QFI IRM Model Solutions
Fall 2023

1. Learning Objectives:

1. The candidate will understand and be able to identify and describe types of risk present in investment management.

2. The candidate will understand and be able to apply different approaches to measuring and assessing risk exposures.

Learning Outcomes:

(1a) Identify and describe the various kinds of risks, including strategic, market, credit, operational, liquidity etc.

(2d) Understand the difference between real world and risk neutral processes and select appropriate market risk models.

(2e) Understand, evaluate, and apply credit risk models.

Sources:

QFII-104-14: Correlation: Pitfalls and Alternatives

QFII-110-15: The Devil is in the Tails: Actuarial Mathematics and the Subprime Mortgage Crisis

Quantitative Enterprise Risk Management, Hardy & Saunders Ch 3: Risk Measures

Commentary on Question:

This question tests candidates' knowledge on risk identification techniques and application of ILN and GARCH models and their extensions. It also tests candidates' ability to perform calculations using the GARCH model. To receive full credit, candidates needed to provide support for their analysis whether in words or mathematical support.

Solution:

(a)

(i) Identify two risk identification techniques that would be the most suitable to use with the above participants.

(ii) Assess the approach of using a standardized, one-time questionnaire with the above participants.

1. Continued

Commentary on Question:
Candidates performed well on identifying suitable risk identification techniques but did not perform as well on assessing the approach of using a standardized questionnaire.

(i) The Gap Analysis should be applied to the junior staff and the senior staff so that we can form a clear picture of desired and actual levels of risk exposure. The Delphi techniques can be applied to the experts to comment on the risks anonymously and independently.

(ii) A standardized questionnaire can allow results to be analyzed quantitatively. However, this will clearly have the effect of limiting the possible responses. Free text is suggested to capture this information but follow up surveys are needed based on the response of the initial survey.

(b)

(i) List four important stylized facts about equity price movements

(ii) Evaluate how well ILN addresses these four facts

(iii) Evaluate how well GARCH addresses these four facts

Commentary on Question:
Candidates overall did well on this question. Most candidates were able to list the relevant facts and explain whether they did or did not apply to the ILN and GARCH models.

(i)
(1) Volatility is stochastic not constant
(2) High volatility periods are fairly rare, but tend to be clustered together
(3) When the absolute value of the log-return is high, the return is more likely to be negative than positive
(4) When volatility does move from low to high, it is far more likely to be precipitated by a sudden drop in prices than a sudden increase in prices

(ii) ILN does not comply with any of the stylized facts. For ILN, the volatility is constant, and therefore there is no randomness and no clustering. The log-returns over each non-overlapping period are independent. The log-returns are symmetric about the mean, so jumps up are just as likely as jumps down. Finally, with constant volatility, there is no leverage effect.

1. Continued

(iii) The GARCH model incorporates stochastic volatility, through the dependence on Y_t. It also incorporates volatility clustering, through the $(Y_{t-1} - \mu)^2 \; and \; \sigma_{t-1}$. The GARCH model has no mechanism for incorporating the leverage effect, where higher volatility clusters are associated with market crashes or failures. Under the GARCH model, high volatility periods are equally likely to be instigated by a random jump up in the stock price as a random jump down.

(c)

(i) Explain whether you should use the risk-neutral or real-world measure for this calculation.

(ii) Calculate the probability that the EI will drop by more than 20% in a month.

(iii) Calculate after how many months will the expected variance equal to the long-term variance (within 6 decimals).

After reviewing, you realized that the junior staff had made a mistake in his calibration, and the $a_1 = 0.15 \, , b = 0.75$.

(iv) Describe qualitatively how these new parameters will impact the results of the model.

Commentary on Question:
Candidates had mixed performance on this question. In part (i) many candidates could identify to use the Real-World measure but provided limited support. In part (ii), candidates could often calculate Y0 but many could not calculate the other needed values. Some candidates did not apply natural log when calculating the return and receive partial credits. In part (iii), most candidates calculated the long-term volatility but were unable to determine the amount of time needed to converge to the long-term volatility. In part (iv), most candidates correctly identified the impact of one parameter update but not all candidates provided accurate discussion of both.

(i) You should use real-world measure for this calculation. The real-world measure captures the true underlying dynamics of price movements and is more suitable for assessing tail risk or analyzing risk mitigation strategies.

1. Continued

(ii) $Y_0 = \ln(400/450) = -0.11778$

$$\sigma_1^2 = a_0 + a_1(Y_0 - \mu)^2 + b\sigma_0^2$$

$= 0.00065 + 0.1 * (-0.11778 - 0.003)^2 + 0.6 * 0.0036$

$= 0.004269$

$S_1 = 400 * 0.8 = 320$

$Y_1 = \ln(320/400) = -0.22314$

$$Z = \frac{Y_1 - \mu}{\sigma} = \frac{-0.22314 - 0.003}{\sqrt{0.004269}} = -3.46121$$

Using the Normal Distribution, Probability = 0.000269

(iii) Long-Run Variance = $\frac{0.00065}{1-(0.1+0.6)} = 0.002167$

$$\mathrm{E}_0[\sigma_t^2] = a_0\left(\frac{1-(a_1+b)^{t-1}}{1-(a_1+b)}\right) + (a_1+b)^{t-1}\sigma_1^2$$

Applying the equation above recursively, the value reaches required precision at t=23.

1. Continued

t	sigma
1	0.004269
2	0.003638
3	0.003197
4	0.002888
5	0.002671
6	0.00252
7	0.002414
8	0.00234
9	0.002288
10	0.002251
11	0.002226
12	0.002208
13	0.002196
14	0.002187
15	0.002181
16	0.002177
17	0.002174
18	0.002172
19	0.00217
20	0.002169
21	0.002168
22	0.002168
23	0.002167

(d) (*0.5 points*) Critique the junior staff's suggestion.

Commentary on Question:
Most Candidates correctly identified that GARCH is not scalable but failed to provide the correct explanation.

The junior actuary is not correct. GARCH is not scalable. If we generate daily returns following a GARCH process, and observe the resulting weekly returns, we are observing the sum of dependent random variables, with no simple analytic form for the variance or the distribution.

(e) (*1.5 points*) Propose an equivalent process that addresses the CRO's needs and describe its distribution in terms of your model's parameters.
(Equation is not required. A detailed explanation of the process would suffice.)

1. Continued

Commentary on Question:
Most Candidates did poorly on this question. Some candidates were able to correctly identify the use of risk-neutral measure and the use of risk free rates and received partial credits.

