Advanced Long-Term Actuarial Mathematics

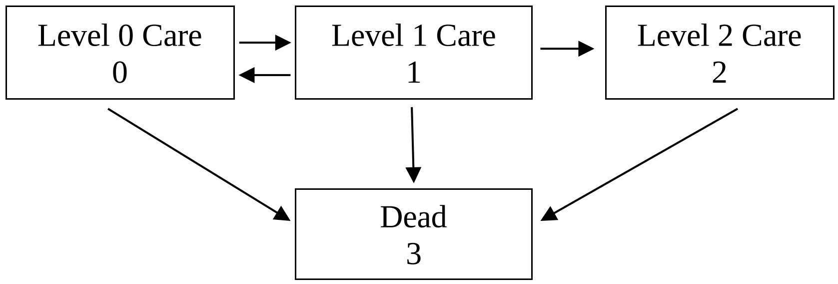
Sample Questions

The sample questions in this Study Note are designed to help candidates in their preparation for the Advanced Long-Term Actuarial Mathematics exam. Many of these questions are taken from the written answer sections of past MLC or LTAM exams.

Version Dated 24 January 2023.

Question 1

(*8 points*) A long-term care provider offers three care levels. Transitions are modeled as a Markov multiple-state model. Transitions and states are shown in the following diagram:



(a) (*1 point*) Write down Kolmogorov's forward differential equations with the associated boundary conditions for this model for:

(i) ${}_t p_x^{10}$

(ii) ${}_t p_x^{11}$

(b) (*3 points*) Estimate ${}_1 p_{80}^{10}$ using Euler's forward method, a step size of $h = \frac{1}{3}$, and the transition intensities and probabilities in the table below:

t	${}_t p_{80}^{11}$	μ_{80+t}^{01}	μ_{80+t}^{03}	μ_{80+t}^{10}	μ_{80+t}^{12}	μ_{80+t}^{13}	μ_{80+t}^{23}
0	1.00000	0.10000	0.02981	0.08000	0.15000	0.05962	0.11924
$\frac{1}{3}$	0.90346	0.10000	0.03082	0.08000	0.15000	0.06164	0.12328
$\frac{2}{3}$	0.81652	0.10000	0.03186	0.08000	0.15000	0.06373	0.12746
1	--	0.10000	0.03294	0.08000	0.15000	0.06589	0.13178

Question 1 (Continued)

(c) (*4 points*) You are given the following annuity values at 5%:

x	$\bar{a}_x^{00}$	$\bar{a}_x^{01}$	$\bar{a}_x^{02}$	$\bar{a}_x^{11}$	$\bar{a}_x^{10}$	$\bar{a}_x^{12}$	$\bar{a}_x^{22}$
80	5.5793	1.3813	0.6109	3.0936	1.6719	1.7206	4.4712
85	4.8066	1.0396	0.3403	2.6723	0.8834	1.0883	3.2367

You are also given:

- ${}_5p_{80}^{00} = 0.53880$ ${}_5p_{80}^{01} = 0.17327$ ${}_5p_{80}^{02} = 0.06956$
- $i = 0.05$
- Residents pay a service fee of 8000 per year continuously while in Level 0 or Level 1 Care.
- Level 2 Care costs are paid at a continuous rate of 30,000 per year for lives age 80 to 85, and 40,000 per year for lives older than 85.

A person, who is age 80, is currently in Level 0 Care.

(i) Calculate the expected present value of the person's future Level 0 and Level 1 service fees.

(ii) Calculate the expected present value of the person's future Level 2 Care costs.

(MLC, Fall 2014, Question 4.)

Question 2

(*6 points*) You are given the following excerpt from a triple decrement table:

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$	$d_x^{(3)}$
60	1000	$d_{60}^{(1)}$	60	45
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

You are also given the following information about the decrements:

- $\mu_{60+t}^{(1)} = 1.2t$ for $0 \le t \le 1$
- Decrement 2 happens exactly halfway through the year.
- Decrement 3 happens at the end of the year.

(a) (*2 points*) Calculate $q_{60}'^{(2)}$.

(b) (*2 points*) Calculate $d_{60}^{(1)}$.

Now suppose instead that Decrement 2 occurs at the start of the year, and that each $q_{60}'^{(i)}$ remains unchanged.

(c) (*2 points*) State with reasons the effect (increase, decrease, no change, cannot be determined) that this change would have on the following probabilities:

(i) $q_{60}^{(1)}$

(ii) $q_{60}^{(2)}$

(iii) $q_{60}^{(3)}$

(MLC, Spring 2015, Question 2.)

Question 3

(*10 points*) For a special fully discrete two-year term insurance on an individual, age 60, you are given:

(i) The death benefit is 1000 plus the return of gross premiums paid with interest at 6%.

(ii) The following double decrement table, where decrement (*d*) is death and decrement (*w*) is withdrawal:

x	$q_x^{(d)}$	$q_x^{(w)}$
60	0.06	0.04
61	0.12	0.00

(iii) There are no withdrawal benefits.

(iv) $i = 0.06$

(v) G denotes the annual gross premium.

(vi) L_0 denotes the insurer's loss at issue random variable for the individual's policy.

(a) (*4 points*) Calculate the values in the following table. Express L_0 in terms of G where appropriate:

Event	Value of L_0, Given that the Event Occurred	Probability of Event
Death in year 1		
Withdrawal in year 1		
Death in year 2		
Neither death or withdrawal		

Question 3 (Continued)

(b) (*4 points*)

(i) Show that $E\left[L_0\right] = a - b\,G,$ where *a* is 150 to the nearest 10 and *b* is 1.58 to the nearest 0.01. You should calculate *a* to the nearest 1 and *b* to the nearest 0.001.

(ii) Show that $Var\left(L_0\right) = c\,G^2 + d\,G + e,$ where *c* is 0.5 to the nearest 0.1, *d* is 480 to the nearest 10, and *e* is 116,000 to the nearest 1000. You should calculate *c* to the nearest 0.01, *d* to the nearest 1, and *e* to the nearest 100.

(c) (*2 points*) The insurer expects to issue 200 such policies to insureds with independent future lifetimes. The premium for each policy is $G = 130$.

Let L_{agg} denote the insurer's aggregate future loss random variable at issue for these 200 policies.

Calculate $\Pr\left(L_{agg} > 0\right)$ using the normal approximation without continuity correction.

(MLC, Fall 2015, Question 4.)

Question 4

(*10 points*) A life insurer issues a 5-year term insurance, with premiums payable half-yearly, to a person, who is age 60. The death benefit of 100,000 is paid at the end of the half-year of death.

You are given:

(i) Expenses are 10% of each semi-annual premium.

(ii) $i = 0.05$

(iii) Mortality follows the Standard Ultimate Life Table; the force of mortality of the Standard Ultimate Life Table is:

$\mu_{x+t} = A + Bc^{x+t}$, where $A = 2.2 \times 10^{-4}$, $B = 2.7 \times 10^{-6}$, $c = 1.124$

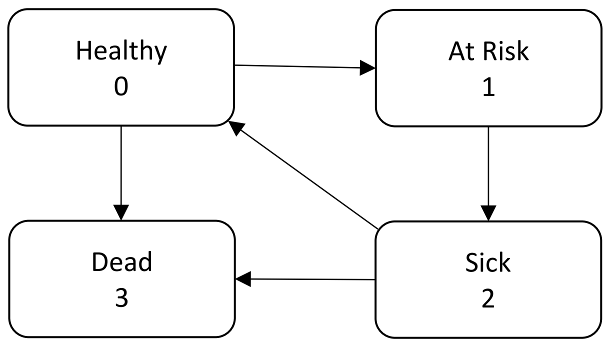
(iv) The gross premium is 270 per half-year.

(a) (*3 points*)

(i) Show that $100,000\ {}_{0.5}q_{64} = 260$ to the nearest 10. You should calculate the value to the nearest 1.

(ii) Show that the gross premium policy value at time 4 is 30 to the nearest 10. You should calculate the policy value to the nearest 0.1.

In the final year of the policy, the insurer decides to re-evaluate the gross premium policy value due to a virus that is spreading through the community. The insurer uses the following multiple state model.



Question 4 (Continued)

A Healthy person will move to State 1, At Risk, if exposed to the virus. Every person moving into the At Risk state will be tested for the virus. If the result of the test is positive, the person moves to State 2, Sick; otherwise, the person will move back to the Healthy state.

The transition intensities for all *x* are:

$\mu_x^{01} = 0.06$

μ_x^{03} is the force of mortality of the Standard Ultimate Life Table, as in (iii) above.

$\mu_x^{10} = 20.0; \quad \mu_x^{12} = 80.0$

$\mu_x^{20} = 4.0; \quad \mu_x^{23} = 5\mu_x^{03}$

You are also given that the insured is Healthy at time 4, and that premiums are payable conditional on being in State 0, 1, or 2.

(b) (*2 points*) Calculate the probability that the insured will remain in the Healthy state throughout the final year of the policy, ${}_1p_{64}^{\overline{00}}$.

(c) (*3 points*) The insurer re-evaluates the gross premium policy value at time 4 using the model above, with the same interest rate and expense assumptions as in part (a).

You are given the following probabilities:

x	${}_{0.5}p_x^{00}$	${}_{0.5}p_x^{01}$	${}_{0.5}p_x^{02}$	${}_{0.5}p_x^{03}$	${}_{0.5}p_x^{13}$	${}_{0.5}p_x^{23}$
64.0	0.98661	0.00059	0.01016	0.00264	0.00605	0.00700
64.5	0.98646	0.00059	0.01016	0.00279	0.00640	0.00740

Calculate the revised gross premium policy value for the insured's policy at time 4.

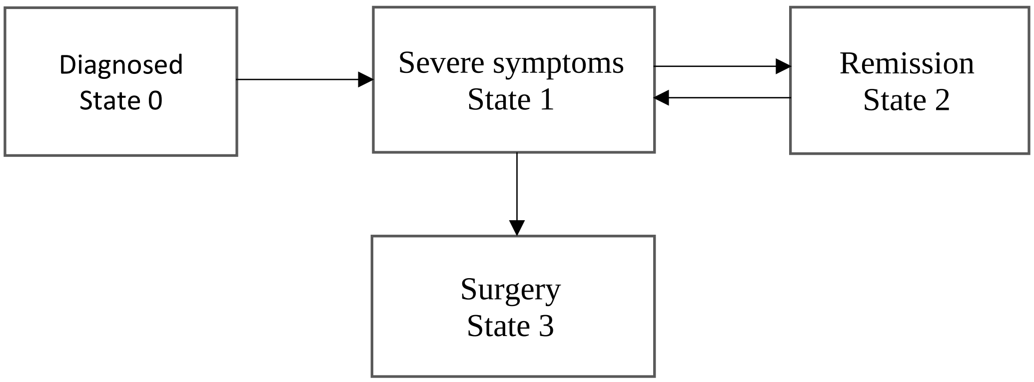
(d) (*2 points*) Calculate the expected time, in days, of each sojourn in the At Risk state.

(LTAM, Spring 2021, Question 1)

(Part (a) is covered by the FAM-L exam.)

Question 5

(*9 points*) You are analyzing costs associated with a medical condition. You model the progression of the condition after a new diagnosis using the following 4-state model.



You are given the following information. Time *t* is measured in years from the date of diagnosis.

(i) $\mu_t^{01} = a + be^{ct}$ where $a = 0.4, b = 2.0,$ and $c = 0.7$;

(ii) $\mu_t^{12} = 0.2$; $\mu_t^{21} = 0.4$; $\mu_t^{13} = 0.1$

(iii) $\delta = 0.04$

(iv) $\bar{a}_0^{01} = 2.930$; $\bar{A}_0^{01} = 1.5185$

(a) (*3 points*)

(i) Show that, in this case, $\dfrac{\bar{A}_0^{03}}{\bar{a}_0^{01}} = \mu_0^{13}$.

(ii) Calculate the value of $\bar{a}_t^{\overline{11}}$.

(iii) State with reasons whether $\bar{a}_t^{\overline{11}}$ is bigger, smaller or the same as $\bar{a}_t^{11}$.

Question 5 (Continued)

(b) (*4 points*) The cost of nursing care in State 1 is 12,000 per year, incurred continuously. The cost of surgery is 22,000. There are no nursing care costs in State 0 or State 2.

(i) Calculate the expected present value of the cost of surgery for a newly diagnosed patient.

(ii) Calculate the expected present value of the nursing care costs for a newly diagnosed patient.

(iii) COVID-19 precautions result in additional costs at a rate of 8,000 per year for the first 4 months of each period in State 1.

Write down an integral expression for the additional expected present value, using continuous sojourn annuities.

(c) (*2 points*) As a result of reorganization, the value of μ_t^{13} is increased to 0.2. All other transition intensities stay the same.

(i) State with reasons whether the expected present value of the cost of nursing care will increase, decrease, or stay the same.

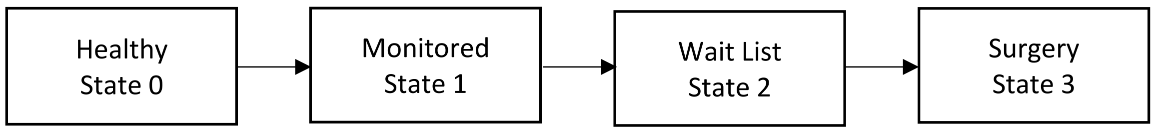
(ii) State with reasons whether the expected present value of the cost of surgery will increase, decrease, or stay the same.

(LTAM, Fall 2021 Form A, Question 3)

Question 6

(*9 points*) You are analyzing waiting times and expenses associated with optometric procedures. The analysis is based on the multiple state model illustrated below.

An individual who is diagnosed with a specific eye condition moves into State 1, where they are Monitored. When the condition progresses, they move onto a Wait List for surgery (State 2). Surgery (State 3) is performed once their name reaches the top of the wait list. The time spent on the wait list is random, depending on the progression of the disease and the availability of surgical resources.



You are given the following information:

(i) $\mu_{x+t}^{01} = a + be^{c(x+t)}$ where $a = 0.004, b = 0.015,$ and $c = 0.005;$

(ii) $\mu_{x+t}^{12} = 0.2; \quad \mu_{x+t}^{23} = 0.4.$

(iii) $\delta = 0.05$

(a) (*2 points*) Calculate the probability that a Healthy life age 60 will be diagnosed with the eye condition before age 65.

(b) (*1 point*) Show that ${}_t p_x^{12} = \left(e^{-0.2t} - e^{-0.4t}\right)$.

(c) (*1 point*) Calculate the probability that a life currently being Monitored (State 1) will have Surgery (State 3) within 5 years.

(d) (*2 points*)

(i) Show that $\bar{a}_{x:\overline{5}|}^{11} = 2.85$ to the nearest 0.01. You should calculate the value to the nearest 0.001.

(ii) Show that $\bar{a}_{x:\overline{5}|}^{12} = 0.87$ to the nearest 0.01. You should calculate the value to the nearest 0.001.

(iii) Show algebraically that $\bar{a}_{x:\overline{5}|}^{11} + \bar{a}_{x:\overline{5}|}^{12} + \bar{a}_{x:\overline{5}|}^{13} = \bar{a}_{\overline{5}|}$

Question 6 (Continued)

(e) (*3 points*) The health authority is considering two alternatives for improving care for those diagnosed with the eye condition. One criterion for deciding which course of action to choose is based on a quality of life (QL) index, as follows:

- In State 1, $QL = 0.5$ per year.
- In State 2, $QL = 0.2$ per year
- In State 3, $QL = 1.0$ per year.

The authority will choose one of the following options:

Option A: Open a new surgical center. This would increase μ_x^{23} to 1.0 without changing any other transition intensities.

