Session 046: Longevity Pooling and Tontines

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Session 046: Longevity Pooling and Tontines

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Monday, October 28 1:45PM – 3:00PM ET

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Introduction to Tontines

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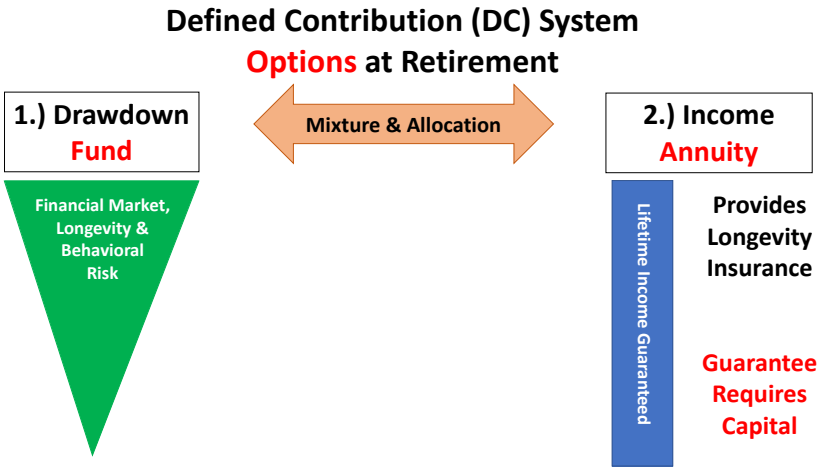
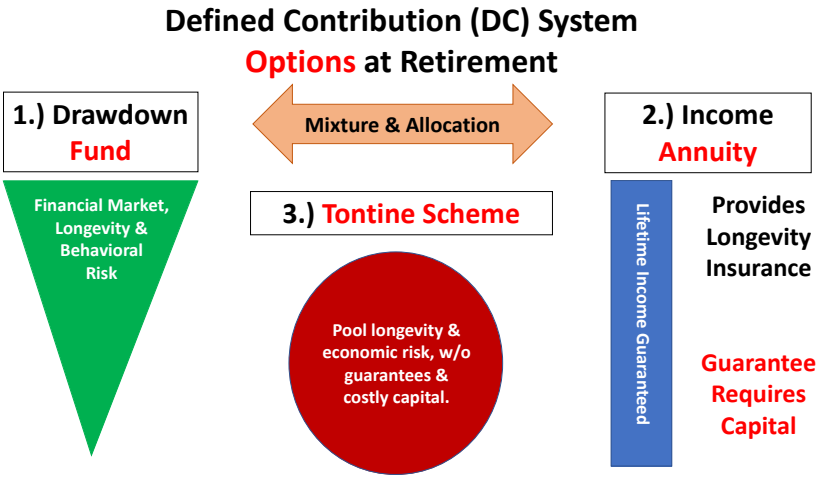
Outline

- Introduction – Tom Salisbury
- Tontines in History and Literature – Franz Blaha
- Tontine vs TIAA CREF? What should Dad have done? – Nicola Blaha
- Optimal Tontines - Tom Salisbury

Current Options



Why not add a third option?



What is a tontine?

- Like an annuity, you
  - Purchase an annuity up front
  - and then receive income for life
- Unlike an annuity,
  - what you receive each month is not guaranteed
  - Instead it is the total payout to the tontine pool is guaranteed, but whatever amount that is gets split among the survivors.

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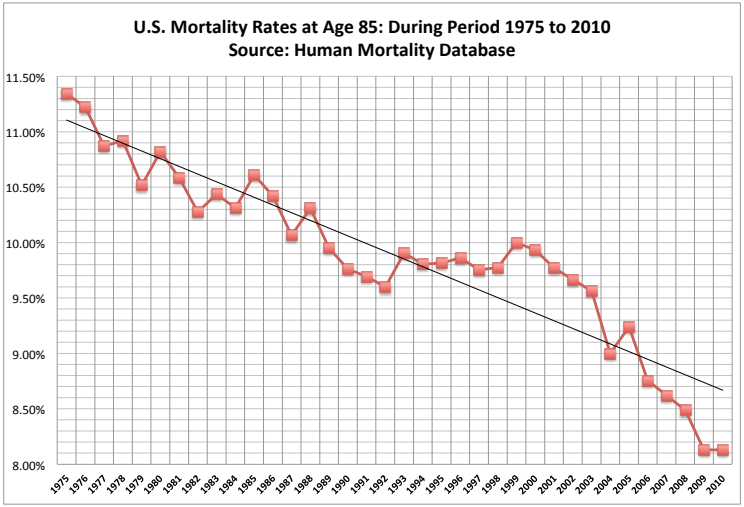
- From the 17th to 19th century, one of the most popular long-term investments, more so than annuities.
- Made illegal in U.S. in early 20th century, after incidents of fraud or predatory terms.

Why might we bring them back?

- People buy annuities to hedge longevity risk, but this comes in 2 flavours;
  - idiosyncratic longevity risk (will I live longer than others?)
  - systematic longevity risk (will everyone live longer than expected? eg due to medical breakthroughs)
- If it is expensive enough to insure against the latter, we may prefer to hedge the first, but retain the second.
- Or might want to diversify and split our money between them.
  ie. hedge some systematic longevity risk but not all.

Systematic longevity risk

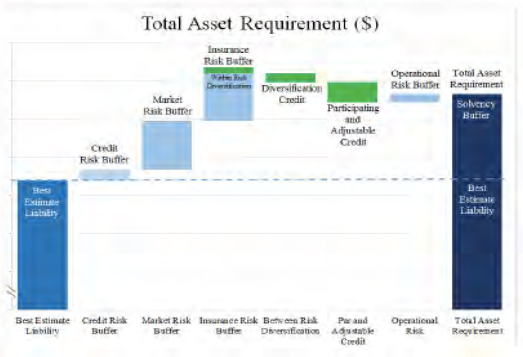


A low cost alternative

- For example, some estimates show Solvency II capital requirements increasing annuity prices by up to 20% because of systematic longevity risk.
- Tontines have **NO** systematic longevity risk.
  And are cheap to implement and manage:
  - Once set up, no actuarial calculations required to hedge it;
  - Can hedge with a simple (static) bond portfolio.
  - No regulatory capital requirement.
  - No risk that changes in longevity $\Rightarrow$ issuer insolvency.

Cost of an annuity's longevity guarantee

The following graph illustrates the new life insurance standard approach QIS6 components and their aggregation within a total solvency buffer.[10]



In fact, they're already here

- Swedish national pension plan is essentially a tontine
- Jurisdictions without an established annuity market have people looking at starting them (eg RSA)
- Mercer Australia's *LifetimePlus* is a variant.
- And there are entrepreneurial versions.
  Here's one from Gibraltar:

And are in the news

Tontine design

- Tontines may differ in how fast/slow they pay out.
- Historical tontines were often FLAT: yearly *total payout* doesn't change from year to year. Therefore, as survivor pool declines, the *payout per person* rises. But pushing income to advanced ages is far from optimal.
- Call a tontine NATURAL: if yearly *total payout* declines in proportion to survival probabilities ${}_tp_x$ (mimicking annuities). So *payout per person* stays stable.
- We'll see later that this design is *optimal* for individuals with *logarithmic utility* (risk aversion $\gamma = 1$).

Tontines in History and Literature

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What is a “tontine”?

Pierre Magnan (1922-2012)

- *Élégie pour Laviolette (2010)*
- *Magnan’s last novel*
- *Set in the mountains of Haute Provence*

- Et autrement, dit-elle, est-ce que vous savez ce que c’est qu’une tontine?
Je tombai the nues. Jamais je n’avais entendue ce mot ni ce qu’il pouvait signifier et je lui le dit.

- Il m’en parlait tout le temps. Il me disait: << Quand je serais mort, je suis le plus jeune, tu auras ma tontine. >>.