We need the equivalent Q-measure process to be risk-neutral in the period from *t*-1 to *t*, which means that

$$\mathrm{E}_{\mathbb{Q}}[e^{Y_t}|\mathcal{F}_{t-1}] = e^r,$$

where *r* is the risk free rate of interest. We can achieve this, retaining the original GARCH
variance process, with the distribution

$$\begin{aligned} Y_t|\mathcal{F}_{t-1} &\sim_{\mathbb{Q}} N(r - \tfrac{\sigma_t^2}{2}, \sigma_t^2) \\ &\Rightarrow Y_t|\mathcal{F}_{t-1} = r - \tfrac{\sigma_t^2}{2} + \sigma_t \varepsilon_t, \\ \sigma_t^2 &= a_0 + a_1(Y_{t-1} - (r - \tfrac{\sigma_t^2}{2}))^2 + b\sigma_{t-1}^2, \end{aligned}$$

where *a*0, *a*1, and *b* have the same values under the Q-measure as under the P-measure. This Q-measure is no longer a regular GARCH process, because of the term in $\sigma t2$ in the equation
The process is called a GARCH-M, or GARCH-in-Mean process.

(f) (*1 point*) Propose a variation of the GARCH model to address the CRO's concern, and justify your response.

Commentary on Question:
Most Candidates correctly identified the use of TGARCH but failed to provide the sufficient explanation and support.

TGARCH model (for Threshold GARCH) introduces an additional term in the variance equation if the previous period log-return was negative.

$$\text{TGARCH variance:} \quad \sigma_t^2 = a_0 + (a_1 + \gamma \min(Y_{t-1}, 0))(Y_{t-1} - \mu)^2 + b\sigma_{t-1}^2.$$

We expect $\gamma < 0$, so that there is more of a jump in volatility when *Yt*-1 is negative than when it is positive, creating the leverage effect that we identified in the data.

2. Learning Objectives:

2. The candidate will understand and be able to apply different approaches to measuring and assessing risk exposures.

Learning Outcomes:

(2c) Analyze and evaluate the use and misuse of correlation, integrated risk distributions and copulas.

Sources:

QFII-104-14 Correlation: Pitfalls and Alternatives

Quantitative Enterprise Risk Management, Hardy & Saunders, 2022, Chapter 6

Commentary on Question:

Commentary is listed underneath each question component.

Solution:

(a) Critique the CRO's statement: "correlation is not an appropriate dependency measure".

Commentary on Question:

This question tests candidate's understanding of problems with linear correlation. Candidates performed well in this question. Full marks were awarded for identifying at least four disadvantages of linear correlation.

Correlation works well only when there is a linear relationship between the two variables being studied. Market risk and credit default risk generally do not exhibit a linear relationship. Therefore, the CRO's statement is correct, as linear correlation has the following problems:

- Correlation is simply a scalar measure of dependency and does not tell us the dependence structure of risks.
- Possible values of correlation depend on the marginal distributions of the risks. All values between –1 and 1 are not necessarily attainable.
- Perfectly positively (negatively) dependent risks do not necessarily have a correlation of 1, and perfectly negatively dependent risks do not necessarily have a correlation of –1.
- A correlation of zero does not indicate independence of risks.
- Correlation is not invariant under transformations of the risks.
- Correlation is only defined when the variances of the risks are finite. It is not an appropriate dependence measure for very heavy-tailed risks where variances appear infinite.

2. Continued

(b) Describe these two types of relationships.

(i) Concordant

(ii) Discordant

Commentary on Question:
Candidates overall performed well on this question. Most candidates were able to provide a clear description.

Let (X*, Y*) be a bivariate random variable that has the same joint distribution as (X, Y) and is independent of (X, Y).

(i) X and Y are concordant when $(X - X^*)(Y - Y^*)$ tend to be positive. That is, larger values of X are associated with larger values of Y, and smaller values of X with smaller values of Y.
(ii) X and Y are discordant when $(X - X^*)(Y - Y^*)$ tend to be negative. That is, larger values of X are associated with smaller values of Y, and vice versa.

(c) Calculate three types of correlation of X_1 and X_2

(i) Pearson's

(ii) Spearman's rank

(iii) Kendall's rank

Commentary on Question:
Candidates performed very well on part (i) and part (ii) but had difficulties in part (iii). A common calculation error is a mixed use of population variance (covariance) and sample covariance (variance) in the correlation formula.

(i) Pearson's: 23.62%

CORREL(X_1 , X_2) would be the most simple Excel formula to use.

Alternatively, one can also calculate the covariance $\mathrm{Cov}(X_1, X_2)$ and standard deviation σ_1 and σ_2 first and then obtain the correlation by the formula $\frac{\mathrm{Cov}(X_1,X_2)}{\sigma_1\sigma_2}$. Note that one should either use population covariance COVARIANCE.P(…) and population standard deviations STDEV.P(…), or sample covariance COVARIANCE.S(…) and sample standard deviations STDEV.S(…). Mixing them up will yield incorrect results.

2. Continued

(ii) Spearman's rank: 10.30%

One can first obtain the ranks of X_1 and X_2 respectively by using the Excel formula RANK(…), then calculate Pearson's correlation of the two arrays of ranks below:

Rank(X_1)	Rank(X_2)
7	8
5	4
2	1
9	9
3	2
6	5
10	3
4	7
1	10
8	6

(iii) Kendall's rank: 15.56%

$$t_k = \frac{\text{number of concordant pairs} - \text{number of discordant pairs}}{\frac{1}{2}n(n-1)}$$

One way to count the concordant and discordant pairs is to construct the following table, where the concordant pairs will receive a value of 1, and the discordant pairs will receive a value of –1.

	X1	2.1%	10.2%	20.1%	-4.4%	19.7%	5.5%	-7.5%	10.6%	30.4%	-1.7%
	X2	-0.5%	2.0%	7.8%	-1.8%	3.8%	1.0%	2.4%	0.0%	-1.9%	0.5%
X1	X2										
2.1%	-0.5%										
10.2%	2.0%	1									
20.1%	7.8%	1	1								
-4.4%	-1.8%	1	1	1							
19.7%	3.8%	1	1	1	1						
5.5%	1.0%	1	1	1	1	1					
-7.5%	2.4%	-1	-1	1	-1	1	-1				
10.6%	0.0%	1	-1	1	1	1	-1	-1			
30.4%	-1.9%	-1	-1	-1	-1	-1	-1	-1	-1		
-1.7%	0.5%	-1	1	1	1	1	1	-1	-1	-1	

2. Continued

Using the top left highlighted cell as an example, the formula is:

$$\text{sign}\Big((10.2\% - 2.1\%) \times \big(2.0\% - (-0.5\%)\big)\Big) = \text{sign}(0.2025\%) = 1$$

where

$$sign(z) = \begin{cases} -1 & \text{if } z < 0 \\ 0 & \text{if } z = 0 \\ 1 & \text{if } z > 0 \end{cases}$$

The sum of all 1's and –1's in the table above is 7, which is essentially $(\text{number of concordant pairs} - \text{number of discordant pairs})$ in t_k.