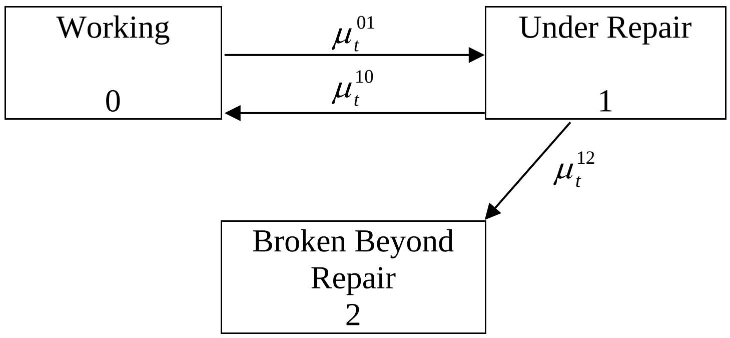
Option B: Improve care for those in States 1 and 2. This would improve the QL index to 0.55 in State 1, and to 0.3 in State 2.

Calculate whether the health authority should select Option A or Option B, based on the increase in the expected present value of the 5-year QL of lives currently in State 1, using a force of interest of $\delta = 0.05$.

(LTAM, Fall 2021 Form B, Question 3)

Question 7

(*7 points*) An actuarial student in your department is using the following 3-state Markov model to price a one-year consumer warranty for a television.



The product has the following features:

(i) The warranty is sold at the time that a consumer purchases the television and pays a replacement cost of 1000 at the end of the half-year during which the television becomes Broken Beyond Repair.

(ii) Premiums are payable at the beginning of each half-year, but are waived if the television is not Working at the time a premium is due.

(iii) All televisions are in the Working state at time of purchase.

(a) (*2 points*) Write down the Kolmogorov forward differential equations with associated boundary conditions (initial conditions) for ${}_tp_0^{00}$, ${}_tp_0^{01}$, and ${}_tp_0^{02}$ under this model.

The forces of transition in this model for a television purchased *t* years ago are:

$$\mu_t^{01} = 0.5 + 0.6t$$

$$\mu_t^{10} = 0.2$$

$$\mu_t^{12} = 2^t$$

The student has applied the Forward Euler approximation to the Kolmogorov forward differential equations with a step size of $h = 0.5$ to calculate probabilities for this model. You have verified that, using this approach, ${}_{0.5}p_0^{00} = 0.75$.

Question 7 (Continued)

(b) (*2 points*) Using the student's approach:

(i) Calculate ${}_{0.5}p_0^{01}$.

(ii) Show that the probability that a television will become Broken Beyond Repair within a year of purchase is 0.18 to the nearest 0.01. You should calculate the probability to the nearest 0.001.

(c) (*2 points*) Using the probabilities from the student's approach and $i^{(2)} = 8\%$:

(i) Calculate the actuarial present value at issue of the replacement cost payments for this policy.

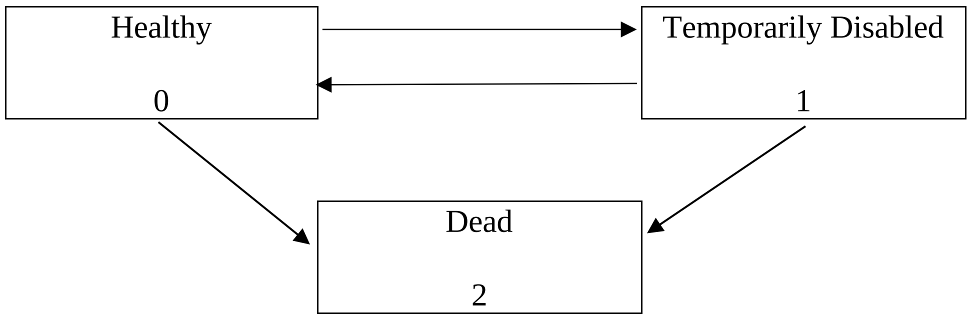
(ii) Calculate the semi-annual net premium for this policy.

(d) (*1 point*) Suggest a change to the student's approach that would improve the accuracy of the probability calculations.

(MLC, Spring 2014, Question 2)

Question 8

(*9 points*) You are using the following 3-state Markov model to price a 10-year disability insurance product.



(a) (*2 points*) Show that for this model, $\sum_{j=0}^{2} \bar{a}_{x:\overline{10|}}^{0j} = \bar{a}_{\overline{10|}}$.

The product has the following features:

- The product is issued to individuals age *x* who are in the Healthy state.
- The product pays a continuous disability benefit at a rate of 1000 per year while the insured is in the Temporarily Disabled state.
- The product pays a death benefit of 10,000 at the moment of death.
- Net premiums are payable continuously while the insured is in the Healthy state.

You are also given the following information:

$\delta = 0.1$	$\bar{a}_{x:\overline{10|}}^{00} = 4.49$	$\bar{a}_{x:\overline{10|}}^{02} = 1.36$	$\bar{A}_{x:\overline{10|}}^{02} = 0.3871$
$\mu_{x+t}^{01} = 0.04$	$\mu_{x+t}^{02} = 0.02t$	$\mu_{x+t}^{10} = 0.05$	$\mu_{x+t}^{12} = 0.04t$

(b) (*2 points*) Show that the net premium rate for this policy is 970 per year to the nearest 10. You should calculate the rate to the nearest 1.

Let ${}_tV^{(i)}$ denote the net premium policy value for a policy in state *i* at time *t*. You are given:

$${}_3V^{(0)} = 1304.54 \qquad {}_3V^{(1)} = 7530.09$$

(c) (*2 points*) Calculate $\frac{d}{dt}\,{}_tV^{(0)}$ at $t = 3$.

Question 8 (Continued)

(d) (*3 points*) Your company is considering adding an additional feature to this product. Under this additional feature, the insurer would return the sum of the premiums paid at the end of 10 years without interest if no benefits were paid during the life of the policy.

Calculate the increase in the net premium rate payable continuously for the product as a result of including this feature.

(MLC, Spring 2015, Question 1)

Question 9

(*7 points*) On 1/1/2009, a person purchased a 10-year hybrid long term care and life insurance product. The product has the following features:

(i) The product uses a reimbursement approach where the caregiver is reimbursed up to a maximum rate of 2000 per month when the insured requires assistance to perform 2 or more activities of daily living (ADLs), of the 6 commonly used ADLs.

(ii) The product has a 3 month waiting period and a 6 month off period.

(iii) The sum insured for the life insurance portion of the product is 100,000.

(iv) Premiums are payable at a continuous rate of 150 per month while the insured is active (able to perform at least 5 of 6 ADLs).

The insured's experience during the term of this policy was as follows:

- From 1/1/2012 to 6/30/2012, the insured was only able to perform 4 ADLs, and required outpatient care costing 1000 per month.
- From 3/31/2013 to 11/30/2013, the insured was only able to perform 3 ADLs, and required inpatient care costing 2500 per month.
- For the remainder of the time from 1/1/2009 through 6/30/2018, the insured was able to perform at least 5 ADLs.
- The insured died on 7/1/2018.

(a) (*1 point*) List the six ADLs in common use for long term care products.

(b) (*2 points*) Draw a sketch of a Markov model that could be used to model this hybrid insurance product.

(c) (*2 points*) Calculate the amount of the death benefit payable upon the insured's death, assuming:

  (i) The long term care and life insurance benefits are combined using the "return of premium" approach.

  (ii) The long term care and life insurance benefits are combined using the "accelerated benefit" approach.

Now suppose that instead of using a 6 month off period, the insurer uses a 12 month off period.

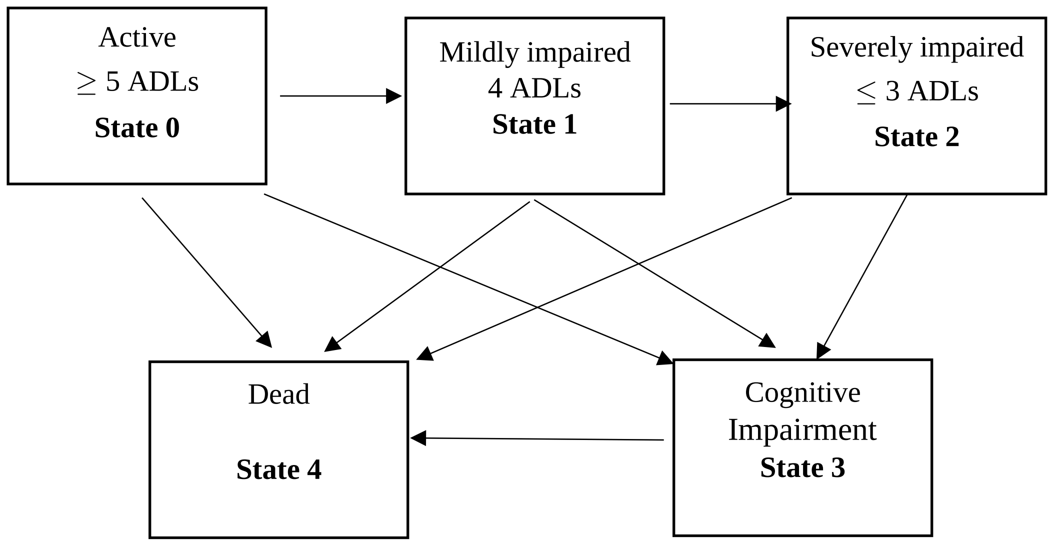
(d) (*2 points*) Calculate the change in total benefits paid as a result of this change, assuming:

  (i) The long term care and life insurance benefits are combined using the "return of premium" approach.

  (ii) The long term care and life insurance benefits are combined using the "accelerated benefit" approach.

Question 10

(*10 points*) You are using the following multiple state Markov model to analyze a long term care product which provides coverage until the death of the insured. Assume all relevant transition intensities exist.



You are given the following information:

(i) The insurance pays a benefit of 3000 per year, payable continuously, while the policyholder is in State 2.

(ii) Policyholders may opt to have benefits start immediately on transition into State 2 or, for a lower premium, may select a 6-month waiting period before benefits begin.

(iii) Benefits are valued at a force of interest of $\delta = 0.05$

(iv) Transition intensities for all ages $x \geq 90$ are

$$\mu_x^{12} = 0.10 \quad \mu_x^{13} = 0.04 \quad \mu_x^{14} = 0.10$$
$$\mu_x^{23} = 0.04 \quad \mu_x^{24} = 0.30 \quad \mu_x^{34} = 0.20$$

(a) (*2 points*) Derive the Kolmogorov forward differential equation for ${}_t p_x^{12}$.

(b) (*1 point*) Show that ${}_t p_{90}^{12} = e^{-0.24t} - e^{-0.34t}$

(c) (*2 points*) Calculate the expected present value of the LTC benefit in State 2 for a life currently age 90 and in State 1, assuming benefits begin immediately on transition.

Question 10 (Continued)

(d) (*1 point*) $\bar{a}^{\overline{22}}_{90:\,\overline{0.5}|} = 0.45$ to the nearest 0.01. Calculate $\bar{a}^{\overline{22}}_{90:\,\overline{0.5}|}$ to the nearest 0.001.

(e) (*2 points*) Calculate the expected present value of the LTC benefit in State 2 for a life currently age 90 and in State 1, assuming a waiting period of 6 months between transition to State 2 and the start of the benefit payments.

(f) (*2 points*) Suggest two reasons why an insurer uses waiting periods in long term health products.

Question 11

(*12 points*) An individual, who is now 65 years old, is entering a Continuing Care Retirement Community (CCRC) under a full life care (Type A) contract. The individual will move into an Independent Living Unit (ILU). The individual pays a one-time fee of F immediately on entry and a level monthly fee of M at the start of each month that she is in the CCRC, including the first.

You are given:

(i) The entry fee, F, is equal to 25% of the expected present value of all future costs.

(ii) The CCRC operates three types of accommodation; they are listed here, with the monthly costs incurred at the beginning of the month by the CCRC for each resident in each category:

Independent Living Unit (ILU): 3,000

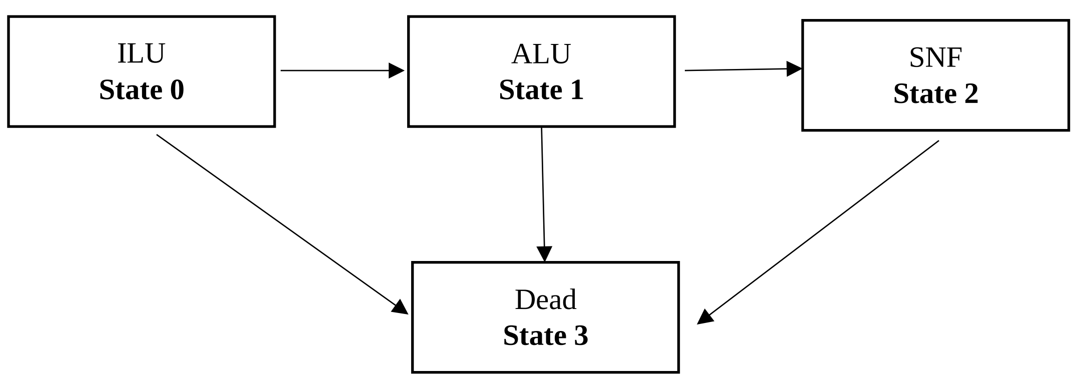
Assisted Living Unit (ALU): 7,500

Specialized Nursing Facility (SNF): 15,000

(iii) $i = 0.05$

(iv) The monthly fee M is determined so that the expected present value of the monthly costs at entry is equal to the expected present value of the fees at entry, including the entry fee.

(v) The CCRC uses the following multiple state model to determine the fee structure.



(vi) The following actuarial functions have been evaluated for the model at $i = 0.05$

x	$\ddot{a}_x^{(12)00}$	$\ddot{a}_x^{(12)01}$	$\ddot{a}_x^{(12)02}$	$\ddot{a}_x^{(12)11}$	$\ddot{a}_x^{(12)12}$	$\ddot{a}_x^{(12)22}$	$A_x^{(12)03}$
65	11.4106	1.3570	0.3745	11.8352	0.7979	10.6905	0.3601
70	9.5210	1.7037	0.4942	10.1754	0.9960	9.1961	0.4294

Question 11 (Continued)

(a) (*3 points*)

(i) You are given that *F* is 150,000 to the nearest 1000. Calculate *F* to the nearest 10.

(ii) Calculate *M*.

(b) (*3 points*)

(i) Calculate ${}_5V^{(0)}$, the policy value five years after entry, assuming the insured is in state 0.

(ii) Calculate ${}_5V^{(1)}$, the policy value five years after entry, assuming the insured is in state 1.

(c) (*2 points*) You are given that

$${}_{\frac{1}{12}}p^{00}_{69\frac{11}{12}} = 0.94937 \qquad {}_{\frac{1}{12}}p^{01}_{69\frac{11}{12}} = 0.00906 \qquad {}_{\frac{1}{12}}p^{02}_{69\frac{11}{12}} = 0.00003$$

Calculate ${}_{4\frac{11}{12}}V^{(0)}$.

The CCRC introduces an option under which 50% of the initial fee is refunded at the end of the month of death. The revised entry fee, *F*, will be equal to 25% of the expected present value of all future costs plus the refund. The revised monthly fee M is determined so that the expected present value of the monthly costs plus the expected present value of the refund benefit are equal to the expected present value of M plus the expected present value of F.

(d) (*4 points*)

(i) Calculate the revised entry fee assuming the individual selects the refund option.

(ii) Calculate the revised monthly fee assuming the individual selects this option.

(iii) State with reasons whether ${}_5V^{(0)}$ will increase, decrease or stay the same under the 50% refund contract.