- << Tontine: convention par laquelle plusieurs personnes stipulent qu’au décès de chacune d’elles, le capital lui appartenant, reviendra aux survivants ou à leurs héritiers et ainsi de suite jusqu’au dernier.>>

Origin and History of Tontines

Lorenzo de Tonti (c1602 -1683)

Provincial governor in the Kingdom of Naples, had to flee to Paris around 1650 for having backed an uprising against the Spanish rulers of Naples. Changed his name to Laurans de Tonty. Supported at first by Cardinal Mazarin, a fellow Italian, but apparently fell in disfavor with Jean-Baptiste Colbert, who succeeded Mazarin upon his death in 1661.

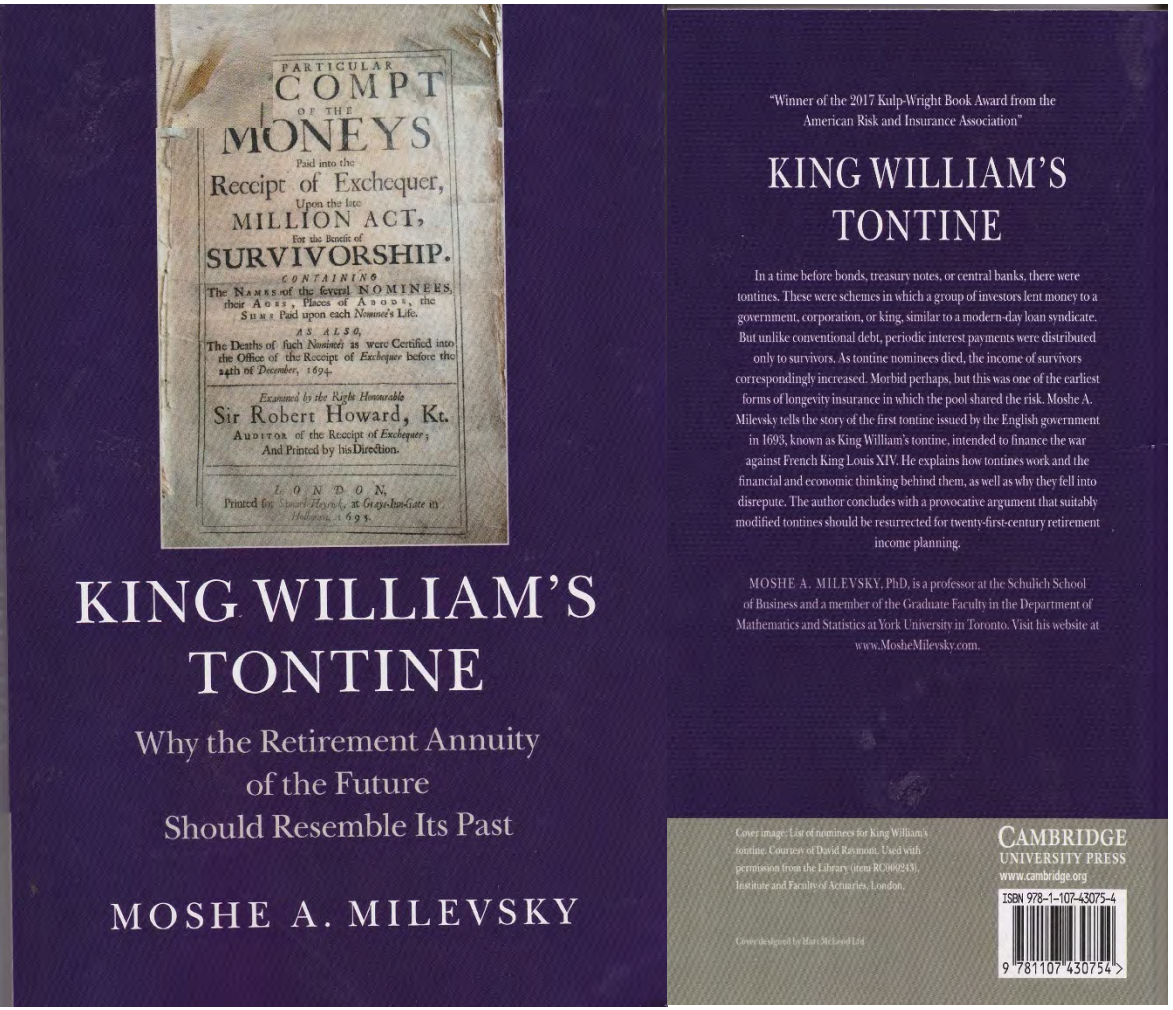
Proposed a novel method of raising funds for Frances perpetual wars with England which later became know as a "tontine," for obvious reasons.

Voltaire's *The Age of Louis XIV* (1751) tells us that in 1649, "*an Italian, named Tonti, […,] then invented a new loan upon life annuities, chargeable on the national revenues, and divided into shares and classes; the income of each proprietor that died to be shared among the survivors. Hence similar loans came to be named after the inventor.*" (Quoted from *Gentleman's Magazine,* vol 2 (1790)

It was initially rejected by the Parliament, a decision Tonti appealed, apparently vehemently, so much so that Tonti ended up spending 7 years in the Bastille (1668-75). He died in 1684, poor and in disgrace, 31 years after his first proposal for a tontine had been rejected by the French parliament, and 5 years before the first fully subscribed tontine was launched. (1689) see Coudy, Julien, La « Tontine Royalle » sous le Règne de Louis XIV, *Revue historique de droit français et étranger (1922-), Quatrième série, Vol. 34 (1957), pp. 127-147.*

The term « tontine » for such an annuity is first used in France in 1696, in the advertisement for the fourth such enterprise.

King William's Tontine (1695)



- The Sacred Scroll for all things "Tontine"
- Very engagingly written, with plenty of information, and the origins, the development, the decline, and the potential resurgence of tontines "thinking" in the retirement annuity of the future.

The Structure of Tontines

- An annuity, with the benefit of survivorship, or a loan raised on life annuities with the benefit of survivorship. Thus, an annuity is shared among a number of investors, on the principle that the share of each, at his death, is enjoyed by the survivors, until at last the whole goes to the last survivor, or to the last few, according to the terms on which the money is advanced.
- In 1833, Charles Compton, with the benefit of being able to look back on 150 years of tontine schemes, described a tontine as

> an annuity, after a certain rate of interest, is granted to a number of persons, divided into classes, according to their respective ages; so that annually the whole fond of each class is divided among the survivors of that class, till at last it falls to one, and upon the extinction of that life reverts to the power by which the tontine was created, and which thereby becomes security for the due payment of annuities.

Charles Compton, *A Treatise on Tontine*, London, 1833

Component parts:

1. An **issuer**, either public or private, who needs to raise funds for either a governmental or private enterprise (e.g., a war, large infrastructure projects, or a business undertaking). This entity also determines the parameters for this loan, such as the size of the shares, a subdivision into classes, interest rates, how the tontine is wound up, keeps records, or designates an agency to do so on its behalf.
2. **Investors** who provide the funds with their purchase of one or several shares.
3. **Shareholders** who receive the periodic interest.
4. **Nominees** to whose lives the annuity payments are linked.

In most tontines, though not all, 2. 3. and 4. are the same person.

CLASSIFICATION OF TONTINES

USE OF FUNDS	Money is loaned by the entire group and members are considered *creditors*	Members invest in an active business and members are *shareholders*
Tontine Type 1A Tonti's original scheme	Survivors share periodic constant interest payments, but loan principal is forfeited	Survivors shared periodic variable dividends, but original share capital is forfeited *
Tontine Type 1B	Survivors share periodic constant interest payments, and winner(s) get original principal. *	Survivors shared periodic dividends; winner(s) inherit ownership of business
Tontine Type 2 King William's design	Entire loan is amortized over a maximum lifespan and survivors shared a declining cash flow stream	Business assets are systematically liquidated and cash + dividends are distributed to survivors
Tontine Type 3 Winner takes all! Riskiest and suboptimal.	Surviving winners get compound interest plus original principal. No periodic cash flow or income.	Dividends are reinvested (not distributed) and the winner(s) inherit ownership of an ongoing business.

