

# Fundamentals of Actuarial Mathematics Exam—July 2024

The Fundamentals of Actuarial Mathematics exam is a three-and-one-half hour exam that consists of 40 multiple-choice questions and is administered as a computer-based test (CBT). For additional details on CBT, please refer to Exam Rules.

The FAM exam will consist of a short-term half (FAM-S), and a long-term half (FAM-L), each with 20 questions. Up to the July 2024 sitting, candidates can write FAM-S or FAM-L if they are missing half of the FAM exam due to transition. FAM-S and FAM-L will both be one-and-three-quarters hour exams with 20 questions. Otherwise, candidates must write the entire FAM exam. For those writing the entire exam there will be 20 questions from each of the short-term and long-erm portions, presented in a random order.

A variety of tables are available below for candidate use and will be provided to the candidate at the examination. Only tables relevant for the examination being taken will be provided. These include values for the abridged inventories of discrete and continuous probability distributions, and life and decrement tables as appropriate for each half. Candidates will not be allowed to bring copies of the tables into the examination room. The CBT environment will include a normal distribution calculator.

Check the Updates section on this exam's home page for any changes to the exam or syllabus.

In the learning outcomes, weights have been provided to indicate the relative emphasis on different sections. The ranges of weights shown are intended to apply to the large majority of exams administered. On occasion, the weights of topics on an individual exam may fall outside the published range. Candidates should also recognize that some questions may cover multiple learning outcomes.

Each multiple-choice problem includes five answer choices identified by the letters A, B, C, D, and E, only one of which is correct. Candidates must indicate responses to each question on the computer.

As part of the computer-based testing process, a few pilot questions will be randomly placed in the exam (both paper and pencil and computer-based forms). These pilot questions are included to judge their effectiveness for future exams, but they will NOT be used in the scoring of this exam. All other questions will be considered in the scoring. All unanswered questions are scored incorrect. Therefore, candidates should answer every question on the exam. There is no set requirement for the distribution of correct answers for the multiple-choice preliminary examinations. It is possible that a particular answer choice could appear many times on an examination or not at all. Candidates are advised to answer each question to the best of their ability, independently from how they have answered other questions on the examination.

Since the CBT exam will be offered over a period of a few days, each candidate will receive a test form composed of questions selected from a pool of questions. Statistical scaling methods are used to ensure within reasonable and practical limits that, during the same testing period of a few days, all forms of the test are comparable in content and passing criteria. The methodology that has been adopted is used by

many credentialing programs that give multiple forms of an exam. Because this is a new exam, results for the first several administrations will not be instantaneous. Results will be released on the SOA website about 8 weeks after each testing window ends.

# **LEARNING OUTCOMES – SHORT-TERM (FAM-S)**

The syllabus for the short-term section of the examination provides an introduction to modeling and covers important actuarial methods that are useful in modeling. It will also introduce students to the foundational principles of ratemaking and reserving for short-term coverages. A thorough knowledge of calculus, probability, and mathematical statistics is assumed.

## 1. Topic: Insurance and Reinsurance Coverages (7.5-12.5%)

#### **Learning Objectives**

The Candidate will understand the key features of insurance and reinsurance coverages.

## **Learning Outcomes**

- a) Define and apply the concept of insurable risk.
- b) Identify different types of short-term insurance coverage including auto, homeowners, liability, health, disability, and workers compensation.
- c) Identify the types of coverage modifications for short-term insurance.
- d) Perform calculations assessing the impact of coverage modifications.
- e) Perform calculations of the loss elimination ratio and the effect of inflation on losses.
- f) Identify the operation of basic forms of proportional and excess of loss reinsurance and understand their impact on reserving and pricing.
- g) Determine the allocation of claim amounts paid by the insurer and reinsurer under various forms of reinsurance.

#### 2. Topic: Severity, Frequency, and Aggregate Models (12.5-15%)

# **Learning Objectives**

The Candidate will understand the characteristics of and uses for commonly used severity, frequency, and aggregate models.

# **Learning Outcomes**

The Candidate will be able to, for severity models:

- a) Calculate moments and percentiles.
- b) Identify the role of scale and shape parameters in continuous models.
- c) Recognize classes of distributions and their relationships.
- d) Characterize distributions by existence of moments.

The Candidate will be able to, for frequency models:

- e) Identify the role of parameters for the (a,b,0) and (a,b,1) classes of distributions.
- f) Recognize the (a,b,0) and (a,b,1) classes of distributions and their relationships.
- g) Perform calculations for the (a,b,0) and (a,b,1) classes of distributions.
- h) Identify appropriate distributions for a given application.

