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Quantifying Market Competitiveness and Predicting Business Mix Impacts due to Rate Regulation

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Scope

- This paper concerns optimal mixes of business at the firm and state levels under varying competitive conditions
- Applies to lines where triangulation of losses is feasible
  - "Triangulation" – historical losses are formed into loss triangles
  - This could include weather events where losses occur with sufficient frequency
  - Modeled losses can be incorporated for catastrophic or infrequent losses (hurricanes, umbrella, etc.)
    - Note – modeled losses are artificial and hence create significant modeling risk
- Applies directly to Group Life, Accident Insurance, Long Term Disability and Healthcare as triangulation is feasible
- It can also be modified to other lines such as Individual Life Insurance but we will not be discussing those differences

Appeal

- The method should appeal to:
  - Enterprise Risk Management (ERM)
  - Strategic Planning
  - Rate Regulatory Actions
  - Responses to Rate Regulation

Three Questions

1. For a given geographical area, what is a company's optimal premium mix which will maximize its profits?
   A. Additionally, when should a company exit a line of business in that area?
2. How can a state regulator test the market's competitiveness for a given line of business?
   A. Is the market better served with a rate regulation policy that is "file and use" (competitive locations) or "prior approval" (less competitive)?
3. Given a regulatory restriction on profit provisions, how can the future changes in the mix of business in the state be estimated to ensure proper capacity?

➡ The method introduced in this paper will be able to provide answers to all three of these questions

Components of the Approach

- Incorporates the idea of Mean-Variance Optimization (from modern portfolio theory)
- Introduces a new concept – Insurance Risk Triangle
- Determines covariance of profits between lines
- Develops a maximization function
- Solves for maximization through the use of eigenvalues & eigenvectors (matrix algebra)

Mean-Variance Optimization

A brief review –

- Want to find the optimal portfolio mix that will minimize the risk and obtain a target profit
- Essentially, we will create an optimization problem, then solve it

Find: Percent of portfolio to invest in each option

Minimize: Portfolio's risk

Constraints:
Portfolio Return ≥ Target
Sum of investment = 100% of Portfolio
Each investment ≥ 0% (no shorting)

Mean-Variance Optimization (cont)

A simple example –

- Based on 10 years of investment returns, we determine expected future returns (i.e. mean):

ABC	XYZ	R-Free
11.62%	11.98%	2.03%

- From historical returns, we construct a covariance matrix:

	ABC	XYZ	R-Free
ABC	0.1027	0.0393	0.0001
XYZ	0.0393	0.0728	0.0001
R-Free	0.0001	0.0001	0.0000

- Calculate the variance of the entire portfolio (w = weight):

$$\text{Portfolio Variance} = \sum_{l=1}^{3}\sum_{k=1}^{3} w_l w_k cov(R_l, R_k)$$

$$\begin{aligned} = \; & w_{ABC}(0.1027w_{ABC} + 0.0393w_{XYZ} + 0.0001w_{R\text{-}Free}) \\ & + w_{XYZ}(0.0393w_{ABC} + 0.0728w_{XYZ} + 0.0001w_{R\text{-}Free}) \\ & + w_{R\text{-}Free}(0.0001w_{ABC} + 0.0001w_{XYZ} + 0.0000w_{R\text{-}Free}) \end{aligned}$$

Mean-Variance Optimization (cont)

- Now we can create an optimization problem and then solve it:

Find: w_{ABC} , w_{XYZ} , $w_{R\text{-}Free}$

Minimize:

$$w_{ABC}(0.1027w_{ABC} + 0.0393w_{XYZ} + 0.0001w_{R\text{-}Free})$$
$$+ w_{XYZ}(0.0393w_{ABC} + 0.0728w_{XYZ} + 0.0001w_{R\text{-}Free})$$
$$+ w_{R\text{-}Free}(0.0001w_{ABC} + 0.0001w_{XYZ} + 0.0000w_{R\text{-}Free})$$