Therefore,

$$t_k = \frac{7}{\frac{1}{2} \times 10 \times (10-1)} = \frac{7}{45} = 15.56\%$$

(d) Describe the advantages and disadvantages of rank correlation.

Commentary on Question:
Candidates performed well on this question. Full marks were awarded for identifying both advantages and disadvantages of rank correlation.

Advantages:

- Compared with linear correlations, all values between –1 and 1 are attainable under rank correlation.
- Perfectly positively (or negatively) dependent risks have a rank correlation of 1 (or –1).
- Rank correlation is invariant under transformations of the risks.
- Rank correlation is defined even when the variances of the risks appear infinite.

Disadvantages:

- Rank correlation is still a scalar measure of dependency and does not provide us with the full picture of the dependence structure of risks.
- A correlation of zero does not indicate independence of risks.

(e) Explain the advantages of the copula approach to model the tail risk.

Commentary on Question:
Candidates performed as expected on this part. Most candidates identified at least one advantage of using the copula approach in the context of tail risk modelling.

2. Continued

Unlike correlations that produce just a single number, copulas can describe the full dependency structure between two variables, providing additional information on how market risk and credit default risks are correlated in stressed scenarios.

Additionally, copulas disentangle the marginal distributions from the joint dependence structure. Consequently, we can choose specific copulas to capture the tail dependence, which is crucial for understanding how risks interact with each other in a recessionary cycle.

3. Learning Objectives:

2. The candidate will understand and be able to apply different approaches to measuring and assessing risk exposures.

3. The candidate will understand and be able to apply the components of an effective risk management system to investment portfolio management and enterprise management.

Learning Outcomes:

(2c) Analyze and evaluate the use and misuse of correlation, integrated risk distributions and copulas.

(2e) Understand, evaluate, and apply credit risk models.

(3a) Identify and describe various approaches for managing portfolio risks including VaR/ ES methods, risk budgeting, position limits, etc.

Sources:

Credit Risk Modeling, Bolder, Chapters 1 -4

Commentary on Question:

Commentary is listed underneath each question component.

Solution:

(a) Compute the 97.5% VaR of L

Commentary on Question: *This question tests candidates' understanding of computing the loss distribution for an independent credit portfolio and the resulting VaR. Overall, candidates performed well in this question. Some candidates applied the normal approximation to compute the VaR, and no credits were given to such an approach.*

The loss distribution of the portfolio can be computed via

Case	Default	Loss	PMF	CDF
1	$\mathbb{I}_{\mathcal{D}_1} = \mathbb{I}_{\mathcal{D}_2} = 0$	0	0.8*0.9=0.72	0.72
2	$\mathbb{I}_{\mathcal{D}_1} = 1, \mathbb{I}_{\mathcal{D}_2} = 0$	10	0.2*0.9=0.18	0.9
3	$\mathbb{I}_{\mathcal{D}_1} = 0, \mathbb{I}_{\mathcal{D}_2} = 1$	20	0.8*0.1=0.08	0.98
4	$\mathbb{I}_{\mathcal{D}_1} = 1, \mathbb{I}_{\mathcal{D}_2} = 1$	30	0.2*0.1=0.02	1

Thereby, $VaR_{97.5\%}$ =20 because the third case is the first time the CDF exceeds 0.95.

3. Continued

(b) Recalculate the 97.5% VaR of L, using the threshold model approach

Commentary on Question: *This question mainly tests candidates' understanding of computing the loss distribution for a dependent credit portfolio constructed based on the threshold approach. Overall, candidates did not perform well in this question. Many candidates did not know how to compute the joint default probability under the threshold model.*

Note that $d_1 = \Phi^{-1}(p_1) = -0.84162$ and $d_2 = \Phi^{-1}(p_2) = -1.28155$ are the default thresholds for the two assets. We have the following default probabilities:

- $P\left(\mathbb{I}_{\mathcal{D}_1} = \mathbb{I}_{\mathcal{D}_2} = 0\right) = P(y_1 > d_1, y_2 > d_2) = P(y_1 \leq -d_1, y_2 \leq -d_2) = 0.7515$
- $P\left(\mathbb{I}_{\mathcal{D}_1} = 1, \mathbb{I}_{\mathcal{D}_2} = 0\right) = P(y_1 \leq d_1, y_2 > d_2) = P(y_1 \leq d_1) - \Phi_2(d_1, d_2; \rho)$
- $P\left(\mathbb{I}_{\mathcal{D}_1} = 0, \mathbb{I}_{\mathcal{D}_2} = 1\right) = P(y_1 > d_1, y_2 \leq d_2) = P(y_2 \leq d_2) - \Phi_2(d_1, d_2; \rho)$
- $P\left(\mathbb{I}_{\mathcal{D}_1} = 1, \mathbb{I}_{\mathcal{D}_2} = 1\right) = P(y_1 \leq d_1, y_2 \leq d_2) = \Phi_2(d_1, d_2; \rho)$

Hence, the loss distribution of the portfolio can be computed via

Case	Default	Loss	PMF	CDF
1	$\mathbb{I}_{\mathcal{D}_1} = \mathbb{I}_{\mathcal{D}_2} = 0$	0	0.7515	0.7515
2	$\mathbb{I}_{\mathcal{D}_1} = 1, \mathbb{I}_{\mathcal{D}_2} = 0$	10	0.2-0.0515=0.1485	0.9
3	$\mathbb{I}_{\mathcal{D}_1} = 0, \mathbb{I}_{\mathcal{D}_2} = 1$	20	0.1-0.0515=0.0485	0.9485
4	$\mathbb{I}_{\mathcal{D}_1} = 1, \mathbb{I}_{\mathcal{D}_2} = 1$	30	0.0515	1

Finally, $VaR_{97.5\%}$ =30 because the fourth case is the first time the CDF exceeds 0.95.

(c) Describe the difference between the VaR values based on the independence model and the threshold model.

