ILA LAM Model Solutions Fall 2022

1. Learning Objectives:

1. The candidate will understand, evaluate and use stochastic, generalized linear, multi-state, projection and transition matrix models. The candidate will demonstrate an understanding of their underlying methodologies, strengths, limitations, and applications.

Learning Outcomes:

(1a) With respect to stochastic models:

- Explain and apply the stochastic modeling methodology, including measurement metrics (e.g., CTE).
- Describe and apply the theory and uses of real world versus risk neutral assumptions.
- Describe and apply the techniques of Monte Carlo simulation (including variance reduction and importance sampling).
- Describe and evaluate Random Number Generator models, and explain their uses, advantages, and theory.
- Describe and evaluate how stochastic models may be used to understand mortality and policyholder behavior risks and inform the use of reinsurance.
- Describe the technique of nested stochastic projections and explain why they are needed, and evaluate implementation issues.

Sources:

LAM-135-19: Stochastic Modeling, Theory and Reality from and Actuarial Perspective Beware of Stochastic Model Risk!, Stroman, Risk & Rewards, SoA, Aug 2019

Commentary on Question:

Candidates generally did well in this question. Stronger candidates were able to identify the appropriate source material and applied them correctly in the context of the questions.

Solution:

(a) With respect to the Monte Carlo simulation:

- (i) Calculate the Monte Carlo estimate of the call option
- (ii) Calculate the Monte Carlo sampling standard error of the estimated call option price

- (iii) Evaluate the reasonableness of the estimated call option price
- (iv) Explain why a company would want to use a variance reduction technique

Commentary on Question:

Candidates did well in part (i) with the correct calculations for the Monte Carlo estimate of the call option. Only a handful of candidates were able to calculate the standard error correctly in part (ii). Candidates generally did well in parts (iii) and (iv)

(i)

 $\begin{array}{l} S(0) = 100 \\ K = 100 \\ r = 2\% \ (back-calculated \ using \ information \ provided) \\ \sigma = 0.2 \\ T = 1 \end{array}$

To calculate r, use this formula S(T)=S(0) * exp[(r- $\sigma^2/2$) * T + σ * ϵ * T^{1/2}]

	epsilon	S(T)	c ^a
1	-1.7701	70.19	0.00
2	-0.7908	85.37	0.00
3	-0.3590	93.07	0.00
4	0.8190	117.80	17.45
5	-0.5828	89.00	0.00
6	0.7156	115.39	15.09
7	-0.3182	93.83	0.00
8	0.9314	120.48	20.07
9	1.2897	129.43	28.85
10	0.5930	112.59	12.34

a – Calculation of present value of Call Payout (=exp(-r*T)*max(S(T)-K,0))

Estimated call price c = average of column c = 9.38

(ii)

Sample Error = Standard Deviation of column c / SQRT (Sample Size) = 10.7480 / SQRT (10) = 3.40

(iii)

Black-Scholes Model = N(d₁) St– N(d₂) Ke^{-rt}, where d₁ = [ln(S(t)/K) + (r + $\sigma^2/2$)*t]/ σ *SQRT(t) d₂ = d₁ - σ *SQRT(t)

Black-Scholes call = 8.92

The calculated Monte Carlo (MC) price should converge to the Black-Scholes price at a sufficient N. One way to assess the reasonableness of the results is to compare the MC price with the Black-Scholes call price. The Black-Scholes price is 8.92. Comparing with the Black-Scholes price, the MC price of 9.38 seems reasonable. This can further be improved by increasing the number of Monte Carlo simulations.

(iv)

Monte Carlo usually requires a large number of iterations to achieve a certain degree of accuracy. This can be very computationally intensive. Variance reduction techniques can greatly reduce computing time while increase the accuracy of the estimate

- (b) Your company would like to explore the use of a nested stochastic model to project equity returns.
 - (i) Explain how a nested stochastic model could be implemented
 - (ii) Discuss the advantages and disadvantages of two possible methods that could be used to manage the run-time of the model

Commentary on Question:

Most candidates were able to provide some high level overview of the nested stochastic model in part (i), and were able to identify two methods to improve run-time in part (ii)

(i)

In a stochastic model, projections are derived from values projected over time using a set of randomly generated economic scenarios.

