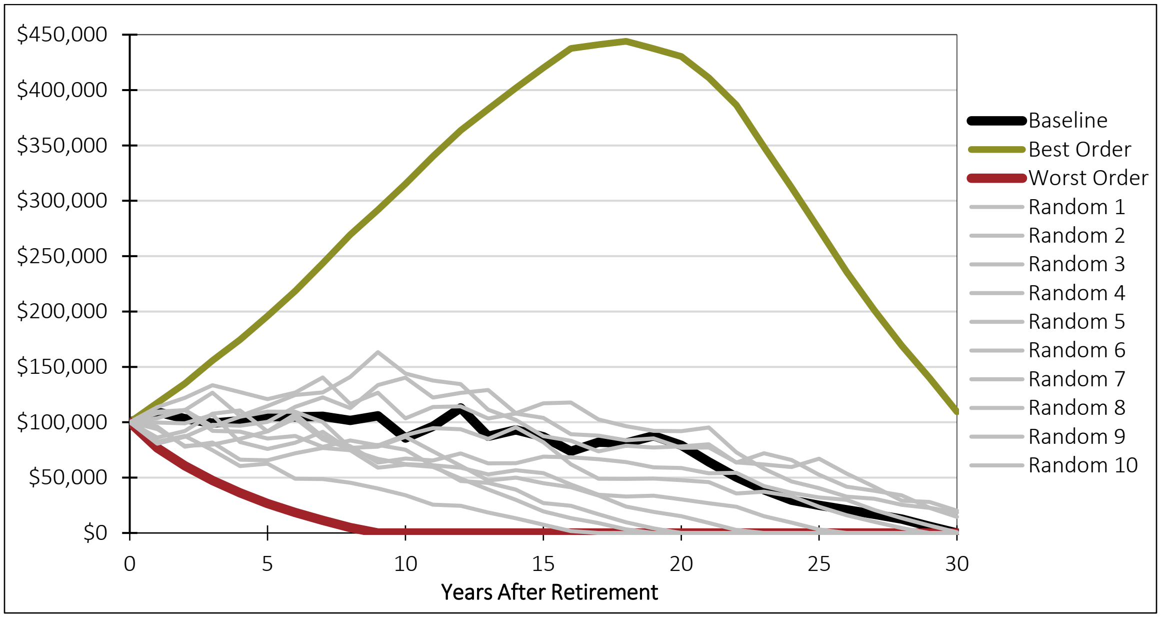


Figure 17 (and Figure 16) are formulated under the assumption of fixed withdrawals. While basing withdrawals on a percentage of the account balance would limit sequencing risk, it will have an impact nevertheless as withdrawals are still being made.

A.3 Longevity Risk

Longevity risk relates to the failure to sustain a desired standard of living due to living longer than planned. Longevity risk manifests as income risk through the retirement savings account being eroded to a level at which the desired income cannot be supported as the retiree ages. As discussed in Section 1.3, longevity risk should be viewed in the context of the trade-offs surrounding uncertain mortality, in particular the potential reduction in expected income that arises from accepting lower income to hedge against living longer. Lower income may come in the form of either reducing withdrawals from the retirement savings account with a view to spread income over a longer period, or the impact on expected income from purchasing a lifetime income stream such as a life annuity.[127]

[127] Life annuities act to reduce expected income due to being based around fixed income and the impact of loadings. The impact of these influences may be mitigated through variable annuities and pooled arrangements. See Section 2.1 for further discussion.

The implications of longevity risk for income risk tend to become more important than investment risk later in retirement. This is because uncertainty over how much longer the retirement savings account needs to last begins to outweigh the influence of investment risk as the retiree ages. Collie (2016) presents an analysis of the relative magnitude of investment and longevity risk with reference to confidence intervals around the income that might be generated from available assets at varying ages. Under his assumptions, the relative magnitude of longevity risk becomes "equivalent to a fairly substantial level of investment risk" beyond age 80.

A.4 Inflation Risk

Inflation risk, or cost-of-living risk, relates to whether the income generated might be insufficient to sustain the desired standard of living due to increased prices for items purchased. Inflation risk should be related back to the investment strategy, rather than considered in isolation. A retiree is hedged from inflation risk to the extent that return on investments and the income generated from any lifetime income streams adjust with inflation. Any incomplete adjustment leaves the retiree exposed to inflation risk.

From this perspective, the following aspects may be considered:

- The extent to which investments offer inflation hedging might be taken into account in forming the investment strategy. For instance, nominal annuities are exposed to inflation risk, whereas inflation-protected annuities offer an inflation hedge. Long duration nominal bonds are also exposed to inflation risk, whereas short duration fixed income may offer some protection to the extent that short-term interest rates adjust with inflation. While inflation appears to hurt equities in the short term, equities may offer some inflation hedging over the long term.

- The inflation expectations built into asset prices matters. Whether inflation deviates from market expectations is more relevant than its actual level. For example, the income support provided by long duration bonds will be enhanced if inflation turns out to be lower than expected as they will generate higher real returns than anticipated, and vice versa. The exception is inflation-protected assets that adjust in value based on the actual rather than expected inflation.

- Access to inflation hedging may come at a cost of lower real returns, which may inhibit the ability to sustain a desired level of standard of living. For example, US treasury inflation-protected securities (TIPs) provide inflation-protected cash flows but have been known to trade on negative real yields, for instance through most of 2020 and 2021. Low-to-negative real yields also bring high sensitivity of bond prices to yield changes, which can heighten exposure to sequencing risk if the bond needs to be sold to fund income.

- The relevant inflation rate is the change in the price of a basket of goods and services desired by the retiree. Problems emerge when cost of the desired items grows at a faster pace than the general rate of inflation that is either priced into investment markets or used to adjust income streams. In this regard, inflation risk is more relevant for non-discretionary items such as food and rent, where there is typically less scope to reallocate spending from high priced to low priced items.

A.5 Other risks

Table 13 recognizes other risks that could impact on capacity to generate income. The risks listed tend to be more context dependent than the four key risk drivers above but can nevertheless be significant at times.

Table 13
OTHER RISK DRIVERS

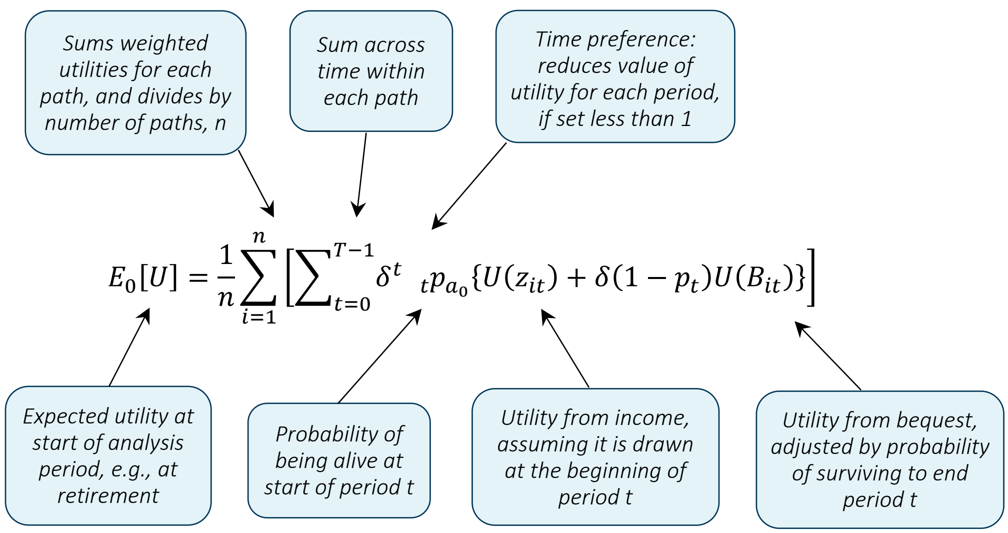
Risk Driver	Nature
Public policy risk	• Possibility that the rules and regulations change in a way that adversely impacts the capacity to generate income
Counterparty risk	• Product provider fails to deliver on promises, perhaps due to bankruptcy, leading to either loss of asset value or failure to continue to provide income • Particularly pertinent for annuities, where providers are relied on to deliver an income stream that potentially spans decades
Large, unplanned expenditure, e.g., health, aged care	• Can result in income risk by diverting assets away from income generation • Might be seen as a type of 'cost-of-living' risk, albeit driven by spending needs rather than inflation in the prices of goods and services
Decline in functional status	• Cognitive decline with aging is the primary risk • May result in income being impaired through poor decision making, or vulnerability to financial deception
Family risk	• Disruption stemming from loss of partner, or family members needing assistance

APPENDIX B: Utility Functions – Technical Details

This Appendix provides guidance on how utility functions may be applied and presents selected formulae.[128] The focus is evaluating income under loss aversion and risk aversion utility functions. A summary of the treatment of bequests and other utility functions is also addressed for completeness, while noting that this Primer focuses on retirement income and the examples presented exclude bequest motives. The general formula for expected utility including bequests is initially presented, reflecting that seen in the academic literature. Restricted formulas for utility from income alone are subsequently provided, including those under loss aversion and risk aversion utility functions. Estimation of various components of the formula and alternative treatments are then discussed.