Commentary on Question: *This question mainly tests candidates' understanding of the difference between independent model and threshold model, as well as how the induced dependence impacts the joint default probability. Overall, candidates performed well in this question. Most candidates could identify that the VaR under the threshold model is larger than that under the independence model, although the some candidates provided inappropriate reasoning.*

The VaR of the threshold model is greater than that of the independence model. This is because the threshold model assumes a positive dependence between the defaults of two bonds, thus the probability of both defaulting becomes higher in comparison with that of the independence model. This indicates that ignoring the positive dependence of default events may cause an under-estimation of the credit risk inherent in a portfolio.

4. Learning Objectives:

1. The candidate will understand and be able to identify and describe types of risk present in investment management.

3. The candidate will understand and be able to apply the components of an effective risk management system to investment portfolio management and enterprise management.

Learning Outcomes:

(1c) Identify behavioral risks and explain how they factor into investment management.

(3c) Understand and evaluate model and parameter risks.

Sources:

Quantitative Enterprise Risk Management, Hardy & Saunders:
Ch. 14: Model Risk and Governance
Ch. 19: Behavioral Risk Management

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a)

(i) Identify the two main sources of model risk.

(ii) List the main purposes of model governance.

Commentary on Question:

Candidates did well on this section, with most getting full credit on the first part of the question. Some candidates struggled with identifying key purposes of model governance. Only two model governance functions were required to receive full credit in (a)(ii).

(i) Model Risk can be from misused model or from incorrect models.

(ii) The main purposes of model governance are:

- Managing different life stages
- Managing inventory
- Assessing the materiality of the model being used

(b)

(i) List four methods for constructing proxy models.

(ii) Identify two risks of using proxy models.

4. Continued

Commentary on Question:
Most candidates received partial credit for each subpart. To receive full credit on (b)(ii) candidates needed to call out both that the simplifications may only capture limited information, but also that proxy models are susceptible to relationships breaking down.

(i)

1. Representative scenarios – run a smaller set of scenarios that captures the same risks as the full set.
2. Parametric curve fitting – use the full set of simulations to fit a function that generates an output based on the set of important variables.
3. Non-parametric curve-fitting – use machine learning techniques or other advanced techniques to approximate the more sophisticated model; use that model prospectively in the form of an approximating function.
4. Replacing elements of full model with simplified model points – for example, using a smaller set of representative policies instead of the entire population of policies the insurer sells to/has inforce.

(ii)

1. Representative scenarios or simplifactions may capture only limited information or fail to capture tail risk; approximating functions may have limited domain of applicability.
2. Proxy models are also susceptible to relationships breaking down, which can happen very quickly.

(c)

(i) Explain the conditions when standard errors can be used to estimate parameter uncertainty.

(ii) Describe the key advantage of Bayesian methods over MLE for assessing parameter uncertainty.

(iii) Describe the steps to simulate values from the predictive distribution using the Markov Chain Monte-Carlo (MCMC) algorithm given a prior p and likelihood function L.

Commentary on Question:
Most candidates struggled with this part, particularly the more technical subparts (i) and (iii). To receive full credit on (i), candidates needed to call out both the normality assumptions and the necessary sample size. For (iii), candidates often failed to describe each of the required steps from prior to simulation.

4. Continued

(i)

- Large sample sizes ensure an MLE estimate that is usually asymptotically normally distributed, and a robust standard error.
- Therefore, the standard errors can be used to construct a confidence interval around the MLE estimate of the parameter.

(ii) Bayesian methods provide the full range of potential parameters values along with associated probabilities, not just point estimates.

(iii)

1. Generate N samples parameter sets (θ_i) from the posterior $\pi = c \times L(\boldsymbol{\theta}) \times p(\boldsymbol{\theta})$ (only 1 pt if posterior function is not defined)
2. Generate N uniform variates U_j from the uniform distribution on (0,1), where j = 1, 2, …, N.
3. Convert the uniform random variates to values from the predictive distribution by calculating values X_j such that the CDF of each X_j using $\boldsymbol{\theta_i}$ is U_i
   i.e., $F(X_j | a = a_j, \theta = \theta_j) = U_j$

(d) Critique the appropriateness of the model characteristics above.

Commentary on Question:
Candidates did fairly well on this question; most received at least partial credit. Many did not appropriately evaluate on the third point, where suitable data proxies can be supportable given the lack of recent data and economic intermingling.

Point1: Weekly projection is to be too fine-grained for the use, adding unnecessary complexity. Monthly data would likely be more appropriate.

Point 2: Neither pair is an appropriate benchmark. Canada is not emerging and neither Canada nor Mexico are in SE Asia; both economies are well understood. Currency pairs for countries similar in size, population, region, and other economic metrics should be used.

Point 3: This method may be appropriate if the two rates are highly correlated.

(e) Define the following behavioral biases:

(i) Confirmation Bias

(ii) Framing Bias

(iii) Anchoring Bias

4. Continued

Commentary on Question:
Candidates did well on this question. To receive full credit candidates needed to provide a clear definition, not just incidental information on its importance or prevalence. Without such definitions, the biases are not sufficiently distinguished from each other.

(i) The tendency to ignore or downplay information that is contradictory to a previously held view, and to assign extra weight to information that supports this prior view. It may arise due to overconfidence of a manager in their beliefs, as they discount any evidence of their lack of competence.

(ii) Framing a choice in terms of a positive outcome is more likely to achieve agreement than framing it in terms of a negative outcome, which can distort the resulting determination. The decisions we make are impacted by the frame in which choices are presented.

(iii) An individual's decision is overly influenced by an initial piece of information. The bias is persistent even when the decision-maker is aware of the potential for anchoring, and even when the initial information has little or no value.

5. Learning Objectives:

3. The candidate will understand and be able to apply the components of an effective risk management system to investment portfolio management and enterprise management.

Learning Outcomes:

(3a) Identify and describe various approaches for managing portfolio risks including VaR/ ES methods, risk budgeting, position limits, etc.

(3b) Explain the best practices of investment risk management.

(3c) Understand and evaluate model and parameter risks.

Sources:

The Top Ten Operational Risks: A Survival Guide for Investment Management Firms and Hedge Funds, Miller and Lawton, 2010

Quantitative Enterprise Risk Management, Hardy & Saunders, Ch. 9: Short Term Portfolio Risk

QFII-119-19: Chapter 3 of The Known, the Unknown, and the Unknowable in Financial Risk Management: Measurement and Theory Advancing Practice

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a) Identify which "Top Ten" operational risk is addressed by each practice.