A nested stochastic introduces another layer of economic scenarios when it is needed to stochastically derive a variable. Common terms are the inner and outer scenarios when the inner paths are embedded on each other scenario. The point in time at which the inner paths begin is defined as a node.

(ii)

The following is a list of reasonable responses. And credit is awarded for other reasonable responses.

Reduce number of model points by grouping individual policies into model cells.

It may be difficult to achieve high levels of compression. Repeated testing of compression is an ongoing process that must be managed with care.

Reduce the number of inner paths

Focusing on the tail of the distribution allows for efficiency when a CTE measure is used. This could lead to loss of important data outside of the tail region. We must use careful actuarial judgement here because the selected paths may not be the same across all nodes.

Reduce the number of nodes and **Reduce the number of outer scenarios** are other reasonable answers

Others include examples of variance reduction methods and grid processing.

- (c) Critique the following statements:
 - A. For the underlying index, the volatility parameter is an average of 65 years of data; therefore, the credibility of such a long sample period means it is reasonable to set σ to 20% for all nodes in the model.
 - B. The relation between interest rates and equity returns has been proven beyond statistical doubt. Consequently, in reserve calculations the expected return on equities should exceed the risk-free rate by an expected risk premium at every time step in every scenario.
 - *C.* Stock prices are distributed normally and therefore the company can use the stock price volatility as a parameter in a stochastic model.

Commentary on Question:

Responses for this section vary. Only a handful of candidates were able to obtain full credits for identifying all the mistakes in the statements and provide reasons to back up their critiques.

A.

Regardless of calibration period, models using a single value of a volatility do not fit historical market prices well. The fitting of actual market prices using the Normal distribution results in an "implied volatility surface", which is an array of different values depending on strike price and tenor.

В.

The first sentence is false. The relationship has **NOT** been proven beyond statistical doubt. The lack of statistical proof does not mean such a relationship does not exist. For real-world scenarios, equity returns will exceed the risk-free rates by a risk premiums, whereas all assets under risk-neutral scenarios would earn the risk-free rates.

C.

Stock returns are assumed to be normally distributed and **NOT** stock prices. If stock prices where normally distributed, there would be negative stock prices. Even though it is assumed returns are normally distributed, in practice, there is a much fatter tail and stronger central peak. Often a single volatility parameter is used in modelling stock however it is more appropriate to use an implied volatility.

2. Learning Objectives:

2. The candidate will understand and be able to assess issues and concerns common to actuarial models and their development and management.

Learning Outcomes:

- (2b) Explain and apply the technique for the compression of model data using the "Cluster Analysis Spatial Approach".
- (2c) Describe and evaluate best practices for actuarial model governance over process and controls
- (2g) Describe best practices documentation and governance over assumptions used in models
- (2j) Describe and evaluate considerations around the governance of expert judgment in actuarial modelling

Sources:

The Importance of Centralization of Actuarial Modeling Functions, Part 1: Focus on Modularization and Reuse, The Modeling Platform, Nov 2019

Model Validation for Insurance Enterprise Risk and Capital Models, 2014 (excluding Appendices)

Assumption Governance, The Actuary, Jan 2021

Commentary on Question:

To receive full credit, Candidates need to provide their critique with justification from cost effective and controlled environment point of views. A lot of candidates mentioned the divide and conquer fallacy. Without clearly explanation of the divided and conquer fallacy from the cost effective and controlled environment perspective, only partial credit will be granted. Many candidates recommended alternative approaches to the statements as a better solution rather than noting the issues and challenges with the proposed approach in the statements

Solution:

(a) You oversee new product modeling at ABC Life Insurance Company. ABC is looking to develop a cost effective and controlled environment for modeling.

Critique the following two statements:

- A. To save time and effort a new product will be developed using a decentralized approach where a copy of an existing model will be created for the new product and maintained separately.
- B. To avoid replicating errors in the existing model, new products will always need to be developed from first principles.

A.

Do not recommend this approach

Not cost effective: when starting from copying, there are multiple groups effectively maintaining the same functionality and solving similar problems Not controlled: with multiple models requiring maintenances, it becomes difficult to keep all under control

В.