Constraints:

$$0.1162w_{ABC} + 0.1198w_{XYZ} + 0.0203w_{R\text{-}Free} \geq 10\%$$ *Target a 10% return

$$w_{ABC} + w_{XYZ} + w_{R\text{-}Free} = 100\%$$
$$w_{ABC} \geq 0,\ w_{XYZ} \geq 0,\ w_{R\text{-}Free} \geq 0$$

- Could perform this operation through Solver in Excel
  - Results: $w_{ABC} = 26.1\%$ $w_{XYZ} = 54.9\%$ $w_{R\text{-}Free} = 19.0\%$

Covariance of Insurance Profits

- Similar to Mean-Variance Optimization, we need a covariance matrix of profits

- From the paper:

$$Cov(R_i, R_j) = PLR_i \text{ x } PLR_j \text{ x } Cov(\varepsilon_i, \varepsilon_j)$$

- Key – Use the Error Triangles

Covariance of Insurance Profits (cont)

- Example:

ERROR TRIANGLES

LINE 1

	Development Period				
PY	0-12	12-24	24-36	36-48	48-60
1	0.26236	-0.08004	0.04082	0.01980	0.00391
2	0.27625	-0.07146	0.03637	-0.03637	
3	0.26236	-0.15700	0.03922		
4	0.18924	-0.27625			
5	0.26236				

LINE 2

	Development Period				
PY	0-12	12-24	24-36	36-48	48-60
1	0.03279	0.14953	0.04082	0.01980	0.00391
2	0.09531	0.16908	0.03637	-0.03637	
3	0.08338	0.04082	0.03922		
4	0.14927	0.04310			
5	0.07205				

Covariance by Age

LINE 1 \ LINE 2	0-12	12-24	24-36	36-48
0-12	-0.1116%	0.1719%	-0.0017%	-0.0390%
12-24	-0.3675%	0.5493%	-0.0027%	-0.0241%
24-36	-0.0066%	-0.0046%	0.0005%	0.0125%
36-48	-0.1756%	-0.0549%	0.0125%	0.1578%

Matrix Sum

0.1163%

Covariance by Line

	Line 1	Line 2
Line 1	1.7723%	0.1163%
Line 2	0.1163%	-0.0198%

Optimization Function

- What are we looking to maximize?
  - Profits → but still need to minimize variance

U/W Profit
+ Inv Inc

$$f(\text{Line Allocation}) = \frac{\text{Expected Profit of Lines}}{\text{Variance of Lines}}$$

Covariance Matrix

$$\boxed{f(w_1,..w_n) = \frac{\sum_{k=1}^{k=n} w_k E(R_k)}{\sum_{l=1}^{l=n}\sum_{k=1}^{k=n} w_k w_l \operatorname{cov}(R_k, R_l)}}$$

* Note – this can be written as a matrix problem

Eigenvalues & Eigenvectors

- Review:

  Given matrix A, non-zero vector v, and number λ:

  If $Av = \lambda v$, then v is the eigenvector
  λ is the associated eigenvalue

- Why are these used?
  - It provides a simpler method for the calculations – no need to perform linear programming (e.g. Solver)

- How can these be calculated?
  - There are a number of mathematical/statistical software packages available
  - We are using the R programming language

Optimal Premium Mix

THEOREM 1: Under perfectly competitive insurance markets, a necessary (but not sufficient) consequence is that the company is naturally mean-variance optimized and thus, the business mix of a firm is optimal.

- Only addresses the situation where perfectly competitive markets exist
- Will prove helpful shortly

Optimal Premium Mix (cont)

- A company writes several lines in a state with risk (volatility) and profits (expected returns). We optimize the total risk/profit (mean-variance) for the portfolio. Covariance matrix of risk loads is calculated.

$$f(w_1,..w_n) = \frac{\sum_{k=1}^{k=n} w_k E(R_k)}{\sum_{l=1}^{l=n} \sum_{k=1}^{k=n} w_k w_l \operatorname{cov}(R_k, R_l)}$$

Optimal Premium Mix (cont)

- Redefine this as a matrix problem
- Define matrices

$w' = [w_1, w_2, \ldots, w_n]$ (weights)

$\Sigma = \{Cov(R_k, R_l)\}_{kl}$ (covariance)