The Candidate will be able to, for aggregate risk models:

- i) Define collective and individual risk models and calculate their mean and variance.
- j) Use the log-normal or normal approximation to approximate the aggregate distribution.
- k) Calculate probabilities using the convolution method.
- I) Calculate the expected payment for stop-loss insurance.

- m) Calculate Value at Risk and Tail Value at Risk.
- n) Determine whether a given risk measure has certain desirable properties.

## 3. Topic: Parametric and Non-Parametric Estimation (5-10%)

# **Learning Objectives**

The Candidate will understand and be able to estimate parameters for parametric models.

# **Learning Outcomes**

The Candidate will be able to:

- a) Estimate the parameters for severity and frequency distributions using Maximum Likelihood Estimation for:
  - Complete, individual data
  - Complete, grouped data
  - Truncated or censored data

# 4. Topic: Introduction to Credibility (2.5-5%)

# **Learning Objectives**

The Candidate will understand the concepts of credibility and be able to apply certain types of credibility in some practical settings.

# **Learning Outcomes**

- a) Understand the concept of credibility.
- b) Perform calculations using limited fluctuation (classical) credibility.

#### 5. Topic: Pricing and Reserving for Short-Term Insurance Coverages (7.5-12.5%)

#### **Learning Objectives**

The Candidate will be able to use basic methods to calculate premiums and reserves for short-term insurance coverages.

# **Learning Outcomes**

The Candidate will be able to:

- a) Describe and apply techniques for estimating outstanding claims, using the following methods:
  - Expected Loss Ratio
  - Chain-Ladder
  - Bornhuetter-Ferguson
- b) Understand the objectives of ratemaking and the data used for ratemaking.
- c) Calculate the adjustments to ratemaking data, including development, trend and adjusting premium to current rate levels.
- d) Understand how expenses and the profit and contingencies loading are used in ratemaking.
- e) Calculate overall average rates and rate changes using the loss cost and loss ratio methods.

## 6. Topic: Option Pricing Fundamentals (2.5-7.5%)

#### **Learning Objectives**

The Candidate will be able to value simple options and derivatives using risk neutral expected present values, under the binomial and Black-Scholes models.

#### **Learning Outcomes**

- a) Identify the cash flows and characteristics of puts and calls.
- b) Apply the binomial option pricing model to calculate the price of a simple European-style derivative on a single non-dividend paying asset.
- c) Apply the Black-Scholes formula to calculate the price and delta hedge of a simple Europeanstyle derivative on a single non-dividend paying asset.
- d) Apply put-call parity.

# LEARNING OUTCOMES – LONG-TERM (FAM-L)

The syllabus for the long-term section of the examination develops the candidate's knowledge of the theoretical basis of contingent payment models and the application of those models to insurance and other financial risks. A thorough knowledge of calculus, probability, mathematical statistics and interest theory is assumed.

#### 7. Topic: Insurance Coverages and Retirement Financial Security Programs (2.5-7.5%)

## **Learning Objectives**

The Candidate will understand the key features of insurance coverages and retirement financial security programs.

## **Learning Outcomes**

The Candidate will be able to:

- a) Define and apply the concept of insurable interest.
- b) Identify the long-term insurance coverages (life, health), annuities, and defined benefit and defined contribution pension plans.

## 8. Topic: Mortality Models (7.5-12.5%)

# **Learning Objectives**

The Candidate will understand key concepts concerning parametric and non-parametric mortality models for individual lives.

## **Learning Outcomes**

- a) Understand parametric survival models, life tables, and the relationships between them.
- Given a parametric survival model, calculate survival and mortality probabilities, the force of mortality function, and moments of the curtate and complete future lifetime random variable.
- c) Identify and apply standard actuarial notation for future lifetime distributions and moments, including select and ultimate functions.
- d) Given a life table, calculate survival and mortality probabilities, the force of mortality function, and moments of the curtate and complete future lifetime random variable, using appropriate fractional age assumptions where necessary.
- e) Understand and apply select life tables.
- f) Identify common features of population mortality curves.

#### 9. Topic: Parametric and Non-Parametric Estimation (2.5-7.5%)

# **Learning Objectives**

The Candidate will understand and be able to estimate parameters for parametric and non-parametric survival models.

# **Learning Outcomes**

The Candidate will be able to:

- a) Use Maximum Likelihood Estimation to estimate log-likelihood functions for various laws of mortality
- b) Apply Kaplan Meier and Nelson Aalen methods to estimate empirical survival functions using censored and truncated lifetime data.
- c) Calculate approximate standard errors of the parameter/probability estimates.
- d) Construct linear and non-linear confidence intervals (as appropriate) for parameters/estimates.