Commentary on Question:

Nearly all candidates did well, providing a pertinent "Top Ten" risk corresponding to each statement. The most common are listed below, but others may be suitable.

1. **Playbooks** – formal workflows reduce the risk of critical procedural errors/omissions through more standardized, consistent approaches throughout the company.
2. **Reconciliation Gaps** – having other truly independent sources of the same information is a good validation of accuracy and reliability.
3. **Novices, Apprentices, & Soloists** – training and cross-training is beneficial in keeping knowledge/skills current and diversifying highly specialized roles at the company.
4. **Dropped Batons** – this is the risk of issues (e.g., communication, timing, etc.) arising from hand offs from one person/team to another. The practice of repeating instructions confirms the information has been correctly received.

5. Continued

(b)

(i) Calculate the 90% VaR roundtrip flight time from City A to City B assuming a normal distribution with its mean and standard deviation estimated from the sample data presented in Excel spreadsheet.

(ii) Calculate the smoothed empirical 90% VaR of the roundtrip flight time.

(iii) Explain whether one-sided or two-sided hypothesis testing is more appropriate for testing the 90% VaR.

(iv) Calculate significance level (p-value) for which the normal distribution approximation of the 90% VaR roundtrip flight time would be rejected based on the provided sample data. Use a one or two sided test depending on your answer from part (iii).

(v) Determine whether approximating total flight time with a normal distribution is reasonable for this data.

Commentary on Question:
Candidates did very well on subpart (i). Of the candidates that did not receive full credit, the most frequent oversight was treating the sample data as being the entire population when estimating the standard deviation.

Candidates also did well on subpart (ii). Common mistakes were not recognizing the provided data was not ordered, using values from subpart (i) instead of directly from the empirical data, determining the 91% VaR, or neglecting to interpolate.

Subpart (iii) was a steppingstone to subpart (iv) that proved moderately challenging for candidates. A large proportion of candidates incorrectly argued a one-sided test is more appropriate since the VaR metric is one-sided.

Very few candidates managed to perform a reasonable determination of the significance level for subpart (iv).

Despite the hurdles of subparts (iii)-(iv), some candidates were able to provide reasoned explanations for subpart (v) to earn credit.

(i) Mean: $\bar{x}$ = AVERAGE(*data range*) = 485.42
Standard Deviation: σ = STDEV.S(*data range*) = 2.17
$VaR_{90} = \bar{x} + \Phi^{-1}(0.9) * \sigma$ = NORM.INV(0.9, $\bar{x}$, σ) = **488.20**

5. Continued

(ii) 90% * (100 + 1) = 90.9
90^{th} Largest Value = 488.40
91^{st} Largest Value = 488.66
Interpolate 90.9^{th} Largest: 488.40 * 0.1 + 488.66 * 0.9 = **488.63**

(iii) A **two-sided hypothesis test** is more relevant to the company since having too little fuel or too much fuel are detrimental. The company is penalized in both circumstances.

(iv) There were 12 data points higher than the 488.20 VaR_{90} from subpart (i). The expected number of exceptions is 10% * 100 = 10.

One-Sided Hypothesis Test, if chosen in subpart (iii):
1–BINOM.DIST(12-1,100,10%,1) = **29.7%**

Two-Sided Hypothesis Test, if chosen in subpart (iii):
1–[BINOM.DIST(12-1,100,10%,1)–BINOM.DIST(2*10-12,100,10%,1)]
= **61.8%**

(v) The high p-value of 61.8% [or 29.7% for one-sided] means the null hypothesis of the distribution being normal is not likely to be rejected. Therefore, approximating with a normal distribution is reasonable. Some caution is warranted since the sample size is relatively small and not necessarily representative, also there does not appear to be a left-tail to the distribution.

(c) For roundtrip flight from City C to City D,

(i) Calculate the expected costs due to insufficient fuel per roundtrip flight.

(ii) Calculate the expected minutes holding excess fuel per roundtrip flight.

(iii) Determine the largest value of X such that the additional cost of carrying extra fuel is worth the financial risk of insufficient fuel costs on average.

Commentary on Question:
Overall, candidates performed poorly on part (c). Performance on subparts (i) and (ii) were similar, with (ii) being more challenging given it reverses the usual tail consideration. Most candidates that calculated expected shortfalls neglected to consider the averages are conditional, and thus require weighting. Many candidates who attempted responses were able to receive partial credit from subpart (iii) by simply recognizing the solution involves dividing subpart (i) by subpart (ii).

5. Continued

(i) Expected Shortfall (ES) = $\mu + \varphi[\Phi^{-1}(0.9)]/(1-0.9) * \sigma = 305.26$ min
VaR = NORM.INV(0.9, $\mu = 300$, $\sigma = 3$) = 303.84 min
ES – VaR = 305.26 min – 303.84 min = 1.42 min
Multiply by 10% since these are averages given higher than 90% VaR.
Multiply by \$1M cost per minute: 0.142 min * \$1M = **\$142,029.53**

(ii) Expected Shortfall (ES) = $\mu - \varphi[\Phi^{-1}(0.1)]/(1-0.1) * \sigma = 299.42$ min
*[Alternatively, leveraging subpart (i), ES = (μ – 305.26 min * 0.1)/0.9]*
VaR – ES = 303.84 – 299.42 = 4.43 min
Multiply by 90% since these are averages given lower than 90% VaR.
4.43 min * 90% = **3.99 min**

(iii) X * 3.99 min = \$142,029.53
X = \$142,029.53 / 3.99 min = **\$35,625.98/min**

(d)

(i) Explain whether a fractal or a Gaussian distribution is more appropriate to model fuel costs.

(ii) Calculate the likelihood that the average roundtrip fuel cost between C and D will exceed \$288k under either a fractal distribution with alpha (exponent) of 3 or Gaussian with mean of \$1.98/gal, depending on which was recommended in part d(i).

Commentary on Question:
Most candidates correctly selected the fractal distribution, but were challenged to provide sufficiently well-reasoned explanations that weren't contradictory, tautological, or applied equally to both distributions. And while very few candidates received full credit for subpart (ii), many received partial credit.

(i) It's possible for a single observation to disproportionately affect the total since there are no theoretical limits to the price of fuel. A single disruptive event affecting supply chains or runaway inflation could send prices soaring. This is more aligned with "wild randomness" rather than "mild randomness" and be better described with fractal distributions.