Do not recommend this approach

Not cost effective: starting from scratch is usually risky and very expensive. There is a bias to underestimate time and effort needed. Not controlled: no matter the good intentions of starting with a clean slate, there is a good chance that reality will set in and the new model will start having all the same blemishes as all the other models in the organization

- (b) ABC is growing quickly and it is becoming more challenging to manage assumption governance requirements. You have been asked to replace the current compliance focused framework with a strategic assumption governance framework.
 - (i) List four beneficial byproducts of having a strategic assumption governance framework.
 - (ii) Recommend improvements to each of the following assumption governance requirements, if necessary. Justify your recommendations.
 - A. The top priority of assumption governance is compliance with regulatory requirements.
 - B. All models must have consistent assumptions.
 - *C. Each assumption must be reviewed annually.*
 - D. Assumptions should only be passed to modelers for implementation after they have been approved by all relevant stakeholders.

Commentary on Question:

(i) Full credit will be given for every 4 out of the 5 items in the list. Many candidates were able to get full credit. Many candidates provided the full list of 5 points

(ii) In general, most candidates provided good responses to most of the statements statement B: many candidates confused consistent assumptions with the same assumptions. Very few referred to ASOP 56

Statement C: most candidates were able to recommend the risk-based approach

Statement D: only a few candidates clarified that the stakeholders should include the modelers and note the importance of communication and collaboration across all of the stakeholders.

- (i) Fostering communication among subject experts Leveraging the best technology and data Controlled, accurate implementation Documentations and defensible processes Satisfying management, regulator, and auditor requirements
- (ii) A.

Although compliance is an important outcome, it should not be the stated objective, as that rarely achieves the desired outcome.

Governance needs to be deeply embedded in the organizations culture to encourage discipline.

Recommend achieving a strategic goal, such as set the best assumptions possible, as the top priority of assumption governance.

В.

ASOP 56 requires that consistency in assumptions across models be considered and explained, not mandated.

Recommend allowing inconsistency, as long as it is appropriately explained. This allows actuaries to make expert decisions but requires them to think deeply about assumption consistency. C.

When companies review and set assumptions annually, they encounter time and resource constraints.

Recommend risk-based approach that evaluates each assumption to define its risk level, and let risk level dictates the level of review. Assumptions with highest financial impact and most complexity get more scrutiny. D.

Modelers need to know how assumptions are developed, not just know what assumptions are. Data and assumption owners need to understand the models to better formulate the assumptions that are best for the purpose.

Recommend regular and consistent feedback loop and collaboration between experts in data, assumptions, and modeling

(c) Your manager has suggested to use the interest rate model developed by the American Academy of Actuaries (the Academy model) for the pricing of new products.

Assess if this is reasonable. Justify your response.

Commentary on Question:

This question is not very well done. Many candidates provided very generic responses to the use of an interest rate model for pricing which was not answering the question. Most candidates did not understand any specifics about the American Academy of Actuaries interest model which was required to answer the question for full credit

It will not be reasonable:

- The academy model is stochastic and can be difficult to use for pricing purpose
- The academy model is designed to replicate the general behavior of interest rates, but not periodically calibrated to reflect the latest market condition
- The academy model is intended to be used to measure the volatility of insurer financial results in the outlier scenarios—the ones at the tails of the distribution.
- However, in pricing, the greatest probability weight is placed on the bulk of scenarios in the middle of the distribution

3. Learning Objectives:

3. The candidate will understand the principles of Asset-liability Management ("ALM"), and be able to describe and evaluate various techniques for addressing the mitigation of risk.

Learning Outcomes:

- (3a) With respect to Asset-Liability Models:
 - Describe and apply the fundamental elements of the theory and practice of ALM in an insurance company, including assessing the dangers of mismatched assets and liabilities.
 - Describe and demonstrate how ALM can be used to identify and manage product and asset risks, including:
 - Major product risks for which ALM can be a useful tool for their management.
 - Using ALM as a means to manage interest rate risk, equity risk, and risks from optionality.
 - Describe how common insurance contracts and variations generate embedded options in an insurer's balance sheet, and assess basic strategies for managing exposures created by such embedded options.
 - Describe and apply the basic concepts of cash flow matching, immunization, duration/convexity matching, segmentation.
 - Describe and apply Key Rate Durations (KRD) and their use in evaluating interest rate sensitivities of portfolios, including understanding the derivation of KDRs, the profiles of KDRs for selected major asset types, and assessing KRDs in a portfolio context.
 - Describe and evaluate the Goldman Sachs' ALM/Strategic Asset Allocation approach for integrating ALM into an enterprise's risk and financial management framework.
 - Describe and evaluate ALM modeling considerations in the context of modeling risk aggregation, dependency, correlation of risk drivers and diversification.
- (3b) With respect to asset adequacy analysis and cash flow testing, describe and evaluate actuarial practice with respect to:
 - Modeling and selecting assets and related assumptions (incl. modeling assets with contingent cash flow risks).
 - Handling liability cash flow contingencies and risks.
 - Setting up projection model parameters and assumptions.
 - Describe how Interest Rate Forwards and Futures and Swaps can be used in ALM, and apply the mathematics in given situations.