B.1 General Formula for Expected Utility

The general formula below describes how expected utility is calculated by first generating an expected 'per period' utility for each path *i* based on evaluating 'outcomes' using a utility function and applying time-weights, with expected utility then estimated by averaging across all paths. 'Outcomes' may comprise of both income[129] while alive and any bequest upon death. The formula assumes income occurs at the beginning and bequests occur at the end of the period, and that every path occurs with equal probability. It is possible to weight the paths by the probability that they occur: this may be useful for scenario analysis.



$$E_0[U] = \frac{1}{n}\sum_{i=1}^{n}\left[\sum_{t=0}^{T-1}\delta^t \; {}_tp_{a_0}\{U(z_{it}) + \delta(1-p_t)U(B_{it})\}\right]$$

Variables: $E_0[U]$ is expected utility at *t* = 0 for a retiree aged a_0
$i = 1,2,\dots,n$ are the paths simulated
$t = 0,1,\dots,T-1$ are the periods simulated in each path
δ is the time preference parameter, typically less than 1
p_t is probability of survival from age a_{0+t} until age a_{0+t+1}
${}_tp_{a_0}$ is probability of survival from age a_0 until age a_{0+t}
$z_{i,t}$ is the income in path *i* during period *t*

[128] This Appendix draws on Warren (2019), Butt, Khemka and Warren (2022) and Warren (2022).
[129] The literature typically refers to consumption, although this might be taken a synonymous with income assuming the latter is spent.

$U(z_{i,t})$ is the utility from income in path *i* during period *t*
$U(B_{i,t})$ is the utility from bequest in path *i* paid during period *t*

In situations where utility is based entirely on income (i.e., bequests are ignored), a scaling adjustment may be applied to the survival probabilities so they sum to one.[130] This translates total expected utility into a 'per period' expected utility, which can be converted directly into a certainty equivalent income measure, i.e., *RAI* and *CRADT*. The formula below describes this approach.

$$E_0[U] = \frac{1}{n}\sum_{i=1}^{n}\left[\sum_{t=0}^{T-1} \delta^t \frac{{}_tp_{a_0}}{\sum_{t=0}^{T-1} {}_tp_{a_0}} U(z_{it})\right]$$

Probability of surviving to time t, scaled by sum of survival probabilities over the projection horizon T. Ensures weights sum to 1.

Utility arises from income only

Where analysis is based on a fixed planning horizon of T periods without considering survival probabilities, utility from income in each period might be equally weighted by $\frac{1}{T}$ rather than by ${}_tp_{a_0}$, prior to imposing any time preference.

The above approach envisages what is known as 'time-separable' utility functions, which facilitate adding up utility over time and entail relatively straightforward calculations. This simple adding-up is not feasible under some more complex formulations that allow for various dynamics, such as recursive utility functions[131] or dynamic programming, which are treated as beyond the scope for this Primer. Time separable utility functions should suffice in many cases.[132]

The structure of utility $U(z_{it})$ under the income target and income optimization objectives is now shown in Appendix B.2 and B.3, respectively.

B.2 Income Target: Loss Aversion Utility Using the Prospect Theory Value Function

The value function from prospect theory, which this Primer applies as the chosen utility function under the income target objective, appears as the set of formulas below. Which of three formulas is applied depends on whether that income is above, equal to or below the income target ($\overline{z_t}$). The function produces utility values that are positive above the target, negative below the target, and zero when equal to the target.

$z_{it} > \overline{z_t}$ *(gain):* $\quad U(z_{it}) = (z_{it} - \overline{z_t})^{\propto}$

$z_{it} = \overline{z_t}$*:* $\quad U(z_{it}) = 0$

$z_{it} < \overline{z_t}$ *(loss):* $\quad U(z_{it}) = -\lambda\left((\overline{z_t} - z_{it})^{\beta}\right)$

[130] Dividing by the sum of the survival probabilities is feasible as it effectively scales all utility values by a constant, bearing in mind that only the relative values and not the levels of the utility scores are relevant.
[131] Under some utility functions, expected utility needs to be calculated recursively, i.e., starting at the end, and adding backwards. In this case, the pattern of outcomes through time matters, and not just their overall level.
[132] This issue is discussed by Warren (2019, pp 26-27).

Variables: $\overline{z_t}$ is the income target during period *t*
λ is the loss aversion parameter, i.e., weighting on losses
$\propto$ is the curvature parameter on gains
β is the curvature parameter on losses

The absolute difference between income and the target are moderated by 'curvature' parameters ($\propto$, β) as income moves away from the target. Any below-target values are further adjusted by a loss aversion or 'weighting' parameter (λ), which applies a multiplier to the curvature-adjusted scores on losses. If the curvature parameters on gains and losses are the same, then a loss aversion parameter of (say) two will attach a negative score to income of -$X below that is double the positive score on income +$X above the target. Scores under this utility formulation are reported as dollar values.

The curvature parameters typically used under prospect theory are less than one, implying a diminishing rate of change in utility when moving away from the target. Income gains thus yield diminishing marginal utility. The reverse occurs for losses, where the utility decrease from larger losses is proportionately less than smaller losses. This element implies risk-seeking when income is below target. That is, an individual faced with below-target income gambles to get back to the target, as a lessening of the loss brings more utility than increasing the loss by the same amount. [133]

Constant Risk-Adjusted Difference from Target

It is useful to locate the constant (i.e., riskless) income stream that delivers the equivalent expected utility as the stochastic (i.e., risky) set of outcomes. However, two complications arise under loss aversion utility. First, different equations need to be applied depending on whether expected utility is a positive number (i.e., above target) or a negative number (i.e., below target). Second, it is feasible that the income target may vary over time. To deal with the latter, a constant risk-adjusted difference from target' ($CRADT$) can be estimated as the notional risk-free difference between income and the target over retirement that delivers the equivalent utility as the risky income stream. If the income target is constant, $CRADT$ can be added to the target to arrive at an estimate of risk-adjusted income (RAI) if desired.

The formulas below outline the calculation of $CRADT$ depending on the zone that $E_0[U]$ falls into. Note that this formula assumes that $E_0[U]$ is average per-period expected utility as described in Appendix B.1, which assumes no bequest motive.

$$E_0[U] > 0: \qquad CRADT = E_0[U]^{1/\propto}$$

$$E_0[U] = 0: \qquad CRADT = 0$$

$$E_0[U] < 0: \qquad CRADT = -\left(\frac{-E_0[U]}{\lambda}\right)^{1/\beta}$$

B.3 Income Optimization: Risk Aversion Utility Function

The formula below for risk aversion (i.e., power) utility is applied directly to the level of income. It has one parameter, being the co-efficient of relative risk aversion (ρ).