(ii) Average Flight Time x 60 gal/min = 300 min x 60 gal/min = 18,000 gal.
Fuel cost to reach \$288k = \$288k / 18,000 gal = \$16/gal.

<u>For Fractal Distribution, if chosen in subpart (i):</u>
(\$16/gal) / (\$8/gal) = 2.
The odds will go down by a factor of $2^{\alpha} = 2^3 = 8 \rightarrow$ **1 in 40** or **0.025**

5. Continued

For Gaussian Distribution, if chosen in subpart (i):
\$8/gal is the 80^{th} percentile since $0.8 = 1 - 1/5$.
$\$8/gal = \$1.98/gal + \Phi^{-1}(0.8) * \sigma \rightarrow \sigma = \$7.153/gal.$
$1 - \Phi(\$16/gal; \mu = \$1.98/gal, \sigma = \$7.153/gal) =$ **0.025** or **1 in 40**

6. Learning Objectives:

4. The candidate will understand and be able to apply different approaches to mitigate investment risks using derivatives.

Learning Outcomes:

(4a) Explain and implement techniques used to mitigate market risks.

(4b) Understand interest rate derivatives and use them to mitigate interest rate risk.

Sources:

Investment Risk Management, Baker, Filbeck, Chapter 26: Swaps

Commentary on Question:

Overall, candidates performed poorly on this question. Many candidates did not attempt the workbook calculations in parts b and d.

Solution:

(a) Explain why neither loan offered by the bank is suitable for New Company based on the current MIR.

Commentary on Question:

Candidates performed below average in this straightforward question. Candidates that arrived at the correct conclusion without explaining how they got there received partial credit. Candidates are reminded that questions that ask to explain something expect an explanation in the answer.

Prefer variable loan given expected cashflows, however:

Variable loan analysis

If not successful, lose 100 basis points (Inflow vs Outflow)
If successful, breakeven.
Doesn't make sense if the best scenario is breakeven.

Fixed loan analysis

If not successful, lose 75bp (50% chance)
If successful, gain 25bp (50% chance)
Doesn't make sense if expected value is a loss.

(b) Determine the total savings of two companies that could be realized through a swap arrangement in basis points, based on the rates given. (Remember to remove the service fee.)

Commentary on Question:

Candidates performed below average in this part. Candidates that showed their work but arrived at the incorrect solution are awarded partial credits as appropriate. Candidates that arrived at the incorrect solution but did not show their work received no credits.

6. Continued

Rate Savings: 25 basis point less 5 bp fee = 20bp
Explanation: Spread on both fixed loans is 100 bp. Spread on both variable loans is 125 basis points. Difference is 25bp

(c)

(i) The loans to be taken by each company.

(ii) The notional amounts of the swap and, all the swap parameters and their possible ranges.

Commentary on Question:
Candidates performed poorly on this part. Very few candidates managed to explain the swap well.

Loans:
Loans Big Co take 5 million loan at fixed rate and 5 million loan at variable rate. (Note to grader: Need to fund entire project)
Loans New Co takes out 5 million fixed rate loan (6%).

Swap:
Big Co and New Co swap 5 million notional amount to remove risk.
Big Co receives variable rate and pays fixed rate at (3.5%-5%) + margin.
New Co is pays variable rate and receives fixed rate.

Margin can vary from 152.5 - 172.5 (Half fee is paid by each party) (150+2.5 = 152.5; 175-2.5 = 172.5)

Or,
Big Co receives MIR+X and pays 5%.
New Co pays MIR+X and receives 5%.
X is between 152.5 and 172.5.

As long of the swap is described well and correctly full credit should be granted. Candidates may present it as net settlement or keep the legs separate while still answering the question.

(d) Determine the following cash flows (if applicable) for Big Company, New Company, and your company, based on your expectations.

(i) The projects

(ii) The loans

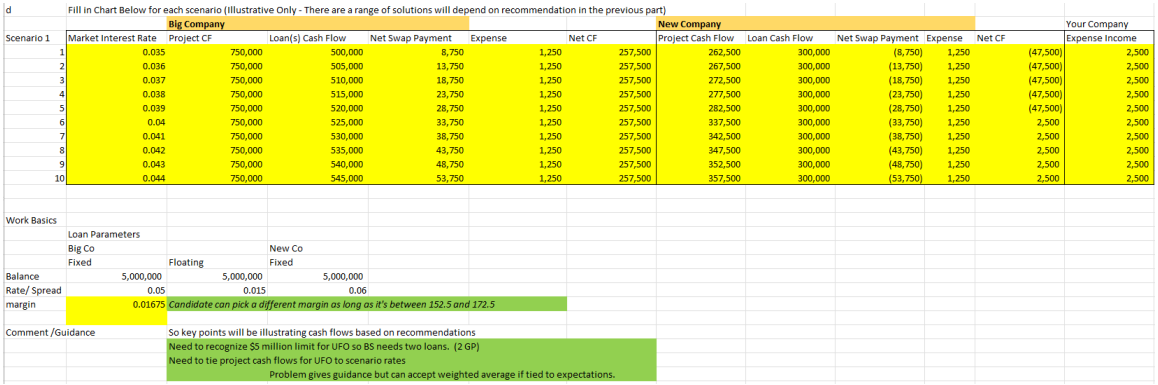
6. Continued

(iii) The swap that you proposed

(iv) The swap service fee/income

(v) The net cash flows

Commentary on Question:
Candidates performed poorly on this part. The majority of candidates did not attempt this question, and almost all that did did not explain their work.



d	Fill in Chart Below for each scenario (Illustrative Only - There are a range of solutions will depend on recommendation in the previous part)											
		Big Company					New Company					Your Company
Scenario 1	Market Interest Rate	Project CF	Loan(s) Cash Flow	Net Swap Payment	Expense	Net CF	Project Cash Flow	Loan Cash Flow	Net Swap Payment	Expense	Net CF	Expense Income
1	0.035	750,000	500,000	8,750	1,250	257,500	262,500	300,000	(8,750)	1,250	(47,500)	2,500
2	0.036	750,000	505,000	13,750	1,250	257,500	267,500	300,000	(13,750)	1,250	(47,500)	2,500
3	0.037	750,000	510,000	18,750	1,250	257,500	272,500	300,000	(18,750)	1,250	(47,500)	2,500
4	0.038	750,000	515,000	23,750	1,250	257,500	277,500	300,000	(23,750)	1,250	(47,500)	2,500
5	0.039	750,000	520,000	28,750	1,250	257,500	282,500	300,000	(28,750)	1,250	(47,500)	2,500
6	0.04	750,000	525,000	33,750	1,250	257,500	337,500	300,000	(33,750)	1,250	2,500	2,500
7	0.041	750,000	530,000	38,750	1,250	257,500	342,500	300,000	(38,750)	1,250	2,500	2,500
8	0.042	750,000	535,000	43,750	1,250	257,500	347,500	300,000	(43,750)	1,250	2,500	2,500
9	0.043	750,000	540,000	48,750	1,250	257,500	352,500	300,000	(48,750)	1,250	2,500	2,500
10	0.044	750,000	545,000	53,750	1,250	257,500	357,500	300,000	(53,750)	1,250	2,500	2,500