#### 10. Topic: Present Value Random Variables for Long-Term Insurance Coverages (10-15%)

#### **Learning Objectives**

The Candidate will be able to perform calculations on the present value random variables associated with benefits and expenses for long term insurance coverages.

# **Learning Outcomes**

- a) Identify the present value random variables associated with life insurance, endowment, and annuity payments for single lives, based on annual, 1/m-thly and continuous payment frequency.
- b) Calculate probabilities, means, variances and covariances for the random variables in Topic 10(a), using fractional age or claims acceleration approximations where appropriate.
- c) Understand the relationships between the insurance, endowment, and annuity present value random variables in Topic 10(a), and between their expected values.
- d) Calculate the effect of changes in underlying assumptions (e.g., mortality and interest).
- e) Identify and apply standard actuarial notation for the expected values of the random variables in Topic 10(a).

## 11. Topic: Premium and Policy Value Calculation for Long-Term Insurance Coverages (15-20%)

## **Learning Objectives**

The Candidate will be able to use and explain the premium and policy value calculation processes for long-term insurance coverages.

## **Learning Outcomes**

The Candidate will be able to:

- a) Identify the future loss random variables associated with whole life, term life, and endowment insurance, and with term and whole life annuities, on single lives.
- b) Calculate premiums based on the equivalence principle, the portfolio percentile principle, and for a given expected present value of profit, for the policies in Topic 11(a).
- c) Calculate and interpret gross premium, net premium and modified net premium policy values for the policies in Topic 11(a).
- d) Calculate the effect of changes in underlying assumptions (e.g., mortality and interest).
- e) Apply the following methods for modelling extra risk: age rating; constant addition to the force of mortality, constant multiple of the rate of mortality.

## Readings - Short-Term (FAM-S):

Loss Models: From Data to Decisions, (Fifth Edition), 2019, by Klugman, S.A., Panjer, H.H. and Willmot, G.E., Wiley, ISBN: 978-1-119-52378-9

Chapter 3 (Sections 1, 2, 4.1, 5)

Chapter 4

Chapter 5 (Sections 3, 4)

Chapter 6

Chapter 8 (Sections 1-5)

Chapter 9 (Sections 1, 2, 3.1, 3.2, 7, 8.1, 8.2)

Chapter 11 (Sections 1-4)

Chapter 12 (Sections 1-3)

Chapter 16

Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance (Fifth Edition), 2022 by Brown and Lennox, ACTEX, ISBN: 978-1-64756-787-3 [Candidate may also use Fourth Edition, 2015, (same chapters) ACTEX, ISBN: 978-1625424747]

Chapter 1 (background only)

Chapter 2

Chapter 3 (Sections 1-6.4)

Chapter 4 (Sections 1-8.1)

Chapter 5 (Section 3 (background reading only), Section 5)

• FAM-25-18 *Individual Health Insurance* (Second Edition), 2015, by Bluhm and Leida, Chapter 2, Sections 2.1, 2.9

Actuarial Mathematics for Life Contingent Risks, Third Edition Dickson, C.M.D., Hardy, M.R., Waters, H.R. (2020), Cambridge University Press ISBN: 978-1-108-47808-3. Exercises are considered part of the required readings.

Chapter 16

## Readings - Long-Term (FAM-L):

Actuarial Mathematics for Life Contingent Risks, Third Edition Dickson, C.M.D., Hardy, M.R., Waters, H.R. (2020), Cambridge University Press ISBN: 978-1-108-47808-3. Exercises are considered part of the required readings.

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Chapter 1
Chapter 2
Chapter 3 (except Sections 3.11,3.12)
Chapter 4
Chapter 5
Chapter 6
Chapter 7 (Sections 1-3 [except 2.4, 2.5], 7, 8)
Chapter 18 (Sections 1-5)
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#### Other Resources - Short-Term (FAM-S):

- Tables for FAM-S
- Notation and Terminology used on FAM-S
- Sample Questions and Solutions for FAM-S

Corrections and Comments for Loss Models, Fifth Edition

# Other Resources - Long-Term (FAM-L):

Tables for FAM-L

<u>Excel Workbook for FAM-L Tables</u> (These spreadsheets were used to develop the tables used for the FAM-L exam and is provided for educational purposes only. The workbook will not be available at the FAM-L exam.)

- Notation and Terminology used on FAM-L
- Sample Questions and Solutions for FAM-L

Note: The texts, notation and terminology notes, and any study notes will not be available with the examination booklet. A copy of the Tables will be available.