$r' = [r_1, r_2, \ldots, r_n]$ (profits)

$$C = \begin{pmatrix} r_1, r_1, \ldots, r_1 \\ \ldots\ldots\ldots.. \\ r_n, r_n, \ldots, r_n \end{pmatrix}$$ (matrix of profits)

$i' = [1, 1, \ldots, 1]$ (array of 100%)

Optimal Premium Mix (cont)

- Now the optimizing function becomes:

$$f(w) = \frac{w' C\, w}{w' \Sigma\, w}$$

* subject to w'i = 1

- Which associates with the matrix:

$$\Sigma^{-1} C$$

Optimal Premium Mix (cont)

THEOREM 2: Assuming that Σ is a non-singular, positive definite, the maximum of $f(w)$ with respect to $w \in R^n \setminus \{0\}$ is given by the largest eigenvalue, λ, of the matrix $\Sigma^{-1}C$ and is attained by the eigenvector, ξ, associated with the largest eigenvalue of $\Sigma^{-1}C$.

- In other words, by rewriting the optimization function as a matrix problem, we can use Eigen values and vectors to solve for the maximum

Optimal Premium Mix (cont)

- The largest eigenvalue identifies the maximum solution
- The associated eigenvector is the weights we are solving for
  - Eigenvector will need to be rebalanced, $\frac{\xi}{\xi' i}$
- Negative values in the Eigenvector imply the need to exit the line
  - Remove the line and re-determine the weights

Optimal Premium Mix (cont)

Hypothetical Example:

5 lines of business for company X:

	Permissible Loss Ratio	Expense Ratio	Underwriting Profit Provision	Investment Income Offset	Profit
Line 1	72.0%	30.0%	-2.0%	5.1%	3.1%
Line 2	65.0%	30.0%	5.0%	-2.4%	2.6%
Line 3	62.0%	30.0%	8.0%	-0.1%	7.9%
Line 4	60.0%	30.0%	10.0%	3.3%	13.3%
Line 5	70.0%	30.0%	0.0%	8.5%	8.5%

Optimal Premium Mix (cont)

- C-Matrix

3.1%	3.1%	3.1%	3.1%	3.1%
2.6%	2.6%	2.6%	2.6%	2.6%
7.9%	7.9%	7.9%	7.9%	7.9%
13.3%	13.3%	13.3%	13.3%	13.3%
8.5%	8.5%	8.5%	8.5%	8.5%

- Next, we calculate the profit covariance matrix, Σ
  - We show the calculation of covariance of lines 1 and 2 using insurance risk triangles

Optimal Premium Mix (cont)

LINE 1 INSURANCE RISK TRIANGLE

Pol Yr	0	12	24	36	48	60	72	84	96	108
2000	46,626,088	48,962,669	49,330,414	49,160,522	49,161,441	49,060,192	49,008,980	48,998,750	48,993,364	49,017,743
2001	54,401,695	56,455,375	56,539,405	56,577,908	56,481,454	56,379,874	56,316,201	56,302,856	56,369,719	
2002	51,737,143	50,138,872	49,772,205	49,722,349	49,664,590	49,577,476	49,549,720	49,551,433		
2003	55,136,470	53,019,269	52,558,149	52,329,009	52,294,807	52,308,224	52,290,406			
2004	58,883,995	57,905,328	57,304,304	57,228,641	56,989,183	56,967,681				
2005	67,767,614	66,627,556	66,772,152	66,919,289	66,732,202					
2006	57,902,520	57,596,782	57,295,053	57,111,064						
2007	64,464,615	64,046,763	62,294,141							
2008	88,926,002	88,314,038								
2009	79,038,753									

LINE 1 ERROR TRIANGLE

Pol Yr	0 - 12	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108
2000	0.04890	0.00748	-0.00345	0.00002	-0.00206	-0.00104	-0.00021	-0.00011	0.00050
2001	0.03706	0.00149	0.00068	-0.00171	-0.00180	-0.00113	-0.00024	0.00119	
2002	-0.03138	-0.00734	-0.00100	-0.00116	-0.00176	-0.00056	0.00003		
2003	-0.03916	-0.00874	-0.00437	-0.00065	0.00026	-0.00034			
2004	-0.01676	-0.01043	-0.00132	-0.00419	-0.00038				
2005	-0.01697	0.00217	0.00220	-0.00280					
2006	-0.00529	-0.00525	-0.00322						
2007	-0.00650	-0.02775							
2008	-0.00691								