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[133] The suitability of the assumption of risk seeking in the realms of losses might be questioned for retirees who rely on income for living expenses. An alternative course of action might be to accept some shortfall versus target and aim to secure as much income as possible, rather than taking any risk of an even larger shortfall. The latter would imply a curvature parameter on below-target income in excess of one, which if applied, would most probably require some adjustment to the loss aversion parameter to those typically used.

$$U(z_{it}) = \frac{z_{i,t}{}^{(1-\rho)}}{1-\rho} \qquad \textit{Note: if } \rho = 1 \textit{ then, } U(z_{it}) = ln\left(z_{i,t}\right)\text{, i.e., log utility}$$

Variables: ρ is the risk aversion parameter

Risk Adjusted Income
The formula below estimates certainty equivalent income, denoted here as risk-adjusted income (RAI), assuming no bequest motive. Where a bequest motive exists, RAI might be estimated through numerical methods, i.e., estimating utility for a range of constant income values under the assumptions governing the analysis, then search for the constant income that matches expected utility for the stochastic outcomes.

$$RAI = (1-\rho)E_0[U] > 0^{\left(\frac{1}{1-\rho}\right)} \qquad \textit{Note: if } \rho = 1 \textit{ then, } RAI = exp\left(E_0[U] > 0\right)$$

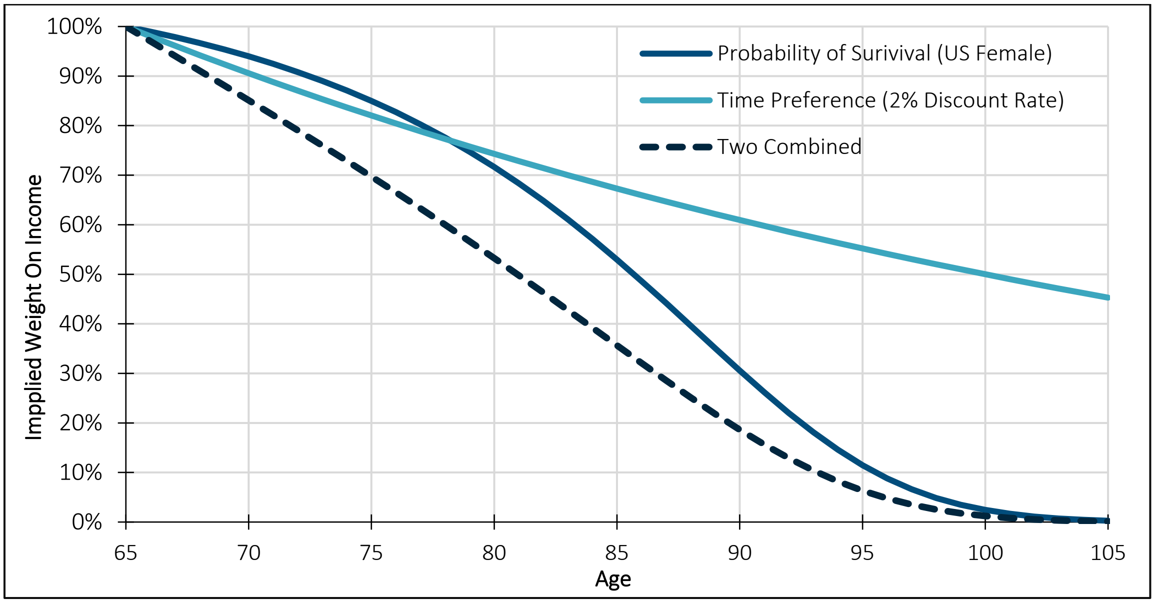
B.4 Note on Time Weighting

Section 4.2.5 discussed the application of time weighting by applying either survival probabilities and/or discounting by time preference. Figure 18 reveals the potential effect for a US female of age 65 of applying survival probabilities,[134] a discount rate of 2%, or both combined. The series plotted are the multiplier that would be applied to utility from projected income from age 66 through to age 105. Applying a probability of survival can have a substantial impact especially at older ages, acting to reduce utility scores at age 75 to 85% of their raw value, then to 53% at age 85, 11% at age 95, and close to 0% by age 105. If combined with a 2% discount rate, these percentages decline to 70%, 36%, 6% and even closer to 0%. Bearing in mind that what matters is the relative utility scores assigned to various outcomes, applying time weights can result in income earlier in retirement becoming more influential relative to income at very old ages in determining overall expected utility and hence the ranking of strategies. It effectively takes the stance that income earlier in retirement is highly preferred and more highly valued, in a large part because it is more likely to be enjoyed.

[134] Probability of death at each age is sourced from the Social Security Administration 'Period Life Table, 2019' found at https://www.ssa.gov/oact/STATS/table4c6.html, and accumulated to generate the cumulate chance of survival up to age 105.

Figure 18

IMPLIED WEIGHTS ON INCOME AT VARIOUS AGES (EVALUATED AT AGE 65)



B.5 Illustration of Utility Calculations for Income Streams

Table 14 illustrates the calculation of expected utility and related certainty equivalents $CRADT$ and RAI, applying the formulas set out above. This simplified analysis is based on three income paths (A, B, C) spanning three periods (1, 2, 3). Equal weights and survival probability weights (as appearing near the top of the table) are both applied in aggregating over the three periods, with equal weighting representing a fixed planning horizon. The two weighting schemes generate differing values for expected income, distance from target and utility, as well as $CRADT$ and RAI. Explanatory text bubbles are provided to assist with interpretation. Table 14 shows that converting a series of income paths into utility estimates using the formulas presented further above is relatively straightforward and can be readily performed in a spreadsheet package.

Table 14
UTILITY CALCULATION EXAMPLE

	Periods			Weighting of Periods		Assumptions	
	1	2	3	Equal	Survival		
Weights							
Probability of Survival	90%	50%	10%				
Survival Weights	60.00%	33.33%	6.67%		100%		
Equal Weights	33.33%	33.33%	33.33%	100%			
Income Paths ($)							
A	27,500	30,000	30,000	29,167	*28,500*		
B	25,000	25,000	20,000	23,333	*24,667*		
C	22,500	20,000	10,000	17,500	*20,833*		
Expected Income	*25,000*	*25,000*	*20,000*	***23,333***	***24,667***		
Loss Aversion Utility							
Difference from Target							
A	2,500	5,000	5,000	4,167	*3,500*	Income Target	25,000
B	0	0	-5,000	-1,667	*-333*		
C	-2,500	-5,000	-15,000	-7,500	*-4,167*		
Expected Difference	*0*	*0*	*-5,000*	***-1,667***	***-333***		
Utility Calculations							
A	978	1,799	1,799	1,525	*1,306*	Parameters:	
B	0	0	-4,048	-1,349	*-270*	Loss aversion	2.25
C	-2,200	-4,048	-10,645	-5,631	*-3,379*	Curvature on gains	0.88
Expected Utility	*-407*	*-750*	*-4,298*	***-1,818***	***-781***	Curvature on losses	0.88
Constant Risk-Adjusted Difference from Target (*CRADT*, $)				-2,014	-771		
Risk Aversion Utility							
Utility Calculations							
A	-0.000036	-0.000033	-0.000033	-0.000034	*-0.000035*	Parameters:	
B	-0.000040	-0.000040	-0.000050	-0.000043	*-0.000041*	Risk aversion	2
C	-0.000044	-0.000050	-0.000100	-0.000065	*-0.000050*		
Expected Utility	*-0.000040*	*-0.000041*	*-0.000061*	***-0.000047***	***-0.000042***		
Risk-Adjusted Income (*RAI*, $)				21,054	23,844		

Rescaling of survival probabilities so weights sum to 100%

Weights used for expected value calculations

Weighted 'per period' expected utility …

… converted into a certainty equivalent

Risk aversion (power) utility numbers meaningless in isolation

Weighted 'per period' expected utility …

… converted into a certainty equivalent

Survival probabilities place more weight on outcomes earlier in retirement

B.6 Bequests

An explanation of the bequest term is provided for completeness, noting that the prime focus of this Primer is retirement income. The academic literature largely focuses on bequests under risk aversion (power) utility coupled with survival probability weightings. After addressing bequests from this perspective, a suggestion is made on how bequests might be treated under loss aversion utility.