Work Basics			
	Loan Parameters		
	Big Co		New Co
	Fixed	Floating	Fixed
Balance	5,000,000	5,000,000	5,000,000
Rate/ Spread	0.05	0.015	0.06
margin	0.01675	*Candidate can pick a different margin as long as it's between 152.5 and 172.5*	
Comment /Guidance		So key points will be illustrating cash flows based on recommendations	
		Need to recognize $5 million limit for UFO so BS needs two loans. (2 GP)	
		Need to tie project cash flows for UFO to scenario rates	
		Problem gives guidance but can accept weighted average if tied to expectations.	

Net Swap column needs to be explained.

(e) Explain why the companies may prefer to work directly with a larger swap broker instead of your company.

Commentary on Question:
Candidates performed poorly in this part. In the context of this question, the swap broker acts as an intermediary to match counterparties in the legs of the swap. Many candidates mentioned that the use of a swap broker lowers transaction fees or that it reduces counterparty risk, which is not true.

Biggest reason would to be able to address default costs.
Big Co also could bring up the notional from 5 to 10 million without adding additional risk if parameters remained the same.
New Co would no longer necessarily be negotiating the spread difference and would only be looking at the bid-ask spreads which hopefully would give them a better shift on funding.

7. Learning Objectives:

1. The candidate will understand and be able to identify and describe types of risk present in investment management.

4. The candidate will understand and be able to apply different approaches to mitigate investment risks using derivatives.

Learning Outcomes:

(1a) Identify and describe the various kinds of risks, including strategic, market, credit, operational, liquidity etc.

(1c) Identify behavioral risks and explain how they factor into investment management.

(4a) Explain and implement techniques used to mitigate market risks.

(4b) Understand interest rate derivatives and use them to mitigate interest rate risk.

Sources:

Fixed Income Securities: Valuation, Risk, and Risk Management, Pietro Veronesi, Chapter 6

Quantitative Enterprise Risk Management, Hardy & Saunders, Chapter 2 and 19

Commentary on Question:

This question tests candidates' knowledge on identifying various risks given a situation, as well as using various financial instruments such as futures and forwards to mitigate market risks. It also tests candidates' ability to identify behavior risks and explain how they impact investment decision. In addition, the calculation part seeks to test candidates' ability to compute forward rates and the Profit and Loss at various durations. To receive full credits, candidates are expected to provide support (whether in words or in mathematical terms) for the final answer.

Solution:

(a) Identify the risks that Company XYZ is facing after losing the lawsuit.

Commentary on Question:

Candidates generally did well on this question. Most candidates were able to identify at least two risks and provided brief explanations to receive full marks. Some candidate only identified the risks without any explanation and therefore, only received partial credits.

7. Continued

Reputational Risk

Since the company loses a lawsuit, it will hurt its public image and have potential impact from the reputational risk such as lower sales, tighten regulation, etc.

Credit Risk

There is a risk that the buyer doesn't have the capability to pay the XYZ cash in 6 months.

(b) Describe the bias held by the coworker and the risks associated with the bias.

Commentary on Question:
Candidates overall did well on this question. Most candidates were able to quote a certain type of bias based on the syllabus and to provide explanations based on the case given. However, only a few were able to accurately state the risks associated with the bias, which was asked through the second part of the question. Therefore, majority of the candidates received partial credits in this question.

Framing Bias – The coworker only emphasizes the gain the company can make but downplay the lawsuit settlement the company has to pay.

This bias may encourage the company to do the same actions that leads to more lawsuits in future.

(c) Determine the minimum bid price your company should accept from Company ABC that will allow the company to pay the lawsuit, assuming no interest between April 1 and July 1.

Commentary on Question:
Candidates generally did poorly in this question. Some candidates were able to correctly calculate the forward rate of 4.61% and therefore, received partial credits.

Because future price is \$95.39, then
$f_4(t, T_1) = (100\text{-}95.39)/100 = 4.61\%$

Min. price to accept = $\dfrac{100}{1+ f_4(t,T_1)/4}$ = \$98.8606

7. Continued

(d)

(i) Calculate the profit or loss of the futures contract.

(ii) Assess whether your firm will be able to pay the lawsuit on July 1, 2008.

Commentary on Question:
Candidates generally performed poorly on this calculation question. Some candidates were able to get some parts of calculation correct and received partial credits.

(i) Total profit from futures: 0.25 x ($97.2912 – $95.3900) = $0.4753

(ii) The firm receive enough money that is needed to pay for the lawsuit. In fact, the firm actually receives more than what is needed ($100M) and therefore was buying too many futures contracts.

P&L from futures = $0.4753
Proceeds from Company ABC = $98.8606
Total = $99.3359
The total amount available to the firm after 90 days is:
99.3359 x (1 +3.7088%/4) = $100.2569 million

(e)

(i) Identify the shortcoming of hedging with futures that is referred to by your coworker.

(ii) Explain one method to overcome this shortcoming.

Commentary on Question:
Candidates generally performed well on this question, especially the first part. The question aims to test candidates' knowledge on identifying the shortcoming of hedging with futures based on the results of the part (d), which is the tailing of the hedge. Many candidates generally described the short comings of futures in general, and only received partial credits. For part (ii), candidates who simply stated that future should be used but did not provide any explanation did not receive any point.

7. Continued

(i) Tailing of the hedge

The firm is buying too many futures contracts, because the cash flows arising from the futures position accrue over time, which implies the need of the firm to take into account the time value of money between the time at which the cash flow is realized and the maturity of the hedge position. This will call for a reduction in the position in futures.

(ii) Tailing Tailing the hedge - It is a technique used to optimize the hedge ratio with the passage of time.

For example, one way to adjust the position in futures is to find a "tailing factor" through which we adjust the number of futures contract in order to return to the desired levels. Tailed hedge = untailed hedge x tailing factor.