Optimal Premium Mix (cont)

LINE 2 INSURANCE RISK TRIANGLE

Pol Yr	0	12	24	36	48	60	72	84	96	108
2000	106,899,696	107,350,642	107,594,689	107,725,044	107,735,964	107,829,442	107,876,347	107,908,361	108,007,128	108,002,516
2001	111,601,714	111,065,524	110,825,320	110,877,586	111,151,159	111,084,670	111,025,941	111,003,944	111,026,514	
2002	118,155,250	116,693,238	116,120,634	116,186,452	116,056,376	115,908,761	115,849,317	115,799,339		
2003	120,253,173	116,453,383	115,003,098	114,579,948	114,279,658	114,200,316	114,078,314			
2004	121,428,756	116,942,681	115,243,327	114,561,218	114,156,392	113,967,227				
2005	123,305,650	119,168,530	117,872,337	117,333,081	116,907,745					
2006	122,456,951	121,024,652	120,195,487	119,443,446						
2007	127,734,926	127,202,366	126,394,911							
2008	128,204,161	127,825,667								
2009	135,788,483									

LINE 2 ERROR TRIANGLE

Pol Yr	0 - 12	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108
2000	0.00421	0.00227	0.00121	0.00010	0.00087	0.00043	0.00030	0.00091	-0.00004
2001	-0.00482	-0.00217	0.00047	0.00246	-0.00060	-0.00053	-0.00020	0.00020	
2002	-0.01245	-0.00492	0.00057	-0.00112	-0.00127	-0.00051	-0.00043		
2003	-0.03211	-0.01253	-0.00369	-0.00262	-0.00069	-0.00107			
2004	-0.03764	-0.01464	-0.00594	-0.00354	-0.00166				
2005	-0.03413	-0.01094	-0.00459	-0.00363					
2006	-0.01177	-0.00687	-0.00628						
2007	-0.00418	-0.00637							
2008	-0.00296								

Optimal Premium Mix (cont)

Error Covariance Matrix by Age

	0 - 12	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96
0 - 12	0.03046%	0.01390%	0.00588%	0.00659%	0.00286%	0.00220%	0.00134%	0.00042%
12 - 24	0.00128%	0.00231%	0.00115%	0.00086%	0.00066%	0.00042%	0.00026%	0.00021%
24 - 36	-0.00075%	-0.00011%	0.00004%	0.00001%	-0.00008%	0.00000%	-0.00006%	-0.00015%
36 - 48	0.00185%	0.00069%	0.00036%	0.00018%	0.00012%	0.00002%	0.00003%	0.00006%
48 - 60	-0.00168%	-0.00066%	-0.00029%	-0.00019%	-0.00005%	-0.00005%	-0.00001%	-0.00001%
60 - 72	-0.00053%	-0.00021%	-0.00007%	-0.00008%	-0.00002%	-0.00002%	-0.00001%	0.00000%
72 - 84	-0.00010%	-0.00004%	0.00000%	-0.00002%	-0.00001%	0.00000%	0.00000%	0.00000%
84 - 96	-0.00059%	-0.00029%	-0.00005%	0.00015%	-0.00010%	-0.00006%	-0.00003%	-0.00005%

sum 0.07%

The covariance matrix by line is then assembled similarly for all lines:

Error Covariance Matrix by Line

	Line 1	Line 2	Line 3	Line 4	Line 5
Line 1	0.12%	0.07%	0.24%	0.38%	0.26%
Line 2	0.07%	0.08%	0.20%	0.37%	0.21%
Line 3	0.24%	0.20%	0.62%	1.02%	0.67%
Line 4	0.38%	0.37%	1.02%	1.91%	1.08%
Line 5	0.26%	0.21%	0.67%	1.08%	0.72%