The formula below is commonly used in the literature to represent the utility from leaving a bequest:

$$U(B_{it}) = b\,\frac{\left(\left(\frac{\phi}{1-\phi}\right)z_b + B_{i,t}\right)^{1-\rho}}{1-\rho}, \quad \text{where } b = \left(\frac{\phi}{1-\phi}\right)^{\rho}$$

Variables: $U(B_{it})$ is the utility generated by a bequest paid in path *i* during period *t*
$B_{i,t}$ is the bequest amount paid in path *i* during period *t*
$z_b \geq 0$ is an income threshold above which bequests are treated as 'luxury goods'
ϕ is a parameter within the range [0,1] that controls strength of the bequest motive
ρ is the risk aversion parameter

Under the formula above, $\phi = 0.5$ leads to a multiplier on the bequest term of 1-times so that it is valued equally to the income stream (prior to application of survival probabilities and time preference). The bequest motive strengthens as ϕ increases toward one.[135] This formula can be interpreted as establishing a trade-off between any income above the income threshold and bequests. Understanding this trade-off is made complicated by income being experienced as a stream of outcomes over time, whilst bequests are a one-off outcome notionally experienced upon death. See Lockwood (2018) for discussion of the above formula; and Cocco, Gomes and Maenhout (2005) for intuition around the trade-offs and the relative weights placed on bequests and consumption.

A bequest motive might be incorporated into loss aversion utility by setting a bequest target (i.e., a $\bar{B}$), perhaps reflecting the 'number of years of income' that the retiree desires to leave as a bequest. Utility from bequests would then be estimated using the same formulas as outlined above for loss aversion utility, with the bequest target operating as the reference point. Setting $\bar{B} = 0$ treats any bequest as an incidental gain by applying the curvature for gains. Under a strong bequest motive, $\bar{B}$ would be set at greater than zero, so that failure to achieve the bequest target is evaluated as a loss to be traded off against income.[136]

B.7 Other Utility Functions

With a multitude of alternative utility functions available, the aim here is to recognize and briefly describe those that may be useful for modelling retirement income strategies. The relationship to the three income objectives described in Section 1.1 is described where relevant.

- Hyperbolic absolute risk-aversion (HARA) – This is a general family of functions that includes power, quadratic, exponential and log utility, among others. It accommodates setting absolute

[135] The literature has often applied ϕ approaching one, with Lockwood (2018) using ϕ *=0.96* for instance. This literature generally aims to calibrate models to fit data containing bequests, with a view of establishing how a bequest motive might explain observed behavior. A high ϕ is typically required to make bequests sufficiently meaningful so that the model fits the data, as the application of survival probabilities and time preference combine to reduce the weighting on bequests given that death is more likely later in retirement.

[136] For example, assuming $\bar{z}$ of \$25,000 and $\bar{B}$ of \$150,000 = 6$\bar{z}$ would imply any bequest below \$150,000 would be evaluated as a loss. It would create an incentive to limit $z_{i,t}$ to less than \$25,000 in order to build B until the utility loss associated with $z < \bar{z}$ and $B < \bar{B}$ are equalized. This formulation follows the intuition expressed in Cocco, Gomes and Maenhout (2005).

and relative risk aversion as either increasing, decreasing or constant in wealth. Power utility assumes constant relative risk aversion, implying that preferences are directly proportional to the overall level of wealth, i.e., the level of wealth does not impact on decisions. The HARA family supports a wider range of possibilities under the income optimization objective, such as allowing for decreasing risk aversion as wealth increases.

- Income (consumption) floor – This function is part of the HARA family. Effectively it applies the power utility function to income in excess of a floor and so can capture both income optimization and income floor objectives. In broad terms, it behaves similarly to a dislocated power utility function, with a heightened utility penalty being applied as consumption approaches the floor. This function is one way to incorporate the influence of an income floor directly into the utility function,[137] and is used by some researchers.[138]

- Epstein-Zin[139] – This utility function is relatively widely used in academic studies as an alternative to power utility for income optimization. The function decouples the preference for earlier versus later resolution of uncertainty ('elasticity of intertemporal substitution') from risk aversion related to volatility in outcomes. Epstein-Zin utility is calculated recursively, meaning that simple aggregation of utility values over time is not feasible.

- Habit persistence – This function evaluates utility relative to a dynamically estimated reference level and might be applied under an income target objective where the target varies over time. One application in a retirement setting might be to tie the income target level to recent income, thus capturing a desire to sustain income as currently experienced and in particular avoid any downward income shifts. Another application would be to define the reference level based on the income of peers.[140]

- Safety-first – This minimizes the risk of falling short of a reference point while placing no value on exceeding it. It might be applied under an income target objective where there is an overarching desire to achieve an income target as far as it is possible, along with no concern with above-target income.[141]

[137] The floor needs to be set below the feasible income range, otherwise utility becomes undefined. This function induces a strong desire to avoid income falling towards the floor, and locking in the floor if possible through purchasing lifetime income streams. A consumption floor with a lower risk aversion parameter has quite similar properties to power utility with a higher risk aversion parameter.
[138] For example, see Iskhakov, Thorp and Bateman (2015).
[139] Epstein and Zin (1989).
[140] These two applications align with internal habit formation and external habit formation, respectively, as discussed by Grishchenko (2010).
[141] Roy (1952) proposed the safety-first criterion. The parameters of Blake, Wright and Zhang (2013) as used here for high loss aversion are similar to safety-first preferences, given the relatively large discount placed on above-target income by a curvature parameter of 0.44.

APPENDIX C: Potential Metrics and their Calculation

This Appendix presents a wider range of metrics than the 'primary' metrics suggested and applied within the body of this Primer. It also provides formulae. The list is reasonably comprehensive, although not all known metrics are covered. Other overviews of available retirement metrics include MacDonald et al (2013) and Callil, Danziger and Sneddon (2018). Table 15 summarizes the metrics, arranged into the five classes as identified in Section 4.3.

Table 15

OVERVIEW OF SELECTED RETIREMENT METRICS

Metric Class	Purpose and Link to Income Objectives	Formats *(refer to Appendix B and C for estimation details)*
Distribution of income	Characterizes expected level and variability of income over time: • More relevant under income optimization, under which income tends to vary over time • Under income target, may reveal how far income may fall once retirement savings account is exhausted • Income percentiles most useful for portraying entire distribution over time	1. *Expected income* - Over projection horizon# - At selected ages 2. *Standard deviation of income* - Over projection horizon# - At selected ages 3. *Income percentiles* - At selected ages
Likelihood of income shortfall	Risk of income falling below a reference level: • Most relevant under income floor and income target, where an explicit reference level exists • Under income optimization, might be applied by using expected income as a reference level, although this may be difficult to interpret	1. *Probability of shortfall* - At any time over projection horizon# - At selected ages 2. *Expected years of shortfall* - Unconditional* over full projection horizon# - Conditional* on shortfall occurring
Magnitude of income shortfall	Captures size of potential shortfall in income below a reference level: • More relevant under income floor and income target, as explicit reference level exists • Under income optimization, might be applied by using expected income as a reference level, although this may be difficult to interpret • Reporting minimum income conveys where income would fall if assets were exhausted	1. *Expected shortfall* (in income per period) - Unconditional* over full projection horizon# - Conditional* on shortfall occurring - For selected ages 2. *Minimum income* - Sum of social security, lifetime income streams, etc. - If constant in real terms, single value may suffice - If varies over time, estimate value at selected ages
Remaining assets *(Retirement savings account balance might be used as proxy, e.g., examples in this Primer)*	Examining the remaining assets is useful as: • Reveals amount that can be flexibly accessed, including for unplanned spending or bequests • May indicate inefficient strategy, if excessive • Potential age that assets are exhausted is of interest under an income target, to indicate when income declines to minimum levels	1. *Value of remaining assets* - Expected value at selected ages - Percentiles at selected ages 2. *Expected age of exhaustion* 3. *Probability of exhaustion of retirement savings* - Over projection horizon# - At selected ages
Utility-based (i.e., certainty equivalents)	Summarizes entire distribution of outcomes: • More interpretable by converting expected utility into certainty equivalent income	1. *Risk aversion utility:* - Risk-adjusted income (*RAI*) 2. *Loss aversion utility* - Constant Risk-Adjusted Difference from Target (*CRADT*) - Given constant target: *RAI = Target plus CRADT*

Projection horizon could either a fixed planning horizon (e.g., 30-years) or to an unlikely age of survival (e.g., 110).