Sources:

LAM-140-19: Asset Adequacy Analysis Practice Note, 2004, questions: 3, 5, 10-16, 18-20, 27, 29-31, 39, 42-60, 65-68, 71-82, 85 & 89

LAM-131-19: Life Insurance Accounting, Asset/Liability Management Ch 22

Commentary on Question:

The goal of this question is to test the candidates' ability to utilize asset knowledge, particularly the modeling of assets for cash flow testing and the management of asset/liability risk.

Solution:

(a) Critique each of the following statements:

- A. To capture varying credit losses observed historically, the models assume lower default rates in higher interest rate scenarios.
- B. To be conservative, prepayments of CMOs and MBSs are not modeled; prepayments would increase cash flows and could only improve results.
- C. Only 5% of the bonds in the portfolio are callable and many include make-whole provisions, so they are modeled as non-callable for simplicity.
- D. The reinvestment strategy is modeled as a simple reinvestment portfolio of AAA-rated 20-year bonds.
- *E.* Investment expenses are not modeled explicitly; instead, 10bps are deducted from the asset earned rate to represent investment expenses.
- F. The cash flow testing models will be used for BAX's ALM analysis.

Commentary on Question:

Many candidates had challenges with this question, particularly on parts A, C, E and F.

- A. Few candidates were able to state that credit losses do not have any strong link with interest rate levels, which was directly addressed by Q51 of the practice note LAM-140-19.
- C. To fully justify the simplifying assumption, candidates should opine on the significance of 5% and note the risk mitigation from "make whole" provisions. If 5% is claimed to be significant, one should defend with a valid argument, e.g. possibility of having large gain/loss despite small proportion.

- E. Investment expenses developed as a number of basis points is a common industry practice in asset modeling. Many candidates did not consider modeling practicality and criticized the use of reasonable assumptions without providing valid justifications.
- F. To earn full credit, candidates should note that CFT model is appropriate and also discuss that changes are likely needed to align with ALM purposes. A number of candidates provided comparison between CFT and Gross Premium Valuation method, without relating it back to the question and failed to address the statement directly.
- A. This statement is not appropriate.

Studies have not established a strong link between the shape or level of yield curves and credit losses; therefore it is common to assume that defaults and recoveries do not vary by interest rates. There is, however, a strong correlation between default rates and credit spreads and the general economic conditions.

B. This statement is not appropriate.

The amount and timing of prepayments can have significant effects on future cash flows with negative impacts to results. As interest rates decline, prepayments will likely increase, and those cash flows will be reinvested in lower-yielding assets. As interest rates rise, there is a risk of fewer prepayments (extension risk), and the company can't reinvest in higher interest environment with those originally anticipated cashflows.

C. This statement is appropriate.

If callable bonds are an insignificant portion of the portfolio, the impact of the call feature can be excluded from testing. 5% is likely insignificant. In addition, given many callable bonds carry a "make whole" provision, this means the bond issuer will pay an amount to the bond holder to compensate for the loss due to the call, and it is common to model these as non-callable.

D. This statement is not appropriate.

While it is acceptable to use a small, simplified portfolio of bonds to represent the reinvestment strategy, it is likely not appropriate to assume all of the bonds are of the same quality (and very high quality) and term. The reinvestment portfolio should represent the company's actual investment strategy, which should be developed to support the profile (e.g. duration) of BAX Life's underlying liabilities.

E. This statement is appropriate.

It is a common practice to not explicitly model investment expenses and instead subtract a fixed basis point amount from earned interest rates. However, the bps assumption should not be an arbitrary number; the actuary should consult with the investment team on the assumption and/or reconcile the modelled investment expenses against the actual expense experience to ensure reasonability.