8. Learning Objectives:

5. The candidate will understand the importance of risk culture and governance.

Learning Outcomes:

(5a) Explain the importance of risk culture and ethics in an investment firm.

(5b) Identify sources of unethical conduct and explain the role of a fiduciary.

Sources:

Parts (a) – (c) Investment Ethics, Peck, Sarah, 2011 Ch. 1-2

Part (d) Investment Ethics, Peck, Sarah, 2011 Ch. 1-2 and 3

Commentary on Question:

This question tested the candidate's knowledge and understanding of Risk Culture and Governance and identification of risk types. In general, candidates did well on this question.

Solution:

(a) Critique the above arrangement.

Commentary on Question:

This part distinguishes candidates' understanding of Soft Dollars. Most candidates addressed the first two bullet points correctly. Few candidates received full credit for the last two bullet points.

1. The broker violated the rule of making use of soft dollars by using commissions to pay for the travel expenses.
2. Soft Dollars can only be used to purchase research, per SEC rules.
3. Best execution: Fiduciary role of the asset manager means investment professional have an obligation to minimize the cost of trading to clients.
4. The asset manager is violating his responsibility for best execution by allowing soft dollars to be embedded into the trading commissions.

(b)

(i) Identify any unethical behavior in this scenario.

(ii) Recommend three possible remedial actions.

Commentary on Question:

Candidates did well on this question. However, few candidates received full credit because the majority of candidates missed that the client should also understand the transaction.

8. Continued

(i)

1. This violates the principle of ethical understanding.
2. You are obligated not to knowingly engage in an investment transaction either you or your client do not sufficiently understand.

(ii)

Possible remedial actions include:

1. Acknowledge your lack of expertise and recommend that your client seeks other expert advice.
2. Actively learn the product and hedging mechanics and become an expert in this area.
3. Ensure your client understands the investment strategy and the connection with the product.

(c) Critique the asset manager's actions regarding the IPO.

Commentary on Question:
Candidate performance was mixed for the first part, whereas for the second part, it was poor.

1. Broker should not limit to large and profitable clients.
2. Broker should not call due to the difference in timing. Clients should be informed simultaneously, as in email.
3. Fair way to distribute shares is on pro rata basis, based on account value, rather than equal amounts.

Excluding your client is not appropriate.

4. Your client needs to be informed, as all clients for which the IPO meets the investment objectives should be given the opportunity to invest.
5. However, it is appropriate to exclude certain clients if the IPO does not meet the investment objectives.
6. Excluding client before considering suitability of IPO investment is inconsistent with the principles of fair dealing and responsible investing.

(d) Identify any unethical behavior in using the above approach.

Commentary on Question:
*Candidates did well on this question. Most candidates properly identified three (**names**, **fees,** and **selected time period**) out of four unethical behaviors. Few candidates received full credit by pointing out that including only **current clients** in past performance is not ethical.*

8. Continued

Unethical behaviors:

1. Client confidentiality is being breached by mentioning the **names** of your clients.
2. Past performance is being misrepresented by including only **current** clients (terminated clients excluded) and only the performance over **selected time periods**.
3. **Fees** are not being disclosed but must be, otherwise, your manager can misuse/distort the information in investment.

9. Learning Objectives:

5. The candidate will understand the importance of risk culture and governance.

Learning Outcomes:

(5a) Explain the importance of risk culture and ethics in an investment firm.

(5b) Identify sources of unethical conduct and explain the role of a fiduciary.

(5e) Understand and apply the lessons learned from risk management failures.

Sources:

Investment Ethics, Peck, Sarah, 2011 - chapters 1-3, 9

Commentary on Question:

Candidates performed well on this question, particularly part c, which accounted for half the points. The question dealt with ethics as they relate to providing investment services and how fund performance may be reported.

Solution:

(a) Assess whether the company should continue providing the enhanced service to the pension fund.

Commentary on Question:

Candidate performance was fair for this question. Some candidates did not clearly identify that the offering of enhanced treatment on investment recommendations and IPOs to only the pension fund is not fair and that it should either be disseminated to all clients or none at all.

It is important for the fund managers to deal fairly and objectively with all clients when providing investment analysis, making recommendations, and taking investment actions.

However, treating clients fairly does not mean that they can't provide differentiated, personal, specialized and in-depth service to clients who are willing to pay for premium services. An example of the service that could continue to be provided is the investment research publications.

At the same time, different levels of services cannot disadvantage or negatively affect other clients and should be disclosed. The disclosure must be made available to all current & potential clients. The enhanced treatment on investment recommendations and IPOs negatively affects other clients so should be ended.

9. Continued

(b) Paula discovers that one of the traders purchased 50,000 shares of stock X and 2,000 call options for their personal trading account, as traders regularly disclose their personal investments. Shortly thereafter, that trader used $50 million from one of the fund's accounts to purchase more than 1 million shares of stock X.

Assess the actions of the trader.

Commentary on Question:
Candidates performed well on this question. The impact of the timing of the trade and the possible market manipulation to the benefit of the trader was well understood.

These trades in her personal account would not be acceptable.

The trader's personal investments are tracking with its client's investments, so there appears to be no direct conflict of interest.

However, the timing of the trade is at question here. There is an attempt at market manipulation using client's investments to benefit personal holding.

In this case, the trader is front-running the client's investments. Investment transactions for clients must have priority over investment transactions in which the trader is the beneficial owner.

Disclosure is not deemed to be a cure of front-running.

(c) Critique:

(i) The methodology used to calculate the performance metric of the fund.

(ii) The claim made by the company in its marketing brochure.

Commentary on Question:
Candidates performed well on this question. In general, the analysis and critique of the data provided was done well.

The returns are currently reported gross of fees since the fee is not incorporated into the return calculation. Returns should be reported net of fees because gross of fees gives the appearance of overperformance.

The terminated account should not be removed from the composite returns, as cherry picking of accounts can result in misleading performance reporting. This is especially true if the account is significant for the overall fund, which is the case here, as determined by the large proportion of assets belonging to Client A.

9. Continued

The average composite returns are calculated using equal weighting of the fund returns, which is not appropriate. A weighted average is more representative of performance because it includes size as determinant of performance.

While the company's claim is true, the fund manager is comparing the fund performance against the broader market (large-cap) returns. This is misleading as the performance should be benchmarked against a benchmark suitable for a mid-cap growth strategy.

Ethical reporting of performance measures should include the risk relative to the benchmark as well. The company should provide risk-adjusted return metrics such as Sharpe and/or Treynor ratio.