* Unconditional refers to average across all periods and provides an indication of expected shortfall. Conditional refers to average for the periods where shortfall occurs and provides an indication of the magnitude of shortfall that may be experienced once it happens.

C.1 Metric Calculations

This Appendix sets out the formulae for metrics identified in Table 15. See Appendix B for the formulae for the utility-based metrics.

C.1.1 Comment on Weighting Methods

The metrics presented below apply equally-weighting to each path.[142] However, differing time weights are applied *within* each path for projection over a fixed planning horizon (e.g., 30-years) and projection to an unlikely age of survival (e.g., 110) in the following manner:

- *Fixed planning horizon* – Each period within a path is equally weighted for metrics that involve either income streams or shortfalls that occur over the course of the path, such as expected shortfall and years of shortfall. For metrics comprising a single point value, such as the probability of shortfall and probability of asset exhaustion, the metric represents the probability of experiencing the 'failure' of concern during the planning horizon.

- *Unlikely age of survival* – Metrics for each path are estimated by applying survival probabilities to the values. The effect for income streams and shortfall is to weight the values by the likelihood that the retiree will be alive to experience them. Applying survival weights turns the probability of shortfall and asset exhaustion into a joint probability, i.e., the probability of experiencing the 'failure' and being alive.

C.1.2 Definitions and Notation

Below is a list of the notation that is used across all metrics, which is consistent with the notation used in Appendix B.

- a_0 denotes the current age.
- z_{ij} is the income in the i^{th} simulated path $(i = 1, \ldots, n)$ in the t^{th}period $(t = 0,1, \ldots, T-1)$, where $T-1$ represents the final period simulated. This may represent a fixed planning horizon of T years or a maximum age before death of a_{0+T}.
- p_t denotes the probability of survival from age a_{0+t} until age a_{0+T+1} .
- ${}_tp_{a_0}$ denotes the probability of survival from age a_0 until age a_{0+t}.
- $\mathbb{I}(X < Y)$ denotes an indicator function that takes a value 1 when $X < Y$ and 0 otherwise.
- $\overline{z_t}$ denotes the income target in the t^{th}period.
- S_i denotes the first age where $z_{it} < \overline{z_t}$ in the i^{th} simulated path (note, if no shortfall happens then S_i takes a value of a_{0+T}). (We assume that if shortfall happens once, it continues thereafter.)
- f_{it} denotes remaining assets (i.e., retirement savings account balance) in the i^{th} simulated path in the t^{th}period.
- b_i denotes the age at which the retirement savings account is exhausted in the i^{th} simulated path.

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[142] The calculations will need to be adjusted where varying probabilities are attached to the paths, e.g., under scenario analysis, under which each outcome would be weighted by the probability of the path occurring.

C.2 Distribution of Income

C.2.1 Expected Income

The expected income (EI) over a fixed planning horizon is:

$$EI = \frac{1}{T \times n} \sum_{i=1}^{n} \sum_{t=0}^{T-1} z_{it}$$

Expected income at selected ages can be calculated by taking the average of the income over the n simulations at the selected age $a_0 + t$:

$$EI_t = \frac{1}{n} \sum_{i=1}^{n} z_{it}$$

C.2.2 Survival Weighted Expected Income

We calculate the survival weighted expected income $(SWEI)$ as:

$$SWEI = \frac{1}{n \times \sum_{t=0}^{T-1} \ {}_tp_{a_0}} \sum_{i=1}^{n} \sum_{t=0}^{T-1} \ {}_tp_{a_0} \times z_{it}$$

C.2.3 Standard Deviation of Income

Let EI be the expected income over a fixed planning horizon, the standard deviation of income (SDI) over the planning horizon is:

$$SDI = \left(\frac{1}{T \times n} \sum_{i=1}^{n} \sum_{t=0}^{T-1} (z_{it} - EI)^2 \right)^{\frac{1}{2}}$$

If EI_t is the expected income at a selected age $a_0 + t$, then the standard deviation of income at that age (SDI_t) is:

$$SDI_t = \left(\frac{1}{n} \sum_{i=1}^{n} (z_{it} - EI_t)^2 \right)^{\frac{1}{2}}$$

C.2.3 Survival Weighted Standard Deviation of Income

To calculate the survival weighted standard deviation of income $(SWSDI)$, first we define the expected income along a single simulation (EI_i):

$$EI_i = \frac{1}{T} \sum_{t=0}^{T-1} z_{it}$$

Then $SWSDI$ is:

$$SWSDI = \left(\frac{1}{n \times \sum_{t=0}^{T-1} {}_t p_{a_0}} \sum_{i=1}^{n} \sum_{t=0}^{T-1} {}_t p_{a_0} \times (z_{it} - EI_i)^2 \right)^{\frac{1}{2}}$$

C.2.4 Income Percentiles

Income percentiles for a selected age $a_0 + t$ can be calculated by ordering the z_{it}'s across i and identifying the desired percentile.

C.3 Likelihood of Income Shortfall

C.3.1 Probability of Shortfall

The probability of shortfall (PoS), over a fixed planning horizon is defined as:

$$PoS = \frac{1}{n} \sum_{i=1}^{n} \min \left(1, \sum_{t=0}^{T-1} \mathbb{I}(z_{it} < \bar{z}_t) \right)$$

The probability of shortfall at selected age $a_0 + t$, PoS_t , is defined as:

$$PoS_t = \frac{\sum_{i=1}^{n} \mathbb{I}(z_{it} < \bar{z}_t)}{n}$$

C.3.2 Survival Weighted Probability of Shortfall

The survival weighted probability of shortfall ($SWPoS$) is calculated as:

$$SWPoS = \frac{1}{n} \sum_{i=1}^{n} {}_{s_i} p_{a_0}.$$

C.3.3 Expected Years of Shortfall

The expected years of shortfall (YoS) is defined as:

$$YoS = \frac{1}{W} \sum_{i=1}^{n} \sum_{t=0}^{T-1} \mathbb{I}(z_{it} < \bar{z}_t)$$

where $W = n$ if unconditional YoS is required, and $W = \sum_{i=1}^{n} \min\left(1, \sum_{t=0}^{T-1} \mathbb{I}(z_{it} < \bar{z}_t)\right)$ if conditional YoS is required.

C.3.4 Survival Weighted Expected Years of Shortfall

The survival weighted years of shortfall ($SWYoS$) is calculated as:

$$SWYoS = \frac{1}{W_1} \sum_{i=1}^{n} \left(\frac{1}{W_2} \sum_{t=0}^{T-1} {}_t p_{a_0} \times \mathbb{I}(z_{it} < \bar{z}_t) \right)$$

where $W_1 = n$ and $W_2 = 1$ if unconditional YoS is required, and $W_1 = \sum_{i=1}^{n} \min(1, \sum_{t-0}^{T-1} \mathbb{I}(z_{it} < \bar{z}_t))$ and $W_2 = {}_{s_i} p_{a_0}$ if conditional YoS is required.

C.4 Magnitude of Income Shortfall

C.4.1 Expected Shortfall

The expected shortfall (ES) over a fixed planning horizon is defined as:

$$ES = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{W}\sum_{t=0}^{T-1}(\bar{z}_t - z_{it})\mathbb{I}(z_{it} < \bar{z}_t)\right)$$

where $W = T$ if unconditional ES is required, and $W = \sum_{t=0}^{T-1}\mathbb{I}(z_{it} < \bar{z}_t)$ if conditional ES is required.