F. This statement is appropriate.

CFT models are often good to leverage for ALM purposes because they already contain granular asset and liability data, and well-considered assumptions. However, as CFT models are primarily regulatory-inspired, modifications are often necessary in order to produce desirable ALM metrics.

(b) Evaluate the change in BAX Life's interest rate risk profile for each of the proposed product design changes.

Commentary on Question:

Overall candidates did well on this question, many were able to adequately demonstrate the understanding of risks associated with the proposed changes.

Many candidates were able to describe the immediate effects of the proposed changes, e.g. policyholders surrender early, bonds are called. However, it is also necessary to discuss the "so-what" impact to BAX to earn full credit.

The proposed higher guaranteed rate exposes BAX to the risk that interest rates fall below the level anticipated to achieve the expected profit margin, or even fall below guaranteed rate with BAX suffering a loss. In a decreasing interest rate environment, the policyholders are more likely to retain their policy if the guaranteed rate is higher than what they can earn elsewhere. This creates "reinvestment risk" as assets mature while policies remain in-force, and maturing assets must be reinvested at lower interest rates.

The shorter surrender charge period introduces "capital value risk" in a rising interest rate environment. When interest rates increase, policyholders are more likely to lapse after the surrender charge period to seek other higher yielding alternatives. This forces BAX to sell the assets backing the product at depressed market values.

Choosing to invest in callable bonds rather than non-callable introduces "option risk", where in decreasing interest rate environment the assets are more likely to be called as bond issuer seeks to refinance at lower rates. This forces BAX to take cash proceeds and reinvest at lower interest rates.

- (c) Critique each of the following statements related to Asset Liability Management:
 - A. Each product should be analyzed individually so we can understand the total investment risk of the product.
 - B. When developing new Universal Life products, we are only able to manage the investment risk through design features such as lower guaranteed credited rates and higher surrender charges.
 - C. The best way to achieve immunization is to match the effective duration of each of our asset segments with the effective duration of their corresponding liability segments.

Commentary on Question:

Candidates performed well on this question. Many demonstrated solid understanding of the benefits of holism, the management of investment risk, and the limitations of effective duration.

For B, it is important to discuss risk management through investment strategy (or asset side of risk management in general); listing reinsurance as an alternative would not earn full credit.

- A. While it is a good idea to understand risks at a product level, BAX should also look at risks holistically at an enterprise level. By using holism, one could analyze the potential benefit achieved through product diversification and avoid overstating the total investment risk.
- B. Product design can help manage risk, but certainly is not the only method. Risks can also be managed from the asset side through a sound investment strategy that matches the cash flow profile of the underlying liabilities.
- C. Matching the effective duration can be very limiting, as it requires constant rebalancing and assumes a small, parallel shift in yield curves. Convexity matching should be considered to protect against a wider range of interest rate movements. Key rate duration matching is also a more superior technique; it protects against non-parallel shifts in the yield curve by calculating durations at multiple points on the curve.

4. Learning Objectives:

4. The candidate will understand the basic design and function of Economic Scenario Generators and Equity Linked Insurance Models.

Learning Outcomes:

- (4a) With respect to Economic Scenario Generators:
 - Describe the need for ESGs and explain the structure of ESG models and components.
 - Describe and apply basic default free interest rate models, including one-factor continuous time models.
 - Assess the propriety of a particular ESG model and related assumptions for particular applications.

Sources:

LAM-148-19: Introduction to Economic Scenario Generators - Selecting and Specifying ESGs Economic Scenario Generators: A Practical Guide, 2016, Ch. 1, 2, 4.1, 5, 6, 9, 10, 11.1 & 11.3

LAM-139-19: Simulation of a Guaranteed Minimum Annuity Benefit, Freedman, 2019; Excel Model - Stochastic Simulation of a GMAB Option (Accompanies Simulation of a GMAB) Investment Guarantees Ch 6, Hardy, 2003

Commentary on Question:

Commentary listed underneath question component.