Tontines and the Public Imagination

- For some 200 years, Tontines became the most prevalent and popular source of the public financing of large projects.
- Despite that fact, many tontines were undersubscribed, more so in England than in France and other European countries, so that issuers had to supplement tontines with more traditional life annuities to raise enough money for their projects.
- In addition, tontines were beset with record keeping problems, such as the timely and accurate registration of deaths and the verification of identities.

Last Man (Woman) Standing

- Early tontines severely miscalculated longevity. In fact, modern mortality tables were a byproduct of the development of tontines.
- In the first of 14 French State tontines, the last survivor was Charlotte Barbier, who had turned her investment of 300 livres into 73,000 livres by the time she died at the age of 96.

Famous Tontine Financed Projects

1. Kew Bridge (1759)£ 16,500 raised by a tontine @ £100/share, repaid by a toll of 1p for each pedes-trian and 1s6p for a horse-drawn carriage.

2. Richmond Bridge (1774) Money raised from the sale of shares at £100 each in two tontine schemes, the first for £20,000 and the second for £5,000. The issue was financed by a toll of 1/2p per pedestrian to 2s 6p for a carriage and six horses.

4. The Tontine Hotel, Telford, Shropshire (1780). Built by a group of investors with the ownership finally devolving to the last survivor, to accommodate the expected mass of tourists going to see the Ironbridge, the first bridge built from cast and now a World Heritage site.

5. The Tontine Coffee House, Wall Street, New York City (1793)

Precursor of the New York Stock Exchange, built by the sale of 203 shares @ $200/each by a group of businessmen to trade stock, and to conduct other trade, political, and social business, including after-hour gaming.

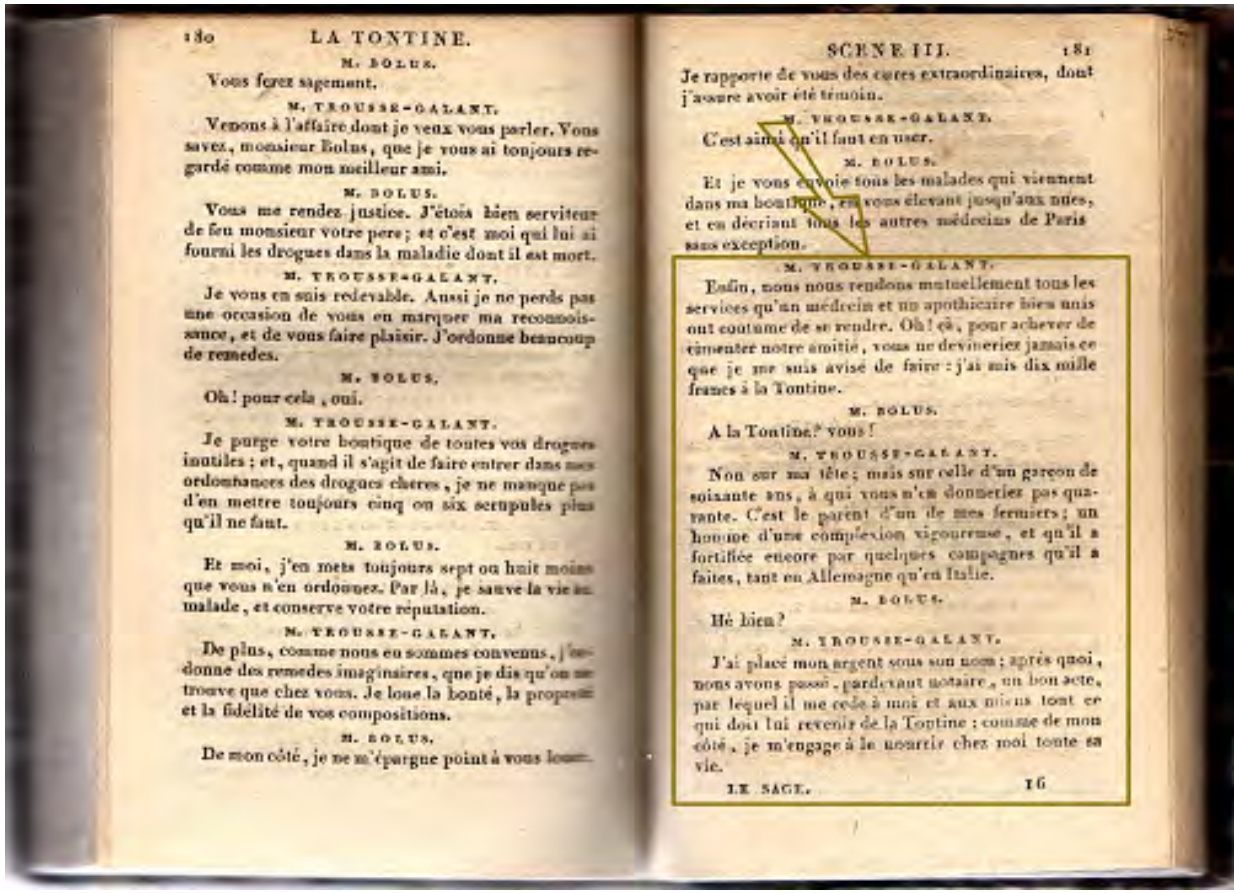
TONTINES IN LITERATURE

Alain-René Lesage (1668-1747)

La Tontine (1708, first performed 1732)

One-act comedy.

- First mention of a tontine in literature, although the plot is a satire on greed and medical malpractice, as well as the collusion between a doctor and a pharmacist to create business for each other (what a novel idea!!!)
- A doctor wants to marry off his daughter to his pharmacist collaborator by buying a substantial share in a Tontine in her name as her dowry. He also engages to use all his medical arts to keep the nominee, a healthy-looking sixty-year-old alive long enough to cash in.
- First written only 15 years after Kind William's Tontine was launched in 1693, and only 19 years after the first French tontine (1689).



180 LA TONTINE.

M. BOLUS.

Vous ferez sagement.

M. TROUSSE-GALANT.

Venons à l'affaire dont je veux vous parler. Vous savez, monsieur Bolus, que je vous ai toujours regardé comme mon meilleur ami.

M. BOLUS.

Vous me rendez justice. J'étois bien serviteur de feu monsieur votre pere; et c'est moi qui lui ai fourni les drogues dans la maladie dont il est mort.

M. TROUSSE-GALANT.

Je vous en suis redevable. Aussi je ne perds pas une occasion de vous en marquer ma reconnoissance, et de vous faire plaisir. J'ordonne beaucoup de remedes.

M. BOLUS.

Oh! pour cela, oui.

M. TROUSSE-GALANT.

Je purge votre boutique de toutes vos drogues inutiles; et, quand il s'agit de faire entrer dans mes ordonnances des drogues cheres, je ne manque pas d'en mettre toujours cinq ou six scrupules plus qu'il ne faut.

M. BOLUS.

Et moi, j'en mets toujours sept ou huit moins que vous n'en ordonnez. Par là, je sauve la vie au malade, et conserve votre réputation.

M. TROUSSE-GALANT.

De plus, comme nous en sommes convenus, j'ordonne des remedes imaginaires, que je dis qu'on ne trouve que chez vous. Je loue la bonté, la propreté et la fidélité de vos compositions.

M. BOLUS.

De mon côté, je ne m'épargne point à vous louer

SCENE III. 181

Je rapporte de vous des cures extraordinaires, dont j'assure avoir été témoin.

M. TROUSSE-GALANT.

C'est ainsi qu'il faut en user.