Question 12

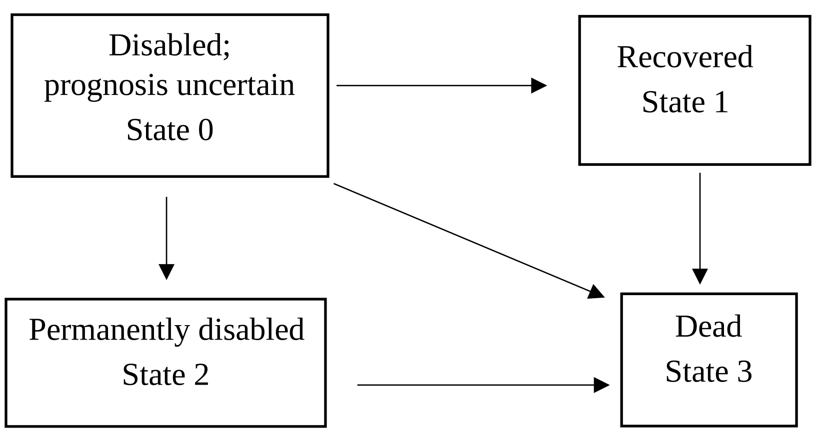
(*10 points*) Person G, who is age 45, has recently suffered a disabling injury on the job. G's prognosis is uncertain. His annual salary before the accident was 100,000

Person G receives a structured settlement from MRH Insurance. The settlement is a life annuity, starting immediately, and payable continuously at a rate of 90,000 per year while G is disabled. An additional annuity of 20,000 per year is payable continuously while G's prognosis is uncertain for up to two years, to offset medical expenses.

(a) (*1 point*) State two reasons why structured settlements often use an annuity format rather than a lump sum.

(b) (*1 point*) State two possible reasons why the long term annuity would replace less than 100% of G's pre-injury earnings.

MRH uses the following multiple state model:



You are given:

(i) MRH calculates policy values equal to the expected present value of future benefits.

(ii) $i = 0.04$

You have calculated the following table of annuity values and transition probabilities:

x	$\bar{a}_x^{00}$	$\bar{a}_x^{01}$	$\bar{a}_x^{02}$	$\bar{a}_x^{11}$	$\bar{a}_x^{22}$	${}_2p_x^{00}$	${}_2p_x^{01}$	${}_2p_x^{02}$
45	0.559	5.540	7.161	19.948	10.703	0.0301	0.2766	0.6164
47	0.559	5.420	7.104	19.528	10.623	0.0301	0.2766	0.6162

Question 12 (Continued)

(c) (*2 points*) Show that the expected present value of future benefits at $t = 0$, the start date for the annuity payments, is 705,700 to the nearest 100. You should calculate the value to the nearest 1.

(d) (*4 points*)

(i) Show that ${}_2V^{(0)}$, the policy value at $t = 2$ if G is in State 0 at that time, is 689,700 to the nearest 100. You should calculate the value to the nearest 1.

(ii) Calculate ${}_2V^{(2)}$, the policy value at $t = 2$ if G is in State 2 at that time.

(iii) Show that the expected present value at $t = 0$ of the policy value at $t = 2$ is 564,000 to the nearest 1,000. You should calculate the value to the nearest 10.

(iv) Calculate the expected present value at $t = 0$ of the payments during the first two years.

The chief actuary of MRH reviews your assumptions. The chief actuary asks you to redo your calculations, increasing μ_{45+t}^{01} by 0.01 for all *t*, with no other changes.

(e) (*2 points*)

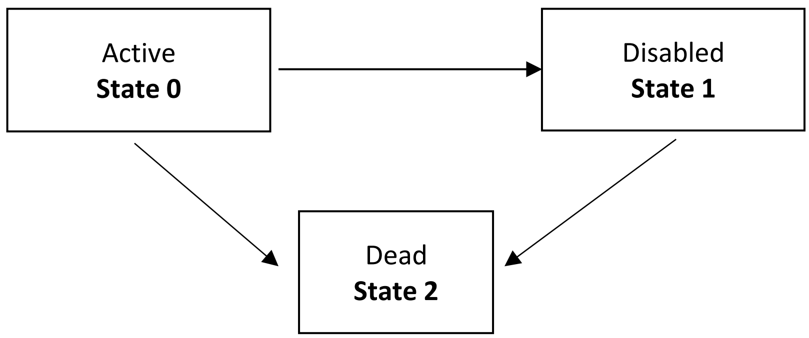
(i) State with reasons whether the value of ${}_2V^{(2)}$ will increase, decrease, or stay the same as a result of this change.

(ii) State with reasons whether the expected present value, at age 45, of the policy value at the end of two years will increase, decrease or stay the same as a result of this change.

(LTAM, Spring 2019, Question 2)

Question 13

(*9 points*) An insurer issues a whole life insurance of 50,000 to (55), with a waiver of premium on disability. You are evaluating this policy using the following model:



You are given:

(i) The gross premium G is paid at the start of each year if the insured is Active.

(ii) The death benefit is payable at the end of the month of death.

(iii) Commissions are 5% of each premium paid. There are no other expenses.

(iv) $i = 0.05$

(v)

x	$A_x^{(12)\,02}$	$A_x^{(12)\,12}$	$\ddot{a}_x^{00}$
55	0.480	0.531	8.832
65	0.634	0.681	5.416

(vi) ${}_0L^g$ denotes the present value of the gross loss-at-issue random variable.

(vii) ${}_tV^{(j)}$ denotes the gross premium policy value at time t, for a life then in State j.

(a) (*2 points*) The gross premium is determined such that $\mathrm{E}[{}_0L^g] = -\,0.1G$. Show that $G = 2{,}900$ to the nearest 100. You should calculate G to the nearest 1.

(b) (*2 points*) Show that ${}_{10}V^{(0)}$ is 16,800 to the nearest 100. You should calculate the policy value to the nearest 1.

Question 13 (Continued)

You are also given:

(i) $_{10\frac{1}{12}}V^{(1)} = 34{,}110$

(ii) $_{10\frac{2}{12}}V^{(1)} = 34{,}170$

(iii) The following probabilities:

x	$_{\frac{1}{12}}p_x^{01}$	$_{\frac{1}{12}}p_x^{02}$	$_{\frac{1}{12}}p_x^{12}$
65	0.00461	0.00293	0.00493
$65\frac{1}{12}$	0.00467	0.00295	0.00497

(c) (*3 points*)

(i) Show that $_{10\frac{1}{12}}V^{(0)} = 19{,}500$ to the nearest 100. You should calculate the value to the nearest 1.

(ii) Calculate $_{10\frac{2}{12}}V^{(0)}$.

(d) (*2 points*) To increase sales, the insurance company raises its commission rate to 15% of the first premium and 8% of each renewal premium paid. The gross premium is not changed.

(i) Without further calculation, state with reasons whether the value of $_{10\frac{2}{12}}V^{(0)}$ will increase, decrease or stay the same as a result of these changes.

(ii) Without further calculation, state with reasons whether the value of $_{10\frac{2}{12}}V^{(1)}$ will increase, decrease or stay the same as a result of these changes.

(LTAM, Fall 2019, Question 2)

Question 14

(*10 points*) For a fully discrete three-year term insurance of 100,000 on a person, age 40, you are given:

(i) The person's future mortality follows a double decrement model, where:

- Decrement 1 is death from Disease 1
- Decrement 2 is death from any cause except Disease 1

(ii) $\mu_{40+t}^{(1)} = A + Bc^{40+t}$ for all $t \geq 0$, where $A = 0.0001$, $B = 1.075 \times 10^{-5}$, $c = 1.12$

(iii) $\mu_{40+t}^{(2)} = 3\mu_{40+t}^{(1)}$, for all $t \geq 0$.

(iv) $i = 0.05$

(a) (*2 points*) Show that $p_{40}^{(\tau)} = 0.995$ to the nearest 0.001. You should calculate the value to the nearest 0.0001.

You are also given that ${}_2p_{40}^{(\tau)} = 0.99027$ and ${}_3p_{40}^{(\tau)} = 0.98462$.

(b) (*2 points*)

(i) Show that the expected present value at issue of the person's death benefits is 1400 to the nearest 100. You should calculate the value to the nearest 1.

(ii) Show that the net annual premium for the person's insurance is 490 to the nearest 10. You should calculate the premium to the nearest 0.1.

The person purchases a policy rider that pays an additional 50,000 at the end of the year of death, if death is due to Disease 1.

(c) (*4 points*)

(i) Write down an integral expression for ${}_tq_{40}^{(1)}$, in terms of the dependent survival probabilities, ${}_sp_{40}^{(\tau)}$, and the force of mortality from Disease 1, $\mu_{40+s}^{(1)}$.

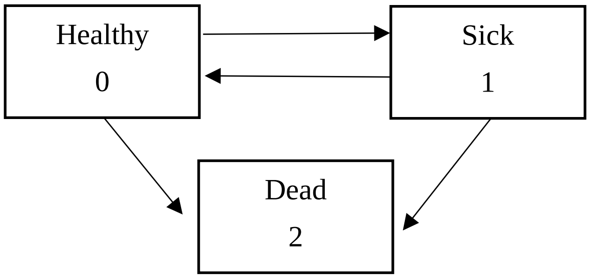
(ii) Using your expression from part (c) (i), prove that ${}_tq_{40}^{(1)} = \frac{1}{4}\,{}_tq_{40}^{(\tau)}$.

(iii) Show that the net premium for the Disease 1 rider is 60 to the nearest 10. You should calculate the premium to the nearest 0.1.

(d) (*2 points*) Calculate the net premium policy value at time 1 for the policy, including the Disease 1 rider.

Question 15

(*9 points*) You are using the following model to price a 10-year Disability Income Insurance policy issued to (x).



The product has the following features.

(i) Policyholders are healthy at the issue date.

(ii) A benefit of 20,000 is payable immediately on death within 10 years.

(iii) A continuous benefit of 1000 per year is paid while the policyholder is in the Sick state, during the 10-year term.

(iv) Premiums are payable continuously while the life is in the Healthy state.

(v) Maintenance expenses are paid at a continuous rate of 60 per year while the insured is alive.

(vi) Commissions are payable continuously at a rate of 5% of gross premium.

(vii) No benefits, premiums or expenses are payable beyond 10 years.

You are also given:

(viii) $\mu_{x+t}^{01} = 0.03$; $\mu_{x+t}^{02} = 0.02$; $\mu_{x+t}^{10} = 0.01$; $\mu_{x+t}^{12} = 0.05$, for $t > 0$

(ix) $\delta = 0.07$.

(x) $\bar{a}_{x:\overline{10}|}^{00} = 5.844$; $\bar{a}_{x:\overline{10}|}^{01} = 0.684$; $\bar{A}_{x:\overline{10}|}^{01} = 0.175$; $\bar{A}_{x:\overline{10}|}^{02} = 0.151$.

Question 15 (Continued)

(a) (*2 points*) Show that the gross premium rate, calculated using the equivalence principle, is 738 to the nearest 1. You should calculate the value to the nearest 0.1.

(b) (*2 points*) Calculate the probability that the life dies without becoming sick, within the 10-year term.

The insurer offers a new product with the following features:

- For any period of sickness that starts within the 10-year term, the benefit of 1000 per year is paid until death or recovery, even if that occurs after the 10-year term.
- There is an elimination period of 1 year.
- All other policy features and assumptions are the same as given above.

(c) (*1 point*) Define the term **elimination period** in the context of a disability income insurance product.

(d) (*2 points*) Show that the expected present value of a 1-year continuous sojourn annuity in State 1 is $\bar{a}^{\overline{11}}_{x:\overline{1|}} = 0.94$ to the nearest 0.01. You should calculate the value to the nearest 0.0001.

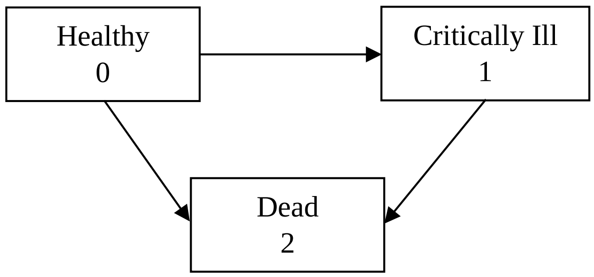
(e) (*2 points*) Calculate the premium for this new policy, using the model and parameters given above.

(LTAM, Spring 2020, Question 4)

Question 16

(*10 points*) An insurer issues a 20-year term insurance to (50) which pays a death benefit of 500,000, and an additional 100,000 on diagnosis of a critical illness (CI).

The insurer uses the Markov model illustrated below to value the CI benefit.



You are given the following information:

(i) Premiums are payable continuously while the policyholder is healthy.

(ii) The annual net premium rate is 4,850.

(iii) Death and CI benefits are payable immediately on transition.

(iv) $i = 0.05$.

(v) ${}_tV^{(j)}$ denotes the net premium policy value at time t, conditional on being in State j at that time.

(vi) The following values from the model illustrated above:

x	$\bar{a}_x^{00}$	$\bar{A}_x^{01}$	$\bar{A}_x^{02}$	$\bar{A}_x^{12}$	${}_{10}p_x^{00}$	${}_{10}p_x^{01}$	${}_{10}p_x^{11}$
60	10.1729	0.34249	0.39077	0.47904	0.75055	0.13135	0.75283
70	6.5690	0.49594	0.54335	0.62237			

Question 16 (Continued)

(a) (*3 points*)

(i) Show that $\bar{A}^{02}_{60:\overline{10}|} = 0.0902$ to the nearest 0.0001. You should calculate the value to the nearest 0.00001.

(ii) Show that ${}_{10}V^{(0)} = 21{,}850$ to the nearest 50. You should calculate the value to the nearest 1.

(iii) Show that ${}_{10}V^{(1)} = 95{,}700$ to the nearest 10. You should calculate the value to the nearest 1.

(b) (*3 points*) You are given that $\mu^{01}_{60} = 0.00818$; $\mu^{02}_{60} = 0.00724$; $\mu^{12}_{60} = 0.01811$.

(i) Calculate $\frac{d}{dt}{}_{t}V^{(0)}$ at $t = 10$.

(ii) Calculate $\frac{d}{dt}{}_{t}V^{(1)}$ at $t = 10$.

(iii) Explain why ${}_{t}V^{(1)}$ is decreasing while ${}_{t}V^{(0)}$ is increasing at $t = 10$.

The policy terms are revised for new policies such that the CI benefit is paid six months after diagnosis, conditional on the policyholder surviving to that time, and conditional on the diagnosis occurring within the 20-year term.

(c) (*2 points*) Write down an integral expression for the expected present value at issue of the revised CI benefit.

(d) (*2 points*) One reason for the change is that mortality is very high in the first year after a CI diagnosis.

(i) Define the Markov property.

(ii) Explain whether high mortality in the period immediately after a CI diagnosis is consistent with the Markov property.

(LTAM, Spring 2022, Question 2)

Question 17

(*6 points*) For (x) and (y) with independent future lifetimes, you are given that $q_x = 0.2$ and $q_y = 0.1$.

(a) (*1 point*) Explain in words the meaning of the probability described by the symbol q_{xy}.

You are also given that mortality within integral ages follows a uniform distribution of deaths assumption for each of (x) and (y) individually.

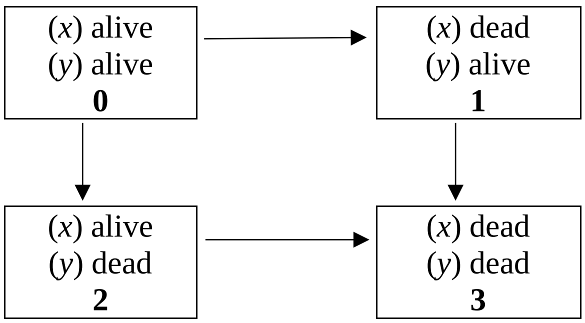
(b) (*2 points*) Sketch the graph of ${}_s p_x$ as a function of s for $0 \le s \le 1$. You should mark numerical values on each axis.

(c) (*3 points*) Show that ${}_s q_{xy} = s q_{xy} + g(s) q_{\overline{xy}}$ for $0 \le s \le 1$, where $g(s)$ is a function of s that you should specify.

(MLC, Fall 2014, Question 3)

Question 18

(*11 points*) The mortality of a couple, (x) and (y), is modeled using the Markov multiple-state model described in the following diagram:



You are given:

- $\mu^{01}_{x+t:y+t} = A + Bc^{x+t}$ $\quad \mu^{02}_{x+t:y+t} = A + Bc^{y+t}$ $\quad \mu^{13}_{y+t} = D + Bc^{y+t}$ $\quad \mu^{23}_{x+t} = E + Bc^{x+t}$
- $A = 0.0001 \quad B = 10^{-5} \quad c = 1.12 \quad D = 0.00015 \quad E = 0.0002$

(a) (*1 point*) State with reasons whether (x) and (y) have independent future lifetimes under this model.