The expected shortfall (ES_t) at selected age $a_0 + t$ is defined as:

$$ES_t = \frac{1}{W}\sum_{i=1}^{n}(\bar{z}_t - z_{it})\mathbb{I}(z_{it} < \bar{z}_t)$$

where $W = n$ if unconditional ES_t is required, and $W = \sum_{i=1}^{n}\mathbb{I}\left(z_{ij} < \bar{z}_j\right)$ if conditional ES_t is required.

C.4.2 Survival Weighted Expected Shortfall

The survival weighted expected shortfall ($SWES$) is defined as:

$$SWES = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{W}\sum_{t=0}^{T-1}\ {}_tp_{a_0}(\bar{z}_t - z_{it})\mathbb{I}(z_{it} < \bar{z}_t)\right)$$

where $W = \sum_{t=0}^{T-1}\ {}_tp_{a_0}$ if unconditional $SWES$ is required, and $W = \sum_{t=0}^{T-1}\ {}_tp_{a_0}\mathbb{I}(z_{it} < \bar{z}_t)$ if conditional $SWES$ is required.

C.4.3 Minimum Income

Minimum income for any period *i* is simply the sum of any income in that period from guaranteed income sources such as social security and defined benefit pensions and any purchased income streams.

C.5 Remaining Assets

C.5.1 Expected Account Balance

The expected account balance (AB_j) at age $a_0 + t$ is calculated as:

$$AB_t = \frac{1}{n}\sum_{i=1}^{n} f_{it}$$

C.5.2 Remaining Account Balance - Percentiles

Percentiles of remaining account balance for a selected age j can be calculated by ordering the f_{it}'s across i and identifying the desired percentile.

C.5.3 Expected Age of Account Exhaustion

Expected age of account exhaustion ($EoAE$) is defined as:

$$EoAE = \frac{1}{n}\sum_{i=1}^{n} b_i$$

Percentiles rather than expected age of exhaustion can be calculated by ordering the b_i's and identifying the desired percentile.

C.5.4 Probability of Account Exhaustion

The probability of account exhaustion ($PoAE$) over a fixed planning horizon is defined as:

$$PoAE = \frac{1}{n}\sum_{i=1}^{n}\left(\mathbb{I}(f_{iT} = 0)\right)$$

The probability of account exhaustion at age $a_0 + t$, $PoAE_j$ is defined as:

$$PoAE_t = \frac{\sum_{i=1}^{n} \mathbb{I}(f_{it} = 0)}{n}$$

C.5.5 Survival Weighted Probability of Account Exhaustion

The survival weighted probability of exhaustion ($SWPoAE$) is calculated as:

$$SWPoAE = \frac{1}{n}\sum_{i=1}^{n} {}_{b_i}p_{a_0}$$

APPENDIX D: Additional Detail on Illustrative Examples

D.1 Modelling Assumptions

The key assumptions underpinning the examples appearing in this Primer are listed below. They are primarily chosen to support the illustrations, including to induce clear differentiation between strategies (e.g., by imposing low risk appetite). Nevertheless, they are selected to sit within a plausible range.

- *Analysis in 'real' terms* – It is implicitly assumed that all quantities are indexed by a common inflation rate. Values as reported are expressed in dollars as at date of retirement.
- *Modelling frequency* – Yearly, with income occurring at the beginning of the year
- *Growth assets* – returns simulated from lognormal distribution:
  - Expected real return (i.e., geometric mean) of 6% per annum net of any fees charged
  - Standard deviation of 15%
- *Defensive assets* – treated as risk-free asset with constant real return of 1% per annum
- *Life annuities*
  - Implicitly inflation-protected, i.e., real annuities
  - Annuitization factor estimated with reference to risk-free return and 8% loading
- *Survival probabilities* – Australian Life Tables 2015-17 for females without mortality improvement, sourced from Australian Government Actuary; death occurs with certainty at age 110
- *Income floor objective* – analysis designed to ensure floor is never breached:
  - Section 4: Life annuity purchased to secure the income floor
  - Section 5.4 and 5.5: Income floor set equal to available guaranteed income streams
- *Withdrawal strategy from the retirement savings account: income target objective*
  - Draw-to-target, until retirement savings account exhausted
  - Provision for excess withdrawals once 'affordable' withdrawal amount exceeds 1.5-times the draw-to-target amount
  - See Section 2.2 for further details
- *Withdrawal strategy from the retirement savings account: income optimization objective*
  - 'Affordable' withdrawal amount, estimated as notional annuitization of account balance at each age, assuming a time horizon equal to halfway between life expectancy and the number of years remaining until age 110
  - Assumed Interest Rate (AIR) for annuitization equal to the expected return on the retirement account less 1%, subject to a minimum of the risk-free rate of return
  - See Section 2.2 for further details
- *Utility function for income target objective*
  - Loss aversion utility function (see Section 4.2 and Appendix B.3)
  - Parameterized for low risk appetite, in line with 'high loss aversion' curve using the Blake, Wright and Zhang (2013) parameters as described in Section 4.2.
- *Utility function for income optimization objective*
  - Risk aversion utility function (see Section 4.2 and Appendix B.4)
  - Parameterized for low risk appetite with coefficient of relative risk aversion of five, in line with 'high risk aversion' curve plotted in Section 4.2.

D.2 Complete Set of Metrics for Strategy Selection Examples of Section 5

Table 16 and Table 17 provide a complete set of 'primary' metrics as recommended in Section 4.3 for the illustrative examples appearing in Section 5.4. Bolded numbers that were considered for use after initial analysis.

Table 16

STRATEGY MENU – INCOME TARGET OBJECTIVE, HIGH WEALTH RETIREE

Strategy	1	2	3	4	5
Life annuity allocation	100%	**50%**	Nil	25%	Nil
Retirement savings account (growth/defensive)	Nil	**25/75**	25/75	75/25	75/25
Withdrawal (draw-to-target plus excess)	Target + XS	**Target + XS**	Target + XS	Target + XS	Target + XS
Assumptions About Retiree					
Income floor	$30,000	**$30,000**	$30,000	$30,000	$30,000
Income target	$60,000	**$60,000**	$60,000	$60,000	$60,000
Risk appetite	Low	**Low**	Low	Low	Low
Assets at retirement (age 65)	$700,000	**$700,000**	$700,000	$700,000	$700,000
Guaranteed income:					
Social security + defined benefit pension	$30,000	**$30,000**	$30,000	$30,000	$30,000
Metrics					
Expected age of account exhaustion	65	**102**	99	106	106
Probability of shortfall vs. income target					
Age 70 (Survival probability 97%)	0%	**0%**	0%	0%	0%
Age 75 (Survival probability 91%)	0%	**0%**	0%	0%	0%
Age 80 (Survival probability 81%)	0%	**0%**	0%	0%	0%
Age 85 (Survival probability 66%)	0%	**0%**	0%	3%	3%
Age 90 (Survival probability 42%)	0%	**2%**	5%	8%	9%
Age 95 (Survival probability 18%)	0%	**16%**	31%	13%	14%
Age 100 (Survival probability 4%)	0%	**46%**	63%	19%	20%
Age 105 (Survival probability 0.5%)	0%	**69%**	83%	24%	26%
Minimum income	$61,630	**$45,815**	$30,000	$37,907	$30,000
Life annuity component	$31,630	**$15,815**	$0	$7,907	$0
Utility-based metrics#					
Constant risk-adjusted difference from target (CRADT)	$1,630	**($87)**	($374)	($203)	($313)
Risk-adjusted income (*target + CRADT*)	$61,630	**$59,913**	$59,626	$59,797	$59,687
Median account balance					
Age 70 (Survival probability 97%)	-	**$318,124**	$627,586	$538,256	$714,537
Age 75 (Survival probability 91%)	-	**$284,167**	$549,743	$554,951	$734,919
Age 80 (Survival probability 81%)	-	**$241,464**	$453,903	$537,953	$711,950
Age 85 (Survival probability 66%)	-	**$196,601**	$350,821	$521,926	$688,952
Age 90 (Survival probability 42%)	-	**$148,369**	$238,293	$473,390	$628,117
Age 95 (Survival probability 18%)	-	**$89,684**	$106,280	$382,237	$508,971
Age 100 (Survival probability 4%)	-	**$25,618**	-	$270,897	$360,976
Age 105 (Survival probability 0.5%)	-	**-**	-	$142,726	$190,630