M. BOLUS.

Et je vous envoie tous les malades qui viennent dans ma boutique, en vous élevant jusqu'aux nues, et en décriant tous les autres médecins de Paris sans exception.

M. TROUSSE-GALANT.

Enfin, nous nous rendons mutuellement tous les services qu'un médecin et un apothicaire bien unis ont coutume de se rendre. Oh! çà, pour achever de cimenter notre amitié, vous ne devineriez jamais ce que je me suis avisé de faire: j'ai mis dix mille francs à la Tontine.

M. BOLUS.

A la Tontine? vous!

M. TROUSSE-GALANT.

Non sur ma tête; mais sur celle d'un garçon de soixante ans, à qui vous n'en donneriez pas quarante. C'est le parent d'un de mes fermiers; un homme d'une complexion vigoureuse, et qu'il a fortifiée encore par quelques campagnes qu'il a faites, tant en Allemagne qu'en Italie.

M. BOLUS.

Hé bien?

M. TROUSSE-GALANT.

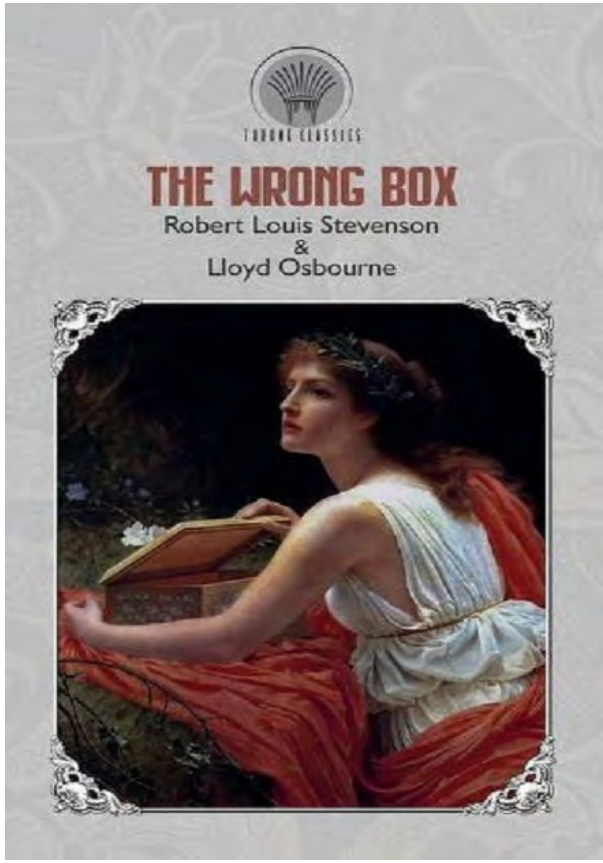
J'ai placé mon argent sous son nom; après quoi, nous avons passé, pardevant notaire, un bon acte, par lequel il me cede à moi et aux miens tout ce qui doit lui revenir de la Tontine; comme de mon côté, je m'engage à le nourrir chez moi toute sa vie.

LE SAGE. 16

R.L. Stevenson & Lloyd Osbourne

The Wrong Box (1889)

- When the novel opens, 2 brothers are the only survivors of a “tontine” which originally had 40 investors when it was started some 60 years before.
- The novel is a romantic comedy of errors and mistaken identities, in which the tontine is the only ray of hope for the family for redeeming themselves for past flawed business decisions.
- Despite all suspicions and plots, both brothers are still alive at the end. The older brother’s son a lawyer, will eventually get the tontine, after having settled his uncle’s debts.



http://robert-louis-stevenson.org/works/the-wrong-box-1889/

Henry Hawley Smart (1833-1893)

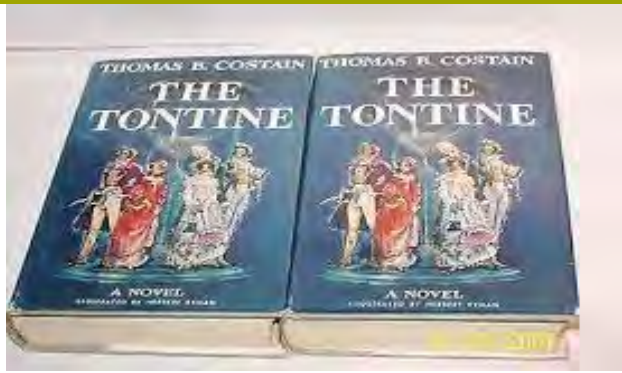
The Great Tontine (1881)

- Based on a fictional tontine to build a modern opera house in London. An issue of £ 160,000 (1600 shares@ £100) with an annual interest of 5% (£ 8,000) Minimum age of nominee=60)
- Involves 4 families and their various fortunes of the next 25years or so.
- The plot jumps quickly to the time when only 4 nominees are still alive.
- Resorts to the almost stereotypical device in tontine novels of having dead nominees “revived” by impersonators, often actors.

Thomas B. Costain (1885 – 1965)

The Tontine (2vols) (1955)

A monumental historical novel, held together very loosely by a fictional "Waterloo" tontine, which takes 60 years to wind up.



- Paints a sweeping picture of English social, economic, and cultural life during the Industrial Revolution by following the lives of three generations of 2 rival families.
- While the Waterloo Tontine (for the benefit of the British veterans of the Napoleonic Wars) serves as a loose thread, the novel could well have worked without it.
- There are the usual tontine shenanigans towards the end. The final survivor and beneficiary is a character the reader has been manipulated to empathize with, whereas the ruthless robber baron gets his just deserts.
- Probably the tontine novel most deserving of literary praise

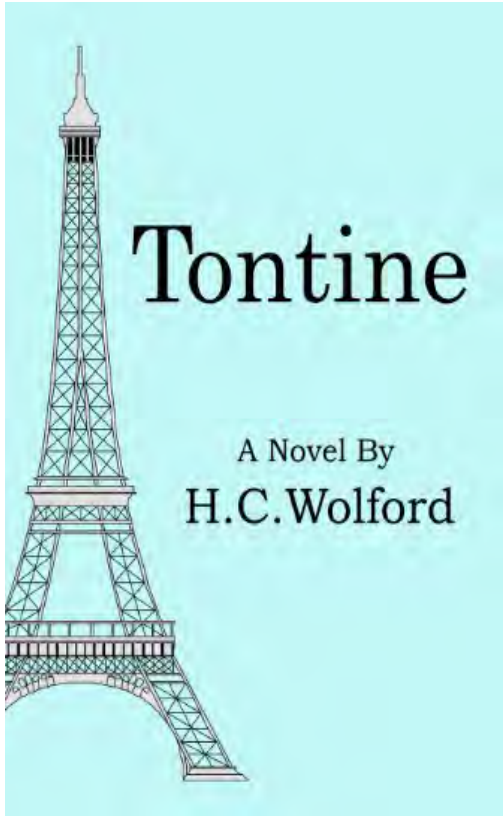
The Bad and the Ugly

H.C. Wolford *Tontine* (1996)

A platoon of 10 American GIs in France during the last days of WWII take a jewel-encrusted chalice from a group of fleeing SS men. At the suggestions of a French woman whom they have befriended, they form a "tontine," with the chalice to go to the last survivor. 20 years later, they are gruesomely murdered, one after the other.

- "What is a tontine?" asked Tinker.

  'How should I say it? Well, the parties give the bank the money or property to hold. All of the parties sign a contract which agrees that the property goes to the last surviving person . … you all sign a document, the tontine."
- Don't read this book near an open flame! This book sucks badly. The End (anonymous reviewer on Amazon).