(b) (*3 points*)

(i) Write down the Kolmogorov forward differential equation for ${}_t p^{00}_{xy}$, and give the associated boundary condition.

(ii) Starting from the equation in (i), prove that

$$ {}_t p^{00}_{xy} = \exp\left\{ -\int_0^t \left(\mu^{01}_{x+r:y+r} + \mu^{02}_{x+r:y+r} \right) dr \right\}. $$

A couple, who are ages $x = 50$ and $y = 55$, purchases a special single premium, deferred joint and last survivor annuity. The annuity will pay 50,000 per year while both are alive, and will pay 30,000 per year while only one is alive. Payments are continuous, and begin 10 years after the annuity purchase.

If neither life survives the deferred period, then a sum of 3 times the single premium is paid immediately on the second death.

There are no other benefits.

Question 18 (Continued)

You are given:

(i) ${}_{10}p^{00}_{50:55} = 0.86041 \quad {}_{10}p^{01}_{50:55} = 0.04835 \quad {}_{10}p^{02}_{50:55} = 0.08628$

(ii) $\bar{A}^{03}_{50:55:\overline{10}|} = 0.003421$

(iii) $\bar{a}^{00}_{60:65} = 8.8219 \quad \bar{a}^{01}_{60:65} = 1.3768 \quad \bar{a}^{02}_{60:65} = 3.0175$

(iv) $\bar{a}^{11}_{65} = 10.1948 \quad \bar{a}^{22}_{60} = 11.8302$

(v) $i = 0.05$

(c) (*3 points*)

(i) Show that the expected present value of the future benefits at time 10, if both lives survive to time 10, is 573,000 to the nearest 1000. You should calculate the value to the nearest 100.

(ii) Calculate the single net premium for the policy.

(d) (*4 points*)

(i) Determine ${}_{10}V^{(0)}$, ${}_{10}V^{(1)}$ and ${}_{10}V^{(2)}$ for the policy.

(ii) Write down Thiele's differential equation for the policy value at t, $t \geq 10$, assuming both lives survive to t.

(iii) Using Euler's forward method, with a step size of $h = 0.5$, calculate the policy value required at 10.5 assuming both lives are alive at that time.

(MLC, Fall 2015, Question 2)

Question 19

(*11 points*) Person P and Person R, each age 40, buy a fully discrete, last survivor insurance with a sum insured of 100,000

You are given:

(i) Premiums are payable while at least one life is alive, for a maximum of 20 years.

(ii) Mortality of each follows the Standard Ultimate Life Table (SULT).

(iii) $i = 0.05$

(iv) With independent future lifetimes, $\ddot{a}_{40:40:\overline{20}|} = 12.9028$.

(a) (*2 points*) Show that the annual net premium assuming that the future lifetimes are independent is 620 to the nearest 10. You should calculate the value to the nearest 1.

(b) (*1 point*) State two reasons why couples may have dependent future lifetimes.

The insurer decides that premiums and policy values for this policy will be determined using a mortality model incorporating dependency.

You are given the following information about this model:

(i) The future lifetimes for the first 20 years are not independent.

(ii) If both lives survive 20 years, it is assumed that the future lifetimes from that time will be independent, and will follow the Standard Ultimate Life Table.

(iii) The mortality of each of P and R, individually, follows the Standard Ultimate Life Table, whether the other is alive or dead.

(iv) $\ddot{a}_{40:40:\overline{10}|} = 8.0703$; $\ddot{a}_{40:40:\overline{20}|} = 12.9254$; ${}_{20}E_{40:40} = 0.35912$; ${}_{100}E_{50:50} = 0.59290$.

(v) $A_{\overline{50:50}} = 0.13441$

(vi) ${}_{10}p_{40:40} = 0.9866$, ${}_{10}p_{\overline{40:40}} = 0.9980$

Question 19 (Continued)

Use the dependent mortality model for the rest of this question.

(c) (*3 points*)

(i) Show that $A_{40:40} = 0.158$ to the nearest 0.001. You should calculate the value to the nearest 0.0001.

(ii) Show that ${}_{10}E_{40:40} = 0.606$ to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(iii) Show that $\ddot{a}_{50:50:\overline{10}|} = 8.02$ to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(d) (*1 point*) Show that the annual net premium is 645 to the nearest 5. You should calculate the value to the nearest 0.1.

(e) (*3 points*) Let ${}_kL$ denote the net future loss random variable at time *k* for the insurance.

(i) Calculate $E\left[{}_{10}L\right]$ given that only P is alive at time 10.

(ii) Calculate $E\left[{}_{10}L\right]$ given that both P and R are alive at time 10.

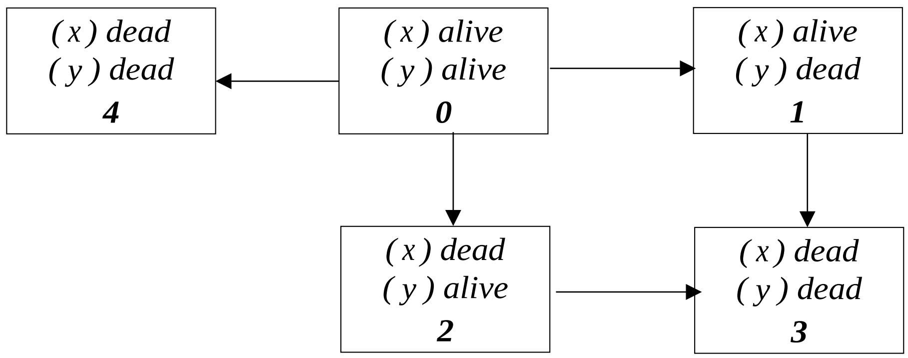
(iii) Calculate $E\left[{}_{10}L\right]$ given that at least one of P and R is alive at time 10.

(f) (*1 point*) Because the insurer is not informed of the first death for these policies, the actuary decides that they should hold the same reserve for all policies in force. Explain which, if any, of the policy values in part (e) would be a suitable value for the time 10 reserve for each in-force policy.

(LTAM, Spring 2019, Question 4)
(This is Question 4 from the Spring 2019 Written Answer exam.)

Question 20

(*10 points*) An insurer uses the following model for joint life policies.



You are given the following additional information

(i) μ_w^* is the force of mortality for a life age w under the Standard Ultimate Life Table.

(ii) $\mu_{x+t:y+t}^{01} = \mu_{y+t}^* - 0.0005$; $\mu_{x+t:y+t}^{02} = \mu_{x+t}^* - 0.0005$; $\mu_{x+t:y+t}^{04} = 0.0005$

(iii) $\mu_{x+t:y+t}^{13} = 1.2\mu_{x+t}^*$; $\mu_{x+t:y+t}^{23} = 1.05\mu_{y+t}^*$

(iv) $i = 0.05$

(v) For $x = 40$ and $y = 50$, $\bar{a}_{x:y:\overline{10|}}^{00} = 7.8487$ and $\bar{A}_{x:y:\overline{10|}}^{03} = 0.00789$

(a) (*2 points*) Describe two ways that dependency is incorporated in this model.

(b) (*3 points*)

(i) Calculate ${}_{10}p_{50}^{23}$.

(ii) Calculate ${}_{10}p_{x:y}^{00}$ for $x = 40$ and $y = 50$.

(c) (*3 points*) A couple, (x) who is age 40 and (y) who is age 50, buys a special 10-year joint life insurance policy with the following features:

- Level premiums are payable continuously while in State 0.
- A death benefit of 100,000 is payable immediately on the second death.
- If the deaths are simultaneous, the death benefit is increased to 300,000.

Calculate the annual net premium rate.

Question 20 (Continued)

(d) (*2 points*) State with reasons whether each of the following would be higher, lower or stay the same if $\mu^{23}_{x+t:y+t}$ is increased to $1.15\mu^{*}_{y+t}$, assuming all other transition intensities are as given above.

(i) $\bar{a}^{00}_{x:y:\overline{10}|}$

(ii) $\bar{A}^{03}_{x:y:\overline{10}|}$

(iii) $\bar{a}_{x|y}$

(iv) The annual net premium for the contract in (c).

(This is Question 3 from the Fall 2019 Written Answer exam.)

Question 21

(*10 points*) A fully discrete, joint life 10-year endowment insurance policy, with sum insured 100,000, is issued to two independent lives, both aged 70. Premiums are payable annually until the earlier of the first death or the contract maturity.

You are given:

(i) Acquisition expenses are 400 plus 50% of the first year's premium.

(ii) Renewal expenses are 2% of each subsequent premium.

(iii) Mortality follows Standard Ultimate Life Table.

(iv) $i = 0.05$

(v) The gross premium is $G = 10{,}658$.

(vi) L_0 is the gross loss at issue random variable.

(a) (*1 point*) Calculate the probability that both lives survive to the end of the contract.

(b) (*2 points*) Show that the expected loss at issue for the policy is $-4{,}470$ to the nearest 10. You should calculate the value to the nearest 1.

(c) (*3 points*)

(i) Show that ${}^{2}A_{70:70:\overline{10}|} = 0.44$ to the nearest 0.01. You should calculate the value to the nearest 0.001.

(ii) Show that the standard deviation of L_0 is 30,000 to the nearest 10,000. You should calculate the value to the nearest 10.

Question 21 (Continued)

(d) (*2 points*) Now consider a portfolio of 100 identical, independent policies issued to couples both aged 70, with independent future lifetimes. Calculate the probability that the aggregate loss at issue is positive, using the Normal approximation to the aggregate loss distribution.

(e) (*2 points*) In a sub-portfolio of 10 identical, independent policies issued to couples with lives both age 70, the age at death of the 10 couples' lives turned out to be the following:

	Policy Number									
	1	2	3	4	5	6	7	8	9	10
Age at death of first to die	75.2	78.5	80.5	82.1	83.5	83.8	83.9	91.3	91.5	95.8
Age at death of second to die	77.2	93.1	81.7	90.1	92.3	95.0	96.3	91.7	94.2	100.8

Without further calculation, state whether the value of the total profit realized in this sub-portfolio will be greater or less than the expected value of the profit for the sub-portfolio. Justify your answer.

(This is Question 1 from the Spring 2020 Written Answer exam.)

Question 22

(*10 points*) D and M are each aged 50, and have independent future lifetimes. They purchase a fully discrete last survivor whole life insurance of 100,000 . You are given:

(i) Mortality for each life follows the Standard Ultimate Life Table (SULT).

(ii) $i = 0.05$

(iii) Premiums of G per year are payable until the second death.

(iv) Premiums are payable semi-annually for the first 20 years, then annually thereafter.

(v) G is calculated using the equivalence principle.

(vi) Commissions are 10% of each premium.

(vii) Other acquisition expenses, payable at issue, are 70% of G.

(viii) The two-term Woolhouse formula is used to value single and joint life annuities.

(a) (*1 point*) Show that the probability that exactly one of D and M survives 20 years, is 0.14 to the nearest 0.01. You should calculate the probability to the nearest 0.0001.

(b) (*2 points*) Show that $\ddot{a}^{(2)}_{50:50:\overline{20|}}$ is 12.92 to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(c) (*2 points*) Show that G is 850 to the nearest 10. You should calculate the value to the nearest 0.1.

(d) (*2 points*)

(i) Show that the policy value at time 20, if both lives are alive, is 22,400 to the nearest 100. You should calculate the value to the nearest 1.

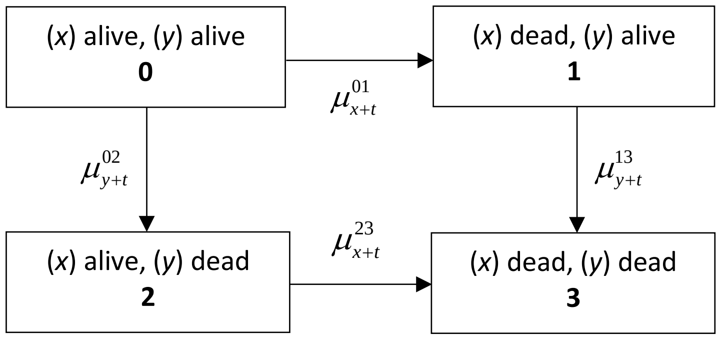
(ii) Calculate the policy value at time 20 if only one of the lives is alive at that time.

(e) (*3 points*) Calculate the policy value at time 19.5 if both D and M are alive at that time. Assume a constant force of mortality between ages 69 and 70 for the individual lives.

(This is Question 4 from the Fall 2020 Written Answer exam.)

Question 23

(*9 points*) Assume the following 4-state joint life model of mortality.



(a) (*1 point*) Describe the annuity that has actuarial value $\bar{a}_{xy}^{01}$.

(b) (*2 points*) Write down the Kolmogorov forward differential equations, with boundary conditions, for ${}_t p_{xy}^{00}$ and ${}_t p_{xy}^{01}$.

(c) (*3 points*) You are given the Euler-Maclaurin-Woolhouse approximation formula

$$\int_0^\infty g(t)dt \approx \sum_{k=0}^{\infty} g(k) - \frac{1}{2} g(0) + \frac{1}{12} g'(0) \ .$$

Use this approximation, with $g(t) = {}_t p_{xy}^{00} e^{-\delta t}$, to prove that

$$\bar{a}_{xy}^{00} \approx \ddot{a}_{xy}^{00} - \frac{1}{2} - \frac{1}{12}\left(\mu_x^{01} + \mu_y^{02} + \delta\right).$$

Hint: Use the Kolmogorov forward differential equations, and the chain rule.

Question 23 (Continued)

(d) (*3 points*) Z and A are both 70 years old. They purchase a last survivor annuity, with a level benefit payable continuously.

The single net premium is 1,000,000.

You are given the following information:

(i) The mortality of each life follows the Standard Ultimate Life Table.

(ii) Z and A have independent future lifetimes.

(iii) $i = 0.05$

(iv) $\mu_{70} = 0.009881$

(v) $\bar{a}_{70|70} = 2.0317$

Calculate the level annual rate of benefit, using the Euler-Maclaurin-Woolhouse approximation.

(This is Question 6 from the Spring 2021 Written Answer exam.)

Question 24

(*7 points*) ABC Insurance issues a 10-year endowment insurance policy to Person A who is 60 and Person B who is 50. The sum insured is 200,000 payable at the earliest of the end of the year of the first death and the end of 10 years. Premiums are paid annually in advance.

You are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) $i = 0.05$

(iii) Person A and Person B have independent future lifetimes.

(iv) Gross premiums are calculated using the equivalence principle.

(a) (*1 point*) Show that the net premium is 15,780 to the nearest 10. You should calculate the value to the nearest 1.

(b) (*2 points*) Show that ${}^{2}A_{50:60:\overline{10}|} = 0.39$ to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(c) (*1 point*) Calculate the standard deviation of the net Loss at Issue random variable.

(d) (*1 point*) Expenses are 25% of the first year's premium plus 10% of all renewal premiums. Calculate the gross premium using the equivalence principle.

(e) (*2 points*) Following a major economic upheaval, the interest rate has increased to 10.25% per year. Note that $1.05^2 = 1.1025$.

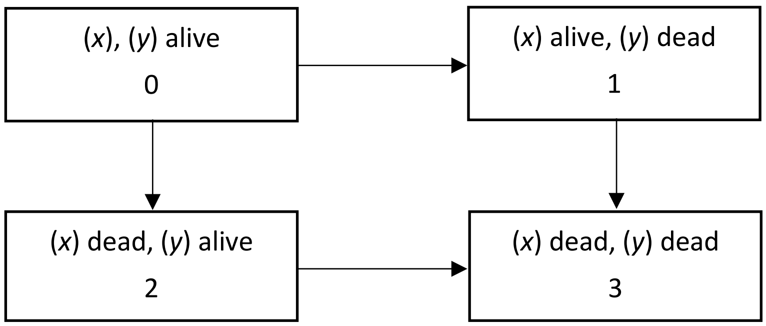
(i) Calculate the percentage reduction in the gross premium, as a result of the change in the interest rate.