Survival probabilities are applied in estimating utility-based metrics

XS = 'excess'

Table 17

STRATEGY MENU – INCOME TARGET OBJECTIVE, LOW WEALTH RETIREE

Strategy	1	2	3	4	5
Life annuity allocation	100%	50%	Nil	25%	**Nil**
Retirement savings account (growth/defensive)	Nil	25/75	25/75	75/25	**75/25**
Withdrawal (draw-to-target plus excess)	Target + XS	Target + XS	Target + XS	Target + XS	**Target + XS**
Assumptions About Retiree					
Income floor	$20,000	$20,000	$20,000	$20,000	**$20,000**
Income target	$40,000	$40,000	$40,000	$40,000	**$40,000**
Risk appetite	Low	Low	Low	Low	**Low**
Assets at retirement (age 65)	$300,000	$300,000	$300,000	$300,000	**$300,000**
Guaranteed income:					
Social security	$20,000	$20,000	$20,000	$20,000	**$20,000**
Metrics					
Expected age of account exhaustion	65	78	83	89	**93**
Probability of shortfall vs. income target					
Age 70 (Survival probability 97%)	100%	0%	0%	0%	**0%**
Age 75 (Survival probability 91%)	100%	2%	0%	2%	**0%**
Age 80 (Survival probability 81%)	100%	99%	8%	24%	**12%**
Age 85 (Survival probability 66%)	100%	100%	86%	52%	**36%**
Age 90 (Survival probability 42%)	100%	100%	100%	67%	**54%**
Age 95 (Survival probability 18%)	100%	100%	100%	77%	**65%**
Age 100 (Survival probability 4%)	100%	100%	100%	83%	**71%**
Age 105 (Survival probability 0.5%)	100%	100%	100%	86%	**76%**
Minimum income	$33,556	$26,778	$20,000	$23,389	**$20,000**
Life annuity component	$13,556	$6,778	$0	$3,389	**$0**
Utility-based metrics#					
Constant risk-adjusted difference from target (CRADT)	($6,444)	($5,443)	($4,550)	($2,806)	**($2,276)**
Risk-adjusted income (*target + CRADT*)	$33,556	$34,557	$35,450	$37,194	**$37,724**
Median account balance					
Age 70 (Survival probability 97%)	-	$97,920	$230,546	$189,234	**$264,672**
Age 75 (Survival probability 91%)	-	$39,549	$152,952	$145,057	**$221,608**
Age 80 (Survival probability 81%)	-	-	$64,689	$82,246	**$162,070**
Age 85 (Survival probability 66%)	-	-	-	$9,707	**$90,598**
Age 90 (Survival probability 42%)	-	-	-	-	**-**
Age 95 (Survival probability 18%)	-	-	-	-	**-**
Age 100 (Survival probability 4%)	-	-	-	-	**-**
Age 105 (Survival probability 0.5%)	-	-	-	-	**-**

Survival probabilities are applied in estimating utility-based metrics

XS = 'excess'

Table 18

STRATEGY MENU – INCOME OPTIMIZATION OBJECTIVE, HIGH WEALTH RETIREE

Strategy	1	2	3	4	5
Life annuity allocation	100%	50%	Nil	**25%**	Nil
Retirement savings account (growth/defensive)	Nil	25/75	25/75	**75/25**	75/25
Withdrawal	Affordable	Affordable	Affordable	**Affordable**	Affordable
Assumptions About Retiree					
Income floor	$30,000	$30,000	$30,000	**$30,000**	$30,000
Risk appetite	Low	Low	Low	**Low**	Low
Assets at retirement (age 65)	$700,000	$700,000	$700,000	**$700,000**	$700,000
Guaranteed income: Social security + defined benefit pension	$30,000	$30,000	$30,000	**$30,000**	$30,000
Metrics					
Expected income#, +	$61,630	$59,591	$57,553	**$69,747**	$72,452
Median income					
Age 65 (Survival probability 97%)	$61,630	$58,361	$55,093	**$64,490**	$65,444
Age 70 (Survival probability 97%)	$61,630	$58,975	$56,321	**$65,792**	$67,180
Age 75 (Survival probability 91%)	$61,630	$59,624	$57,618	**$67,445**	$69,383
Age 80 (Survival probability 81%)	$61,630	$60,034	$58,438	**$68,357**	$70,600
Age 85 (Survival probability 66%)	$61,630	$60,257	$58,884	**$69,074**	$71,555
Age 90 (Survival probability 42%)	$61,630	$59,883	$58,136	**$68,738**	$71,107
Age 95 (Survival probability 18%)	$61,630	$58,579	$55,528	**$66,285**	$67,836
Age 100 (Survival probability 4%)	$61,630	$56,275	$50,921	**$61,600**	$61,589
Age 105 (Survival probability 0.5%)	$61,630	$52,692	$43,754	**$53,882**	$51,300
Minimum income	$61,630	$45,815	$30,000	**$37,907**	$30,000
Life annuity component	$31,630	$15,815	$0	**$7,907**	$0
Utility-based metrics: # Risk-adjusted income (*RAI*)	$61,630	$59,410	$56,824	**$64,733**	$64,172
Median account balance					
Age 70 (Survival probability 97%)	-	$325,611	$651,222	**$509,530**	$679,374
Age 75 (Survival probability 91%)	-	$296,854	$593,708	**$489,571**	$652,761
Age 80 (Survival probability 81%)	-	$258,873	$517,745	**$446,285**	$595,046
Age 85 (Survival probability 66%)	-	$216,031	$432,062	**$391,428**	$521,903
Age 90 (Survival probability 42%)	-	$166,902	$333,804	**$319,509**	$426,012
Age 95 (Survival probability 18%)	-	$114,753	$229,506	**$231,101**	$308,134
Age 100 (Survival probability 4%)	-	$65,130	$130,259	**$138,326**	$184,435
Age 105 (Survival probability 0.5%)	-	$24,204	$48,408	**$54,519**	$72,691

Survival probabilities are applied in estimating these metrics

+ Expected income can be higher than median income due to income being positively skewed

Table 19

STRATEGY MENU – INCOME OPTIMIZATION OBJECTIVE, LOW WEALTH RETIREE

Strategy	1	2	3	4	5
Life annuity allocation	100%	50%	Nil	25%	**Nil**
Retirement savings account (growth/defensive)	Nil	25/75	25/75	75/25	**75/25**
Withdrawal	Affordable	Affordable	Affordable	Affordable	**Affordable**
Assumptions About Retiree					
Income floor	$20,000	$20,000	$20,000	$20,000	**$20,000**
Risk appetite	Low	Low	Low	Low	**Low**
Assets at retirement (age 65)	$300,000	$300,000	$300,000	$300,000	**$300,000**
Guaranteed income:					
Social security	$20,000	$20,000	$20,000	$20,000	**$20,000**
Metrics					
Expected income#, +	$33,556	$32,682	$31,808	$37,034	**$38,194**
Median income					
Age 65 (Survival probability 97%)	$33,556	$32,155	$30,754	$34,782	**$35,190**
Age 70 (Survival probability 97%)	$33,556	$32,418	$31,280	$35,339	**$35,934**
Age 75 (Survival probability 91%)	$33,556	$32,696	$31,836	$36,048	**$36,879**
Age 80 (Survival probability 81%)	$33,556	$32,872	$32,188	$36,439	**$37,400**
Age 85 (Survival probability 66%)	$33,556	$32,967	$32,379	$36,746	**$37,809**
Age 90 (Survival probability 42%)	$33,556	$32,807	$32,058	$36,602	**$37,617**
Age 95 (Survival probability 18%)	$33,556	$32,248	$30,941	$35,551	**$36,216**
Age 100 (Survival probability 4%)	$33,556	$31,261	$28,966	$33,543	**$33,538**
Age 105 (Survival probability 0.5%)	$33,556	$29,725	$25,894	$30,235	**$29,129**
Minimum income	$33,556	$26,778	$20,000	$23,389	**$20,000**
Life annuity component	$13,556	$6,778	$0	$3,389	**$0**
Utility-based metrics: #					
Risk-adjusted income (*RAI*)	$33,556	$32,621	$31,564	$35,226	**$35,226**
Median account balance					
Age 70 (Survival probability 97%)	-	$139,548	$279,095	$218,370	**$291,160**
Age 75 (Survival probability 91%)	-	$127,223	$254,446	$209,816	**$279,755**
Age 80 (Survival probability 81%)	-	$110,945	$221,891	$191,265	**$255,020**
Age 85 (Survival probability 66%)	-	$92,585	$185,170	$167,755	**$223,673**
Age 90 (Survival probability 42%)	-	$71,529	$143,059	$136,932	**$182,577**
Age 95 (Survival probability 18%)	-	$49,180	$98,360	$99,043	**$132,058**
Age 100 (Survival probability 4%)	-	$27,913	$55,825	$59,283	**$79,044**
Age 105 (Survival probability 0.5%)	-	$10,373	$20,746	$23,365	**$31,153**