Liam Manning, *Tontine (2018)*

After two days in Sicily to familiarise themselves with each other, six professional killers from around the world are brought to the city of Rome. In the sun-choked streets of the Eternal City, there is just one rule. Be the only one standing when time is up five days later at the Trevi Fountain. Five million dollars awaits the winner. An ingenious micro-location implant is all each has, to discover if, and when, they are the hunter or the hunted. (Back Cover blurb)

- 'Have you ever heard of Tontine (sic!), Giuseppe?'
  'Tontine?' Gupo was confused. ' Isn't that the name of an American Red Indian with the cowboy who wore a mask ? The Alone Ranger?' […]
  'Tontine, Giuseppe, was originally a form of private insurance entered into by a group of people. Usually friends or colleagues, particularly army associates, who would pay premiums into a pot. Over time the pot would naturally grow with the premiums and interest. However, if someone died, their share would remain in the pot amongst that of the remaining shareholders. And over the years the pot would grow substantially larger, while the group would grow smaller.'
- Once more, the tontine ends badly for all participants, who have been pawns in a "tontine" played out for the entertainment of some wealthy paying customers. The "winner' is the widow of a police officer who has been killed investigating the scheme.
- 4 mostly positive Amazon and Goodreads(!) reviews.

Tontines in Literature, a review

- A tontine à la Tonti or King William is not at all suitable as a topic or a structural frame for literature.
- It is slow to develop, involves a host of characters, and can by nature not create suspense until the very end, thus not permitting much character development and requiring substantial interruptions in the chronology of the plot.
- Therefore, respectable novels that focus on traditional tontines are either very long (Cobain's *The Tontine* runs over a 1000 pages in 2 volumes) or focus entirely on the "endgame," abandoning character development in the interest of creating suspense.
- As a consequence, most tontine novels are Type 3 in Milevsky's classification scheme.

Winding up

- The primary purpose of all tontines has traditionally been to raise money for many different purposes.
- Despite this mercenary intent (or because of it) almost all proposed tontines included the notion that tontines could provide security for shareholders in their old age, albeit at some expense to the estate for potential heirs. The ideal purchasers of tontine shares would be childless persons of moderate means.
- Early critics of tontines focused on the ghoulish aspects of tontines, and the possibilities that they might induce fraud and/or violent crime.
- Unfortunately, the overwhelming majority of Tontine literature has focused on the unsavory aspects of tontines, despite clear evidence to contrary.
- The renewed interest in tontines as retirement instruments has been caused by the spreading anxiety among many people of outliving their retirement savings. Modern tontines are and should be designed to allay those fears.

References

Note: There are currently at least 2 dozen novels available through Amazon that feature the term "tontine" in their title.

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Tontine vs TIAA CREF? What should Dad have done (had a Tontine existed)?

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Assumptions for Dad

Tontine Assumptions

- We can assume a Natural Tontine or γ = 1 (without loss of utility if his risk aversion is something else)
- Age = 66
- Gender = Male
- Mortality Table = A2000 Table plus 100% of Scale G
- n = 400 investors
- i= 4.35% (risk-free rate as of 1/1/2009)
- Assume a Binomial Distribution where Number of Survivor ~ Bin(400, *p)*

TIAA CREFF Assumptions

- Dad invested $540,000 in a TIAA Cref Account
  - $112,000 in a Transfer Payout Annuity paid out over 10 years @ 5.5%
  - $428,000 in a Market Fund

Year	E(N)	Actual N	Actual Monthly Payout	Actual Annual Payout		Year	E(N)	Actual N	Actual Monthly Payout	Actual Annual Payout
2009	400	400	$ 3,331	$ 39,973		**2020**	329	329	$ 3,331	$39,973
2010	396	396	$ 3,331	$ 39,973		**2021**	320	320	$ 3,331	$39,973
2011	391	391	$ 3,331	$ 39,973		**2022**	310	310	$ 3,331	$39,973
2012	386	386	$ 3,331	$ 39,973		**2023**	300	300	$ 3,331	$39,973
2013	381	381	$ 3,331	$ 39,973		**2024**	289	289	$ 3,331	$39,973
2014	375	375	$ 3,331	$ 39,973		**2025**	277	277	$ 3,331	$39,973
2015	368	368	$ 3,331	$ 39,973		**2026**	265	265	$ 3,331	$39,973
2016	361	361	$ 3,331	$ 39,973		**2027**	252	252	$ 3,331	$39,973
2017	354	354	$ 3,331	$ 39,973		**2028**	239	239	$ 3,331	$39,973
2018	346	346	$ 3,331	$ 39,973		**2029**	224	224	$ 3,331	$39,973
2019	338	338	$ 3,331	$ 39,973		**2030**	210	210	$ 3,331	$39,973

Payments stay flat due to the pool dying off exactly as expected.

"Actual" vs Expected Survivors

At the 50th Percentile

Year	E(N)	Actual N	Actual Monthly Payout	Actual Annual Payout		Year	E(N)	Actual N	Actual Monthly Payout	Actual Annual Payout
2009	400	400	$ 3,331	$ 39,973		**2020**	329	339	$ 3,235	$38,821
2010	396	398	$ 3,309	$ 39,709		**2021**	320	330	$ 3,228	$38,739
2011	391	395	$ 3,299	$ 39,591		**2022**	310	321	$ 3,221	$38,656
2012	386	391	$ 3,291	$ 39,493		**2023**	300	311	$ 3,214	$38,570
2013	381	386	$ 3,284	$ 39,404		**2024**	289	300	$ 3,207	$38,483
2014	375	381	$ 3,277	$ 39,319		**2025**	277	289	$ 3,199	$38,391
2015	368	375	$ 3,270	$ 39,237		**2026**	265	277	$ 3,191	$38,294
2016	361	369	$ 3,263	$ 39,155		**2027**	252	264	$ 3,183	$38,194
2017	354	362	$ 3,256	$ 39,073		**2028**	239	250	$ 3,174	$38,087
2018	346	355	$ 3,249	$ 38,990		**2029**	224	236	$ 3,164	$37,973
2019	338	347	$ 3,242	$ 38,906		**2030**	210	221	$ 3,154	$37,850

Payments go DOWN due to the pool dying off slower than E(N).

"Actual" vs Expected Survivors

At the 90th Percentile

Year	E(N)	Actual N	Actual Monthly Payout	Actual Annual Payout		Year	E(N)	Actual N	Actual Monthly Payout	Actual Annual Payout
2009	400	400	$ 3,331	$ 39,973		**2020**	329	319	$ 3,433	$41,195
2010	396	393	$ 3,353	$ 40,240		**2021**	320	310	$ 3,441	$41,288
2011	391	387	$ 3,363	$ 40,362		**2022**	310	300	$ 3,449	$41,383
2012	386	381	$ 3,372	$ 40,464		**2023**	300	289	$ 3,457	$41,481
2013	381	375	$ 3,380	$ 40,558		**2024**	289	278	$ 3,465	$41,582
2014	375	369	$ 3,387	$ 40,648		**2025**	277	266	$ 3,474	$41,690
2015	368	361	$ 3,395	$ 40,737		**2026**	265	253	$ 3,484	$41,805
2016	361	354	$ 3,402	$ 40,826		**2027**	252	240	$ 3,494	$41,925
2017	354	346	$ 3,410	$ 40,915		**2028**	239	227	$ 3,505	$42,055
2018	346	338	$ 3,417	$ 41,006		**2029**	224	213	$ 3,516	$42,195
2019	338	329	$ 3,425	$ 41,099		**2030**	210	198	$ 3,529	$42,348

Payments go UP due to the pool dying off faster than E(N).