(ii) State with reasons whether the percentage reduction in the premium for a 10-year term joint life insurance on Person A and Person B would be greater or less than the change in (i) above.

(This is Question 5 from the Fall 2021 Form A Written Answer exam.)

Question 25

(*8 points*) ABC Insurance uses the following model for calculating premiums and policy values for joint life insurance policies.



The forces of transition are as follows, where μ_z^s is the force of mortality at age z under the Standard Ultimate Life Table (SULT).

$$\mu_{x:y}^{01} = \mu_y^s - 0.005; \qquad \mu_{x:y}^{02} = \mu_x^s; \qquad \mu_x^{13} = \mu_x^s + 0.0025; \qquad \mu_y^{23} = \mu_y^s + 0.005$$

(a) (*1 point*) State with reasons whether the future lifetimes of (x) and (y) are independent under this model.

(b) (*1 point*) Show that ${}_tp_{x:y}^{00} = e^{\lambda t}\,{}_tp_x^s\,{}_tp_y^s$, where ${}_tp_x^s$ and ${}_tp_y^s$ are from the SULT, and where $\lambda > 0$ is a constant which you should specify.

In the following table, you are given annuity values calculated using SULT mortality, at different forces of interest, and assuming independent future lifetimes for joint life functions.

δ	3.50%	3.75%	4.00%	4.25%	4.50%
$\bar{a}_{40:50}^s$	19.3199	18.6530	18.0216	17.4235	16.8564
$\bar{a}_{40}^s$	22.4243	21.4880	20.6129	19.7939	19.0268
$\bar{a}_{50}^s$	20.0391	19.3174	18.6358	17.9917	17.3825

(c) (*1 point*) Determine $\bar{a}_{x:y}^{00}$ at $\delta = 4\%$, for $x = 40$ and $y = 50$, based on the ABC Insurance model.

Question 25 (Continued)

(d) (*3 points*)

(i) Write down an integral expression for the expected present value of a payment of 1 immediately on the first death of (x) and (y), using the multiple state model notation.

(ii) Show that, for $x = 40$ and $y = 50$, at $\delta = 4\%$, the expected present value of a payment of 1 immediately on the first death, under the ABC Insurance joint life mortality model, is 0.227 to the nearest 0.001. You should calculate the value to the nearest 0.00001.

(e) (*1 point*) ABC Insurance issues a fully continuous joint life policy of 500,000 on (x), who is 40, and (y), who is 50.

Calculate the annual rate of net premium for this policy using the ABC Insurance joint life mortality model, with $\delta = 4\%$.

(f) (*1 point*) Exactly 10 years after issue (y) dies while (x) survives. At that time, (x) uses the sum insured to provide a continuous annuity paying X per year while she survives. The annuity amount is determined using the equivalence principle with $\delta = 4\%$.

Calculate X.

(This is Question 4 from the Spring 2022 Written Answer exam.)

Question 26

(*8 points*) An insurer issues fully discrete whole life insurance policies to 10,000 lives, each age 45, with independent future lifetimes.

The death benefit for each policy is 100,000. Gross premiums are determined using the equivalence principle. You are given the following information:

	Pricing and Policy Value Assumptions	Policy Year 1 Actual Experience	Policy Year 2 Actual Experience
Interest	5%	7%	Same as pricing
Expense at Start of the Year	75% of premium + 100 per policy in the first year; 10% of premium + 20 per policy thereafter	75% of premium + 105 per policy	Same as pricing
Settlement Expense	200 per policy	Same as pricing	220 per policy
Mortality	Standard Ultimate Life Table	Same as pricing	10 deaths

(a) (*2 points*) Show that the gross premium for each policy is 1020 to the nearest 10. You should calculate the premium to the nearest 1.

(b) (*2 points*) Calculate the gross premium policy value for a policy inforce at the end of policy year 1.

(c) (*3 points*) For each of interest, expense and mortality, in that order, calculate the gain or loss by source in policy year 1 on this block of policies.

(d) (*1 point*) Explain the sources and direction of any gains or losses in policy year 2. Exact values are not necessary.

(This is a modified version of Question 4 from the Spring 2014 Written Answer exam.)
(Part (a) and part (b) are covered by the FAM-L exam.)

Question 27

(*11 points*) For a special 3-year term life insurance issued to (50) with a premium refund feature, you are given:

(i) The death benefit is 100,000.

(ii) The premium refund feature refunds the last premium payment, without interest, at the end of the 3-year term if the insured is still alive.

(iii) The mortality rates are:

x	q_x
50	0.00592
51	0.00642
52	0.00697

(iv) Pre-contract expenses are 155.

(v) Commissions are 5% of each premium.

(vi) The hurdle rate is 14%.

(vii) The reserves of this policy have been set to:

t	${}_tV$
0	0
1	400
2	800

(viii) The annual premium for this policy is 1100.

(ix) The earned interest rates are:

Year 1	Year 2	Year 3
0.01	0.02	0.03

(a) (*1 point*) Show that the expected profit in policy year 2 for a policy in force at the start of year 2 is 37 to the nearest 1. You should calculate your answer to the nearest 0.01.

(b) (*4 points*) Calculate the profit vector of this policy.

Question 27 (Continued)

(c) (*3 points*) Calculate the profit signature and Net Present Value (NPV) of this policy.

(d) (*3 points*) Rank from low to high the Internal Rate of Return (IRR) of the following products, explaining your order.

Product A: The special 3-year term life insurance described above.

Product B: A 3-year term life insurance policy with the following profit signature: $[-155, 0, 0, 210]$

Product C: The same special 3-year term life insurance as Product A, except that the reserves of the product have been set to:

t	${}_tV$
0	0
1	300
2	800

(This is a modified version of Question 5 from the Fall 2014 Written Answer exam.)

Question 28

(*10 points*) You are performing a profit test on a 50,000, fully discrete 10-year term life insurance policy issued to a healthy life age 60.

The annual premium is waived when the insured is sick.

You are given the following information regarding the policy value basis:

(i) A Markov model with three states: Healthy (0), Sick (1), and Dead (2) is used to value the policy.

(ii) The annual probability transition matrix for an insured age $60+k, k=0,1,\cdots,9$ is:

$$\begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0.90-0.01k & 0.05 & 0.05+0.01k \\ 0.70-0.01k & 0.20 & 0.10+0.01k \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

(iii) The annual gross premium is 5000 from the start of the third year.

(iv) Premiums in the first two years are lower.

(v) Reserves are gross premium policy values.

(vi) Issue expenses are 300 per policy.

(vii) Maintenance expenses are 150 incurred at the start of each year, including the first, for all policies in force.

(viii) $i = 6\%$

(ix) The following actuarial present value functions, calculated at $i = 6\%$:

k	$A^{02}_{60+k:\overline{10-k}\rvert}$	$A^{12}_{60+k:\overline{10-k}\rvert}$	$\ddot{a}^{00}_{60+k:\overline{10-k}\rvert}$	$\ddot{a}^{01}_{60+k:\overline{10-k}\rvert}$	$\ddot{a}^{10}_{60+k:\overline{10-k}\rvert}$	$\ddot{a}^{11}_{60+k:\overline{10-k}\rvert}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
2	0.46667	0.49680	4.7328	0.2533	3.3340	1.4060
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Question 28 (Continued)

(a) (*2 points*) Calculate ${}_2p_{60}^{01}$.

${}_kV^{(j)}$ is the gross premium policy value at the end of year k, for a policy then in state *j*.

(b) (*4 points*)

(i) Show that ${}_2V^{(0)}$ is 420 to the nearest 10. You should calculate the value to the nearest 1.

(ii) Show that ${}_2V^{(1)}$ is 8880 to the nearest 10. You should calculate the value to the nearest 1.

(iii) You are given that ${}_3V^{(0)} = 1788$. Calculate ${}_3V^{(1)}$.

You are given the following additional information regarding the profit test for this policy:

- The earned rate is 5.7%.
- The hurdle rate is 8%.
- Pre-contract expenses are 200.
- Maintenance expenses are 60 at the start of each year, including the first, for all policies in force.
- Mortality and morbidity are the same as in the policy value basis.
- Reserves are gross premium policy values.
- There are no withdrawals.
- The profit signature values for $t = 1, 2$ are $\Pi_1 = 84.74$ and $\Pi_2 = 80.35$

(c) (*4 points*)

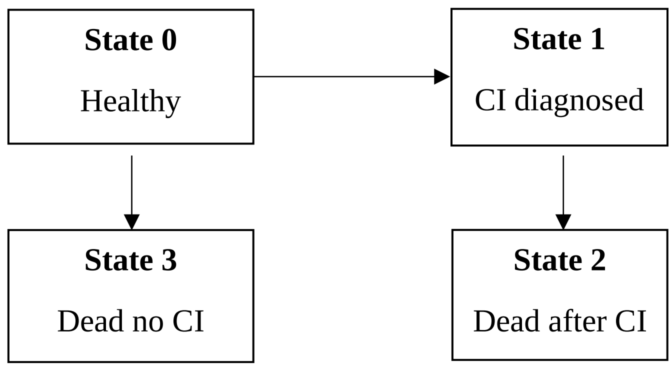
(i) Show that Π_3 is 70 to the nearest 10. You should calculate the value to the nearest 0.1.

(ii) Calculate the Discounted Payback Period.

(This is Question 1 from the Fall 2018 Written Answer exam.)

Question 29

(*8 points*) A 5-year term life insurance policy with a partially accelerated Critical Illness (CI) rider is sold to a Healthy life age x. The policy is to be profit-tested using the multiple state model:



The policy pays 600,000 at the end of the year of CI diagnosis, if the policyholder is still alive at the end of that year; 1,000,000 at the end of the year of death, if the policyholder was in State 0 at the start of that year; and 500,000 at the end of the year of death if the policyholder was in State 1 at the start of that year.

Annual premiums of 30,000 are payable while the policyholder is in State 0.

You are given the following profit test assumptions:

(i) One-year transition probabilities at age $x+k$, for k=0,1, 2, are:

$$p_{x+k}^{01} = 0.01 + 0.002k, \quad p_{x+k}^{02} = 0.008 + 0.003k, \quad p_{x+k}^{03} = 0.004, \quad p_{x+k}^{12} = 0.25$$

(ii) The state-dependent reserves are:

t	2	3	4
${}_tV^{(0)}$	12,000	9,000	5,000
${}_tV^{(1)}$	280,000	210,000	120,000

(iii) Pre-contract expenses are 500.

(iv) Commissions are 5% of each premium including the first.

(v) Maintenance expenses in State 1 are 100 at the beginning of the year.

(vi) The insurer earns interest on investments of 6% per year.

(vii) The hurdle rate is 10% per year.

Question 29 (Continued)

(a) (*3 points*) Show that the emerging profit in year 3, conditional on being in State 0 at the start of the year, is 4,900 to the nearest 100. You should calculate your answer to the nearest 1.

(b) (*2 points*) Show that the emerging profit in year 3, conditional on being in State 1 at the start of the year, is 14,200 to the nearest 100. You should calculate your answer to the nearest 1.

(c) (*2 points*) Calculate the profit signature value for year 3, Π_3.

(d) (*1 point*) You are given the first three values of the profit signature:

$$\Pi_0 = -500 \quad \Pi_1 = -770 \quad \Pi_2 = 3536$$

Calculate the partial net present value for the first 3 years, NPV(3).

(This is Question 5 from the Spring 2019 Written Answer exam.)

Question 30

(*9 points*) An insurer issues a 20-year deferred annuity contract to (65). Level annual premiums will be paid for a maximum of 10 years. On survival to age 85, an annuity of 36,000 per year is paid annually in advance. If the life dies after 10 years, but before the first annuity payment, the premiums paid are returned without interest, at the end of the year of death. There is no payment on death in the first 10 years.

You are given the following premium assumptions:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) Initial expenses are 1000 plus 30% of the premium.

(iii) Expenses of 5% of the premium are payable with the second and subsequent premiums.

(iv) Renewal expenses of 100 per year are payable annually in advance throughout the term of the contract, starting in the second year.

(v) $i = 0.05$

(vi) Premiums are calculated using the equivalence principle.

(a) (*2 points*) Show that the premium is 10,260 to the nearest 10. You should calculate the premium to the nearest 0.1.

The insurer profit tests the policy. You are given the following assumptions for the profit test:

(i) Mortality rates are 90% of the mortality rates in the Standard Ultimate Life Table.

(ii) For the first five years of the contract, 3% of policyholders who survive to each year end lapse at that time. There are no lapses after the first five years.

(iii) Pre-contract expenses are 7000.

(iv) Maintenance expenses start at 70 at the start of the first year, and increase at a rate of 2% per year.

(v) Premium expenses are 5% of each premium, including the first.

(vi) The insurer’s funds earn 7% interest each year

Question 30 (Continued)

(vii) The hurdle rate is 10%.

(viii) Reserves at times 0, 1, 11, 12, 29 and 30 are

$$ {}_{0}V = 500 \qquad {}_{1}V = 10{,}150 $$
$$ {}_{11}V = 143{,}035 \qquad {}_{12}V = 151{,}210 $$
$$ {}_{29}V = 155{,}745 \qquad {}_{30}V = 146{,}275 $$

(b) (*4 points*) Calculate Pr_t for t = 0, 1, 12 and 30.

(c) (*3 points*) The insurer calculates the NPV for each policy issued as 8860.
Calculate the NPV at the start of the second year of future cashflows for each policy in force at that time.

(This is Question 6 from the Fall 2019 Written Answer exam.) (Part (a) is covered by the FAM-L exam.)

Question 31

(*10 points*) A life insurer issues a single premium whole life annuity-immediate, payable annually, with a 10-year guarantee, to B who is age 65.

The basis for calculating the premium and policy values is as follows:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) $i = 0.05$

(iii) Commission is 2000 at issue.

(iv) Other acquisition expenses are 500.

(v) Maintenance expenses of 25 per year are payable with the annuity payments.

(vi) The premium is determined using the equivalence principle.

The single premium is 100,000.

(a) (*2 points*) Show that B's annuity payment is 7550 to the nearest 10. You should calculate the payment amount to the nearest 0.1.

The insurer conducts a profit test of this contract using the following three states:

State 0: Payments in progress; annuitant alive
State 1: Payments in progress; annuitant dead
State 2: Payments ceased; annuitant dead

You are also given the following assumptions for a profit test of the contract.

- Interest on insurer's funds: 6%
- All other assumptions follow the premium and policy value basis.
- Reserves are gross premium policy values.

Question 31 (Continued)

(b) (*1 point*) Write down values for the following probabilities:

(i) p_{73}^{01}

(ii) p_{73}^{02}

(iii) p_{73}^{12}

(c) (*2 points*) Show that the reserve at time 8 for a policy in State 0 is ${}_8V^{(0)} = 76{,}100$ to the nearest 100. You should calculate the value to the nearest 1.

(d) (*1 point*) Show that the reserve at time 8 for a policy in State 1 is ${}_8V^{(1)} = 14{,}100$ to the nearest 100. You should calculate the value to the nearest 1.

(e) (*4 points*)

(i) Show that $\mathrm{Pr}_9^{(0)} = 760$ to the nearest 10. You should calculate the value to the nearest 0.1.

(ii) Show that $\mathrm{Pr}_9^{(1)} = 140$ to the nearest 10. You should calculate the value to the nearest 0.1.

(iii) Calculate Π_9.

(This is Question 5 from the Spring 2020 Written Answer exam.) (Part (a) is covered by the FAM-L exam.)

Question 32

(*10 points*) NED Life issues a fully discrete whole life insurance of 100,000 on *(50)* with a return of premium benefit subject to the following conditions:

- If death occurs in the first 10 years, no premiums are returned.
- If death occurs after 10 years, all premiums, including the first 10, are returned without interest at the end of the year of death.

You are given:

(i) The gross premium is 2,000.

(ii) Premium expenses, payable at the beginning of the year, are 80% of the first year's premium and 5% of premium in subsequent years.

(iii) Termination expenses of 1,000 are paid at the end of the year of death.

(iv) Mortality follows the Standard Ultimate Life Table.