Solution:

- (a) Critique the following statements from their report:
 - A. The ESG produces thousands of scenarios covering a wide distribution of projected interest rate and equity growth paths so it is ideal for risk management applications.
 - B. The pricing department follows industry-leading practices to calibrate their ESG to current market conditions, producing near-perfect replication of option prices.
 - *C.* The ESG output is regularly tested to verify the absence of arbitrage opportunities.
 - D. If a new ESG is built for risk management, then for the sake of consistency between the inner loops and outer loops of nested scenarios the company should also use the new ESG for pricing.

Commentary on Question:

The question requests analysis of ESG characteristics in a risk management context, which demands a real-world (RW) ESG instead of a risk-neutral (RN) ESG. Some credits would be given if acknowledging the appropriateness of these ESG characteristics in a pricing context, but the candidates need to address the risk management context in order to earn full credit.

A) Risk neutral ESGs are a math trick to solve partial differential equations to calculate the value of an option. RN ESGs generate scenarios that seem unreal.

In risk management we care about what really happen in different scenarios. The ESG should generate the types of events that result in the outcomes that we care about.

The purpose of a real-world simulation is to capture market dynamics, risks and returns in a way that an insurance company or other financial institution might experience them. Real-world simulations enable the exploration of the what-if questions asked by management as it tries to gauge the likelihood of future events and their business impact.

B) This is appropriate for pricing purposes. Because the market-consistent model calibration process is designed to reproduce the prices of traded derivatives, the ultimate calibration is dependent on both the pricing date and the set of traded derivatives used to calibrate the model. The validation associated with the model calibration is based on how well the model reproduces the market values of the universe of traded derivatives used to calibrate the model.

But the context here is risk management and risk management applications, in contrast, require that ESGs be capable of producing dynamics that are representative of the possible future paths of economic variables. Commonly referred to as "real-world" calibrations, they enable managers to ask what-if questions as they try to gauge the likelihood of future events and their impact on its business.

C) No-arbitrage is absolutely essential to have in a neutral ESG because that was a fundamental assumption in its development.

But for risk management purposes often the no-arbitrage condition severely restricts your ability to generate appropriate real-world behaviors. For a real world ESG, if the model is complex enough that you can't reliably determine with the nature of the model while living within a single scenario, then formal no-arbitrage may not be necessary. And by dropping this requirement you may facilitate a much richer menu of behaviors in your model.

D) Keep using risk-neutral scenarios to price the assets at each node, since the assets do not have closed form formula.

The risk management and hedging of variable annuities is a standard example in which real-world and risk-neutral scenarios are applied together. For such an application that involves nested stochastic (i.e., inner loops) the state of the world or node is simulated under the real-world measure, and cash flows are priced in that state of the world using risk-neutral scenarios projected forward from the current node of the simulation.

- (b) Calculate the expected profit and loss for the product under each of the following scenarios:
 - (i) The underlying equity index earns 5% per year
 - (ii) The underlying equity index earns -5% per year

Commentary on Question:

None

See "ILALAM Fall22 Calcuation_Q4.xlsx" for detailed calculations

Calculation of the Account Value (AV):

AV(EOP) = AV(BOP) + Interest - Rider Fee

where,

Interest = AV(BOP) * AV Growth %

Rider Fee = [AV(BOP) + Interest] * Ride fee %

(i) Calculation of net income under the 5% scenario

Profit = Rider Fee - Hedging Cost + Hedging Payout – GMAB Claim Cost

Where:

Hedging Cost = AV(BOP) * 25/1000

Hedging Payout = 0

GMAB Claim Cost = 0

(ii) Calculation of net income under the -5% scenario

Profit = Rider Fee Income - Hedging Cost + Hedging Payout – GMAB Claim Cost

Where:

Hedging Cost = AV(BOP) * 25/1000

Hedging Payout = Loss of AV in down market = MIN [- AV(BOP) * AV Growth %, 0]

GMAB Claim Cost = AV (t= 3) - GMAB(t= 3)

Where:

GMAB(t = 0) = 100,000

GMAB (t) = MAX [GMAB(t-1), AV(t)]

(c) Your manager has expressed a concern that the proposed product is too risky and despite the proposed risk mitigation the company could be exposed to significant claim costs if markets decrease.

Recommend changes to the hedging strategy to address your managers concerns.

Commentary on Question:

Most of the candidates had trouble with this question, possibly due to the wording of the question.

The manager's statement is incorrect. The hedging strategy perfectly offsets all downside risk. In question (b) calculation, Fee Income + Hedge Payout + Claims cost = 0