"Actual" vs Expected Survivors

At the 10th Percentile

End of Year	Total Annual Payout	Rate of Return	Remaining Balance		End of Year	Total Annual Payout	Rate of Return	Remaining Balance
2009	$ 25,100	14%	$ 459,652		2020	$ 25,000	8.00%	$ 318,148
2010	$ 25,100	14%	$ 500,859		2021	$ 25,000	8.00%	$ 309,777
2011	$ 25,100	0%	$ 484,084		2022	$ 25,000	8.00%	$ 300,936
2012	$ 31,600	6%	$ 481,611		2023	$ 25,000	8.00%	$ 291,596
2013	$ 59,600	18%	$ 496,447		2024	$ 25,000	8.00%	$ 281,730
2014	$ 64,100	5%	$ 463,224		2025	$ 25,000	8.00%	$ 271,309
2015	$ 82,100	0%	$ 387,372		2026	$ 25,000	8.00%	$ 260,301
2016	$ 42,100	9%	$ 377,994		2027	$ 25,000	8.00%	$ 248,673
2017	$ 57,100	17%	$ 379,791		2028	$ 25,000	8.00%	$ 236,390
2018	$ 41,600	-6%	$ 329,512		2029	$ 25,000	8.00%	$ 223,415
2019	$ 48,100	15%	$ 324,464		2030	$ 25,000	8.00%	$ 209,710

- Based on Dad's actual returns from 2009-2019
- Assumed an 8% of future returns
- Assuming $25,000/year in future payouts, funds run out in 2041.

Dad's Experience with his TIAA CREF Account vs. A Tontine

How Did Dad do?

- **Dad did very well since the stock market has been very good to him. He still has a balance of $350,000 in his TIAA Cref.**
- **His total payout so far has been $501,000**
- **The 50% Percentile Tontine Payout would have been $440,000**
- **The 90% Percentile Tontine Payout would have been $433,000**
- **The 10% Percentile Tontine payout would have been $447,000**

Year	50th Percentile Payout	90th Percentile Payout	10th Percentile Payout	TIAA Cref Payout
2009	$ 39,973	$ 39,973	$ 39,973	$ 25,100
2010	$ 39,973	$ 39,709	$ 40,240	$ 25,100
2011	$ 39,973	$ 39,591	$ 40,362	$ 25,100
2012	$ 39,973	$ 39,493	$ 40,464	$ 31,600
2013	$ 39,973	$ 39,404	$ 40,558	$ 59,600
2014	$ 39,973	$ 39,319	$ 40,648	$ 64,100
2015	$ 39,973	$ 39,237	$ 40,737	$ 82,100
2016	$ 39,973	$ 39,155	$ 40,826	$ 42,100
2017	$ 39,973	$ 39,073	$ 40,915	$ 57,100
2018	$ 39,973	$ 38,990	$ 41,006	$ 41,600
2019	$ 39,973	$ 38,906	$ 41,099	$ 48,100

How Will Dad Do?

Dad plans on withdrawing $25,000 per year from now on.

The Tontine payouts would more than meet that objective.

Year	50th Percentile Payout	90th Percentile Payout	10th Percentile Payout	TIAA Cref Payout
2020	$ 39,973	$ 38,821	$ 41,195	$ 25,000
2021	$ 39,973	$ 38,739	$ 41,288	$ 25,000
2022	$ 39,973	$ 38,656	$ 41,383	$ 25,000
2023	$ 39,973	$ 38,570	$ 41,481	$ 25,000
2024	$ 39,973	$ 38,483	$ 41,582	$ 25,000
2025	$ 39,973	$ 38,391	$ 41,690	$ 25,000
2026	$ 39,973	$ 38,294	$ 41,805	$ 25,000
2027	$ 39,973	$ 38,194	$ 41,925	$ 25,000
2028	$ 39,973	$ 38,087	$ 42,055	$ 25,000
2029	$ 39,973	$ 37,973	$ 42,195	$ 25,000
2030	$ 39,973	$ 37,850	$ 42,348	$ 25,000

Notes for Dad

Advantages of Tontine Annuities

- Mortality Credits
- Income is relatively stable
- Payouts would have supported lifestyle with some planning
- Income goes until death
- No risk of funds running out

Disadvantage of Tontine Annuities

- Less flexibility than the TIAA CREF Account
- No payout to beneficiary upon death

Notes for Dad (continued)

Advantages of the TIAA CREF

- Flexibility of payouts
- Remaining balance (if any) can go to a beneficiary

Disadvantage of the TIAA CREF

- Risk of funds running out during lifetime
- Market Timing is a factor

Daughter's Proposal

Diversify!!!

- For example, put 50% in the Tontine and 50% in the TIAA Account
- To a certain degree, Dad did diversify with more of a bond fund with some guaranteed income.
- But why not put use the Tontine payout and the Bond Fund payout first, and then withdrawal any excess from the Market Fund.

Optimal Tontines

Sources

Will discuss results from three papers, all joint with M.A. Milevsky

- *Optimal retirement income tontines* – IME (2015): mathematically analyzes how to optimally design a tontine for a homogeneous population
- *Equitable retirement income tontines: mixing cohorts without discriminating* – Astin (2016): how to adjust when the population is heterogeneous?
- *Annuities versus tontines in the 21st century* – SoA research report (2018). Also with G. Gonzalez and H. Jankowski): how would they have done in practice?

Related academic proposals

- There are related proposals: eg pooled annuities. eg Stamos 2008, Donnelly-Guillen-Nielsen 2013.
- These yield a somewhat higher utility, but are more complicated to implement – payout depends not just on observed survival rate, but on its history (path dependence)
- The simplicity of a tontine makes it easier to explain to purchasers, and easier for purchasers to plan around. For us, this is worth a (very small) utility premium.
- And mathematically we know a lot more about tontines than about pooled annuities.
- Actuarial Research Centre has a nice webinar: `https://www.youtube.com/watch?v=2HVrmlFMFZk`

How should we design a tontine?

Consider the following tontine contract.

- n investors
- each puts \$1 in a pool at time 0
- All are age x, single gender.
- Pool grows continuously at the risk-free rate r.
- Fixed total payout at rate $d(t)$
- Total payout is split equally among survivors, all of whom share the same probability ${}_tp_x$ of survival to time t.
- $N_t = \#$ of survivors at time $t \sim \text{Binomial}(n, {}_tp_x)$
- Individual payout rate: $\dfrac{nd(t)}{N_t}$

Tontine design means choosing $d(t)$. **NOT FLAT**.

Questions

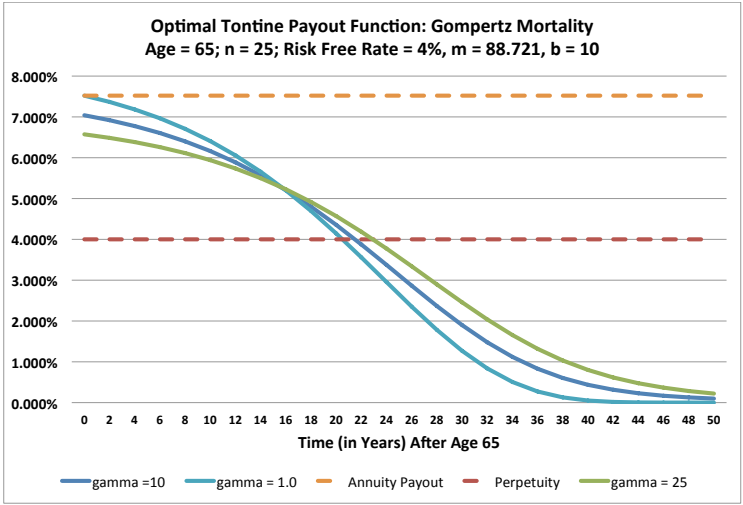
- What payout function $d(t)$ is optimal?
- How does that $d(t)$ behave?
- How does the utility from a tontine compare to that from an ordinary annuity?
- Why might one prefer a tontine over an annuity?
- How would this actually have performed, in practice?
- What if purchased by a heterogeneous population, or at different times?