(v) $i = 0.05$

(vi) L^g is the gross loss at issue random variable.

(vii) $(IA)_{60} = 6.63303$

(viii) ${}_kV^g$ denotes the gross premium policy value at the end of year k.

(a) (*1 point*) Show that if $T_{50} = 11.8$, then L^g is 53,420 to the nearest 10. You should calculate the value to the nearest 1.

(b) (*2 points*) Calculate $\mathrm{E}[L^g]$.

(c) (*2 points*) Show that ${}_{10}V^g$ is 20,070 to the nearest 10. You should calculate the value to the nearest 1.

Question 32 (Continued)

NED Life had 1000 such policies in force at the end of 15 years. You are given:

(i) ${}_{15}V^{g} = 34{,}333.78$ and ${}_{16}V^{g} = 37{,}480.51$

(ii) During year 16:

- There were 7 deaths.
- i = 0.052
- Premium expenses were 4% of premium.
- Termination expenses were 2,000 per death.

(d) (*4 points*)

(i) Calculate the total gain in the 16th year.

(ii) Calculate the gain by source in the following order: expenses (E), interest (I) and mortality (M).

(e) (*1 point*) An actuary at NED Life was asked to calculate the gain by source. He wasn't sure which order to use, so he calculated the gain based on all six possible orders. Referring to the sources of gain in order by their first letters, the order you did in part (d) was EIM.

Without further calculation, state, using the three-letter ids, which, if any, of the other 5 orders had the same gain from interest as EIM. Justify your response.

(This is Question 1 from the Fall 2021 Form A Written Answer exam.)
(Part (a), (b) and (c) are covered by the FAM-L exam.)

Question 33

(*10 points*) A whole life insurance policy on (50) has an initial sum insured of 200,000 payable at the end of the year of death. At the beginning of each policy year after the first, the sum insured will increase by 5,000. The insured will pay level annual premiums payable for at most 20 years.

You are given:

(i) Mortality follows the Standard Ultimate Life Table

(ii) $i = 0.05$

(a) (*2 points*)

(i) Show that $(IA)_x = A_x + v\,p_x\,(IA)_{x+1}$

(ii) Use this relationship to show that if $(IA)_{50} = 5.8255$, then $(IA)_{51} = 5.9$ rounded to the nearest 0.1. You should calculate the value to the nearest 0.001.

Annual gross premiums are calculated using the equivalence principle based on expenses of:

- 1000 on the payment of the death benefit.
- 15% of the first premium.
- 5% of all other premiums.

(b) (*2 points*) Show that the annual gross premium is 5,500 to the nearest 100. You should calculate the value to the nearest 1.

(c) (*2 points*) Show that the gross premium policy value at the end of the second policy year is 10,080 to the nearest 10. You should calculate the value to the nearest 1.

Question 33 (Continued)

(d) (*4 points*) You are given:

- The gross premium policy values at times 10 and 11 are ${}_{10}V^g = 63,208$ and ${}_{11}V^g = 71,217$.
- At the start of the 11th policy year, at time $t = 10$, there were 1000 identical, independent policies in force.
- During the 11th year, the experience was as follows:
  - The interest rate earned was 6%.
  - There were six deaths.
  - Expenses of 6% of premiums, plus 1100 for each death benefit, were incurred.

(i) Show that the surplus earned on the portfolio in the 11th policy year, per policy in force at the start of the year, was 160 to the nearest 10. You should calculate the value to the nearest 1.

(ii) Calculate the gain by source, in the order: (1) Interest; (2) Mortality; (3) Expenses.

(This is Question 1 from the Fall 2021 Form B Written Answer exam.)
(Part (a), (b) and (c) are covered by the FAM-L exam.)

Question 34

(*10 points*) XYZ Insurance Company sells two year term policies to lives age *x*. The death benefit of 1,000,000 is payable at the end of the year of death. Premiums are payable annually.

The insurer holds reserves of 0 for this policy at time $t = 0$ and at time $t = 1$.

The assumptions used for profit testing this policy are:

(i) Precontract underwriting expenses are 3,000 per policy.

(ii) Commissions are 20% of premium in year 1 and 5% of premiums each year thereafter. The extra 15% of premium commissions in the first year is treated as a pre-contract cash flow for profit testing purposes.

(iii) Maintenance expense is 100 per policy at the beginning of every year including the first year.

(iv) The earned interest rate is 7%.

(v) Mortality is $q_{x+t} = 0.05 + 0.05t$ for $t = 0,1,2$

(vi) At the end of the first year 10% of the surviving policyholders withdraw.

(vii) There are no cash values.

(viii) $\Pr_1 = 31,213$.

XYZ uses a gross premium of 80,000 in its first profit test for this two year term policy.

(a) (*1 point*) Determine $\Pr_0$.

(b) (*2 points*) Determine $\Pr_2$.

(c) (*1 point*) Determine the profit signature for this policy.

(d) (*1 point*) The NPV for this policy using a risk discount rate of 20% is negative. Calculate the NPV to the nearest 1.

(e) (*1 point*) Calculate the internal rate of return for this policy.

Question 34 (Continued)

(f) (*1 point*) Your boss questions how you can have a positive IRR but a negative NPV. Explain how this is possible.

(g) (*2 points*) Calculate the NPV if the lapse rate increases to 15%.

(h) (*1 point*) Policies where profits increase as the lapse rate increases are called lapse supported policies. Explain why such policies are risky to the insurance company.

(This is Question 2 from the Fall 2021 Form A Written Answer exam.)

Question 35

(*10 points*) XYZ Insurance Company sells three-year term policies with a death benefit of 1,000,000 payable at the end of the year of death, to lives age *x*. Premiums are payable annually.

The insurer holds reserves for the policy equal to net premium policy values, based on an interest rate $i = 0.05$ and mortality of $q_{x+t} = 0.10 + 0.10t$ for $t = 0,1,2$.

The net premium for this policy is 177,313.20. The net premium policy value at the end of the first year, ${}_1V^n$, is 95,754.

The assumptions used for profit testing this policy are:

(i) The pre-contract underwriting expense is 4,000 per policy.

(ii) Commissions are 20% of premium in year 1 and 5% of premium each year thereafter. The extra 15% of premium in commissions in the first year is treated as a pre-contract cash flow for profit testing purposes.

(iii) Maintenance expense is 100 per policy at the beginning of every year including the first year.

(iv) The earned interest rate is 7%.

(v) Mortality is equal to the policy value mortality.

(vi) At the end of each of the first two years, 10% of the surviving policyholders withdraw.

(vii) There are no cash values.

(a) (*1 point*) List four ways that profit tests are applied in practice.

(b) (*1 point*) Show that the net premium policy value at the end of the second year is 108,400 to the nearest 10. You should calculate the value to the nearest 1.

Question 35 (Continued)

XYZ uses a gross premium of 210,000 in its first profit test for this three-year term policy. Using this premium, $\mathrm{Pr}_2 = 37{,}766$ and $\mathrm{Pr}_3 = 29{,}347$.

(c) (*2 points*)

(i) Determine Pr_0.

(ii) Determine Pr_1.

(d) (*3 points*)

(i) Determine the profit signature for this policy.

(ii) Calculate the NPV for this policy using a risk discount rate of 12%.

(iii) Calculate the profit margin for this policy.

(e) (*3 points*) XYZ wants to achieve a profit margin of 10% on this policy. Calculate the gross premium that XYZ should charge to meet that objective.

(This is Question 2 from the Fall 2021 Form B Written Answer exam.)

Question 36

(*8 points*) An insurer offers **"**immediate needs annuities**"** (INAs) to lives age 80 or older who are moving permanently into long term care facilities (LTCFs). The annuity may be purchased as a single premium annuity-due, or as a single premium deferred annuity-due**,** with a 2-year deferral period. The annuity payments are paid directly to the care home to cover the resident's costs.

(a) (*2 points*) You are given the following excerpt from a select and ultimate mortality table used to price the INA. The selection age is the age at which the life moves into an LTCF. The selection period is 1 year.

$[x]$	$l_{[x]}$	l_{x+1}	$x+1$
94	1200	700	95
95	1000	550	96
96	750	385	97
97	450	195	98
98	220	50	99
99	75	0	100

(i) Calculate the 1-year mortality probability for (95) assuming that she moved into an LTCF at age 95.

(ii) Calculate the 1-year mortality probability for (95) assuming that she moved into an LTCF at age 94.

(iii) Describe how the selection effect in this model differs from that of typical select and ultimate tables used for pricing life insurance.

Question 36 (Continued)

(b) (*4 points*) You are given the following additional information for a profit test of an annuity-due contract issued to a 95-year old who is just moving into an LTCF.

- For a single premium of 110,000 the annuity payment is 50,000 at the start of each year including the first, conditional on survival.
- Pre-contract expenses are 300.
- Expenses of 100 are incurred with each annuity payment. There are no other expenses.
- Reserves for the first two years are: ${}_0V = 0$; ${}_1V = 105,000$; ${}_2V = 80,000$
- The insurer earns 7% per year.
- The hurdle rate is 12% per year.

You are also given the following partially completed profit test table, which is missing the entries for the first two policy years:

t	${}_{t-1}V$	Premium	Annuity	Expenses	Interest	Expected End-year reserve	Pr_t
0				--			--
1	--	--	--	--	--	--	--
2	--	--	--	--	--	--	--
3	80,000	0	50,000	100	2,093	31,656	337
4	62,500	0	50,000	100	868	12,846	422
5	50,100	0	50,000	100	0	0	0

(i) Determine the missing entries for the first three rows of the table (missing entries are indicated by --).

(ii) Show that the net present value of the contract, is 6,710 to the nearest 10. You should calculate the value to the nearest 1.

(iii) Show that the profit margin is 6.1% to the nearest 1%. You should calculate the profit margin to the nearest 0.001%.

Question 36 (Continued)

(c) (*3 points*) The deferred annuity contract is identical to the annuity-due contract, but with no payment in the first two years, and with a lower single premium payable at issue. The time 1 reserve for the deferred annuity contract is ${}_1V = 54{,}900$. All other reserves and expenses are identical to the annuity-due contract.

Calculate the premium required under the deferred annuity contract that generates the same profit margin as the annuity-due.

(d) (*1 point*) The insurer offers the deferred annuity at a single premium of 28,500. Describe one advantage and one disadvantage of the deferred annuity, compared with the annuity-due, from the perspective of the purchaser of the annuity.

(This is Question 6 from the Spring 2022 Written Answer exam.)
(Part (a) and (d) are covered by the FAM-L exam.)

Question 37

(*7 points*)

(a) (*1 point*) List three reasons why employers sponsor pensions for their employees.

A benefit plan provides a retirement benefit if the employee lives to age 65, and a death benefit if the employee dies prior to age 65.

- The retirement benefit is an annual whole life annuity-due of 3% of the final 3-year average salary for each year of service.

- The death benefit is a lump sum payable at the end of the year of death equal to two times the employee's annual salary in the year of death.

You are given:

- A person started the employment on January1, 1990 at exact age 38 with a starting salary of 50,000.

- The company gives salary increases of 3% on January 1 each year.

- Employees can terminate employment only by retirement at 65 or death.

You are given the following:

- $q_{62+k} = 0.08 + 0.01k$ for $k = 0,1,2,3$

- The expected present value on January 1, 2017 of an annuity-due of 1 per year payable annually to the person, if they survive, will be 4.7491.

- $i = 0.04$

(b) (*3 points*) Calculate the actuarial present value on January 1, 2014 of the person's death benefit.

(c) (*3 points*) Calculate the actuarial present value on January 1, 2014 of the person's retirement benefit.

(This is a modified version of Question 7 from the Spring 2014 Written Answer exam.)

Question 38

(*9 points*) A company sponsors a defined benefit pension plan. Key attributes of the pension plan are:

- Benefits are payable as a single life annuity, paid at the beginning of each month.
- The annual benefit payable at age 65 is calculated as 2% of the final 3-year average salary up to 100,000 multiplied by years of service, plus 3% of the final 3-year average salary over 100,000 multiplied by years of service. The monthly benefit is the annual benefit divided by 12.
- You are given:

  (i) $i = 5\%$

  (ii) $A_{65}^{(12)} = 0.470$

An individual joined the company on January 1, 2009 at exact age 40. The individual is planning to retire on January 1, 2034 at age 65. The salary for 2014 is 80,000 and the individual receives annual raises of 4% each January 1.

(a) (*2 points*) Assuming the individual works until age 65, show that the monthly accrued benefit at age 65 is 8050, to the nearest 10. You should calculate the benefit to the nearest 1.

(b) (*1 point*) Calculate the replacement ratio at age 65.

(c) (*2 points*) Calculate the expected present value of the benefit at age 65.

In addition to accruing a benefit in the defined benefit plan, the individual will contribute a constant percentage of their annual salary to a defined contribution plan.

You are also given:

(iii) Contributions are made at the start of each year, beginning on January 1, 2014.

(iv) Contributions earn an investment return of 7% per year.

(v) The contribution balance is converted to a monthly life annuity using assumptions (i) and (ii) above.

(d) (*4 points*) Calculate the percentage of each year's salary that the individual needs to contribute so that their total replacement ratio is 80% at age 65.

(This is Question 2 from the Fall 2014 Written Answer exam.)

Question 39

(*6 points*) A person begins work at age 40 at ABC Life on January 1, 2014, with a starting salary of 30,000. The person will switch jobs to XYZ Re at some time before age 55 at the then-current salary and will remain at XYZ Re until retirement.

- Both companies offer 2% annual salary raises on January 1 of each year.
- The annual retirement benefit at ABC Life is 900 per year of service.
- The annual retirement benefit at XYZ Re is 3% of the final 3-year average salary for each year of service.

The person will retire on the 65th birthday, and will receive retirement benefits from both companies.

(a) (*2 points*) Assume the person stays at ABC Life for 9.5 years and then switches to XYZ Re. Show that the replacement ratio would be 63% to the nearest 1%. You should calculate the replacement ratio to the nearest 0.1%.

(b) (*1 point*) Calculate the maximum length of time that the person can remain at ABC Life and still attain a replacement ratio of at least 65%.

The person switches to XYZ Re after seven years on January 1, 2021 (and gets the annual raise). Later, on January 1, 2029, XYZ Re decides to stop all benefit accruals. No further benefits accrue after this date. On retirement, the benefit in respect of service before 2029 will be based on the final average salary as at January 1 2029.

On January 1, 2029, the person purchases a 10-year deferred whole life annuity due, with premiums payable annually during the deferred period. The annuity payments, combined with the accrued retirement benefits, will give the person a replacement ratio of 65%.

You are given:

- The person's mortality follows the Standard Ultimate Life Table
- $i = 5\%$

(c) (*3 points*) Calculate the annual net premium for the annuity.

(This is a modified version of Question 7 from the Spring 2015 Written Answer exam.)

Question 40

(*8 points*) On December 31, 2015, an individual, who is age 55, has 25 years of service. The individual's salary in 2015 was 50,000.

The annual accrued benefit as of any date is 1.6% of the three-year final average salary as of that date, multiplied by years of service as of that date. The pension is payable only to retired participants as a monthly single life annuity, with the first payment due at retirement.

The valuation assumptions are as follows:

- There are no benefits paid upon death.
- There are no benefits paid upon disability prior to age 60.
- Exits from employment follow the Standard Service Table, except that all lives surviving in employment to age 61 retire at that time.
- There are retirements at age 60 that occur at exact age 60 while other decrements at age 60 occur throughout the year after the retirement at exact age 60 and prior to exact age 61. Those that retire or become disabled between exact age 60 and exact age 61 are assumed to retire at exact age 61.
- After retirement, mortality follows the Standard Ultimate Life Table.
- Annuities are valued using the 2-term Woolhouse formula.
- The individual has received a 3% salary increase on January 1 of each year for the last five years.
- Future salaries are expected to increase by 3% each year on January 1.
- All contributions are paid on January 1 each year.
- $i = 0.05$

(a) (*1 point*) A pension plan may be classified as a Defined Contribution plan or a Defined Benefit plan. State which type of plan the individual has and briefly describe the other type of plan.