Survival probabilities are applied in estimating these metrics

+ Expected income can be higher than median income due to income being positively skewed

Table 20

OPTIMAL STRATEGIES – INCOME TARGET OBJECTIVE

	High Wealth ($700,000)			Low Wealth ($300,000)		
Strategy	Optimal	Selected from Menu	Difference	Optimal	Selected from Menu	Difference
Life annuity allocation	100%	50%	50%	0%	0%	0%
Retirement savings account (growth/defensive)	Nil	25/75	-25/+25	100/0	75/25	25/-25
Withdrawal (draw-to-target plus excess)	Target + XS	Target + XS		Target + XS	Target + XS	
Assumptions About Retiree						
Income floor	$20,000	$20,000		$20,000	$20,000	
Income target	$60,000	$60,000		$40,000	$40,000	
Risk appetite	Low	Low		Low	Low	
Savings at retirement (age 65)	$700,000	$700,000		$300,000	$300,000	
Guaranteed income:						
Social security + defined benefit pension	$30,000	$30,000		$20,000	$20,000	
Metrics						
Expected age of account exhaustion	n.a.	102		96	93	
Probability of shortfall vs. income target						
Age 65 (Survival probability 97%)	0%	0%	0%	0%	0%	0%
Age 70 (Survival probability 97%)	0%	0%	0%	2%	0%	2%
Age 75 (Survival probability 91%)	0%	0%	0%	13%	12%	2%
Age 80 (Survival probability 81%)	0%	0%	0%	31%	36%	-5%
Age 85 (Survival probability 66%)	0%	2%	-2%	44%	54%	-10%
Age 90 (Survival probability 42%)	0%	16%	-16%	51%	65%	-14%
Age 95 (Survival probability 18%)	0%	46%	-46%	57%	71%	-14%
Age 100 (Survival probability 4%)	0%	69%	-69%	62%	76%	-14%
Minimum income	$61,630	$45,815	$15,815	$20,000	$20,000	-
Life annuity component	$31,630	$15,815	$15,815	-	-	-
Utility-based metrics:#						
Constant risk-adjusted difference from target (CRADT)	**$1,630**	($87)	$1,717	**($1,979)**	($2,276)	$297
Risk-adjusted income (*Target + CRADT*)	**$61,630**	$59,913	$1,717	**$38,021**	$37,724	$297
Median account balance						
Age 70 (Survival probability 97%)	-	$284,167	($284,167)	$254,583	$221,608	$32,975
Age 75 (Survival probability 91%)	-	$241,464	($241,464)	$210,592	$162,070	$48,522
Age 80 (Survival probability 81%)	-	$196,601	($196,601)	$162,720	$90,598	$72,122
Age 85 (Survival probability 66%)	-	$148,369	($148,369)	$95,940	-	$95,940
Age 90 (Survival probability 42%)	-	$89,684	($89,684)	$5,780	-	$5,780
Age 95 (Survival probability 18%)	-	$25,618	($25,618)	-	-	-
Age 100 (Survival probability 4%)	-	-	-	-	-	-

Survival probabilities are applied in estimating utility-based metrics

XS = 'excess'

Table 21

OPTIMAL STRATEGIES – INCOME OPTIMIZATION OBJECTIVE

	High Wealth ($700,000)			Low Wealth ($300,000)		
Strategy	Optimal	Selected from Menu	Difference	Optimal	Selected from Menu	Difference
Life annuity allocation	34.6%	25%	9.6%	16.9%	0%	16.9%
Retirement savings account (growth/defensive)	100/0	75/25	25/-25	100/0	75/25	25/-25
Withdrawal strategy	Affordable	Affordable		Affordable	Affordable	
Assumptions About Retiree						
Income floor	$30,000	$30,000		$30,000	$30,000	
Risk appetite	Low	Low		Low	Low	
Savings at retirement (age 65)	$700,000	$700,000		$300,000	$300,000	
Guaranteed income:						
Social security + defined benefit pension	$30,000	$30,000		$20,000	$20,000	
Metrics						
Expected income[#, +]	$74,598	$69,747	$4,851	$40,617	$38,194	$2,423
Median income at selected ages						
Age 65 (Survival probability 100%)	$67,872	$64,490	$3,382	$36,955	$35,190	$1,765
Age 70 (Survival probability 97%)	$68,982	$65,792	$3,190	$37,559	$35,934	$1,625
Age 75 (Survival probability 91%)	$70,391	$67,445	$2,946	$38,326	$36,879	$1,448
Age 80 (Survival probability 81%)	$71,042	$68,357	$2,684	$38,681	$37,400	$1,280
Age 85 (Survival probability 66%)	$71,448	$69,074	$2,374	$38,902	$37,809	$1,092
Age 90 (Survival probability 42%)	$71,015	$68,738	$2,277	$38,666	$37,617	$1,049
Age 95 (Survival probability 18%)	$68,543	$66,285	$2,259	$37,320	$36,216	$1,105
Age 100 (Survival probability 4%)	$64,173	$61,600	$2,574	$34,941	$33,538	$1,402
Age 105 (Survival probability 0.5%)	$56,527	$53,882	$2,645	$30,777	$29,129	$1,649
Minimum income	$40,947	$37,907	$3,040	$32,294	$30,000	$2,294
Including life annuity	$10,947	$7,907	$3,040	$2,294	-	$2,294
Utility-based metrics:[#]						
Risk-adjusted income	**$66,615**	$64,733	$1,883	**$36,270**	$35,226	$1,044
Median account balance						
Age 70 (Survival probability 97%)	$448,071	$509,530	($61,460)	$243,981	$291,160	($47,180)
Age 75 (Survival probability 91%)	$434,187	$489,571	($55,384)	$236,420	$279,755	($43,334)
Age 80 (Survival probability 81%)	$399,475	$446,285	($46,810)	$217,519	$255,020	($37,501)
Age 85 (Survival probability 66%)	$353,252	$391,428	($38,175)	$192,351	$223,673	($31,322)
Age 90 (Survival probability 42%)	$292,438	$319,509	($27,070)	$159,237	$182,577	($23,340)
Age 95 (Survival probability 18%)	$214,434	$231,101	($16,667)	$116,762	$132,058	($15,296)
Age 100 (Survival probability 4%)	$131,487	$138,326	($6,839)	$71,597	$79,044	($7,447)
Age 105 (Survival probability 0.5%)	$52,384	$54,519	($2,134)	$28,524	$31,153	($2,630)

Survival probabilities are applied in estimating these metrics

[+] Expected income can be higher than median income due to income being positively skewed

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Acknowledgments

The authors' deepest gratitude goes to those without whose efforts this project could not have come to fruition: the volunteers who generously shared their wisdom, insights, advice, guidance, and arm's-length review of this Primer prior to publication. Any opinions expressed may not reflect their opinions nor those of their employers. Any errors belong to the authors alone.

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