Formula Sheet Used on Exam ALTAM

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ALTAM Formula Sheet

Interest Functions

$$\alpha(m) = \frac{id}{i^{(m)}d^{(m)}} \text{ and } \beta(m) = \frac{i - i^{(m)}}{i^{(m)}d^{(m)}}$$

Makeham's Law

$$\mu_x = A + Bc^x \quad \text{and} \quad {}_tp_x = \exp\left(-At - \frac{B}{\log c}c^x\left(c^t - 1\right)\right)$$

Three-term Woolhouse's Formula in a single decrement context

$$\ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m} - \frac{m^2-1}{12m^2}\left(\delta + \mu_x\right)$$

Three-term Woolhouse's Formula in a multiple state context

$$\ddot{a}_x^{(m)ii} \approx \overline{a}_x^{ii} + \frac{1}{2m} + \frac{\mu_x^{i\bullet} + \delta}{12m^2} \text{ where } \mu_x^{i\bullet} = \sum_{j \neq i} \mu_x^{ij}$$

$$\ddot{a}_x^{(m)ij} = \overline{a}_x^{ij} - \frac{\mu_x^{ij}}{12m^2} \quad i \neq j$$

Note that in the "three-term" equation for $\ddot{a}_x^{(m)ij}$, where $i \neq j$, the second term is equal to zero.

GMMB Embedded Option Value

Assume management charges of *m* per year payable continuously, a GMMB of *kP*, that $S_0 = 1$, and let F_t denote the fund value immediately after any expense deduction at *t*.

$$p(t) = kPe^{-r(n-t)}\Phi\left(-d_2(t)\right) - P\xi S_t\Phi\left(-d_1(t)\right) = kPe^{-r(n-t)}\Phi\left(-d_2(t)\right) - F_t e^{-m(n-t)}\Phi\left(-d_1(t)\right)$$

$$\pi(t) = {}_{n-t}p_{x+t}\, p(t)$$

$$d_1(t) = \frac{\log\left(F_t e^{-m(n-t)}/kP\right) + (r + \sigma^2/2)(n-t)}{\sigma\sqrt{n-t}} \quad \textit{and} \quad d_2(t) = d_1(t) - \sigma\sqrt{n-t}$$