(b) (*3 points*)

(i) Show that the actuarial accrued liability for the individual at December 31, 2015 using the Projected Unit Credit (PUC) funding method is 220,000 to the nearest 1000. Calculate the actuarial accrued liability to the nearest 1.

(ii) Calculate the normal cost for 2016 using PUC.

Question 40 (Continued)

(c) (*3 points*)

(i) Show that the actuarial accrued liability for the individual at December 31, 2015 using the Traditional Unit Credit (TUC) funding method is 186,000 to the nearest 1000. Calculate the actuarial accrued liability to the nearest 1.

(ii) Calculate the normal cost for 2016 using TUC.

(d) (*1 point*) For the individual, the normal cost during 2016 using PUC is less than the normal cost during 2016 using TUC. Explain why this will be true for all employees near retirement.

Question 41

(*10 points*) A corporation offers its employees a Final Salary Defined Benefit pension with an accrual rate of 2% per year of service. The Final Pensionable Salary is the salary earned in the final year of employment. The pension is paid as a single life monthly annuity-due with a 10-year guarantee.

Employees contribute 6% of pay at the end of each month throughout their period of employment.

An employee of the corporation is about to retire at age 65. The employee was hired at age 30 and has worked continuously since then. Their starting salary was 40,000 per year.

You are given the following additional information:

(i) Salaries increase monthly at a rate of 3.6% per year compounded monthly.

(ii) The two term Woolhouse Formula is used for valuing the retirement pension.

(iii) Mortality is assumed to follow the Standard Ultimate Life Table.

(iv) The valuation interest rate is 5%.

(v) Members have the option of taking a lump sum at retirement equal to the expected present value of their retirement pension, using the valuation assumptions.

(a) (*2 points*) Show that the employee's monthly pension is 8050 to the nearest 10. You should calculate the value to the nearest 0.1.

(b) (*2 points*) Calculate the expected present value of the employee's retirement pension at age 65.

(c) (*2 points*) Show that the accumulated value of the employee's contributions at retirement, assuming interest of 9.6% per year compounded monthly, is 995,500 to the nearest 100. You should calculate the value to the nearest 10.

(d) (*1 point*) Assume that the employee takes the lump sum option. State with reasons whether the internal rate of return (IRR) that the employee has earned on their contributions is more than or less than 9.6% per year compounded monthly.

Question 41 (Continued)

(e) (*1 point*) Employees have the option to take a higher monthly pension at retirement without a guarantee period. The revised benefit has the same expected present value at age 65 as the standard benefit.
Calculate the revised monthly payment.

(f) (*2 points*)
(i) Define adverse selection.

(ii) Explain how adverse selection might impact the pension plan's costs based on the employees' selection of annuity with guarantee, annuity without guarantee, and lump sum.

(This is Question 6 from the Fall 2018 Written Answer exam.)

Question 42

(*8 points*) Person D is a member of a pension plan that offers an annual retirement pension of 2% of the member's career average earnings for each year of service.

At the valuation date, D is exactly 63 years old, has 30 years of service, and their total past earnings are 2,500,000. D's salary in the year following the valuation date will be 160,000 if they work for the full year, and are paid in level monthly payments.

You are given:

(i) The pension is paid as a monthly life annuity-due.

(ii) Members can retire at any time after age 60 without actuarial reduction of benefits.

(iii) There are no benefits other than the age retirement pension.

(iv) The Traditional Unit Credit approach is used for valuation and funding.

(v) The valuation uses the Standard Service Table.

(vi) All lives retiring before age 65 are assumed to exit exactly half-way through the year of age.

(vii) Mortality after retirement follows the Standard Ultimate Life Table with Uniform Distribution of Deaths.

(viii) $i = 0.05$

(ix) $\ddot{a}_{63.5}^{(12)} = 13.514, \quad \ddot{a}_{64.5}^{(12)} = 13.231, \quad \ddot{a}_{65}^{(12)} = 13.086$

(a) (*2 points*) Show that the Actuarial Liability for D is 595,000 to the nearest 1000. You should calculate the value to the nearest 1.

(b) (*3 points*) Show that the Normal Contribution for D is 36,000 to the nearest 1000. You should calculate the value to the nearest 10.

Question 42 (Continued)

(c) (*3 points*) D is eligible for post-retirement health benefits. The benefits are provided through a group insurance policy with premiums paid annually, starting immediately on retirement. You are given the following information:

- Annual premiums are based on the retiree's age last birthday. The first premium is paid immediately on retirement.
- The premium for a new retiree age 60 last birthday is 2000 at the valuation date, $t = 0$.
- For a new retiree age *y* last birthday, retiring *t* years following the valuation date, the initial annual premium is

$$B(y,t) = 2000(1.035)^{y-60}(1.03)^{t} \quad \text{for } y = 60, 61, \cdots, 65, \text{ and } t \geq 0.$$

- $\ddot{a}^{B}_{63.5} = 30.420, \quad \ddot{a}^{B}_{64.5} = 29.124, \quad \ddot{a}^{B}_{65} = 28.487$

Calculate the expected present value of D's supplementary health insurance benefit.

(This is Question 3 from the Spring 2019 Written Answer exam.)

Question 43

(*8 points*) Person X is a member of a defined benefit pension plan which offers an annual pension of 2% of the final one year's salary for each year of service. The benefit is a whole life annuity-due paid monthly. At the valuation date, 1/1/2020, X is 40 years old with 20 years' service. X's salary during the 12 months prior to the valuation was 110,000.

You are given:

(i) All members in service at age 65 retire immediately.

(ii) There are no other retirements or withdrawals (other than death).

(iii) Mortality follows the Standard Ultimate Life Table.

(iv) $i = 0.05$

(v) Salary increases occur annually on 1 January each year.

(vi) Salaries are assumed to increase by 3% each year up to and including age 54, and by 2.5% each year from age 55.

(vii) Woolhouse's 2-term formula is used for monthly annuity functions.

(viii) The benefits are funded using the Projected Unit Credit method.

(a) (*3 points*)

(i) Show that the Actuarial Liability for X's retirement benefits is 323,000 to the nearest 1,000. You should calculate the value to the nearest 1.

(ii) Calculate the Normal Contribution rate for X's retirement benefits.

Question 43 (Continued)

The plan proposes the following enhancement:

- A reversionary annuity benefit to the member's partner at 50% of the member's pension benefit, or
- For members who do not have a partner at their retirement dates, the pension would be increased by 5%.

Assume:

(i) 90% of members have partners at their retirement dates.

(ii) Partners are the same age as the member, for members with partners.

(iii) The mortality of members and their partners follows the Standard Ultimate Life Table.

(iv) Members and their partners have independent future lifetimes.

(b) (*4 points*)

(i) Calculate the revised Actuarial Liability for X's enhanced retirement benefits (including the partner benefit).

(ii) Calculate the revised Normal Contribution rate for X's enhanced retirement benefits.

(c) (*1 point*) The plan actuary is concerned that couples may have future lifetimes which are positively correlated. State with reasons whether the expected present value of the reversionary partner benefits would increase, decrease, or stay the same, in this case.

(This is Question 6 from the Spring 2020 Written Answer exam.)

Question 44

(*8 points*) You act as the valuation actuary for a corporation which sponsors a final average salary defined benefit pension plan for its employees. The age retirement benefit provisions and valuation assumptions for the corporation's plan are described below.

- The accrual rate is 2% per year of service.
- The final average salary is defined as the salary over the final year of employment.
- The Normal Form of pension is a life annuity with no guarantee, paid monthly in advance.
- The normal retirement age is 65.
- Salaries increase each year on 1 January at a rate of 2.5% per year.
- $i = 0.05$
- Mortality of active members and retirees follows the Standard Ultimate Life Table.
- There are no exits prior to retirement at age 65, other than death.
- The two-term Woolhouse formula is used for annuities paid more frequently than annually.
- The plan is funded using the Traditional Unit Credit method.

You are also given the following summary membership data, as of the valuation date, 1 January 2020.

Age	Number of members	Status	Pension in payment	Salary per member in 2019	Years of service per member
35	20	Active	-	45,000	8
60	5	Active	-	62,000	25
70	1	Retired	32,000	--	30

(a) (*2 points*) Calculate the total actuarial liability for the corporation's pension plan as of the valuation date.

(b) (*3 points*) Calculate the normal cost for 2020, expressed as a percentage of the total payroll at the valuation date.

Question 44 (Continued)

(c) (*1 point*) The corporation terminated the employment of all the 35-year-old members on the valuation date.

Calculate the revised normal contribution rate for the corporation, expressed as a percentage of the total payroll of the remaining plan members.

(d) (*2 points*) Without further calculation, state whether the change in the normal contribution rate would be greater or smaller under Projected Unit Credit funding. Justify your answer.

(This is Question 6 from the Fall 2020 Written Answer exam.)

Question 45

(*8 points*) You are given the following information about D, who is a member of a Career Average Earnings pension plan.

(i) On the valuation date, 1/1/2021, D is 63 years old.
(ii) D joined the pension plan on 1/1/1998.
(iii) On 1/1/2021 D's total past earnings are 2,400,000.
(iv) D's salary in 2021, assuming they stay in employment for the whole year, will be 170,000.
(v) All salaries are set on the 1st of January each year.
(vi) Contributions are paid on the 1st of January each year.
(vii) The accrual rate is 2%.
(viii) The normal pension form is a single life annuity-due payable monthly.
(ix) On death in service, the benefit is a lump sum of 4 times the annual pension that the member would have received had they retired at the time of their death.
(x) There are no other benefits.

The valuation assumptions are:

- Decrements from active service follow the Standard Service Table.
- Exits before age 65 occur half-way through the year of age.
- $i = 0.05$
- $\ddot{a}^{(12)}_{63.5} = 13.5139$; $\ddot{a}^{(12)}_{64.5} = 13.2312$; $\ddot{a}^{(12)}_{65} = 13.0870$
- The funding method is Traditional Unit Credit.

(a) (*4 points*)

(i) Calculate the Actuarial Liability for the age retirement pension for D at 1/1/2021.

(ii) Calculate the Normal Contribution for the age retirement pension for D at 1/1/2021.

(b) (*4 points*)

(i) Calculate the Actuarial Liability for the death in service benefit for D at 1/1/2021.

(ii) Calculate the Normal Contribution for the death in service benefit for D at 1/1/2021.

(This is Question 5 from the Spring 2021 Written Answer exam.)

Question 46

(*9 points*) A person (J), who is 57, is a member of a final average salary defined benefit pension plan. The person has 35 years service. You are given the following information. The person's salary was 100,000 last year and this year.

- The accrual rate is 1.8%
- The pension is payable monthly to the plan member from age 65, with a 100% survivor pension paid to the member's partner after the member's death, provided this occurs after age 65.
- There is no benefit on death after withdrawal before age 65.
- The final average salary is the average of the final 2-year's salary before exit.
- Salaries are expected to be frozen for the next year.
- The person's partner (M) is also 57.

You are also given the following valuation assumptions and information:

(i) Decrements follow the Standard Service Table

(ii) $i = 0.05$

(iii) Mortality after exit follows the Standard Ultimate Life Table.

(iv) Withdrawals occur ½ way through the year.

(v) J and M have independent future lifetimes.

(vi) $\ddot{a}_{65}^{(12)} = 13.0870$; $\ddot{a}_{65:65}^{(12)} = 11.2158$

(vii) Uniform distribution of deaths between integer ages for all other fractional age calculations.

(viii) a_x^w represents the value on withdrawal at age x of an annuity of 1 per year paid monthly in advance from age 65, including the survivor's benefit.

(ix) $a_{58.5}^w = 10.5804$; $a_{59.5}^w = 11.1456$

(x) The valuation uses the traditional unit credit funding method.

Question 46 (Continued)

(a) (*2 points*)

(i) Show that, under the Standard Ultimate Life Table, ${}_{7.5}E_{57.5:57.5} = 0.655$ to the nearest 0.001. You should calculate the value to the nearest 0.0001.

(ii) Show that $a_{57.5}^{w} = 10.05$ to the nearest 0.01. You should calculate the value to the nearest 0.001.

(b) (*3 points*) Show that the actuarial liability for the withdrawal benefit is 36,000 to the nearest 1000. You should calculate the value to the nearest 1.

(c) (*2 points*) Calculate the normal contribution for the withdrawal benefit.

(d) (*2 points*) Person J withdraws at age 57.5 and immediately gets divorced. Under the divorce settlement, Person M will receive a pension of *X*/3 for life from age 65, and Person J will receive a pension of *X* for life from age 65. The value of the settlement is equal to the value of the joint and last survivor pension payable had the couple not divorced.

Calculate *X*.

(This is Question 6 from the Fall 2021 Form A Written Answer exam.)

Question 47

(*9 points*) Person M enrolled in a defined contribution (DC) pension plan at age 35. Person M and their employer each contribute 6% of Person M's salary to the pension fund.

M plans for the retirement based on the following assumptions:

- M's initial salary rate is 50,000 per year.
- Salaries increase continuously at an annual rate of 2%, compounded continuously.
- Contributions are invested continuously to a fund earning a 6% per year force of interest.
- M intends to retire on their 60th birthday.

(a) (*2 points*) Show that M's projected DC account balance as of their 60th birthday is 425,000 to the nearest 1000. You should calculate the value to the nearest 1.

(b) (*2 points*) Assume first that, on retirement, M elects to receive a monthly life annuity with a 10-year guarantee. Annuities are priced using the equivalence principle, based on the following assumptions:

- Mortality follows the Standard Ultimate Life Table.
- The annual effective interest rate is $i = 0.05$.
- Woolhouse's two term formula.

(i) Calculate M's projected monthly income in retirement.

(ii) Estimate M's projected Replacement Ratio.

M's spouse, C, is also 60. M and C have independent future lifetimes. When they reach age 60, M's DC account has accumulated to 750,000, after several years of high investment returns.

(c) (*2 points*) Assume that M elects to receive a monthly pension payable while both M and C survive, with a 2/3 survivor's benefit payable after the first death, with no guarantee period. Show that the monthly benefit is 4,200 rounded to the nearest 100. You should calculate the value to the nearest 1.

Question 47 (Continued)

(d) (*1 point*) Person M is also eligible for a Government Pension Plan (GPP) which will provide a benefit of 1000 per month starting at age 65, payable during M's lifetime and with a 50% benefit payable to C after M's death.

Calculate the monthly benefit payable from M's entitlements from both the DC pension plan and GPP eight years after M's retirement under each of the following states:

(i) M and C survive.

(ii) Only M survives.

(iii) Only C survives.

(e) (*2 points*) M elects to receive an integrated pension which will start at X per month at age 60 and will reduce to X–1000 per month at age 65, so as to provide a level benefit before and after the start of M's GPP benefit, assuming M survives to age 65.

Calculate X, assuming the 2/3 survivor pension option.

(This is Question 6 from the Fall 2021 Form B Written Answer exam.)

Question 48

(*9 points*) B is the sole member of a final average salary pension plan which is being valued on January 1, 2022. At the valuation date B is 63 years old and has 12 years of service. B's salary at the valuation date is 72,100. B's salary in 2021 was 70,000.

The pension plan benefits are as follows:

- The annual benefit is 1.6% of final average salary times years of service.
- Final average salary is the salary earned during the 12 months preceding retirement.
- Benefits are reduced by 0.4% for each month retirement precedes age 65.

The valuation at January 1, 2022 uses the projected unit cost (PUC) method with the following assumptions:

(i) Salaries increase by 3% each January 1.

(ii) Retirements before 65 occur in the middle of the year of age.

(iii) Decrements before retirement follow the Standard Service Table.

(iv) $i = 0.05$

(v) Under post-retirement mortality $\ddot{a}^{(12)}_{63.5} = 13.5139$.

(a) (*3 points*)

(i) Show that if B retires on July 1, 2022, the expected present value of B's retirement benefits at that time would be 178,200 to the nearest 50. You should calculate the value to the nearest 1.

(ii) Show that the expected cost of mid-year retirement decrements in 2022 is 16,500 to the nearest 50. You should calculate the value to the nearest 1.