Optimal Premium Mix (cont)

- Next step is to convert the error covariance matrix to a profit covariance matrix, Σ, by multiplying by the PLR for each line
- Solve for all Eigen values/vectors corresponding to $\Sigma^{-1}C$

MAXIMUM

Eigenvalues	15.81454117	-5.60E-16	-1.13E-17	-1.13E-17	1.91E-17
Eigenvectors	1	2	3	4	5
1	0.483060881	0.279984683	-0.3210583	-0.3210583	-0.329742014
2	0.359121275	-0.647776436	-0.430533952	-0.430533952	-0.423579758
3	0.712650327	0.651957714	0.808127965	0.808127965	0.818208988
4	0.172843455	-0.277305496	-0.195912974	-0.195912974	-0.174357789
5	0.316130901	-0.006860464	0.13937726	0.13937726	0.109470574

Optimal Premium Mix (cont)

- The Eigenvector associated with the maximum Eigenvalue needs to be rebalanced:

Line	Eigenvector	Rebalanced
1	0.483060881	23.64%
2	0.359121275	17.57%
3	0.712650327	34.87%
4	0.172843455	8.46%
5	0.316130901	15.47%
Total	2.043806839	100.00%

Optimal Allocation

Testing the Market's Competitiveness

- Shown how to find optimal mix for a single company in a state
- Now move to multiple companies in a state

- Define some terms:
  - s = 1,2,…,m – number of companies
  - p_s – premium for company s
  - q_s – statewide premium weight for company s
    - $q_s = \frac{p_s}{\sum_1^m p_s}$
  - Optimized line allocation for company s = $\frac{\xi_s}{\xi_s' 1}$

Testing the Market's Competitiveness (cont)

THEOREM 3: Assuming that each company is mean-variance optimized, the statewide premium mix, $\eta = \sum_{s=1}^{m} q_s \frac{\xi_s}{\xi_s' i}$, can be determined from the eigenvector of the matrix , $\sum_{s=1}^{s=m} q_s \frac{\Sigma_s^{-1} C_s}{\sum_{k=1}^{k=m} q_k \lambda_k}$, associated with the eigenvalue of 1. Alternatively, the state premium can be obtained directly from, $\eta = \sum_{s=1}^{m} q_s \frac{\xi_s}{\xi_s' i}$, since company premium mixes are known.

Testing the Market's Competitiveness (cont)

- Theorem 3 is simply stating that we can use η as the optimal premium allocation for all companies/lines combinations in a state
  - Determine each company's optimal mix
  - Multiply by each company's statewide premium weight

- Important to note
  - The optimal statewide company/line premium allocation will most likely not be the same as the actual statewide company/line premium allocation

Testing the Market's Competitiveness (cont)

- Can measure how far the actual mix is from the optimal mix
  - "Market Deviation"
    - Create a Deviation Matrix:

$$D = \begin{pmatrix} d_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & d_n \end{pmatrix}$$

$$\text{where } d_k = \frac{\text{Actual Premium Mix for Line k}}{\text{Optimal Premium Mix for Line k}} = \frac{\mu_k}{\eta_k}$$

Testing the Market's Competitiveness (cont)

- Deviation matrix tells us how close the premium mix is to optimal
  - Remember Theorem 1?
    - Under perfectly competitive insurance markets companies will naturally be mean-variance optimized and the business mix will be optimal

d_k's that are close to 1 $\rightarrow$ line is close to perfect competitiveness

Testing the Market's Competitiveness (cont)

THEOREM 4: Suppose that there is at least one company, s, in the industry s > 0 that is not mean-variance optimizing its portfolio for line(s) $\{k_i\}_{0<i\leq n}$. Then a necessary and sufficient condition of perfect market competitiveness for lines $\{k_i\}_{0<i\leq n}$ is existence of $\left\{ d_{k_i} = \frac{\mu_{k_i}}{\eta_{k_i}} = 1 \right\}_{0<i\leq n}$.