Optimal tontines

- Optimal payout rate $d(t)$. depends on the risk aversion $\gamma > 0$ of the pool.
- ie. we maximize the expected utility of lifetime consumption, with CRRA utility $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$
- In other words, receiving payment at some rate c yields utility $u(c)$. We discount these, add them up over a lifetime, and take expectations, giving my overall utility

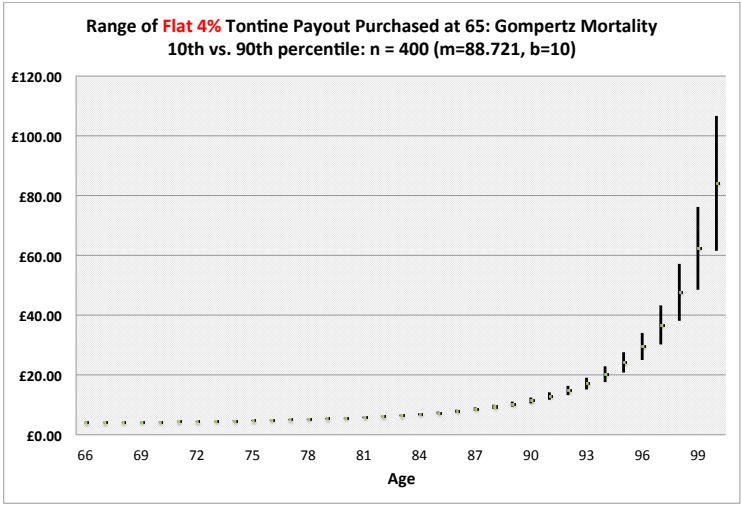
$$\int_0^\infty e^{-rt}{}_t p_x \, E\left[u\left(\frac{nd(t)}{N(t)}\right) \mid \text{I survive to time } t\right] dt$$

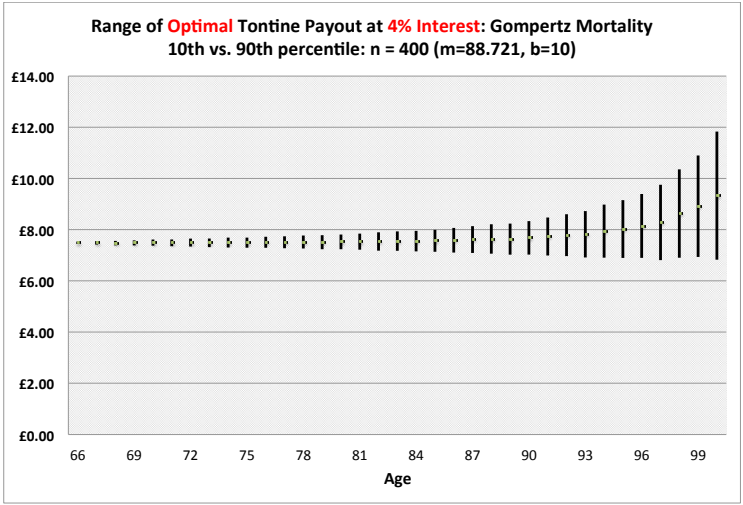
- Then choose $d(t)$ to optimize this.
- Behaviour: regardless of γ, $d(t)$ declines over time.


Optimal Tontine Payout Function: Gompertz Mortality
Age = 65; n = 25; Risk Free Rate = 4%, m = 88.721, b = 10
8.000%
7.000%
6.000%
5.000%
4.000%
3.000%
2.000%
1.000%
0.000%
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
Time (in Years) After Age 65
gamma =10
gamma = 1.0
Annuity Payout
Perpetuity
gamma = 25

Natural tontine

- When $\gamma = 1$ we use utility $u(w) = \log w$ instead.
- Now the optimal payout is $d(t) = d(0)_t p_x$.
- Compare this with the total payout from a fair annuity (per initial \$ invested), of $\frac{1}{a_x} \cdot {}_t p_x$, and get $d(0) = \frac{1}{a_x}$ too.
  ie. it starts out at the fair annuity payout rate.
- We call this tontine the natural tontine.
- An actual vendor wouldn't offer many tontines, one for each γ. We propose the natural tontine as a good product design, if you have to pick a single one. Will come back later to reasons, and the utility loss from using the natural tontine rather than the one that is optimal for your own γ.


Range of Flat 4% Tontine Payout Purchased at 65: Gompertz Mortality
10th vs. 90th percentile: n = 400 (m=88.721, b=10)
£120.00
£100.00
£80.00
£60.00
£40.00
£20.00
£0.00
66
69
72
75
78
81
84
87
90
93
96
99
Age


Range of Optimal Tontine Payout at 4% Interest: Gompertz Mortality
10th vs. 90th percentile: n = 400 (m=88.721, b=10)
£14.00
£12.00
£10.00
£8.00
£6.00
£4.00
£2.00
£0.00
66
69
72
75
78
81
84
87
90
93
96
99
Age

Natural vs Optimal tontines

A natural tontine is optimal only for $\gamma = 1$. If your γ differs, how much utility do you lose by investing in a natural tontine? Equivalently, how much would you need to invest in a natural tontine to achieve the same utility as \$1 in an optimal tontine?

Natural vs. Optimal Tontine			
Certainty Equivalent for $n = 100$			
Age x	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$
30	1.000018	1	1.000215
40	1.000026	1	1.000753
50	1.000041	1	1.001674
60	1.000067	1	1.003388
70	1.000118	1	1.003451
80	1.000225	1	1.009877
$r = 3\%$ and Gompertz $m = 87.25, b = 9.5$			

Annuity loading

- A fair annuity pays individuals at rate $\frac{1}{a_x}$, because of the budget constraint $\int_0^\infty \frac{1}{a_x} e^{-rt}{}_t p_x \, dt = 1$. If it is fair, can prove that it gives higher utility than any tontine.
- But annuities are not fair – they include a loading δ (for capital reserves, hedging costs, etc).
- Assume it pays at rate c with $\int_0^\infty e^{-rt}{}_t p_x c \, dt = 1 - \delta$.
  At what loading δ is a tontine preferable to a loaded annuity?
- Turns out that $n\delta$ (aggregate loading) is stable as n grows.
  Eg. can prove that $1 < \gamma \leq 2 \Rightarrow n\delta < \frac{c_0}{r} - 1$.

Loadings: numerical results

The highest annuity loading δ you are willing to pay if a tontine is available				
LoRA γ	$n = 20$	$n = 100$	$n = 500$	$n = 1000$
0.5	73 b.p.	15 b.p.	3.0 b.p.	1.5 b.p.
1.0	130 b.p.	27 b.p.	5.7 b.p.	2.9 b.p.
1.5	182 b.p.	40 b.p.	8.4 b.p.	4.3 b.p.
2.0	232 b.p.	52 b.p.	11 b.p.	5.7 b.p.
3.0	323 b.p.	75 b.p.	16 b.p.	8.4 b.p.
9.0	754 b.p.	200 b.p.	46 b.p.	24 b.p.
Age $x = 60$, $r = 3\%$, Gompertz Mortality ($m = 87.25, b = 9.5$)				

For these n, the aggregate loading $n\delta$ on a total pool of $\$n$ stays $<$ \$0.84 for reasonable γ ($\gamma \leq 3$). Even at $\gamma = 9$ it $<$ \$2.40

In real life, interest rates aren't constant

So how would it have done using GoC bonds instead (SoA study)?