Question 48 (Continued)

You are also given that the projected accrued liability for B's retirement benefits on January 1, 2023, assuming that B is employed then, is 191,309.

(b) (*2 points*)

(i) Show that the actuarial liability of B's retirement benefits at January 1, 2022 is 167,150 to the nearest 10. You should calculate the value to the nearest 1.

(ii) Calculate the normal cost for B's retirement benefits as of January 1, 2022.

The pension trustees are considering changing to the traditional unit credit (TUC) funding method.

(c) (*2 points*)

(i) State with reasons whether the actuarial liability as of January 1, 2022, under the TUC funding method will be bigger or smaller than the actuarial liability as of January 1, 2022 under the PUC funding method.

(ii) State with reasons whether the normal cost for 2022 under the TUC funding method will be bigger or smaller than the normal cost for 2022 under the PUC funding method.

(d) (*2 points*) You are given the following additional information

- The plan is 100% funded at January 1 2022, under the PUC funding method.
- B retires on January 1, 2023.
- The earned interest rate during 2022 was 5.1%.
- The valuation as of January 1, 2023 uses the PUC funding method, based on the same assumptions as the previous year.
- The normal cost for 2022 was paid in full on January 1, 2022.
- $\ddot{a}_{64}^{(12)} = 13.3735$.

Calculate the gain or loss to the pension plan during 2022.

(This is Question 5 from the Spring 2022 Written Answer exam.)

Question 49

(*8 points*) A life insurance company issues a Type B universal life policy with an Additional Death Benefit of 150,000 to a life age 35.

You are given:

Policy Year	Annual Premium	% Premium Charge	Annual Cost of Insurance Rate Per Thousand	Credited Interest Rate	Account Value at End of Year	Surrender Charge
1	4800	20%	2.31	4%	3647	2500
2	4800	5%	2.46	4%	8166	1500
3	4800	5%	2.62	4%	-	0

(a) (*2 points*) Show that the total death benefit at the end of the third year is 162,800 to the nearest 100. You should calculate the value to the nearest 1.

Now the company wants to carry out a profit test of this contract using the following assumptions:

(i) At the end of the first year, 10% of the in-force policyholders surrender.

(ii) At the end of the second year, 20% of the in-force policyholders surrender.

(iii) At the end of the third year, all remaining in-force policyholders surrender.

(iv) There are no reserves held other than the account value.

(v) The interest earned on all insurer's funds is 8% per year.

(vi) Mortality experience is $q_{35+t} = 0.0015$ for $t = 0, 1$ and 2.

(vii) Pre-contract expenses are 800.

(viii) Percent of premium expenses are 25% in the first year and 7% for years 2 and 3.

(ix) Expenses on death are 500.

(x) The hurdle rate is 10% per year.

Question 49 (Continued)

(b) (*3 points*) Calculate the profit vector.

(c) (*2 points*) Calculate the NPV.

(d) (*1 point*) Calculate the profit margin.

(This is Question 2 from the Fall 2017 Written Answer exam.)

Question 50

(*8 points*) An insurer issues a Type A universal life policy with a face amount of 100,000 and an annual premium of 50,000.

You are given:

Policy Year	Percent of Premium Charge	Annual Expense Charge	COI Rate per 1 of Insurance	Annual Discount Rate for COI	Annual Credited Interest Rate	Corridor Factor
1	20%	75	0.025	4.5%	6.50%	1.5
2	8%	75	0.030	4.5%	5.75%	1.4

(a) (*3 points*) Show that the account value at the end of two years is 91,000 to the nearest 1000. You should calculate the account value to the nearest 1.

The policyholder surrenders the policy at the end of the second year, when she is age 60. The policy does not have a surrender charge. She uses the surrender value as a net single premium to purchase a special last-survivor life annuity-due with her husband, who is 10 years older than the policyholder.

This annuity provides the following payments at the beginning of each quarter:

- For the first ten years, a guaranteed payment of Q
- After the first ten years, a payment of Q if the policyholder is alive
- After the first ten years, a payment of $0.6Q$ if the policyholder is dead, but her husband is alive.

You are given that the net single premium and reserves for this annuity are calculated based on the following information and assumptions.

(i) The future lifetimes of the couple are independent.

(ii) Mortality follows the Standard Ultimate Life Table.

(iii) $i = 0.05$

(iv) The two-term Woolhouse formula

Question 50 (Continued)

(b) (*3 points*) Show that Q is 1630 to the nearest 10. You should calculate Q to the nearest 1.

You are given that the policyholder's husband dies during during the first 10 years of the annuity.

(c) (*1 point*) Calculate the net premium reserve immediately prior to the payment of Q at the start of the eleventh year.

In the eleventh year, Person S decides that they no longer needs the annuity and asks the company to pay the reserve. The insurance company refuses to pay the full reserve.

(d) (*1 point*) Explain why the insurance company would not agree to S's request.

(This is Question 3 from the Spring 2017 Written Answer exam.)

Question 51

(*10 points*) Person D buys a Type B universal life contract of 100,000. You are given:

(i)

Policy Year k	Annual Premium	Annual Cost of Insurance Rate per 1000 of Insurance	Percent of Premium Charge	Annual Expense Charge	Surrender Charge
1	1000	––	60%	––	––
2	P_2	2	10%	10	200
3	P_3	3	10%	10	100
$k \geq 4$	P_k	k	5%	10	0

(ii) The credited interest rate is $i = 0.06$.

(iii) D's account value at the end of year 1 is 165.

(iv) Except as indicated, there are no deaths or surrenders.

(a) (*2 points*) Show that if P_2 were 1000, D's account value at the end of year 2 would be 920 to the nearest 10. You should calculate the account value to the nearest 1.

(b) (*2 points*) D's account value at the end of year 3 can be expressed as $aP_2 + bP_3 + c$. Calculate a, b, and c.

Question 51 (Continued)

(c) (*4 points*) In year 2, D pays a premium of 1000 with probability 0.6, or 200 with probability 0.4.

If D paid 1000 in year 2, then in year 3 D will pay either 1000 with probability 0.6, or 200 with probability 0.4.

If D paid 200 in year 2, then in year 3 D will pay either 1000 with probability 0.2, or 200 with probability 0.8

(i) Calculate the expected death benefit payable at the end of year 3, if D dies then.

(ii) Calculate the expected surrender benefit payable at the end of year 3, if D surrenders the contract then.

(d) (*2 points*) D's identical twin, M, buys a contract identical to D's. If M pays 1000 every year, M's account value at the end of year 10 will be 5114.

M will pay premiums of 1000 in 9 of the first 10 years. M will pay no premium in one year, with the year of no premium equally likely to be year 3 or year 10.

Calculate M's expected surrender value at the end of year 10.

(This is Question 5 from the Fall 2015 Written Answer exam.)

Question 52

(*9 points*) For universal life insurance policies with a death benefit of 100,000 plus the account value issued to independent lives age 50, you are given:

(i)

Policy Year	Annual Premium	Percent of Premium Charge	Annual Cost of Insurance Rate per 1000	Annual Expense Charge	Surrender Charge
1	2500	5%	5.00	30	$0.8 * AV_1$

(ii) The credited interest rate is $i = 0.06$.

(a) (*2 points*) Show that the cash value at the end of policy year 1 is 400 to the nearest 10. You should calculate the value to the nearest 1.

You are also given the following information for profit-testing these policies:

(i) The expenses for each policy are:

- Pre-contract expenses of 200
- Annual maintenance expenses of 120 at the beginning of each year including the first year
- Surrender benefit expenses of 100
- Death benefit expenses of 200

(ii) $q_{50} = 0.004$

(iii) Death benefits are payable at the end of the year of death.

(iv) At the end of the first year, 10% of the policyholders in force are expected to surrender.

(v) The earned rate is 11%.

(vi) The hurdle rate is 14%.

Question 52 (Continued)

(b) (*3 points*) Show that the expected profit emerging at the end of year 1 is 400 to the nearest 10. You should calculate the value to the nearest 1.

(c) (*1 point*) Calculate NPV(1), the actuarial present value of expected profits through year 1.

You have calculated that the Net Present Value (NPV) for the policy is 2000. Your supervisor suggests that the 10% surrender assumption is too low. He recommends assuming 20% of the policies in force surrender at the end of year 1.

(d) (*3 points*) Calculate the revised NPV using the new surrender assumption.

(This is Question 6 from the Spring 2015 Written Answer exam.)

Question 53

(*10 points*) For a Type A universal life policy issued to (50), you are given:

(i) The face amount is 100,000.

(ii) All cash flows occur at policy anniversaries.

(iii) The policyholder pays an initial premium of 15,000.

(iv) The cost of insurance (COI) is calculated based on 120% of the mortality in the Standard Ultimate Life Table. The interest rate for discounting the net amount at risk, i^q, is 0.04.

(v) The expense charge is 1% of premium.

(vi) The credited interest rate for policy year 1 is 5%.

(vii) The corridor factor in year 1 is 2.2.

(viii) The surrender charge in policy year 1 is 5% of the premium paid.

(a) (*1 point*) Explain why the corridor factor requirement exists for universal life insurance.

(b) (*5 points*)

(i) Calculate the COI in policy year 1 assuming there is no corridor factor requirement.

(ii) Calculate the COI in policy year 1 based only on the corridor factor (as if the face amount were 0).

(iii) Determine the COI in policy year 1.

Question 53 (Continued)

(c) (*2 points*) Calculate the account value, additional death benefit, and cash value at the end of policy year 1.

(d) (*2 points*) The policy contains a no-lapse guarantee providing term insurance of 100,000 until age 90.

If the expected present value of the guaranteed insurance coverage is greater than the account value, the company holds a reserve for the no-lapse guarantee equal to the difference. The expected present value is based on the Standard Ultimate Life Table at 5% interest with no expenses.

The account value at the end of policy year 20 is 20,000.

Calculate the reserve for the no-lapse guarantee at the end of year 20.

(This is an edited version of Question 6 from the Fall 2014 Written Answer exam.)

Question 54

(*10 points*) For Type B universal life insurances with additional death benefit 100,000 on lives age 60, you are given the following policy details:

Policy Year k	Annual Premium	Credited Interest Rate	Percent of Premium Expense Charge	Annual Expense Charge	Annual Cost of Insurance Rate per 1000	Surrender Charge
1	5000	5%	60%	200	6.00	600
2	1000	5%	10%	50	7.47	200
3	–	–	–	–	–	0

You are also given the following profit testing assumptions:

Policy Year k	Dependent Probabilities		Earned Interest Rate	Commissions as Percent of Premium	Other Expenses
	$q^{(d)}_{60+k-1}$	$q^{(w)}_{60+k-1}$			
1	0.004	0.100	6%	90%	100
2-10	$0.003 + 0.001k$	0.050	6%	5%	10

Expenses are incurred at the start of each policy year. Your company uses account values as reserves. The account value at the end of year 2, for a policy in force, is 1500.

(a) (*2 points*) Show that the account value at the end of year 1, for a policy in force, is 1300 to the nearest 100. You should calculate the account value to the nearest 1.

Your company discounts expected profits at 12%. The expected profits for the first three years are:

Policy Year k	Expected profits at the end of the year for a policy in force at the start of the year
1	−1206
2	374
3	400

(b) (*3 points*) Calculate the partial Net Present Value up, to time 3, $NPV(3)$.

Question 54 (Continued)

(c) Your company's pricing actuary asks you to redo the profit test using the end of year cash values as reserves, with no other changes.

(i) (*3 points*) Calculate the revised expected profit for policy year 2, per policy in force at the start of year 2.

(ii) (*1 point*) Explain why the expected profit in policy year 2 decreases due to this change.

(iii) (*1 point*) State with reasons whether the total present value of profits in all years will increase, decrease, or stay the same due to this change.

(This is Question 6 from the Spring 2014 Written Answer exam.)

Question 55

An insurer issues a 3-year equity-linked insurance policy to (x). The policyholder pays a single premium of 5000, and an additional policy fee of 350. The insurer invests 100% of the premium in a separate account, with management charges of 2.5% of the fund value deducted at the start of each year including the first.

The policy offers a Guaranteed Minimum Death Benefit and a Guaranteed Minimum Maturity Benefit of 100% of the initial premium (excluding the policy fee).

The insurer stress tests the profitability of the contract using the following assumptions:

- Pre-contract expenses are 200
- Renewal expenses are 30 at the start of the 2nd and 3rd years.
- Fund management expenses are 2% of the policyholder's fund value at the start of the first year, and 1% of the policyholder's fund value at the start of the 2nd and 3rd years, before management charge deductions in each case.
- $q_{x+t} = 0.05 + 0.01t$ for $t = 0, 1, 2$.
- There are no lapses
- Interest of 5% per year is earned on the insurer's funds.
- No reserve is held for the contract guarantees in the insurer's assets.
- The insurer uses a risk discount rate of 10% per year.
- The return on the assets in the policyholder's fund for each of the three years (before any management charge deductions) are:

Year 1	–14%
Year 2	– %
Year 3	+ 4%

(a) Show that the projected fund value at time 3 is 3940 to the nearest 10. You should calculate the value to the nearest 1.
(b) Calculate the profit vector for the contract.
(c) Show that the net present value of this contract, based on the stress test assumptions given, is –600 to the nearest 10. You should calculate the value to the nearest 1.

The insurer can purchase options on the underlying fund assets to hedge the guarantees, using the following parameters:

(i) Volatility, $\sigma = 0.25$
(ii) The risk free rate is 4% per year compounded continuously
(iii) Management charge deductions are 2.5% per year

(d) Show that the price of a 3-year put option guaranteeing a return of premium is 0.136 per unit of premium, to the nearest 0.001. You should calculate the value to the nearest 0.0001.

(e) You are given the following market prices at time 0 for 1 and 2 year put options on the underlying fund assets, based on the pricing assumptions above, and a premium of 5000.

Term	Put Option Price
1-year	446.7
2-year	590.8

Assume that mortality follows the profit test assumptions above. Show that the total cost at issue of hedging the guaranteed minimum death benefit and the guaranteed minimum maturity benefit is 665 to the nearest 5. You should calculate the value to the nearest 0.1.

(f) Calculate the net present value of the contract under the same stress test as in (a), but assuming that the insurer hedges the guarantees by purchasing options at a cost of 665 at the start of the first year.

(g) Describe one advantage and one disadvantage for the insurer of purchasing put options to hedge the guarantee costs.

Question 56

A 10-year equity-linked policy is issued to (x). The policy offers a guaranteed minimum maturity benefit of 100% of the initial single premium. After an initial expense charge of 5% of the premium, the remainder is invested in separate fund. The insurer deducts a 0.2% management charge from the policyholder's fund at the end of each month, if the policyholder survives.

You are given:

- An investment at $t = 0$ of 1 in the underlying asset will have value S_t at t, where S_t follows a geometric Brownian motion, with volatility σ, and $S_0 = 1$.
- $\text{BSP}(K) = \text{E}_0^Q\left[e^{-10r}\left(K - S_{10}\right)^+\right]$ is the Black-Scholes option price formula for a put option on an investment of 1 in the underlying stock at time 0, with strike price K.
- There are no exits other than death.

(a) Show that the risk-neutral value of the GMMB option at issue is
$\pi(0) = P\left({}_{10}p_x\ \xi\, \text{BSP}(K^*)\right)$
You should state any assumptions you need, and you should define the parameters ξ and K^* in terms of the information given about this specific contract.

(b) Given $\sigma = 0.25$, $x = 60$, $P = 100{,}000$, Standard Ultimate Life Table mortality, and $r = 0.04$, show that $\pi(0) = 17{,}100$ to the nearest 100.

(c) Assume that the insurer constructs a hedge portfolio at $t = 0$ based on the Black Scholes option valuation. The insurer does not rebalance the hedge until one month later. You are given that in the first month, the underlying asset values fell by 3%. Calculate the value of the hedge portfolio at the end of the first month, before rebalancing.

(d) You are given that $a_{60:\overline{10}|}^{(12)} = 8.679$. Calculate the cost of the option as a monthly deduction from the fund, assuming no lapses.