Testing the Market's Competitiveness (cont)

Example

Observed Written Premiums and Weights (q_s)

	Written Premium	Weight
Company 1	10,400,000	16.4%
Company 2	20,000,000	31.4%
Company 3	5,200,000	8.2%
Company 4	28,000,000	44.0%
Total	63,600,000	100.0%

Testing the Market's Competitiveness (cont)

- Using the weights, q_s, we determine the optimal premium allocation by line & company, $\sum_{s=1}^{m} q_s \frac{\xi_s}{\xi_s' i}$

Statewide Premium Mix

	Company 1	Company 2	Company 3	Company 4	Computed Statewide
Line 1	11.44%	23.39%	19.64%	29.62%	23.87%
Line 2	13.04%	14.88%	14.53%	31.38%	21.82%
Line 3	30.89%	22.42%	19.64%	7.41%	16.97%
Line 4	21.57%	12.08%	34.76%	21.98%	19.84%
Line 5	23.07%	27.23%	11.43%	9.60%	17.50%
Total	100%	100%	100%	100%	100%

Testing the Market's Competitiveness (cont)

- Next, calculate the deviance matrix using actual statewide premium and the computed optimal statewide mix

Market Deviance

	Optimal Statewide Mix	Actual Statewide Mix	Deviance
Line 1	23.87%	25.17%	1.054
Line 2	21.82%	19.44%	0.891
Line 3	16.97%	18.35%	1.081
Line 4	19.84%	23.77%	1.198
Line 5	17.50%	13.26%	0.758
Total	100%	100%	

Lines 1 & 3 are closest to perfect competitiveness

Restrictions on Profit

- Suppose a regulation is introduced that limits the profit provision underlying the rates
  - Will the optimal mix of business be affected?
  - Will this adversely affect the capacity of a line of business in the state?

Restrictions on Profit (cont)

- Through the use of matrices, the impact of rate regulations can be easily investigated
  - The C-matrix (profits) entries are restricted to the capping limit
  - New optimal premium mixes are calculated
  - Apply deviation matrix to create predicted mixes

Restrictions on Profit (cont)

Example

- Profit caps of 5% for lines 2 & 3 are being reviewed for impacts to premium mix

Profit Provisions underlying Rates

	Company 1	Company 2	Company 3	Company 4
Line 1	4.87%	4.96%	4.95%	5.50%
Line 2	5.74%	5.11%	5.86%	5.69%
Line 3	4.94%	5.20%	5.18%	4.59%
Line 4	5.96%	5.05%	4.83%	5.50%
Line 5	5.95%	4.83%	4.09%	4.30%

Capped Profit Provisions

	Company 1	Company 2	Company 3	Company 4
Line 1	4.87%	4.96%	4.95%	5.50%
Line 2	5.00%	5.00%	5.00%	5.00%
Line 3	4.94%	5.00%	5.00%	4.59%
Line 4	5.96%	5.05%	4.83%	5.50%
Line 5	5.95%	4.83%	4.09%	4.30%

Restrictions on Profit (cont)

- Using the techniques described earlier, new optimal premium mix is calculated

	Optimal Statewide Mix (Uncapped)	Optimal Statewide Mix (Capped)
Line 1	23.87%	24.15%
Line 2	21.82%	20.15%
Line 3	16.97%	17.29%
Line 4	19.84%	20.35%
Line 5	17.50%	18.06%
Total	100%	100%

Restrictions on Profit (cont)

- Apply deviation matrix to create the predicted statewide mix

	Actual Statewide Mix	Optimal Statewide Mix (Capped)	Deviance	Predicted Statewide Mix (Capped)
Line 1	25.17%	24.15%	1.054	25.47%
Line 2	19.44%	20.15%	0.891	17.95%
Line 3	18.35%	17.29%	1.081	18.70%
Line 4	23.77%	20.35%	1.198	24.38%
Line 5	13.26%	18.06%	0.758	13.68%
Total	100%	100%		100%

Restrictions on Profit (cont)

- Predicted effects of the rate caps
  - More than 1.5% decline in writings in line 2
  - Small increase in line 3 writings
  - Lines 1, 4 and 5 each pick up a share of the offset from line 2 shifts