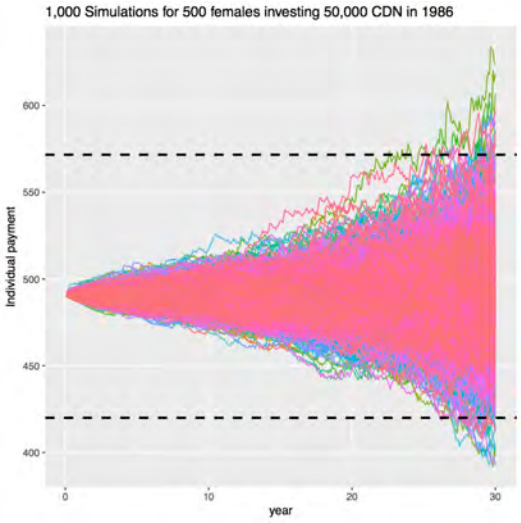
Table 3a. Initial Tontine Dividend Versus Annuity Income for a $50,000 Investment

Year	GoC Bond Yield		Monthly Annuity Income		Initial Tontine Income	
(January)	10-Year	30-Year	Male	Female	Male	Female
1986	9.61%	9.22%	$510.5	$475.9	$526.5	$486.4
1987	8.66%	7.30%	$509.1	$476.6	$488.4	$446.3
1988	9.84%	7.80%	$536.5	$504.7	$527.9	$485.2
1989	9.95%	8.46%	$537.6	$505.3	$532.2	$489.7
1990	9.27%	9.50%	$514.9	$481.9	$511.7	$471.0
1991	9.94%	11.50%	$525.9	$494.2	$532.3	$493.5
1992	8.38%	9.35%	$478.9	$446.4	$475.6	$438.9
1993	8.11%	7.87%	$453.4	$423.5	$462.2	$424.8
1994	6.93%	7.51%	$411.5	$378.4	$420.6	$384.6
1995	9.09%	9.06%	$459.4	$428.7	$492.4	$454.1
1996	7.21%	7.77%	$410.1	$377.8	$428.2	$391.7
1997	6.75%	7.48%	$384.9	$354.7	$410.8	$375.4
1998	5.52%	5.89%	$364.9	$333.9	$371.4	$334.7
1999	4.90%	5.15%	$347.0	$315.2	$348.1	$311.7
2000	6.36%	5.99%	$373.1	$341.1	$391.0	$353.4

Details

- For the most part, tontines yield more. But not much more.
- Annuity payouts are life-only (figures courtesy of CANNEX)
- Tontine payouts use actual GoC bond yields and mortality from IAM1983 plus 100% of Scale G.
- In reality, tontine payouts could be slightly less, because of costs and return on capital for the issuing corporation. But the costs should be lower than for the competing annuities. On the other hand, if we're willing to use (or mix with) corporate bonds, that would raise tontine payouts.
- Biggest uncertainty: how to roll over bond yields past the tenors available in the market. We didn't model that, and just extrapolated the yield curve.

1986 tontine: simulation



Heterogeneity

- The above analytics assumed a homogeneous population.
- In fact, we think a strong case can be mode for segregating different populations, and only issuing homogeneous tontines. The utility loss due to a small population size is modest, and the simplicity of the resulting product makes it easy for consumers to understand and plan around.
- But if the sample is small enough, a case can be made for pooling a heterogeneous group.
  Can that be done fairly, and without loss of utility?

Equitability

- If there are different cohorts participating in the same tontine, we may quote them different prices at which to buy tontine units. Funds paid out are then split in proportion to the number of units owned.
- Call a tontine *equitable* if the present value of what each person receives (per $ invested) is equal.
  [Can't $= 1$ though, as funds will remain when the last dies.]
- The mathematical question is: given a payout function $d(t)$, and cohort parameters (ith cohort has n_i individuals, age x_i and investing w_i), will equitable prices exist? And will they be unique (up to a constant multiple)?
- If exist, are unique, and usually can calculate equitable prices. But in extreme cases sometimes equity is infeasible.

Examples

- For example, a flat tontine with \$10K from a 30 year old and \$10K from a 100 year old will **never** be equitable.
- Or a natural tontine with a 65 year old investing \$1 and another 65 year old investing \$1M.
- But with less extreme heterogeneity, CAN usually achieve equity in most cases where $d(t)$ is tailored to multiple cohorts, eg where $d(t)$ is proportional to the mean number of surviving units.
- In many cases, a well designed equitable tontine will increase utility of both cohorts by combining them.
- It is even possible to allow new participants to join in, in an equitable way.

Take-away messages

- Practitioners and academics are thinking through alternatives to annuities and standard pensions, that will insure idiosyncratic longevity risk, but not systematic longevity risk.
- It may be appealing to allocate some of one's nest egg (though not necessarily all) to such products.
- Have tried to show how these could be cheaper and easier to maintain, and can provide reasonable performance if designed carefully.
- Have tried to illustrate some of the issues to keep in mind in designing such products.
- In some form, Tontine thinking will probably influence how we ensure sustainable income in retirement, going forward.

Fine print: precise formulation

- Assume CRRA utility $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$ $[\gamma > 0]$, initial pool of n.
- Let ζ = the individual's lifetime, and $N(t)$ = # of survivors at time t. Choose the payout rate $d(t)$ to optimize

$$E\Big[\int_0^\zeta e^{-rt} u\Big(\frac{nd(t)}{N(t)}\Big)\, dt\Big] = \int_0^\infty e^{-rt} E\Big[1_{t<\zeta} u\Big(\frac{nd(t)}{N(t)}\Big)\Big]\, dt$$

$$= \int_0^\infty e^{-rt}\, {}_tp_x\, E[u\Big(\frac{nd(t)}{N(t)}\Big) \mid \zeta > t]\, dt = \int_0^\infty e^{-rt} \frac{d(t)^{1-\gamma}}{1-\gamma} \beta({}_tp_x)\, dt$$

where $\beta(p) = \beta_{n,\gamma}(p) = pE[(\frac{n}{N})^{1-\gamma}]$ and $N \sim 1 + \text{Bin}(n-1, p)$.

- Budget constraint: $\int_0^\infty e^{-rt} d(t)\, dt = 1$

(any balance absorbed by the vendor upon last death)

Fine print: form of optimal withdrawals

- Euler-Lagrange $\Rightarrow \exists$ a Lagrange multiplier λ such that the optimal $d(t)$ satisfies $e^{-rt}d(t)^{-\gamma}\beta({}_tp_x) = \lambda e^{-rt}$ for every t. In other words, $d(t) = D({}_tp_x)$, where $D(p) = D(1)\beta(p)^{1/\gamma}$.
- ie. the only place the mortality model enters is determining the constant $D(1)$, which then satisfies $D(1) = \left[\int_0^\infty e^{-rt}\beta({}_tp_x)^{\frac{1}{\gamma}}\,dt\right]^{-1}$.

  This allows us to make analytic arguments using the [quite tractable] function $\beta(p)$, and prove conclusions which will hold regardless of the underlying mortality model.
- Maximal utility is then

$$\int_0^\infty e^{-rt}\frac{(D(1)\beta({}_tp_x)^{\frac{1}{\gamma}})^{1-\gamma}}{1-\gamma}\beta({}_tp_x)\,dt = \frac{1}{1-\gamma}\Big(\int_0^\infty e^{-rt}\beta({}_tp_x)^{\frac{1}{\gamma}}\,dt\Big)^\gamma$$

Fine print: properties

- $d(t) \downarrow$ with t, and $\to 0$ when $t \to \infty$
- $d(t) \to \frac{1}{r}$ as $\gamma \to \infty$ (flat tontine).
- $\beta(p) < p^\gamma$ if $0 < \gamma < 1$ and $> p^\gamma$ if $\gamma > 1$.
  Plugging the latter into expressions for the annuity and tontine utilities gives a proof that (unsurprisingly)

$$\text{Annuity Utility} > \text{Tontine Utility}$$

Fine print: equitability

- We can prove a necessary and condition for existence of equitable prices:

$$\int_0^\infty e^{-rt} d(t) \Big(\prod_{i \notin A} {}_t q_{x_i}^{n_i}\Big)\Big(1 - \prod_{i \in A} {}_t q_{x_i}^{n_i}\Big)\, dt$$
$$< \frac{1}{w} \sum_{k \in A} n_k w_k \int_0^\infty e^{-rt} d(t) \Big(1 - \prod_i {}_t q_{x_i}^{n_i}\Big)\, dt.$$

  for every set A of cohorts, where w is the total investment.

- Meaning: split the population in two, and pay the 2nd group only after the first group all die. If this is a good deal for group 2, then equity is impossible.